Project 2. Optimize a 3D Truss.

CEE 201L. Uncertainty, Design, and Optimization
Department of Civil & Environmental Engineering
Duke University


Introduction

In this project you will first use numerical optimization to design a 3D truss. You will assess the effects of uncertainty on the performance of the optimized truss by carrying out a safety analysis. The project has seven tasks in two parts:

Part 1

Task 1. Conceptualize the design of your truss with sketches.

Task 2. Parameterize the design in terms of the some of the node coordinates, and the bar cross sections.

Task 3. Make use of a provided analysis code that computes truss bar forces and node displacements of the truss.

Task 4. Make use of numerical optimization in order to minimize the weight of the truss such that the truss does not have excessive displacement and such that the bars do not buckle or yield.

Part 2

Task 5. Study the effect of relaxing constraints on the solution.

Task 6. Investigate the sensitivity of the cost to variations in the design parameters, and

Task 7. Determine the safety of your optimized design.

Design Requirements

In this project each group will conceptualize, analyze, and optimize a 3D truss. The coordinates of the reaction points and the coordinates and directions of the applied loads are shown in Figure 1. The bars shall be made of steel: mass density $\rho = 7.86 \times 10^{-9} \text{T/mm}^3$, elastic modulus $E = 200,000 \text{N/mm}^2$, and yield stress $S_y = 250 \text{N/mm}^2$. Units of N, mm, s, and T are consistent ... 1 N = (1 T)(1 mm/s$^2$). Each truss bar, $b$, is to be a circular tube (outer diameter $D_{o,b}$ and inner diameter $D_{i,b} = 0.9D_{o,b}$). The truss is to be supported at nodes 1, 2, 3, 4, 9, 10, 11, and 12 at the coordinates shown on the $x - y$ plane. The truss is to support its own self-weight, and external forces at nodes 5, 6, 7, and 8, as shown. The design shall specify:
• the \((x_n, y_n, z_n)\) coordinates of the nodes in your truss;
• the diameter of each bar \((D_{o,b})\).

The mass of material used (in Tons) shall be minimized such that the following criteria are satisfied:

• Safety against yielding shall be satisfied in each bar with a safety factor of 1.2 on the loads and a safety factor of 0.9 on the yield strength of the bars; and

• Safety against buckling shall be satisfied in each bar with safety factor of 1.2 on the loads and a safety factor of 0.8 on the buckling strength of the bars.

• Each bar shall be no longer than 20 m.

• The maximum deflection of any node is less than 0.2 m in any direction.

Figure 1. Configuration of the reaction nodes and load nodes. The vertical (downward, \(-z\)) forces have a value of \(P_z = 10\) kN: 40 kN total downward force. The horizontal \((+y)\) forces have a value of \(P_y = 5\) kN: 20 kN total.

Task 1: Sketch design alternatives

Provide one sketch of a possible truss for each member of the group. All design alternatives must:

• inter-connect the eight reaction nodes with the four load-bearing nodes with some network of bars in which the bars could be no longer than the allowable length; and

• have no un-stable nodes. In 3D, a stable node connects three or more truss elements that are not co-planar. In other words, at least three bars must connect to each node and the bars may not all lie within the same plane. Many stable trusses are a tessellations of tetrahedra.
Figure 2. *Stable prisms built of tetrahedra.* Upper left: triangular prism of three tetrahedra, six nodes and twelve bars. Upper right: rectangular prism of five tetrahedra, eight nodes and eighteen bars.

*Stable prisms not built of tetrahedra:* The triangular prism on the lower left has the blue/yellow diagonal reversed. It is not an assembly of three tetrahedra. Each node in this prism is supported by four bars. The rectangular prism on the lower right has the blue/red, yellow/red and green/red diagonals reversed. It is not an assembly of tetrahedra. Pairs of diagonals are parallel. Each node in this prism is supported by four or five bars.

All prisms in this figure are made of triangular panels.

In general diagonals should be arranged to carry tension whenever possible.

