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# Split awards, procurement, and innovation

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and

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*In many procurement settings, it is possible for a buyer to split a production award between suppliers. In this article, we develop a model of split-award procurement auctions in which the split choice is endogenous. We characterize the set of equilibrium bids and allocations for optimizing agents in an environment in which suppliers are fully informed about each other's costs. Split-award equilibria simultaneously exhibit strong collusive features and cost-efficiency properties. Despite the former property of the equilibria, upstream investment considerations may lead a buyer to prefer a split-award auction format to a winner-take-all auction format.*

## 1. Introduction

■ Auctions are used to award contracts for a variety of product and service requirements in the public (e.g., defense systems and municipal services) and private sectors (e.g., input supply and franchising). These auctions can result in a sole-source award, in which a single producer provides all of the required production, or in a split award, in which production is divided between two or more firms. Split-award auctions have been employed by buyers to procure a wide range of complex technologies, including missiles by the U.S. government and computer chips by IBM. With the notable exceptions of Wilson (1979) and Bernheim and Whinston (1986), research on auctions has focused on winner-take-all (or unit) auctions.<sup>1</sup>

Our major concern in this article is the price performance of split-award procurements.<sup>2</sup> Can a buyer expect to pay a price that closely reflects the underlying costs of the suppliers,

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<sup>1</sup> Winner-take-all auctions explicitly exclude the possibility of a divided production award, whereas split-award auctions allow for divided production awards as well as sole-source awards.

<sup>2</sup> In the last section of the article, we consider the indirect benefits of maintaining dual-source production (e.g., enhancing the prospect of increased future competition). Our model suggests that bidders will be able to capture any value the buyer may attribute to these indirect benefits.

TABLE 1 A Split-Award Bidding Equilibrium

Award	Costs		Bid Prices	
	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>
50/50 Split	4	4	6	6
Sole Source to <i>A</i>	10	0	12	—
Sole Source to <i>B</i>	0	10	—	12

or will strategic bidding allow suppliers to earn supranormal profits?<sup>3</sup> To address this issue, we develop a formal model of split-award auctions and characterize the equilibrium bidding behavior.

The most striking feature of split-award auctions is the existence of bidding equilibria that involve the efficient or cost-minimizing split of the total production award. These equilibria have the properties that the price to the buyer is maximized and that the individual's and joint suppliers' profits are maximized relative to all other equilibria. Thus, not only do split-award auctions fail to promote competition, they effectively present bidders with an invitation for implicit price collusion.<sup>4</sup>

In Table 1 we provide a simple example of a split-award equilibrium in which two bidders, *A* and *B*, support a "collusive" outcome. As indicated, the buyer can make one of three possible awards. Each supplier submits a sealed bid with price offers for its share of the split award and for a sole-source award. In the equilibrium, the buyer pays 12, each supplier earns a profit of 2, and the 50/50 split is selected by the buyer. Neither supplier has an incentive to reduce its sole-source price to win the entire production award, and the buyer's price is well above the supply cost. While simple in structure, this example captures a number of features that are general properties of split-award auctions.

The ability to support collusive prices does not arise from any limitations on the range of splits (divisions of the total award). To see this, add to the example the possibility of two additional splits of 25/75 and 75/25. The 50/50 outcome with a price of 12 can still be supported as long as both suppliers offer a "high" bid price (e.g., 12) for their 25% share. This example illustrates that a single bidder can effectively veto any non-sole-source split by submitting a relatively high price for its share of the split, thereby rendering the split unattractive to the buyer. Later, we will demonstrate that the sole-source prices provide competitive pressure on the bidding and establish a ceiling on equilibrium split-award prices.

In the next section, we present a bidding model of split-award auctions. The model structure corresponds to one method employed by the government to procure defense technologies: the government specifies a set of "step-ladder" (split-award) quantities; contractors submit sealed bids that specify prices for those quantities; and the government then chooses a split of the total requirement that minimizes the procurement price.<sup>5</sup>

<sup>3</sup> The potential that split-award auctions raise for noncompetitive bidding has attracted some attention in the defense management literature (e.g., Meeker (1984) and Boger and Liao (1988)). Bidding in such auctions is seen as involving "reverse competition," in which suppliers are content to obtain a small share of the total award at a relatively high price and do not bid aggressively for larger shares.

<sup>4</sup> Collusive equilibria have also been found by Wilson (1979) for a share auction in which, when translated into our setting, the buyer must pay each supplier the same price per unit, where the price is set to equate supply and demand. In our model, the buyer can act as an optimizing agent, selecting any combination of price bids that will minimize the total procurement price. Allowing the buyer to act with this freedom eliminates the collusive equilibria that can occur in Wilson's complete information case.

<sup>5</sup> Ralph Winter suggested to us that the model might also be used to explore pricing in exclusive dealing arrangements. For example, in Mathewson and Winter (1987) two upstream wholesalers can offer a single retailer prices for an exclusive dealing arrangement (sole-source bid) or a competitive arrangement (split-award bid).

We develop a complete characterization of the bidding equilibria, along with the associated set of equilibrium split-award outcomes and payoffs, in Sections 3 and 4. The split-award auction is set in a framework that involves a general reduced-form cost function and two suppliers who are fully informed about each other's costs. The share of production awarded to each supplier is a choice variable of the buyer. The bidding results can be applied in a straightforward way to explore a variety of structural models. We present an example that features production economies and process innovation, a setting in which the choice of the split has endogenous consequences that involve downstream strategic interaction between the suppliers.

Section 5 presents a comparison between the split-award and the winner-take-all formats for procurement auctions and highlights the impact of the choice of format on upstream investment. An example is provided in which the split-award format induces prebid innovation that would not occur under a winner-take-all format. We conclude in Section 6 with a discussion of related policy issues.

## 2. The model

■ A buyer must procure a given quantity of  $x$  units. There are two potential suppliers, labelled  $D$  (developer) and  $S$  (second source); any division of  $x$  between the suppliers is feasible. The buyer minimizes the procurement costs, and the suppliers maximize profits.

In a split-award auction, each supplier submits a sealed bid that specifies prices for varying splits of the total award of  $x$ . To simplify the notation, all variables are expressed in terms of the share of  $x$  that is awarded to one of the bidders. Letting  $\alpha \in [0, 1]$  denote the share of  $x$  that supplier  $D$  produces, a bid is a function  $P_i: [0, 1] \rightarrow \mathcal{R}$ . Thus, for a pair of bids,  $(P_D, P_S)$ , a split of  $\alpha$  implies that  $D$  produces  $\alpha x$  units for a payment of  $P_D(\alpha)$ , while  $S$  produces  $(1 - \alpha)x$  units for  $P_S(\alpha)$ . Bids are not required to be smooth (e.g., continuous or differentiable).

