Quality, Upgrades, and Equilibrium in a Dynamic Monopoly Model

James Anton and Gary Biglaiser

Duke and UNC

November 5, 2010

Introduction

- What do we know about dynamic durable goods monopoly?

Most work is on a good of a single quality- lit on Coase Conjecture

Most goods do not fit single quality good framework.

Examples

- ‘Upgrade’ Goods
  - Software:
    - Operating Systems (Microsoft),
    - Applications (Scientific Word, Adobe)
  - Commercial Airplane Manuf (Boeing, Airbus)
  - Defense Systems: Planes, Ships
  - Cellular Networks
- ‘Independent’ Goods
  - Computer, Television, Car

This paper studies upgrade goods

- Important market features
  - Infinite horizon environment
  - Firm - new quality increments to sell in the future
  - Firm - can offer any bundles of quality increments
  - Buyer - private information
  - Consumers need previous quality increments for next increment to be valuable (upgrade)

Key questions:

- What determines the equilibrium division of surplus?
- Will the market be efficient?

Answers hinge on buyer credible threat

- Rests on the ability of seller to tempt a buyer to buy (jump ahead of market) when others do not

Preview of Main Result:

- Market with homogeneous buyers- cleanest environment to understand pricing
- For efficient equilibrium: Any division of surplus between the one period flow value of a good and its PDV is an equilibrium for ANY discount factor
- For high enough discount factors, inefficient equilibrium exist, and buyers always get positive surplus and seller more than flow value
- Key: Growth in surplus + buyer implicit coordination leads to possible loss in market power. It gives buyers credible threat.
- Examples: Microsoft ME and Vista

Policy Implications:

- US-Microsoft anti-trust case of the late 1990s - Did Microsoft have monopoly power?
- Fudenberg and Tirole (2000) “Both sides in US vs. Microsoft agree that Microsoft’s pricing of Windows does not correspond to short-run profit maximization by a monopolist.”
- Schmalensee (Microsoft)- fear of entry- limit pricing argument
- Fisher and Rubinfeld (Government) - network effects of consumers having the same operating systems
- Other economists - buy Microsoft’s application programs
- We offer a different interpretation

Outline for rest of talk:

- Model
- Benchmarks
- Efficient Equilibria
- Inefficient Equilibria
- Discussion
- Conclusion

Model

Infinite horizon $t = 1, 2, ..., \infty$

Monopolist:

- A new quality increment in each period
- No commitment to future pricing decisions
- Production costs are 0
- Can offer any feasible set of qualities in a period
- Maximizes discounted (δ) profit

Buyers

- Buyers:
- Measure one of identical consumers $[0, 1]$
- v flow value of a unit of quality
- For a quality increment to be valuable in a period, buyer must possess all previous increments (upgrade structure)
- Common discount factor δ
- Maximize expected discounted utilities:
  - value from quality - payments

Information and Timing

- All cost and valuations are known
- Any price or bundle is available to any consumer (no conditioning on *individual* behavior)
- All players know aggregate quality shares

Timing

- Each period - firm offers bundles and prices for the bundles
- Buyers then decide which, if any, bundles to purchase

Interpretation

- Value flow v - marginal utility and quality increment
- Discount factor δ - time preference and innovation frequency

Example of Path

Period 1 - Seller offers Unit 1 for p_1 purchased

$\Rightarrow$ flows of p_1 for seller and $v - p_1$ for each buyer

Period 2 - Units $\{1, 2\}$ feasible, seller makes no offer

$\Rightarrow$ flows of 0 for seller and v for each buyer

Units $\{1, 2, 3\}$ feasible, seller offers bundle $\{2, 3\}$ for p_3, buyers purchase

