Delay as Agenda Setting*

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Abstract

We examine a dynamic multi-party decision-making process in which two players face a set of unrelated proposals and each player can allocate resources to influence whether a proposal is accepted, rejected or delayed. Delay is strategically interesting when decision makers with asymmetric preferences face multiple proposals and have limited resources to influence outcomes. A delayed decision means that a proposal becomes part of the subsequent agenda, thereby altering resource allocation. The opportunity to delay leads the players to act against their proposal preferences. We characterize delay equilibria and identify when players act to increase delay and "pin" an opponent to a recurring proposal, or to reduce delay and "focus" the other player on a different proposal.

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1 Introduction

Delay in multiplayer decision processes is ubiquitous. As a justification for more informed decision making, delay would be important, yet unremarkable. But when attempts to influence current decisions affect the prospects of delay, delay becomes a strategic channel through which current influence resource allocations affect future resource allocations. When multiple proposals come to the table over time, delay changes the future agenda of proposals faced by the players. Further, a player can exploit delay to alter the allocation of future influence resources by the other players.

One category of settings where strategic delay takes place involves decision making by firms, committees in professional partnerships, and academic institutions. Here, for example, yes, no, or delay decisions regarding project approval, investments, and promotions are frequently made during meetings that follow a relatively fixed schedule. In such settings, even apparently independent decisions are linked through resource constraints (e.g. attention) that limit the amount of influence interested parties exert for or against the items on the meeting agendas. In these cases, the portfolio of items on the agenda for a given meeting and the preferences of the various committee members over those items determine the effort exerted to influence any individual item outcome. A proposal delayed from a previous meeting may change the overall allocation of influence exerted across the items scheduled for the subsequent meeting and, hence, the decision outcomes. Knowing this, members may have a strong incentive to either delay or to resolve prior meeting decisions—even regarding proposals that are quite unimportant to them—to alter the subsequent allocation of resources so as to improve outcomes on items of greater importance.

Delay can result from a variety of factors. Familiar examples include a purposeful choice to postpone a decision, re-emergence of proposals that had previously been rejected, lack of follow-through on an affirmative decision,[1] or because resolution of a decision requires action by an outside

[1] In corporate settings, a failed proposal that can be reintroduced is an example of a decision that lacks commitment. For example, rejected proposals of subordinates are sometimes quietly maintained in hope that changed circumstances will allow the proposal to be revisited. Burgelman (1991) [7], for example, argues that the RISC processor project at Intel was kept alive despite the company's explicit strategy of not pursuing such a processor. Our model can also be used to explore the dynamics of whether a proposal is placed on the agenda in the first place. Proposals that do not make it on the agenda have not been officially killed and can therefore be interpreted as "delayed." In the political system,

player (e.g., a decision to make a bid for an acquisition is resolved if the bid is accepted, but may not be if rejected). The possibility of delay raises the prospect that it can be strategically exploited. In the multiple decision context which this paper addresses, the allocation of attention or influence resources across those decisions constitutes a core factor influencing outcomes, leading to questions such as: Why is delay strategically valuable and who will initiate it? Does conflict lead to more delay? How does the order of proposals considered affect delay, efficiency and outcomes?

The structure of our model isolates the effect of delay while allowing for endogenous interaction. We abstract from the specifics of various decision structures and build a spartan strategic ark involving two players, two decisions, and two periods. Players seek to maximize their two-period payoff and, in each period, a player allocates a stock of non-storable influence resources (e.g., attention) across the proposals currently up for a decision. In the first period, each proposal may be resolved, either via acceptance or rejection, or it may be delayed. Resource allocations determine the likelihood of each outcome. If players act in concert to support (oppose) a proposal, then the acceptance (rejection) likelihood is increased. Importantly, delay prospects are also impacted and opposing (reinforcing) resource choices by players will increase (decrease) the probability that a proposal is delayed. That is, disagreement makes delay more likely. A delayed proposal is deferred to the second period where players again allocate resources and all proposals on the table are permanently resolved.

Our main result establishes existence and identifies when equilibrium necessarily entails player choices that are driven by strategic delay incentives. Suppose that player interests are aligned on one proposal but in conflict on a second one. Strategic delay then emerges whenever one player is concerned primarily about the alignment proposal while the other is concerned primarily about conflict proposal. In this case, each player will take initial actions that are against their proposal preferences: one player will reverse position to act against the other player on the alignment proposal while the other player moves to support the choice of the first player on the conflict proposal. These reversals reflect an intriguing set of strategic incentives among the players to structure the subsequent agenda by exploiting delay possibilities.

Two main tactics emerge: *focusing* and *pinning*. Our main result shows that they emerge simulta-

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rejection of legislation may be viewed as a decision that lacks commitment as such bills are frequently reintroduced in subsequent legislative sessions. Similarly, passed bills are also open to reshaping or even repeal ("Nothing ever gets settled in this town" George Shultz, former U.S. Secretary of State, quoted in R. W. Apple, "A Lesson from Shultz," *NY Times,* December 9, 1986).

neously in equilibrium with one player adopting a focusing strategy and the other a pinning strategy. To understand these effects, it is helpful to analyze first a sequential version of the full model in which only one proposal is on the agenda in the first period and only acceptance or delay (but not rejection) are the possible outcomes. This sequential version removes the within-period strategic effects that arise in the full model and allows us to concentrate on across-period effects that turn out to be at the core of focusing and pinning behavior. When the alignment proposal comes first, we find that pinning emerges. When the conflict proposal is first, focusing emerges.

In equilibria characterized by pinning, one player expends first-period resources against proposal preferences to increase the probability that the first-period proposal is delayed. The pinning player pursues this strategy when the second-period proposal involves conflict and the first-period proposal as compared to the second-period proposal is relatively less important to the pinning player. Pinning increases the probability that the rival's resources will continue to be allocated to the first proposal, leaving less resources for the rival to contest the pinning player on the second proposal.

If delaying on the first proposal is sometimes valuable, then resolving that proposal should be valuable in other circumstances. Suppose, for example, the players' interests align on the second proposal. Then it may be optimal for a player to decrease the likelihood of delay to free the other player's resources for (supportive) use on the second proposal. We refer to this dual of pinning as *focusing.* The focusing incentive can be sufficiently strong that a player will support the other player's efforts to secure acceptance of the initial proposal, even though that player prefers rejection of the proposal. In effect, the focusing player sacrifices one proposal to focus the other player's attention and resources on the other proposal. Focusing has the desirable feature of avoiding waste associated with the offsetting use of resources and provides a time-consistent explanation for logrolling which does not rely on reputations or other considerations beyond the immediate interaction. A necessary condition for focusing is that preferences directionally align on the second-period proposal, while a necessary condition for pinning requires conflict on the second-period proposal. When these necessary conditions are met, we show that focusing and pinning occur when the players differ significantly regarding which proposal is relatively more important. Furthermore, as noted above, pinning and focusing also emerge in a model in which both proposals are considered in the first period but can be delayed to the second period.

An intriguing property of focusing and pinning equilibria is that an ostensibly more attractive proposal may be strategically disadvantageous. Given the choice between two versions of a second-period proposal, the version that provides both players a weakly higher utility payoff if it is adopted may encounter a significantly lower chance of adoption due to the players' endogenous actions. For example, a focusing outcome will prevail with the lower-utility proposal while the higher-utility proposal leaves agents following basic proposal preferences. Since focusing increases the chances of adoption, the equilibrium payoff is higher with the lower-utility proposal. This outcome reflects the implicit 'threat' regarding future action that the higher-utility proposal embodies, namely, that a player will devote more effort to supporting that proposal. When this is a proposal on which both players interests directionally align, the other player loses the incentive to focus since strong support on the alignment proposal will be forthcoming anyway should delay occur. In contrast, the lower-utility proposal implies weaker support on the alignment proposal and the other player has an incentive to focus and remove the competing proposal from the future period.

A general point emerging from the analysis of both the sequential and full model is that there is frequently strategic value to resolution in a world with possible delay and multiple decisions. Even if a current proposal is of limited concern to one player, delaying that proposal's resolution creates a claim on future resources for the other player. Therefore, a player may be quite willing to expend significant resources to secure resolution of a proposal of no direct importance. The mere fact of resolving a proposal may carry more significance than the specific manner of the resolution.[2]

Pinning and focusing activities exist in a wide range of situations. As noted previously, perhaps the most common setting involves go versus no-go decisions in which more than one (perhaps unrelated) decision may be considered in a given committee meeting. Consider, for example, a situation in which a personnel committee makes or delays personnel decisions during its regularly scheduled meetings, such as in up-or-out promotion decisions (common in professional service firms) or the hiring of new employees. In advance of the meetings, committee members allocate attention and effort across the items on a meeting's agenda given the importance of those decisions to the member. Thus, the overall level of support for a member's key candidate depends on whether other supporters of that candidate have even more important personnel choices coming up in the same meeting. Given the possibility of delay and then distraction of one's allies, a member may find it expedient to work to resolve a

[2] Delay is representative of a class of decision problems involving endogenous commitment. Deferring a decision until the next decision point is equivalent to no commitment, while a decision, if irreversible, constitutes full commitment. In the former case, the delayed decision becomes part of the subsequent decision agenda and potentially changes each decision maker's allocation of influence activity.

choice in a previous meeting that alone would be of little intrinsic interest. This behavior corresponds to focusing.[3] Pinning, in contrast, corresponds to behavior that increases the strategic potential for delay (e.g., by raising questions that require additional work and are deferred to a later meeting).

Delayed actions and deferred decisions are commonplace in business, politics, and even personal life. Elements of pinning, for example, are also found in litigation between horizontal competitors in which the first-period decision involves expenditures on litigation and the second-period decision involves subsequent expenditures relevant to the ongoing competition. Litigation expenditures influence the duration and outcome of the legal dispute. Resolution or non-resolution, in turn, affects the marginal value of investments in competition through, for example, the impact of uncertainty regarding the outcome of litigation on the players' abilities to attract complementary investments from third parties.[4] Similarly, firms that compete across multiple product or geographic markets frequently pin each other's resources by opening second fronts in markets that are important to their rivals. Although, such market-based competition lacks the explicit timing of a final resolution point, one can interpret it as an attempt to delay a rival's entry into a key market.[5]

[3] Focusing, which requires an issue over which no conflict exists, is most applicable to settings involving some level of explicit or implicit cooperation, e.g., a business joint venture or a political coalition. Both types of organizations almost invariably navigate proposals over which there are areas of agreement and disagreement. By sequencing the consideration of the proposals to create incentives for one party to focus the other, both parties can more easily support each other's key proposals without the need for other enforcement mechanisms. For example, in presidential and parliamentary political systems with modest party discipline, influence activities involve keeping party members in line with party positions regarding the current legislative agenda. Where party discipline is very strong, keeping party members in line is easy and a key use of party leadership resources involves allocating leadership attention to issues that require negotiation prior to their introduction.

[4] The protracted litigation between AMD and Intel regarding AMD's access rights to Intel intellectual property as a former second source for the 286 microprocessor can be interpreted as Intel pinning AMD to more expensive R&D and customer development under conditions of uncertainty. This litigation finally settled after the access to the rights was no longer competitively significant.

[5] MacMillan, van Putten, and McGrath (2003) [16] call this class of tactics "feints" in which an attack in one market diverts resources from another market. They describe how Philip Morris attacked R.J. Reynold's U.S. position in premium cigarettes and, thereby, diverted RJR's resources away from important Eastern European markets. The authors also discuss a competitive interaction between Gillette and Bic in which Gillette, by withdrawing from the disposible lighter market, induced Bic to invest more in the disposible lighter market and to pull resources out of disposible razors.

A related application concerns political dynamics. In this regard, an illustrative example is provided by the events just after the Watergate scandal. In early 1974, amid pressure from the Watergate scandal which would ultimately scuttle his presidency, an embattled Richard Nixon proposed a comprehensive national health insurance plan in his State of the Union address. President Nixon's support gave national health care insurance legislation, a traditionally Democratic issue, a real chance for passage, but Democratic support did not materialize and a plan was never passed. The timing of Administration's formal support coupled with the bill's substantive attractiveness to the Democrats have led some observers to interpret Nixon's proposal as strategic.[6] If the Democrats wanted genuine progress on such legislation, they would need to devote resources and attention to health care. Had they pursued this course of action, Congressional action on Watergate would likely have been delayed, perhaps through the remaining two years of Nixon's term, and would certainly have been reduced in intensity. Relative to our model, this example has several subtle features that involve sorting out pinning and focusing behavior by the various players. In Section 4, we interpret how the Nixon side involves pinning while on the Democratic side focusing behavior emerges.

Little, if any, research has used analytical models to explore the effects of limited attention and delay on organizational decision making. Our analysis connects research on influence activity to that of agenda setting. The influence activity models of Milgrom and Roberts (1988 [18], 1990 [19]) focus on the design of incentives for agents who, given the incentive structure, optimally split their time across current production and influence activities that impact all of the agents' payoffs. Our interest in strategic decision delay and their interest in organizational design lead to quite different models. We build a dynamic model to explore deferred decisions, but do not address various optimal organizational designs that structure the nature of the intra and inter-period decision-maker interactions.[7] In our dynamic decision setting, for example, agents sometimes allocate influence against interest which does

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[6] Light (1991; p. 256) [15] states, for example, that "struggling to distract a Democratic Congress from the Watergate crisis, Nixon offered national health insurance as a last-second bargain to save his Presidency." National health care insurance legislation was more attractive to the Democratic party than the Republican party, though Nixon had supported it as well. The leading Democrat advocating national health care insurance was Ted Kennedy who had run for President in 1972. Ultimately, the fate of the Nixon NHI proposal was resolved early when Kennedy failed to support it. The 1974 failure to pass a national health care insurance plan was reportedly Kennedy's "biggest regret." (Washington Post, August 28, 2009)

[7] See also, Dessein and Santos (2006 [10]), Gilligan and Krehbiel (1987 [12]), and Powell (2015 [23]).

not occur in the static allocation setting.

