An Example of Infringement with Damages that Include Lost Licensing Revenues

Supplementary Material for "Finding Lost Profits: An Equilibrium Analysis of Patent Infringement Damages" by J. Anton and D. Yao

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In "Finding Lost Profits" we analyze cases of infringement in which the "lost profits" calculation does not anticipate any lost licensing revenues but is based only on lost profits from market competition with imitation. We do, however, argue that equilibrium infringement will sometimes still occur in a setting where lost profits include foregone licensing revenues. Here we provide an example of equilibrium infringement when lost licensing revenues are considered a portion of lost profits.

In this example the court employs a lost profit award that "anticipates" a licensing outcome so the damage measure refers to the profit of i under the licensing arrangement. Suppose that i offered a per unit license at $\rho = \bar{c} - c$, but j refused and perfectly imitates at a cost c. What is the "infringement" outcome? We need these payoffs to decide whether j should accept or refuse the license offer and therefore show that infringement is an equilibrium outcome.

For reference, the output and prices in a Cournot equilibrium are

$$q^i\left(c^i, c^j\right) = \frac{1}{3}\left[1 - 2c^i + c^j\right], \qquad q^j\left(c^i, c^j\right) = \frac{1}{3}\left[1 - 2c^j + c^i\right], \qquad p\left(c^i, c^j\right) = \frac{1}{3}\left[1 + c^i + c^j\right] \tag{1}$$

To begin, let us conjecture an outcome in which j "passively" infringes. In the licensing revenue case, passive infringement involves an imitator quantity that results in no lost profit damages. This means we need to find quantities (q_i, q_j) for which each player is at a best response and i earns a payoff at the reference licensing level of

$$\pi_L^i = \pi^i\left(c, \bar{c}\right) + (\bar{c} - c)q^j\left(c, \bar{c}\right). \tag{2}$$

We propose the following quantities:

$$q_i^* = \sqrt{\pi_L^i}, \qquad q_j^* = 1 - c - 2\sqrt{\pi_L^i}. \tag{3}$$

At these quantities, we have

$$\Pi_i^* = (p^* - c)\, q_i^* = \left[1 - q_i^* - q_j^* - c\right] q_i^* \tag{4}$$

$$= \left[1 - c - \sqrt{\pi_L^i} - \left(1 - c - 2\sqrt{\pi_L^i}\right)\right]\sqrt{\pi_L^i} \tag{5}$$

$$= \pi_L^i. \tag{6}$$

Thus, the market payoff for i equals the license level. The payoff for j is then (since damages are zero)

$$\Pi_j^* = (p^* - c)\, q_j^* = \sqrt{\pi_L^i}\left[1 - c - 2\sqrt{\pi_L^i}\right]. \tag{7}$$

We want this passive infringement profit to exceed the payoff to j from accepting the license offer:

$$\Pi_j^* \quad > \quad \pi^j\left(c, \bar{c}\right) \tag{8}$$

$$\Leftrightarrow \quad \sqrt{\pi_L^i}\left[1 - c - 2\sqrt{\pi_L^i}\right] > \frac{1}{9}\left(1 - 2(c+\rho) + c\right)^2. \tag{9}$$

where $\rho = (\bar{c} - c)$. We can work with this condition directly by plugging in our candidate parameters.

Now consider the following values for the parameters: $c = 0, \bar{c} = .25$ and $\rho = \bar{c} - c = .25$. As in our model, we also allow for a parameter γ which is the probability that the court finds infringement. Under passive infringement, lost profits are zero so γ does not matter for the profit comparison, though it will matter for the best response conditions. Inserting the numerical values, the inequality (9) above reduces to

$$3\sqrt{31}\left(1 - \frac{\sqrt{31}}{6}\right) > 1 \qquad \Leftrightarrow \qquad 1.2 > 1. \tag{10}$$

We now have to verify the best response conditions. Define the damage measure at an arbitrary (q_i, q_j) pair by

$$LP\left(q_i, q_j\right) = Max\left\{0, \pi_L^i - (p-c)q_i\right\}. \tag{11}$$

Then the payoff function for i is

$$\Pi^i\left(q_i, q_j\right) = \begin{cases} (p-c)q_i & if \quad LP\left(q_i, q_j\right) = 0 \\ (p-c)q_i + \gamma LP\left(q_i, q_j\right) & if \quad LP\left(q_i, q_j\right) > 0 \end{cases} \tag{12}$$

Given that j is at q_j^*, we find that i has a unique (with $\gamma < 1$) optimizing choice at our candidate q_i^* from (3). Higher i output makes lost profits (LP) strictly positive and the payoff is below π_L^i. Lower i output means we still have LP at zero but the market payoff is lower. This is because i essentially follows the Cournot best response function.

The payoff for j is

$$\Pi^j\left(q_i, q_j\right) = \begin{cases} (p-c)q_j & if \quad LP\left(q_i, q_j\right) = 0 \\ (p-c)q_j - \gamma LP\left(q_i, q_j\right) & if \quad LP\left(q_i, q_j\right) > 0 \end{cases} \tag{13}$$

Checking for j's best response is trickier. Given q_i^*, if j produces below q_j^* then LP is still zero and

$$\frac{\partial \Pi_j}{\partial q_j} = 1 - c - q_i^* - 2q_j \tag{14}$$

and the payoff is concave in this lower output region. For the best response to be at q_j^*, we need the above derivative to be non-negative at q_j^*:

$$\frac{\partial \Pi_j}{\partial q_j}\Big|_{q_j \uparrow q_j^*} = 1 - c - q_i^* - 2q_j^* = 3\sqrt{\pi_L^i} - (1-c) \geq 0 \tag{15}$$

Now consider higher j quantities. Then,

$$\frac{\partial \Pi_j}{\partial q_j} = 1 - c - (1+\gamma)q_i^* - 2q_j^* \tag{16}$$

$$\frac{\partial \Pi_j}{\partial q_j}\Big|_{q_j \downarrow q_j^*} = 1 - c - (1+\gamma)q_i^* - 2q_j^* = (3-\gamma)\sqrt{\pi_L^i} - (1-c) \leq 0. \tag{17}$$

Now, let $\gamma = 1$ and return to the parameter values $c = 0, \bar{c} = .25$ from above. The above best response conditions for j (they are at a kink in the j payoff) are always satisfied for these quantities as the inequalities (15) and (17) reduce to

$$\begin{aligned} 3\sqrt{\pi_L^i} - (1-c) \quad &\geq \quad 0 \geq (3-\gamma)\sqrt{\pi_L^i} - (1-c) && (18) \\ &\Leftrightarrow \quad 3\frac{\sqrt{31}}{12} > 1 > 2\frac{\sqrt{31}}{12} && (19) \\ &\Leftrightarrow \quad 1.4 > 1 > 0.938. && (20) \end{aligned}$$

Intuitively, j always infringes passively with the high license fee and perfect detection. Aggressive infringement turns out to not to be an equilibrium outcome here.