The costs of the suppliers at an arbitrary  $\alpha \in [0, 1]$  are given by  $C_D(\alpha)$  and  $C_S(\alpha)$ . These cost functions are interpreted as the expected present discounted value of all costs incurred by a bidder, conditional on the split  $\alpha$ . For instance,  $C_D$  and  $C_S$  may consist of the expected direct production costs plus the investment costs associated with process innovation.<sup>6</sup>

The profits of each bidder are defined by

$$\Pi_i(\alpha) = P_i(\alpha) - C_i(\alpha), \quad \alpha \in [0, 1], \quad i = D, S.$$

We assume that  $C_D(0) = 0$  and  $C_S(1) = 0$ . Since neither supplier can expect to receive a payment for zero production, we set  $P_D(0) = 0$  and  $P_S(1) = 0$ . Furthermore, we restrict our attention to bids for which  $P_i \geq C_i$  for each bidder. This restriction rules out equilibrium payoffs that are supported by threats involving negative profits. Suppliers are assumed to be fully informed about each other's costs when they bid.<sup>7</sup>

The joint cost function of the suppliers is given by

$$B(\alpha) = C_D(\alpha) + C_S(\alpha), \quad \alpha \in [0, 1].$$

Given the cost functions,  $C_D$  and  $C_S$ , the joint cost function as defined by  $B$  provides a simple cost-efficiency criterion for comparing various splits. Clearly, whenever  $B(\alpha) < B(\hat{\alpha})$ , there exists a reallocation such that all parties would benefit from a switch from the split  $\hat{\alpha}$  to the split  $\alpha$ .

<sup>6</sup>  $C_D$  and  $C_S$  can also be defined to incorporate the possibility of repeated auctions. See Anton and Yao (1987).

<sup>7</sup> This assumption allows us to abstract from complications associated with uncertainty about relative costs. The assumption does not appear to be essential to the qualitative results of the model; an example with asymmetric cost information between the suppliers is provided in Section 4.

Given a pair of bids,  $(P_D, P_S)$ , the buyer determines the outcome of the split-award auction as follows. The total payment by the buyer at an arbitrary split of  $\alpha$  is given by

$$G(\alpha) = P_D(\alpha) + P_S(\alpha), \quad \alpha \in [0, 1].$$

The buyer selects a split to minimize the procurement costs, so

$$\alpha \in \underset{a \in [0,1]}{\operatorname{argmin}} \{ G(a) \} \quad (1)$$

is the buyer's optimal choice.<sup>8</sup> In (1), the buyer is making an *ex post* comparison of the submitted bid prices, which does not require any information about the suppliers' costs.

When there are ties and the award choice from (1) is not unique, the appropriate tie-breaking rule is to select an  $\alpha$  randomly from among those splits in the set  $\operatorname{argmin} G(\cdot)$  with the smallest  $B(\cdot)$  value. As discussed in Milgrom (1986), this procedure mimics the outcome that occurs when bid prices are discrete instead of continuous because a lower-cost bidder will avoid a tie by reducing his bid by a small amount.

If the awarded split is  $\alpha = 0$  or  $\alpha = 1$ , it is called a sole-source outcome. A split-award outcome refers to a case in which  $\alpha \in (0, 1)$ , in which both suppliers produce a positive amount.

A Nash equilibrium is a pair of bids,  $(P_D^*, P_S^*)$ , that are mutual best responses for the suppliers. If  $\alpha^*$  satisfies (1) and the tie-breaking rule for these bids, then it is an equilibrium outcome. The best-response property means that neither bidder can increase his profits above their realized values of

$$\Pi_i^* = P_i^*(\alpha^*) - C_i(\alpha^*), \quad i = D, S$$

by altering  $P_i^*$  while taking his opponent's bid as given. Note that a bidder can change the prices at many splits simultaneously by altering a bid. For example, by raising his price at  $\alpha^*$  and by lowering his price at some  $\hat{\alpha}$ , a bidder can try to induce the buyer to select  $\hat{\alpha}$ . Finally, let

$$g^* = G(\alpha^*) = P_D^*(\alpha^*) + P_S^*(\alpha^*)$$

denote the equilibrium procurement price.

A number of economic situations are consistent with the above model. These situations are specified by describing the factors that determine  $C_D$  and  $C_S$ . We explore these possibilities in later sections by considering the roles of production economies, process innovation after  $\alpha$  is awarded, and prebid R&D. While the determination of  $C_D$  and  $C_S$  may be quite complicated, the equilibrium structure of a split-award auction is completely determined by  $B$ , the joint production cost of the suppliers.

### 3. Properties of equilibrium bids

■ This section provides a simple method for finding the set of Nash equilibrium bids and associated outcomes. Equilibrium split-award outcomes are then examined in the following section.

□ **Calculating equilibrium bids.** The bid prices for  $\alpha$  at zero and one,  $P_S(0)$  and  $P_D(1)$ , correspond to the sole-source outcomes in which one supplier produces all  $x$  units. Since an opponent's sole-source bid price places a ceiling on the prices for alternative splits that a bidder can induce the buyer to accept, the sole-source prices are pivotal in determining the equilibrium bids, profits, and the procurement price. A necessary condition for a par-

<sup>8</sup> If a minimum does not exist, use the infimum.

ticular split to be an equilibrium outcome is that the procurement price for this split equals each of the sole-source prices.

*Lemma 1 (Price equivalence).* Suppose that  $(P_D^*, P_S^*)$  is a Nash equilibrium, and let  $g^*$  be the associated total price to the buyer. Then, the equilibrium bids satisfy  $g^* = P_D^*(1) = P_S^*(0)$ .

*Proof.* Suppose the bids do not. Then, one of the bid prices must be larger than  $g^*$  as, by definition,  $g^*$  is the price to the buyer at his optimal choice. If  $P_S^*(0) > g^*$ , then the outcome of zero is not optimal for the buyer. Let  $\epsilon = P_S^*(0) - g^*$ , and consider  $D$ 's bid, defined by

$$P_D(\alpha) = \begin{cases} P_D^*(\alpha) + \epsilon/2, & \alpha \neq \alpha^*, \quad \alpha \neq 0 \\ P_D^*(\alpha) + \epsilon/3, & \alpha = \alpha^*, \end{cases}$$

and  $P_D(0) = 0$ , where  $\alpha^*$  is an optimal choice for the buyer in the original equilibrium. Given the other supplier's bid,  $D$ 's bid guarantees that  $\alpha^*$  is the buyer's unique choice and also that  $D$ 's profits are higher than they are with  $P_D^*$ . Thus,  $P_D^*$  is not a best response, and we have a contradiction. The case for  $P_D^*(1) > g^*$  is analogous. *Q.E.D.*

In equilibrium, the buyer is always indifferent with respect to at least two outcomes—and at least three if  $\alpha^* \in (0, 1)$ , a split-award outcome. The intuition is simple. If the price equivalence property (Lemma 1) fails to hold, then one bidder's sole-source price is strictly greater than  $g^*$ . The other bidder can then increase his profit by raising all of his bid prices slightly (except, of course, at the sole-source price of the opponent) without disturbing the original  $\alpha^*$  choice of the buyer.

