

Finding “Lost” Profits: An Equilibrium Analysis of Patent Infringement Damages

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We examine the impact of patent infringement damages in an equilibrium oligopoly model of process innovation where the choice to infringe is endogenous and affects market choices. Under the lost profits measure of damages normally employed by U.S. courts, we find that infringement always occurs in equilibrium with the infringing firm making market choices that manipulate the resulting market profit of the patent holder. In equilibrium, infringement takes one of two forms: a “passive” form in which lost profits of the patent holder are zero and an “aggressive” form where they are strictly positive. Even though the patentee’s profits are protected with the lost profits damage measure, innovation incentives are reduced relative to a regime where infringement is deterred.

1. Introduction

Patent law encourages innovation and the dissemination of knowledge by providing exclusivity in exchange for knowledge disclosures. The value of exclusivity derives from two penalties imposed on infringing parties: injunctions that stop subsequent use and damages in compensation for previous use. Because it is common for litigation to conclude after an infringer has been in the market for some time, expected damages play an important role in establishing incentives for innovation.

Since 1946 U.S. courts have largely adopted a compensatory approach to awarding damages to the patent holder as a result of patent infringement.

[Damages] have been defined by this Court as “compensation for the pecuniary loss he (the patentee) has suffered from the infringement, without regard to the question whether the defendant has gained or lost by his unlawful acts.” They have been said to constitute “the difference between his pecuniary condition after the infringement, and what this condition would have been if the infringement had not occurred.” (*Yale Lock Mfg. Co. v. Sargent*, 117 U.S. 536, 552, 1886).¹

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1. *Aro Mfg. Co. v. Convertible Top Replacement Co.*, 377 U.S. 476 at 507 (1964).

The ideal damage award under this approach is the “lost profits” of the patentee which are determined by calculating patentee profits that would have occurred absent infringement.²

In this article, we examine how this lost profits measure of damages affects competition, infringement, and the incentives for innovation in a market competition between the patentee and a potential infringer. This examination involves determining a reference level for lost profits based on market competition that is consistent with equilibrium competitive choices. Evidentiary and information concerns are suppressed to permit a focus on the implications of the ideal lost profits measure. We address two questions. First, when will damages based on lost market profits deter infringement? Second, if infringement is not deterred, how are innovation incentives impacted by the lost profits approach?

We focus on a process innovation that allows the patentee to lower its costs relative to a noninnovating firm. Given the patent, the noninnovating firm chooses whether to imitate (and risk infringement). The market setting is a critical element in our analysis since the subsequent market outcomes are determined endogenously with behavior incorporating the consequences of the lost profits damage measure. We assume a best case for the enforcement regime: whenever infringement is discovered, the court assesses accurately the associated damages and litigation does not involve transactions costs.³ We then incorporate the equilibrium outcomes for infringement and market competition into a patent race to assess how the lost profit damage measure affects innovation incentives.

Despite the long-standing interest in the question of patent damage measures, except for Schankerman and Scotchmer (2001), there has been little equilibrium analysis of the effects of those measures on market competition and innovation. Our article complements the contribution of Schankerman and Scotchmer by considering endogenous competition with both infringing and noninfringing options. Schankerman and Scotchmer, in contrast, only consider competition through infringement. That perspective is a natural extension of their analysis of vertical licensing relationships—the primary concern of their article—and is often appropriate for horizontal settings. But when a noninfringing option exists, the logic underlying the finding of no infringement of Schankerman and Scotchmer breaks down and infringement can result. We explain the difference in results and contrast the damage measures in Section 7.⁴

2. When such calculations cannot be reasonably made, damages are based on a calculation of royalties on infringer's actual sales. See Section 7.2 for an analysis of this case.

3. We do, however, allow for uncertainty in whether the infringement is detected. This allows for weak versus strong patent protection rights and, thus, a relative assessment of incentives in the benchmark case of perfect detection.

4. Our purpose is to illustrate how analyzing market structure illuminates the lost profits legal damages. In Section 7.2, we analyze alternative damage approaches (reasonable royalties and disgorgement of infringer profits) and find similar results. See also Kaplow and Shavell (1996), Blair and Cotter (1998) and Schankerman and Scotchmer (2001) for comparisons of various liability and damage approaches.

In the presence of even a weak noninfringing substitute, we find that a damage measure based exclusively on lost profits of the patentee (as a result of infringement) and excluding lost licensing revenues always results in infringement in equilibrium.⁵ Infringement occurs because a noninnovating firm always has the strategic option of passive infringement in which the patent is imitated, but market choices are made to avoid lost profits (e.g., the imitator mimics choices associated with the noninfringement outcome and takes profit gains only via lower costs). By definition, the patent holder suffers no lost profit, and so the noninnovating firm strictly prefers to infringe. In some cases, infringement takes a more aggressive form where the noninnovating firm and patentee choose market positions that push lost profits to a strictly positive level.

In equilibrium, under both forms of infringement the patentee receives the same net profits as if no infringement had taken place. Thus, one might expect that the lost profits measure preserves the incentive (reward) for innovative efforts. We find, however, that basing damages on lost profits reduces the incentive to innovate relative to the benchmark case (no infringement). The explanation lies with the effect of the damage measure on infringer payoffs. In equilibrium, infringement always occurs and, at a minimum, a loser (noninnovating firm) in a patent race will have a (valuable) passive infringement option. Thus, as *ex ante* innovation incentives are based on the profit differences between being the patentee and being the infringer, overall innovation incentives will be reduced.

We present the model in Section 2. Patent holder and imitator incentives are examined in Sections 3 and 4, respectively, and the equilibrium market outcomes and infringement choices are derived in Section 5. We then examine innovation incentives in a patent race in Section 6 and conclude in Section 7 with a discussion of our results and an extension of the results to alternative damage rules. Proofs are in the Appendix.

2. The Model

Our model consists of an innovator with a patented cost-reducing process innovation, firm i , and a potential infringer (imitator), firm j . Both firms produce a homogeneous good but with potentially different costs. Market competition is concluded before infringement damages, if any, are awarded to the patent holder. The firms are risk neutral and maximize expected profits. We focus on a strategic setting where quantities are chosen simultaneously.⁶ Prior to

5. U.S. courts generally exclude lost licensing revenues in their calculation of lost profits. Including lost licensing revenues reduces the relative attractiveness of the infringement choice versus the no infringement, licensing alternative and infringement will no longer always obtain.

6. Similar results obtain if quantity choice is sequential because the imitator still has the option of passive infringement. One could also formulate the analysis in terms of price setting, differentiated goods, and a product innovation.

innovation, the status quo has the two firms competing with constant marginal costs of $\bar{c}$ in a market with linear demand⁷

$$P(q) = \alpha - \beta q. \quad (1)$$

Thus, the prior status quo involves the traditional Cournot equilibrium outcome.

Now suppose that firm i has obtained a patent for an innovation that allows it to produce at cost c , where $c < \bar{c}$. Firm j has the option of remaining with the old technology and producing at cost $\bar{c}$ without any risk of infringement. Denote the option for j of no imitation by $\mathcal{N}$. When j chooses $\mathcal{N}$, we have quantity competition between firm i at cost c and firm j at cost $\bar{c}$; let $\pi_i^{\mathcal{N}}$ and $\pi_j^{\mathcal{N}}$ denote the resulting profit outcomes for the firms. In the calculation of lost profits, these are the reference profit levels that correspond to the hypothetical involving the market outcome in the event that no infringement had occurred.