Agenda setting models in the economics and the formal political science literatures (see, e.g. Plott and Levine 1978 [22]) explore the effects of decision order. Agenda-setting models also typically focus on the influence of decision order when decision payoffs and outcomes are linked across decisions. By contrast, our focus on the impact of deferred commitments on the allocation of influence does not require any outcome or payoff link across decisions. Stated alternatively, we focus on across-meeting decisions whereas most of the agenda setting literature focuses on within-meeting decisions. Furthermore, the value of changing the agenda mechanism in our model operates by changing the allocation of influence resources of other parties. A recent exception is Chen and Eraslan (2017 [8]) who examine a sequential model of agenda setting in which an incumbent chooses which issue to resolve. We focus, instead, on strategic action by both players in the initial period and examine how these choices are influenced by the anticipated consequences of delay. For example, delay allows one player to pin the other to a key unresolved proposal.

Some of the tensions between conflict and cooperation among decision making parties which we address in this paper are explored in other articles. In the evolution of cooperation literature, for example, Skaperdas (1992) [25] examines how the structure of marginal productivity in the use of resources can lead to varying degrees of equilibrium cooperation in a two-player static model where each player divides resources between joint production of a good and increasing the probability that it will "win" the value of that product. Of course, with a single good and a static model, the use of strategic delay to manipulate a rival's use of resources across issues does not arise. In Bonatti and Rantakari (2016) [3], strategic delay arises when the decision on project adoption is deferred until a second project is available. In this event, a rival's project is already available and delay influences both the type of project and the effort level chosen by the delaying agent. Pinning behavior in our model shares the feature of increased delay but the economic motivation differs as pinning is driven by the benefit derived from inducing a rival to continue to expend effort on the delayed proposal. In the bargaining literature, delay is a central issue in non-cooperative models. In a complete-information setting where two bargaining "pies" arrive at different times, Acharya and Ortner (2013) [1] show that equilibrium involves a delay on division of the first pie. When the players have asymmetric valuations for each pie, the efficiency gain in the bargaining outcome from having more than one pie to divide leads to delay on dividing a portion of the initial pie. Again, while pinning in our model features increased delay, the opposing actions of players typically lead to an efficiency loss rather than a gain.

In the next two sections we develop and analyze our sequential model. Section 4 examines the full model and establishes that strongly asymmetric and partially conflicting preferences necessarily result in an equilibrium with simultaneous pinning and focusing. In Section 5 we discuss the organizational context of our analysis as well as applications and limitations. Section 6 concludes.

2 Model

Our model consists of two players, A and B, who, over two periods, independently allocate their respective attention resources to influence the outcomes of two unrelated proposals, X and Y. Two players is usually considered the smallest number needed for decision conflict; two periods is the minimal time structure that can capture the effects of delay; while two decisions is the smallest number that forces an allocation of attention.

We begin with a sequential model in which one proposal is on the agenda in the first period while the other is added in the second period. The allocation choices in the first period result in the proposal being accepted or delayed to the second period, while in the second period the allocation choices result in proposals being either accepted or rejected. We denote by $X \to Y$ and $Y \to X$ the sequential models in which proposals X and Y, respectively, are first on the agenda. These simple sequential models are sufficient for focusing and pinning to emerge in equilibrium and also allow us to examine the implications of such strategic actions for the preferred ordering of the proposals. When two proposals are on the agenda, we have the additional complication that focusing and pinning may occur simultaneously with player A attempting to focus player B on one proposal at the same time player B is attempting to pin the player A on the other proposal. For this reason, we begin with sequential models where only one proposal is on the agenda in period one. In Section 4 we examine a model in which both proposals are on the agenda in period one and each proposal may be accepted, rejected or delayed.

The advantages and disadvantages of delaying a decision depend on the nature of conflict between the two players. We focus on the strategically interesting case of *partial conflict* in which the players' preferences conflict on proposal X, but align on proposal Y. Within this general context, of course, substantial variation may exist regarding the relative intensities of conflict or alignment. We assume that player A has a utility $u_X < 0$ when proposal X is accepted and $u_Y > 0$ when proposal Y is accepted. Player B's utilities upon acceptance are similarly represented by $v_X > 0$ and $v_Y > 0$.

Condition 1 *$v_X > 0 > u_X$ (Conflict on X), $u_Y > 0$ and $v_Y > 0$ (Alignment on Y).*

Utility for the rejection of a proposal is normalized to 0. Consequently, the acceptance utilities are more precisely viewed as the incremental utility or disutility of accepting versus rejecting the proposal. Additionally, we make three simplifying assumptions: no discounting occurs; the utility associated with each proposal is independent of the outcome of the other proposal; and the preferences of each player are known to the other player. Our model abstracts away from informational concerns and focuses on the importance of delay for resource allocation under complete information. Each player maximizes the two-period sum of expected utility.

In each period a player allocates resources (e.g., attention or effort) to influence the outcome of proposals on the agenda. We model each player as choosing probability influence increments. A player supports (opposes) a proposal when she chooses a positive (negative) probability increment. We assume that each player's allocations have a direct effect which is linear and additive and that influence is neither cumulative nor storable across periods. This simple structure has the advantage of isolating the across-period strategic effects since additivity eliminates strategic interaction in a static single-period setting. Thus, when proposal i is on the agenda and player A chooses a_i and B chooses b_i, proposal i is accepted with probability

$$p_i = z + a_i + b_i \tag{1}$$

where z is a shift parameter reflecting exogenous factors that affect the probability of acceptance.[8] We differentiate first-period and second-period actions by using lower case and upper case proposal-identifying subscripts, respectively.

When the agenda only contains proposal i, A chooses a_i while B chooses b_i. When the agenda contains both proposals, player A chooses probability increments a_X and a_Y and B chooses b_X and b_Y. To reflect the assumption that total influence is constrained in a multi-proposal setting, we assume that there is a probability influence frontier g.

Condition 2 *Influence choices for player A (similarly for B) satisfy $|a_Y| \leq g(|a_X|)$ for $a_X \in [-\bar{p}, \bar{p}]$, where the probability frontier g satisfies (i) g is decreasing and concave over the interval $[0, \bar{p}]$, with $g(0) = \bar{p}$ and $g(\bar{p}) = 0$, (ii) g is symmetric around the 45° line: $a_Y = g(a_X) \Leftrightarrow a_X = g(a_Y)$, and (iii) $g'(0) = 0$ and $g'(\bar{p}) = -\infty$.*

[8] For example, in a hierarchical setting with two subordinates seeking to influence a superior, z would constitute the superior's initial bias regarding the decisions at issue.

Under this resource constraint, the maximum probability influence on a single proposal is equal to $\bar{p}$. The advantage of g's concave frontier structure is that influence allocation choices in a multiple-proposal setting will be interior to the interval $(-\bar{p}, \bar{p})$. Symmetry of g with respect to the 45° line comparably situates each proposal. In the first period, Condition 2 reduces to the requirement that influence choices on the single proposal lie in $[-\bar{p}, \bar{p}]$. Resource constraints along the lines of Condition 2 are common in political and decision-making formulations. Levy, Razin and Young (2021) [14], for example, employ a quadratic–form budget constraint (analogous to $g(|a_X|) = \sqrt{\bar{p}^2 - |a_X|^2}$) in their study of dynamic political competition.

Finally, we assume that uncertainty cannot be eliminated.

Condition 3 $2\bar{p} < z < 1 - 2\bar{p}$ *(feasible influence choices never lead to deterministic outcomes).*

Thus, for any choices a_i and b_i on proposal i, the probability p_i is always in $(0, 1)$. In the sequential agenda model, the first period has no reject possibility, so the probability of delaying proposal i is $1 - p_i$. In the second period, there is no delay state, so the probability of rejection is also $1 - p_i$. In Section 4, we relax these assumptions with a richer accept, reject and delay structure that allows for both proposals to be on the agenda in the first period and for each proposal to be accepted, rejected or delayed in the first period.

2.1 The Static Equilibrium Benchmark

The optimal second-period actions provide building blocks for the dynamic analysis and a benchmark case in which strategic interaction is absent. These actions depend on whether delay occurred in the first period. We begin with the simplest case of no delay. If only the aligned proposal Y remains in period 2 (i.e., given the $X \to Y$ agenda, X was accepted), then the players have directionally common interests as u_Y and v_Y are both positive. Clearly, each player will choose $\bar{p}$ such that the likelihood of accepting Y is maximized. The resulting payoffs associated with Y are then $U_Y = u_Y (z + 2\bar{p})$ and $V_Y = v_Y (z + 2\bar{p})$. If, instead, only the conflict proposal remains (i.e., Y was accepted under the $Y \to X$ agenda), then the players have opposing interests since $u_X < 0 < v_X$ and they will take offsetting actions ($a_X = -\bar{p}$, $b_X = \bar{p}$). This results in payoffs of $U_X = u_X z$ and $V_X = v_X z$.

Now consider the two-proposal case which arises whenever the first-period proposal has been delayed. We solve for a Nash equilibrium in which each player allocates their own probability influence across each of the two proposals. Given a choice by player A, say a_X and a_Y, player B's problem is to choose influence levels to $\max_{(b_X,b_Y)} v_X\left[z+a_X+b_X\right]+v_Y\left[z+a_Y+b_Y\right]$ over feasible influence levels relative to the probability frontier. Since player A's actions only have an additive effect on this payoff, the optimal choice by player B is given by

$$-\frac{v_X}{v_Y}=g'(b_X^*) \tag{2}$$

on proposal X and $b_Y^*=g\left(b_X^*\right)$ on proposal Y. Player A faces a similar problem except that A will seek to oppose proposal X. Thus, $-a_X\in[0,\bar{p}]$ and the solution is

$$\frac{u_X}{u_Y}=g'(-a_X^*) \tag{3}$$

on proposal X and $a_Y^*=g\left(-a_X^*\right)$ on proposal Y. The magnitude of a choice depends only on the preference intensity, defined by $u\equiv\left|\frac{u_X}{u_Y}\right|$ and $v\equiv\left|\frac{v_X}{v_Y}\right|$. Optimal choices equalize the probability trade-off and utility trade-off between X and Y. As the preference intensity for X rises, an increase in u or v, the magnitude of the action on X rises, while that for Y falls. The sign of a choice always follows the sign of the utility effect.

These choices constitute the Nash equilibrium for the static game. It is precisely because the other player's action does not impact the marginal benefit of one's own action that the two players optimize independently of each other and there is no strategic interaction. Critically, however, each player's payoff does depend on the other player's actions. This is the channel for dynamic strategic effects in our model: anticipating that another player will support or oppose a proposal that remains unresolved, an incentive exists to take action today to influence the other player's future move. To analyze this channel, we need the payoff outcomes for the simple static Nash equilibrium and we define $U_{XY}=u_X\left(z+a_X^*+b_X^*\right)+u_Y\left(z+a_Y^*+b_Y^*\right)$ and $V_{XY}=v_X\left(z+a_X^*+b_X^*\right)+v_Y\left(z+a_Y^*+b_Y^*\right)$.

2.2 Dynamic Equilibrium Choice

The static equilibrium strategies described above are also the optimal second-period equilibrium strategies. We now turn to the first-period actions. Consider the $X\rightarrow Y$ sequence ($Y\rightarrow X$ is analogous). From our analysis of the static case, we have the continuation payoffs for the players across the two possible states according to whether proposal X was delayed to the second period. The probabilities of each state are given by: $\{Y\}$ with p_x and $\{X,Y\}$ with $(1-p_x)$. The payoffs for the players at a candidate set of period 1 choices are then given by the sum of the expected period 1 and 2 payoffs:

$$U^a \equiv \left[u_X+U_Y\right]\left(z+a_x+b_x\right)+U_{XY}\left(1-\left(z+a_x+b_x\right)\right) \tag{4}$$

$$V^b \equiv \left[v_X+V_Y\right]\left(z+a_x+b_x\right)+V_{XY}\left(1-\left(z+a_x+b_x\right)\right) \tag{5}$$

The incentives for players A and B to allocate influence in period one are, respectively:

$$\frac{\partial U^a}{\partial a_x} = u_X + U_Y - U_{XY} \tag{6}$$

$$\frac{\partial V^b}{\partial b_x} = v_X + V_Y - V_{XY}. \tag{7}$$

We are now ready to examine equilibrium choices and strategic delay in period 1.

3 Player Preferences and Strategic Delay

We first analyze focusing in a $X \to Y$ agenda and then analyze pinning in a $Y \to X$ agenda.

3.1 Partial Conflict and Focusing ($X \to Y$ Agenda)

The first step is to determine the optimal second-period allocations of attention. This was done in the static benchmark analysis above. The next step is to analyze the optimal first-period allocations. The linear structure of influence implies that the objective functions for players A and B are maximized by allocating all influence to support proposal X if and only if $u_X + U_Y - U_{XY} > 0$ and $v_X + V_Y - V_{XY} > 0$, respectively. See (6) and (7).