Now, consider the incentive of a supplier to alter his bid. Let  $(P_D, P_S)$  be an arbitrary pair of bids, and let  $\alpha$  be a candidate outcome. If a bidder, say  $D$ , can find a price,  $\hat{p}$ , for some other outcome,  $\hat{\alpha}$ , such that  $\hat{p} + P_S(\hat{\alpha}) < P_D(\alpha) + P_S(\alpha)$  and  $\hat{p} - C_D(\hat{\alpha}) > P_D(\alpha) - C_D(\alpha)$ , then the buyer faces a lower total price at  $\hat{\alpha}$ , and  $D$  earns greater profits. The existence of such a  $\hat{p}$  is equivalent to the fulfilling the equality  $P_S(\alpha) - P_S(\hat{\alpha}) > C_D(\hat{\alpha}) - C_D(\alpha)$ . Since  $\Pi_S = P_S - C_S$ , we have

$$\Pi_S(\alpha) + [C_D(\alpha) + C_S(\alpha)] > \Pi_S(\hat{\alpha}) + [C_D(\hat{\alpha}) + C_S(\hat{\alpha})].$$

The same argument applies to bidder  $S$ .

Because joint production costs are  $B = C_D + C_S$ , we see that if the deviation condition

$$\Pi_j(\alpha) + B(\alpha) > \Pi_j(\hat{\alpha}) + B(\hat{\alpha}), \quad j = S, D \quad (2)$$

holds, then supplier  $i$  can profitably induce the buyer to switch from  $\alpha$  to  $\hat{\alpha}$ . Note that the profit level of an opponent and the joint cost of production together determine this incentive.

We use (2) and Lemma 1 to construct a procedure for calculating the equilibrium bids. Formally, a pair of bids,  $(P_D^*, P_S^*)$ , is a Nash equilibrium if and only if, for all  $\alpha \in [0, 1]$  and  $i = D, S$ ,

$$g^* \leq P_D^*(\alpha) + P_S^*(\alpha) \quad \text{with } < \text{ if } B(\alpha) < B(\alpha^*), \quad (3)$$

$$\Pi_i(\alpha) + B(\alpha) \geq \Pi_i^* + B(\alpha^*), \quad (4)$$

and

$$g^* = P_D^*(1) = P_S^*(0), \quad (5)$$

where  $g^*$  and  $\Pi_i^*$  refer to the values at  $\alpha^*$ .

The conditions are clearly necessary. Condition (3) states that  $\alpha^*$  is an optimal choice for the buyer and that the tie-breaking rule is satisfied, while Condition (5) follows directly

from Lemma 1. Finally, Condition (4) ensures that neither bidder can profitably deviate from  $P_i^*(\cdot)$  by using one of the simple alternative bids discussed above (Condition (2)).

Now consider the sufficiency of Conditions (3), (4), and (5). Clearly, (3) ensures that  $\alpha^*$  is the award outcome at the bids  $(P_D^*, P_S^*)$ . It remains to show that each bidder is at a best response. From (5), we see that neither bidder can alter his bid and unilaterally force the buyer to pay more than  $g^*$ . Since each bidder must take  $g^*$  as given (due to the sole-source price offered by his opponent), the only relevant alternative bids are those which induce the buyer to switch to an outcome of  $\alpha \neq \alpha^*$ . Any such bid for  $i$  must entail a price that, given  $P_j^*(\cdot)$ , results in a total price at  $\alpha$  for the buyer that is below  $g^*$ . From above, we know that (4) ensures that such a price offer is not profitable.<sup>9</sup>

Bernheim and Whinston (1986) developed a model of menu auctions under complete information in which bidders submit offers for each item (or action) in a set of choices that is available to an auctioneer. While the economic focus is different, the formal structure of the equilibria in split-award auctions is similar in a number of ways to the structure of equilibria in these menu auctions. For example, by relaxing the bounds on bid prices in a menu auction, one can obtain a version of our Lemma 1 as a limiting case.

**□ Efficiency and sole-source outcomes.** If a sole-source award is the equilibrium outcome, then one supplier produces all  $x$  units, and the joint costs are  $B(0) = C_S(0)$  for  $\alpha = 0$  and  $B(1) = C_D(1)$  for  $\alpha = 1$ . The sole-source supplier will be the supplier with the lower costs. Suppose, without loss of generality, that this is bidder  $D$ .

*Proposition 1.* Suppose  $B(1) < B(0)$ . Then,  $\alpha^* = 1$  is an equilibrium outcome, and  $D$  is the sole-source supplier. Equilibrium bids satisfy

$$g^* = B(0) = P_S^*(0) = P_D^*(1),$$

$$\Pi_D^* = B(0) - B(1), \quad \text{and} \quad \Pi_S^* = 0.$$

Furthermore, if  $B(1) < B(\alpha)$  for all  $\alpha \in [0, 1)$ , then  $\alpha^* = 1$  is the unique equilibrium outcome.

*Proof.* See the Appendix.

The intuition for the sole-source equilibrium is straightforward. The low-cost supplier wins the total award and earns a profit equal to his cost advantage over the high-cost supplier, who earns zero profits; the buyer pays a total price equal to the production cost of the high-cost supplier. The buyer faces the same price at the outcomes of zero and one, and the tie-breaking rule selects  $\alpha = 1$ , since  $B(1) < B(0)$ . This is appropriate, since bidder  $D$  can always offer a sole-source price that is arbitrarily close to  $B(0)$ .<sup>10</sup>

Proposition 1 indicates that a sole-source outcome is the unique equilibrium outcome whenever sole-source production is efficient. The proposition also shows, however, that a sole-source equilibrium exists regardless of the structure of the joint production costs at split-award outcomes ( $0 < \alpha < 1$ ). This reflects an important feature of the bids in a split-

<sup>9</sup> While our analysis deals with the case of two suppliers, many of the equilibrium properties described above also hold when there are  $M$  suppliers. For example, a version of Lemma 1 applies to boundary points in the  $M - 1$  dimensional simplex such that, in equilibrium, no bidder can unilaterally raise the price paid by the buyer. Split-award outcomes will still hinge on the behavior of joint costs in the interior and on the boundary of the simplex.

