e-Fulfillment: Vertical vs. Horizontal Cooperation

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One of the biggest challenges all Internet retailers face is the e-fulfillment process. This differs from the traditional retail demand fulfillment process. There, customers visit a physical store and obtain products from the store's inventory directly. The Internet stores need to consider the avenue through which products are delivered to customers. One common approach is drop-shipping, in which the e-tailer sells products to consumers at a retail price and then ships them directly from the supplier (or its warehouse) at a wholesale price. The drop-shipping supply chain structure, which is employed by many catalog companies, is widely adopted by Internet retailers. A survey of online retailing (Eretailing World 2000) indicates that 30.6 percent of pure e-tailers use drop-shipping as their primary way to satisfy demand. However, there is an emerging trend where e-tailers direct their customer demand to traditional retailers (so-called demand referral), and thus outsource the demand fulfillment process to traditional retailers. While drop-shipping seeks vertical cooperation in the supply chain, demand referral seeks horizontal cooperation. We investigate the virtues and shortcomings of each kind of fulfillment approach.

(CUSTOMER ACQUISITION; INVENTORY ALLOCATION; COORDINATION)

1 Introduction

One of the biggest challenges all Internet retailers face is the e-fulfillment process. This differs from the traditional retail demand fulfillment process. There, customers visit a physical store and obtain products from the store's inventory directly. The Internet stores need to consider the avenue through which products are delivered to customers. As startup companies, most pure Internet retailers find it difficult to initiate or cover the huge investment in inventory management. Thereby, virtual demand fulfillment, drop-shipping, is popular among Internet retailers[1]. With drop-shipping, the Internet retailer obtains customer orders and then requests a manufacturer or supplier to ship the product directly to customers. By seeking this approach of vertical cooperation with the upstream supplier, the CD retailer Spun.com saved 8 million in inventory investment[2]. In contrast, CDNow, which owns inventory, has declared bankruptcy because it was unable to cover the cost.

Despite the seemingly attraction of vertical cooperation with upstream partners, other pure Internet retailers have chosen to outsource fulfillment to third party retailers that have long had fulfillment capabilities. With fulfillment outsourcing, the Internet retailers direct online customer demand to traditional retailers. The traditional retailers then fulfill the online orders from their physical stock. In this way, the Internet retailers explore horizontal cooperation with traditional retailers. For instance, 1-800-Flowers.com, Amazon.com (its toy department) employ this strategy. These practices prompt us to ask the following question: between vertical cooperation and horizontal cooperation, which approach is better? The purpose of this paper is to take a first step to understand the drivers of each e-fulfillment model.

Indeed, our study was inspired by the story of Garden.com and its contrast with 1-800-Flowers.com. As described below, while Garden.com adopted a drop-shipping approach, 1-800-Flowers.com uses a fulfillment outsourcing approach. Garden.com eventually went bankrupt, while 1-800-Flowers.com is still thriving. However, taking a closer look at the tradeoffs of each approach reveals no obvious answers to the question of whether the fulfillment outsourcing model is definitively better to Internet retailers.

Garden.com, founded in 1996, was a virtual florist that sold flowers and garden supplies on the Net. Garden did not stock or own inventory but relied on upstream suppliers to fulfill demand. For each unit of realized demand, Garden paid the supplier a wholesale price. Despite its successful marketing (for example, Garden was able to draw more than half a million visitors a month), online order fulfillment created problems and finally put Garden's business to an end. Misaligned incentives between the suppliers and Garden in fulfilling demand was a major cause. On the one

[1] According to *eRetailing World* (2000), 30 percent of pure Internet retailers adopt drop-shipping as their primary order fulfillment approach, compared with 5 percent of multi-channel retailers.

[2] Cheap Tricks. *Forbes*, Feb. 2000.

hand, it was expensive for Garden to acquire and keep a customer, which meant high underage cost. Therefore it was to the benefit of Garden if the suppliers had ample supply. On the other hand, for some of the small suppliers, "Garden was simply another channel to move items that might otherwise be lost" (Johnson 2001), so they did not have the incentive to stock adequately for Garden. Also, given the perishable nature of the products, the suppliers had high overage cost. Another major cause for Garden's failure was high shipping cost since orders were shipped from geographically dispersed suppliers on a per unit basis.