Lemma 1 *Under the $X \to Y$ agenda, player B's optimal first-period allocation b_x is $\bar{p}$. (See Appendix for all proofs).*

Lemma 1 shows that B's strategic and myopic interests coincide. If B were to use negative influence, she would increase the probability that proposal X will be delayed in the first period, which is costly. The effect in the second period would also be negative because there would now be a higher probability that both X and Y are on the agenda in which case B will be opposed by A on proposal X.

Player A has a direct incentive to oppose X in the first period. But if X is off the agenda in the second period, player B will allocate all of its attention to supporting proposal Y which benefits A. This incentive to *focus* player B's attention on proposal Y becomes stronger as A's relative intensity of preference is greater for proposal Y than proposal X.

Definition 1 *(Focusing) A player focuses his rival on proposal j when the player's first-period allocation on proposal i decreases the probability of delay on i relative to the delay probability on i when*

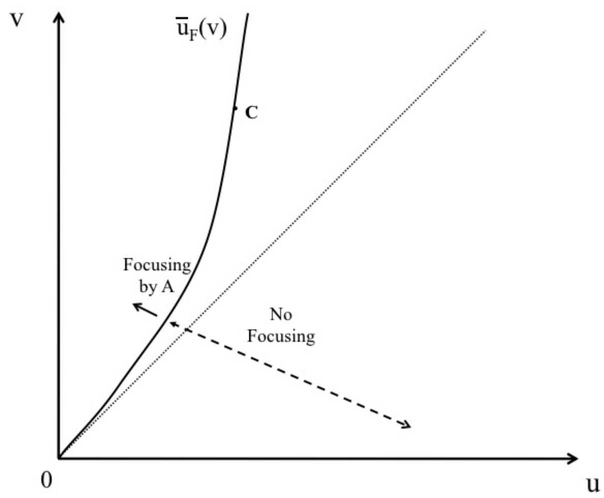

Figure 1: Equilibrium Taxonomy Under $X \to Y$ Agenda

the player chooses his static optimal allocation. A player follows static self-interest when the player chooses a first-period allocation equal to his static optimal allocation.

The incentives for focusing can be usefully characterized as a function of the ratios of the preference intensities for X to Y for each of the two players.

Proposition 1 *Under the $X \to Y$ agenda: (a) For any preference ratio v for player B, there exists a focusing cut-off preference ratio $\overline{u}_F$ below which in equilibrium it is optimal for player A to focus B by selecting $a_x = \bar{p}$ and above which A follows static self-interest by selecting $a_x = -\bar{p}$; B always follows static self-interest by selecting $b_x = \bar{p}$; (b) The focusing cut-off for Player A, $\bar{u}_F(v)$, is increasing in v, satisfies $\bar{u}_F(0) = 0$ and $\bar{u}_F(v) < v$ for $v > 0$, and is bounded above by $2\bar{p}/(1-z)$; (c) Focusing only occurs when both players' preferences are aligned over the proposal that is first introduced in the second period; (d) For $u < \bar{u}_F(v)$, the unique equilibrium involves A focusing B where we have $a_x = \bar{p}$ and $b_x = \bar{p}$; For $u > \bar{u}_F(v)$, the unique equilibrium has both players following static self-interest.*

Proposition 1 establishes that a region of preferences always exists for A and B in which A will uniquely focus B. Let $\mathcal{E}_F$ denote this region, the set of (u, v) with $u \leq \bar{u}_F(v)$. Figure 1 depicts this region in preference-intensity space illustrating focusing and no focusing regions under an $X \to Y$ agenda. These regions are separated by the cutoff function $\bar{u}_F(v)$.

Player A's incentive to focus B depends on both the intensity of preference A has for its key proposal (Y) and the relative gain A gets from focusing–the difference in the payoff to the $\{Y\}$ state and the $\{X, Y\}$ state. Because the $\{X, Y\}$ state payoff depends on B's intensity of preference, as v increases B allocates less resources to proposal Y in the $\{X, Y\}$ state and hence the benefits to A of focusing increase (part b). If, instead, B cared much more about proposal Y than proposal X, the gain to focusing B on proposal Y would not be great, as B would have been relatively focused on Y regardless of any focusing efforts. Thus, as depicted in Figure 1, the focusing region grows with v. Part (c) establishes that preference alignment on proposal Y is necessary for a focusing equilibrium. Focusing increases the probability that the second-period agenda will consist solely of proposal Y. Y alone is attractive under alignment since both players work together, whereas under conflict their efforts will offset.

Focusing is inherently cooperative, but the strategic outcome achieves neither social efficiency nor Pareto-optimality because there is always a positive probability that in the second period both proposals will be on the agenda with the resulting wasteful offset of resources on the conflict proposal X. Focusing is, however, an efficiency improvement over the static benchmark. Clearly, the focusing player is better off since focusing is an optimal strategy. The focused player benefits because of the increased probability that the focused player's favored proposal is accepted in the first period. One interpretation of focusing is that it is endogenous incentive-compatible log-rolling. Focusing also facilitates an agenda design that can support stable coalitions.

We can extend Proposition 1 in several directions. First, in contrast to the necessity of alignment on the second proposal, a focusing equilibrium does not require conflict on the first proposal. This is because the benefit of focusing derives from taking proposal X off the agenda and this benefit is largest when a player has a preference ratio that strongly favors proposal Y. Thus, we find that A will continue to focus B, following a cut-off rule as in Proposition 1, when we have alignment on X with $u_X < 0$ and $v_X < 0$.

Proposition 1 also proves to be robust with respect to the delay structure. In Section 4, we extend the simple accept or delay approach to a framework in which accept, reject and delay are all possible outcomes in the first period. For an $X \to Y$ sequential agenda, we find that a cut-off rule continues to apply and focusing emerges in equilibrium when v is small relative to u, as in Figure 1. The difference is that strategic elements are more pronounced when the delay features interaction effects. The simple incentive structure in (6-7) is then more involved as each player's best response depends on the action of the other player due to interaction effects with the richer delay structure.

Finally, when we move from a single proposal to a multiple proposal first-period setting in Section 4 (X and Y are both on the initial agenda), the first-period choices on X and Y by each player interact through delay channels for each proposal. This leads to a more subtle set of incentives than the base model of this section. The incentive for focusing remains but it is less dramatic. Instead of optimal first-period choices that are cornered, as with $\bar{p}$ and $-\bar{p}$ in our base model, choices will be interior on both proposals.

We now develop a focusing example with a preference structure that we will also use to illustrate pinning and to discuss agenda preferences.

Example 1 *($X \to Y$ Agenda): Suppose that resources are traded off according to* $g(a, \bar{p}) = \sqrt{\bar{p}^2 - a^2}$ *with* $\bar{p} = 0.1$, $z = 0.2$, *and let the preferences be* $u_X = -0.075$, $u_Y = 1$, $v_X = 1$, *and* $v_Y = 0.075$. *Then the equilibrium allocations are*

Eq Alloc	*1st period*	*2nd* $\{Y\}$	*2nd* $\{XY\}$
Proposal X	$a_x = 0.1$	*NA*	$a_X = -.007$
	$b_x = 0.1$	*NA*	$b_X = 0.099$
Proposal Y	*NA*	$a_Y = 0.1$	$a_Y = 0.099$
	NA	$b_Y = 0.1$	$b_Y = 0.007$

In this example both players have strong preferences regarding the outcome of one proposal but not the other, and the primary concern of one player is the secondary concern of the other. The intensity ratio v is large which means that, when faced with both proposals in the second period, B would allocate most of her resources to proposal X. Player A, therefore, receives an incremental benefit from focusing B on proposal Y. Of course, player A's low intensity of preference regarding proposal X makes it less costly to support proposal X against his static interest. In the example, player A's payoff is about five percent greater under focusing than under a myopic (suboptimal static self-interest) strategy. Focusing boosts player B's payoff by about thirty percent.

3.2 Partial Conflict and Pinning ($Y \to X$ Agenda)

Now consider the $Y \to X$ agenda. The analysis is analogous to that for $X \to Y$, except that the proposal order has been reversed. The conflict proposal X is now the *second* proposal and the alignment proposal Y is the first proposal. Note that by Proposition 1c focusing cannot occur with this configuration of preferences.

If player B followed her preferences, she would support proposal Y in the first period. However, dynamic considerations will sometimes cause B to oppose Y in the first period to improve the strategic situation in the second period. Essentially, B works to keep proposal Y on the agenda because if both X and Y are on the agenda in period two, then A will allocate more influence to supporting Y and less influence to opposing B on proposal X. B *pins* A to a proposal that is important to A.

Definition 2 *(Pinning) A player pins her rival to proposal i when the player's first-period action on i increases the probability of delay on i relative to the delay probability on i when the player chooses her static optimal action.*

Player B's decision to resist her static preferences depends on the relative strengths of the incentive to accept proposal Y and the dynamic benefits of pinning player A to proposal Y by delaying it to the second period. B chooses b_y to maximize $(v_Y + V_X)(z + a_y + b_y) + V_{XY}(1 - (z + a_y + b_y))$ where $v_Y + V_X$ captures the value of accepting Y in the first period and V_{XY} the value of delaying Y to the second period. It is clear from the objective function that $V_{XY} > v_Y + V_X$ is a necessary and sufficient condition for $b_y = -\bar{p}$ (which goes against B's preference).

Because there is conflict over X, $V_X = v_X z$. V_{XY}, of course, depends on the optimal static allocations (see (2) and (3)). Following a similar solution approach as above, we establish existence of a unique pinning equilibrium, a comparative static result, and a necessary condition for pinning.

Proposition 2 *Under the $Y \to X$ agenda: (a) For any preference ratio u for Player A, a pinning cut-off preference ratio $\bar{v}_P(u)$ exists above which in equilibrium it is optimal for B to pin A by selecting $b_y = -\bar{p}$ and below which B follows static self-interest $b_y = \bar{p}$; (b) The pinning cut-off for Player B, $\bar{v}_P(u)$, is increasing in u and satisfies $\bar{v}_P(u) > u$ for $u \geq 0$; (c) Pinning can only occur when both players' preferences conflict over a proposal that is first introduced in the second period; (d) For $v > \bar{v}_P(u)$, the unique equilibrium involves B pinning A where we have $b_y = -\bar{p}$ and $a_y = \bar{p}$; For $\bar{v}_P(u) > v > u$, the unique equilibrium has both players following static self-interest.*

The key element here is the relative size of the preference intensities. Existence requires that v is large relative to u: we must have preference intensities in the set $\mathcal{E}_P \equiv \{(u, v) \mid v \geq \bar{v}_P(u)\}$. That is, compared to player A, player B has a stronger relative preference for X. In turn, this relative preference implies that in the static $\{XY\}$ game, the net impact of influence on proposal X will be positive and $a^*_X + b^*_X > 0$ holds. In contrast, with X alone the player allocations cancel out each other. Thus, B pins A by going negative on Y in period 1, acting against (static) interest, to increase the likelihood that proposal Y is alive for the second period. Figure 2 provides a graphical representation of the pinning regions in preference-intensity space. Even as $v \to \infty$, such that player B does not care at all about proposal Y, B will still have an incentive to affect the outcome associated with Y because of that outcome's indirect resource implications for the outcome of proposal X. Thus, one should not be surprised to see a player exert influence activity on a proposal of little relative importance to that player.

The gain to B from delaying proposal Y depends on how A splits his resources in period 2 when both of X and Y are on the agenda. B's gain is largest when $u = 0$ since A will then fully support Y and devote no resources to opposing X. As u rises, A shifts resources from supporting Y to opposing X and, consequently, the gain to B from pinning A will decline. This makes pinning unattractive for a wider range of B preferences and the cutoff, $\bar{v}_P$, must rise with u (part b). Finally, a preference conflict on the second proposal is necessary for a pinning equilibrium (part c).[9]

Here, in contrast to the focusing equilibrium, there is no direct analog of Lemma 1 to guarantee that the other player will always act in accord with preference and support proposal Y. Instead, it is easy to show that Players A and B are symmetric with respect to pinning incentives.[10] Hence, there are two pinning regions in Figure 2. For any pair of u and v utility preference intensities, however, at most one of the players will have an incentive to pin since pinning requires that one intensity be sufficiently greater than the other. When the intensities are comparable in magnitude both players will follow static self-interest (part d).

A pinning strategy is inherently defensive. Therefore, given opposition on the alignment proposal Y in the first period and wasteful offsetting use of resources regarding proposal X in the second period, pinning equilibria are not socially efficient. Finally, pinning equilibria, unlike focusing equilibria, are not Pareto-improving versus static allocations. While the pinning player's expected utility is improved, the pinned player's expected utility declines.

The pinning result in Proposition 2 is robust to the delay structure. As with Proposition 1 and

[9]Pinning can also occur in $Y \to X$ settings involving pure conflict. Suppose player B prefers to accept both proposals whereas player A prefers to reject both proposals. When, for example, B's preference intensity ratio heavily favors proposal X while player A's intensity ratio heavily favors proposal Y, then it is optimal for player B to delay Y. By so doing, B increases the probability that player A will be pinned to Y.

[10]We employ notation that identifies which player is pinning, but there is a common cut-off function, $\bar{u}_P(w) = \bar{v}_P(w)$, for any utility intensity $w \geq 0$. In part (d) of Proposition 2 we need only interchange u and v to identify when A pins B.