<sup>10</sup> When no cost advantage exists, so  $B(0) = B(1)$ , a coin toss decides between  $\alpha^* \in \{0, 1\}$ , and the buyer procures  $x$  at the production cost.

award auction, namely, that each bidder can unilaterally “veto” any interior split by submitting a “high” (relative to the sole-source price) price for that split.<sup>11</sup>

Except for the basic insights regarding efficiency, the sole-source equilibrium is of limited interest whenever split production is efficient. We will demonstrate below that split-award equilibria always Pareto dominate sole-source equilibria from the viewpoint of the bidders.

#### 4. Implicit price collusion and split-award outcomes

■ In this section we characterize the structure of implicit price collusion in split-award auctions. We begin with the question of when a split award is an equilibrium outcome.

□ **Equilibrium outcomes and payoffs.** A split-award outcome is efficient relative to a sole-source outcome when  $B(\alpha) \leq B(1)$ . From Proposition 1, we can see that this is necessary for  $\alpha$  to be an equilibrium outcome. It is also sufficient.

*Proposition 2.* Let  $N = \{\alpha \mid B(\alpha) \leq B(1), 0 < \alpha < 1\}$  be the set of outcomes for which joint production costs are less than sole-source production costs. Then,  $N$  is the set of split-award equilibrium outcomes.

*Proof.* See the Appendix.

Thus, whenever  $B(\alpha) \leq B(1)$ ,  $\alpha$  can be supported as an equilibrium outcome. For a specific  $\alpha^* \in N$ , the following corollary describes the structure of the equilibrium bids, profits, and the price to the buyer.

*Corollary 1.* Let  $\alpha^* \in N$ . If  $(P_D^*, P_S^*)$  is a Nash equilibrium with  $\alpha^*$  as the equilibrium outcome, then the following conditions hold.

(i)  $g^*$ , the total price to the buyer at  $\alpha^*$ , satisfies

$$g^* \in [B(0), B(0) + B(1) - B(\alpha^*)]; \quad (6)$$

(ii)  $\Pi_S^*$ , the profit of  $S$  at  $\alpha^*$ , and  $\Pi_S(0)$  satisfy

$$\Pi_S^* \in [g^* - B(0), B(1) - B(\alpha^*)] \quad (7)$$

and

$$\Pi_S(0) = g^* - B(0); \quad (8)$$

and (iii)  $\Pi_D^*$ , the profit of  $D$  at  $\alpha^*$ , and  $\Pi_D(1)$  satisfy

$$\Pi_D^* = g^* - \Pi_S^* - B(\alpha^*) \quad (9)$$

$$\Pi_D(1) = g^* - B(1). \quad (10)$$

Furthermore, there is an equilibrium for any  $g^*$ ,  $\Pi_D^*$ , and  $\Pi_S^*$  that satisfy Conditions (6)–(10).

*Proof.* See the Appendix.

<sup>11</sup> As the proof of Proposition 1 shows, the supporting prices,  $P_i^*(\alpha)$ , for  $0 < \alpha < 1$  need only satisfy a lower-bound restriction and can be arbitrarily smooth. Given these “high” bid prices from his opponent, a bidder cannot profitably induce a split award. In fact, given the equilibrium bid of his opponent, the only way a bidder can induce the buyer to select an  $\alpha$  for which  $B(\alpha) < B(1)$  is to offer a negative-profit price at  $\alpha$ . Thus, the supporting bid prices in a sole-source equilibrium have the feature that if a bidder is to realize a profit from the “high” bid prices at  $\alpha \neq 1$ , his opponent must deviate using a dominated strategy bid. Because of this feature, a sole-source equilibrium is not proper (Myerson, 1978); the proof involves taking a finite approximation of the set of splits,  $[0, 1]$ , and the set of bid prices,  $\mathcal{R}^+$ .



Corollary 1 establishes the range of payoffs that is associated with a given split-award equilibrium outcome. There are two degrees of freedom here, the price to the buyer,  $g^*$ , which by Condition (9) determines the joint profits,  $\Pi_D^* + \Pi_S^*$ , and the division of the joint profits between the bidders.

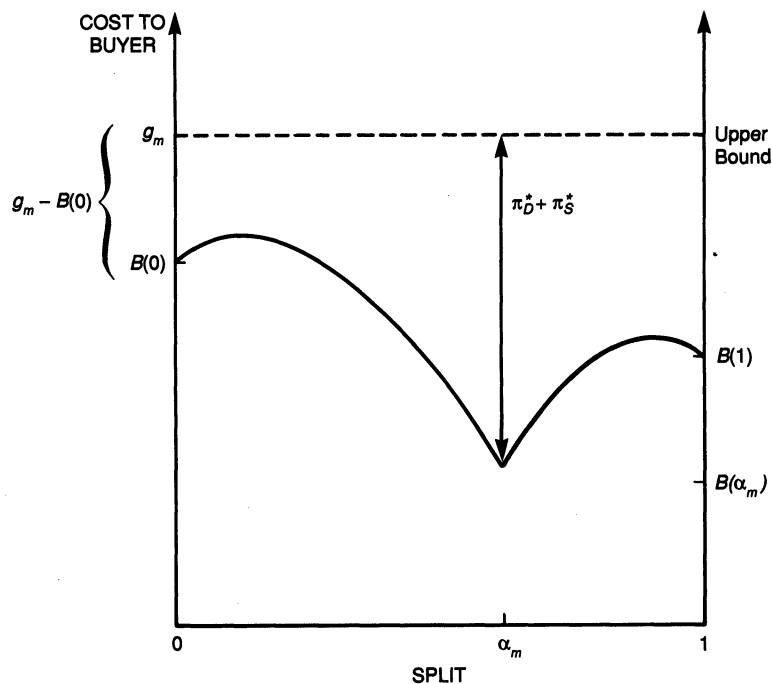
Refer to Figure 1. The equilibrium outcomes are a sole-source award to supplier  $D$  and the split-award outcomes in the neighborhood around the split  $\alpha_m$ . There are no equilibria in which the procurement price is below  $B(0)$ , no matter how small the joint costs are at the split-award outcomes. The reason is that, without risking a negative profit,  $S$  never offers a sole-source price below  $B(0)$  and  $D$  can always increase his profits by raising his bid prices until the total price for the buyer is at least  $B(0)$  for every outcome. The sole-source price of  $S$  cannot police such bids until the price reaches  $B(0)$ . This observation underlines the importance of the sole-source threat in containing prices—the sole-source price is the only limit to the price that a single bidder can impose on the buyer.

