Coordination in Split Award Auctions

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We analyze split award procurement auctions in which a buyer divides full production between two suppliers or awards all production to a single supplier, and suppliers have private cost information. An intriguing feature of split awards is that the equilibrium bids are implicitly coordinated. Because a split award price is the sum of offered split prices, each supplier can unilaterally veto a split award by bidding very high for the split. The need to coordinate is reflected in a split price that does not vary with private information. We also explore conditions under which split award auctions may be preferred to winner-take-all auctions.

I. INTRODUCTION

In the late 1970s the U. S. government began funding the development of a General Electric jet engine as a preliminary to procuring fighter engines competitively instead of through a sole source arrangement with Pratt and Whitney. The ensuing competition was conducted with sealed-bid split award procurement auctions in which both suppliers submitted bids for "splits" of the total requirements of the government as well as for the entire requirement. The government then chose the best bid or combination of bids from those submitted.

Officials associated with this $10 billion "Great Engine War" seemed pleased with the early results of this competition, with some viewing the engine competition as "one of the finest accomplishments of their careers."[1] Serious concerns, however, have been raised about the efficacy of other split award procurements (e.g., the Sparrow AIM-7F missile). For example, after reviewing the price performance of various defense procurement auctions, Beltramo [1983, p. 109] suggested that "split-buy competitions often increase costs" to the government (relative to winner-take-all auctions in which bidders only submit prices for the entire needs of the buyer).[2]

In this paper we explore the price performance of split award

1. See Drewes [1987, p. 3]. In addition, see *The Wall Street Journal* [February 6, 1984, p. 14] and Crocker and Reynolds [1990].
2. See Burnett and Kovacic [1989] for a discussion of dual sourcing and acquisition policy and Anton and Yao [1990] for a discussion of the empirical evidence regarding price performance in defense procurements.

procurement auctions when suppliers have asymmetric information about each other's costs. The model highlights some important features of such auctions and allows us to address the question of why split award auctions perform better than winner-take-all auctions in some contexts, but worse in others. This comparison provides some clues as to the circumstances in which a buyer might find split award auctions desirable.

While our motivation for examining split awards derives from the use of this procurement mechanism in the defense sector, split awards are also widely used by buyers in other sectors. In the public sector, for example, the application of this auction format runs the gamut from high-technology systems for telecommunications to their use in obtaining services such as refuse collection and street cleaning. In the private sector procurements of items such as customized computer chips and commercial aircraft have involved split awards.[3]

In many of the above applications the justification for a split award auction format will hinge on current price performance, which is the focus of our analysis. Additionally, dual sourcing via split award auctions may be justified as a method for dealing with the problems of future buyer dependence on a single (opportunistic) supplier. The adverse effects of this dependence can be avoided if long-term contracts protect the buyer from exploitation or if current price competition reflects the value the bidders attach to the downstream "monopoly." Williamson [1975], however, identifies conditions under which these methods perform unsatisfactorily and suggests vertical integration as an alternative solution. From this economic organization perspective, dual sourcing can be seen as an intermediate alternative in which a buyer "invests" in downstream competitive discipline. Therefore, an understanding of dual source split award auctions may be relevant to the larger question of economic organization, as well as to the more direct problem of price performance in procurement auctions.

We model a procurement auction in which a buyer desires to procure a fixed number of items in a low-price sealed-bid auction

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3. Sellers may also employ a split award format. Among professional sports franchises, for example, the Boston Celtics play home games in Hartford as well as in Boston, and the Green Bay Packers play part of their home schedule in Milwaukee. An interpretation of these examples is that the team (seller) makes a split award by dividing the home schedule between different municipalities (buyers) who bid by offering facilities and amenities.

involving two suppliers.[4] Our model captures the essential features of dual source split award competitions that are run by the U. S. government.[5] Based on private information about costs, each supplier submits bids that consist of a price for supply of the total requirements and a price for supply of a fixed split of the total requirements. The buyer awards production based on the lowest sole source or combination of split award prices. Thus, both sole source and split outcomes are feasible.

An intriguing feature of dual source split awards is that the equilibrium bid prices are implicitly coordinated. Because a split award price is the sum of the prices offered by each supplier, a split award cannot be generated unilaterally—each bidder can always "veto" a split award by offering a high price at the split and a low price for a sole source award.

When split production is cost efficient, bidders choose split and sole source prices to support a split outcome. In contrast to the familiar structure of equilibrium bids in a unit auction (e.g., indivisible good), equilibrium split prices in a split award auction involve pooling: the equilibrium split price does not vary with privately observed cost information. While a range of split prices can be supported as equilibria, a natural focal point is provided by the equilibrium with the highest split price as this is Pareto dominant with respect to the profits of the suppliers.

Relative to procurement settings where the suppliers have very accurate information about each other's costs (e.g., full information), asymmetric cost information has the effect of restricting the degree of equilibrium bidding coordination in a split award auction. This is because under full information the equilibrium bids can support high profits via prices that are structured to account for the incentives of an opponent with known costs; under asymmetric information, however, the equilibrium bids must

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4. In the procurement of high-technology items, it is common to find that there are only two qualified suppliers. For example, the F-15 and F-16 engine competition involved only GE and Pratt and Whitney, the Sparrow air-to-air missile competition was between GD and Raytheon, and the SSN-688 submarine competition was between Electric Boat and Newport News shipyards.

5. The current criteria for an "adequate price competition" for dual sourced defense contracts are spelled out in the May 1989 U. S. Defense Federal Acquisition Regulation (DFAR) Supplement revision. "Adequate" price competition involves prices solicited for various step quantities from at least two suppliers capable of producing full quantity, and awards made to the offeror with the lowest total price or to the two offerors with the lowest combined total price. The document also allows, apparently, quality considerations to affect the award. (See GAO/NSIAD-89-181 for a description.)

account for the incentives of an opponent with a range of possible costs.

This effect of asymmetric information is important for the welfare properties of split award auctions. Under full information the suppliers will prefer a split award format to a winner-take-all format, while the buyer prefers the winner-take-all format. Under asymmetric information the split award format can yield a Pareto improvement for the buyer and the suppliers over the winner-take-all format, because the price to the buyer reflects the restricted bidding coordination under asymmetric information. Thus, we find that a buyer can benefit from a split award format when suppliers have relatively poor information regarding each other's costs.

Previous work relating to split award auctions includes Wilson [1979], Bernheim and Whinston [1986], and Anton and Yao [1989]. Wilson examines bidding under complete and incomplete information in a model of share auctions. The model is not applicable in a procurement context, however, because the buyer is required to select an award such that all bidders receive an identical unit price for their shares of production. In the latter two papers the buyer chooses the award optimally in response to the bids, but the analyses consider only bidding under complete information.

After presenting the model in Section II, we analyze the bidding equilibria of split award auctions under asymmetric information in Section III. In Section IV we examine welfare properties by comparing split award and winner-take-all auction formats. Section V considers the consequences when the buyer employs a reserve price. Section VI concludes. All proofs are contained in the Appendix.

II. THE MODEL

We consider a sealed-bid, low-price split award auction between two suppliers, A and B, each of which has private cost information. The buyer determines the award by selecting the combination of bids that provides the required production at the lowest total price. The suppliers are risk neutral and maximize profits. For simplicity, we present the analysis for the case of ex ante symmetric suppliers.

Suppose that there are three possibilities, $\{0,\sigma,1\}$, for awarding production to A and B, where a given total quantity is required by the buyer. The award outcome 0 (1) corresponds to a sole source award for supplier $A(B)$, where $A(B)$ supplies all of the required units to the buyer. The award σ is a split award where both

suppliers produce a share of the total quantity; one may view σ as a 50-50 split without loss of generality.[6]

Costs for supplier $i = A, B$ are θ_i for sole source production, $\theta_i C$ for split award production, and zero for no award, where θ_i is a privately observed cost parameter and $0 < C < 1$. Thus, joint supply costs are θ_i if i receives a sole source award and $(\theta_A + \theta_B)C$ if the split σ is the award. Each random cost parameter, θ_i, is drawn independently from a distribution with interval support $[\underline{\theta}, \overline{\theta}]$ and a c.d.f. $F(\cdot)$ which has a positive continuous density. Of the three awards $\{0, \sigma, 1\}$, the award with the lowest joint supply costs depends on the realization of (θ_A, θ_B) and the value of C. The relative efficiency of split supply versus sole source supply turns out to be a fundamental determinant of equilibrium bidding behavior.

The sequence of events for the auction is as follows: (1) each supplier i learns its cost parameter θ_i, which is private information for supplier i and is not observed by any other party; (2) both suppliers simultaneously submit a bid specifying two prices, a sole source price p and a split price p_σ; (3) the buyer selects the award by choosing the lowest of the two sole source prices and the sum of the split prices.