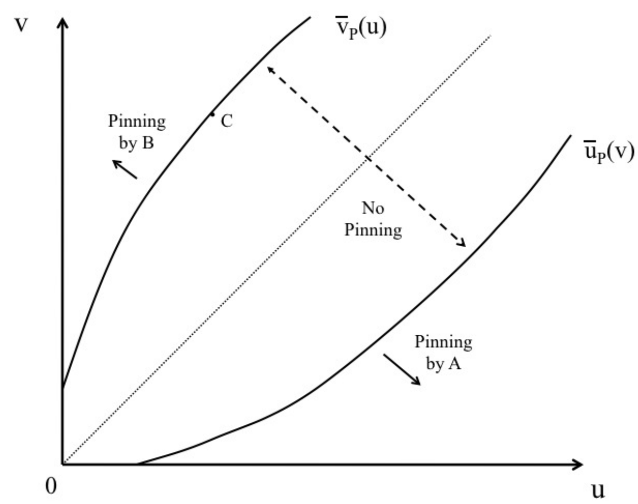

Figure 2: Equilibrium Taxonomy Under $Y \rightarrow X$ Agenda

focusing, we can extend the delay framework (as in Section 4) to allow accept, reject and delay outcomes for the first period of the $Y \rightarrow X$ sequential agenda. A cut-off structure for equilibrium then emerges, similar to that in Figure 2. Pinning behavior also emerges in equilibrium when we move to the multiple proposal first-period setting.

The same preference structure and parameters in Example 1 can be used to illustrate pinning in the $Y \rightarrow X$ agenda. In this $Y \rightarrow X$ agenda example, player B has a weak relative preference for Y and, as such, will pin player A to proposal Y in the second period. The optimal first period allocation of player B is $b_y = -0.1$ which is the opposite of B's static self-interest choice of 0.1. Player A's first-period allocation of $a_y = 0.1$ is consistent with A's static self-interest.[11]

3.3 Preferences and Proposal Pair Content

One key feature of focusing and pinning equilibria was that they were more and less efficient, respectively, compared to a static self-interest benchmark. These results suggest that proposal pairs that are nearly equivalent in terms of both players' preference intensities may differ substantially in terms of expected payoffs. Consequently, there may be a strong incentive for a player to get even a small

[11] The second-period $\{XY\}$ allocations are $a_X = -0.007$, $a_Y = 0.099$, $b_X = 0.099$, and $b_Y = 0.007$. The second-period $\{X\}$ allocations are $a_X = -0.1$ and $b_X = 0.1$.

change in proposal pair content, when possible. The most interesting candidates for such a change would be proposal pairs that result in preference intensity combinations which fall just inside or just outside of the focusing and pinning regions.

In this subsection we first explore the relative attractiveness of two proposal pairings for which the associated preference intensities locate the first pair *at* the focusing boundaries and the second pair *just outside* of the focusing region. We make direct comparisons between the expected payoffs associated with the two proposal pairs, thereby avoiding modeling the process by which a player could obtain a revision of a proposal pair. But the analysis does provide a sense of the incentives for a player to obtain a minor revision to an original proposal pair. Accordingly, we make comparisons between proposal pairs in which the more desirable proposal pair merely involves one player with a lower utility on Y, the alignment proposal, while holding all other utility levels constant.

More formally, each of the proposal pairs compared contain the same proposal X, but they differ in the other proposal with Y^- and Y^+ in pairs 1 and 2, respectively. We first analyze an X to Y agenda pair in which $u_{Y^-} = u_{Y^+}$ but $v_{Y^-} < v_{Y^+}$. Hence, player A's utility from any accepted proposal is the same for both pairs, but player B's utility from an accepted proposal Y is less in the case of pair 2. In Figure 1, pair 2 lies directly south of pair 1 (point C) because a decrease in the proposal Y utility of v_Y means an increase in the preference intensity v. Corollary 1 shows that for preference intensities that place the proposal pair just "south" of the focusing boundary, a proposal pair with the lower utility (v_{Y^-}) will be preferred by B in equilibrium to the proposal pair with the higher utility (v_{Y^+}).

Corollary 1 *Equilibrium payoffs are everywhere continuous in* (u_X, u_Y, v_X, v_Y) *with the exception of (i) the focusing boundary in the* $X \to Y$ *agenda where the payoff of the focused player has an upward jump of* $2\bar{p}[v_X + V_Y - V_{XY}]$ *at* $(\bar{u}_F(v), v)$ *as we cross from self-interest to focusing; and (ii) the pinning boundary in the* $Y \to X$ *agenda where the payoff of the pinned player has a downward jump of* $2\bar{p}[U_{XY} - U_Y - u_X]$ *at* $(u, \bar{v}_P(u))$ *as we cross from self-interest to pinning.*

While the Corollary is agnostic regarding the origins of the comparison pairs, the proposal-pair comparison illustrates how focusing affects the attractiveness of particular proposal-pairs and, therefore, provides a partial guide to understanding settings where proposal content is (locally) endogenous. Furthermore, the specific comparisons encompass a class of revisions which seem quite feasible: the revision to Y, the proposal that both players would like to be accepted, only reduces player B's direct utility value on Y without affecting any other relevant values. The gains from focusing along with continuity in the payoff for player A then help to identify minor utility changes in proposal X that would leave both players better off in equilibrium. As implied by Corollary 1, a similar comparison applies to the $Y \to X$ agenda and pinning.

The Corollary thus describes a threshold effect at each of the focusing and pinning boundaries. With a single proposal on the current agenda, a small change in preferences (proposal content) leads one player to switch positions and this has a large (discontinuous) impact on the other player. At one level, this is not surprising since resources cannot be stored for future use–with a single current proposal, a player will always choose an extreme position.[12] However, the intuition and rationale for the switch do not reflect intense "single-proposal" preferences. Rather, it is the prospect of influencing future events that triggers the switch to focusing and pinning behavior.

This discussion of small changes to proposal content raises the possibility of bargaining. When the ability to strike cooperative agreements is limited, equilibrium outcomes such as focusing or pinning are predicted by our model. In circumstances where bargaining is possible, focusing or pinning outcomes would serve as threat points or disagreement outcomes. In such cases, predictions employing threat points based on proposal preferences will be misleading. One critical element here is the payoff impact on the focused or pinned player that arises as one transitions from preference cases where all players follow their base proposal preferences to the more asymmetric preference cases that lead to focusing and pinning in equilibrium. We find that even with movements in preference space that increase social surplus, focusing and pinning lead to discontinuous decreases in negotiational payoffs for one player and corresponding increases for the other.

3.4 Equilibrium Taxonomy

Figure 3 combines Figures 1 and 2 to reveal the regions in preference-intensity space for which different types of equilibria exist under both the $X \to Y$ and the $Y \to X$ agendas. For the $Y \to X$ agenda, there are three regions: pinning by B (demarcated by $\overline{v}_P(u)$), pinning by A (demarcated by $\overline{u}_P(v)$), and no pinning by either player. Proposition 1c (2c) rules out focusing (pinning) for the $Y \to X$ ($X \to Y$) agenda. Figure 3 illuminates agenda choice consequences because, for a given set of preference intensities, one can better understand whether focusing or pinning will occur.

[12] In the full model of Section 4, both proposals are on the agenda in period 1 and influence action varies smoothly with changes in utility. There, the emergence of focusing and pinning in equilibrium is gradual as players do not jump between $\bar{p}$ and $-\bar{p}$. The larger point regarding efficiency gains from inducing focusing and avoiding pinning is unchanged.

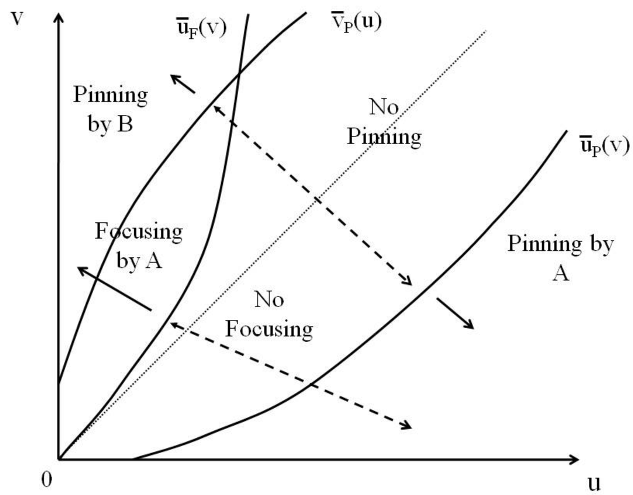

Figure 3: Equilibrium Taxonomy Under Both $X \to Y$ and $Y \to X$ Agendas

In the upper left (northwest) region of Figure 3 we see that a $X \to Y$ agenda results in focusing by A while a $Y \to X$ agenda results in pinning by B. This region is given by $\mathcal{E}_{FP} \equiv \left\{(u,v) \mid v \geq \max\left\{[\bar{u}_F]^{-1}(u), \bar{v}_P(u)\right\}\right\}$. Analysis of the regions where static self-interest prevails is straightforward.

4 A Symmetric Model of Acceptance, Rejection, and Delay

In this section we analyze a model in which both proposals are on the first-period agenda and each proposal may be accepted, rejected or delayed in the first period, effectively eliminating the role of proposal order. The sequential $X \to Y$ and $Y \to X$ models are versions of this more general model. We find that focusing and pinning emerge in the more general model and that for sufficiently asymmetric preferences the equilibrium necessarily involves focusing by one player and pinning by the other player.

Including both proposals in the first period introduces an initial tradeoff between X and Y regarding the optimal use of resources (recall Condition 2 and the g function). To include rejection, we model delay, acceptance and rejection of the proposal as follows: first, with probabilities d_i and $1 - d_i$, respectively, proposal i is delayed to the second period or it is resolved. If resolved, then the proposal is accepted with probability p_i and rejected with probability $1 - p_i$.[13] Delay is modeled as

[13] Given that the sum of the probabilities of the possible decision consequences must sum to one, this particular

$d_i = z_D - \gamma a_i b_i$ where z_D is the exogenous proposal delay component and γ is a scaling factor for the endogenous delay effect caused by conflict or agreement over proposal i. The multiplicative functional form employed here implies that agreement reduces delay while disagreement increases delay.[14] We rule out deterministic outcomes and assume that $\gamma\bar{p}^2 < z_D < 1 - \gamma\bar{p}^2$.

The optimal static equilibrium strategies are, as before, also the optimal second-period equilibrium strategies for this symmetric model. With both X and Y in the first period, we now have four possible agenda states in the second period. The probabilities of each state are given by: $\emptyset$ with $(1-d_X)(1-d_Y)$, only $\{X\}$ with $d_X(1-d_Y)$, only $\{Y\}$ with $(1-d_X)d_Y$, and $\{X,Y\}$ with $d_X d_Y$. The payoff for player A (similarly for player B) at a candidate set of period 1 actions is then given by

$$\begin{aligned} U^a \equiv\ & (1-d_X)d_Y\left[p_X u_X + U_Y\right] + d_X\left(1-d_Y\right)\left[p_Y u_Y + U_X\right] \\ & + d_X d_Y U_{XY} + \left(1-d_X\right)\left(1-d_Y\right)\left[p_X u_X + p_Y u_Y\right]. \end{aligned} \tag{8}$$

The incentives for player A for allocating influence across the two proposals are:

$$\frac{\partial U^a}{\partial a_x} = u_X - u_X d_X + \gamma b_x[-U_X + (U_X - U_Y - U_{XY})d_Y + p_x u_X]$$

$$\frac{\partial U^a}{\partial a_y} = u_Y - u_Y d_Y + \gamma b_y[-U_Y + (U_X - U_Y - U_{XY})d_X + p_y u_Y]$$

with analogous incentives for player B. These incentive effects highlight how delay complicates the tradeoffs when the first period has both proposals on the agenda along with accept and reject outcomes. Compare back to the effects in (6) and (7) for the sequential model. In contrast, the full model reveals that the basic intertemporal tradeoff for focusing and for pinning is mediated by the impact of a rival's choice on delay (the γb_i terms above). Thus, the period-one best response choice for each player will

acceptance-rejection-delay structure distributes the changes in delay probabilities proportionately across accept and reject outcomes.

[14] Most observers have found a positive correlation between the desire to attain decision consensus and delay. Conflict which makes consensus more difficult would then also seem positively correlated with delay. Our delay assumption seems particularly appropriate for environments in which decision makers favor some degree of consensus over pure formal authority or adherence to strict voting rules. For example, in a study of a medical school, Bucher (1970 p. 45 [6]) observed that "most of the opposition to an idea is worked through...or else the proposal dies." The positive relationship between conflict and delay is not, however, uncontroversial. Eisenhardt's (1989) [11] study of decision making speed in microcomputer firms found examples where conflict slowed decisions and where it did not depending on whether the firms valued or did not value consensus, respectively.

depend on the period one choices of a rival and the equilibrium will feature a richer set of interactions involving current-period strategic incentive effects as well as intertemporal incentive effects. Finally, note that concavity or convexity of the objective function depends on the sign of the rival's action.

Consider first the benchmark case of no endogenous delay ($\gamma = 0$). Here, players maximize an objective function strictly analogous to that faced in the second period (static) setting. Hence we have

Lemma 2 *Consider the symmetric model. If delay is exogenous, $\gamma = 0$, then the optimal first-period actions are the same as the corresponding optimal actions in the static equilibrium when proposals X and Y are both on the agenda.*

This result means that the effect of exogenous delay on optimal actions is isolated in the model from the effects of strategic delay. Hence, we can attribute changes in first-period actions relative to the optimal static equilibrium actions as resulting from strategic choices.