Firm j also has the option, denoted by $\mathcal{I}$, of imitating firm i 's patented innovation. However, this entails a risk that the court will find infringement. We assume that imitation allows firm j to reduce costs relative to the prior technology and produce at cost s , where $s < \bar{c}$. A special case involves perfect imitation (where $c = s$), but allowing for cost differences ($s \leq c$) makes it possible to identify the different incentives of the two firms and the results for the special case follow directly from the more general analysis.⁸

If the court finds infringement, the penalty requires that the infringer make a monetary payment to firm i so that firm i earns a net payoff equal to that which would have occurred had no infringement taken place. The penalty is related to market events as follows. Given a choice of $\mathcal{I}$ by firm j , the firms make quantity choices of q_i and q_j , respectively, for i and j . The resulting market price of P is from equation (1), and firm i has a realized market payoff of $(P - c)q_i$. If this is less than $\pi_i^{\mathcal{N}}$, then firm j must pay i the difference. If not, then no damage penalty is assessed. Thus, the damage payment is given by $D(q_i, q_j) \equiv \max\{\pi_i^{\mathcal{N}} - (P - c)q_i, 0\}$. Our penalty assumption corresponds to a lost profits calculation where licensing revenues are not included or would be zero. Where licensing revenues are included, the underlying economics are similar, though more complicated.

We assume that the court finds infringement with probability γ when j chooses $\mathcal{I}$. In practice, the lack of perfect enforcement arises for a number of reasons, including (a) firm j may be able to circumvent the patent, (b) the court may find the patent is invalid, and (c) infringement is not detected.⁹ The size of

7. We assume $\alpha > 2\bar{c}$ so that both firms are active (positive output). The assumption is sufficient to avoid corner cases for outputs in equilibrium outcomes and is easily relaxed.

8. With $s < c$, the follower implements the innovation more effectively than the innovator. A number of studies (e.g., Schnaars 1994) report that second movers (e.g., non-patent holders who imitate) often achieve market dominance.

9. One can argue that the probability is related to the underlying extent of innovation and imitation, so that γ depends on c and s . Also, penalties such as treble damages can be incorporated into γ . As we note below, such relationships are easily incorporated into the analysis. See Lemley (2001) for a general discussion of the patent system.

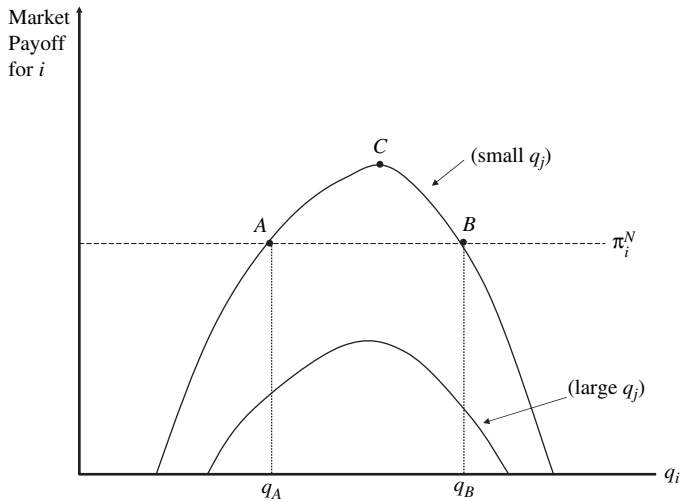


Figure 1. Patent Holder Market Payoff and Lost Profit.

γ effectively indexes the strength of property rights for the patent holder, and we can expect that small values of γ will make $\mathcal{I}$ relatively more attractive for firm j . Given a choice of $\mathcal{I}$, the expected payoffs for each firm are given by

$$\Pi_i(q_i, q_j) = (P - c)q_i + \gamma D \quad (2)$$

$$\Pi_j(q_j, q_i) = (P - s)q_j - \gamma D, \quad (3)$$

at quantity choices q_i and q_j , the market price of $P = P(q_i + q_j)$ and lost profits damages of $D = D(q_i, q_j)$. Given a choice of $\mathcal{N}$, the firms earn $\pi_i^{\mathcal{N}}$ and $\pi_j^{\mathcal{N}}$. We solve for a subgame-perfect equilibrium in which j chooses between $\mathcal{N}$ and $\mathcal{I}$, and then the firms simultaneously choose quantities for market competition. Finally, the court makes an infringement determination with payoffs and damages determined via equations (2) and (3).

3. Market Incentives of the Patent Holder

How should the patent holder (innovator), firm i , choose quantity given that firm j has chosen to imitate? We might expect this to depend on the strength of the patent, indexed by γ , as well as the output expected from j . As a best-response problem, however, the choice of firm i turns out to have a simple strategic structure. The payoff in equation (2) reveals that i is always led to choose a quantity that maximizes the realized market payoff of $(P - c)q_i$ in response to any expected output choice by j . Thus, the prospect of lost profits has no direct impact on the market choice of the patent holder.

Refer to Figure 1. There are two situations for firm i with respect to firm j . If j produces at a relatively high level, then the market payoff for firm i is always below the reference profit for no infringement; this is the lower curve in Figure 1. In this case, lost profits are always positive and equation (2) reduces

to maximizing $(1 - \gamma)$ times the market payoff and γ does not matter for the optimizing choice. In this case, the best response of firm i is easily verified to be $q_i = (\alpha - c - \beta q_j)/(2\beta)$ provided the interior term is positive (and zero when it is not). When infringement is proven, firm i always gets π_i^N , so it is optimal to choose quantity to maximize profits when infringement is not proven. Hence, the simple Cournot best response without regard to damages is optimal.

If j produces at a relatively low level, then we have the situation depicted with the upper curve in Figure 1. Now, depending on i 's response, lost profits may be positive or zero (recall the absolute value restriction on payments from j to i). If i produces below q_A or above q_B , then lost profits are strictly positive and, as before, equation (2) reduces to the (scaled) market payoff. For these quantity cases, i 's payoff is largest at q_A and q_B (both yield π_i^N), so output choices below q_A or above q_B are never a best response for i . For $q_i \in (q_A, q_B)$, the market payoff exceeds π_i^N , and we have $D = 0$ by the definition of lost profits damages. Now equation (2) is identically equal to the market payoff, and hence, an interior choice at point C in Figure 1 is optimal. As before, this reduces to $q_i = (\alpha - c - \beta q_j) / (2\beta)$, and from a strategic point of view, firm i does not respond directly to patent strength (γ) or lost profits.

We formalize this argument as follows. The reference profit level of π_i^N corresponds to the standard Cournot equilibrium outcome for firm i with cost c and firm j with cost $\bar{c}$, and this is given by

$$q_i^N = \frac{1}{3\beta}(\alpha - 2c + \bar{c}) \quad \text{and} \quad \pi_i^N = \beta(q_i^N)^2, \quad (4)$$

$$q_j^N = \frac{1}{3\beta}(\alpha - 2\bar{c} + c) \quad \text{and} \quad \pi_j^N = \beta(q_j^N)^2. \quad (5)$$

Then, we calculate that the maximum value for market profit is above π_i^N whenever q_j is below q_j^N (as is the case with point C in Figure 1). However, as have seen, firm i 's best response is driven by the market payoff, and it does not depend on whether lost profits are positive or zero. Thus, we have

Lemma 1. The best response of firm i to q_j by firm j is given by $\phi_i(q_j) = (\alpha - c - \beta q_j) / (2\beta)$ for all $q_j \leq (\alpha - c) / \beta$ and by $\phi_i(q_j) = 0$ for larger q_j .

Note that the strength of patent rights (γ) and the lost profit reference level have no direct effect on the patent holder's market quantity choice. Instead, the patent holder's objective always reduces to maximizing the market payoff (pure or scaled). Consequently, i 's market decision is only affected indirectly via j 's quantity choice.

4. Market Incentives of the Imitator

The strategic situation of the imitator (infringer) is considerably more complex because of the asymmetry in the payoff functions. Whether lost profits are

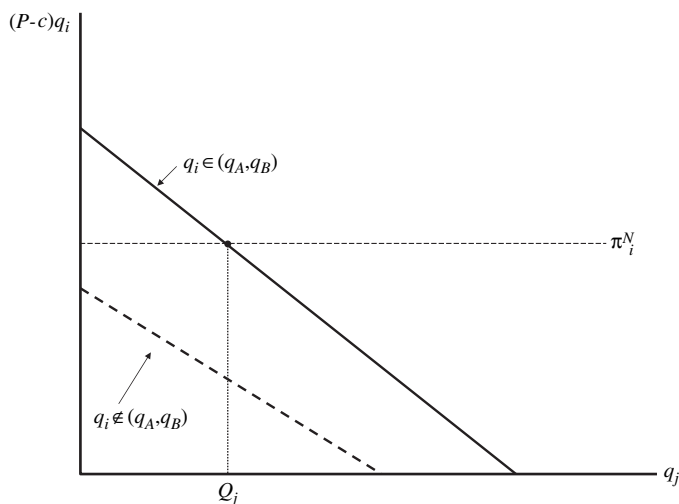


Figure 2. Imitator Output and Lost Profit.

positive or zero, the patent holder's payoff always reduces to a multiple of the market payoff. In contrast, the imitator's payoff involves his own market payoff as well as that of the patent holder whenever lost profits are positive. Moreover, the market choices of both players determine when lost profits are positive. This creates a number of subtle strategic effects.

