SUMMARY OF THE PRINCIPLE OF VIRTUAL WORK
CEE 421L. Matrix Structural Analysis
Fall, 2012

	Strain Energy U	**Internal Virtual Work** $\bar{W}_{\mathrm{I}}$
Axial	$\frac{1}{2}\int \frac{N^2}{EA}\, dl = \frac{1}{2}\sum \frac{N^2 L}{EA}$	$\int \frac{nN}{EA}\, dl = \sum \frac{nNL}{EA}$
Bending	$\frac{1}{2}\int \frac{M^2}{EI}\, dl$	$\int \frac{mM}{EI}\, dl$
Shear	$\frac{1}{2}\int \frac{V^2}{G(A/\alpha)}\, dl$	$\int \frac{vV}{G(A/\alpha)}\, dl$
Torsion	$\frac{1}{2}\int \frac{T^2}{GJ}\, dl$	$\int \frac{tT}{GJ}\, dl$

Principle of Virtual Work:

Virtual work is the work done by a *real force* acting through a *virtual displacement* ...

$$\bar{W} = \sum F_i \bar{D}_i = \bar{U} = \int_V \{\sigma\}^T \{\bar{\epsilon}\}\, dV$$

... or a *virtual force* acting through a *real displacement*.

$$\bar{W} = \sum \bar{F}_i D_i = \bar{U} = \int_V \{\bar{\sigma}\}^T \{\epsilon\}\, dV$$

Temperature: (Statically Determinate Structures)

Axial:
$$\bar{U} = \int n\alpha \left[\Delta T_{\mathrm{t}} - \left(\frac{\Delta T_{\mathrm{t}} - \Delta T_{\mathrm{b}}}{h} \right) h_2 \right] dl$$

Bending:
$$\bar{U} = \int m\alpha \left[\frac{\Delta T_{\mathrm{b}} - \Delta T_{\mathrm{t}}}{h} \right] dl$$

Statically Indeterminate Structures and Superposition:

1. Remove I redundants, R_i, $i = 1, \ldots, I$, where I is the degree of indeterminacy.
2. Solve for the internal forces, M_0, N_0, V_0, in the resulting statically determinate structure (without the redundants), due to the real applied loads.
3. Now, remove all of the real applied loads, and apply I unit virtual loads to the structure in the direction of the redundants, one at a time.
4. Solve for I sets of internal forces, m_i, n_i, v_i, in each of the I different statically determinate systems.
5. Apply superposition for moments, axial forces, and shears.

$$M = M_0 + \sum_{i=1}^{I} R_i m_i \qquad N = N_0 + \sum_{i=1}^{I} R_i n_i \qquad V = V_0 + \sum_{i=1}^{I} R_i v_i$$

6. Write I statements of the principle of virtual work, one for each virtual system, and enforce compatability with respect to support settlement, and relative positions, and solve for the redundants, R_i.