$\Rightarrow$ flows of p_3 for seller and $3v - p_3$ for each buyer

Continue on to later periods

Seller payoff of $p_1 + \delta^2 p_3 + ...$

Buyer payoff of $(v - p_1) + \delta v + \delta^2 (3v - p_3) + ...$

Efficiency and Surplus

For an efficient equilibrium, buyers acquire each unit when first available

Efficiency: Buyers acquire new unit of quality in each period

PDV of flows on efficient path:

$v + \delta 2v + \delta^2 3v + ...$

$= v(1 + \delta + \delta^2 + ...)$ unit 1

$+\delta v(1 + \delta + \delta^2 + ...)$ unit 2

$+ ... = \frac{v}{1-\delta}(1 + \delta + \delta^2 + ...) = \frac{v}{(1-\delta)^2} = S_1$

Equilibria

Markov Perfect Equilibria

- Stationary
- Simple cyclical structure
- Flexible enough to generate entire subgame perfect payoff range for both efficient and inefficient eq.
- State (t, q), t is maximal feasible quality, q highest quality held at start of period
- Players condition strategies on $(t - q)$ "quality gap"
- Implications: Past prices and path of qualities do not matter to players' strategies

Benchmarks

Efficient Allocation and Buyer Extraction

1. Finite Horizon $T > 1$.
    - Does not depend on number of buyers, stationarity, upgrade/independent units
2. Infinite Horizon, Single Buyer, Quality Growth
    - No buyer coordination issue
3. Infinite Horizon, Continuum of Buyers, No Growth
    - Special case of FLT 85

Finite Horizon

- Final Period T: state of the form (T, q_{T-1})
  Upgrade from q_{T-1} to T at extraction price
- Period $T-1$: state of the form $(T-1, q_{T-2})$
  Upgrade from q_{T-2} to $T-1$ at extraction price
  Buyers expect no future surplus increment
  Path to final period state $(T, T-1)$
- Work backwards to period 1
- Efficient path and surplus extraction

1 Buyer and Quality Growth

- No delay in equilibrium - "speed up" argument
- Example - No sale in period 1, then sell 2 units in period 2 for p and cycle

$$\pi_1 = \delta p + \delta^2 \pi_1$$

$$u_1 = \delta(2v - p) + \delta^2 \left[\frac{2v}{1-\delta} + u_1 \right]$$

- Seller can offer unit 1 in period 1 for $\widehat{p}$, buyer accepts and seller increases payoff if

$$v - \widehat{p} + \frac{\delta v}{1-\delta} + \delta u_1 > u_1 \text{ and } \widehat{p} + \delta \pi_1 > \pi_1 \Leftrightarrow$$

$$\frac{v}{(1-\delta)^2} > u_1 + \pi_1$$

- $\Rightarrow$ Efficient path - sell current unit
- $\Rightarrow$ Continuation outcome in any $(t, 0)$ is sale of t units
- $\Rightarrow$ Buyer extracted at price $p_1 = \frac{v}{1-\delta}$

If infinite horizon, continuum, no growth, special case of FLT '85

Benchmark message - Efficiency and Extraction if either finite horizon, finite buyers, and no growth

Basic Results: Flow Dominance

? - If seller offers t units at price $p < vt$ in state $(t, 0)$

All buyers must accept- current surplus, future options

$\Rightarrow$ Lower bounds on seller payoff

$$\pi_1 \geq v + \delta v + ... = \frac{v}{1-\delta}$$

$$\pi_t \geq vt + \delta \pi_1 = vt + \frac{\delta v}{1-\delta}$$

Flow dominance

$$\Rightarrow 0 \leq u_1 \leq \delta S_1$$

extraction $\nearrow$ $\nwarrow$ *static one period monopoly*

Basic Results: Cycles

t-cycle equilibrium- a sale occurs every t periods, and t units are sold in each sale period

Proposition

Every equilibrium is a $t-$cycle equilibrium.