We begin by analyzing the imitator's market incentives when lost profits are positive. Referring back to Figure 1, consider the critical points q_A and q_B where the market payoff for i crosses the reference profit of π_i^N . Fix j 's output at zero for the moment and consider the value of $[P(q_i) - c]q_i$, which corresponds to a standard monopoly payoff function. This is maximized at the monopoly output level of $q^M \equiv (\alpha - c)/(2\beta)$ with corresponding payoff $\pi^M \equiv (\alpha - c)^2/(4\beta)$. With $c < \bar{c}$, we know that $\pi^M > \pi_i^N$ holds. Thus, we can solve for the critical points q_A and q_B where i 's payoff when $q_j = 0$ crosses π_i^N :

$$q_A = \sqrt{\frac{\pi^M}{\beta}} - \sqrt{\frac{\pi^M - \pi_i^N}{\beta}} \quad \text{and} \quad q_B = \sqrt{\frac{\pi^M}{\beta}} + \sqrt{\frac{\pi^M - \pi_i^N}{\beta}}. \quad (6)$$

Consider the imitator's best-response problem if $q_i \leq q_A$ or if $q_i \geq q_B$. Then, as shown with the dashed line in Figure 2, the market payoff for i is necessarily below π_i^N for any choice of q_j :

$$\begin{aligned} [P(q_i + q_j) - c]q_i &= (\alpha - \beta q_i - c)q_i - \beta q_j q_i \leq (\alpha - \beta q_i - c)q_i \quad \text{as } q_j \geq 0, \\ &\leq \pi_i^N \quad \text{as } q_i \notin (q_A, q_B) \quad \text{and} \quad \pi^M > \pi_i^N. \end{aligned}$$

For $q_i \notin (q_A, q_B)$, lost profits are necessarily positive for any (positive) output choice by the imitator.

In contrast, for $q_i \in (q_A, q_B)$, there is a unique corresponding output level for j , denoted by Q_j for the upper solid line in Figure 2, at which lost profits cease being zero and become positive. Solving, we find

$$Q_j(q_i) = \frac{\alpha - c}{\beta} - \frac{\pi_i^N}{\beta q_i} - q_i, \quad \text{for } q_i \in (q_A, q_B). \quad (7)$$

We define Q_j to be zero outside of the interval (q_A, q_B) . The basic properties of Q_j follow directly from equation (7) and are summarized in

Lemma 2. For $q_i \in [q_A, q_B]$, the function $Q_j(q_i)$ satisfies (a) $Q_j(q_A) = Q_j(q_B) = 0$, (b) $Q_j(q_i)$ is strictly concave, (c) $Q_j(q_i)$ has a unique maximum at $q_i = q_i^N$ and $Q_j(q_i^N) = q_j^N$, and (d) the function $q_i + 2Q_j(q_i)$ is strictly increasing for $q_i \in [q_A, q_i^N]$.

Consequently, in this case the imitator can determine through its output choice whether lost profits are zero or, when positive, how large they are. With these preliminary observations in place, we now solve for the best-response function of the imitator.

The first case is that of $q_i \notin (q_A, q_B)$. Since $D > 0$ for all $q_j \geq 0$, we have from equation (3) that $\Pi_j(q_j, q_i) = (q_j + \gamma q_i)[\alpha - \beta q_i - \beta q_j] - s q_j - \gamma c q_i - \gamma \pi_i^N$, upon substituting for P . This is a strictly concave function in q_j , and from the first-order condition, we find the unique optimal choice of

$$\varphi_j(q_i) \equiv \frac{1}{2\beta}[\alpha - s - \beta(1 + \gamma)q_i] \quad \text{if } q_i < \frac{\alpha - s}{\beta(1 + \gamma)} \quad (8)$$

and $\varphi_j(q_i) \equiv 0$ for larger q_i . Thus, as long as the patent holder's output does not force j from the market, the impact of the lost profit penalty depends on the size of γ . As γ rises and property rights become more secure, the imitator becomes more "timid" and reduces output. When property rights vanish, at $\gamma = 0$, $\varphi_j(q_i)$ reduces to the standard Cournot best-response function.

The second case arises when $q_i \in (q_A, q_B)$. A difficulty lies with determining whether j will find it profitable to produce aggressively, thereby inducing a positive lost profits penalty, or keep output low, thereby holding lost profits to zero. As Figure 2 suggests, the output level of $Q_j(q_i)$ is critical for this choice. Formally, we have

Lemma 3. Suppose that $q_i \in (q_A, q_B)$. If $q_i \geq (\alpha - s)/\beta$, then the best response of j is zero. If $q_i < (\alpha - s)/\beta$, then the best response of j is positive and satisfies (a) if $(\alpha - s)/\beta \leq q_i + 2Q_j(q_i)$, then the best response of j is $\varphi_j(q_i) \equiv \frac{1}{2\beta}[\alpha - s - \beta q_i]$, and $D = 0$ at (q_i, q_j) ; (b) if $q_i + 2Q_j(q_i) \leq (\alpha - s)/\beta \leq (1 + \gamma)q_i + 2Q_j(q_i)$, then the best response of j is $Q_j(q_i)$, and $D = 0$ at (q_i, q_j) ; (c) if $(1 + \gamma)q_i + 2Q_j(q_i) \leq (\alpha - s)/\beta$, then the best response of j is $\varphi_j(q_i)$, and $D > 0$ at (q_i, q_j) .

5. Equilibrium Market Outcomes and Infringement

We now characterize the market equilibrium choices for output, given that j has chosen to imitate and risk infringement. Then we examine j 's equilibrium infringement choice.

Proposition 1. Suppose j chose $\mathcal{I}$. If $\varphi_j(q_i^N) > q_j^N$, then the unique equilibrium outcome (given imitation) is at (q_i^*, q_j^*) , where $q_j^* = \varphi_j(q_i^*)$ and $q_i^* = \phi_i(q_j^*)$. Lost profits are strictly positive in equilibrium.

In this case, the lost profit penalty is not sufficient to deter the imitator from driving the patent holder's market profit below the no-infringement reference level. This is an aggressive form of infringement. Solving the equations $q_j^* = \varphi_j(q_i^*)$ and $q_i^* = \phi_i(q_j^*)$, we find

$$q_i^* = \frac{1}{\beta(3-\gamma)}(\alpha - 2c + s),$$

$$q_j^* = \frac{1}{\beta(3-\gamma)}[\alpha(1-\gamma) - 2s + (1+\gamma)c].$$

The imitator produces a large quantity and the patent holder responds by reducing output (relative to the no-infringement outcome of $\mathcal{N}$). On balance, the patent holder's market payoff falls below the reference level and lost profits are strictly positive.

Proposition 2. Suppose j chose $\mathcal{I}$. If $\varphi_j(q_i^N) \leq q_j^N$, then the unique equilibrium outcome (given imitation) is at (q_i^*, q_j^*) , where $q_j^* = q_j^N$ and $q_i^* = q_i^N$. Lost profits are zero in equilibrium.

In this case, we have a passive form of infringement. In fact, the patent holder produces as if no infringement occurred. The imitator, however, is infringing and producing at cost s rather than cost $\bar{c}$. The choice of output by the imitator is specifically set at the level which induces i to respond at the reference output q_i^N . In other words, j produces at its own reference level for no infringement, namely, q_j^N . By doing so, no lost profits are generated. Instead, the imitator takes the gain from infringing completely in the form of reduced production costs for output q_j^N . Of course, the patent holder has a payoff of π_i^N in equilibrium for both cases.