We now show that focusing and pinning occur simultaneously in the symmetric model whenever one player cares primarily about the alignment proposal and the other player cares primarily about the conflict proposal.

Proposition 3 *For players A and B, there exist cut-off preference ratios of $\bar{u}$ and $\bar{v}$, respectively, such that when $u < \bar{u}$ and $v > \bar{v}$, every equilibrium involves (i) A focusing B on proposal Y, by acting against static self-interest on X in period 1, $a_x^* > 0$ and $b_x^* > 0$, and (ii) B pinning A on Y by acting against static self-interest on Y in period 1, $a_y^* > 0$ and $b_y^* < 0$. For $u < \bar{u}$ and $v > \bar{v}$, an equilibrium exists provided that g is sufficiently concave: for $t \in [e, \bar{p}]$, we have*

$$g''(t) < -2\frac{1 - \gamma\bar{p}^2 g'(t)}{\bar{p}[z_D - \gamma\bar{p}^2]}$$

where e is defined by $e = g(e)$.

Proposition 3 highlights preference-intensity settings in which pinning and focusing necessarily occur in equilibrium. In Figure 3, the case of a small u for A and a large v for B corresponds to the regions for focusing and pinning equilibria under the $X \to Y$ and $Y \to X$ agendas, respectively. Weaker forms of focusing and pinning occur when a player does not allocate resources at the static self-interest levels but does maintain a direction of support (or opposition) that is consistent with static self-interest. The Nixon example, described below, provides an example of this weak form.

When the players are asymmetric with respect to their preferred proposals, with $u < \bar{u}$ and $v > \bar{v}$, they each allocate a majority of their influence to their preferred proposal. This means A chooses

$a_y^* > e$ and B chooses $b_x^* > e$, where e is the 45-degree line crossing of the probability resource function g (note that $e > \bar{p}/2$ holds by concavity of g). The sufficient condition, as derived in the Appendix, is needed for the range of large actions, above e, to deal with a technical complication that arises with B, the pinning player. The strong concavity assumption implies that we can rule out any best-response discontinuity (such as jumps to or from 0 and $\bar{p}$, as would be the case if g were too close to a linear function).

Finally, let us consider how this model can be employed to understand the example mentioned in the Introduction with Nixon, health care, and impeachment. The key element for Nixon is to introduce the health care proposal as an agenda item that competes with impeachment for influence resources. As implied by Proposition 3, we now have the potential for simultaneous focusing and pinning when both proposals are on the agenda in period 1. Working directly with the full version of the model is, however, quite involved. Fortunately, we can more easily identify these equilibrium incentives via a simple preference specification applied to two possible sequential agendas. One agenda will then highlight pinning by Nixon and the other will highlight focusing by the Democrats.

Let HC refer to the health care proposal and IP to impeachment proceedings; let A refer to the Democrats and B to Nixon; to illustrate how focusing and pinning emerge, we specify payoffs as

$$u_{HC} = u_{IP} = +1 \qquad and \qquad v_{HC} = 0, v_{IP} = -1$$

so that Democrats put equal weight on health care and impeachment but for Nixon it is rejecting impeachment that is of primary importance. To capture the idea that HC is a potential alignment proposal, consider limiting behavior as $v_{HC} \downarrow 0$. Thus, when the only item on the agenda is HC, we will take the outcome as both players supporting HC, each with influence $\bar{p}$. HC is then the alignment proposal while IP is obviously the conflict proposal.[15]

In the sequential $HC \to IP$ agenda, HC has been placed on the agenda in advance of IP and we are able to capture incentives to influence delay via initial HC choices. In the sequential $IP \to HC$ agenda, by contrast, we capture delay incentives to modify initial IP choices. In this way, we can sort out focusing and pinning behavior in equilibrium by the players.

Consider first the $HC \to IP$ agenda. Employing similar techniques to those in the proof of Proposition 3, we find that the unique equilibrium for this $HC \to IP$ game has $a_{HC}^* = \bar{p}$ and

[15] We can easily allow for an explicit non-zero v_{HC} preference value for (or against) health care by Nixon since what matters is that impeachment is the primary concern. Similarly, once Nixon has placed HC on the agenda, we could instead view player B as Republicans (or add them as a third player).

$b^*_{HC} = -\bar{p}$. Thus, in period 1, Democrats work to support health care while Nixon opposes. Recall that IP is the dominant proposal for Nixon and that, with $v_{HC} \downarrow 0$, there is no direct value from the HC outcome. Strategically, however, the incentive for Nixon is to maximize delay and HC is the method for doing so. This translates into choosing an action that is the opposite of that chosen by the Democrats and, hence, generating the (maximum possible) delay chance of $d_X = z_D + \gamma\bar{p}^2$. This reflects pinning behavior by Nixon. The overall payoffs then work out to be

$$U^a = d_{HC}U_{HC-IP} + (1-d_{HC})[p_{HC}u_{HC} + U_{IP}] = 2z + (z_D + \gamma\bar{p}^2)[2e - \bar{p}]$$

$$V^b = d_{HC}V_{HC-IP} + (1-d_{HC})[0 + V_{IP}] = -z + (z_D + \gamma\bar{p}^2)[\bar{p} - e].$$

If delay occurs, we have $HC - IP$ as the state in period 2. In view of the $u = 1$ and $v = 0$ ratios, we then know that A will choose to divide resources equally in supporting HC and IP with influence e on each proposal, where e is from the 45-degree line crossing of the g function via $g'(e) = -1$; note that $\bar{p}/2 < e < \bar{p}$ due to the concavity of g. In contrast, B uses full influence of $-\bar{p}$ on IP and zero on HC.

Relative to an IP only game, both players prefer the $HC \to IP$ agenda game. However, they do so for very different reasons. For B things are simple. Through delay on HC, player A will be induced to divide influence across both of the HC and IP proposals in period 2. This benefits B and it shows up in the second term of the payoff for B. Compared to the IP only payoff of $-z$, the difference is the positive delay effect: A has been pinned by B and, through delay, will divide resources across two proposals. Player A also benefits in $HC \to IP$ but, in contrast, this occurs via a net effect across both proposals. On the plus side, there are now two opportunities to pass HC, once when it does not delay in period 1 and then in period 2 after delay (reflected by one of the pure z payoff terms and one of the delay weighted e terms). These are the new terms relative to IP only. The downside, however, is that IP is less likely to succeed. As $2e - \bar{p} > 0$, the loss on the IP proposal is outweighed by the gain on HC. [16]

[16] The $HC \to IP$ agenda leads to the perhaps cynical observation that Nixon introduces health care only to oppose it. In this regard, note that the payoff conclusions remain valid even if we replace (or constrain) the initial B choice of $-\bar{p}$ with $+\bar{p}$. Similarly, in the version of the model where both proposals are on the first-period agenda, the HC choice will be less extreme and more so if we specify $v_{HC} > 0$. Finally, as noted, distinguishing between Nixon and Republican actions might be useful.

Consider now the $IP \to HC$ agenda. The interesting strategic element here is a potential focusing incentive in that the prospect of resolving IP creates an opportunity to benefit from alignment on HC. We find that this agenda has a unique equilibrium and B always opposes IP with an action of $-\bar{p}$. But A will, in equilibrium, choose to focus B on HC.[17] This takes the form of reducing support for IP from the static optimal level to the (still positive) level of

$$a_{IP}^* = \frac{1 - z_D}{2\gamma\bar{p}} - (\bar{p} - e) < \bar{p}.$$

Equilibrium payoffs are calculated to be

$$U^a = d_{IP}U_{HC-IP} + (1 - d_{IP})[p_{IP}u_{IP} + U_{HC}] = 2z + a_{IP}^* + \bar{p} + (z_D + \gamma a_{IP}^*\bar{p})[2e - a_{IP}^* - 2\bar{p}]$$

$$V^b = d_{IP}V_{HC-IP} + (1 - d_{IP})[p_{IP}v_{IP} + 0] = -(z + a_{IP}^* - \bar{p}) + (z_D + \gamma a_{IP}^*\bar{p})[a_{IP}^* - e].$$

Both players benefit from the $IP \to HC$ agenda relative to an $IP \to IP$ agenda. For the purposes of considering how adding HC to the agenda alters the situation, the more natural comparison is to the simple $IP \to IP$ game where IP may be resolved or delayed in period 1 and, if delayed, then resolved in period 2; of course, since players oppose and offset each other, the $IP \to IP$ payoffs reduce to z for A and $-z$ for B. With $IP \to HC$, B benefits directly from focusing by A because support for IP is lower in period 1 and also because delay leads to reduced IP support in the $HC - IP$ state. Player A chooses to focus because the benefit of reduced delay is that IP is resolved early and the HC state in period 2 (where there is alignment) becomes more likely.

This is a more subtle form of focusing than the complete influence reversal that arises in Proposition 1. The more general model allows for delay, acceptance and rejection in period 1 along with interaction between player influence choices and the delay probability. As a result, we are able to account for interior focusing (and pinning) levels that do not necessarily involve a change in direction of influence. The common thread, as emphasized in the definitions for focusing and pinning, is the strategic channel of influencing the probability of delay on a proposal. Delay is what matters for the impact on future agendas and this determines the resulting level and direction of equilibrium influence.

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[17] The necessary and sufficient parameter condition for existence of this focusing equilibrium is $1 - z_D - \gamma\bar{p}^2 < \gamma\bar{p}(3\bar{p} - 2e)$. Intuitively, focusing arises when there is significant scope for strategic delay ($\gamma\bar{p}^2$ is not too small). For example, the condition always holds when $\gamma\bar{p}^2 > 1/3$. Otherwise, when the condition fails, A will follow static-self interest on IP.

5 Discussion

In this section we consider how the model provides insight for a broad range of decision settings and, in so doing, address some limitations of the analysis. Throughout this paper we have emphasized strategic opportunities posed by decisions which may be delayed rather than resolved. The strategic use of focusing and pinning to influence delay can be interpreted more broadly as action that increases or decreases the probability of a commitment. Under this commitment interpretation, delay includes "resolved" decisions that are easy to revisit (e.g., a private decision to launch a product with no immediate supporting actions, in contrast to a public commitment to launch a product combined with purchase of specialized assets). In the sequential model p_i (probability of acceptance) would then be a measure of the likelihood of first-period commitment regarding matter i, while in the symmetric model this measure would be $1 - d_i$ (one minus the probability of delay). Modeling more nuanced levels of commitment is potentially an interesting extension.

Our model applies to settings in which players who choose to influence decisions (or that make decisions) have limited resources such as constraints regarding time and attention. Such constraints have been emphasized by the organizational decision-making literature as central to decision making. Simon (1947, p.294) [26], for example, views "[a]ttention...[as] the chief bottleneck in organizational activity" and argues that "the bottleneck becomes narrower and narrower as we move to the tops of organizations..."[18] The importance of attention for organizational decision making has also been highlighted in more political conceptualizations of organizations such as Pfeffer's (1978) [21] micropolitics model or the organized anarchy (garbage-can) model of Cohen, March, and Olsen (1972) [9]. Elaborating upon the latter model, March and Olsen (1979) [17] regard participation in various choice decisions as dependent on organizational obligations, various symbolic aspects of decision making, and rational action regarding the allocation of attention across various alternatives.

[18] Divided attention is a common theme in the decision making literature. Wood and Peake (1998) [28] find, for example, that presidential attention to important unresolved foreign policy issues declines when other foreign policy issues become more prominent. Redman (1973) [24] (pp. 55-57) also delineates numerous examples illustrating the effects of divided attention in the legislative setting. He describes, for example, how an "amendment in committee" strategy for grafting a National Health Service Corps onto another health bill in 1970 was derailed by the U.S. invasion of Cambodia. More recently, attention to health care reform was seen by some legislative aides as "sucking all the oxygen out of the room" and distracting legislative attention away from fully understanding various potential loopholes that lobbyists were introducing in financial reform legislation (Hirsh 2013 [13]).

> "There are almost no decisions that are so important that attention is assured...The result is that even a relatively rational model of attention makes decision outcomes highly contextual....Substantial variation in attention stems from other demands on the participants' time (rather than from features of the decision under study). If decision outcomes depend on who is involved..., if the attention structures are relatively permissive and unsegmented, and if individuals allocate time relatively rationally, then the outcomes of choices will depend on the availability and attractiveness of alternative arenas for activity. The individuals who end up making the decision are disproportionately those who have nothing better to do..." (March and Olsen 1979 [17], pp. 46-47).[19]

While March and Olsen's comment regarding the influence of the idle reflects an element of whimsy, it reflects a serious undercurrent regarding the use of resources that are non-storable. Our model adopts the starkest version of attention resources: there is no marginal cost of use up to a fixed maximum. As such, our model is directly applicable when the benefit or cost of the less important decision outcome exceeds the marginal cost associated with influence activities. Because our results depend on relative rather than absolute proposal preferences, this zero marginal cost-of-use assumption is not particularly limiting. Alternatively, one could treat the marginal cost of effort as a filter that limits the number of decisions that are sufficiently important to attract the attention of parties with significant influence resources.