A bidding strategy for a supplier is a pair of $F(\cdot)$-measurable functions $(p, p_\sigma): [\underline{\theta}, \overline{\theta}] \to \mathscr{R}_2^+$, so that a supplier of type θ bids a sole source price of $p(\theta)$ and a split price of $p_\sigma(\theta)$. All information except for the realized value of the cost parameters is common knowledge between the suppliers. The buyer's selection of the award with the lowest total price is mechanical, involving only a comparison of submitted bid prices; thus, the analysis of equilibrium bidding strategies is independent of the information structure with respect to the buyer. This information structure and the strategic options available to the buyer are examined after the analysis of equilibrium bidding.

In the case of a sole source award, the low bidder's payoff is the low bid minus the cost of supplying the required units, while the high bidder receives a payoff of zero. In the case of a split award, each supplier produces its share of the total and receives a payment from the buyer equal to its offered split price, so that each supplier's payoff is its split price minus the cost of supplying its share.

In a split award auction a bidder has the strategic option of

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6. In practice, buyers often specify a set of possible awards (e.g., percentage splits of 100/0, 80/20, etc.) for the auction. Our results continue to hold with a larger set of possible awards, including the unit interval [0,1].

varying its own sole source price and split price so as to influence the probabilities of a sole source award and a split award. For instance, to increase the likelihood of receiving a sole source award, a bidder may lower its sole source price while holding its split price constant. A bidder can also make a sole source award more likely by raising its split price while holding its sole source price steady, as this will make a choice of σ less attractive to the buyer. Simultaneous changes are also a strategic option.

We study symmetric Bayesian-Nash equilibria (hereafter, bidding equilibria) of the split award auction described above. Conditional on the realized type, θ_i, a bidder i is at a best response to an opponent who bids according to $\{p(\hat{\theta}_j), p_\sigma(\hat{\theta}_j)\}$ if no changes in the sole source and split prices submitted by i lead to higher expected profits (where the expectation is over the opponent's type). In equilibrium a bid of $\{p(\theta_i), p_\sigma(\theta_i)\}$ is a best response for each θ_i against $\{p(\hat{\theta}_j), p_\sigma(\hat{\theta}_j)\}$.

In view of the three possible award outcomes of the auction—a sole source award for A or B or a split award—there are potentially three classes of equilibria. First, the equilibrium may involve a sole source award for all cost-type realizations. Second, the equilibrium may involve sole source and split awards, with the award depending on the cost types that are realized. Finally, it may be that the split award obtains for all cost types. We refer to these, respectively, as winner-take-all (WTA), hybrid, and σ equilibria.

The price performance and bidding structure of split award auctions when splits are the outcome (as observed in the missile and engine-war cases) is the focus of this paper. Consequently, we concentrate on the analysis of σ equilibria. WTA equilibria, not surprisingly, are essentially identical to the familiar equilibrium of a standard unit auction; therefore they provide a useful benchmark for the evaluation of σ equilibria and are discussed in Section IV. Hybrid equilibria, which have many features in common with σ equilibria, involve a number of technical complications and are not treated in any depth in this paper.

We now turn to the analysis of equilibrium bidding.

III. EQUILIBRIUM BIDDING

This section characterizes bidding equilibria which have the property that the split σ is always the award. Two fundamental points emerge from the analysis of σ equilibria. First, there is a close relationship between the cost efficiency of the split award

relative to the sole source awards and the existence of σ equilibria. Second, implicit coordination is reflected in the equilibrium bids.

We begin the analysis by developing necessary and sufficient conditions for σ equilibria and derive equilibrium bids (Propositions 1 and 2). Afterwards, we assess the nature and extent of equilibrium bidding coordination.

A. σ Equilibria

A σ equilibrium is defined as an equilibrium bidding strategy $[p(\cdot),p_\sigma(\cdot)]$ in which the bid prices lead the buyer to select σ as the award (with probability one). Formally, this definition requires that the bids satisfy $p_\sigma(\theta_A) + p_\sigma(\theta_B) \leq \min \left[p(\theta_A),p(\theta_B)\right]$ for almost every θ_A and θ_B. The split σ is then an optimal choice for the award by the buyer as the total price for σ is lower than either sole source price.

Several necessary conditions for bid prices in a σ equilibrium follow from this definition. The most remarkable feature is that a σ equilibrium always involves "pooling" at a constant split price: the bid price at the split does not vary with privately observed cost information. The reason is simple. Because the bid prices in a σ equilibrium always induce the buyer to choose a split award, the equilibrium profit of each type θ is $p_\sigma(\theta) - \theta C$. Now consider a candidate σ equilibrium where split prices vary with cost types so that $p_\sigma(\theta_1)$ is greater than $p_\sigma(\theta_2)$ for some θ_1 and θ_2 types. But then θ_2 has an obvious strategy for increasing profits from the equilibrium value of $p_\sigma(\theta_2) - \theta_2 C$: submit the same bid prices as the type θ_1. Since the buyer chooses a split award at these prices, θ_2 would then receive the larger payment of $p_\sigma(\theta_1)$ and earn higher profits, thus upsetting the candidate σ equilibrium. Clearly, the only way to eliminate this bidding incentive is to pool at a single split price.[7]

The pooling property and two additional necessary conditions are formalized in Proposition 1 (all proofs are in the Appendix).

PROPOSITION 1. Suppose that $[p(\cdot),p_\sigma(\cdot)]$ is a σ equilibrium. Then

(1) $p_\sigma(\theta)$ is constant over $\theta \in [\underline{\theta},\bar{\theta}]$; denote it by p_σ:

(2) $$\bar{\theta}C \leq p_\sigma \leq \underline{\theta} (1 - C);$$

(3) $$2p_\sigma = \inf_\theta p(\theta).$$

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7. Pooling also arises in hybrid equilibria. In these equilibria all cost types that receive a split award with positive probability submit bids with the same split price.

The pooling property is condition (1). The equilibrium split price p_σ is bounded above and below as described in condition (2). The upper bound on the split price in condition (2) arises from the bidding incentive to undercut the price to the buyer of $2p_\sigma$, in an attempt to capture a sole source award. For this deviation to be unprofitable, it must be that $p_\sigma - \theta C \geq (2p_\sigma - \epsilon) - \theta$ for all θ and an arbitrarily small $\epsilon > 0$. Simplifying this yields $\theta(1 - C) > p_\sigma - \epsilon$. Thus, $\underline{\theta}(1 - C)$ is the necessary upper bound, reflecting the fact that type $\underline{\theta}$ has the strongest incentive to try to capture a sole source award. Under more general cost structures the upper bound is determined by the cost type with the smallest cost differential between the split and sole source awards.[8] Finally, condition (3) must hold, or else the slack between the price to the buyer at the split, $2p_\sigma$, and the sole source prices would allow some type to charge a split price above p_σ without moving the award from σ.

The price bounds in condition (2) have a simple interpretation in terms of efficiency. Define dual source efficiency (DSE) by the parameter condition,

(4) $$\bar{\theta} C < \underline{\theta}(1 - C).$$

When (4) holds, joint supply costs at the split $(\theta_A + \theta_B)C$ are less than sole source costs, θ_A or θ_B, for all cost types. As (4) is a necessary condition, a σ equilibrium results in the efficient production award.

Many of the settings where split award auctions have been employed, such as the missile and engine procurements, appear to be inconsistent with DSE from a long-run perspective due to the presence of production economies. Given existing production capabilities, however, short-run production costs may be lowest when the award is divided between the suppliers (U-shaped cost curves), while long-run costs are declining with scale. Even with sufficient lead time, capital expenditures to achieve these lower production costs (at higher scale) may not be economically justified if the suppliers have concerns about future demand (e.g., project cancel-

8. Let costs at the split be $C(\theta)$, an increasing function of θ, and define $\theta_m \equiv$ argmin $|\theta - C(\theta)|$. The same logic underlying the proof of condition (2) then shows that $\theta_m - C(\theta_m)$ replaces $\underline{\theta}(1 - C)$ in (2). With multiplicative costs θ and θ_m coincide. The cost structure of θC greatly simplifies the exposition. Nothing of substance is lost as long as the above distinction between low absolute costs and a low cost differential is maintained when interpreting the results. In addition, we shall henceforth ignore the nuisance case of $\bar{\theta} C = \underline{\theta}(1 - C)$ in which the interval $[\bar{\theta} C, \underline{\theta}(1 - C)]$ collapses to a point.

lation in the defense procurement case).[9] DSE is less likely to hold when experience effects contribute to declining long-run costs and the suppliers anticipate stable demand in future procurement rounds, since current production experience would then have additional value to the suppliers.[10] Thus, the relevant supply costs at the time of bidding include the anticipated effects of downstream interactions.[11]

Condition (4) also proves to be crucial for the existence of σ equilibria.[12] From condition (2) the set of possible equilibrium split prices is bounded above by $\theta(1 - C)$. In Proposition 2 we show that when (4) holds there exists an equilibrium with the highest split price $\theta(1 - C)$.

