CEE 251L

Uncertainty, Design and Optimization

Spring 2026

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Lecture:	Mo We Fr 12:00 - 12:50		130 Wilkinson
Recitation:	Th 4:40 - 5:55		130 Wilkinson
Office Hours:	HPG: Tu 3:30 - 4:30	and by appointment	133 Hudson
	FJ: We 1:15 - 2:30	and by appointment	220 Hudson
	BB: Th 3:00 - 4:00	and by appointment	220 Hudson

Textbook: A.H.S. Ang and W.H. Tang, *Probability Concepts in Engineering*, Wiley, 2007. Notes and assignments are available through the course website.

Website: http://www.duke.edu/~hpgavin/cee251/

Prerequisite: EGR 201L. Solid Mechanics or ECE 110L. Fundamentals of ECE

Academic Integrity: *The Duke Community Standard* applies.

Your Super-Powers: are accommodated.

Grading: ∞ participation: (5%); 8 homework (20%); 2 projects (25%); 1 quiz: (10%); 1 presentation (10%); 1 take home final (30%);

BULLETIN DESCRIPTION

Principles of design as a creative and iterative process involving problem statements, incomplete information, conservative assumptions, constraining regulations, and uncertain operating environments. Parameterization of costs and constraints and formulation of constrained optimization problems. Analytical and numerical solutions to constrained optimization problems. Evaluation of design solutions via sensitivity and risk analysis. Application to design problems in civil and environmental engineering.

COURSE OBJECTIVES AND MEASURABLE OUTCOMES

Students successfully completing CEE 251L will be able to:

1. Identify, formulate, and solve engineering problems by:
   (a) formulating constrained optimization problems from broad design goals,
   (b) interpreting and assessing designs provided by solutions to constrained optimization problems, and
   (c) identifying possible obstacles to practical and successful implementations.
2. Design systems, components, and processes to meet desired needs within realistic constraints, (including economic, environmental, social, and safety constraints) by:
   (a) mathematically modeling the system to be designed,
   (b) defining design attributes to parameterize the design, and
   (c) intelligently iterating on the design to meet the desired objectives without violating constraints.

This course measures the students' progress in meeting the above objectives by requiring students to:

1. understand procedures for numerical optimization and to apply these methods to diverse problems (homework and projects);
2. interpret constrained optimization as method for structural analysis using energy methods (homework and midterm exam);
3. model uncertainties involved in system design and to propagate these uncertainties through optimized designs (homework and projects);
4. work in teams to design feasible and cost-effective systems to meet specific design goals (projects);
5. write programs (using `Python`) to analyze candidate systems and to utilize optimization-solvers in order to efficiently converge on optimal designs (projects); and
6. present methods and communicate findings in written project reports (projects).

COURSE POLICIES

(a) **The Duke Community Standard**
Duke University is a community dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Citizens of this community commit to reflect upon and uphold these principles in all academic and non-academic endeavors, and to protect and promote a culture of integrity.

To uphold the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

(b) **Super-powers:** Students with *super-powers* may seek accommodation approval via the SDAO portal. Please schedule approved services a week in advance of tests or quizes via the SDAO portal.

(c) **Attendance:** Your engaging participation will make this course much more fun! Be on time. Taking notes on paper improves focus and retention. Keep your phones off and your laptops closed.

(d) **Communication:** We will use e-mail for out-of-class communications related to the course.
Start the subject line of every email to me with `CEE 251:`

(e) **Coding:** Some homework assignments and all projects involve coding in `Python`. The TA's and HPG can help with `Python`. Here is a `Python` tutorial for beginners and a more comprehensive `Python` tutorial. Include *your* code and any plots to your homework and project PDFs prior to submitting your work.

(f) **Collaboration and Academic Integrity:** Collaboration with other students enrolled in the course is encouraged on homeworks and is required on projects. Present your own solutions in a way that makes sense to you. Copying will be considered a violation of the Duke Community Standard. The TAs may not solve homework problems for you. On each assignment, write and sign the reaffirmation,

"I have adhered to the Duke Community Standard in completing this assignment."

(g) **Neatness:** 15 of 100 points on each assignment are allocated to neatness.