In terms of decision-making settings, our model would apply to decision making both by committee and within a hierarchy.[20] Consider a committee structure. While committee decision making typically involves more than two players, the impact of different preferences on the incentives to allocate influence resources to affect delay in committee decision settings is arguably captured in our two-player model. In our model a player A with unbalanced preferences has an incentive to take a strategic action against myopic interest when another player B also has unbalanced preferences. Other involved players who have more balanced preferences have an incentive to take actions consistent with their static self-interest. The actions of these "other" committee members can then be interpreted as

[19] See Bendor, Moe, and Shotts (2001) [2] for a critical review of the research program addressing the garbage-can model of organizational decision making.

[20] See Persico 2004 [20] and Visser and Swank 2007 [27] for research that focuses on information issues in committee decisionmaking.

being captured by z, the exogenous probability parameter.

Next consider an extension of the two-player model to accommodate N decision makers each of whom may have unbalanced preferences. We conjecture that equilibria exist in such models which involve multiple players taking focusing or pinning actions while others act with static self-interest. When expanding from a two-player setting to multiple-player settings, one must account for a more complex preference set. Recall that two factors determine whether a player will focus (or pin): the relative intensity of own preferences and the incremental value of such a strategic action relative to the baseline of acting with static self-interest. Incremental value depends on the anticipated actions which the other players will take in the single-proposal-only and in the multiple-proposal states. One can propose a multi-player equilibrium and then check deviations by examining each player's incentives based on their respective preferences and the "net" actions implied by the equilibrium for the other players. The additive separability inherent in the model's structure facilitates such an analysis.

Hierarchical decision making represents the other extreme in which a single person is the decision maker. Within this context, each of the two players in our model can be interpreted as taking actions to influence the ultimate decision maker. Subordinates commonly have considerable latitude regarding the influence and attention they devote to any given decision. Bower (1970) [4], for example, describes strategy choice as a resource-allocation process in which a firm's strategy emerges from a decision making system in which upper management primarily controls organizational level decisions (e.g., a firm's overall direction or its culture) but implicitly relies on the judgment of middle managers who compete over project-level decisions. Decision making from this perspective is seen as "decidedly multilevel and multiperson," (Bower, Doz, and Gilbert 2005, p.13) [5]. In this interpretation z would constitute the bias of the decision maker.

6 Conclusion

> "In a minute there is time [f]or decisions and revisions which a minute will reverse."
> (*The Love Song of J. Alfred Prufrock*, T.S. Eliot 1915)

When the outcome of a decision does not involve real commitment, the decision remains either explicitly on the agenda because the decision was deferred or implicitly on the agenda because the decision is reversible (e.g., 2010 Affordable Care Act). Important but reversible decisions continue to attract decision making attention thereby affecting future influence allocations and, therefore, future outcomes. Consequently, anticipating such future effects, decision makers may alter their allocation of current resources. Such decision dynamics prompt two closely-related strategies: taking actions against myopic interest to pin a rival's future attention to a proposal carried over from the current round or taking actions against myopic interest to remove a distracting proposal and focus a rival's future attention on a particular proposal. These strategic actions emerge in equilibrium when decision participants have strong relative preferences for one proposal over another. Strategies of pinning and focusing also alter the value of having one proposal precede another proposal. The analysis, therefore, has implications for across-meeting agenda setting, rather than for the more commonly analyzed problem of within-meeting agenda setting.

There is much room to extend the theoretical analysis to multiple participants with varying resources as well as to consider additional issues. In addition to exploring the effect of deferring decisions empirically, other arguably interesting avenues would be to examine the effect of related decisions in which adoption of one proposal changes the utilities associated with other proposals and to further explore design of proposal content to take dynamic advantage of decision participant preferences. Finally, allowing for incomplete information regarding preferences on proposals would naturally lead to a role for signaling and reputations.

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Appendix

The following simple result characterizes the optimal static actions for any configuration of player preferences. We make frequent use of this result in subsequent proofs, including those for Propositions 1 and 2.

Lemma A1 Let $u \equiv \left|\frac{u_X}{u_Y}\right|$ and $v \equiv \left|\frac{v_X}{v_Y}\right|$ denote the preference intensities. Then, the strategies in the static Nash equilibrium when X and Y are on the agenda are given by

i) $g'(|a_X|) = -u$ and $g'(|b_X|) = -v$

ii) $g(|a_X|) = |a_Y|$ and $g(|b_X|) = |b_Y|$

iii) $sgn(a_i) = sgn(u_i)$ and $sgn(b_i) = sgn(v_i)$ for $i = X, Y$.

Proof: We prove the results for Agent A; the proof for B involves a simple change of labels. Property (iii), $sgn(a_i) = sgn(u_i)$, is trivial. If $u_i > 0$ but $a_i < 0$, then $u_i a_i < 0 < -u_i a_i$ and $-a_i > 0$ is a better choice for A. Similarly, if $u_i < 0$ but $a_i > 0$, then $-a_i < 0$ is again a better choice. For (ii), $g(|a_X| = |a_Y|$, suppose not. Then, by feasibility, we have $g(|a_X|) > |a_Y|$. If $u_Y > 0$, then a choice of a_X and $\hat{a}_Y = g(|a_X|)$ yields a higher payoff. Similarly, if $u_Y < 0$, then using the slack in resources to set $\hat{a}_Y = -g(|a_X|)$ increases the payoff. Because the objective, $u_X a_X + u_Y a_Y$, is linear and the constraint set, $g(|a_X| \geq |a_Y|$ for $0 \leq |a_X| \leq \bar{p}$, is symmetric, Properties (ii) and (iii) of Lemma A1 imply that we can solve A's choice problem for any (u_X, u_Y) by first solving the problem for the case of $u_X > 0$ and $u_Y > 0$ and then making an adjustment of sign on the optimal influence choices. Thus, for $u_X > 0$ and $u_Y > 0$ the choice problem of A reduces to

$$\max\left[u_X a_X + u_Y g(a_X)\right] \qquad s.t. \qquad 0 \leq a_X \leq \bar{p}$$

This is a continuous objective on a compact set and therefore has a solution. Since g is strictly concave, the solution is uniquely determined by the first-order condition $u_X + u_Y g'(a_X) = 0$. By part (iii) of Condition 2 for g, the solution is interior. For reference, we use $a_X(u)$ and $a_Y(u)$ to denote the solution for any ratio $u \equiv |u_X/u_Y| > 0$. Comparative statics are straightforward. Defining $G(u) \equiv [g']^{-1}(-u)$, these are given by $a'_X(u) = -1/g''(G(u)) > 0$ and $a'_Y(u) = u/g''(G(u)) < 0$. Finally, note that $ua'_X(u) + a'_Y(u) = 0$ (Envelope Theorem).■

Proof of Lemma 1: By Condition 1, we have $V_X = zv_X$ since $a_X = -\bar{p}$ and $b_X = \bar{p}$ are the optimal choices. Similarly, $V_Y = (z + 2\bar{p})v_Y$, since $a_Y = b_Y = \bar{p}$. Finally, $V_{XY} = v_X\left(z_X + a_X^* + b_X^*\right) + v_Y\left(z_Y + a_Y^* + b_Y^*\right)$, by the optimal static choices (denoted by *) from Lemma A1 when X and Y are both on the agenda. Player B chooses b_x to maximize $(z + a_x + b_x)(v_X + V_Y) + [1 - (z + a_x + b_x)]V_{XY}$. Clearly, since the objective is linear, $b_x = \bar{p}$ iff $v_X + V_Y > V_{XY}$. Simplifying, this inequality reduces to

$$v_X + (z + 2\bar{p})v_Y > (z + a_X^* + b_X^*)\, v_X + (z + a_Y^* + b_Y^*)\, v_Y$$

This is valid because (1) $1 > z + a_X^* + b_X^*$, by Condition 3; (2) $z + 2\bar{p} \geq z + a_Y^* + b_Y^*$ by $\bar{p} \geq a_Y$ and $\bar{p} \geq b_Y$, and (3) each of $v_X > 0$ and $v_Y > 0$ holds by Condition 1.■

Proof of Proposition 1, Part A: Player A chooses a_x to maximize $\{(z+a_x+b_x)(u_X+U_Y)+[1-(z+a_x+b_x)]U_{XY}\}$. The solution is $a_x = \bar{p}$ iff $u_X + U_Y > U_{XY}$ (it is $a_x = -\bar{p}$ when $u_X + U_Y < U_{XY}$). Substituting for U_Y and U_{XY} with the optimal static actions, rearranging terms, and dividing by $u_Y > 0$, we have

$$u_X + U_Y > U_{XY} \Leftrightarrow (2\bar{p} - a_Y^* - b_Y^*) + \frac{u_X}{u_Y}(1 - z - a_X^* - b_X^*) > 0.$$

Now, using the definitions of $u \equiv -\frac{u_X}{u_Y} > 0$ and $v \equiv \frac{v_X}{v_Y} > 0$, and writing the the optimal choices in the $\{XY\}$ state in terms of the solutions to the first-order conditions from Lemma A1, that is $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, our condition for $a_x = \bar{p}$ becomes

$$h(u,v) \equiv \left[2\bar{p} - a_Y(u) - b_Y(v)\right] - u[1 - z + a_X(u) - b_X(v)] > 0.$$

We claim that, for any $v > 0$, the function $h(u, v)$ is (1) decreasing in u, (2) positive at $u = 0$, (3) negative as $u \to \infty$, and, hence, (4) there $\exists!\ u \ni h$ crosses 0. To show (1), differentiate h w.r.t. u and apply the envelope theorem, $a_Y'(u) + ua_X'(u) = 0$, to find $h_u = -[1 - z + a_X(u) - b_X(v)] < 0$, as follows from Condition 2 for interior probabilities. For (2), let $u \to 0$ and note that $a_X(u) \to 0$ and $a_Y(u) \to \bar{p}$, so that $h(0, v) = \left[\bar{p} - b_Y(v)\right] > 0$. For (3), letting $u \to \infty$ in $h(u, v)$ and noting $a_X(u) \to \bar{p}$ and $a_Y(u) \to 0$, we see $h(u, v) \to -\infty$. Then, (4) follows by continuity and $h(u, v)$ crosses zero one time at a unique $u = \bar{u}_F(v) \in (0, \infty)$. Thus, $h(u, v) > 0$ holds for $0 < u < \bar{u}_F(v)$ and then $a_x = \bar{p}$, while $h(u, v) < 0$ holds for $u > \bar{u}_F(v)$ and then $a_x = -\bar{p}$.

Part (b): To verify that $\bar{u}_F(v)$ is increasing, simply note that $\bar{u}_F'(v) = -h_v/h_u$, the ratio of partials for h. From above, we know $h_u < 0$. Calculating, we find $h_v = ub_X'(v) - b_Y'(v) > 0$. Hence, $\bar{u}_F'(v) > 0$ holds. To verify that $\bar{u}_F(0) = 0$, observe that $b_x(0) = 0$ and $b_y(0) = \bar{p}$ so that $h(u, 0) = \bar{p} - a_Y(u) - u[1 - z + a_X(u)]$. At $u = 0$, we have $a_X(0) = 0$ and $a_Y(0) = \bar{p}$. This implies $h(0, 0) = 0$ and, hence, $\bar{u}_F(0) = 0$. To show that $\bar{u}_F(v) < v$ for any $v > 0$, it is sufficient to show that $h(v, v) < 0$ since this implies h crosses zero to the left of v. Simplifying $h(u, v)$ at $u = v$, we have

$$h(v, v) = 2[\bar{p} - b_Y(v)] - v[1 - z].$$

Note that the (scaled) payoff function $[vb_X(v) + b_Y(v)]$ has value $\bar{p}$ at $v = 0$ and is increasing with derivative $b_X(v) > 0$ (Envelope Theorem). Thus, we have $\bar{p} \leq vb_X(v)+b_Y(v)$ and, hence, $2[\bar{p}-b_Y(v)] \leq 2vb_X(v)$. Since $1 - z > 2\bar{p}$ by Condition 3 and $\bar{p} \geq b_X(v)$, we have $v[1 - z] > 2vb_X(v)$ and, thus, the inequality for $h(v, v) < 0$ is valid. Finally, for the upper bound on $\bar{u}_F(v)$, write $h(\bar{u}_F(v), v) = 0$ as (suppressing arguments) $\bar{u}_F = [2\bar{p} - a_Y - b_Y]/[1 - z + a_X - b_X]$. Since a_Y and b_Y are non-negative and $a_X(\bar{u}_F) < b_X(v)$ for $\bar{u}_F < v$, the upper bound of $\bar{u}_F < 2\bar{p}/(1 - z)$ follows directly.

Part (c): Suppose Y is a conflict proposal. In a focusing equilibrium, player A chooses $a_x = \bar{p}$ against own interest based on $u_X < 0$. The choice $a_x = \bar{p}$ is optimal iff $u_X + U_Y - U_{XY} > 0$. Substituting for U_Y and U_{XY}, noting that $\hat{a}_Y + \hat{b}_Y = 0$ [where $\hat{a}_Y$ and $\hat{b}_Y$ are the optimal actions when only (the conflict) proposal Y is on the second-period agenda] as players A and B choose oppositely in Y, and rearranging terms yield $u_X + U_Y - U_{XY} > 0 \Leftrightarrow$

$$u_X[1 - (z + a_X^* + b_X^*)] - (a_Y^* + b_Y^*)u_Y > 0. \tag{9}$$

To show that alignment in Y is necessary, we show that the first order condition for focusing (9) cannot hold when players A and B conflict on proposal Y. There are two cases for conflict (A) $u_Y > 0 > v_Y$ and (B) $u_Y < 0 < v_Y$.