Why? pure 'speed-up', but must have $\tau < t$

No implication of a sale in every period

Argument breaks down when $\tau = t > 1$

- Feasibility
- Not necessarily optimal for an individual buyer to accept the offer

Speed-Up Intuition:

Suppose t is date of first sale but only $\tau < t$ units

In $t-1$, seller offers τ for price $\hat{p} = v\tau + \delta p - \epsilon$

Individual buyer accepts even if others reject:

$$(v\tau - \hat{p}) + \delta v\tau + \delta^2 u(t+1,\tau) > 0 + \delta\,(v\tau - p) + \delta^2 u(t+1,\tau)$$

Seller offer successfully speeds up path

$$\hat{p} + \delta\pi_{t-\tau} = \hat{p} + \delta^2\pi_{t-\tau+1} > 0 + \delta p + \delta^2\pi_{t-\tau+1}$$

Efficient Equilibria

New quality units sold immediately at price p_1

equilibrium path of $(1, 0) \rightarrow (2, 1) \rightarrow (3, 2) \rightarrow \ldots$

Need to specify continuation payoffs

propose '*cash-in*' support off-equilibrium-path

in $(\tau, 0)$ have sale of τ units at price p_τ

It must be optimal for the seller to offer τ at p_τ

versus delay or partial cash-in

Buyer strategies follow simple cut-off rule:

accept σ units in state $(\tau, 0)$ iff $p \leq p(\sigma, \tau)$

Must hold for all $\tau \geq 2$ and cut-off rules $p(\sigma, \tau)$ for all $\sigma \leq \tau$ (and $\tau = 1$)

Example 1: Efficient Equ. (constant utility support)

- Equ. path - sell new unit at price $p_1 \Rightarrow$ payoffs

$$\pi_1 = p_1 \left(1 + \delta + \delta^2 + ...\right) = \frac{p_1}{1-\delta}$$

$$\begin{aligned} u_1 &= (v - p_1) + \delta\,(2v - p_1) + \delta^2\,(3v - p_1) + ... \\ &= \frac{1}{1-\delta}\left(\frac{v}{1-\delta} - p_1\right) \end{aligned}$$

- Support - prices rise by $\frac{v}{1-\delta} \Rightarrow u \equiv u_1 = u_2 = ...$
  If delay, then seller is residual claimant of growth
- Delay incentive $\Rightarrow u(1-\delta) < v$
  v is loss from delay (surplus) and $(1-\delta)\,u$ is gain

- ? Why buyers refuse $\hat{p} = p_1 + \epsilon$, if all others reject $\Rightarrow \delta u_2$
  If individual buyer accepts: $v - \hat{p}$ today plus option

  $$\max\left\{\frac{v}{1-\delta}, u\right\} = \frac{v}{1-\delta}$$

  So must have

  $$\delta u > \left[\frac{v}{1-\delta} - p_1\right] = (1-\delta)u \Leftrightarrow \delta > \frac{1}{2}$$

- Interpret - coordinate on share of first unit surplus
  $\Rightarrow$? why not coord on Units $2, 3, \ldots$

Summary: Example 1

- Constant utility support $\Rightarrow$ seller residual claimant
- Delay incentive limits buyer payoff
  Coord on rejecting high prices for positive payoff
  Extraction is special case where $u = 0$
  Positive buyer payoffs in equilibrium
- ? Potential for coord on future surplus

Efficient Equ - Analysis

- Buyer Strategies (symmetric): cut-offs $p(\sigma, \tau)$

If seller offers σ units upgrade for p in state $(\tau, 0)$

When all other buyers accept, individual payoff

$$v\sigma - p(\sigma,\tau) + \delta\left[\frac{v\sigma}{1-\delta} + u_{\tau+1-\sigma}\right] \text{ (accept)} \qquad 0 \text{ (reject)}$$

Thus, equ. $\Rightarrow$ upper bound

$$\frac{v\sigma}{1-\delta} + \delta u_{\tau+1-\sigma} \geq p(\sigma,\tau)$$

Fall behind path $\Rightarrow$ zero (inessential)