You may use these .m-files: *ThreeTetrahedra.m* and *FiveTetrahedra.m* to rotate the prisms,
Figure 3. Stable prisms with non-triangular panels: six nodes with twelve elements (top); and of eight nodes with sixteen elements (bottom). Configurations are described by node locations of the numbered nodes (blue), \(XYZ\) and the truss element connectivity of the numbered elements (black, green, violet) \(C\). You will of course recognize these prisms as tensegrity geometries ☺️.

\[
XYZ = \begin{bmatrix}
5.0 & 0.0 & 0 \\
-2.5 & 4.3 & 0 \\
-2.5 & -4.3 & 0 \\
-4.3 & 2.5 & 16 \\
0.0 & -5.0 & 16 \\
4.3 & 2.5 & 16 
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 2 \\
2 & 3 \\
3 & 1 \\
4 & 5 \\
5 & 6 \\
6 & 4 \\
1 & 6 \\
2 & 4 \\
3 & 5 \\
1 & 4 \\
2 & 5 \\
3 & 6 
\end{bmatrix}
\]
Optimize a 3D truss.

Sketch the configurations to be considered with the $x, y, z$ coordinate system provided in Figure 1. Since this is a 3D system, make sketches of each candidate design from three perspectives ($x - y$, $x - z$, $y - z$), and possibly a skew perspective. Use a straight edge!

Consider one design alternative from each group member and, as a group, make a preliminary assessment of each design in terms of:

- the two “must-have’s” itemized above,
- complexity and aesthetics, and
- any other attributes you may wish to consider.

Score each alternative, rank the set of alternatives, discuss the ranking, and select one alternative for design optimization. Your group need not select the alternative with the highest ranking.

**Task 2: Formulate an optimization-oriented design analysis**

The trusses optimized in this project will be analyzed for their bar forces and node displacements using MATLAB software linked below. The results of this analysis provides the information required for a quantitative assessment of each and every candidate design, in terms of the design objective (amount of material used), and the various constraints for safety, serviceability, and so on.

To do so you must parameterize the truss design in terms of a set of design variables $p$ that specify the geometry of the truss (the location coordinates of some nodes) and the outer diameter of the truss bars.

**Parameterize the design for optimization**

1. Number the nodes and number the elements for your selected design alternative.

2. Write out the connectivity matrix $C$ (listing the pair of nodes that each element connects, sorted by the element number). The connectivity matrix for the example in Figure 3 is provided in the figure. (Element 1 links nodes 1 and 2, element 8 links nodes 2 and 4 Element $j$ connects node $C_{i,1}$ to node $C_{i,2}$ ). The truss configuration is fully specified by its node location coordinates and its connectivity matrix.

3. List the set of unknown design parameters in the problem. This list could include some of the nodes coordinates. Some node coordinates can be specified in terms of other node coordinates in order to reduce the number of independent design parameters in order to specify any kind of symmetry you would like to preserve. Referring to the example shown in Figure 3, if nodes 1, 2, and 3 are at pre-specified locations, and if the height of nodes 4, 5, and 6 is specified as $z_4 = z_5 = z_6 = h$, and if all elements
are to have the same cross section, then the set of \((n = 7)\) design parameters for this problem would be

\[ p = [x_4, y_4, x_5, y_5, x_6, y_6, D_o] \]

Further, if the top triangle is to be an equilateral triangle, centered at \((0, 0, h)\) and if compression bars are to have a different diameter than tension bars, then the list of design parameters could be reduced to four variables

\[ p = [x_4, y_4, D_{ot}, D_{oc}] \]

since \((x_5, y_5)\) and \((x_6, y_6)\) can be found from \((x_4, y_4)\) using geometry and the fact that the top triangle is horizontal and equilateral. Simplifying the optimization problem by reducing the list of design variables restricts the kinds of topologies the optimized design can achieve to \textit{supposedly} desirable topologies, and simplifies the optimization process.

Formulate the optimization-oriented design analysis

Given:

- the specified fabrication material and the nominally assumed values for its mass density \(\rho\), elastic modulus \(E\), and yield strength \(S_y\),
- the specified values for the nominal loads \(P_z\) and \(P_y\),
- the specified inner bar diameter, \(D_i\), in terms of the outer bar diameter \(D_o\), \(D_i = 0.9\ D_o\),
- your sketches of the system to be analyzed for optimization, in which some of the node location coordinates are design parameters,
- your geometric relationships specifying all (3D) node location coordinates in terms of the design parameters and any constants you choose to constrain the geometry. For the example in Figure 3, these relationships might be \(r = \sqrt{x_4^2 + y_4^2}\), \(\theta = \arctan(y_4/x_4)\); \((x_4, y_4, z_4) = (x_4, y_4, h)\); \((x_5, y_5, z_5) = (r \cos(\theta + 2\pi/3), r \sin(\theta + 2\pi/3), h)\); \((x_6, y_6, z_6) = (r \cos(\theta - 2\pi/3), r \sin(\theta - 2\pi/3), h)\);
- any refinement you may wish to implement for the bar outer diameters. In the example of Figure 3, elements 10, 11, 12 might be known to carry compression, and so should have a larger bar diameter than the tensile elements (why?), so \(D_{o,b} = D_{oc}\) for \(b = (10, 11, 12)\), \(D_{o,b} = D_{ot}\) for all other bars, and the bar diameters are now parameterized by two design variables, \(D_{ot}\) and \(D_{oc}\).
- your connectivity matrix \(C\), in which row \(i\) of \(C\) has the node numbers of the two nodes connected by element \(i\),
- and a set of numerical trial values for the design parameters, for example,

\[ p = [x_4, y_4, D_{ot}, D_{oc}] = [-1, 1, 0.1, 0.5] \]
Optimize a 3D truss.

one may compute the:

- matrix of node location coordinates
- vector of inner and outer bar diameters, $D_o$ and $D_i = 0.9D_o$
- vector of bar cross section areas, $A = \pi(D_o^2 - D_i^2)/4$
- vector of bar cross section second moment of areas, $I = \pi(D_o^4 - D_i^4)/64$
- vector of bar lengths, $L = \sqrt{(x(C_{b1}) - x(C_{b2}))^2 + (y(C_{b1}) - y(C_{b2}))^2 + (z(C_{b1}) - z(C_{b2}))^2}$
  (which is generously computed for you by the provided analysis code)
- vector of bar forces $T$ (also computed by the provided analysis code)
- total mass of material used in the truss, $M = \rho A^T L$
- compressive bar critical buckling strengths, $P_{cr} = \pi^2EI/L^2$
- vector of bar tensile yield strengths, $T_y = SyA$

inequalities expressing the safety criteria for bar buckling and bar yielding and the geometric criteria for bar lengths in terms of the bar tensions $T$, the cross section properties $A$ and $I$, the bar lengths $L$, the material yield strength, $S_y$, the elastic modulus $E$, and safety factors, $\phi_L$ for loads, $\phi_Y$ for yield strength, and $\phi_B$ for buckling strength. Write the inequality in a nondimensionalized comparison to zero. For example, the inequality

$$\phi_L \cdot L \leq \phi_R \cdot R$$

stating that the factored load $\phi_L \cdot L$ may not exceed the factored resistance $\phi_R \cdot R$, can be expressed as a comparison to zero

$$\phi_L \cdot L - \phi_R \cdot R \leq 0,$$

or as a non-dimensionalized comparison to zero

$$\frac{\phi_L \cdot L}{\phi_R \cdot R} - 1 \leq 0.$$

In other words, with candidate values for your system design parameters, you may compute a value for the design objective (e.g., the volume of material in the truss) and values for a set of constraints to satisfy other performance criteria (e.g., the bars do not yield or buckle).

A computational framework that can calculate all of the items in the list above is readily applicable to numerical optimization, (e.g., the Nelder-Mead method) in order solve the design optimization that minimizes the amount of material used while ensuring that the optimized system is safe.

Cool.
The downloadable .m-function `truss_3dg.m` and example `truss_3dg_test.m` show how a 3D truss can be analyzed and plotted. Download them now and run the `truss_3dg_test.m` code to be sure it all works. Study the well-commented script `truss_3dg_test.m`. You’ll be using many parts of this script in this project. The method implemented in `truss_3dg` is called matrix structural analysis, and is the topic of CEE 421L. The course notes `truss-design-analysis.pdf` provide a template for implementing an optimization-oriented analysis for a truss. In Task 3, you will put all these concepts together in detail.

**Task 3: Implement an optimization-oriented design analysis**

In this project you will use `NMAopt.m` and `SQPopt.m` to optimize your truss. Re-acquaint yourself with the use of these codes by reviewing `optim-examples.pdf`.