- Use pencil, preferably a mechanical pencil.
- Write neatly and clearly.
- Write your first and last name, the course number, the assignment number and the due-date in the upper right corner of the first page. Write the page number on each page (e.g., 3/6, means page 3 of 6)
- Write out each problem statement. (i.e., Given=. . . , Find=. . . , Collaborators=. . .)
  For "Collaborators:" list the people with whom you worked on the problem.
  If there are no collaborators just write "none".
- Use a straight edge (your Duke ID card, a ruler, or a triangle) to draw straight lines.
- Present solutions to problems in the same order as listed in the assignment.
  Begin every problem on a new page that it can fit on the same page.
- For partial credit, describes your thinking in words.
- Draw a box around your final answer. Give the units for numerical answers.
- Scan your solution set. Use: PDF, b&w, 300 dpi, and high contrast settings.
  Name your `.pdf` file ... `cee251-MyName-HW#-2026.pdf`

(h) **Submission of work:** Name your `.pdf` file ... `cee251-MyName-HW#-2026.pdf` Upload your homework assignments and project reports to the CEE 251L Canvas site (under "Grades") by on the specified due date and time. Please set your document scan settings to generate reasonably sized files ... less than 5MB. Low quality scans will be penalized 5 neatness points.

(i) **Short-term illness:** If illness or injury prevents you from attending class or meeting a deadline, read the university policy on short-term and long-term health issues, complete this secure on-line incapacitation form, *and* meet with or contact me within 48 hours to figure out a plan for your support and accommodation.

(j) **Extensions:** Every student can get one automatic *one-work-day homework extension.* To obtain the extension, send me (HPG) an e-mail with the subject line: `CEE 251: <your name> extension request` at least 24 hours before the deadline. No explanation is required.

No extensions are available on projects or the final.

(k) **Extra-curricular and co-curricular activities:** In most cases extra-curricular and co-curricular activities conflicting with course commitments can be resolved easily. Just let me know beforehand.

(l) **Late work:** Is not accepted except for illness or athletic notifications.

(m) **Grading:** The TAs will grade your solutions to homework assignments. I will grade the projects and the take-home exams. See me (not the TA) about any grading issues before the last day of class.

(n) **Team Project Peer Evaluation (required):** For each project each student is *required* to anonymously indicate the percent contribution of each member of their team, including themselves, by submitting a *Project Cross Evaluation Form*. The individual project grade g_i for team member i depends on the team project grade G and the contributions c_{ij} of team member i as reported anonymously by team member j,

$$g_i = \min \left[100 \ , \ G \ \left(\frac{n}{100} \bar{c}_i \right)^{\left(\frac{1}{4} \frac{\max(\bar{\boldsymbol{c}})}{\min(\bar{\boldsymbol{c}})} \right)} \right] \quad , \quad \bar{c}_i = \frac{1}{m} \sum_{j=1}^{m} c_{ij}$$

where the team has n members and m team members submit project cross-evaluation forms. For example, if a team earns a “B” (a grade G of 85), the grades for individual team members are:

evaluated student	*anon. evaluations*			*avg. contribution*	*individual grade*
	$j=1$	$j=2$	$j=3$	$\bar{c}_i$	g_i
$i=1$ Alex	30	20	25	25	75
$i=2$ Bryce	30	30	30	30	81
$i=3$ Chris	40	50	45	45	97
	100	100	100	100	

The form also asks for a few sentences justifying your assessment of each group member’s contribution, including yourself.

(o) **Homework answer keys:** will be posted on Canvas after homeworks are graded.