In contrast, **1-800-Flowers.com**, which does not carry its own inventory either, relies on horizontal partners, the local florists, to fulfill demand. 1-800-Flowers.com attracts and receives customer orders through the Internet, but refers a specific order to the nearest local florist. 1-800-Flowers, who takes the order, gets a percentage of the sale, and the florist, who fulfills the order, gets the rest. There are two apparent advantages for this kind of alliance. One is savings in shipping cost, because the florists receive shipments from the suppliers in truckloads. Even though the online demand is delivered to the customer individually, due to proximity, the total shipping cost is much less than in the drop-shipping model. The other advantage is savings in demand risk pooling, because the local florists stock for both demands (in store and online). However, horizontal cooperation does not solve the issue of incentive misalignment. The retailers who bear the inventory risk of perishable products do not want to stock high. The Internet retailer who bears additional marketing expense and thus high underage cost expects a high stocking level at the retailer.

To summarize, while both vertical and horizontal cooperation in fulfilling demand help the Internet retailers avoid investment in setting up warehouse and obtaining inventory management skill, neither one is free of costs and risks. With drop-shipping, most small suppliers do not have sophisticated inventory management systems. They may not stock adequately either(as in Garden's case), which will deteriorate their partner's service quality. Moreover, the Internet retailer often suffers a loss of profit margin when having items shipped individually from geographically dispersed suppliers. With fulfillment outsourcing, the horizontal partners - local retailers - are experienced in inventory management, and are closer to the market. However, the Internet retailer is exposed to *rationing* because the conventional retailer has its own demand as well. Finally, both fulfillment approaches share a same dilemma – the Internet retailer is responsible for customer acquisition, and the supply chain partner is responsible for inventory procurement and order fulfillment. Since the marketing and operations functions are performed by two different firms, inefficiencies (under-spending on marketing, in addition to understocking) may arise that result in suboptimal system performance. Motivated by the concerns about the different fulfillment models, we aim to address the following research questions.

1. Under vertical/horizontal cooperation, what are the factors that affect each firm's decisions

and performance?

2. To an e-tailer (we refer to the Internet retailer as e-tailer, the traditional retailer as retailer hereafter), when is one fulfillment model preferred to the other?

3. Under horizontal cooperation, what are the effects of the retailer's different rationing policies?

Neither drop-shipping nor fulfillment outsourcing supply chains have been extensively studied in the literature. Drop-shipping has been described qualitatively in the marketing literature, see Scheel (1990) and the references therein. Netessine and Rudi (2004) study coordination issues in a drop-shipping supply chain. They show that the decentralized optimal solutions (e-tailer's marketing expense and the wholesaler's stocking quantity) are system-wide suboptimal. They propose a contract to coordinate such a supply chain.

Related to fulfillment outsourcing is the so-called referral supply chain, in which a referral website attracts customers by providing product information and then directs intended buyers to enrolled retailers. Chen *et.al.* (2002) analyze the effect of referral services on retail markets and examine the contractual arrangements they should use in selling their services. They find that the role of a referral website as a price discrimination mechanism leads to lower online prices. In the fulfillment outsourcing model we consider, we assume exogenous prices but consider inventory issues. There also exists a subtle difference between referral and fulfillment outsourcing. With referral, the Internet site only lists the partners' product information and thus is solely an information intermediary. With the latter, the e-tailer could choose to sell its "own" product on its website but outsource the fulfillment process to its partners. To the best of our knowledge, our work is among the first to compare the advantages and disadvantages of the two e-fulfillment approaches.