The methods implemented in these optimization functions keep track of the parameters, the cost function, and the constraints. The algorithms use this information to determine what the next guess at the optimal value of the parameters should be, based on the information it has developed by trying several different values of the parameters. As information about the problem accumulates with each successive iteration, the trial parameter values get closer and closer to the true set of optimal parameters (at least in theory). The optimization functions `NMAopt` and `SQPopt`, will make use of your group’s function `ABC_truss_analysis` that you will write in Task 4. Your function `ABC_truss_analysis` will be passed to `NMAopt` and `SQPopt` in single quotes, to indicate that the variable is a function name. The initial guess of the optimal parameters is specified in the MATLAB vector `p_init`. The lower and upper bounds of the parameters are given in the MATLAB vectors `p_min` and `p_max`. You will need to specify the values for the vectors `p_init`, `p_min`, and `p_max`. Set the max and min values of the joint coordinates to limit the scope of where the nodes could be located. If you allow the nodes to be just about anywhere, you could get a very strange looking truss, or a truss that is not stable. On the other hand, make sure that the max values for the bar diameters are not too small to start with. If you don’t allow the bars to be big enough, the optimization may not converge to a feasible solution. Think about the physical size of the problem to guide your decision about good values for the max and min parameter values. You can relax these parameter bounds once the optimization is running well. The initial parameter values do not need to satisfy the constraints, but it helps if they do. The optimal parameter values are returned in the vector `p_opt`.

Download .m-files at the end of `truss-design-analysis.pdf`. Just to make sure you have all the require .m-files on your computer, run `truss_opt`. It should complete without error. Copy these files to your group’s files. Name them: `ABC_truss_opt.m`, `ABC_truss_analysis.m`, and `ABC_truss_safety.m` where you replace ABC with the initials of the members of your group.

As a guide to editing your group’s .m-files, note that lines marked with * will require some editing.
Optimize a 3D truss.

ABC_truss_opt.m

Edit ABC_truss_opt.m to:

1. use units of Newtons, millimeters, Tons, and seconds (and N/mm², etc.)

2. assign values of the constants in the problem. For example, in this problem some of the constants (and their values) are: \( g = 9810 \text{ mm/s}^2 \), \( \rho = 7.86 \times 10^{-9} \text{ T/mm}^3 \), \( E = 200,000 \text{ N/mm}^2 \), \( S_y = 250 \text{ N/mm}^2 \), and values of \( P_z \) and \( P_y \) in units of Newtons, as given in the caption of Figure 1. \( \phi_L = 1.2 \), \( \phi_Y = 0.9 \), and \( \phi_B = 0.8 \); Your group may wish to combine constants in this list, such as with a vector \( \text{P\_nominal} \) containing the nominal values of \( P_y \) and \( P_z \).

3. assign values to the vectors and matrices used by truss_3dg.m: \( g \) (gravity vector); \( XYZ \) (initial guess for node location coordinates); \( \text{TEC} \) (truss element connectivity, \( C \)); \( \text{RCT} \) (node reactions); \( \text{EAp} \) (modulus, area, density); \( \text{P} \) (external loading); and \( D = [] \); (prescribed node displacements – there are none).

Unlike the example provided in truss_3dg_test.m, in your analysis, some of the elements of \( XYZ \) and \( \text{EAp} \) will be design variables, or will be computed from the design variables. Elements of these matrices will be updated within your group’s .m-function, ABC_truss_analysis.m.

4. collect constants (\( \text{P\_nominal}, \text{Sy\_nominal}, E, g, XYZ, \text{TEC}, \text{RCT}, \text{EAp}, P, D, Lmax, \phi_L, \phi_Y, \phi_B, \text{draw\_truss} \)) into a set called ... constants. Put the variable \( \text{draw\_truss} \) in last element of the set of constants.

If \( \text{draw\_truss} = 0 \) then the truss is analyzed without drawing the truss, to save time.
If \( \text{draw\_truss} = 1 \) then a truss is drawn for you without analysis.
If \( \text{draw\_truss} = 2 \) then a truss is drawn for with analysis.

5. extract the values for the initial guess of node locations from \( XYZ \) into your vector of design parameters \( \text{p\_init} \).

6. append initial guess(es) for bar diameter(s) to \( \text{p\_init} \).

7. assign upper- and lower- limits for the parameter values, \( \text{p\_min} \) and \( \text{p\_max} \), based on your initial guess for the parameter values \( \text{p\_init} \).

