Desiccation shrinkage of non-clayey soils: a numerical study

Liang Bo Hu1, Hervé Péron2, Tomasz Hueckel3,*,† and Lyesse Laloui2

1Department of Civil Engineering, University of Toledo, Toledo, OH, USA
2Laboratory of Soil Mechanics, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland
3Department of Civil and Environmental Engineering, Duke University, Durham, NC, USA

SUMMARY

A mesoscale model of desiccation of soil based on the evolution of the pore system idealized as bimodal is numerically examined. A simplified evolution of the model reveals a series of characteristics that qualitatively agree with the observed macroscopic experimental findings. The principal mechanism is deemed to be driven by the surface evaporation and water outflow generating a pore pressure gradient resulting in the shrinkage mainly of the largest pores. The amount of shrinkage is a function of (negative) pore pressure and is controlled by the compressibility of the solid matrix. The numerical model includes also the ensuing partial saturation stage initiated by the air entry simulated as a scenario with a moving phase interface inside the pore. The proposed model can be extended beyond the two-mode porosity soils, to include the multi-modal porosity, or its statistical representation. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Macroscopic desiccation shrinkage experiments on wet soils indicate that most of the drying shrinkage occurs while soil is still saturated [1]. Shrinkage practically stops simultaneously with the air entrance into the soil, when the water content is high (above 20% for the soils tested by the authors). The remaining drying process occurs with a much-reduced shrinkage rate, but almost entirely via desaturation [2–6]. Still, the specific mechanisms of shrinkage limit and air entry value are still not well understood. Hu et al. [7] (this Journal, this issue) have examined microscopic data of the pore system evolution as represented by the mercury porosimetry results and postulated corresponding mechanisms based on the pressure (suction) development in the vessels and a critical water pressure at the moment of air entry.

The proposed model is based on macroscopic drying experiments [8–10], which indicate that at least two distinctly different stages develop in soils during drying, prior and post shrinkage limit, separated by the air entry. Physically, the pore system is represented by a two-vessel system, with the vessel diameters corresponding to two chosen principal mode pores. The first stage of drying consists in a Poiseuille flow of water in deformable pore vessels driven by the evaporation flux imposed at the boundaries (by the humidity difference). Air entry is interpreted either as a meniscus plunging or a subsurface cavitation in water, which are ‘physically indistinguishable’ phenomena [11]. The latter, implies reaching the tensile water stress equal to water tensile strength, which notoriously depends on the presence of dissolved air microbubbles or solid impurities in water. In the post air entry stage, evaporation proceeds according to a classical scenario of the receding liquid–vapor interface from the

*Correspondence to: Tomasz Hueckel, Department of Civil and Environmental Engineering, Duke University, Durham, NC, USA.
†E-mail: hueckel@duke.edu

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open vessel end, incrementally emptying the filled part of the vessel. This stage is characterized by a marginal water flow and a minimal vessel deformation, while it is dominated by a displacement of the phase transition interface. The process strongly depends on the vessel size and hence vessel compressibility, which controls the liquid pressure at the interface. It is at variance with microscopic models of unsaturated soils, which focus on inter-particle bridges, as opposed to pore vessels as here.

The proposed model is numerically evaluated to examine an interplay of the deformation of the pore vessels and the corresponding water loss. The role of the relevant mechanisms is assessed at the microscopic pore scale whereas the evolution of the macroscopic properties is obtained by averaging. The results are discussed in the context of the microscale and macroscale experiments to interpret various stages of desiccation shrinking process.

2. OUTLINE OF THE MICROSCOPIC MODEL

The model discussed in [7] is based on a geometrically bimodal porosity deduced from mercury intrusion porosimetry (MIP) evolution results.

The initial pore system is envisioned as a bundle of parallel straight thick-walled vessels with a cylindrical cavity of two initial sizes called small (SV) and large (LV) vessels. Their internal diameters as well as the external size of the vessels are approximated to match the observed size of modal pores and their respective porosity of soils. A representative elementary volume (REV) is chosen as a single cylindrical thick-walled prismatic parallelepiped soil block of a unit length containing one single cylindrical LV located centrally and a series of parallel cylindrical SVs, as shown in Figure 1. For the considered Bioley clayey silt [8], there are 12 SV for one LV. Three further assumptions are also made: (i) stress—irreversible strain relationship for the solid material of the vessel is linear during loading whereas the material is rigid during unloading; (ii) the external perimeter of the REV is approximated as a circular cylinder to take advantage of the axial symmetry of the corresponding boundary value problems; and (iii) deformation of the REV is approximated as a superposition of deformations of centrally located LV and 12 identical individual SVs, also centrally located.