Figure 3 illustrates the two cases for the equilibrium outcome. When φ_j is large, as with the upper solid line, the equilibrium is at point A, where φ_j and ϕ_i intersect. When it is small, the equilibrium is always at point B, where the level of lost profits is at zero.

An important question concerns which of these two cases applies in relation to the underlying structural parameters of the model: property rights (γ), the level of innovation (c), and the efficacy of imitation (s). From Propositions 1

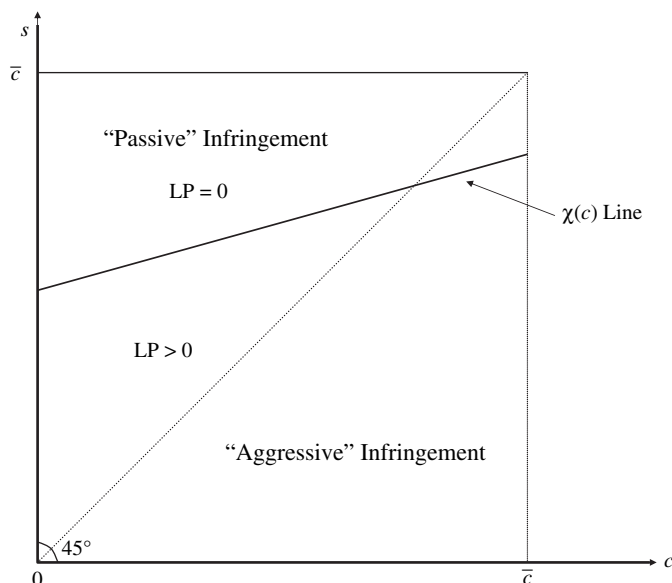


Figure 4. Equilibrium Infringement Outcomes.

imitation is always profitable. This is because j pursues the passive infringement strategy to produce the same output and receive the same market price as with no imitation, but production costs are lower and there is no lost profit penalty.

Infringement is also profitable when lost profits are positive in equilibrium. Lost profits are positive in equilibrium when j increases quantity above the passive infringement quantity. Since j can always generate profit improvement using passive infringement, aggressive infringement will only be used if it provides yet greater rents. Therefore, no matter what the strength of property rights, the lost profits damage criterion will necessarily trigger infringement. Formally, we have

Proposition 4. The imitator earns strictly greater profits from $\mathcal{I}$ than from $\mathcal{N}$. Thus, in equilibrium, the imitator always chooses to infringe.

6. Innovation Incentives

An objective of the patent system and of intellectual property rights regimes more generally is to encourage innovation. At the same time welfare is affected by market allocation given innovation. Determining the optimal damage measure thus depends on the effect of the measure on both dynamic and static competition.¹¹ Because there is little consensus on the right way to model

11. Furthermore, such an assessment would need to examine innovation incentives that lie outside of or are substitutes for the legal system (see, e.g., Boldrin and Levine 2003).

innovation, we address a more modest question: how do our results for the lost profit damage measure relate to innovation incentives?¹²

A natural benchmark is to consider innovation incentives relative to a setting in which infringement is completely deterred and the patent holder earns the reference payoff of π_i^N . In the equilibrium under lost profits damages, we always have infringement by the imitator but, significantly, whether infringement is passive or aggressive, the patent holder continues to earn an equilibrium payoff of π_i^N . With the same reward to a patent, it is tempting to conclude that innovation incentives are not distorted relative to the benchmark.

This intuition, however, is misleading. Although the reward to innovating is the same, the reward to not innovating is different. Specifically, the incentive to invest in R&D will be affected by the prospect of the reward to “failure,” namely, the option to imitate and infringe on a patented innovation. In equilibrium, this option always has positive value (Proposition 4). Thus, relative to the benchmark of no infringement, the lost profits damage criterion creates a free-rider incentive. Failure has its reward too.

Innovation incentives can be explored in a variety of R&D contexts. Let us examine the incentive to innovate in a standard “memoryless” (Poisson) patent race in continuous time with two ex ante symmetric firms, $k = 1, 2$, and interest rate r (our treatment is a variation on Reinganum [1983]). Each firm invests at the expenditure rate x_k and succeeds at innovation with instantaneous probability $h(x_k)$; the Appendix develops further the technical requirements. In the absence of a success by either firm, each earns the status quo flow profit of $\bar{\pi} \equiv (\alpha - \bar{c})^2 / (9\beta)$. The first to achieve success, denoted by firm i , patents the innovation, assumes the role of the patent holder, and earns a payoff with present discounted value of V_i . The other firm becomes the imitator, denoted by firm j , and receives a discounted payoff of V_j . In the benchmark case, we have $V_i = \pi_i^N / r$ and $V_j = \pi_j^N / r$. The question of innovation incentives can be examined by computing the comparative static of equilibrium R&D with respect to V_j .

The intertemporal payoff to R&D of x_1 when the rival is at x_2 is given by

$$\begin{aligned} U_1(x_1, x_2) &= \int_0^\infty [h(x_1)V_i + h(x_2)V_j + \bar{\pi} - x_1] e^{-rt} e^{-[h(x_1)+h(x_2)]t} dt \\ &= \frac{h(x_1)V_i + h(x_2)V_j + \bar{\pi} - x_1}{r + h(x_1) + h(x_2)}, \end{aligned}$$

as follows from standard reasoning in patent-race models. It is straightforward to show that a unique equilibrium exists and is symmetric. The comparative static result is then

Proposition 5. The equilibrium level of R&D in the patent race is decreasing in V_j , the payoff to the imitator.

12. Ayres and Klemperer (1999) suggest a method for incrementally improving the tradeoff between dynamic and static efficiencies by inducing some limited infringement through increased use of ex post “make whole” damages over preliminary injunctions.

To interpret Proposition 5, note that lost profit damages involve an increase in V_j relative to the benchmark setting of no infringement. By increasing the payoff of the firm that fails to patent, R&D incentives are reduced in the patent race and both firms invest less in R&D.¹³ To map the payoffs of V_i and V_j into the lost profits model, the simplest interpretation is that the duration of the infringement fight effectively runs for the market life of the innovation. Then, firm i continues to earn $V_i = \pi_i^N/r$ while firm j earns $V_j = \Pi_j(q_j^*, q_i^*)/r > \pi_j^N/r$.¹⁴

Proposition 5 is specific to a patent race or tournament structure. Consider instead a pure nontournament structure in which only one firm can invest in patentable cost-reducing R&D. Noting that the payoff π_i^N in equation (4) rises as costs fall, it is clear that the investment incentives of the firm under lost profits damages and under full deterrence of infringement are the same. An interesting extension would be to explore R&D incentives in a setting involving both tournament and nontournament dimensions.

7. Discussion

7.1 The Lost Profits Reference Level and Licensing

In this article, we illustrate how damages based on lost profits in the market lead to infringement. The critical element provided by an explicit market structure is that an infringer has the (valuable) strategic option of choosing market actions that are designed to manipulate the resulting equilibrium level of lost market profits. This approach best captures situations in which the innovator and imitator are horizontal competitors and for antitrust reasons are, therefore, likely to have significantly limited or possibly even completely foreclosed licensing alternatives.

If licensing were feasible and easily negotiated, arguably the appropriate lost profits measure would be the greater of the innovator's market profits from exclusive use and market profits plus license revenues from sharing the patent with the imitator.¹⁵ With respect to the latter, Schankerman and

13. A similar comparative static holds in an alternative model involving a two-stage game where firms invest in period 1 and then receive $\bar{\pi}$, V_i , or V_j in period 2. Also, Reingnaum (1982) reports a similar comparative static for follower payoffs in a patent race with knowledge accumulation and imperfect patents (imitation).