As a preliminary step, consider how sole source prices $p(\cdot)$ relate to bidding incentives in a σ equilibrium with a split price of p_σ. A bidder i has the strategic option of bidding a split price, say $\hat{p}_\sigma$, and a sole source price, say $\hat{p}$, where $\hat{p} < \hat{p}_\sigma + p_\sigma$. This bid essentially "vetoes" the award σ, as the total price of $\hat{p}_\sigma + p_\sigma$ is unattractive to the buyer, and the bidder will receive a sole source award when $\hat{p}$ is below the opponent's sole source price, $\hat{p} < p(\theta_j)$. This strategy would net profits of $(\hat{p} - \theta_i)$ times the probability of $\hat{p} < p(\theta_j)$. For this strategy to be unprofitable (relative to σ-equilibrium profits of $p_\sigma - \theta C$), equilibrium sole source prices cannot be "too high."

To formulate the precise bound on sole source prices, define the function,

$$G(\theta,\rho) \equiv \rho + \frac{\rho - \bar{\theta}CF(\theta)}{1 - F(\theta)} \qquad \text{for } \rho \geq \bar{\theta}C,$$

9. This scenario—increasing average costs with current capabilities and a reluctance to increase capacity which would lower average costs—may characterize Boeing's economic situation in 1989 with respect to commercial aircraft production. At that time it was reported that Boeing, despite an enormous order backlog and rosy demand predictions, was reluctant to expand capacity because of fears of a sudden downturn. Instead, Boeing relied on overtime and expanded subcontracting to meet demand. See *The Economist* [November 11, 1989, p. 79].

10. See Demsetz [1968] and Riordan and Sappington [1987] for discussions of unit auctions when production economies are dominant.

11. Additional examples of factors that bear on the likelihood of DSE include random start-up delays which lead to compressed production schedules, insurance against firm-specific problems such as strikes, and spillovers across suppliers arising from innovative activities. Anton and Yao [1989] develop a supply cost example in which process innovation and spillovers offset production economies and DSE holds. Preference considerations such as distributing political benefits in the case of government or maintaining the prospect of "future" competition may also be important.

12. In cases where (4) does not hold but σ is still the efficient award for a subset of the range of cost types, hybrid equilibria exist.

which, by differentiation, is strictly increasing in θ. As $G(\theta, p_\sigma) = 2p_\sigma$, this bounding function lies above $2p_\sigma$, the price to the buyer at the split in a σ equilibrium. As long as sole source prices satisfy $p(\theta) \leq G(\theta, p_\sigma)$, the deviation bid described above is unprofitable. With this anchor for sole source prices, we can state sufficient conditions for σ equilibria.

PROPOSITION 2. Suppose that (4) holds: $\bar{\theta}C < \underline{\theta}(1 - C)$. Let $p_\sigma \in [\bar{\theta}C, \underline{\theta}(1-C)]$, and $p(\cdot)$ be continuous and strictly increasing with $\underline{p}(\theta) = 2p_\sigma$ and $p(\theta) \leq G(\theta, p_\sigma)$ for all θ. Then $\{\underline{p}(\cdot), p_\sigma\}$ is a σ equilibrium.

Equilibrium bids induce the buyer to select σ as the award since all types (except for $\underline{\theta}$) submit a sole source price in excess of $2p_\sigma$. In equilibrium the auction profit of a type θ supplier is $p_\sigma - \theta C$, unconditional auction profits are $p_\sigma - E\{\theta|C$, and the buyer pays $2p_\sigma$. Although the buyer never chooses a sole source award in equilibrium, the variation of sole source prices with θ and the upper bound of $G(\theta, p_\sigma)$ play an important role in supporting the equilibrium: the upper bound on sole source prices makes it unprofitable to pursue strategies that result in a sole source award.

Proposition 2 establishes that the interval $[\bar{\theta}C, \underline{\theta}(1 - C)]$ is the range of split prices for the set of σ equilibria. Combined with Proposition 1, we then have the split price of $\underline{\theta}(1 - C)$ as the highest price over all σ equilibria. Denote this highest-price σ equilibrium as the Σ equilibrium (which is unique up to the sole source prices), and let $P_\Sigma \equiv 2\underline{\theta}(1 - C)$ denote the total price to the buyer. Because equilibrium profits are $p_\sigma - \theta C$, the Σ equilibrium is Pareto dominant over all σ equilibria with respect to the auction profits of the suppliers (pointwise in θ and, hence, in expectation as well). Thus, the Σ equilibrium is a natural focal point for the suppliers.

Additional perspective on high versus low split prices in the $[\bar{\theta}C, \underline{\theta}(1 - C)]$ interval is provided by the structure of the supporting sole source prices. Note that as long as $p(\theta) \geq \theta$, sole source prices lead to nonnegative profits when a bidder receives a sole source award. Because $p(\cdot)$ supports the buyer's choice of σ, sole source awards are "off the equilibrium path," and $p(\theta) \geq \theta$ is not required in a Bayesian-Nash equilibrium. Nevertheless, it may be unreasonable to expect the suppliers to submit negative-profit supporting sole source prices. The tension here for σ equilibria is that the nonnegative profit condition places a lower bound of θ on sole source prices while the requirement that bidding deviations be

unprofitable places an upper bound of $G(\theta,p_\sigma)$ on sole source prices. Therefore, the condition $G(\theta,p_\sigma) \geq \theta$ ensures that p_σ can be supported with nonnegative-profit sole source prices. For low split prices, including $\bar{\theta}C$, this condition fails.[13]

At higher split prices the condition becomes progressively easier to satisfy because a higher split price implies higher equilibrium profits, and this weakens the incentive to deviate from the equilibrium bids.[14] For $p_\sigma = \underline{\theta}(1 - C)$, the sole source deviation condition and the nonnegative profit bid condition can be satisfied in a number of ways. For instance, if $\underline{\theta}(1 - C) > \bar{\theta}/2$, then these conditions hold for any $F(\cdot)$ distribution. Thus, at least for the multiplicative cost structure assumed in our model, equilibria with higher split prices seem to be more reasonable than equilibria with lower split prices.

In summary, then, Propositions 1 and 2 provide necessary and sufficient conditions that characterize the set of σ equilibria. Within this set the Σ equilibrium has the highest price to the buyer, $P_\Sigma = 2\underline{\theta}(1 - C)$, and is Pareto dominant for the bidders. We now turn to the structure of implicit bidding coordination at the Σ equilibrium.

B. Assessing the Extent of Bidding Coordination

Thus far, we have concentrated on the structure of equilibrium bidding when cost information between suppliers is asymmetric. The asymmetric information assumption captures an essential feature of procurement environments such as the "engine war," in which innovation is an important dimension of the competition and where the production process is in flux. In "later" stages of competition or in environments with stable technology, however, suppliers might best be viewed as having full information about each other's costs. In this section we examine the relationship between split award auctions under conditions of asymmetric information and full information. This comparison brings out the

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13. To see this, consider the lowest split price of $p_\sigma = \bar{\theta}C$, and note that $G(\theta,\bar{\theta}C) = 2\bar{\theta}C$ collapses to a constant for all θ. From $\bar{\theta}C < \underline{\theta}(1 - C)$ we then see that $G(\theta,\bar{\theta}C)$ falls below θ, the sole source cost, as θ approaches $\bar{\theta}$. By continuity, this also holds for any split price sufficiently close to $\bar{\theta}C$.

14. For this reason, $G(\theta,p_\sigma)$ increases with p_σ. To see how the condition works, consider the highest split price, $p_\sigma = \underline{\theta}(1 - C)$, and simplify $G(\theta,p_\sigma) \geq \theta$ to obtain

$$2\underline{\theta}(1 - C)+[\underline{\theta}(1 - C) - \bar{\theta}C](F(\theta)/(1 - F(\theta))) \geq \theta.$$

Under (4), this condition always holds for θ values sufficiently close to θ and $\bar{\theta}$. The condition holds globally in θ as long as the c.d.f. $F(\theta)$ does not increase too slowly with θ.

qualitative differences in bidding coordination and, hence, price performance, that can be expected between these procurement environments.

Two features stand out in the structure of equilibrium bids when there is asymmetric information between the suppliers. First, there is the "pooling" feature of split prices: realized costs of the suppliers are not reflected in the equilibrium split price. Second, the highest split price of $\theta(1 - C)$ in the Σ equilibrium depends on specific common-knowledge elements of the overall cost structure.

In contrast, in a setting with full information the equilibrium bid prices depend directly on the realized cost types. Moreover, there is a precise sense in which these bid prices are coordinated. We begin with a brief analysis of the full information setting. We then use these results to show that asymmetric information has the effect of forcing the suppliers to adopt a set of bid strategies that correspond to a limiting case of the bidding coordination under full information.

In a split award auction with full information, realized cost types are common knowledge between the suppliers. Given realizations θ_A and θ_B, each supplier i makes a strategic bid choice $(p_i, p_{i\sigma})$ that may reflect knowledge of θ_j, the realized cost type of the opponent, as well as θ_i. This information structure leads to the following result, which is an extension of the results in Anton and Yao [1989].