The mathematical formulation and solution simulates two stages of the desiccation process: first, the pore space evolution during the saturated drying under the assumption that the vessels are unconstrained kinematically and remain filled with water; and second, an unconstrained desaturation stage, in which the air penetration is characterized by a cavitation critical water pressure. The involved mechanisms are discussed in detail in the companion paper [7], where the corresponding mathematical formulation is represented by Equations (6)–(15). Results of the simulations are presented for the respective stages as follows.

3. RESULTS OF THE SIMULATIONS FOR THE SATURATION STAGE

Results are first presented for large and small pores separately. The numerical values of the stiffness modulus of the solid \( H = 125 \) kPa (from the suction-void ratio data in [1]), and those of pore water viscosity are chosen the same for the analyses of the LV and the SVs. The length of both tubes is 30 cm (hence, \( L = 15 \) cm), taken as the length of the sample in the macroscopic experiment of Peron et al. [6].

![Figure 1. Schematics of a single deformable tube with a suction-driven outflow.](image-url)
It is clearly realized that in a 1D model such as this, the suction driven pore vessel deformation consists in a radial closure. Hence, the shortening of the length of the vessels is not considered. Both types of vessels are subjected to the same external negative water pressure variation, resulting from the same flux of water vapor (e.g., [12]). The imposed boundary pressure evolution for an SV at $x = L$ is plotted in Figure 2. In the same figure, the pressure evolution at $x = L/4$ and $x = L/2$ are also plotted, showing a relatively uniform increase in negative pressure. Clearly, the pressure difference (gradient) along the tube is rather small, growing to about 5% in 10 h, as evident in almost identical pressure profiles for both small and large vessels controlled by the same imposed outside pressure (Figure 3 and Figure 4). Still, the pressure gradient yields a sufficient flow to produce a significant reduction of the inner cavity of the pore.

The evolution of the vessel radii at different locations in SV and LV is shown in Figures 5 and 6, respectively. These figures show the scale of the difference between the response of the two modal pores in terms of closure of the inner cavity: in the first 10 h of the first stage, the SV inner radius closes over 0.08 μm at the internal boundary with the original value of 0.5 μm, whereas the LV radius closure amounts to a much more substantial 0.72 μm of the original 1.5 μm (Figure 7). The profiles of the inner cavity along the axis for each pore type are shown in Figure 8. The relative shrinking of the pore entrance for a single LV is almost triple of that of a single SV, indicating that the deformation of

![Figure 2. Evolution of pore pressure in small vessels (SV) at different locations of the tube.](image)

![Figure 3. Profile of pore pressure in SV along the tube.](image)
Figure 4. Profile of pore pressure in large vessels (LV) along the tube.

Figure 5. Evolution of the inner radii in SV ($a_0$ is the initial radius.)

Figure 6. Evolution of the inner radii in LV.
an individual LV is much greater than SV. Physically, the difference is clearly due to the difference in thickness of the tube solids. Notably, the difference would be even more marked if an actual, higher value were considered for compressibility of the material surrounding the small pores, in effect containing one large pore and other small pores. The resulting cumulative volume loss by a single LV and SV is shown in Figure 9. Water loss from a single SV is nearly 15 times less than that from an LV. However, it should be noted that because there is a 12 : 1 ratio between the number of SVs and the number of LVs per a REV, their total contributions while still dominated by large pores, are at least comparable.

On the mechanics side of the problem, it is interesting to note that because of the same value of the externally applied negative pressure for both types of tubes, their pressures are very similar throughout almost the entire history of the drying process in the saturated range. It has to be realized however, that the two modal tubes have a drastically different overall stiffness because of the differences in their wall-thickness. This indeed produces such a dramatically different response in terms of the deformation of the tubes.

It should be pointed out that Fung’s approximation adopted here [7], while well established in transport phenomena studies of deformable vessels, does not take into account the Poisson’s coefficient effect, and yields a stiffer overall response compared with the exact plane strain solutions for axisymmetric expansion or compression of tubes [13], in particular at large strains. As our task is to find leading mechanisms and variables affecting the pore shrinkage while maintaining the numerical representation as simple as possible, Fung’s approximation has still a merit for the present purpose.
4. RESULTS OF THE SIMULATIONS FOR THE POST-SATURATION STAGE

4.1. Air entry

The post saturation process starts when the pore vessel water reaches a threshold value $p^*_c$, which occurs at the boundary of the wet soil body, that is, at the exit of individual LV and SV pores. As the threshold values are inversely proportional to the vessel radius (see Equation (10) in [7], in which the value of $\tau_s = 72$ mN/m is adopted), they vary differently with the deformation process for LVs and SVs. On the other hand, the amount of suction in the vessels needed for water withdrawal becoming exponentially higher with the increasing deformation, it relatively easily reaches a threshold value prescribed by Equation (11) in Hu et al. [7], see Figure 10, which for LV is 194 kPa and for SVs is 280 kPa.