Another related stream of literature is on channel conflict and channel coordination. Examples in the operations management literature include Ahn *et al.* (2002), Cattani *et al.* (2006), Chiang *et al.* (2003), Tsay and Agrawal (2000), Tsay and Agrawal (2004a) and reviews by Cattani *et al.* (2004) and Tsay and Agrawal (2004b). Examples in the marketing literature (on dual-channel distributions) include Balasubramanian (1998), Bell *et al.* (2002), Hendershot and Zhang (2004), and Rhee and Parker (2000) . Although in our model we consider two different channels, the Internet and conventional, we focus on retail and e-tail collaboration on demand fulfillment and do not explicitly consider channel conflict.

The rest of the paper is organized as follows. Section 2 describes the general model setting. Section 3 analyzes vertical cooperation (a drop-shipping supply chain, and a traditional supply chain for comparison purpose). Horizontal cooperation is detailed in Section 4. Section 5 compares vertical and horizontal cooperations under various inventory rationing policies. Coordination issues are discussed in Section 6. Section 7 extends the model to multi-period settings and concludes the paper.

2 The Model Setting

We consider a retailer and an e-tailer, both facing independent stochastic demand. A common supplier offers one kind of product to the two firms at a wholesale price w per unit. The supplier incurs a unit production cost c. We assume single-period demand (extension to multi-period model is discussed in Section 7). The traditional supply chain consists of the supplier and the retailer. Before the selling season the retailer places an order with the supplier. We assume that the supplier has sufficient capacity and therefore produces exactly what the retailer orders. In the drop-shipping setting comprising the supplier and the e-tailer, the former needs to produce and stock a quantity for the upcoming shipping requests from the e-tailer. While in the setting of horizontal cooperation, the retailer again replenishes from the supplier, in anticipation of the e-tailer's and its own demand. It is further assumed that unfulfilled demand is lost. Detailed events under each scenario will be presented when it comes to the particular model.

We denote the retailer's random demand by Y, which has a probability density function (pdf) g and a cumulative distribution function (cdf) G.

To distinguish the e-tailer from the retailer, we assume that the e-tailer makes additional marketing effort over the retailer. The level of this effort is a decision variable of the e-tailer. This is because pure e-tailers do not have a physical presence and, as startup companies, do not have brand recognition or customer loyalty. Pure e-tailers' customer acquisition costs are about twice as much of the companies' budgets as the traditional retailers' [3]. Based on the above assumption, we denote the e-tailer's demand by $\Theta(a) + X$, where a is the e-tailer's marketing expense and X is a random variable with a pdf f and a cdf F. Furthermore, $\Theta(a)$ has the following properties.

$$\frac{d\Theta(a)}{da} \geq 0, \quad \frac{d^2\Theta(a)}{d(a)^2} \leq 0,$$

which are supported by empirical evidence from the marketing literature (Simon and Arndt 1980). This demand form is widely used in the operations and economics literature (see Petruzzi and Dada 1999), in which only the mean demand depends on a, and the uncertainty is captured by the random part X. Note that here the retailer's marketing expense is normalized to equal zero.

Finally, since our interest is in the impact of the marketing and operations functions on different supply chains, we assume that retail/e-tail prices are exogenous.

We compare two kinds of cooperation of the three firms, which are described below.

[3] Online retailers less merry in '00. Computer World, August 14, 2000.

Vertical Cooperation

In this setting, there exist two separate supply chains – a traditional supply chain with a supplier and a retailer and a drop-shipping supply chain with a supplier and an e-tailer.

In the traditional supply chain, the retailer orders a newsvendor quantity, Q_r, at w per unit and sells the product at p_r per unit. As mentioned earlier, we assume the supplier has sufficient capacity. Therefore, the supplier's decision is not involved (i.e., the supplier's production quantity equals the retailer's order quantity Q_r). At the end of the selling season, excess demand is lost and left-over inventory (at the retailer) is disposed of. Furthermore, since Q_r is shipped from the supplier to the retailer in truckloads, unit shipping cost is normalized to equal zero. Mathematically, The retailer's objective is to maximize

$$\Pi_r = E\left[p_r \min\left(Q_r, Y\right) - wQ_r\right]. \tag{1}$$

The supplier's profit is $\Pi_s = (w - c)Q_r$.