PROPOSITION 3. Suppose that (4) holds: $\bar{\theta}C < \theta(1 - C)$. Let $p_i = (\theta_A + \theta_B)(1 - C)$ and $p_{i\sigma} = \theta_j(1 - C)$ for $i = A, B$ and $j \neq i$. Then $\{(p_A, p_{A\sigma}), (p_B, p_{B\sigma})\}$ is a σ equilibrium under full information for each θ_A and θ_B in the interval $[\underline{\theta}, \bar{\theta}]$. Furthermore, over the set of all equilibria under full information, this equilibrium results in the highest price to the buyer and is Pareto dominant with respect to the auction profits of the suppliers.

Denote this full-information equilibrium by Σ (θ_A, θ_B) so as to highlight the dependence of the bids on the realized (θ_A, θ_B) types. There are clear parallels with the asymmetric-information results. First, there is the close relationship between efficiency and existence as dual source efficiency, condition (4), is pivotal in both cases: in equilibrium a split award must be the efficient award, and if it is efficient, then a σ equilibrium exists. Another parallel is the existence of an equilibrium with the highest price to the buyer that is also Pareto dominant for the bidders.

The bids in the $\Sigma(\theta_A, \theta_B)$ equilibrium reveal that each supplier submits bid prices that are based on the realized cost of the other supplier. Furthermore, the supporting sole source price submitted by i is structured to remove the incentive of an opponent with cost type θ_j to deviate and capture a sole source award. This equilibrium coordination in the bidding relies extensively on full information about the other supplier's costs. Equilibrium bids result in the same total price of $(\theta_A + \theta_B)(1 - C)$ at each award in $[0, \sigma, 1]$, and so σ is an optimal choice of the buyer.[15]

At these bid prices each bidder i is indifferent between an award of σ or a sole source award for i. Given the sole source price of j, bidder i would have to accept smaller profits to induce the buyer to select i for a sole source award. The key element that makes these positive profits possible is dual source efficiency. The profits available at a sole source award are smaller than the profits at the split because the split is efficient relative to the sole source awards. Thus, by employing sole source prices that exactly offset the incentive of an opponent with known costs, the suppliers are able to share in the efficiency gains at the split and support positive equilibrium profits.

Equilibrium bidding coordination of this sort is restricted when there is asymmetric information about costs. In both cases, an equilibrium with σ as the award entails a set of sole source prices that remove an opponent's incentive to deviate and capture a sole source award. Under full information a supplier can tailor his bid to account for the known cost type of the opposing supplier. Under asymmetric information, equilibrium bids must ensure that all types have no incentive to deviate to a sole source outcome.

To see the extent to which coordination is limited by this requirement, recall that $P_\Sigma = 2\bar{\theta}(1 - C)$ is the highest split price in a σ equilibrium under asymmetric information. From Proposition 3 we see that over (θ_A, θ_B) the lowest $\Sigma(\theta_A, \theta_B)$-equilibrium price is associated with $\theta_A = \theta_B = \bar{\theta}$, and that this price is also $2\bar{\theta}(1 - C)$. Thus, asymmetric cost information has the effect of restricting the equilibrium bidding coordination to the extent that P_Σ coincides with the lowest realized price under full information.

The bidding incentives of low-cost types are responsible for limiting P_Σ to $2\bar{\theta}(1 - C)$. At any higher split price the equilibrium

unravels because the low-cost types will underbid to capture a sole source award. In effect, then, the equilibrium price of P_Σ reflects the discipline imposed on bidding coordination by the incentives of these cost types even though the equilibrium price is independent of the realized cost types.

IV. WELFARE PROPERTIES

We now turn to the welfare properties of split award auctions with respect to the price to the buyer and the profits of the suppliers. After reviewing results for winner-take-all (WTA) auctions, we compare the relative performance of the two auction formats under asymmetric and full information. The main conclusion we draw from the welfare comparison is that the split award format can generate a Pareto improvement over a WTA format for the buyer and sellers when asymmetric information is present.[16]

A. Winner-Take-All Auctions

Consider a low-price, sealed-bid, WTA auction in which each supplier bids a price for a sole source award (i.e., split awards are not possible with a WTA format). The following proposition summarizes results for such an auction under conditions of asymmetric and full information. As these results are well established in the literature, the proofs are omitted (see Milgrom [1986] and McAfee and McMillan [1987] for recent surveys).

PROPOSITION 4. Under asymmetric information the equilibrium bid in a WTA auction is given by $p(\theta) = \theta + [1 - F(\theta)]^{-1} \times \int_\theta^{\bar{\theta}} [1 - F(t)] dt$, and the lower-cost supplier wins the award. Under full information the equilibrium bid in a WTA auction is given by $p_i = \theta_j$ when $\theta_j \geq \theta_i$ and $p_i = \theta_i$ when $\theta_j < \theta_i$ for $i = A, B$ and $j \neq i$, and the lower-cost supplier wins the award at a price equal to the realized cost of the other supplier. The expected price to the buyer (over $\bar{\theta}_A$ and $\bar{\theta}_B$) is the same under full and asymmetric information.

Thus, the buyer makes a sole source award to the lower-cost supplier in the equilibrium outcome of a WTA auction. In contrast to a σ equilibrium, the equilibrium price in a WTA auction depends on the cost realizations.

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16. The welfare comparisons apply to the basic auction formats studied in this paper, and while suggestive, the results are not directly applicable to optimal auction mechanisms.

In Section II we noted that a split award auction can have a WTA equilibrium. There is a simple correspondence between WTA equilibria of split award auctions and equilibria of WTA auctions. The existence of WTA equilibria (sole sourcing is always the outcome) in a split award auction is a direct consequence of the implicit veto power each bidder can exercise over the split award: by submitting a bid with a split price that is very high relative to the sole source price, a bidder can unilaterally make a choice of σ unattractive to the buyer. Given such a bid from an opponent, a bidder's own split price becomes an irrelevant strategic option.

Sole-source prices are given by the standard bidding strategy for the WTA auction (see Proposition 4), and as long as an opponent submits a bid with a high split price, a bidder is at a best response when he also submits a high split price. Because of this correspondence, all of the analysis and results that follow on the WTA auction can also be interpreted as a comparison between WTA equilibria and σ equilibria of a split award auction.[17] In the discussion below, Σ equilibrium refers to the Σ equilibrium in the split award auction, and WTA equilibrium to the equilibrium in the WTA auction.

B. The Effect of Information with a Given Auction Format

Now consider the welfare properties of each auction format from the viewpoint of the buyer and the sellers as information varies. In subsection III.B we compared split award σ equilibria under conditions of full information and asymmetric information. We found that the price under asymmetric information was independent of the cost draws and, moreover, was equal to the lowest price that can obtain under full information, the Σ (θ_A, θ_B)-equilibrium price when $\theta_A = \theta_B = \underline{\theta}$. Thus, the buyer will always prefer the asymmetric information environment. Because equilibrium prices are higher, sellers will always prefer the full-information setting.

Prices in WTA auctions are described in Proposition 4. The expected price to the buyer is not affected by information conditions. In addition, it is easy to show that ex ante WTA profits are identical under full and asymmetric information.

17. To make the correspondence, take the sole source price from Proposition 4, and let p_σ denote the split price. Any $p_\sigma > \underline{\theta}(1 - C)$ is sufficient: if an opponent adopts the bidding strategy $[p(\cdot), p_\sigma]$, then in order for σ to be the award with positive probability, a bidder must submit a bid with a split price of, say $\hat{p}_\sigma$, that satisfies $\hat{p}_\sigma + p_\sigma < p(\underline{\theta})$. This implies, however, that $\hat{p}_\sigma - \theta C < p(\underline{\theta}) - p_\sigma - \theta C < 0$ for all θ and therefore any such bid is dominated.

C. The Effect of Auction Format in a Given Information Setting

Consider first the comparison of split award and WTA formats under asymmetric information. The WTA expected price (over $\hat{\theta}_A$ and $\hat{\theta}_B$) is $P_{WTA} = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(t)^2 \, dt$. The fundamental difference between P_Σ and P_{WTA}, the expected prices to the buyer under a split award format (Σ equilibrium) and a WTA format (WTA equilibrium), is that P_Σ does not depend on the shape of $F(\cdot)$, the distribution of cost types. To see the implications of this, compare two distributions, F_1 and F_2, both with support $[\underline{\theta}, \bar{\theta}]$, but let F_1 have almost all of the probability mass in the upper end of the support while F_2 has almost all of the mass in the lower end. This means that $F_2(\theta) > F_1(\theta)$ and, intuitively, F_2 is likely to generate low-cost types and F_1, high-cost types. The expected price in the Σ equilibrium, $P_\Sigma = 2\underline{\theta}(1 - C)$, is the same under F_1 or F_2. In sharp distinction, the WTA expected price will be much higher with F_1 than with F_2 (as follows from $F_2(\theta) > F_1(\theta)$). Thus, we see that the buyer can expect better price performance from the Σ equilibrium relative to the WTA equilibrium when the distribution of costs is skewed to the high end versus the low end.