14. One can extend this in a number of ways. For instance, imagine that the infringement suit is resolved over a period of time T during which the firms earn payoffs of π_i^N and $\Pi_j(q_j^*, q_i^*)$. At T , the payoffs switch to π_i^N and π_j^N with probability γ (j is found to have infringed) and to $(\alpha - 2c + s)^2/(9\beta)$ and $(\alpha - 2s + c)^2/(9\beta)$ with $1 - \gamma$ (no infringement).

15. A basic licensing option, which typically invokes less antitrust concern than would unconstrained licensing, involves a lump sum payment in return for an unrestricted right to employ the patented technology in production. Suppose such a licensing negotiation occurs in our model prior to the infringement choice. Under such a license, competition would result in the standard Cournot outcome (with the patent holder at cost c and the noninnovator at cost s). It is straightforward to show that conflicting incentives will often make a mutually acceptable license payment infeasible. For an example, take demand with $\alpha = \beta = 1$, perfect infringement detection with $\gamma = 1$, and costs with $\bar{c} = 1/4$ and $c = s = 1/8$. Note that it is also possible to (at least theoretically) avoid antitrust issues by setting a per-unit license so as to ensure that prices do not increase because of the license. For more on licensing and market structure, see Anand and Khanna (2000), Gans and Stern (2000), and Hoppe et al. (2006).

Scotchmer (2001) point out that appropriate license revenues cannot be pinned down as they depend on legal damages which, in turn, depend on licensing revenues. From this perspective, our measure might appear to be at the low end of the feasible interval of lost profits since we explicitly exclude possible licensing revenues. But our measure can be interpreted as consisting of the expected market profit from sharing plus an implicit license. Our reference level, exclusive-use profits under airtight property rights, is larger than expected (no license) profits under weaker property rights. The difference between the two profit levels can then be viewed as a profit from an implicit license. For example, when γ , the parameter in our model that captures the strength of property rights, is considerably below one, our reference profit, π_i^N , exceeds the no-license expected profit level and includes a healthy license fee.

By contrast, the upper end of the lost profit reference level includes a sharing of revenues from a license based on both efficient production and the coordination of output.¹⁶ This reference level is employed by Schankerman and Scotchmer to explore vertical relationships. When their main model is adapted to horizontal relationships, they find that imitators never infringe (off-the-equilibrium path) a result directly opposite to what we find. What accounts for this difference?

The difference in reference levels—including increased gains to licensing—decreases the relative attractiveness of infringement but is not sufficient to always deter infringement. It is easy to construct examples using the S-S reference level in which infringement will occur when an imitator is more effective at implementing the innovation than the innovator ($s < c$) or when licensing might otherwise have occurred (so that $s = c$), but with competitively determined output.¹⁷

An important difference is that the S-S model and our model analyze different cases. The S-S model does not allow for noninfringing competition by the imitator ($q_j^N \equiv 0$ in our setting), effectively making the imitator's non-infringement profit and quantity equal to zero.¹⁸ Our model allows for non-infringing competition so that some imitator production is the fallback. Thus, the innovator's "but for" infringement reference level is based on profits obtained through an asymmetric duopoly competition, not through "monopoly" use. This difference allows the imitator some strategic latitude to choose a quantity that avoids damages by maintaining the innovator's but for profit

16. For example, suppose the patent involved a trivial cost reduction in a setting with horizontal competitors. Then the whole of the but for profit increase comes from a reduction of competition through a coordinated output reduction. It is just this type of possibility that the antitrust laws on licenses between competitors circumscribe.

17. An example of equilibrium infringement when the lost profits reference level includes foregone licensing revenues is available from the authors.

18. In the S-S model, the profits of each firm are treated as parameters. See Aoki and Hu (1999) for an analysis of the impact of litigation on licensing and innovation and Baker and Mezzetti (2001) for an analysis of joint ventures and enforcement.

level while still benefiting from a decrease in production costs. Such passive infringement is ruled out by assumption in S-S.¹⁹

Both our model and the S-S model focus on the implications of lost profit damage rules under ideal conditions where the but for level of profit is known. In practice, evidentiary considerations and murkiness in the appropriate application of economics to the creation of a hypothetical benchmark have led courts to be flexible regarding patent damage awards.²⁰ Historically, lost profits have been awarded without allowing for the price-reducing effects of competition from an infringer. A number of recent decisions, however, have explicitly incorporated this notion of “price erosion” in lost profit damages calculations.²¹ Note also that courts calculating lost profit damages typically focus on an exclusive-use reference profit and not on a more expanded reference that includes licensing.

7.2 Alternative Damage Measures: Reasonable Royalties and Disgorgement

Under conditions where the but for hypothetical is inherently difficult to establish, use of lost profits has been discouraged by the courts in favor of reasonable royalties in which the patent holder is awarded a royalty on the sales of the infringing product.²²

Our basic infringement results are robust to a version of reasonable royalty damages in which the damages consist of some fraction of the cost savings attributable to using the patented innovation times the number of infringing

19. S-S’s no-infringement result adapted to our setting appears to require the assumption that joint infringement profits (before damage payments) under competition are less than the no-infringement profits of the innovator and the imitator. Under passive infringement and a cost-reducing technology, this assumption will not hold. See our Propositions 1 and 2.

20. Merges (1997, 1080–1) notes that “the trend in patent law damages since the 1980s has been to allow patentees more and more latitude to describe the second-order effects of the infringer’s entry and presence in the market for the patented good. . . . Almost all of these damages require the patentee to spin a narrative entitled ‘what life would have been like without the infringer.’ . . . By the same token, it would seem self-evident that courts should invite evidence of second-order responses by infringers under the (increasingly ornate) hypothetical scenarios being spun by patentees. . . .”

21. The inclusion of price erosion has been accepted by the courts when there is evidentiary support for price erosion. For example, the U.S. Federal Circuit noted that “lost sales and price erosion damages are inextricably linked. . . .” *Crystal Semiconductor Corp. v. Trittech Microelectronics International Inc.*, 246 F.3d 1336 (Fed. Cir. 2001) and earlier that “the award of damages [incorporating price erosion] was consistent with our precedent.” *In re Mahurkar* 71 F.3d 1573 at 1576 (Fed. Cir. 1995). Note that our infringement and decreased innovation results are robust to the absence of price erosion because the passive infringement option still remains.

22. When lost profits cannot be satisfactorily estimated, Section 284 of the Patent Act specifies that damages will be no less than that calculated based on reasonable royalties. *Panduit Corp. v. Stahl Bros. Fibre Works Inc.*, 575 F.2d 1152 (6th Cir. 1978) laid out four conditions necessary for damages to be awarded based on lost profits. When these conditions are not met, courts typically use a royalties-based approach for assessing damages. In recent years, however, the Federal Circuit has shown some willingness, where evidentiary problems are less severe, to deviate from the strict conditions of Panduit though some recent precedent is still “not reconciled.” *Micro Chemical, Inc. v. Lextron, Inc.*, 318 F.3d 1119 (Fed. Cir. 2003).

units.²³ Under this reasonable royalty rule, damages are a fraction ρ of the cost savings times the quantity of infringing units or $D_R = \rho(\bar{c} - s)q_j$. If j chooses to infringe, then the payoffs to i and j at quantities q_i and q_j , respectively, are given by

$$\Pi_i^R(q_i, q_j) = [P - c]q_i + \gamma\rho(\bar{c} - s)q_j,$$

$$\Pi_j^R(q_i, q_j) = [P - (1 - \gamma\rho)s - \gamma\rho\bar{c}]q_j.$$

Since q_i has no direct impact on the damage payment, firm i always chooses a quantity that maximizes the realized market payoff. Thus, the patent holder's best response is again given by $q_i = \phi_i(q_j) = (\alpha - c - \beta q_j)/(2\beta)$, as in Lemma 1. For the imitator, observe that D_R implies that j 's payoff coincides with that of a Cournot duopolist who has a cost of $\hat{c} \equiv (1 - \gamma\rho)s + \gamma\rho\bar{c} < \bar{c}$. Thus, j 's best response follows $q_j = (\alpha - \hat{c} - \beta q_i)/(2\beta)$. The economic intuition for the resulting outcome is then clear. Relative to the $\mathcal{N}$ outcome with i at cost c and j at cost $\bar{c}$, the only difference under R is that there is an outward shift in the best response of the imitator due to $\hat{c} < \bar{c}$. As a result, j will increase quantity while i reduces quantity in the resulting equilibrium. Solving the best-response conditions simultaneously, it is straightforward to verify that