8. converge to a roughly-optimal design using NMAopt and then refine the design using SQPopt, as shown in truss_opt.m.
ABC_truss_analysis.m

Delete the three groups lines of ABC_truss_analysis.m starting with with `% vvv` and ending with `% ^ ^ ^`. This will remove the “equilibrium-matrix” based analysis of 2D trusses. Edit the lines marked with * to:

1. break out the parameter vector param into individual design variables;
2. break out the vector constants into individual constants; This vector provides values for quantities like (Pn, Syn, E, g, XYZ, TEC, RCT, EAp, P, D, Lmax, phi_L, phi_Y, phi_B, draw_truss) and possibly others that your group would include;
3. update the elements of XYZ with node coordinate values from your parameter vector.
4. update the elements of the matrix P with values from the constants vector P_nominal.
5. assign values for inner and outer bar diameters for each bar from values from the parameter vector, and computes values for the cross section area, A, and the moment of inertia, I, for each bar
6. use truss_3dg.m to solve for the vector of truss node deflections, D, truss bar tensions, T, and bar lengths L (and optionally plot the truss in 3D)
7. calculate the total mass of steel of the truss in units of metric Tons (as a scalar value);
8. calculate the yield strength of each bar, T_y in Newtons;
9. calculate the buckling strength of each bar, P_cr in Newtons;
10. calculate the (column) vector of safety constraints (yielding, buckling, including factors of safety) and geometric constraints (bar lengths). For a truss with nB bars, the first nB constraints should be for safety against yielding, the next nB constraints should be for safety against buckling, and the last nB constraints could be for bar length.

**Task 4: Optimize the design objective while satisfying constraints**

To check your 3D truss model, start by using draw_plots = 1; to simply get a drawing of your truss model. Confirm that the nodes are where you intended them to be, and that the nodes are connected with bars, as you intended them to be.

Do not proceed with optimization until the truss model looks good.

Once the truss model looks good, proceed with optimization.

If an optimization is successful, check the optimal parameter values. If any node value is optimized to an upper or lower bound

The Lagrange multipliers are returned by SQPopt in the vector lambda of length (m + 2n). The first m Lagrange multipliers correspond to the m constraint equations. The next n Lagrange multipliers correspond to the p_{lb} ≤ p constraints, and the last n Lagrange multipliers correspond to the p ≤ p_{ub} constraints.
Optimize a 3D truss.

Pointers

(a) Figure 1 shows twelve nodes, eight of the nodes are reaction nodes, four of the nodes carry external loads. The loaded nodes are not reaction nodes. The \((x, y, z)\) location coordinates of these eight nodes are not design parameters. The location of these nodes may not be modified in the design optimization.

(b) The only reaction nodes available to you are nodes 1,2,3,4,9,10,11,12 as shown in Figure 1. These nodes are fixed, in all three axes.

(c) If your truss has 4 nodes in addition to the nodes with prescribed coordinates (the reaction nodes and the loaded nodes), then the design parameters will will include the twelve values of the \((x, y, z)\) location coordinates of these four nodes, and the remaining design parameters will give the diameters of the bars. ...Lines 32-36 of `truss_analysis.m` would change to something like ...

\[
x13 = \text{param}(1); \quad \% \text{x-coordinate of joint 13}
y13 = \text{param}(2); \quad \% \text{y-coordinate of joint 13}
z13 = \text{param}(3); \quad \% \text{z-coordinate of joint 13}
x14 = \text{param}(4); \quad \% \text{x-coordinate of joint 14}
y14 = \text{param}(5); \quad \% \text{y-coordinate of joint 14}
z14 = \text{param}(6); \quad \% \text{z-coordinate of joint 14}
x15 = \text{param}(7); \quad \% \text{x-coordinate of joint 15}
y15 = \text{param}(8); \quad \% \text{y-coordinate of joint 15}
z15 = \text{param}(9); \quad \% \text{z-coordinate of joint 15}
x16 = \text{param}(10); \quad \% \text{x-coordinate of joint 16}
y16 = \text{param}(11); \quad \% \text{y-coordinate of joint 16}
z16 = \text{param}(12); \quad \% \text{z-coordinate of joint 16}
\text{Do} = \text{param}(13); \quad \% \text{outer diameters of all bars}
\]

(d) Bars connecting two fixed nodes can not stretch or compress and therefore do not carry tension or compression. There’s no need to connect two fixed nodes with a bar.