For the bidders the expected profit for a cost-type θ in the Σ equilibrium is independent of the distribution of costs. In the WTA equilibrium expected profits for a type θ are $(p(\theta) - \theta)$ multiplied by the probability of being the low source bidder, so $\Pi_{WTA}(\theta) = \int_{\theta}^{\bar{\theta}} [1 - F(t)] \, dt$. This implies that $\Pi_{WTA}(\bar{\theta}) = 0$, whereas in the Σ equilibrium type $\bar{\theta}$ earns positive profits. In the WTA equilibrium each type θ earns higher WTA profits when the opponent is more likely to have higher costs. Thus, the bidders (at each θ) can expect better performance from the Σ equilibrium relative to the WTA equilibrium when the cost distribution is skewed to the low end.

These insights lead directly to sufficient (and necessary) conditions under which all participants prefer the Σ equilibrium to the WTA equilibrium.

PROPOSITION 5. Suppose that a Σ equilibrium exists. Then, under asymmetric information
(i) the Σ-equilibrium price is strictly less than the WTA-equilibrium price iff $2\underline{\theta}(1 - C) < \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)^2 \, d\theta$, and
(ii) the Σ-equilibrium profits are strictly greater than the WTA-equilibrium profits for all θ iff $E\{\hat{\theta}\} < 2\underline{\theta}(1 - C)$.

The conditions of (i) and (ii) in Proposition 5 hold for a variety of distributions and C values (e.g., Uniform on [0.75,1] with $C = 0.4$).

Proposition 5 shows that, in procurement environments with asymmetric cost information, the split award auction format can lead to a Pareto improvement relative to the WTA auction format. Asymmetric information is pivotal for this result: under full information the buyer and the suppliers necessarily have conflicting rankings of the two formats. To see this, consider the comparison under full information.

Over all split award auction equilibria under full information, Proposition 3 states that the highest-price σ equilibrium, $\Sigma(\theta_A,\theta_B)$, involves the highest price to the buyer and is also Pareto dominant for the suppliers. A simple price and profit comparison reveals that the full-information WTA equilibrium described in Proposition 4 involves a lower price to the buyer and lower profits for the suppliers than the Σ (θ_A,θ_B) equilibrium of the split award auction.

Recalling from Section III that price and profits are higher in split award auctions under full information than under asymmetric information (comparing Σ (θ_A,θ_B) and Σ equilibria), it is clear that a split award format under full information produces the highest price and profits over the four possible combinations of auction formats with information settings. Together, Propositions 4 and 5 provide the conditions that determine the ranking across formats of price and profits for the buyer and suppliers under asymmetric information.

D. Discussion

The 2 × 2 comparison of price and profits across the auction formats and information conditions leads to the following conclusion: while the buyer and supplier necessarily have conflicting preferences between a split award and WTA format under full information, in an environment with asymmetric information all parties can benefit from a split award format over a WTA format.

The reason the buyer and the suppliers have diametrically opposed interests between WTA and Σ (θ_A,θ_B) under full information is simple. In the Σ (θ_A,θ_B) equilibrium, full information regarding (θ_A,θ_B) allows for extensive equilibrium coordination of the bids, leading to high profits for both suppliers and a high price to the buyer. In contrast, in the WTA equilibrium, with (θ_A,θ_B) known, bidding cannot support profits for both suppliers. The lower-cost supplier receives a sole source award in the WTA

equilibrium, and the price to the buyer is held to the larger of θ_A and θ_B, which is always less than the Σ (θ_A, θ_B) price. Thus, under full information the buyer prefers WTA, while the suppliers prefer Σ (θ_A, θ_B).

Now consider how asymmetric information alters the comparison. By restricting the degree of bidding coordination, asymmetric information allows the Σ-equilibrium price to the buyer to fall below the WTA-equilibrium price. This does not imply, however, that the suppliers prefer the profits of the WTA equilibrium: the efficiency gains associated with the split award, as compared with the sole source awards, still allow all supplier types to earn higher profits in the Σ equilibrium. Thus, by restricting the extent of bidding coordination in a split award auction, asymmetric information allows the buyer to share in the efficiency gains that are denied to the buyer under full information.

V. BUYER STRATEGIES AND RESERVE PRICES

In this section we examine the effects of one set of buyer strategies—the use of reserve prices—on equilibrium outcomes in split award auctions with asymmetric information. The benefits a buyer can derive from reserve prices depend on the amount of information available to the buyer as well as the valuation the buyer places on the procurement in question. We begin the analysis under the simplifying assumption that the buyer has available all of the information that is common knowledge between the suppliers. The usefulness of reserve prices under weaker assumptions is discussed at the end of this section.

In general, a (preannounced) reserve price is a commitment by the buyer to reject a bid if the bid price exceeds the reserve price. For a split award auction with the three potential awards in $\{0, \sigma, 1\}$, one can imagine different reserve prices for each award and each bidder.[18] We explore the effects of a common reserve price of r across all three awards. Under a common reserve price a sole source award is rejected if the sole source price is above r, and the split award is rejected if the sum of split prices exceeds r. If all awards are rejected, then no award is made, and no payments are exchanged.

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18. For example, if the buyer establishes a reserve price of r for sole source awards, then the analogue of the conditions in Proposition 5 can be derived using standard results for unit auctions with a reserve price.

Consider the effect of a preannounced r on bidding when a Σ equilibrium exists. A relatively high reserve price $(r > P_\Sigma)$ is potentially binding only at the supporting sole source prices. This relaxes the upper-bound condition, $G(\theta, \underline{\theta}(1 - C)) \geq p(\theta)$, as the reserve price now provides an effective upper bound on sole source prices and mitigates the incentive to deviate and capture a sole source award. Thus, a relatively high reserve price reinforces the bidding structure of the Σ equilibrium without altering the payoffs.

An intermediate region, $2\bar{\theta}C \leq r \leq P_\Sigma$, simply results in a truncation of the set of σ equilibria to equilibria with split prices between $2\bar{\theta}C$ and r. The split is always selected as the award, and the reserve price replaces P_Σ as the highest equilibrium split price.

The interesting case, $r < 2\bar{\theta}C$, occurs when the reserve price is relatively low. Because r is less than $2\bar{\theta}C$ a σ equilibrium will not exist (see Proposition 1). In the Appendix we identify a class of equilibria that exists when r is less than $2\bar{\theta}C$. In these equilibria the award outcome is σ when both suppliers have relatively low costs, a sole source award when one supplier has much lower costs than the other, and no award when both have relatively high costs (see the Appendix for the formal analysis).

For such equilibria a reduction in r reduces the price for the buyer when an award is made. This establishes a tradeoff between the lower price when an award is made and the increased likelihood of not making an award, which is analogous to the tradeoff in a unit auction.

The choice of an optimal value for r depends on how the buyer assesses this tradeoff and, consequently, on the information that is available to the buyer. Under the assumption that the model structure is common knowledge among the buyer and the suppliers (only the realized cost types are private information), a reserve price of $r \leq 2\bar{\theta}C$ will be selected. This is because the buyer can push the reserve price down to $2\bar{\theta}C$ without any risk of not making an award. Reserve prices below $2\bar{\theta}C$ involve the above tradeoff, and the choice of r is a straightforward constrained minimization problem.

For many procurement settings, it is unreasonable to expect that the cost structure is common knowledge among the buyer and the suppliers. Buyers often cite inferior cost information as the reason for employing an auction, as this allows the procurement to be carried out via price comparisons. In the context of our model, for example, the suppliers may both know the actual value of C,

owing to better information regarding disturbances that influence the shape of short-run costs, while the buyer may view C as stochastic. The ability of the buyer to target the reserve price to the actual cost structure is then reduced because equilibrium prices (e.g., $P_\Sigma = 2\theta(1 - C)$) are now stochastic from the perspective of the buyer. In such a setting, the choice of an optimal reserve price depends on assessing the likelihood of outcomes involving no award, sole source awards at the WTA price, awards at the binding reserve price, and awards at the price P_Σ.[19]

VI. CONCLUSION

Auctions allow a buyer to bring competitive pressures to bear on suppliers even when the buyer knows little or nothing about the cost structure of the suppliers. When a number of units are being procured, the buyer can employ a WTA auction or a split award auction. A critical factor in this choice is the relative price performance of the two formats.

In this paper we have analyzed the price performance and developed the properties of split award auctions that allow one to begin to identify settings that favor the use of these auctions. In contrast to the situation with WTA auctions, implicit coordination of equilibrium bids by suppliers with private cost information is feasible in split award auctions.