$$q_i^R = \frac{1}{3\beta}(\alpha - 2c + \hat{c}) < q_i^{\mathcal{N}},$$

$$q_j^R = \frac{1}{3\beta}(\alpha - 2\hat{c} + c) > q_j^{\mathcal{N}}.$$

For profits, we have $\Pi_j^R(q_i^R, q_j^R) = \beta(q_j^R)^2 > \beta(q_j^{\mathcal{N}})^2 = \pi_j^{\mathcal{N}}$, and it is always optimal for the imitator to infringe. Essentially, infringement provides an option to operate with lower costs, and, in equilibrium, the reasonable royalty damage payment only partially offsets the resulting cost gains at the resulting market price and quantities. The resulting profit impact on the patent holder, however, is more subtle. While the market price and q_i^R both decline relative to $\mathcal{N}$, implying a decline in market revenue, the damage payment enters as an offsetting positive term. When $\gamma\rho$ is sufficiently small, j is sufficiently aggressive that we have $\Pi_i^R(q_i^R, q_j^R) < \pi_i^{\mathcal{N}}$. In this case, innovation incentives are necessarily reduced since the reward to innovation now falls while, as before, the reward to failure rises. For larger $\gamma\rho$, however, the reduced innovation incentive is partially mitigated since $\Pi_i^R(q_i^R, q_j^R) > \pi_i^{\mathcal{N}}$.

The reasonable royalty fallback directly impacts the problem posed by an infringer's strategic response to damages based on the pure market lost profits discussed above. If the royalties fallback was developed primarily to make the damage calculation simpler, it clearly has a salutary effect in partially patching

23. This method was employed, for example, in *Hanson v. Alpine Valley Ski Area, Inc.*, 718 F.2d 1075 (Fed. Cir. 1983) which upheld a magistrate's award of 1/3 of the savings from the patented method times the expected use. Relative to lost profits, such a reasonable royalty calculation requires less information.

a loophole left by a pure market lost profits doctrine. Since reasonable royalties typically leave an infringer with positive incremental profits, our notion of passive infringement extends to this situation with the reinterpretation that, instead of free infringement, the passive infringer is choosing a favorable (implicit) licensing deal over aggressive infringement with its lost profit potential damages.

Our model can also be adapted to an alternative approach based on disgorgement of infringer profits (see, e.g., the discussion in Blair and Cotter [1998]).²⁴ Disgorgement, like lost profits, is determined with respect to a reference level of profits absent infringement, though with disgorgement the focus is on infringer profits. The disgorgement damage function is $D_D = \max\{(P - s)q_j - \pi_j^N, 0\}$. Suppose that j chooses to infringe and consider output pairs at which D_D is positive. With γ indexing the strength of property rights, the payoffs are then

$$\Pi_i^D(q_i, q_j) = [P - c]q_i + \gamma[(P - s)q_j - \pi_j^N],$$

$$\Pi_j^D(q_i, q_j) = (1 - \gamma)[P - s]q_i + \gamma\pi_j^N.$$

We see that the imitator payoff now reduces to a scale multiple of the market payoff. Thus, the best response of the imitator follows ϕ_j (for cost s) in complete analogy to the patent holder's behavior under lost profits. For the patent holder, however, an increase in q_i not only impacts the market payoff (in the standard way) but also reduces the market price and, hence, the margin on disgorgement damages from j . Because of this added effect, the patent holder becomes more timid and the best response shifts inward relative to that under N . Thus, relative to N , we find that the imitator increases output while the patent holder reduces output. Further, the imitator always finds infringement to be profitable, as $\Pi_j^D(q_i^D, q_j^D) > \pi_j^N$.²⁵ As with reasonable royalties, however, the profit impact on the patent holder depends inversely on the strength of property rights, with similar implications for innovation incentives.

The analysis of these alternative damage measures illustrates that infringement is induced when a damage rule shifts the imitator's best-response function toward higher output while the innovator's best-response function remains unaffected or shifts toward lower output. These features characterize the lost

24. Blair and Cotter (1998) argue that the courts should adopt a property right damage regime (see, e.g., Calabresi and Melamed [1972] and Kaplow and Shavell [1996] for general discussions on property and liability rules) more akin to that used for trade secrets: injunctions plus damages which should be the greater of lost profits of the patentee and the incremental profits earned by the infringer. Effectively, Blair and Cotter would like the law to move back in the direction of the restitution theory for damages. They propose using lost profits when the infringer is less efficient (this is greater than disgorgement and has a greater deterrence effect) and disgorgement with more efficient infringers (this is greater than the reasonable royalties). This approach was commonly used before the 1946 patent act revision.

25. This is readily verified from the solutions to the best-response conditions: $q_i^D = \frac{1}{(3-\gamma)\beta}[(1-\gamma)\alpha - 2c + (1+\gamma)s] < q_i^N$ and $q_j^D = \frac{1}{(3-\gamma)\beta}[\alpha - 2s + c] > q_j^N$.

profits, reasonable royalties, and disgorgement damage rules that we have examined.

7.3 Incomplete Information

An important element in our analysis is the attractive strategic option for the infringer to avoid lost profits damages by maintaining its preinfringement quantity level. This option makes infringement a dominant strategy and is the linchpin for our analysis. How robust is the passive infringement option to incomplete information?

In practice, costs are likely to be a source of private information. The problem is that the infringer will not know what quantity would obtain given the patent and no infringement until it has some market experience competing against the patentee. Once it has this information, the passive infringement option can be used without direct knowledge of a rival's costs because only knowledge of previous period quantity (with the patentee producing using the innovation and the infringer without that innovation) is needed. Private costs could, however, affect the boundary of the parameter region in which the passive infringement strategy is employed. This would occur through its effect on market choices, directly in market competition and indirectly in terms of the infringement damages that are anticipated.

7.4 Weak Property Rights

The reference levels used for damages assessment are further complicated as one moves away from a binary view of infringement to a view in which firms choose from a continuum of product or process choices that have some level of associated infringement risk. Incorporation of weak property rights in our model provides some insight into this more complex problem. Proposition 3 shows that the strength of property rights impacts the attractiveness of passive versus aggressive infringement in a natural way. An interesting problem that we did not address is whether passive infringement endogenously affects a patent holder's awareness of infringement since sales levels will not suggest infringement.²⁶ Also, with weak property rights, the innovator has a stronger incentive to choose secrecy over patenting in order to deny a competitor useful cost-reducing knowledge.²⁷ In fact, from a "weak property rights" perspective, so-called infringement is not unambiguously bad since it is not at all obvious that the patent holder should have exclusivity with respect to the patented technology. In this case, overdeterrence of infringing innovation becomes a more serious concern.

In brief summary, the inherent complexity of market interactions when intertwined with legal sanctions does not mean that one should necessarily throw up one's hands and declare a reasonable royalty. It should, however,

26. Crampes and Langinier (2002) address monitoring investments regarding infringement.

27. See Anton and Yao (2004) for a treatment of weak property rights and imitation under incomplete information. See also Anand and Galetovic (2000) on R&D, Baccara and Razin (2002) on market structure, and Ponce (2002) on disclosure, in settings with weak property rights.

force an equilibrium analysis of the underlying structure of the competitive interaction.

Appendix

The proofs for Lemmas 1 and 2 and for Proposition 3 are straightforward and therefore omitted.