4.2. Partially saturated stage

The post-saturated process is represented by a complete set of relationships of a Stefan boundary value problem described by Equations (13)–(16) in Hu et al. [7]. We simplify the computation further by assuming the following: (i) the critical cavitation pressure value as constant and corresponding to its value at the moment of the air entry; (ii) the gas-filled part of the vessel is assumed to be in the rigid state; hence, in this stage, not deforming further in this pore segment; (iii) the vapor flux term in the right hand side (the second term) of Equation (16) is assumed to remain constant, and is obtained by equating the left hand side to zero; thus, it is numerically equal to the water flux at the moment of air entry. In the present work, the problem is solved using a finite difference scheme [14] coded in MATLAB (MathWorks, Natick, MA, USA).

Evolution of the water pressure distribution along the tube axis and evolution of the interface position during desaturation stage are shown for an LV in Figures 10 and 11 in which $s$ denotes the position of water–vapor interface. It can be seen that when the water pressure reaches the value of $p^*_c$ at the tube end at time $T = 37,299$ s, the interface starts to migrate from the end boundary toward the center of the tube. Meanwhile, the (negative) water pressure in the (saturated) tube segment continues to evolve, decreasing (algebraically) further until it reaches a uniform distribution at the value of the critical (cavitation) pressure everywhere, hypothesized as constant (radius independent). The change in pressure is accompanied by further deformation. This occurs at a continuously moving interface at which the boundary condition is applied. Interestingly, this process is initially quite slow, but soon becomes very fast, and in the performed simulation, it ends within 42 min with the interface at a 1/8th of the length inside the tube. After that, the water pressure gradient is equal to zero in the entire liquid-filled segment. Consequently, the flow of liquid water ceases, as does the deformation.
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Figure 10. Evolution of the pressure profile in LV during unsaturated phase (Inset: the pressure distribution at the onset of desaturation, $T = 37,299$ s).

Figure 11. Motion of the interface along the tube axis in an LV.

Further water removal proceeds exclusively through evaporation across the interface, which moves also relatively fast (slightly more than 2-1/2 h in the simulation). After the interface reaches the mid-section of the tube, the tube is considered as empty of fluid. Naturally, an assumption of a varying critical cavitation pressure, dependent on the evolving pore size could lead to a more accurate simulation. However, the presented results suggest, in accordance with the macroscopic experimental observation, that after the air entry, the suction varies only a little during a long range of continuous drying, at only a slight further deformation.

Summarizing the process in each tube, it composes of three stages: (i) Poiseuille flow with an increasing negative pore pressure in the entire tube; (ii) invasion of air followed by the liquid–vapor interface motion with the simultaneous Poiseuille flow in the central portion of the tube until the pressure gradient ceases, followed by (iii) the motion of the liquid–vapor interface as a sole mechanism of water removal, at a constant liquid pressure and deformation.

Both large and small pore tubes start shrinking at the same time, immediately upon the application of the evaporation flux. Although the small vessels individually produce a smaller flux, in a REV, there are 12 of these vessels versus one LV; hence, their total flux is about half of that effected by the single large vessel. The difference in the inner cavity size between LV and SV also uniquely determines the cavitation pressure value at which the two modal tubes enter the unsaturated tube stage. Clearly, water
in SVs, cavitates at a much lower (algebraically) pressure (higher suction), and hence, much later than in LVs. Hence, the gaseous stage starts in SVs at a later time.

The performed simulations provide an insight into the overall interplay in the pore space evolution within the REV as presented in Figure 1. An averaging method is used to assess the overall response. To start with, the estimate of the total pore volume loss, \( \Delta V_t \), in a REV is made by combining the pore volume loss by LVs and SVs

\[
\Delta V_t = \Delta V_{LVs} + \Delta V_{SVs}
\]

where the volume loss for LVs and SVs, \( \Delta V_{LVs} \) and \( \Delta V_{SVs} \), respectively, are calculated as \( \Delta V_{LVs} = n_{LV} \times \Delta V_{LV} \) and \( \Delta V_{SVs} = n_{SV} \times \Delta V_{SV} \). \( \Delta V_{LV} \) and \( \Delta V_{SV} \) are the volume losses for an individual LV and SV, respectively, \( n_{LV} \) and \( n_{SV} \) are numbers of the respective vessels in the REV. In the system discussed (and presented) in Figure 1, \( n_{LV} = 1, n_{SV} = 12 \).