Case A ($u_X < 0$ and $u_Y > 0 > v_Y$): Consider (9). Substitute with $u = -u_X/\ u_Y > 0$ and simplify with the solutions to the first-order conditions, $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), -b_Y(v))$, to see that (9) holds iff

$$[b_Y(v) - a_Y(u)] - u\,[1 - (z - a_X(u) + b_X(v))] > 0$$

This expression is strictly decreasing in u since the partial (applying the Envelope Theorem) is $-\,[1 - (z - a_X(u))] < 0$. At $u = 0$, the expression reduces to $b_Y(v) - \bar{p} < 0$. Hence, the expression is never positive, which is a contradiction.

Case B ($u_X < 0$ and $u_Y < 0 < v_Y$) : Consider (9). Substitute with $u = u_X/\ u_Y > 0$ and simplify with the solutions to the first-order conditions, $(a_X^*, a_Y^*) = (-a_X(u), -a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, to see that (9) holds iff

$$[b_Y(v) - a_Y(u)] - u\,[1 - (z - a_X(u) + b_X(v))] > 0.$$

Since this is the same expression as in Case A, we have a contradiction.

Part (d): By Lemma 1, B always chooses $b_x = \bar{p}$. Since A chooses a_x according to the cut-off function, $\bar{u}_F(v)$, the equilibrium result follows directly. Finally, note that in the knife-edge case when

(u, v) lies on the focusing boundary we have $u_X + U_Y - U_{XY} = 0$; any $a_x \in [-\bar{p}, \bar{p}]$ is then optimal and part of an equilibrium.■

Proof of Proposition 2, Part (a): The partial conflict pinning assumptions are: $v_X > 0 > u_X$, $u_Y > 0$ and $v_Y > 0$. Player B chooses b_y to maximize $(z + a_y + b_y)(v_Y + V_X) + [1 - (z_Y + a_y + b_y)]V_{XY}$. Thus, $b_y = -\bar{p}$ iff $v_Y + V_X < V_{XY}$. Substituting for V_X and V_{XY} with the optimal static actions, rearranging terms and dividing through by $v_Y > 0$, we have

$$v_Y + V_X < V_{XY} \Leftrightarrow 1 - [z + a_Y^* + b_Y^*] - \frac{v_X}{v_Y}(a_X^* + b_X^*) < 0.$$

Using $u \equiv -\frac{u_X}{u_Y} > 0$ and $v \equiv \frac{v_X}{v_Y} > 0$, and writing the the optimal choices in the $\{XY\}$ state in terms of the solutions to the first-order conditions, that is $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, our condition for $b_y = -\bar{p}$ becomes

$$k(v, u) = 1 - [z + a_Y(u) + b_Y(v)] - v\,[b_X(v) - a_X(u)] < 0.$$

Next, we claim that, for any $u > 0$, the function $k(v, u)$ is (1) increasing in v for $v < u$ and decreasing in v for $v > u$, (2) positive at $v = 0$, (3) negative as $v \to \infty$, and, hence, (4) there $\exists!\ v \ni k$ crosses 0. To show (1), differentiate k w.r.t. v and apply the envelope theorem, $b_Y'(v) + v b_X'(v) = 0$, to find $k_v = a_X(u) - b_X(v)$. From the proof of Lemma A1, we know that $a_X(u) \gtreqless b_X(v)$ as $u \gtreqless v$ since both are increasing in the utility intensity. Then, (1) follows directly. For (2), let $v \to 0$ and note that $b_X(v) \to 0$ and $b_Y(v) \to \bar{p}$, so that $k(0, u) = 1 - [z + a_Y(u) + \bar{p}] > 0$, by Condition 2. For (3), letting $v \to \infty$ in $k(v, u)$ and noting $b_X(v) \to \bar{p}$ and $b_Y(v) \to 0$, we see $k(v, u) \to -\infty$. Then, (4) follows by continuity and $k(v, u)$ crosses zero one time at a unique $v = \bar{v}_P(u) \in (0, \infty)$. Thus, $k(v, u) > 0$ holds for $0 < v < \bar{v}_P(u)$ and then $b_y = \bar{p}$, while $k(v, u) < 0$ holds for $v > \bar{v}_P(u)$ and then $b_y = -\bar{p}$. Note that, by property (1), for a given u, the maximum of k over all $v \geq 0$ occurs at $v = u$. Since $k(u, u) > 0$, it follows that k crosses zero in v to the right of $v = u$ and we therefore have $\bar{v}_P(u) > u$. Finally, note that $k(0, 0) > 0$, so that we have $\bar{v}_P(0) > 0$.

Part (b): Implicit differentiation of $k(\bar{v}_P, u) = 0$ yields $\bar{v}_P'(u) = -k_u/k_v$, the ratio of partials. We know $k_v < 0$ holds when $k(\bar{v}_P, u) = 0$. Also, we easily find that $k_u = -a_Y'(u) + v a_X'(u) > 0$. Hence, $\bar{v}_P'(u) > 0$. Finally, $\bar{v}_P(u) > u$ was shown just above.

Part (c): Suppose X is an alignment proposal. In a pinning equilibrium player B chooses $b_y = -\bar{p}$ against own interest based on $v_Y > 0$. We know $b_y = -\bar{p}$ is optimal when the condition $v_Y + V_X - V_{XY} < 0$ holds. Substituting for V_X and V_{XY} and rearranging terms $v_Y + V_X - V_{XY} < 0 \Leftrightarrow$

$$v_Y[1 - (z + a_Y^* + b_Y^*)] + v_X(\hat{a}_X + \hat{b}_X) - v_X(a_X^* + b_X^*) < 0 \tag{10}$$

where $\hat{a}_X$ and $\hat{b}_X$ are the optimal actions where only proposal X is on the second-period agenda. There are two cases of alignment for X:

Case 1 ($u_X > 0, v_X > 0$): This implies $\hat{a}_X = \hat{b}_X = \bar{p}$. But then $v_Y[1-(z+a_Y^*+b_Y^*)] + v_X(2\bar{p} - a_X^* - b_X^*) > 0$, which contradicts (10).

Case 2 ($u_X < 0, v_X < 0$): This implies $\hat{a}_X = \hat{b}_X = -\bar{p}$. Then $v_Y[1-(z+a_Y^*+b_Y^*)] - v_X(2\bar{p} + a_X^* + b_X^*) > 0$, which again contradicts (10).

Hence, pinning cannot occur with alignment over the second proposal.

Part (d): A completely symmetric argument shows that Player A also has a cut-off value, denoted by $\bar{u}_P(v)$ and it is defined by the condition $k(\bar{u}_P, v) = 0$. The claim regarding a pinning equilibrium now follows directly. For $v > \bar{v}_P(u)$, we know Player B optimally chooses $b_y = -\bar{p}$. Because $\bar{v}_P(u) > u$, we see that $v > u$ holds. We then have $\bar{u}_P(v) > v > u$ and Player A optimally chooses $a_y = \bar{p}$.■

Proof of Corollary 1: Continuity away from the pinning or focusing boundary follows trivially from continuity of the period-2 actions in u and v. For $X \to Y$ and the focusing boundary, we calculate the payoff difference via

$$\begin{aligned} V_F^b - V_{SI}^b &= [(z+2\bar{p})(v_X + V_Y) + (1-z-2\bar{p})V_{XY}] - [z(v_X+V_Y) + (1-z)V_{XY}] \\ &= 2\bar{p}[v_X + V_Y - V_{XY}] \end{aligned}$$

which is strictly positive, by Lemma 1. For $Y \to X$ and the pinning boundary, a similar calculation yields the difference as stated in the Corollary for $U_P^a - U_{SI}^a$. To see that this is strictly negative, simplify to obtain $(1-z-a_Y^*-b_Y^*)u_Y > (a_X^*+b_X^*)u_X$. The left-hand side is positive by feasibility and $u_Y > 0$ while the right-hand side is negative since $(a_X^*+b_X^*) > 0$ holds as $\bar{v}_P(u) > u$ and $u_X < 0$.■

Proof of Lemma 2: $\gamma = 0$ implies that $d_X = d_Y = z_D$. $U^a = z_D^2 U_{XY} + z_D(1-z_D)[U_X + p_Y u_Y] + (1-z_D)z_D[p_X u_X + U_Y] + (1-z_D)^2[p_X u_X + p_Y u_Y]$ which, after rearranging terms and simplifying gives $U^a = z_D[U_X + U_Y] - z_D^2[U_X + U_Y - U_{XY}] + (1-z_D)[p_X u_X + p_Y u_Y]$. Similarly, $V^b = z_D[V_X + V_Y] - z_D^2[V_X + V_Y - V_{XY}] + (1-z_D)[p_X v_X + p_Y v_Y]$. Maximizing U^a and V^b involves solving $\max\limits_{a_X, a_Y}\{p_X u_X + p_Y u_Y\}$ and $\max\limits_{b_X, b_Y}\{p_X v_X + p_Y v_Y\}$ with solutions that are the same as those for the static actions when both proposals X and Y are on the agenda.■

Proof of Proposition 3: To begin, we simplify U^a from the text and the analogous expression for V^b by collecting terms to obtain

$$\begin{aligned} U^a &= d_X d_Y U_{XY} + (1-d_X)d_Y U_Y + d_X(1-d_Y)U_X + (1-d_X)p_X u_X + (1-d_Y)p_Y u_Y \\ V^b &= d_X d_Y V_{XY} + (1-d_X)d_Y V_Y + d_X(1-d_Y)V_X + (1-d_X)p_X v_X + (1-d_Y)p_Y v_Y. \end{aligned}$$

The values when only X or only Y are on the agenda in period 2 are unchanged from before; also, values for U_{XY} and V_{XY} are determined by the preference ratios. We prove the proposition by taking limits as $u \to 0$ and $v \to \infty$. Since the relevant terms involve strict inequalities, our result holds in a neighborhood of these limiting values. For convenience, adopt the normalization of $u_Y = v_X = 1$ and let $u_X \uparrow 0$ while $v_Y \downarrow 0$. Then the limiting values for the $\{X, Y\}$ state in period 2 are $U_{XY} = (z + \bar{p})$ and $V_{XY} = (z + \bar{p})$, since the actions of A follow $a_X(u) \to 0$ and $a_Y(u) \to \bar{p}$ while those of B follow $b_X(v) \to \bar{p}$ and $b_Y(v) \to 0$. Substituting in the payoffs above and simplifying yield

$$\begin{aligned} U^a &= d_Y [z + 2\bar{p} - d_X\bar{p}] + (1 - d_Y)p_Y \\ &= z + a_Y + b_Y + [z_D - \gamma a_Y b_Y] [2\bar{p} - a_Y - b_Y - \bar{p} (z_D - \gamma a_X b_X)] \end{aligned}$$

and

$$\begin{aligned} V^b &= d_X [z + d_Y\bar{p}] + (1 - d_X)p_X \\ &= z + a_X + b_X + [z_D - \gamma a_X b_X] [\bar{p} (z_D - \gamma a_Y b_Y) - a_X - b_X] . \end{aligned}$$

We can now employ a revealed preference argument to show that at any best response we have $a_Y \geq 0$ for A and $b_X \geq 0$ for B. For A, fix any given (b_X, b_Y) by B and compare the payoff U^a at (a_X, a_Y) where $a_Y > 0$ to that at $(a_X, -a_Y)$. Note that when (a_X, a_Y) is feasible then so is $(a_X, -a_Y)$. The payoff is larger with a_Y if and only if

$$2a_Y \{1 - z_D - \gamma b_Y [2\bar{p} - b_Y - \bar{p}d_X]\} > 0.$$

Since $a_Y > 0$ we need only show the bracketed term is positive. By feasibility, we have $1 - z_D > \gamma\bar{p}^2$ so it is sufficient to show $\bar{p}^2 > b_Y [2\bar{p} - b_Y - \bar{p}d_X]$. Over all $w \in [-\bar{p}, \bar{p}]$, the function $w [2\bar{p} - w - \bar{p}d_X]$ is strictly concave with an interior maximum at $w = \bar{p}(2 - d_X)/2$ where the function assumes its maximum value of $(\bar{p}(2 - d_X)/2)^2$. Our sufficient condition then reduces to $4 > (2 - d_X)^2$, which is clearly valid since we always have $d_X \in (0, 1)$. Thus, A will never choose $a_Y < 0$ in any best response.

The proof that $b_X \geq 0$ in any best response of B is similar and therefore omitted.

The following properties are straightforward to verify:

A1: $\dfrac{\partial U^a}{\partial a_X} = \gamma\bar{p}b_X d_Y \geq 0$ if $b_X \geq 0$ (strict if $b_X > 0$);

A2: $\dfrac{\partial U^a}{\partial a_Y} = 1 - z_D - \gamma b_Y [\bar{p}(2 - d_X) - 2a_Y - b_Y] > 0$ if $b_Y \leq 0$;

B1: $\dfrac{\partial V^b}{\partial b_X} = 1 - z_D - \gamma a_X [\bar{p}d_Y - a_X - 2b_X] > 0$ if $a_X \geq 0$ and $b_X \geq 0$;

B2: $\dfrac{\partial V^b}{\partial b_Y} = -\gamma\bar{p}a_Y d_X \leq 0$ if $a_Y \geq 0$ (strict if $a_Y > 0$).