Proof of Lemma 3. For $q_i \in (q_A, q_B)$, the payoff to j is given by

$$\Pi_j = \begin{cases} [P(q_i + q_j) - s]q_j \\ [P(q_i + q_j)(q_j + \gamma q_i) - sq_j - \gamma c q_i - \gamma \pi_i^N] \end{cases} \quad \text{as } q_j \leq Q_j(q_i).$$

This is a strictly concave objective in q_j , but it has a kink at $q_j = Q_j(q_i)$. Calculating, we find

$$\frac{\partial \Pi_j}{\partial q_j} = \begin{cases} \alpha - s - \beta q_i - 2\beta q_j \\ \alpha - s - \beta(1 + \gamma)q_i - 2\beta q_j \end{cases} \quad \text{as } q_j \leq Q_j(q_i).$$

By continuity and strict concavity, we know a unique solution exists. Since $\partial \Pi_j / \partial q_j|_{q_j=0} = \alpha - s - \beta q_i \leq 0 \Leftrightarrow q_i \geq (\alpha - s)/\beta$, we see that $q_j = 0$ is the solution for this case. Otherwise, in cases (a) and (c), we see that the first-order condition holds with equality, $\partial \Pi_j / \partial q_j = 0$, at the specified q_j values. In case (b), only the kink value of $q_j = Q_j(q_i)$ satisfies the first-order condition. ■

Proof of Propositions 1 and 2. We establish Propositions 1 and 2 via a sequence of claims. First, we claim that for any cost pair (c, s) and any $\gamma \in (0, 1)$, there is no equilibrium in which j chooses to produce on ϕ_j . Recall from Lemma 3(a) that ϕ_j is potentially a best response for j when $q_i \in (q_A, q_B)$. We know from Lemma 1 that the best response for i is always on ϕ_i . Now, recall the reference outcome $\mathcal{N}$, specified in equations (4) and (5). In $\mathcal{N}$, i follows the best response ϕ_i while j follows the best response of $\phi_j^{\mathcal{N}} \equiv \max \{0, (\alpha - 2\bar{c} - \beta q_i)/(2\beta)\}$; the equilibrium outcome for $\mathcal{N}$ has i producing $q_i^{\mathcal{N}}$, and this output level also satisfies $q_i^{\mathcal{N}} = \arg\max Q_j(q_i)$. Noting that $\phi_j > \phi_j^{\mathcal{N}}$, because $s < \bar{c}$, we see that the intersection of ϕ_i and ϕ_j occurs at a quantity $\hat{q}_i$ that is strictly below $q_i^{\mathcal{N}}$; the corresponding quantity $\hat{q}_j$ for j is strictly above $Q_j(\hat{q}_i)$. But this implies lost profits are positive at $(\hat{q}_i, \hat{q}_j)$, and this implies that $\phi_j(q_i)$ is not the best-response choice for j . Formally, we have $\hat{q}_i < q_i^{\mathcal{N}}$ and, by Lemma 2(d), this implies $\hat{q}_i + 2Q_j(\hat{q}_i) < q_i^{\mathcal{N}} + 2Q_j(q_i^{\mathcal{N}}) = (\alpha - \bar{c})/\beta < (\alpha - s)/\beta$, where the final two steps follow from simplifying and observing that $s < \bar{c}$. Thus, Lemma 3(a) cannot apply at $\hat{q}_i$.

Next, we claim there is no equilibrium in which $q_i > q_i^{\mathcal{N}}$. Suppose, by way of contradiction, that such an equilibrium does exist. Denote it by $(\hat{q}_i, \hat{q}_j)$. Then j must be at a best response to $\hat{q}_i$ and this must be in the set $\{Q_j(\hat{q}_i), \phi_j(\hat{q}_i)$,

$\varphi_j(\hat{q}_i)\}$, by Lemma 3. We can rule out any $\hat{q}_i \geq (\alpha - c)/(2\beta)$ since the best response of i to any q_j is always below this level (Lemma 1). Then, in this equilibrium, we must have $(\alpha - c)/(2\beta) \geq \hat{q}_i > q_i^N$. We can now rule out Q_j as a best response for j : ϕ_i , the best response for i intersects Q_j only one time, and this occurs at q_i^N . We can rule out ϕ_j for j from the first claim above. This leaves only φ_j . Now, since ϕ_i lies strictly below Q_j for $\hat{q}_i \in (q_i^N, (\alpha - c)/(2\beta)]$, we see that $\hat{q}_j = \phi_i^{-1}(\hat{q}_i) < Q_j(\hat{q}_i)$ and $D = 0$ holds. By Lemma 3, j is not at a best response with φ_j when $D = 0$. This establishes the second claim.

We now prove Proposition 1. First, note that j will not play Q_j in any equilibrium: from $\varphi_j(q_i^N) > Q_j(q_i^N) = q_j^N > 0$, we have $(\alpha - s)/[\beta(1 + \gamma)] > q_i^N$ by definition of φ_j . We know from the text that j plays φ_j when $q_i \leq q_A$. When $q_A < q_i \leq q_i^N$, we have $(\alpha - s)/\beta > (\alpha - s)/[\beta(1 + \gamma)] > q_i^N \geq q_i$. Thus, consider which case of Lemma 3 applies:

$$\begin{aligned} \varphi_j(q_i^N) > q_j^N &\Leftrightarrow \frac{1}{2\beta}[\alpha - s - \beta(1 + \gamma)q_i^N] > Q_j(q_i^N) \\ &\Leftrightarrow (\alpha - s)/\beta > (1 + \gamma)q_i^N + 2Q_j(q_i^N). \end{aligned}$$

Thus, Lemma 3(c) applies at $q_i = q_i^N$. By Lemma 2(d), $(1 + \gamma)q_i + 2Q_j(q_i)$ is strictly increasing in q_i over $[q_A, q_i^N]$; therefore, Lemma 3(c) applies over this entire interval and j plays φ_j as a best response. Thus, we have shown that i must play ϕ_i , j must play φ_j , and any equilibrium must have $q_i \leq q_i^N$. It is then routine algebra to solve for the (unique) intersection, yielding $q_i^* = (\alpha - 2c + s)/[\beta(3 - \gamma)]$ and $q_j^* = [\alpha(1 - \gamma) - 2s + (1 + \gamma)c]/[\beta(3 - \gamma)]$, and verify directly that $q_i^* \in (0, q_i^N)$ and $q_j^* > q_j^N$.

We now prove Proposition 2. From $\varphi_j(q_i^N) < Q_j(q_i^N) = q_j^N$, we find that j cannot play φ_j in any equilibrium since the (unique) intersection of ϕ_i and φ_j now occurs at a q_i above q_i^N . Thus, j must play Q_j in any equilibrium, and it only remains to verify that (q_i^N, q_j^N) are best responses. This is trivial for i since $\phi_i(q_j^N) = q_i^N$. For j , we need to verify that Lemma 3(b) applies. Note that $q_i^N \in (q_A, q_B)$ is clearly valid. Next, we must show $(\alpha - s)/\beta > q_i^N$. This reduces to $[2(\alpha + c) - \bar{c}]/3 > s$; this is valid since the left-hand side is increasing in c and positive at $c = 0$, by $\alpha > 2\bar{c}$. Finally, we must verify that $q_i^N + 2Q_j(q_i^N) \leq (\alpha - s)/\beta \leq (1 + \gamma)q_i^N + 2Q_j(q_i^N)$. The inequality on the right is implied directly by $\varphi_j(q_i^N) < q_j^N$. Substituting directly for q_i^N from equation (5) and evaluating with Q_j via equation (7), this reduces to $s < \bar{c}$. Hence, j is at a best response with $Q_j(q_i^N) = q_j^N$. ■