Figure 12 presents contributions of the LV and of the whole population of SVs, shown together with the cumulative effects in terms of the total volume loss of the pore space. Notably, the contributions from the small and large vessels vary. At the beginning, about 2/3 of the pore space is lost, and at this stage, also 2/3 of the water removal is provided by the large pores. At the end of the process, pore volume losses from LVs and all SVs are about equal. Note, that the visible end of the monotonic part of the process for each pore population (a and b in Figure 12), corresponds to the end of the strictly pore fluid pressure driven deformation. This stage is followed by a very brief transition stage (aa’ and bb’) of the continued fluid flow and simultaneous interface motion, subsequently followed by the sole interface motion stage, which does not induce any further deformation. Notably, for the considered case, the onset of desaturation of small pores is delayed about 4 h relative to LV desaturation onset. In the single LV, the air entry occurs around \( t = 37,299 \) \( s = 10.36 \) h (marked as a in Figure 12), and there is a very short period of continued fluid flow and deformation in the tube (Figure 10) until \( t = 39,299 \) \( s = 10.91 \) h (marked as a’ in Figure 12). For a single SV, the two corresponding instants are \( t = 50,117 \) \( s = 13.92 \) h (marked as b) and \( t = 56,233 \) \( s = 15.62 \) h (marked as b’), respectively. Thus, the shrinkage phase of the LV after air entry takes about 33 min, whereas the same for the SV takes nearly 2 h.

The cumulative pore volume loss graph (Figure 12) mirrors all these points with consecutive decreases in the rate of the pore volume loss. This curve is compared with the experimental data points of the pore volume loss obtained from the MIP study and from the strain measurements under the assumption of the rigid grain solids [1, 8], referred to as the REV.

Given a highly schematic nature of this simulation, the result is fairly encouraging. Clearly, the model in the second stage predicts more pore volume loss than in the experiments. To visualize in a

![Figure 12](https://example.com/figure12.png)

Figure 12. The pore volume loss in a representative elementary volume (REV) (Figure 1) for LVs (dash line); SVs (dot dash line); LVs+SVs (solid line); result calibrated from the mercury intrusion porosimetry (MIP) study (squares) in [16]; result calibrated from experiments in [1] (triangles).
more suggestive way the pore space evolution during drying, the history of the LVs and SVs is shown for the instants at LL, SL and of dry material in terms of model pores in the modal pore distribution in Figure 13. It is to be compared with the results of the MIP and its modal representation in Figure 4 in [7]. The simulated vessels in Figure 13 can be seen to migrate in the porosimetry chart because they are uniquely identified by the model as the original LVs unlike in the MIP chart in Figure 7 in [7]. Nevertheless, it is seen that their volume fraction drops to 3.9% compared with 5% in the MIP at shrinkage limit and becomes significantly smaller, but barely evolves any further after SL. However, the pore sizes get significantly smaller during the transition from LL to SL; on the other hand, the original SVs decrease in their porosity fraction. However, if they are counted together with the shrunk ex-LVs, they amount to 17% of porosity fraction at SL. Although it is realized that in MIP the small pores increased in their fraction, it is noted that the reported volume fractions are not corrected for the volumetric strain, or decrease of the total volume of the drying material. It is concluded that given the strong assumption of the linearity of the stress–strain behavior of solids (which has a strong locking tendency in reality), the model modestly overestimates the volume of the collapsing LVs.

Similarly to Figure 12, Figure 14 presents the pore volume evolution in terms of void ratio change. Furthermore, Figure 15 provides the total void ratio history during drying. Figures 12, 13 and 15
Figure 15. Evolution of void ratio in a REV (Figure 1) for simulation based on LVs+SVs (solid line); result calibrated from the MIP study (squares) in [16]; result calibrated from experiments in [1] (triangles).

Figure 16. Evolution of volumetric water content in a REV (Figure 1), simulated based on LVs and SVs. Indicate that before the ultimate shrinkage limit is reached, a significant part of the volume change still takes place in the SVs, although with a much slower rate than during the stage dominated by LV shrinkage. Although the simulation does not completely match numerically the experimental data, a good reproduction of the main characteristics of shrinkage is evident.