Building on these properties, we can now show that i) in any best response to (b_X, b_Y) where $b_X \geq 0$, A always chooses such that $a_X \geq 0$, and ii) in any best response to (a_X, a_Y) where $a_Y \geq 0$, B always chooses such that $b_Y \leq 0$. We prove i) and omit the proof of ii), which is similar. There are two cases: $b_X > 0$ and $b_X = 0$. For $b_X > 0$, compare U^a at $a_X > 0$ and at $-a_X < 0$ for given (b_X, b_Y) and a_Y. Then the payoff at $a_X > 0$ is larger if and only if $2\gamma a_X b_X \bar{p} d_Y > 0$. As $b_X > 0$ is given, $a_X > 0$ is by construction, and $d_Y \in (0, 1)$, we are done. For the case of $b_X = 0$, the payoff is independent of a_X since U^a simplifies to $U^a = d_Y [z_Y + 2\bar{p} - z_D\bar{p}] + (1 - d_Y)p_Y$. Now, if $b_Y \leq 0$, then we see from property A2 above that U^a is strictly increasing in a_Y. Hence, $a_Y = \bar{p}$ is optimal and, therefore, $a_X = 0$. If $b_Y > 0$, then observe that $\partial^2 U^a / \partial a_Y{}^2 = 2\gamma b_Y > 0$ and U^a is convex. The optimal choice of a_Y by A must then be one of the endpoints, either $\bar{p}$ or $-\bar{p}$. Comparing U^a at these two choices, we find that $\bar{p}$ is optimal if $\bar{p}^2 > b_Y[\bar{p}(2 - z_D) - b_Y]$. The maximum value for the right-hand side is $[\bar{p}(2 - z_D)/2]^2$, which occurs at $b_Y = \bar{p}(2 - z_D)/2$, and this is clearly less than $\bar{p}^2$. Thus, A will never choose $a_X \leq 0$ in a best response and i) is established.

Summarizing, we have shown that in any equilibrium we necessarily have: $a_X \geq 0$, $a_Y \geq 0$, $b_X \geq 0$, and $b_Y \leq 0$. From this pattern, we now show that all of these inequalities are strict in equilibrium and, furthermore, that each agent does at least e on their dominant proposal, where e is defined by $g(e) = e$, where g crosses the 45° line. Note that $g'(e) = -1$ and $g(e) > \bar{p}/2$.

We begin with B. Since we have $a_X \geq 0$ and $b_X \geq 0$ in equilibrium, property B1 implies that V^b is strictly increasing in b_X. This implies that $b_Y = -g(b_X)$ in any equilibrium. To see why, recall that $b_Y \leq 0$ holds in equilibrium. If we ever had $b_Y < -g(b_X)$ then the slack could be used to increase b_X and this would lead to a strict increase in V^b. Next, substituting with $b_Y = -g(b_X)$ for b_Y in V^b, the resulting variation with b_X is given by

$$\frac{\partial V^b}{\partial b_X} - g'(b_X)\frac{\partial V^b}{\partial b_Y}.$$

At $b_X = 0$, $g'(b_X) = 0$ holds (note that $\partial V^b / \partial b_Y$ is bounded by 1 in magnitude). Since $\partial V^b / \partial b_X > 0$ from B1, we see that B always chooses $b_X > 0$ in equilibrium. Incorporating $b_X > 0$, a similar argument allows us to conclude that $a_Y = g(a_X)$ and that $a_X > 0$ also holds. In turn, we can then show $a_Y > 0$ and $b_Y < 0$.

To show that each of a_Y and b_X exceed e, it is straightforward to substitute with g and reduce each of U^a and V^b to a function of only b_X and a_Y. We can then show that each of these functions is strictly increasing (in the action on the player's dominant proposal) over the interval $[0, e]$. For A we calculate

$$\begin{aligned}
U^a(a_Y, b_X) &= z + a_Y - g(b_X) + [z_D + \gamma a_Y g(b_X)]\,[\bar{p}(2 - z_D) - a_Y + g(b_X) + \gamma \bar{p} g(a_Y) b_X] \\
\frac{\partial U^a}{\partial a_Y} &= [1 - z_D + \gamma \bar{p}(2 - z_D) g(b_X) + \gamma g(b_X)^2] - 2\gamma g(b_X) a_Y \\
&\quad + \gamma \bar{p} z_D b_X g'(a_Y) + \gamma^2 \bar{p} b_X g(b_X)[g(a_Y) + a_Y g'(a_Y)] \\
\frac{\partial^2 U^a}{\partial {a_Y}^2} &= -2\gamma g(b_X) + \gamma \bar{p} z_D b_X g''(a_Y) + \gamma^2 \bar{p} b_X g(b_X)[2g'(a_Y) + a_Y g''(a_Y)] < 0
\end{aligned}$$

Since U^a is concave in a_Y, we need only show that $\partial U^a / \partial a_Y$ is positive at $a_Y = e$ for all $b_X \in [0, \bar{p}]$ to conclude that A chooses $a_Y > e$ in any best response. Evaluating and simplifying, we have

$$\left(\frac{\partial U^a}{\partial a_Y} \right|_{a_Y = e} = 1 - z_D + \gamma[\bar{p}(2 - z_D) g(b_X) + g(b_X)^2 - 2e g(b_X) - \bar{p} z_D b_X]$$

where we have used the properties $g(e) = e$ and $g'(e) = -1$.

Differentiating the above expression with respect to z_D yields $-1 - \gamma\bar{p}[g(b_X) + b_X] < 0$ and, therefore, the expression is bounded below by the value at $z_D = 1 - \gamma\bar{p}^2$, which is the maximum feasible value for z_D. Substituting with $z_D = 1 - \gamma\bar{p}^2$ in the original expression and simplifying, it is then sufficient to show

$$\bar{p}^2 + \bar{p}(1 + \gamma\bar{p}^2) g(b_X) + g(b_X)^2 - 2g(b_X)e - \bar{p}(1 - \gamma\bar{p}^2) b_X > 0$$

This expression is increasing in γ since $\bar{p}^3[g(b_X) + b_X] > 0$ and, therefore, it is bounded below by the value at $\gamma = 0$. It is then sufficient to show

$$\bar{p}^2 + \bar{p} g(b_X) + g(b_X)^2 - 2g(b_X)e - \bar{p} b_X > 0$$

This expression is increasing in $\bar{p}$ since $2\bar{p} + g(b_X) - b_X > 0$ and is therefore bounded below by the value at $\bar{p} = 0$. As a result, it is sufficient to show $g(b_X)[1 - 2e] > 0$. Since feasibility implies $e < \bar{p}$ and $\bar{p} < 1/2$, we are done. This establishes that $\partial U^a / \partial a_Y > 0$ at $a_Y = e$ for all $b_X \in [0, \bar{p}]$.

To show that B always chooses a b_X that exceeds e, we calculate

$$\begin{aligned}
V^b(b_X, a_Y) &= z + g(a_Y) + b_X + [z_D - \gamma g(a_Y) b_X]\,[\bar{p} z_D - g(a_Y) - b_X + \gamma \bar{p} a_Y g(b_X)] \\
\frac{\partial V^b}{\partial b_X} &= [1 - z_D - \gamma \bar{p} z_D g(a_Y) + \gamma g(a_Y)^2] + 2\gamma g(a_Y) b_X \\
&\quad + \gamma \bar{p} z_D a_Y g'(b_X) - \gamma^2 \bar{p} a_Y g(a_Y)[g(b_X) + b_X g'(b_X)] \\
\frac{\partial^2 V^b}{\partial {b_X}^2} &= 2\gamma g(a_Y) + \gamma \bar{p} z_D a_Y g''(b_X) - \gamma^2 \bar{p} a_Y g(a_Y)[2g'(b_X) + b_X g''(b_X)]
\end{aligned}$$

V^b is not necessarily concave in b_X and the proof that $\partial V^b/\partial b_X > 0$ for $b_X \in [0, e]$ and $a_Y \in [0, \bar{p}]$ is more complicated than that for agent A. To begin, differentiating $\partial V^b/\partial b_X$ with respect to z_D yields $-1 - \gamma\bar{p}g(a_Y) + \gamma\bar{p}a_Y g'(b_X) < 0$. Hence, $\partial V^b/\partial b_X$ is decreasing in z_D and, therefore, bounded below by the value at $z_D = 1 - \gamma\bar{p}^2$, the maximum feasible value for z_D. Substituting and simplifying in $\partial V^b/\partial b_X$, it is sufficient to show

$$\bar{p}^2 - \bar{p}(1-\gamma\bar{p}^2)g(a_Y) + g(a_Y)^2 + 2g(a_Y)b_X + \bar{p}(1-\gamma\bar{p}^2)a_Y g'(b_X) - \gamma\bar{p}a_Y g(a_Y)[g(b_X) + b_X g'(b_X)] > 0.$$

We claim this expression is increasing in γ. Differentiating with respect to γ, we need to show

$$\bar{p}^2 g(a_Y) - \bar{p}^2 a_Y g'(b_X) - a_Y g(a_Y)[g(b_X) + b_X g'(b_X)] > 0.$$

This last expression is positive at $b_X = 0$ since $\bar{p}(\bar{p} - a_Y)g(a_Y) > 0$ and it is increasing in b_X since, differentiating with respect to b_X, we have

$$-\bar{p}^2 a_Y g''(b_X) - a_Y g(a_Y)[2g'(b_X) + b_X g''(b_X)] > 0.$$

Thus, we have shown the sufficient condition is increasing in γ.

As a result, the sufficient condition is bounded below by the value at $\gamma = 0$ and, in turn, it is now sufficient to show

$$\bar{p}^2 - \bar{p}g(a_Y) + g(a_Y)^2 + 2g(a_Y)b_X + \bar{p}a_Y g'(b_X) > 0$$

for $b_X \in [0, e]$ and $a_Y \in [0, \bar{p}]$. Observe that this last condition is increasing in $\bar{p}$ since, by differentiation in $\bar{p}$, we have $2\bar{p} - g(a_Y) + a_Y g'(b_X) > 0$, as follows from $g'(b_X) \geq -1$ for $b_X \leq e$. Hence, $\bar{p} = 0$ provides a lower bound for the sufficient condition and we need only show $g(a_Y)^2 + 2g(a_Y)b_X > 0$, which clearly holds. We have thus established that V^b is increasing in b_X for $b_X \in [0, e]$ and that a best response by B will necessarily involve an action above e.

Existence of Equilibrium: we provide a simple pure-strategy existence result. To begin, note that the players have symmetric best-responses to extreme choices. It is straightforward to verify that the best-response of A to $b_X = 0$ is $a_Y = \bar{p}$ and, similarly, that the best-response of B to $a_Y = 0$ is $b_X = \bar{p}$. At the other extreme, the best response of A to $b_X = \bar{p}$ is interior and solves the first-order condition

$$0 = \left(\frac{\partial U^a}{\partial a_Y}\right|_{b_X = \bar{p}} = 1 - z_D + \gamma\bar{p}^2 z_D g'(a_Y).$$

Similarly, in response to $a_Y = \bar{p}$, the best-response problem for B is identical to that of A once we substitute $a_Y = \bar{p}$ in V^b.

As noted above, U^a is concave and A has a continuous best-response function that always exceeds e and is characterized by the unique solution to the first-order condition at any $b_X \in (0, \bar{p}]$, with $a_Y = \bar{p}$ in response to $b_X = 0$. It can be shown that A's best response is decreasing in b_X for $b_X \in [0, e]$ but this need not hold at larger b_X values.

The complication with B is that V^b is not necessarily concave. As is easily verified, V^b is convex in b_X when $a_Y = 0$ and it is concave when $a_Y = \bar{p}$. To proceed, note first that

$$\frac{\partial}{\partial a_Y} \frac{\partial^2 V^b}{\partial {b_X}^2} = 2\gamma g'(a_Y) + \gamma \bar{p} z_D g''(b_X) - \gamma^2 \bar{p}[g(a_Y) + a_Y g'(a_Y)][2g'(b_X) + b_X g''(b_X)] < 0$$

holds for $a_Y \geq e$. To see this, note that the first two terms are clearly negative. For the last term, we know that the function $[g(t) + tg'(t)]$ has value 0 at $t = e$ where $g(e) = e$ and $g'(e) = -1$, with derivative $[2g'(t) + tg''(t)] < 0$. Thus, the last term is negative for $t > e$, and the claim is established. With the second partial declining in a_Y, we know that it will be negative at $(b_X, a_Y) \in [e, \bar{p}]^2$ if it is negative at $a_Y = e$. Hence, for $e \leq t \leq \bar{p}$,

$$\frac{1}{\gamma} \left(\left. \frac{\partial^2 V^b}{\partial {b_X}^2} \right|_{b_X = t, a_Y = e} \right) = 2e + \bar{p} z_D e g''(t) - \gamma \bar{p} e^2 [2g'(t) + tg''(t)] < 0 \qquad \Longleftrightarrow$$

$$2 \frac{1 - \gamma \bar{p} e g'(t)}{\bar{p}[z_D - \gamma \bar{p} e]} < -g''(t)$$

is a sufficient condition for concavity of V^b for $b_X \in [e, \bar{p}]$. Finally, recalling that $e > \bar{p}/2$, and substituting for e in the above expression to bound the numerator from above and the denominator from below, we arrive at the simpler but more stringent condition in Proposition 3. With this sufficient condition in place, we have V^b concave for $e \leq b_X \leq \bar{p}$. As a result, B now has a continuous best-response function, characterized by the solution to the first-order condition. We know from above that every best response of B is above e.

It follows directly from continuity and the common values of A and B in response to 0, e, and $\bar{p}$ that the best-response functions cross each other and an equilibrium exists.$\blacksquare$