(e) Once you see the plot of the 3D truss generated by `truss_3dg`, carefully scrutinize the figure by rotating the view angle to check that the truss as modeled in the code conforms to your expectations. If it does not, re-check the truss node location matrix `XYZ` and the truss element connectivity matrix `TEC`.

(f) Use different values for the lower and upper parameter bounds `p.lb` and `p.ub` for each parameter. To start with, don’t make the in-bound ranges very large. If an optimized parameter is bounded by its `p.lb` or `p.ub` value, expand the range for those bounded values and re-optimize; you’ll probably get a more optimal (lighter) design.

(g) Before optimizing, adjust the initial values of the node location coordinates, to get the plotted figure to look reasonable. Starting the optimization from a weird-looking configuration is challenging for numerical optimization algorithms.
(h) Once you see that the plot generated by the MATLAB analysis looks right, you may set `draw_truss=0;` in your function `ABC_truss_opt.m` to get the optimization to run faster, or you may leave `draw_truss=1;` to experience how cool and informative it is to visualize how the optimization algorithms modify your truss geometry from iteration to iteration.

(i) Your truss must be stable. In a stable 3D truss, every (non-reaction) node has at least three bars connected to it, in three independent directions. Every non-reaction node must be connected to at least three bars that are not co-planar. In an unstable truss, nodes can move without stretching or compressing bars. If your truss is unstable, the MATLAB analysis of the truss gives an error ...

**Warning:** Matrix is singular, close to singular or badly scaled. Results may be inaccurate. RCOND = NaN.

One way to fix this might be to re-arrange the bars so that all the panels are triangular. Or you may need to add bars. Alternatively, your truss configuration might be fine, but it might be too flexible to support the given loads. One way to fix this may be to increase the bar diameters in your initial design. Or you may need to add bars.

(j) Each bar may have its own outer diameter. Or you may choose to design the truss to be built from bars with just a few diameters. For example, you could specify one group of bars to have a diameter $D_{o1}$, a second group of bars to have a diameter $D_{o2}$ and a third group of bars to have a diameter $D_{o3}$.

(k) The more design parameters there are, the more flexibility the optimization algorithm has to fine-tune the solution, and to converge to a more optimal solution (a lighter truss), but the longer the optimization process will take, and the more sensitive the optimization could be to changes in the initial guesses. Also, the more finely-tuned a system is optimized, the less robust it’s performance might be to changes in operating environments.

(l) If in your analysis code, you specify a load as a negative number, apply the negative sign in the code wherever you use the variable, and change the value of the number to be positive. This pointer is not important for the design optimization but will be important for the safety analysis in Task 7.

(m) Do not ever edit any of the optimization codes downloaded from the course webpages, e.g., `NMAopt.m`, `avg_cov_func.m`, `truss_3dg.m`. They are already perfect.
Part 1 Project Report Contents

1. A (one page) introduction describing the goals of your project design, the conceptual approach you took to developing a parameterized design, the benefits of optimization-oriented design analysis, and the numerical methods taken to optimize the design ... as if you were describing your project to a BME sophomore.

2. Include the candidate sketches from Task 1, with a brief explanation of why the designer of each candidate thinks the design is good. Include the assessment table from Task 1 with an explanation of why the selected candidate design was chosen.

3. Include the MATLAB figure showing the initial guess for your selected truss configuration, showing labels for node numbers.

4. List the design parameters for your design and their optimal values. Include the MATLAB figure of your optimized truss showing truss bar forces.

5. What is the optimal cost in units of metric Tons? (Remember to specify units.)

6. Which, if any, of the constraints are binding the optimum solution?
   
   ```matlab
   Active_Constraints = find( g > options(4) )
   Active_Constraint_Values = g(find( g > options(4) ))
   ```

   What do each of these constraints represent, physically?

7. Why might the Lagrange multipliers for buckling be larger than, or smaller than the Lagrange multipliers for yielding? How might that guide a design revision for a lighter truss?

8. Why is the safety factor for buckling strength $\phi_B$ smaller than it is for yielding strength $\phi_Y$?

9. Please share any additional pointers that would have been helpful for Part 1 of this project.

10. Submit your .m-files ABC_truss_analysis.m and ABC_truss_opt.m to gradescope. Do not attach any other code and do not include the code in your project report.