Figure 16 presents the evolution of the water removal, also characterized by visible variable rates during the entire process, particularly around \( t = 4.5 \times 10^4 \text{ s} \). The whole process can be further interpreted by examining other variables such as degree of saturation, as well as the shrinkage curve, discussed subsequently. Figure 17 shows the evolution of degree of saturation, providing the most direct representation of the presence of pore water. The desaturation starts around \( t = 3.7 \times 10^4 \text{ s} \) (approximately 10 h) point a, when the water critical pressure is reached in LPs and air invades the LPs. It appears that the overall drying rate accelerates slightly as the water–vapor interface motion in the LVs starts to affect the overall result. Around \( t = 4.5 \times 10^4 \text{ s} \) (12.5 h, marked as c), the desaturation slows down and so does the water removal. The reason is that at this moment, water is completely depleted in the LVs, and the SVs become the only source of water. Further drying results in the water removal and shrinkage of the SVs alone, which remain saturated till their water cavitation and air invasion around \( t = 5 \times 10^4 \text{ s} \) (13.9 h, marked as b). This is also reflected by the shrinkage...
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Figure 17. Evolution of degree of saturation in a semi-log scale in a REV (Figure 1), simulated based on LVs and SVs.

curve in Figure 22. Prior to the final shrinkage limit (e ≈ 0.35), a short segment is parallel to the saturation line, indicating that the shrinkage volume in this period is equal to the water volume loss (both related to SVs alone).

It should be pointed out that this sequence of events is by no means universal. Rather, it depends on the properties of solids (deformability), pore fluid viscosity, its cavitation threshold (surface tension) and pore size structure. As discussed by Hu et al. [12, 15], soils of different compressibilities and permeated with some other fluids such as ethanol and glycol solutions, with different viscosity, surface tension, and evaporation characteristics, do not necessarily exhibit the same sequence of events in their respective LVs and SVs.

A challenging task is to present the overall pore fluid pressure evolution for the entire system. Clearly, as discussed in the preceding paragraphs, the large pores are invaded by air well before the small pores. Also, the pore water pressure in LVs is assumed to stay at a constant value after the air intrusion whereas the pore water pressure in SVs continues to evolve until it reaches a much higher value of cavitation pressure. In the present simulation, the overall water pressure $\bar{p}$ is obtained based on the average water pressure $\bar{p}^L$ and $\bar{p}^S$ in LVs and SVs weighed by their respective solid–liquid interface surface areas. Hence,

$$\bar{p} = \frac{S^L \bar{p}^L + S^S \bar{p}^S}{S^L + S^S} \tag{2}$$

where $S^L$ and $S^S$ are the original solid–water interface surface areas for LVs and SVs, respectively, calculated as $S^L = 2\pi R_L L$ and $S^L = 12 \times 2\pi R_S L$, with $R_L$ and $R_S$ being radii of large and small vessels, respectively.

The overall (averaged) pore pressure is plotted in Figure 18 against the void ratio. It largely conforms to the macroscopic overall deformability in relation to the negative pressure (suction). This (classically logarithmic) relationship can be further seen in a semi-log scale as in Figure 19. Clearly, around 200 kPa, a slight stiffening effect is observed. This moment marks the initial cavitation and air intrusion into the LVs, as shown in Figure 20. Before this instant (about $t = 3.7 \times 10^4$ s), the soil remains saturated as seen in Figure 21. The next stage is characterized by a subsequent water–vapor phase transition in the LVs and simultaneous shrinkage of the SVs until (a possible, but not unique scenario taking place in our simulation) a complete depletion of water in the LVs, before cavitation and air invasion of the SVs, which is discussed above. The subsequent stage of the water–vapor phase transition in the SVs starts around $t = 4.5 \times 10^4$ s and pore pressure at about 280 kPa. The air enters into the SVs around an overall pressure of 400 kPa at $t = 5 \times 10^4$ s (Figures 17 and 21). Shortly after that, all deformation (shrinkage) stops.

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Figure 18. Void ratio versus pore pressure in a REV (Figure 1), simulated based on LVs and SVs.

Figure 19. Void ratio versus pore pressure in a semi-log scale in a REV (Figure 1), simulated based on LVs and SVs.

Figure 20. Negative pore pressure versus volumetric water content in a REV (Figure 1), simulated based on LVs and SVs.
Equally interesting is the overall shrinkage curve plotted in Figure 22. The initial shrinkage that also contributes to a substantial part of water removal, clearly follows the saturation line (with a slope of 1 : 1). The subsequent stage of shrinkage occurs during the continuous removal of water from both LVs and SVs, whereas only the SVs continue to shrink, thus explaining the deviation from the saturation line. The next segment, cb, discussed earlier at the mesoscale level, represents the shrinking deformation of saturated SVs while the entire LV population is already depleted of water and undeformable. Subsequently, the SV pore water reaches cavitation threshold and SVs are invaded by air. Finally, the SV deformation stops and shrinkage limit is reached when the pressure gradient in SVs becomes zero along the tube. Soon after, also SVs are entirely filled with air, which means that desaturation is completed.