Asymmetric information has the important effect of weakening the ability of the suppliers to coordinate, relative to the full-information case where suppliers can coordinate using each other's known costs. In a stable technological environment where long-time competitors are likely to have good information about a competitor's costs, coordination may be easy to achieve, and split award auctions will perform poorly from the viewpoint of the buyer. However, when innovation is a key competitive dimension, uncertainty introduced by the innovative process makes coordination in split award auctions more difficult and increases the attractiveness of a split award auction format to the buyer. This innovation element—a feature of the pre-bid activities of GE and Pratt and Whitney—may provide a partial explanation for the use

19. In many procurement settings (e.g., military procurement governed by an annual budget cycle), the prospect of no award is so onerous or cost information is so poor that an explicit reserve price is rarely used.

and claimed success of the split award auction format in the "Great Engine War."

APPENDIX

Proof of Proposition 1. We prove (1)–(3) in turn.

(1) $p_\sigma(\theta)$ is constant for all $\theta \in [\underline{\theta}, \bar{\theta}]$. If the bidding strategy $[p(\cdot), p_\sigma(\cdot)]$ is a σ equilibrium, then the equilibrium payoff to a type θ is $p_\sigma(\theta) - \theta C$. Consider two types $\theta \neq \hat{\theta}$, and suppose that $p_\sigma(\theta) > p_\sigma(\hat{\theta})$. The split σ is the outcome with probability one when both bidders adopt $[p(\cdot), p_\sigma(\cdot)]$. Hence, if type $\hat{\theta}$ bids $(p(\theta), p_\sigma(\theta))$, profits will be $p_\sigma(\theta) - \hat{\theta}C > p_\sigma(\hat{\theta}) - \hat{\theta}C$, and $\hat{\theta}$ could not have been at a best response with the split price $p_\sigma(\hat{\theta})$.

(2) $\bar{\theta}C \leq p_\sigma \leq \underline{\theta}(1 - C)$. The left-hand inequality follows upon noting that type $\bar{\theta}$ must earn a nonnegative equilibrium payoff. For the right-hand side consider $\underline{\theta}$ and the equilibrium payoff $p_\sigma - \underline{\theta}C$. Suppose that $\underline{\theta}$ deviates and bids $(\hat{p}, \hat{p}_\sigma) = (2p_\sigma - \epsilon, p_\sigma)$, where $\epsilon > 0$. When the other bidder is using $[p(\cdot), p_\sigma]$, this bid will result in a sole source award w.p.1 for $\underline{\theta}$ and a profit $2p_\sigma - \epsilon - \underline{\theta}$. In equilibrium this deviation cannot be profitable, so $2p_\sigma - \epsilon - \underline{\theta} \leq p_\sigma - \underline{\theta}C$, which reduces to $p_\sigma \leq \underline{\theta}(1 - C) + \epsilon$, and this must hold for all $\epsilon > 0$.

(3) $2p_\sigma = \inf_\theta p(\theta)$. In a σ equilibrium we must have $p(\theta) \geq 2p_\sigma$ for all θ. If strict inequality holds, then any type, say $\underline{\theta}$, can use the bid $(\hat{p}, \hat{p}_\sigma) = (\inf_\theta p(\theta), p_\sigma + 0.5(\inf_\theta p(\theta) - 2p_\sigma))$ which results in the award σ w.p.1 and, as $\hat{p}_\sigma > p_\sigma$, a higher profit than the equilibrium payoff of $p_\sigma - \underline{\theta}C$.

Proof of Proposition 2. Clearly, the buyer selects σ as the award w.p.1 since $p(\underline{\theta}) = 2p_\sigma$ and $p(\cdot)$ is strictly increasing. With this bidding strategy the profit for type θ is given by $p_\sigma - \theta C$. To show that $[p(\cdot), p_\sigma]$ is an equilibrium strategy, we must show that, for each $\theta \in [\underline{\theta}, \bar{\theta}]$, the bid $(p(\theta), p_\sigma)$ is a best response against $[p(\cdot), p_\sigma]$. Consider a bid $(\hat{p}, \hat{p}_\sigma)$ for type θ, where $\hat{p}$ is the sole source price and $\hat{p}_\sigma$ is the split price. There are three cases.

Case 1: $p_\sigma + \hat{p}_\sigma < \hat{p}$. Then θ does not receive a sole source award. Clearly, any bid with $\hat{p}_\sigma < p_\sigma$ is dominated, since $\hat{p}_\sigma = p_\sigma$ earns more profit and any $\hat{p}_\sigma \leq p_\sigma$ results in the award σ w.p.1. For $\hat{p}_\sigma > p(\bar{\theta}) - p_\sigma$, type θ will earn zero profit since the opponent would receive a sole source award w.p.1. This leaves bids with $\hat{p}_\sigma \in [p_\sigma, p(\bar{\theta}) - p_\sigma]$, which earn $\hat{\pi} = (\hat{p}_\sigma - \theta C) \Pr\{p(\hat{\theta}) > \hat{p}_\sigma + p_\sigma\}$. Since $p(\cdot)$ is continuous and monotone, there is a unique $\hat{\theta}$ defined by $\hat{p}_\sigma = p(\hat{\theta}) - p_\sigma$. Then we have $\hat{\pi} = [p(\hat{\theta}) - p_\sigma - \theta C][1 - F(\hat{\theta})]$. The definition of $G(\cdot, p_\sigma)$ and the property $G(\cdot, p_\sigma) \geq p(\cdot)$ imply the

inequality string,

$$p_\sigma[2 - F(\hat{\theta})] - \bar{\theta}CF(\hat{\theta}) \geq p(\hat{\theta})[1 - F(\hat{\theta})]$$

$$p_\sigma \geq [p(\hat{\theta}) - p_\sigma][1 - F(\hat{\theta})] + \bar{\theta}CF(\hat{\theta})$$

$$p_\sigma - \theta C \geq [p(\hat{\theta}) - p_\sigma - \theta C][1 - F(\hat{\theta})] + (\bar{\theta} - \theta)\, CF(\hat{\theta}).$$

Since $\bar{\theta} \geq \theta$, no case 1 deviation by θ earns more than the payoff with $(p(\theta),p_\sigma)$.

Case 2: $p_\sigma + \hat{p}_\sigma > \hat{p}$. Then σ is never the award. If $\hat{p} > p(\bar{\theta})$, then θ earns zero profit; and if $\hat{p} < p(\underline{\theta})$, then this bid is dominated since raising $\hat{p}$ slightly earns more profit while still ensuring a sole source outcome for θ. This leaves bids with $\hat{p} \in [p(\underline{\theta}),p(\bar{\theta})]$ which earn $\hat{\pi} = (\hat{p} - \theta) \Pr \{p(\bar{\theta}) > \hat{p}\}$. Defining $\hat{\theta}$ by $\hat{p} = p(\hat{\theta})$, we have $\hat{\pi} = [p(\hat{\theta}) - \theta][1 - F(\hat{\theta})]$. Subtracting $\hat{\pi}$ from the payoff available with $(p(\theta),p_\sigma)$ results in

$$\left[\frac{p_\sigma + \theta[1 - F(\hat{\theta}) - C]}{1 - F(\hat{\theta})} - p(\hat{\theta})\right][1 - F(\hat{\theta})].$$

To show that this is nonnegative for all $(\theta,\hat{\theta})$ pairs, it is sufficient to show that the first term in the brackets is greater than $G(\hat{\theta},p_\sigma)$ since we know that $G(\hat{\theta},p_\sigma) \geq p(\hat{\theta})$ for all $\hat{\theta}$. Using the definition of $G(\cdot,p_\sigma)$, this reduces to showing that $(p_\sigma + \bar{\theta}C - \theta)F(\hat{\theta}) \geq p_\sigma + \theta C - \theta$. Note that since $p_\sigma \leq \theta(1 - C)$, we have $p_\sigma - \theta(1 - C) \leq 0$. Thus, if $p_\sigma + \bar{\theta}C - \theta \geq 0$, then we are done since $F(\hat{\theta}) \geq 0$. If, instead, $p_\sigma + \bar{\theta}C - \theta < 0$, then we have $(p_\sigma + \bar{\theta}C - \theta)F(\hat{\theta}) \geq p_\sigma + \bar{\theta}C - \theta$, as $1 \geq F(\hat{\theta}) \geq 0$. Since $\bar{\theta} \geq \theta$, $p_\sigma + \bar{\theta}C - \theta \geq p_\sigma + \theta C - \theta$, and we are done. Thus, no case 2 deviation is profitable for θ.

Case 3: $p_\sigma + \hat{p}_\sigma = \hat{p}$. For type $\theta = \underline{\theta}$, the equilibrium bid is of this form. For $\theta > \underline{\theta}$, the bound $\theta(1 - C) \geq p_\sigma$ implies that $\hat{p}_\sigma - \theta C > \hat{p} - \theta$ when $p_\sigma + \hat{p}_\sigma = \hat{p}$, so θ strictly prefers the payoff at the split to the sole source payoff. Therefore, a case 1 bid with $p_\sigma + \hat{p}_\sigma < \hat{p}'$, where $\hat{p}' > \hat{p}$, dominates the case 3 bid with $p_\sigma + \hat{p}_\sigma = \hat{p}$ for type θ.