When all other buyers reject, individual buyer payoff

$$\delta u_{\tau+1} \text{ (reject) } v\sigma - p + \delta \max\left\{\frac{v\sigma}{1-\delta}, u_{\tau+1}\right\} \text{ (accept).}$$

Define '*Net option value*'

$$g(\sigma, u) \equiv v\sigma + \delta \max\left\{\frac{v\sigma}{1-\delta}, u\right\} - \delta u$$

Then, cut-off rules require

$$g(\sigma, u_{\tau+1}) \leq p(\sigma, \tau) \leq \frac{v\sigma}{1-\delta} + \delta u_{\tau+1-\sigma}$$

- "Price Wedge" Always exist
- Buyer Implicit Coordination

$$u_{\tau+1} > \frac{v\sigma}{1-\delta} \Rightarrow g = v\sigma$$

pushes net option value down to flow surplus

- Given buyer responses, seller must find it optimal to offer τ units at price p_τ in state $(\tau, 0)$.
- Seller deviations:
  delay via $\sigma = 0$,
  partial cash-ins via $1 \leq \sigma \leq \tau - 1$
  offer τ upgrade at different price from p_τ.
- Seller optimality requires

$$\pi_\tau \geq p(\sigma, \tau) + \delta \pi_{\tau+1-\sigma}$$

for $\sigma = 0, 1, ..., \tau$

Support Condition - combine buyer and seller

- Recall S_τ is total available surplus.

$$S_\tau = \frac{v\tau}{1-\delta} + \delta S_1 \qquad S_1 = \frac{v}{(1-\delta)^2}$$

- Cash-in support (efficient path) has

$$S_\tau = \pi_\tau + u_\tau$$

- Support conditions that need to be satisfied

$$S_\tau - \delta S_{\tau+1-\sigma} \geq u_\tau - \delta u_{\tau+1-\sigma} + g(\sigma, u_{\tau+1})$$

for all $\sigma \leq \tau$ and all $\tau \geq 1$

Claim:

- We can support maximal range of equ. payoffs

$$\text{all} \qquad u_1 \in [0, \delta S_1]$$

  (recall *flow dominance* bound for seller).
- Introduce *T-Stage Support*

$$u_\tau = v\tau + \delta u_{\tau+1} \qquad \text{for } (u_1, ..., u_T)$$

$$u_\tau = u_T \qquad \text{for larger } \tau$$

- Keeps seller indifferent delay versus cash-in
  ? Why - flow surplus to buyers
- Must truncate eventually: if not, support $\sigma = \tau$ is

$$\begin{aligned} S_\tau - \delta S_1 &\geq u_\tau - \delta u_1 + g(\tau, u_{\tau+1}) \Rightarrow \\ \delta u_1 &\geq v\tau \end{aligned}$$

  at large τ and this will fail.
  ? Why - flow dominance offer

Key Properties for *T-Stage Support*

- At stage T, seller strictly prefers to make cash-in offer

$$u_T < \frac{vT}{1-\delta}$$

- At stages $\tau < T$, buyers willing to pay no more than $v\tau$ (flow value) if others reject