Interestingly, Figure 21 can be viewed as a characteristic curve for the overall material desaturation as represented by our bimodal model for the soil pore system. During its full saturation branch,
from the starting point of the curve to a, there is a substantial suction increase, and major water loss (Figure 16), but no change in saturation, as the water loss in entirely compensated for by the shrinkage strain. The main portion of desaturation, a − c, corresponds to the stage after the air entry to the LV, with the SVs still remaining fully filled and undergoing deformation, which does require an increase of overall suction. The latter practically ceases to grow when water in the SVs reaches the cavitation threshold and air invades also the SVs. The last segment corresponds to the inter-phase interface motion, without much of the change in overall suction in SVs. In conclusion, the process stops at a relatively modest value of the overall suction of about 400 kPa. It needs to be noted that a modal pore vessel representation with additional pore modes at a smaller pore size could no doubt lead to higher values of the overall suction near the completion of the desaturation process.

5. DISCUSSION OF THE SIMULATION RESULTS

The presented mesoscale model is based on a pore system evolution, with the results that appear to be fairly particular and to a degree unexpected. Notably, the pore system is represented here by a highly idealized, bi-modal parallel tubular model. A primary feature of this representation is that it is a 1D model. Also, only two distinct pore modes are selected to represent the entire pore population.

However, this coarse approximation seems to represent fairly faithfully the evolution of the pore space compared with experimental data, as well as the evolution of saturation degree and drying (water depletion) rate. Notably, because of the structure being made of 1D tubes, the mechanism through which the vessel’s contraction contributes to the macroscopic shrinkage is through the vessel’s radial strain. This strain is the only contribution to the pore space reduction in the BVP considered. In the experiments, all three principal strains were observed, and in the central period of the test, they were seen to be isotropic [8, 16]. In the comparisons, only the volume changes were examined, not individual strain components.

It is of interest to follow the evolution of the individual pore vessels separately. Note that LVs’ shrinkage over the entire process is almost 50% in terms of the original radius size, or from 1.5μm to 0.7μm in terms of the absolute radius dimension. This final radius value qualifies the shrunk LV pores as nearly SVs. It reproduces very well the experimentally seen shift in the volume of LVs to the SV volume.

The SVs shrink about 36%, that is, from 0.5μm to 0.32μm. This also mirrors well the experimental data, which show the SVs moving with their average diameter from 0.5μm toward 0.3μm. Overall, given the highly schematic idealization of the pore system, this can be considered a faithful representation.

The presented analysis can be extended by using the actual or a statistical representation of the pore size distribution, assuming a random orientation and length of the tubes. This could provide a more accurate overall performance, removing several of the limitations mentioned above. That is a necessary step to address the individual strain components during desiccation.

An obvious shortcoming of this model appears in Figures 19–21, as it does not predict any further evolution of the (negative) pore water pressure and deformation, practically after air invades all the pores. This is the consequence of the post-saturation scenario that implies that in the unsaturated part of the tube, the entire cross section is free of fluid phase. An alternative scenario, with a more articulated distribution of vapor, fluid, and air including a distributed evaporating surface area within a non-saturated portion of the tube and possible effect of capillary forces is studied elsewhere [17].

In addition to the assumption that the pore structure is bi-modal, another simplification is made by excluding a class of pores below 10−1μm, containing initially about 5% of water volume. The mere fact that these pores are much smaller than current SVs of 0.5 μm would lead to much higher suctions of their critical (cavitation) pressure threshold.

6. SIMULATION RESULTS FOR A CONSTRAINED SOIL BODY AT THE MICROSACLE

Damage of soils during desiccation is considered as a response of soil to an excess of tensile stresses induced by the shrinking body with a constrained kinematics [1, 18]. Hence, it is of great interest to use the presented mesoscale pore model to examine desiccation shrinkage of an externally constrained
soil sample. It is noted that the constraint in the experiments of Péron et al. [1], is indeed imposed in a particular direction, along the sample axis, whereas the shrinkage in the model is perpendicular to the tube axis.

Nevertheless, it is still of interest to see what effect the constraints may have on shrinkage. For that purpose, a single tube is considered with a constraint in a form of a fixed external boundary that does not allow any radial displacement at the outer radius \( r = b_0 \) in the first stage of full saturation of soil. All the elements of the simulation discussed in the previous section are still valid, except that the relationship between the inner perimeter pressure and the inner displacement (the inner radius does change) must be recalculated according to the solution for the tube with a new boundary condition: at \( r = b_0, u = 0 \).