In tandem, the sole-source threats of  $D$  and  $S$  limit the total equilibrium price to  $B(0) + B(1) - B(\alpha^*)$ . To get an intuitive feel for this price ceiling, refer to Figure 1, and consider the equilibrium outcome  $\alpha_m$ . The price ceiling  $g_m$  is the price at which the available joint profits,  $\Pi_D^* + \Pi_S^*$ , just equal the deviation profits of  $g_m - B(0)$  for  $S$  plus  $g_m - B(1)$  for  $D$ . To prevent a sole-source deviation by either bidder, it is necessary to divide the joint profits at the split, so that each bidder gets profits at least as great as the profits from a sole-source deviation. By the definition of  $g_m$ , there are just enough joint profits to avoid a deviation by either bidder.

Now, consider a buyer price that is  $z$  greater than  $g_m$ . Lemma 1 requires that the equilibrium split price and the sole-source bid prices be equal. Thus, to prevent a sole-source deviation to a price just below  $g_m + z$ , each bidder must receive a profit of at least  $z$  plus the profit received at  $g_m$ . To prevent a sole-source deviation, then, joint profits must

FIGURE 1:

THE HIGHEST-PROFIT EQUILIBRIUM OUTCOME



be at least  $2z$  higher than the joint profits at  $g_m$ , but this is impossible because the new split price has only increased by  $z$ . Thus, there is no split-award equilibrium at a price above  $g_m$ .

The above relationships are summarized in Proposition 3. (A formal proof of this proposition is available from the authors.)

*Proposition 3.* Let  $\alpha^* \in N$ . Then, over the range of equilibrium payoffs at  $\alpha^*$ , the highest procurement price occurs at the bids that generate the highest individual profits for each bidder and the highest joint profits.

Thus, over the equilibrium payoffs at  $\alpha^*$ , there is a unique Pareto-optimal (with respect to the profits earned by each bidder) pair of payoffs. Furthermore, it is easy to identify these payoffs because they involve charging the buyer the highest possible equilibrium prices.<sup>12</sup>

□ **Highest-price equilibrium.** Now, consider the question of how the payoffs vary over  $N$ , the set of equilibrium split-award outcomes. Let  $\alpha_m \equiv \underset{\alpha}{\operatorname{argmin}} B(\alpha)$  denote the outcome for which total production costs are minimized. Clearly, from (6), the price

$$g_m \equiv B(0) + B(1) - B(\alpha_m)$$

is the highest equilibrium price that the buyer can pay. This price also leads to the highest values of individual and joint profits for the suppliers.

*Proposition 4.* The maxima for the price, joint profits, and individual profits over all equilibrium outcomes occur at the split-award outcome  $\alpha_m$ , for which the joint costs of the suppliers are minimized.

*Proof.* A formal proof is available from the authors.

Together, Propositions 3 and 4 reveal that a strong form of implicit price collusion can be supported in equilibrium. Over the set of equilibria,  $\alpha_m$  is the unique Pareto-optimal outcome from the viewpoint of the bidders.  $\alpha_m$  is also the worst equilibrium outcome for the buyer.<sup>13</sup>

In this equilibrium, the price to the buyer is  $g_m = B(0) + B(1) - B(\alpha_m)$ . Note that  $B(0)$  is the price the buyer would pay in a winner-take-all auction (the sole-source cost of the less-efficient supplier). Relative to  $D$ , the efficient sole-source supplier, there is an efficiency gain of  $B(1) - B(\alpha_m)$  that arises when the outcome is the split  $\alpha_m$ . None of these gains accrue to the buyer, however, as the equilibrium price is  $B(0)$  plus the efficiency gain.

At this price, both suppliers earn positive profits. To support this price, the equilibrium bids involve sole-source prices of  $g^*$  and split prices of

$$P_S^*(\alpha_m) = B(1) - C_D(\alpha_m)$$

and

$$P_D^*(\alpha_m) = B(0) - C_S(\alpha_m).$$

These prices equate the profits at  $\alpha_m$  with the profits at the sole-source awards for each supplier, so neither opponent has an incentive to capture a sole-source award by reducing

<sup>12</sup> The set of highest-price equilibria is huge, but the bids differ only off the equilibrium path.

<sup>13</sup> While the  $\alpha_m$  outcome is compelling, it is only one equilibrium outcome in a potentially large set. Various normal-form refinements (for discrete games) can be used to reduce the number of equilibria. For example, suppose there are three splits,  $\{0, \alpha, 1\}$ , where  $C_D(1) = C_S(0) = 1$  and  $C_D(\alpha) = C_S(\alpha) = 0$ , and three prices  $\{1/2, 1, 2\}$ . It is straightforward to verify that there are four Nash equilibria (two sole-source and two split-award), but only the split-award equilibrium corresponding to that in Proposition 4 is perfect. In general, with a richer set of  $[0, 1]$  for splits and  $[0, \infty)$  for bid prices, there are fewer dominated strategies, and perfection eliminates only those equilibria that have negative-profit supporting bids; properness is more fruitful. (See footnote 11.)

his sole-source price below  $g_m$ . The split prices also reveal that each supplier earns an equilibrium profit that equals the efficiency gain at  $\alpha_m$  relative to the sole-source cost of the other bidder. Larger efficiency gains thus relax the sole-source threats and allow the suppliers to submit higher bid prices in equilibrium.

□ **Informational considerations.** The assumption that suppliers are fully informed about each other's costs need not be taken literally. When suppliers are fully informed, extensive bidding coordination is feasible. In the highest-price equilibrium, for example, each supplier incorporates the cost information into his calculations in order to structure a bid that accounts for his opponent's incentives. The existence of collusive split-award equilibria, however, does not depend on the suppliers' having full information. Simple examples with asymmetric cost information can be constructed in which common knowledge information about the support (or distribution) of costs is sufficient for split-award bidding equilibria with prices that exceed winner-take-all values to exist.<sup>14</sup>

The model is easily interpreted in terms of a buyer who views  $(C_D, C_S)$  as stochastic, where the bidding is conditional on the  $(C_D, C_S)$  draws of the suppliers. In view of the implicit price collusion that emerges in a split-award auction, the literature on auction design suggests that the buyer will, in general, find it valuable to impose reserve prices or to handicap bidders in order to increase the competitive pressure in the auction. A reserve price in a split-award auction, for instance, effectively functions as an exogenous sole-source price and will restrict price collusion for relatively high  $(C_D, C_S)$  draws. When the buyer lacks a reasonable substitute good, however, establishing a commitment to a reserve price may be difficult.