Proof of Proposition 4. A choice of no imitation by j leads to profits of π_j^N for j . There are two cases. First, when $\varphi_j(q_i^N) \leq Q_j(q_i^N)$ and, by Proposition 2, the equilibrium involves the market quantities from the reference outcome N , we have equilibrium profits for j of

$$\begin{aligned} \Pi_j^* &= (P^N - s)q_j^N - \gamma \text{Max}\{\pi_i^N - (P^N - c)q_i^N, 0\} = (P^N - s)q_j^N \\ &= \pi_j^N + (\bar{c} - s)q_j^N, \end{aligned}$$

and a choice of $\mathcal{I}$ by j is optimal. The second case is that of $\varphi_j(q_i^N) > Q_j(q_i^N)$ where, by Proposition 1, the equilibrium is at (q_i^*, q_j^*) . We must show that $\Pi_j^* = \Pi_j(q_j^*, q_i^*) > \pi_j^N$. Referring back to Figure 3, note that j 's payoff is Π_j^* at point A. At point B, we have $\Pi_j(q_j^N, q_i^N) = (P^N - s)q_j^N = \pi_j^N + (\bar{c} - s)q_j^N > \pi_j^N$. Comparing to j 's payoff at point C, we have $\Pi_j(\varphi_j(q_i^N), q_i^N) > \Pi_j(q_j^N, q_i^N)$, since $\varphi_j(q_i^N) > q_j^N$ is the unique best response for j to q_i^N . Hence, j 's payoff at point C exceeds that at point B. To complete the argument, we compare payoffs at points A and C. Index i 's output between A and C by $x \in [q_i^*, q_i^N]$ and consider j 's payoff of $\Pi_j(\varphi_j(x), x)$ as we move along φ_j between points A and C. Then

$$\begin{aligned} \frac{d}{dx} \Pi_j(\varphi_j(x), x) &= \left(\frac{\partial \Pi_j}{\partial q_j} \varphi_j'(x) + \frac{\partial \Pi_j}{\partial q_i} \right) \bigg|_{(\varphi_j(x), x)} = \frac{\partial \Pi_j}{\partial q_i} \bigg|_{(\varphi_j(x), x)} \\ &= \gamma(\alpha - c - 2\beta x) - \beta(1 + \gamma)\varphi_j(x), \end{aligned}$$

where the first step follows from the envelope theorem (j is at a best response) and the second by direct calculation. Note that

$$\frac{d^2}{dx^2} \Pi_j(\varphi_j(x), x) = \beta[(1 + \gamma)^2/2 - 2\gamma] > 0,$$

which follows from the right-hand side being strictly decreasing in γ and equaling zero at $\gamma = 1$. Hence, by convexity,

$$\begin{aligned} \frac{d}{dx} \Pi_j(\varphi_j(x), x) &\leq \frac{d}{dx} \Pi_j(\varphi_j(x), x) \big|_{(\varphi_j(q_i^N), q_i^N)} \\ &= \gamma(\alpha - c - 2\beta q_i^N) - \beta(1 + \gamma)\varphi_j(q_i^N) \\ &= \beta[\gamma q_i^N - (1 + \gamma)\varphi_j(q_i^N)] < 0. \end{aligned}$$

Thus, $\Pi_j(\varphi_j(x), x)$ is strictly decreasing for $x \in [q_i^*, q_i^N]$, and we have shown that j 's profit at A exceeds that at C. Combining the comparison for A, B, and C, we are done. ■

Proof of Proposition 5. Assume that h satisfies $h' > 0$, $h'' < 0$, $h(0) = 0 = \lim_{x \rightarrow \infty} h'(x)$, and that $h'(0)$ is sufficiently large to rule out an R&D level of zero. Assume that $V_i > \bar{\pi}/r > V_j$ and that $V_i + V_j > 2\bar{\pi}/r$; these assumptions are satisfied for the $\mathcal{N}$ benchmark. Then, the best response for firm 1 (symmetrically for 2) to x_2 is unique, positive, and satisfies the first-order condition:

$$h'(x_1)[h(x_2)(V_i - V_j) + rV_i - \bar{\pi} + x_1] - [r + h(x_1) + h(x_2)] = 0.$$

Let $x_1 = \Phi(x_2)$ denote the best response. It is easy to verify that $\Phi' > 0$; further, an increase in V_j shifts Φ down, so that a larger failure payoff unambiguously reduces the incentive to invest in R&D. From the first-order condition, it is straightforward to show that an equilibrium exists and all equilibria are symmetric. Thus, any equilibrium x^* must satisfy

$$F(x, V_j) \equiv h'(x)[h(x)(V_i - V_j) + rV_i - \bar{\pi} + x] - [r + 2h(x)] = 0.$$

Under the added assumption on h (see Tirole [1988, 416]) that

$$h''(x)[h'(x)(V_i - V_j) + rV_i - \bar{\pi} + x] + h'(x)[-1 + h'(x)(V_i - V_j)] < 0,$$

the equilibrium is unique. It is then straightforward to calculate that dx^*/dV_j and $\partial F/\partial V_j$ have the same sign. Since $\partial F/\partial V_j = -h(x^*)h'(x^*) < 0$, we are done. ■

References

- Anand, Bharat, and Alexander Galetovic. 2000. "Weak Property Rights and Holdup in R&D," 9 *Journal of Economics and Management Strategy* 615–42.
- Anand, Bharat, and Tarun Khanna. 2000. "The Structure of Licensing Contracts," 48 *Journal of Industrial Economics* 103–35.
- Anton, James, and Dennis Yao. 2004. "Little Patents and Big Secrets: Managing Intellectual Property," 35 *Rand Journal of Economics* 1–22.
- Aoki, Reiko, and Jin-Li Hu. 1999. "Licensing vs. Litigation: The Effect of the Legal System on Incentives to Innovate," 8 *Journal of Economics & Management Strategy* 133–60.
- Ayres, Ian, and Paul Klemperer. 1999. "Limiting Patentees' Market Power Without Reducing Innovation Incentives: The Perverse Benefits of Uncertainty and Non-Injunctive Remedies," 97 *Michigan Law Review* 985–1033.
- Baccara, Mariagiovanna, and Ronny Razin. 2002. "From Thought to Practice: Appropriation and Endogenous Market Structure with Imperfect Intellectual Property Rights," Working Paper, Princeton University.
- Baker, Scott, and Claudio Mezzetti. 2001. "Using the Threat of Disclosure to Enforce Knowledge Sharing in Joint Ventures Which Span Multiple Innovation Markets," Working Paper, University of North Carolina.
- Blair, Roger, and Thomas Cotter. 1998. "An Economic Analysis of Damages Rules in Intellectual Property Law," 39 *William and Mary Law Review* 1585–694.
- Boldrin, Michelle, and David Levine. 2003. "The Case Against Intellectual Monopoly," Unpublished Manuscript, UCLA.
- Calabresi, Guido, and A. Douglas Melamed. 1972. "Property Rules, Liability Rules and Inalienability: One View of the Cathedral," 85 *Harvard Law Review* 1089–126.
- Crampes, Claude, and Corinne Langinier. 2002. "Litigation and Settlement in Patent Infringement Cases," 33 *Rand Journal of Economics* 258–74.
- Gans, Joshua, and Scott Stern. 2000. "Incumbency and R&D Incentives: Licensing in the Gale of Creative Destruction," 9 *Journal of Economics and Management Strategy* 485–512.
- Hoppe, Heidrun, Philippe Jehiel, and Benny Moldovanu. 2006. "License Auctions and Market Structure," 15 *Journal of Economic and Management Strategy* 371–96.
- Kaplow, Louis, and Steven Shavell. 1996. "Property Rules Versus Liability Rules: An Economic Analysis," 109 *Harvard Law Review* 713–90.
- Lemley, Mark. 2001. "Rational Ignorance at the Patent Office," 95 *Northwestern University Law Review* 1495–532.
- Merges, Robert. 1997. *Patent Law and Policy: Cases and Materials*. 2nd ed. Charlottesville, Va.: Michie Law Publishers.
- Ponce, Carlos. 2002. "Knowledge Disclosure as Intellectual Property Rights Protection," Working Paper, UCLA.
- Reinganum, Jennifer. 1982. "A Dynamic Game of R and D: Patent Protection and Competitive Behavior," 50 *Econometrica* 671–88.
- . 1983. "Uncertain Innovation and the Persistence of Monopoly," 73 *American Economic Review* 741–8.
- Schankerman, Mark, and Suzanne Scotchmer. 2001. "Damages and Injunctions in Protecting Intellectual Property," 32 *Rand Journal of Economics* 199–220.
- Schnaars, Steven. 1994. *Managing Imitation Strategies*. New York, N.Y.: Free Press.
- Tirole, Jean. 1988. *The Theory of Industrial Organization*. Cambridge, Mass.: MIT Press.