Since the three cases exhaust the possibilities for alternative bids for every type θ, the proposed bidding strategy $\{p(\cdot),p_\sigma\}$ is an equilibrium.

Proof of Proposition 3. By construction, $p_i = p_{i\sigma} + p_{j\sigma} = p_j$ so that the buyer faces the same total price for each award in $\{0,\sigma,1\}$ and σ is an optimal choice (see footnote 15 regarding tiebreaking rules). Thus, the buyer pays the price $P = (\theta_A + \theta_B)(1 - C)$, and profits are given by $\pi_i = \theta_j(1 - C) - \theta_i C$. Note that $\pi_i > 0$ follows from $\bar{\theta}C < \underline{\theta}(1 - C)$. The bids are in equilibrium if each supplier is

at a best response: neither of $i = A, B$ can increase profits by using an alternative bid $(\hat{p}, \hat{p}_\sigma)$. There are three possibilities.

If sole sourcing with j is the outcome when i uses $(\hat{p}, \hat{p}_\sigma)$ and j uses $(p_j, p_{j\sigma})$, then i earns zero profit. If σ is the outcome chosen by the buyer, then we must have $\hat{p}_\sigma + p_{j\sigma} \leq p_j$, and i earns $\hat{\pi}_i = \hat{p}_\sigma - \theta_i C \leq p_j - p_{j\sigma} - \theta_i C$. Substituting the equilibrium bids yields $\hat{\pi}_i \leq \theta_j (1 - C) - \theta_i C$ so that $\hat{\pi}_i$ is no larger than profits from $(p_i, p_{i\sigma})$. If sole sourcing with i is the outcome, then we must have $\hat{p} \leq p_j$, and substitution again yields that $\hat{\pi}_i$ is bounded above by profits from $(p_i, p_{i\sigma})$. Thus, i is at a best response.

Suppose that there is an equilibrium where the buyer pays a higher price than P, say $P + \epsilon$ for $\epsilon > 0$. Each supplier i always has the option of using a bid that offers the buyer a better price, as with the bid $\hat{p} = P + \epsilon - \hat{\epsilon}$ and $\hat{p}_\sigma = P + \epsilon$ for $\hat{\epsilon} < \epsilon$. This bid induces a sole source award for i and profits $\hat{\pi}_i = P + \epsilon - \hat{\epsilon} - \theta_i C$. From $\overline{\theta} C < \theta(1 - C)$ and the definition of P, we see that $\hat{\pi}_i > 0$. Thus, each i must earn positive profits in the equilibrium with price $P + \epsilon$, and therefore, it must be that σ is the award. Also, each i must earn at least $\hat{\pi}_i$ in the equilibrium. From the accounting identity of buyer price equals the sum of supplier profits and costs, we have $P + \epsilon \geq (\hat{\pi}_A + \theta_A C) + (\hat{\pi}_B + \theta_B C)$. Substitution then yields $P + \epsilon \geq 2(P + \epsilon) - 2\hat{\epsilon} - (\theta_A + \theta_B)(1 - C)$. Recalling the definition of P, this reduces to $2\hat{\epsilon} \geq \epsilon$. This must hold for any $\hat{\epsilon} < \epsilon$, but it fails for $\hat{\epsilon} < \epsilon/2$. Therefore, P is the highest equilibrium price.

Now we show that there is no equilibrium where i earns profits greater than $\theta_j (1 - C) - \theta_i C$. Since the equilibrium bids achieve this profit for each i, this equilibrium is Pareto dominant. To prove this, suppose that there is an equilibrium where i earns profits $\hat{\pi}_i > \theta_j (1 - C) - \theta_i C$. Let $\hat{P}$ denote the price to the buyer in this equilibrium, and let $(\hat{p}_i, \hat{p}_{i\sigma})$ denote the bids. Since $\hat{\pi}_i > 0$, it must be that σ is the award or that i receives a sole source award. If i receives a sole source award, then we have $\hat{P} = \hat{\pi}_i + \theta_i > \theta_j (1 - C) - \theta_i C + \theta_i = P$. From above we know that there is no equilibrium with a price $\hat{P} > P$. Thus, the award must be σ. Then $\hat{P} = \hat{p}_{i\sigma} + \hat{p}_{j\sigma}$. Since j can always obtain a sole source award at a price $\hat{P} - \epsilon$, any $\epsilon > 0$, equilibrium requires that $\hat{P} - \epsilon - \theta_j \leq \hat{p}_{j\sigma} - \theta_j C$ and, using $\hat{P} \equiv \hat{p}_{i\sigma} + \hat{p}_{j\sigma}$, this means that $\hat{p}_{i\sigma} - \epsilon \leq \theta_j (1 - C)$ for any $\epsilon > 0$. Thus, letting $\epsilon \to 0$, $\hat{\pi}_i = \hat{p}_{i\sigma} - \theta_i C \leq \theta_j (1 - C) - \theta_i C$, and we are done.

Proof of Proposition 5. Since $P_\Sigma = 2\theta(1 - C)$ and $P_{WTA} = \overline{\theta} - \int_{\underline{\theta}}^{\overline{\theta}} F(t)^2 \, dt$, (i) is trivial. Consider (ii). For the bidders recall that $\Pi_\Sigma(\theta) = \underline{\theta}(1 - C) - \theta C$. Defining the difference in profits

as $\Delta(\theta) = \Pi_\Sigma(\theta) - \Pi_{WTA}(\theta)$, substituting from above, and simplifying yield $\Delta(\theta) = (\theta + \underline{\theta})(1 - C) - \bar{\theta} + \int_{\underline{\theta}}^{\bar{\theta}} F(t)\, dt$. Differentiation reveals that $\Delta(\theta)$ is strictly concave, and by the existence condition for the Σ equilibrium, we have $\Delta(\bar{\theta}) > 0$. Then, by concavity we have $\Delta(\theta) > 0$ for all θ if and only if $\Delta(\underline{\theta}) > 0$. Integrating by parts, we obtain $\Delta(\underline{\theta}) = P_\Sigma - E\{\hat{\theta}\}$. Hence, $\Pi_\Sigma(\theta) > \Pi_{WTA}(\theta)$ for all θ iff $2\underline{\theta}(1 - C) > \bar{E}\{\hat{\theta}\}$.

Equilibrium Bidding with a Reserve Price. Proposition 6 describes equilibrium bidding with a reserve price of $r < 2\bar{\theta}C$ (for the analogue of the highest-price σ equilibrium). The proposition also describes equilibrium bidding for the case when $r \le \underline{\theta}(1 - C)$ but the existence condition for a σ equilibrium, $(\bar{\theta}C \le \underline{\theta}(1 - C))$, fails.

PROPOSITION 6. Suppose that $\underline{\theta}C < r/2 < \underline{\theta}(1 - C)$. The following bidding strategy is an equilibrium for the split award auction with reserve price r:

$$p_\sigma(\theta) = \begin{vmatrix} \dfrac{r}{2} & \theta \le \dfrac{r}{2C} \\[2mm] H_\sigma(\theta) & \theta > \dfrac{r}{2C} \end{vmatrix} \qquad p(\theta) = \begin{vmatrix} r & \theta \le r \\[2mm] H(\theta) & \theta > r, \end{vmatrix}$$

where $H(\cdot)$ and $H_\sigma(\cdot)$ are any $F(\cdot)$-measurable functions satisfying $H(\theta) \ge \theta$ and $H_\sigma(\theta) \ge \max\{h_\sigma(\theta), \theta C\}$, and $h_\sigma(\theta)$ is defined as

$$h_\sigma(\theta) \equiv r - \underline{\theta}C - \left(\dfrac{r}{2} - \underline{\theta}C\right) F\left(\dfrac{r}{2C}\right) \dfrac{1}{F(\theta)}.$$

Proof of Proposition 6. Comparing bid prices, it is easy to see that the buyer is always at an optimal choice with the award pattern in Figure I. Because $\underline{\theta}(1 - C) > r/2$ implies that $r - \underline{\theta} < r/2 - \underline{\theta}C$ for all θ, the incentives of the bidders dictate that any tie between the split and sole source awards is resolved by choosing the split. Proposition 6 applies for all values of $\bar{\theta}C$. The proof is simple when $\bar{\theta}C \le r/2$ because we can apply the proof of Proposition 2 with only minor changes. The proof is more involved when $\bar{\theta}C > r/2$ and, although the logic is straightforward, a number of cases must be considered. The following Lemma is used repeatedly.