   ```matlab
   % Every MATLAB file that you write in this course must be formatted and commented well.
   Comment most lines. Be fastidious about alignment and indenting.
   ```

   Use the downloadable code provided for this project as an example of clean and neat code.
Task 5: Quantify the sensitivity of re-optimized costs to constraint relaxation

To investigate the effect of relaxing constraints, individually decrease the value of the load and increase the value of the material strength (the yield stress) by ten percent (in your MATLAB function `ABC_truss_analysis.m`). Re-run the optimization from Task 4, and tabulate the results of the optimization in terms of parameter values and the cost, using the optimal values from Task 4 as your initial guess. Complete the table below:

<table>
<thead>
<tr>
<th>$P_z$</th>
<th>$P_y$</th>
<th>$S_y$</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>5,000</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>9,000</td>
<td>5,000</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>4,500</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>5,000</td>
<td>275</td>
<td></td>
</tr>
</tbody>
</table>

1. If the point load values are decreased by 10 percent, how much steel can be saved through reduced section areas?
2. If the yield stress is increased by ten percent, how much steel can be saved through reduced section areas?
3. What makes the biggest difference in the optimized cost, reducing the loads by 10 percent, or increasing the yield strength of steel by 10 percent? Why do you think that is the case?

Task 6: Quantify the sensitivity of the cost and constraints to small perturbations in some of the design parameters

Using the optimal solution from Task 4, evaluate the cost and constraints for a set of perturbed parameters. Select three parameters that you think are most important to the design. Call them $p_i$, $p_j$, and $p_k$. These parameters could be a node location coordinate or a bar section dimension. Or they could correspond to the three largest values of the diagonal of the Hessian matrix. If $p^*$ represents the vector of optimal parameters, evaluate the cost and constraints for the six sets of parameters in the table below; note the cost and max constraint for each set.

<table>
<thead>
<tr>
<th>parameter label</th>
<th>set 1</th>
<th>set 2</th>
<th>set 3</th>
<th>set 4</th>
<th>set 5</th>
<th>set 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$</td>
<td>$0.95p_i^*$</td>
<td>$1.05p_i^*$</td>
<td>$p_i^*$</td>
<td>$1.05p_i^*$</td>
<td>$p_i^*$</td>
<td>$1.05p_i^*$</td>
</tr>
<tr>
<td>$p_j$</td>
<td>$p_j^*$</td>
<td>$p_j^*$</td>
<td>$0.95p_j^*$</td>
<td>$1.05p_j^*$</td>
<td>$p_j^*$</td>
<td>$1.05p_j^*$</td>
</tr>
<tr>
<td>$p_k$</td>
<td>$p_k^*$</td>
<td>$p_k^*$</td>
<td>$p_k^*$</td>
<td>$0.95p_k^*$</td>
<td>$p_k^*$</td>
<td>$1.05p_k^*$</td>
</tr>
<tr>
<td>cost</td>
<td>max constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These calculations involve only a re-analysis by re-running your analysis code `ABC_truss_analysis.m` different parameter values; these calculations do not involve re-optimization of the truss.

1. Which of these three design parameters most significantly affects the cost function?
2. Which of these three design parameters most significantly affects max$(g)$?
Optimize a 3D truss.

Task 7: Quantify the propagation of uncertainties on the optimized system's safety

In realistic design situations, any final product will not conform exactly to the design specification. Furthermore, the environment that the product is subject to may be more demanding than was anticipated by the designer. Because of this, the operating environment and the associated performance of the product (in terms of its ability to withstand its operating environment) should be viewed as random variables. In this project, we assume that all of the parameters must be positive, which is quite often a necessary assumption.

To determine the “probability of failure” of the design, we perform many analyses, with each analysis involving a random value for the operating environment (external loads) and material properties (yield strength). Because the loads and the material strength must be positive, the probability distributions of these quantities must conform to a particular class of probability distributions that are defined only for positive values. The log-normal probability distribution is one such distribution. Unlike the symmetric normal (Gaussian) probability distribution, the log-normal distribution function is not defined for negative values. (A negative value for strength would be senseless.) To generate a log-normal random value \( p_i \) with median \( p_m \) and coefficient of variation \( c_P \), use the downloadable .m-functions logn_rnd.m and corr_logn_rnd.m. If in your analysis code, you specified a load as a negative number, apply the negative sign in the code wherever you use the variable, and change the value of the number to be positive.