Hence, a counterpart of Equation (10) in [7] is established based on the solution to the new BVP (following Fung [19], the Poisson’s effect is neglected),

\[
a(x) = \frac{p}{H} \frac{h_0 a_0^2}{a_0^2 + b_0^2} + a_0
\]

It is pointed out that the above expression is an approximate solution, similarly to Fung’s solution. We furthermore drastically simplify the transport and mass conservation equations by neglecting the effect of the evolving radii on the Poiseuille flow, and the effect on the mass flow of the cross-section area change. A PDE for the pore pressure is hence obtained for the externally constrained tube as

\[
\frac{\partial p}{\partial t} = \frac{H a_0 (a_0^2 + b_0^2)}{16 h_0} \frac{\partial^2 p}{\partial x^2}
\]

In contrast to the solution in Section 4.3.2 in [7], the boundary conditions used in this formulation are as follows: \( x = 0, \frac{\partial p}{\partial x} = 0 \) and \( x = \pm L, \kappa \frac{\partial p}{\partial x} = f_{evp} \). The boundary flux at the end boundary is prescribed as an evaporation flux, taken as constant.

To facilitate the comparison between a free shrinking tube and an externally constrained tube, the original Equation (11) in [7] for free shrinking tube is also simplified by introducing the same assumptions resulting in

\[
\frac{\partial p}{\partial t} = \kappa \frac{\partial^2 p}{\partial x^2}
\]

Clearly, the coefficient on the right hand side of the above equation, for simplicity denoted \( \kappa = \frac{H h_0 a_0}{16 \mu} \) plays a role analogous to permeability at the macroscale.

The evolution of the water pressure for both the unconstrained and constrained large pore tubes at the end boundary is shown in Figure 23. Clearly, although the water volume losses are identical, as prescribed by the boundary flux condition, more than a three times higher value of water pressure is needed for the constrained tube to deform it at a given time. This implies also a very similar evolution of the inner radius.

The most important conclusion from this exercise is the analysis of the radial stresses in tube, which are tensile at all time. Figure 24 shows the radial and circumferential stress components for the two types of tubes at \( t = 18,000 \) s. It is evident that the tensile radial stress component (negative in Figure 24) is much larger across the constrained tube. Its maximum occurs at the inner radius. Easily that stress can meet the tensile strength, which is (at macroscale) established at around 5 kPa [16]. According to the earlier elaborated hypothesis, overcoming a tensile stress limit is considered as a condition for cracking. Interestingly, the potential for fracture arises at the pore wall, rather than at the reaction site (external boundary). Hence, it is concluded that the origins of desiccation cracking are to be sought within the drying soil, rather than necessarily at their external boundaries. Furthermore, it may be speculated that the ensuing failure mode consists in spalling of the internal soil layer at the inner wall of the pore. Such a mechanism is common in large scale cavities or galleries in rocks. This conclusion should be verified experimentally. The result is clearly linked to the original formulation of the BVP as a contraction of the tubular pores, and in particular to the constraint articulated as the boundary condition on the entire external perimeter of the tube.
7. CONCLUSIONS

The presented mesoscale model provides a tool for numerical simulations of the drying process of non-clayey soils in its saturated and unsaturated stage. The model is largely based on the evolution of the pore system, idealized as bimodal. The simulations are hence based on idealized, but actual porosimetry data. They indicate that the evolution of a highly simplified physical model reveals a series of characteristics that agree qualitatively with the macroscopically observed experimental findings.

The centerpiece of the model is differentiation of the transport mode in saturated and unsaturated stage. In the saturation stage, the transport of pore fluid toward the perimeter of the drying body produces a contraction of the vessels induced by a negative pore pressure. It appears that the previously unexplored source of an evolving porosimetry offers an opportunity for a physics-based modeling. In particular, a significant reduction in diameter of large pores is seen in the data, compared with a smaller shrinkage of small pores. This is consistent with the difference in their deformability due to size difference of pore walls in the model.

The air entry mechanism is postulated via water (negative) pressure reaching the suction strength understood as an ‘subcutaneous’ cavitation onset of isothermal evaporation. A traditional criterion
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of the meniscus sinking cannot be distinguished from the cavitation. The numerical model simulates
the ensuing post-saturation stage as a scenario involving a moving phase interface inside the large
pore. The fluid flow within the pore vessel is nearly zero at this stage, and all water is removed via
evaporation at the interface. The same occurs to the small pores, but shifted in time. In this particular
simulation, calibrated against the macroscopic experimental findings, the saturated tube stage lasts
12 h, whereas unsaturated stage lasts 2-1/2 h. In the presented scenario, virtually no shrinkage is
associated with latter stage.