In the drop-shipping supply chain, before the selling season the supplier chooses a production quantity Q_e. The e-tailer decides on the customer acquisition expense a. Once the selling season starts, the supplier ships the products directly to customers upon the e-tailer's request. The e-tailer incurs a shipping cost s per unit of items shipped because products are delivered individually to each customer from the supplier. In reality, some e-tailers offer free-shipping while others charge the customers shipping and handling fees. In general, pure online retailers offer competitive total prices. In other words, if shipping charge is imposed, the posted product price is lower. Therefore, we assume that the e-tailer charges the customer a total price p_e and pays third party logistics a shipping cost s. Hence the e-tailer's profit margin is $p_e - w - s$. At the end of the season, residual demand is lost and left-over inventory (at the supplier) has a salvage value that is normalized to equal zero. Given the above sequence of events, the two players' objectives are as follows. Simultaneously, the e-tailer chooses its marketing expense a to maximize

$$\Pi_e = E\left[(p_e - w - s)\min\left(\Theta(a) + X, Q_e\right) - a\right]. \tag{2}$$

The supplier chooses a production quantity Q_e to maximize

$$\Pi_s = E\left[w \min\left(\Theta(a) + X, Q_e\right) - cQ_e\right]. \tag{3}$$

Horizontal Cooperation

Under horizontal cooperation, there exists one supply chain that consists of the supplier, the retailer and the e-tailer, where the e-tailer refers demand to the retailer.

The retailer orders Q_r from the supplier. Due to the same reason of transporting products in batches, unit shipping cost from the supplier to the retailer is normalized to equal zero. Furthermore, even though the e-demand may be delivered to the customers individually, due to geographical proximity, delivery cost is assumed to equal zero.

The arrangement between the retailer and the e-tailer is modeled as follows. Once the e-tailer directs customer traffic to the retailer, the referred demand becomes the retailer's. In other words, the e-tailer generates customer orders (true demand) and refers them to the retailer. The retailer then treats these orders as its own demand. Under this agreement, the retailer pays the e-tailer for all demand generated at τ per unit, no matter whether it has sufficient inventory (alternative order fulfillment mechanisms will also be discussed in Section 4.2). We make this assumption for the following reasons. First, the e-tailer brings additional demand to the retailer, which generally benefits the retailer who already has its own demand stream. As a result, the retailer incurs a penalty cost τ for unmet e-demand. Second, customer acquisition is expensive for the e-tailer and is on a per-customer basis. Therefore, once the e-tailer generates an additional unit of demand in a referral setting, it is reasonable that it gets compensated. We assume that τ is given. Later, we will discuss how τ affects the e-tailer's and the retailer's decisions and profits.

Finally, because the retailer does not differentiate the traditional and e-demand, it is reasonable to assume that prices at both channels are determined by the retailer and thus are the same. We denote this price by p, which is assumed to equal the traditional price p_r. Therefore, the e-tailer chooses a to maximize its expected profit as follows.

$$\Pi_e \quad = \quad E\left[\tau\left(\Theta(a) + X\right) - a\right]. \tag{4}$$

The retailer's problem is to choose Q_r to maximize the following expected profit.

$$\Pi_r \quad = \quad E\left[p \min\left(Q_r, \Theta(a) + X + Y\right) - wQ_r - \tau\left(\Theta(a) + X\right)\right]. \tag{5}$$

In the next section, we analyze the equilibrium solutions under each scenario.

3 Vertical Cooperation

3.1 Equilibrium Solution: The Retailer

In the traditional supply chain, the retailer's problem, given by (1), is a typical Newsvendor problem. The solution, the retailer's optimal order quantity, is characterized by

$$Q_r^T = G^{-1}\left(\frac{p_r - w}{p_r}\right), \tag{6}$$

where the superscript T represents "traditional." Substituting Q_r^T into (1), we get the retailer's optimal profit as follows.

$$\Pi_r^T = p_r \int_0^{G^{-1}\left(\frac{p_r - w}{p_r}\right)} y g(y) dy = p_r \Gamma_Y, \tag{7}$$

where $\Gamma_Y = \int_0^{G^{-1}\left(\frac{p_r - w}{p_r}\right)} y g(y) dy$.

