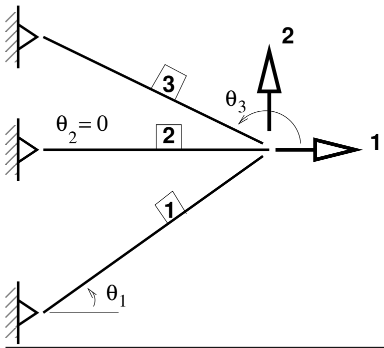
Duke University — CEE 421L. Matrix Structural Analysis

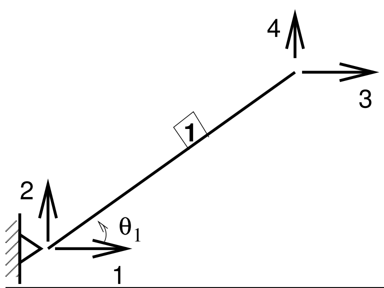
Introduction to Stiffness Matrix Assembly for Trusses

Method 1: Element by Element



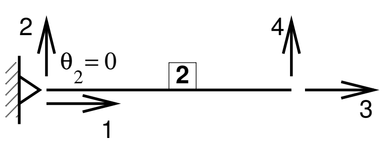
- bar numbers
- un-restrained structural coordinates are the global degrees of freedom
- angles θ imply the element coordinate numbering system

Element 1 Element coordinates 3 & 4 correspond to structural coordinates 1& 2.



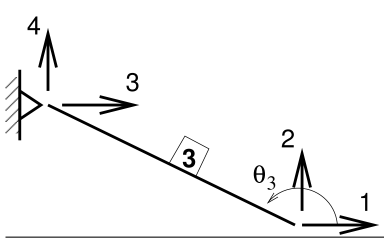
$$\mathbf{K}_1 = \left(\frac{EA}{L}\right)_1 \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & c_1^2 & c_1 s_1 \\ \cdot & \cdot & c_1 s_1 & s_1^2 \end{bmatrix}$$

Element 2 Element coordinates 3 & 4 correspond to structural coordinates 1& 2.



$$\mathbf{K}_2 = \left(\frac{EA}{L}\right)_2 \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & 0 \\ \cdot & \cdot & 0 & 0 \end{bmatrix}$$

Element 3 Element coordinates 1 & 2 correspond to structural coordinates 1& 2.



$$\mathbf{K}_3 = \left(\frac{EA}{L}\right)_3 \begin{bmatrix} c_3^2 & c_3 s_3 & \cdot & \cdot \\ c_3 s_3 & s_3^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

Assemble the structural stiffness matrix, $\mathbf{K_s}$, for the two structural coordinates.

$$\mathbf{K_s} = \begin{bmatrix} \left(\frac{EA}{L}\right)_1 c_1^2 + \left(\frac{EA}{L}\right)_2 + \left(\frac{EA}{L}\right)_3 c_3^2 & \left(\frac{EA}{L}\right)_1 c_1 s_1 + \left(\frac{EA}{L}\right)_3 c_3 s_3 \\ \left(\frac{EA}{L}\right)_1 c_1 s_1 + \left(\frac{EA}{L}\right)_3 c_3 s_3 & \left(\frac{EA}{L}\right)_1 s_1^2 + \left(\frac{EA}{L}\right)_3 s_3^2 \end{bmatrix}$$

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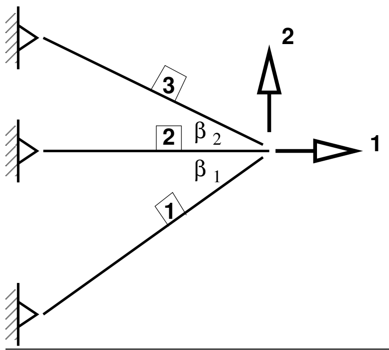
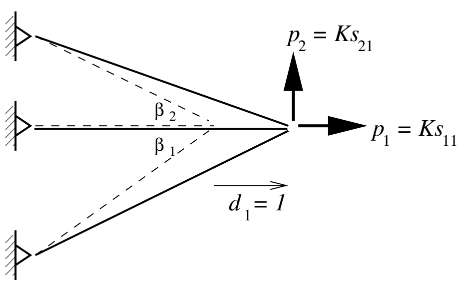
Method 2: Coordinate by Coordinate — column by column



- bar numbers
- un-restrained structural coordinates are the global degrees of freedom
- angles β should be acute, if possible, so that $\cos\beta \geq 0$ and $\sin\beta \geq 0$.

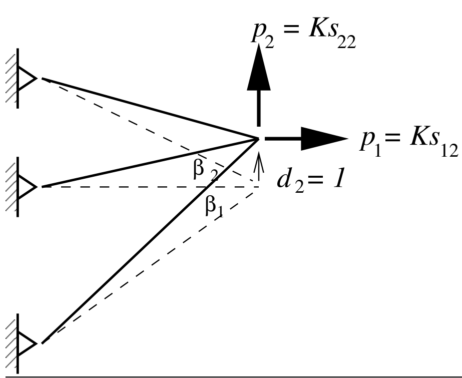
Coordinate 1 Forces required for $d_1 = 1$ (only).



$$\mathbf{K_{s11}} = \left(\frac{EA}{L}\right)_1 \cos^2\beta_1 + \left(\frac{EA}{L}\right)_2 + \left(\frac{EA}{L}\right)_3 \cos^2\beta_2$$

$$\mathbf{K_{s21}} = \left(\frac{EA}{L}\right)_1 \cos\beta_1 \;\sin\beta_1 - \left(\frac{EA}{L}\right)_3 \cos\beta_2 \;\sin\beta_2$$

Coordinate 2 Forces required for $d_2 = 1$ (only).



$$\mathbf{K_{s22}} = \left(\frac{EA}{L}\right)_1 \sin^2\beta_1 + \left(\frac{EA}{L}\right)_3 \sin^2\beta_2$$

$$\mathbf{K_{s12}} = \left(\frac{EA}{L}\right)_1 \cos\beta_1 \;\sin\beta_1 - \left(\frac{EA}{L}\right)_3 \cos\beta_2 \;\sin\beta_2$$

$$\mathbf{K_s} = \left[\begin{array}{cc} \left(\frac{EA}{L}\right)_1 \cos^2\beta_1 + \left(\frac{EA}{L}\right)_2 + \left(\frac{EA}{L}\right)_3 \cos^2\beta_2 & \left(\frac{EA}{L}\right)_1 \cos\beta_1 \;\sin\beta_1 - \left(\frac{EA}{L}\right)_3 \cos\beta_2 \;\sin\beta_2 \\ \left(\frac{EA}{L}\right)_1 \cos\beta_1 \;\sin\beta_1 - \left(\frac{EA}{L}\right)_3 \cos\beta_2 \;\sin\beta_2 & \left(\frac{EA}{L}\right)_1 \sin^2\beta_1 + \left(\frac{EA}{L}\right)_3 \sin^2\beta_2 \end{array}\right]$$

What are the off-diagonal terms if $\beta_1 = \beta_2$?

Does this make sense?