$$u_\tau > \frac{v\tau}{1-\delta}$$

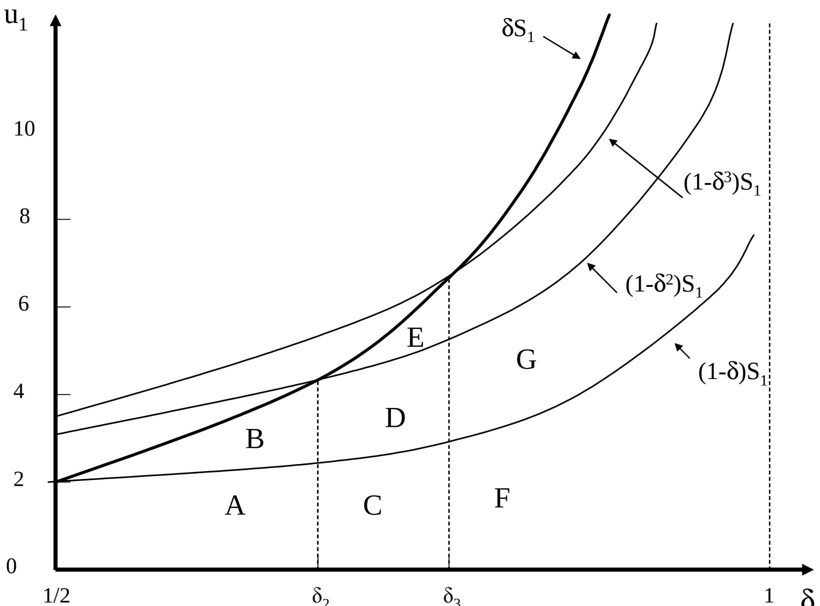
- Need to verify support conditions
- Need to find length T relative to u_1 and δ

Choosing Length T

- Pick utility level between 0 and δS_1.
- If $u_1 \leq (1-\delta)S_1$, then $u_\tau = u_1$ for $\tau > 1$. $\quad [T = 1]$
- If $u_1 \in [(1-\delta)S_1, \delta S_1]$, set u_2 via $u_1 = v + \delta u_2$.
- If $\delta S_1 \leq (1-\delta^2)S_1$, then $u_\tau = u_2$ for $\tau > 2$. $\quad [T = 2]$
- If not, set u_3 via $u_2 = 2v + \delta u_3$.
- Keep following logic until reach T where

$$(1-\delta^{T-1})S_1 \leq \delta S_1 \leq (1-\delta^T)S_1$$

Figure for T-Stage Support



Verifying equ. support conditions:

- If support holds at T it holds at all $\tau > T$
  utility is in constant range; now finite #
- '*Cash-in*' incentive sufficient for $\tau \leq T$
- Then choose T to satisfy '*Cash-in*'

Proposition

Every buyer payoff $u_1 \in [0, \delta S_1]$ can be supported in an efficient equilibrium if $\delta \in [1/2, 1]$.

Corollary

If $\delta \in [1/2, 1]$, then $\pi_1 \in [S_1(1-\delta), S_1]$

- Interpret - as if seller only has static monopoly power
  - each unit sold for price of v
  - no ability to capture future value

Corollary

The buyer share of the surplus, $\frac{u_1}{S1}$ is between 0 and δ.

Discussion: payoffs relative to total surplus.

- Case: $\delta < 1/2$

Can support any buyer payoff from 0 to δS_1

? Why special - when

$$1 > 2\delta$$

$\Rightarrow$ 1 unit now dominates 2 units tomorrow.

Delay and inefficient equilibria

- Every equilibrium is a $t-$cycle equilibrium
- No sales in periods 1 through $t-1$, then sell t units at p_t in period t
- Approach conditions- Minimum $\delta-$ prevent early cash-in
- No delay equ if $\delta < 1/2$
- Buyers must receive positive utility$\Rightarrow$
  if observe delay and bundling, then buyers are *not* extracted
- Sellers must get more than flow payoff
- Thus, payoff bounds are compressed relative to efficient eq.

Discussion

- Bundling in Practice
    - Sometimes the good is just added onto existing version (upgrade)- adding existing programs to a machine
    - Other times, the good is completely replaced - new software version
    - There is not necessarily a technological reason - Microsoft anti-trust case and the browser, PDF for Word or Sci Word
- Generation version with price contingency same as an upgrade version - same set of equilibria
- Results robust to network, lack of compatibility, and adoption costs
  harder to get consumer to jump ahead
- Unbreakable versions - hurts market power Fishman & Rob
- Independent Goods

Future Research

- Price Discrimination
- Innovation
  - Rate of Innovation
  - Scope of IP