COURSE SCHEDULE

Week	Dates	Topic	Reading
		— DESIGN OPTIMIZATION AND UNCERTAINTY —	
1	1/7 -1/9	Design: innovation, analysis, evaluation, iteration, and optimization;	[3]c1
		design parameters, cost functions, safety constraints, and failure probability	[3]c2
		safety factors, uncertainty, and risk. `Python` with VS Code.	[32]
	due 1/14	HW 1: `Python coding skills and debugging`	[20]
2	1/12-1/16	Constrained Optimization: inequality constraints and penalty methods	
		random search and simplex methods	[12] [44] [13] [17]
	due 1/21	HW 2: Constrained Optimization: search methods	
3	*1/19*	*Martin Luther King Day*	MLK: 1963, 1964
	1/21-1/23	Constrained Optimization: LP, QP, gradient, Hessian, and KKT	[14] [15] [16] [17]
		equality constraints, binding and nonbinding inequality constraints	
		Sensitivity of cost to constraint relaxation and parameter variation	
		application to least squares fitting and minimum total potential energy	[19] [22] [21]
	due 1/27	HW 3: Constrained Optimization: gradient methods	
4	1/26-1/30	Uncertainty: Aleatory and Epistemic uncertainty and their sources.	[1]c1
		Axioms: union, intersection, mutually exclusive, collectively exhaustive,	[23] [28]c1,3
		independence, conditional and total probability, Bayes' Rule	[1]c2
	due 2/2	HW 4: Uncertainty: rules of probability	
5	2/2-2/6	Uncertainty: random variables, histograms, and empirical distributions.	[1]c3
		Bernoulli sequences, Binomial, Poisson and Gaussian distributions, return periods	[23]
		uniform, triangular, normal, log-normal, exponential, Rayleigh, Laplace,	[29]p1-21 [28]c4-7
		extreme value distributions, sums & ratios of normal & lognormal variables	[1]c4p160-180
	due 2/9	HW 5: Uncertainty: probability distributions	
6	2/9-2/13	Uncertainty: joint distributions, Monte-Carlo simulation, variance of proportions,	[29]p22-40
		propagating correlated uncertainty.	[1]c5
		least squares fitting and error analysis, parameter and model covariance	[1]c8 [18]p1-14 [28]c10
	due 2/16	HW 6: Uncertainty: monte carlo simulation and regression	
7	2/16-2/20	Uncertainty: sample statistics, method of moments and maximum likelihood,	[24] [1]c6
		confidence intervals, quantiles, hypothesis testing. aleatory and epistemic unc.	[1]s5.2.3 [28]c7-8
	due 2/23	HW 7: Uncertainty: confidence intervals and hypothesis testing	
		— ENGINEERING DESIGN VIA OPTIMIZATION WITH UNCERTAINTY —	
8	2/23-2/27	Design-based engineering analysis of networks and stock-flow systems:	
		trusses, power grids, water supplies, ecology, and basketballs	
9	3/2-3/6	Optimization-based engineering analysis of structures	[42] [43]
	Thu 3/19	quiz in recitation (HW 4 - HW 7)	
	due 3/2	HW 8: Computer aided engineering analysis	
10	*3/9-3/13*	*spring break*	
		— ENGINEERING APPLICATIONS —	
11	3/16-3/20	Minimum Total Potential Energy (with constraints*!*)	[21] [22]
12	3/23-3/27	Project 1: (1.A)Truss (1.B)Power Grid (1.C)Finance (choose A, B, or C)	
13	3/30-4/3	class project work	
	due 4/6	Project 1.	
14	4/6-4/10	Project 2: (2.A)Earthquake (2.B)Water (2.C)Solar Power (choose A, B, or C)	
15	4/13-4/17	class project work	
	due 4/17	Project 2.	
16	4/20-4/22	in class project presentations	
	due 5/1	take home final - due at 5:00 pm	

References

[1] Ang, A.H-S., and Tang, W.H., *Probability Concepts in Engineering – Emphasis on Applications to Civil and Environmental Engineering, 2nd edition,* Wiley, 2007.

[2] Ang, A.H-S., and Tang, W.H., *Probability Concepts in Engineering, V.II – Decision, Risk and Reliability,* Wiley, 1986.

[3] Arora, Jasbir S., *Introduction to Optimum Design*, 2nd ed. Elsevier, 2004. 3rd ed: TA 174 .A76 (2011)

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[13] Gavin, H.P., "The Nelder-Mead Simplex Algorithm in Two Dimensions" CEE 251L. Duke U., 2013

[14] Gavin, H.P. "Linear Programming and Lagrange Multipliers" CEE 251L. Duke U., 2023

[15] Gavin, H.P. and Scruggs, J.T., "Quadratic Programming and Lagrange Multipliers" CEE 251L. Duke U., 2023

[16] Gavin, H.P. and Scruggs, J.T., "Review of Matrix Math" CEE 251L. Duke U., 2012

[17] Gavin, H.P., "An Example of Running Constrained Optimization Codes" CEE 251L. Duke U., 2013

[18] Gavin, H.P., "Fitting Models to Data - Generalized Linear Least Squares and Error Analysis" CEE 251L. Duke U., 2023

[19] Gavin, H.P., "Constrained Linear Least Squares" CEE 251L. Duke U., 2022

[20] Gavin, H.P. "`python` language skills and debugging," 2026.

[21] Gavin, H.P., "Example of the Principle of Minimum Total Potential Energy" CEE 251L. Duke U., 2025

[22] Gavin, H.P., "Minimum Potential Energy in Linear Elastic Solids" CEE 251L. Duke U., 2014

[23] Gavin, H.P., "Summary of some Rules of Probability with Examples" CEE 251L. Duke U., 2013

[24] Gavin, H.P. "Confidence Intervals and Hypothesis Testing" CEE 251L. Duke U., 2024

[25] Gavin, H.P., "Network System Reliability" CEE 251L. Duke U., 2024

[26] Gavin, H.P., "Simple Oscillators" CEE 251L. Duke U., 2014

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[29] Harvey, P.S., Gavin, H.P., and Scruggs J.T., "Probablity Distributions," CEE 251L. Duke U., 2014

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[42] Scruggs, J.T. and Gavin, H.P., "Introductory Optimization Example: design of a two-bar truss," CEE 251L. Duke U., 2022

[43] Scruggs, J.T. Sridhar A. and Gavin, H.P., "Example of an Optimization-Oriented Analysis for Truss Design and Safety Analysis," CEE 251L. Duke U., 2022

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