Additionally, an interesting sequence of differentiated air entry into the two modal pores provides
a possible explanation of the observed behavior. In particular, large pores are completely depleted
of water before smaller pores experience air entry. This sequence by no means is unique. In fact, a
particular sequence in a scenario depends on the size of modal pores and deformability of the solid
material, and in general on surface tension of the pore fluid. Different sequences can be envisioned for
different fluid properties and deformabilities [15].

The critical pressure leading to air entry through whatever mechanism is a crucial parameter for
possible prediction of shrinkage, cracking, and so on. Hence, it deserves much more attention that it
has been given in the past.

It may be stated that in broad terms, the drying shrinkage is ‘practically’ caused by the outflow of
water from a class of the largest pores of the soil driven by the evaporation induced suction, whereas
the shrinkage limit is triggered by the water suction threshold in these pores causing the air entry. Both
are controlled by the compressibility of the porous matrix around the large pores.

Clearly, the proposed model can be extended beyond the two-mode porosity soils, to include the
multimodal porosity, or its statistical representation.

To investigate potential of desiccation cracking, a microscopic model for a constrained deformation
of the tubular pores is examined. As expected, the stresses generated across the constrained tube are
significantly higher, and in particular, the tensile radial stress reaches a value more than three times
higher than that in a free shrinking tube. That obviously increases the potential for tensile cracks. The
tensile strength is met at the inner perimeter of the tube and implies a spalling type of the failure
mode. This conclusion is obviously limited by the simplicity of the BVP considered and needs to be
corroborated experimentally. The question if and how the microscopic cracking occurrence evolves
into a macroscopic crack needs to be further explored. An alternative avenue regarding desiccation
cracking has been pursued elsewhere [20].

The presented model is focused on the mesoscale mechanisms, and hence, a possible quantitative
comparison with the experiments is made by using the averaging method for upscaling the variables
obtained at the mesoscale. A more accurate prediction may demand more sophisticated upscaling
techniques, including a stochastic representation of the pore system configuration. An inclusion in
the model of the additional effect of axial shrinkage of the vessels caused by capillary forces is quite
possible, but would require a much more articulate 3D model and an appropriate homogenization
procedure.

Another major qualitative conclusion from the presented work is that, at least on the mesoscale, the
drying-related response driven by the negative pressure of fluid alone allows one to obtain reasonable
results. The dewatered portion of the pore does not exert any pressure (suction) in this model.
Therefore, after an initial stage of a growing negative pressure, there is a period at which there is a
visible decrease in saturation at modest changes in suction, at a nearly zero shrinkage rate. This is
in disagreement with many suction-induced macroscopic drying experiments, which report very high
suctions at the terminal stages of dewatering. Although such mechanisms may well mirror macroscopic
data of the suction driven experiments, they require a highly non-linear, nearly locking behavior in
circumferential compression to describe the response of pore tubes. Notably, recent experiments with
elementary capillary structures indicate a continuous decline during evaporation of the pressure of
water inside the capillary bridge until zero, followed by an increase of a modest positive pressure
prior to the ultimate rupture of the bridge [21]. There is also lack of experimental evidence of high
microstresses in response to the very high suction that would lead to a widespread damage at the
microscale. Numerical comparisons with an alternative scenario that does include an explicit variable
of diffused capillary forces within a partially dewatered portion of the pore space are expected to be
quite enlightening.
One of the tasks of microscopic modeling is to identify the principal mechanisms and variables responsible for the process observed and measured directly at the macroscale. In the considered case, the principal mechanism is deemed to be the shrinkage of the pores of a significantly large size driven by the water outflow linked to the surface evaporation. The amount of shrinkage is controlled by the compressibility of the solids and is a function of pore pressure (suction). The rate of shrinkage is controlled by the evaporation rate, by the fluid viscosity and by the experimentally measurable variable of the radius of the pores. The shrinkage limit is determined by the developed pore pressure/suction (controlled again by solid compressibility) meeting the water tensile failure (cavitation) pressure criterion. The unsaturated stage occurs practically with no further deformation. Cracking of soils originates via a near cavity spalling. Capillary effects play a minor role in both the saturated and unsaturated stages in the model. Finally, it is often considered that desiccation cracking results from excessive tensile stress in soil [8]. Such tensile stress cannot originate according to the presented scenario from capillary forces, which produce compressive stress in the solid soil structure.

The model requires further investigation of its numerous aspects, to start with the conceptual simplification, 1D character, and rigid plastic material model assumptions as well as disregarding the role of pendular capillary water in the unsaturated stage. An obvious emerging task is to investigate the effect of varying vessel radii on the critical water pressure for air entry defined by the Laplace law. A microscopic criterion for this occurrence is still a point of discussion. Verification of the validity of the present model requires an upscaling procedure to represent the proposed mechanisms at the macroscale.

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