LEMMA. Suppose that b is any number that satisfies $\underline{\theta}C < b < r/2$. Then for any $\theta \in [\underline{\theta}, r/(2C)]$ we have $(b - \underline{\theta}C) \Pr\{p_\sigma(\hat{\theta}) \le r - b\} \le (r/2 - \underline{\theta}C)F(r/(2C))$.
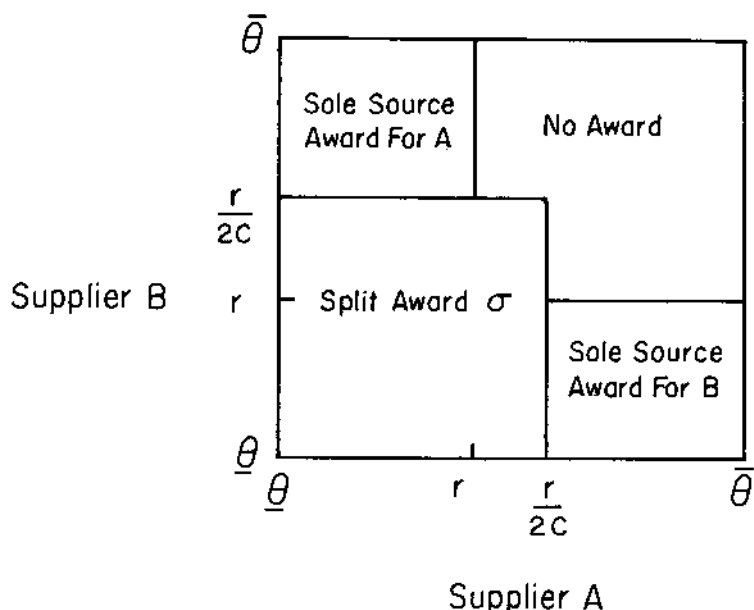
FIGURE I
Award Outcomes in Equilibrium When the Reserve Price is r

Proof of the Lemma. By assumption, $r - b > r/2$. By construction of $p_\sigma(\cdot)$, we then have $p_\sigma(\bar\theta) \le r - b$ for all $\bar\theta \le r/(2C)$. When $\bar\theta > r/(2C)$, we have $p_\sigma(\bar\theta) = H_\sigma(\bar\theta)$. Again, by construction we have $H_\sigma(\bar\theta) \ge \max\{h_\sigma(\bar\theta), \bar\theta C\} \ge h_\sigma(\bar\theta)$, and consequently, $h_\sigma(\bar\theta) \le r - b$ is implied by $p_\sigma(\bar\theta) \le r - b$ when $\bar\theta \ge r/(2C)$.

Using the definition of $h_\sigma(\cdot)$ and noting that $b > \underline\theta C$, we find that the inequality $h_\sigma(\bar\theta) \le r - b$ is equivalent to the inequality

(A1)
$$\bar\theta \le F^{-1}\left(\frac{0.5r - \underline\theta C}{b - \underline\theta C} F\left(\frac{r}{2C}\right)\right).$$

From above, the event $\{\bar\theta | p_\sigma(\bar\theta) \le r - b\}$ occurs iff $\{\bar\theta | (A1)$ holds$\}$ occurs. Consequently, $\Pr\{p_\sigma(\bar\theta) \le r - b\} \le [b - \underline\theta C]^{-1} \times [0.5r - \underline\theta C]F(0.5rC^{-1})$. Now note that $[0.5r - \underline\theta C]/[b - \underline\theta C]$ is a continuous and increasing function of $\underline\theta$ as $\underline\theta$ ranges from $\underline\theta$ to b/C. This establishes the Lemma when $\underline\theta \in [\underline\theta, b/C)$.

When $\underline\theta \in [b/C, r/(2C)]$, we have $b - \underline\theta C \le 0 \le r/2 - \underline\theta C$ and at least one of these inequalities is strict. The Lemma follows immediately for this case since probabilities are nonnegative. This completes the proof of the Lemma.

To verify the equilibrium, we must show that no bid $(\hat{p},\hat{p}_\sigma) \neq (p(\theta),p_\sigma(\theta))$ yields higher expected profits for any θ type. There are three cases for θ.

Case 1: $\theta > r/(2C)$. In the equilibrium, θ never receives an award and always earns zero profits. It is easy to show that any bid $(\hat{p},\hat{p}_\sigma)$ by θ that results in an award will lead to nonpositive profits. Thus, the equilibrium bid for θ is a best response.

Case 2: $r < \theta \le r/(2C)$. In the equilibrium θ earns an expected profit of $(r/2 - \theta C)F(r/(2C))$. Consider a bid $(\hat{p},\hat{p}_\sigma)$ by θ, and begin with the case of $\hat{p} > r$. Since the reserve price is r, θ never receives a sole source award. Now, if $\hat{p}_\sigma > r/2$, then $\hat{p}_\sigma + p_\sigma(\hat{\theta}) > r$ $\forall\hat{\theta}$, and θ never receives a split award, thus netting zero profits. If $\hat{p}_\sigma \le r/2$, then θ receives a split award when $\hat{p}_\sigma + p_\sigma(\hat{\theta}) \le r$, thus netting expected profits of $(\hat{p}_\sigma - \theta C)\Pr\left[p_\sigma(\hat{\theta}) \le r - \hat{p}_\sigma\right]$; $\hat{p}_\sigma = r/2$ maximizes this expected profit, and applying the Lemma, we see that the expected profit is weakly less than the equilibrium expected profit.

It remains to verify that θ cannot benefit when the bid $(\hat{p}, \hat{p}_\sigma)$ involves $\hat{p} \le r$. Note that $\hat{p} - \theta < 0$ by the case hypothesis, and consequently, a sole source award for θ entails a negative profit. Now, if $\hat{p} - r/2 < \hat{p}_\sigma$, then $\hat{p} < \hat{p}_\sigma + r/2 \le \hat{p}_\sigma + p_\sigma(\hat{\theta})$ $\forall\hat{\theta}$, and σ is never the award. Since $p(\hat{\theta}) > r$, when $\hat{\theta} > r$, θ receives a sole source award with positive probability and earns a negative expected profit. If $\hat{p} - r/2 \ge \hat{p}_\sigma$, then we have $\hat{p}_\sigma + r/2 \le \hat{p} \le r$, and, w.p.1, θ receives a split award or a sole source award. Thus, overall negative expected profits are guaranteed if $\hat{p}_\sigma \le \theta C$, as the split profit is nonpositive. When $\hat{p}_\sigma > \theta C$, however, the bid $(\hat{p},\hat{p}_\sigma)$ is dominated by $(p',\hat{p}_\sigma)$, where $p' = \theta$, since this avoids negative sole source profits. As $p' > r$, we have therefore shown that no bid $(\hat{p},\hat{p}_\sigma)$ with $\hat{p} \le r$ can achieve expected profits for θ that are above equilibrium expected profits.

Case 3: $\theta \le r$. Profits for θ are $(r/2 - \theta C)F(r/(2C)) + (r - \theta)(1 - F(r/(2C)))$ in equilibrium. As before, we examine the various cases for $(\hat{p},\hat{p}_\sigma)$. For $(\hat{p},\hat{p}_\sigma)$ with $\hat{p} > r$, the proof is analogous to the parallel bid in case 2.

Now consider $(\hat{p},\hat{p}_\sigma)$ with $\hat{p} = r$. For $\hat{p}_\sigma > r/2$, θ receives a sole source award w.p.1 and earns a profit of $r - \theta$. Since $r/2 < \theta(1 - C)$, we have $r - \theta < r/2 - \theta C$, and therefore, $r - \theta$ is smaller than the equilibrium profit. For $\hat{p}_\sigma < r/2$, θ earns expected profits of $(\hat{p}_\sigma - \theta C)\Pr\left[p_\sigma(\hat{\theta}) \le r - \hat{p}_\sigma\right] + (r - \theta)\Pr\left[p_\sigma(\hat{\theta}) > r - \hat{p}_\sigma\right]$. Applying the Lemma to the first term and noting that $\Pr\left[p_\sigma(\hat{\theta}) > r - \hat{p}_\sigma\right]$ is

bounded above by $1 - F(r/(2C))$ when $\hat{p}_\sigma < r/2$, we see that this expected profit is bounded above by the equilibrium profit. Finally, consider $(\hat{p}, \hat{p}_\sigma)$ with $\hat{p} < r$. For $\hat{p}_\sigma + r/2 > \hat{p}$, θ receives a sole source award w.p.1 and earns $\hat{p} - \theta$ in expected profit. Since $\hat{p} < r$, this is less than equilibrium profits. For $\hat{p}_\sigma + r/2 \le \hat{p}$, θ receives a split award when $p_\alpha(\theta) + \hat{p}_\sigma \le \hat{p}$ and a sole source award otherwise, thus netting expected profits $(\hat{p}_\sigma - \theta C)$ $\Pr\left[p_\alpha(\theta) \le \hat{p} - \hat{p}_\sigma\right] + (\hat{p} - \theta)\Pr\left[p_\alpha(\theta) > \hat{p} - \hat{p}_\sigma\right]$. For the first term, define $p'_\sigma = r - \hat{p} + \hat{p}_\sigma$, and apply the Lemma with p'_σ, which necessarily satisfies $\hat{p}_\sigma < p'_\sigma \le r/2$, to see that the equilibrium bid yields a greater expected profit.

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