Furthermore, to model the randomness of manufacturing variability, a level of quality-control can be presumed so that components that are out of spec (i.e., truss bar cut too long, or too short) are discarded. To model this variability, the design parameters \( (p_1, p_2, \ldots, p_n) \) can be perturbed by adding a small uniformly-distributed random variable.

To determine the probability of failure in a situation in which there is uncertainty both in the as-built condition of the product (random material properties and random design parameters) and in the operating environment (random loads), evaluate the cost function and the constraints 1000 (or more) times, each time use a randomly perturbed set of parameters and randomly perturbed values for the material strength and the loads. Set the median parameter values to be the optimal value you found in Task 4, \( p_m = p^*_i \), and the standard deviation being 1 percent of the mean value, \( c_{P_i} = 0.01 \). Set the median strength value to be the nominal strength of 250 N/mm\(^2\), and the median external loads to be the nominal loads given in Figure 1. The standard deviation of the strength will be 10 percent of the mean, and the standard deviation of the load will be 20 percent of the mean. Furthermore, consider the components of force \( P_y \) and \( P_z \) to be correlated with a correlation coefficient of 0.3.

Exceeding one or more safety constraints (yielding or buckling) constitutes a failure. For each evaluation of the cost and the constraints, count how many evaluations result in a constraint that is positive. The probability of failure is the number of evaluations that result in a constraint violation divided by the total number of evaluations. If you don’t get any constraint violations, then you will need to increase the total number of evaluations, and
re-run the safety analysis.

Edit `ABC_truss_safety.m` by changing the values for `constants`, `opt_params`, and `NSC` and by editing lines marked with `*` for your particular design.

The downloadable `.m`-function `correlation_analysis.m` determines how constraint violations are correlated with errors in the design parameters and uncertainty in the strength $S_y$ and loading $P_y$ and $P_z$.

The Monte Carlo analysis will take several minutes to complete.

1. In computing the failure probability, why are all the safety factors set to 1.0?
2. What is the failure probability for your optimized design?
3. If your failure probability is non-zero, what are the failure mechanisms corresponding to the constraints in violation (the active constraints)?
4. Which (if any) design parameter errors are most positively correlated with the active constraints?
5. Which (if any) design parameter errors are most negatively correlated with the active constraints?
6. Which (if any) load or strength uncertainties are most positively correlated with the active constraints?
7. Which (if any) load or strength uncertainties are most negatively correlated with the active constraints?
8. Using your answers to questions 3-7, how could you modify your design optimization to most effectively reduce the failure probability?
Optimize a 3D truss.

Part 2 Project Report Contents

1. A (one page) introduction describing the importance of uncertainty analysis for systems with uncertain loads and behavior and how Monte Carlo simulation can help quantify uncertainties in system behavior. as if you were describing uncertainty propagation to a BME sophomore.

2. Include completed tables shown in Tasks 5 and 6.

3. Include and describe figures 30 and 31 generated by ABC_truss_safety

4. Answer the questions listed at the end of Tasks 5, 6, and 7.

5. Please share any additional pointers that would have been helpful for Part 2 of this project.

6. Submit your .m-files ABC_truss_analysis.m and ABC_truss_safety.m as email attachments.
   Do not attach any other code and do not include the code in your project report.
   % Every MATLAB file that you write in this course must be formatted and commented well.
   Comment most lines. Be fastidious about alignment and indenting.
   Use the downloadable code provided for this project as an example of clean and neat code.

Grading

(45 points) Correctness of the formulation and implementation of your optimization-orientation analysis. The quality of your code in terms of formatting and commenting. Your ABC_truss_analysis.m must carry out all thirteen steps listed in Task 3 without error.

(5 points) The optimized cost of your truss, in comparison with other group projects.

(5 points) The failure probability of your truss, in comparison with other group projects.

(30 points) A complete report (including the thirteen items from Part 1 and the six items from Part 2).

(15 points) Quality of the report:
   Hand-drawn sketches are drawn with a straight edge and neatly annotated.
   (typed, 11pt or 12 pt, 1.5 space, 1 inch margins, page numbers, figure numbers, figure captions )