The minimum-price rule for selecting a split award thus represents the case of a buyer with very limited commitment power. The feasibility of strategic instruments, such as a reserve price, will depend on the buyer's ability to commit to actions that sometimes conflict with his own *ex post* interest. When bids are considered to be proprietary information and when only the buyer observes the entire set of bids (as is the case in defense procurement) or the buyer is unwilling to delegate source selection to a third party, the scope for establishing such commitments is narrowed.

□ **An example with split-award outcomes.** This example illustrates that split-award outcomes can emerge in the presence of scale economies.

Suppose that the suppliers can invest in a cost-reducing innovation after they are awarded their production shares. In this case,  $C_D$  and  $C_S$  reflect the strategic interaction between  $D$  and  $S$  that occurs after the bidding is completed.

Let  $\alpha$  be an arbitrary split, and suppose each of  $i = D, S$  can invest  $I_i$  dollars in a deterministic cost-reducing innovation. The costs for  $D$  are given by

$$[1 - I_D - \mu I_S] \Gamma(\alpha x) + (AI_D^2)/2,$$

where  $\mu \in [0, 1]$  is an innovation spillover parameter and  $A$  is an investment cost parameter. If no investment occurs, the costs are simply  $\Gamma(\alpha x)$ . The costs for  $S$  are symmetric to those for  $D$ .

The investment pair  $(I_D^*, I_S^*) = (\Gamma(\alpha x)/A, \Gamma((1 - \alpha)x)/A)$  is the unique Nash equilibrium for the process innovation subgame. Define  $C_D$  and  $C_S$  as the values of

<sup>14</sup> Consider a two-bidder model in which procurement is limited to three awards  $\{0, 1/2, 1\}$ , and each supplier has a private cost parameter,  $\theta_i$ , drawn from a common knowledge uniform distribution on  $[0.9, 1]$ . If sole-source costs are equal to  $\theta$  and the split cost equals  $4\theta/9$  for each supplier (so,  $B(1/2) < B(1)$ ), then it is easy to verify that  $P(0) = P(1) = 1$  and  $P(1/2) = 1/2$  is a split-award equilibrium in which both suppliers make positive profits and the buyer price exceeds the price that would occur in a winner-take-all auction.

the costs for each bidder at  $I_D^*$  and  $I_S^*$ . This yields  $C_i(\alpha) = A^{-1} \{ [A - \mu\Gamma_j]\Gamma_i - (\Gamma_i^2)/2 \}$ , where  $\Gamma_D = \Gamma(\alpha x)$  and  $\Gamma_S = \Gamma((1 - \alpha)x)$ . Then,

$$B(\alpha) = A^{-1} \{ A(\Gamma_S + \Gamma_D) - 2\mu\Gamma_S\Gamma_D - (\Gamma_D^2 + \Gamma_S^2)/2 \}.$$

Consider some special cases for  $\Gamma(\cdot)$ . With constant marginal costs,  $\Gamma(x) = \tau x$ , and  $B$  is a quadratic function. When large spillovers occur ( $\mu > 1/2$ ),  $B$  is convex, and the split  $\alpha_m = x/2$  is the profit-maximizing outcome for the bidders. For small spillovers ( $\mu < 1/2$ ),  $B$  is concave, and sole-sourcing is the equilibrium outcome. When  $\mu = 1/2$ ,  $B$  is flat.

If  $\Gamma(\cdot) = \sqrt{\cdot}$ , so scale (or learning) economies exist, similar features for  $B$  arise, although the verification is more tedious.<sup>15</sup> The interesting feature of this case (when  $\mu$  is large) is the interaction between investment and scale economies. With scale economies,  $x$  units can be produced for a lower cost by one supplier than by dividing the award between two, other things being equal. Investment incentives, however, differ across awards. Over the equilibrium outcomes,  $\alpha_m = x/2$  emerges as the efficient outcome. Thus, split-award outcomes can emerge in equilibrium even when scale economies characterize the production costs of both suppliers.

## 5. Auction formats and investment incentives

■ The format for a procurement competition is typically specified by the buyer. In view of the above analysis, a buyer's motivation for adopting a split-award (SA) format is unclear. Given the cost functions ( $C_D$ ,  $C_S$ ), there is no split-award equilibrium in which the buyer pays less than in a winner-take-all (WTA) auction, and there are equilibria in which the buyer pays more.

Because the SA format is dominated by the WTA format in a simple price comparison, a key to understanding why both policies are employed in practice is to identify a dimension along which the buyer benefits from a SA format. One such dimension is the incentive for prebid investment in cost-reducing innovations that each procurement format creates.<sup>16</sup>

We focus here on cost-reducing activities in a frequently encountered situation in which there is an asymmetry in the initial cost positions. For example, when one supplier is responsible for the development of the technology to be produced, as is often the case in defense procurement, or when there are differences in previous production experiences, the lead supplier can be expected to have some cost advantage. In such a case, the follower might, prior to the auction, expend resources to imitate the lead supplier's manufacturing techniques or design technology.

The effects of these actions can render split-award auctions superior to winner-take-all auctions for the buyer. Under a SA format, the follower may anticipate a production award, whereas under a WTA format, no award would be anticipated. Thus, the potential profits in a SA auction can provide an incentive to invest in innovation. When this incentive translates into large innovation expenditures and reductions in joint costs, the resulting price in a SA auction can be less than the price in the corresponding WTA auction.

For instance, consider a slightly modified version of the single-split example in the introduction. Let supplier  $D$  have costs for no production, split production, and sole-source production of 0, 4, and 10, respectively, and let supplier  $S$  have corresponding costs of 0,  $4(1 + \theta)$ , and  $10(1 + \theta)$ ,  $\theta > 0$ . The parameter  $\theta$  indexes the cost disadvantage of supplier

<sup>15</sup> For large  $\mu$ , the investment effect dominates the scale effect on  $B$ , and split awards occur in equilibrium. Further, a camel-hump shape for  $B$  (as in Figure 1) can emerge, and splits near  $\alpha = 0$  and  $\alpha = 1$  are eliminated from the equilibrium set.

<sup>16</sup> Many auctions are announced long before bids are to be submitted. For example, a period for technology transfer (possibly including "learning buys") is often needed before defense contract auctions can be run. During this period, suppliers have an opportunity to improve their relative cost positions.

$S$  and can be decreased prior to bidding by investing in imitative R&D for an expenditure given by  $T(\theta) = 10(\bar{\theta} - \theta)^2$ , where  $\bar{\theta}$  is the initial cost disadvantage.

Letting  $\bar{\theta} = .4$ , it is easy to calculate that the optimal investment is .4, which results in  $\theta = .2$ . Then, using the formulas given in Corollary 1, the buyer pays 13.2 in the highest-price equilibrium, and both suppliers make positive profits. Under the WTA setting, no imitative R&D will be undertaken because  $S$  anticipates that he will never win, and the buyer's price will equal 14,  $S$ 's sole-source cost. Thus, the effect on the equilibrium price of the differential incentive to invest in innovation can lead a buyer to favor SA over WTA auctions.

The above analysis of SA and WTA auctions is a comparison of two specific policies that have been employed by buyers, and consequently, should not be taken as a claim that either policy is optimal for the buyer. A number of potentially superior means of inducing upstream innovation through the design of auctions have been suggested in the literature.<sup>17</sup> The applicability of such schemes depends on the extent of the buyer's commitment powers.

## 6. Related policy issues

■ In this section, we consider two extensions of the basic analysis. First, we examine how the addition of a third bidder might improve price performance for the buyer, and second, we expand our framework to include the potential nonprice benefits of dual-source procurement.

□ **Additional bidders.** In view of the result that implicit price collusion emerges in a split award auction with two bidders, a natural question to ask is, What effect will the addition of a third bidder have on prices? For the buyer, the potential benefit hinges on the additional competitive pressure that this bidder creates with respect to dual-production outcomes.<sup>18</sup>

Because a dual-production award will always exclude one of the suppliers, the rejected bidder must earn zero profits. Any set of bids that allows the rejected bidder to lower his bid, win a partial share of the production, and earn a profit cannot form a bidding equilibrium. This necessary condition implies that the production costs of the least-efficient supplier will limit the price charged by the next-to-the-most-efficient supplier and, consequently, limit the total price to the buyer in any equilibrium involving a dual-source outcome.<sup>19</sup>

Introducing an additional bidder thus provides at least a partial escape from the pricing problems in split-award auctions. This benefit, however, must be weighed against the costs of adding a third bidder. For example, in procurements involving frontier technologies (e.g., defense), the design and development firm is frequently the only natural production source. Qualifying additional firms for production will require substantial expenses for technology transfer and may involve high-priced "learning buys."<sup>20</sup> In such cases, the buyer will find it necessary to subsidize a supplier who does not expect to get a production award; when dual production is cost minimizing, the least-efficient firm in a three-bidder auction is such

<sup>17</sup> Imposing a reserve price or placing a handicap on specific bidders can be used to increase the competitive pressure on bidders, and, in general, an exogenous ability to commit to these auction rules is valuable to the buyer. Laffont and Tirole (1988) and Riordan and Sappington (1989) examined optimal policy for source selection and investment in a winner-take-all setting in which the buyer can employ procurement mechanisms with these features. Also, see Rob (1986) for a discussion of winner-take-all auctions and upstream R&D investment.

<sup>18</sup> When production by only one firm is cost efficient, the equilibrium involves a sole-source outcome at the standard WTA auction price. When production with all three firms active is cost minimizing, it is easy to show that implicit collusion can be supported in an equilibrium with a three-way split and positive profits for all bidders. When dual production (only two firms active) is cost efficient, a three-way split cannot be supported in equilibrium.

<sup>19</sup> For example, consider a setting with three bidders in which dual production is cost efficient, and the two least-efficient firms have identical cost functions. Then, only the most-efficient producer will earn positive profits.

<sup>20</sup> Daly and Schuttinga (1982) reported a \$23 million (in 1972 dollars) cost of initiating competition on the TOW missile program. This cost was roughly half of the reported discounted savings from competition.

a supplier. Thus, the attractiveness of the three-bidder strategy will depend on the trade-off between the expected price reduction and the costs of readying an additional supplier.

□ **Preference for splits.** From a policy perspective, our approach may appear too narrow: by focusing solely on pricing, our model undervalues dual sourcing by excluding its indirect benefits. Justifications for dual sourcing often involve nonprice benefits, such as an increase in the nation's defense mobilization capability or a long-run benefit based on maintaining a potential for future (as yet unspecified) design and production competition.

While the magnitude of these benefits is subject to debate, our model provides a clear prediction regarding the effects of such future benefits on the equilibrium price in a split-award auction. Future benefits translate directly into a preference for dual-source production. To see the impact in our model, one need only adjust the minimum-price selection rule of the buyer to account for this preference. (The buyer is now indifferent between some lower-priced sole-source offer and a higher-priced split offer.)

Because the buyer is willing to pay a premium for a split award, sole-source threats are not as effective in disciplining split prices. In the polar case in which the dual-source preferences of the buyer are known by both bidders, the full value the buyer attaches to dual sourcing is captured by the suppliers through higher split prices in equilibrium.

This effect reinforces our basic conclusion regarding price formation in split-award auctions between two well-informed bidders: such auctions perform very poorly from the viewpoint of the buyer. Implicit bidding coordination by the suppliers not only prevents the buyer from sharing in the direct efficiency gains associated with dual-source production, but also effectively offsets any downstream indirect benefits the buyer may attribute to dual sourcing.

## Appendix

■ The proofs of Propositions 1 and 2 and of Corollary 1 follow.

*Proof of Proposition 1.* First, we solve for the values of  $P_D^*(1)$  and  $P_S^*(0)$  that must prevail if  $\alpha^* = 1$  is an equilibrium outcome. Next, we find bids  $(P_D^*, P_S^*)$  that induce  $\alpha^* = 1$  as the outcome and satisfy the best-response property.

To begin, recall that  $P_S^*(0) = P_D^*(1)$  by Lemma 1. Clearly,  $P_S^*(0) \geq B(0)$ , as  $B(0) = C_S(0)$  and as bids must yield nonnegative profits. If we also show that  $P_S^*(0) \leq B(0)$ , then equality holds, and  $\Pi_D^*$ ,  $\Pi_S^*$ , and  $g^*$  follow from their definitions.

To see that  $P_S^*(0) \leq B(0)$ , recall that  $S$  cannot profitably deviate from  $P_S^*$  in equilibrium. Thus, (4) holds, and  $\Pi_D(1) + B(1) \leq \Pi_D(0) + B(0)$ . Since  $\Pi_D(0) = 0$  and  $\Pi_D(1) = P_D^*(1) - C_D(1) = P_D^*(1) - B(1)$ , the above inequality reduces to  $P_D^*(1) \leq B(0)$ . As  $P_D^*(1) = P_S^*(0)$ , we are done.

Now, consider the bids  $(P_D^*, P_S^*)$ . From above, the sole-source prices  $P_S^*(0) = P_D^*(1) = C_S(0)$  establish  $g^* = C_S(0)$  as the price a bidder must beat in order to upset  $\alpha^* = 1$ . Given this, a pair of bids  $(P_D, P_S)$  forms a Nash equilibrium with outcome  $\alpha^* = 1$  if (3) and (4) hold. (4) holds for each bidder if

$$\Pi_D(\alpha) + B(\alpha) \geq \Pi_D^* + B(1), \quad \alpha \in [0, 1] \quad (A1)$$

$$\Pi_S(\alpha) + B(\alpha) \geq \Pi_S^* + B(1), \quad \alpha \in [0, 1]. \quad (A2)$$

Using the values for  $\Pi_D^*$  and  $\Pi_S^*$  from above and the definition of  $\Pi_i(\cdot)$ , the above inequalities reduce to

$$P_D(\alpha) \geq C_S(0) - C_S(\alpha), \quad \alpha \in [0, 1] \quad (A3)$$

and

$$P_S(\alpha) \geq C_D(1) - C_D(\alpha), \quad \alpha \in [0, 1]. \quad (A4)$$

This places a lower bound on  $(P_D, P_S)$ . The nonnegative profit restriction on bids is also a lower bound, as is (3). Thus, take  $(P_D^*, P_S^*)$  to be two functions on  $[0, 1]$  that lie above these lower bounds, and assume that  $P_D^*(1)$  and  $P_S^*(0)$  equal their stated values. Note that the stated values do not violate the lower bounds.

To show that  $\alpha^* = 1$  is the unique equilibrium outcome, suppose some  $\alpha < 1$  is an equilibrium outcome. By (2), bidder  $D$  can profitably induce the outcome 1 with a price  $p$  if  $\Pi_S(\alpha) + B(\alpha) > \Pi_S(1) + B(1)$ . Since  $\Pi_S(1) = 0$  and  $\Pi_S(\alpha) \geq 0$ , the above inequality is valid by hypothesis. Thus,  $\alpha < 1$  cannot be an equilibrium outcome. *Q.E.D.*

*Proof of Proposition 2.* As noted above, the proof of Proposition 1 shows that if  $B(\alpha) > B(1)$ , then  $\alpha$  is not an equilibrium outcome. We now show that if  $B(\alpha^*) \leq B(1)$ , where  $0 < \alpha^* < 1$ , then  $\alpha^*$  is an equilibrium outcome.

As in the proof of Proposition 1, it is sufficient to exhibit bids  $(P_D, P_S)$  such that the outcome is  $\alpha^*$  and

$$\Pi_D(\alpha) + B(\alpha) \geq \Pi_D^* + B(\alpha^*), \quad \alpha \neq \alpha^* \quad (A5)$$

$$\Pi_S(\alpha) + B(\alpha) \geq \Pi_S^* + B(\alpha^*), \quad \alpha \neq \alpha^*, \quad (A6)$$

where  $\alpha^*$  is a given element of  $N$ . The conditions of Corollary 1 are used to construct values for the bid prices when  $\alpha$  equals zero,  $\alpha^*$ , and one. Then, the two inequality conditions above are used to construct the supporting bid prices.

First, pick values for the cost to the buyer and the profits of  $S$  at  $\alpha^*$ , say  $g^*$  and  $\Pi_S^*$ , such that  $g^* \in [B(0), B(0) + B(1) - B(\alpha^*)]$  and  $\Pi_S^* \in [g^* - B(0), B(1) - B(\alpha^*)]$ .

For these values of  $g^*$  and  $\Pi_S^*$ , construct  $\Pi_D^*$ ,  $\Pi_D(1)$ , and  $\Pi_S(0)$  according to Corollary 1. It is straightforward to verify that (A5) and (A6) hold at  $\alpha = 0$  and  $\alpha = 1$  with these constructed values. For an outcome  $\alpha \notin \{0, \alpha^*, 1\}$ , Conditions (A5) and (A6) reduce to

$$P_D(\alpha) \geq \Pi_D^* + B(\alpha^*) - B(\alpha) + C_D(\alpha) \quad (A7)$$

$$P_S(\alpha) \geq \Pi_S^* + B(\alpha^*) - B(\alpha) + C_S(\alpha), \quad (A8)$$

a pair of lower bounds on bid prices. Thus, take  $(P_D^*, P_S^*)$  to be functions on  $[0, 1]$  such that the lower bounds above, the nonnegative profit lower bounds, and the lower bounds in (3) are satisfied, and the bids take on the constructed values at 0,  $\alpha^*$ , and 1. *Q.E.D.*

*Proof of Corollary 1.* The proof of Proposition 2 demonstrates that the conditions of Corollary 1 are sufficient to construct bids that form a Nash equilibrium and support the outcome  $\alpha^*$ . To prove necessity, suppose that  $\alpha^* \in N$  and  $(P_D^*, P_S^*)$  are the equilibrium bids. Let  $g^*$  denote the cost to the buyer and  $\Pi_S^*$  the profit of  $S$  at  $\alpha^*$ .

Lemma 1 implies that  $\Pi_D(1) = g^* - B(1)$  and that  $\Pi_S(0) = g^* - B(0)$ . Thus, (8) and (10) hold. Since  $\Pi_S^*(0) \geq 0$  by the nonnegativity of the bid profits, the lower bound on  $g^*$  in (6) is established. (9) is an accounting identity. Thus, the upper bound on  $g^*$  in (6) and the upper and lower bounds on  $\Pi_S^*$  in (7) remain to be established.

Since  $P_D^*$  and  $P_S^*$  are equilibrium bids, neither  $D$  nor  $S$  can profitably induce a sole-source outcome, and (4) becomes  $\Pi_S(1) + B(1) \geq \Pi_S^* + B(\alpha^*)$  and  $\Pi_D(0) + B(0) \geq \Pi_D^* + B(\alpha^*)$ . Since  $\Pi_S(1) = \Pi_D(0) = 0$ , these reduce to  $B(1) - B(\alpha^*) \geq \Pi_S^*$  and  $B(0) - B(\alpha^*) \geq \Pi_D^*$ .

Now, consider the upper bound on  $g^*$ . Using (9), we get

$$\begin{aligned} g^* &= \Pi_D^* + \Pi_S^* + B(\alpha^*) \\ &\leq [B(1) - B(\alpha^*)] + [B(0) - B(\alpha^*)] + B(\alpha^*) \\ &\leq B(0) + B(1) - B(\alpha^*), \end{aligned}$$

as follows from the  $\Pi_D^*$  and  $\Pi_S^*$  inequalities.

Consider bounding  $\Pi_S^*$ . The  $\Pi_S^*$  inequality immediately implies the upper bound. For the lower bound, begin with (9) to get

$$\begin{aligned} \Pi_S^* &= g^* - \Pi_D^* - B(\alpha^*) \\ &\geq g^* - [B(0) - B(\alpha^*)] - B(\alpha^*) \\ &\geq g^* - B(0), \end{aligned}$$

as follows from the  $\Pi_D^*$  inequality. *Q.E.D.*

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