3.2 Equilibrium Solution: The E-tailer

Recall that in the drop-shipping supply chain, the e-tailer chooses marketing effort a and the supplier chooses a production quantity Q_e to maximize their profit (2) and (3), respectively. The optimal solution , (a^D, Q_e^D) (where the superscript D denotes "drop-shipping"), is characterized by the following proposition.

Proposition 1. *In the drop-shipping supply chain, the e-tailer's optimal marketing expense, a^D, and the supplier's optimal stocking quantity, Q_e^D, satisfy*

$$\Theta'(a)|_{a^D} = \frac{1}{p_e - w - s} \frac{w}{w - c}, \tag{8}$$

$$Q_e^D = \Theta\left(a^D\right) + F^{-1}\left(\frac{w - c}{w}\right). \tag{9}$$

Proof. Rewrite (2) and (3),

$$\begin{aligned}
\Pi_e &= (p_e - w - s)Q_e - (p_e - w - s) \int_0^{Q_e - \Theta(a)} \left(Q_e - \Theta(a) - x\right) f(x) dx - a \\
\Pi_s &= (w - c)Q_e - w \int_0^{Q_e - \Theta(a)} \left(Q_e - \Theta(a) - x\right) f(x) dx.
\end{aligned}$$

The second-order derivatives of the above functions are

$$\begin{aligned}
\frac{\partial^2 \Pi_e}{\partial a^2} &= (p_e - w - s)\left(\Theta''(a) \int_0^{Q_e - \Theta(a)} f(x) dx - \Theta'(a)^2 f\left(Q_e - \Theta(a)\right)\right) < 0 \\
\frac{\partial^2 \Pi_s}{\partial Q_e^2} &= -w f\left(Q_e - \Theta(a)\right) < 0,
\end{aligned}$$

which imply that the profit functions (2) and (3) are concave in a and Q_e, respectively. The optimal solution can then be derived by setting the first-order derivatives to zero.

For comparison (with horizontal cooperation) purpose, we derive the e-tailer's optimal profit as follows.

$$\Pi_e^D = (p_e - w - s)Q_e^D - (p_e - w - s) \int_0^{F^{-1}\left(\frac{w-c}{w}\right)} F(x) dx - a^D. \tag{10}$$

□

In the traditional supply chain, only price margins affect the order quantity. While in the drop-shipping supply chain, the e-tailer's marketing effort, together with the prices, determines the supplier's stocking quantity. Moreover, the price margins affect the level of marketing expense as well. From Proposition (1), we further observe the following: (i)As the e-tailer's marketing expense increases, the supplier has incentive to stock more. (ii) As the newsvendor fractile $(w-c)/w$ increases, the supplier's production quantity increases. At the same time, $\Theta'(a)$ decreases. Since $\Theta(a)$ is concave, the e-tailer's optimal effort level a^D increases. (iii) As the profit margin, $p_e - w - s$, for the e-tailer decreases, the e-tailer has less incentive to spend on customer acquisition (lower a^D).

4 Horizontal Cooperation

4.1 Equilibrium Solution

Recall that under horizontal cooperation, the e-tailer generates and refers demand to the retailer. Under the agreement of selling all demand, the e-tailer exerts an effort level a to maximize its profit (4). We present the e-tailer's decision and optimal profit in the following proposition.

Proposition 2. *For a given price τ, the e-tailer chooses a customer acquisition level a^R such that $\Theta'(a^R) = 1/\tau$ (where R represents "Referral"). The e-tailer's optimal profit is increasing in τ.*

Proof. It follows that the e-tailer's profit Π_e, (4), is concave in a. Then, from the first order condition, we can calculate the e-tailer's optimal decision, a^R, which is characterized by

$$\Theta'\left(a^R\right) = \frac{1}{\tau}. \tag{11}$$

The e-tailer's corresponding optimal profit function is

$$\Pi_e^R = E\left[\tau\left(\Theta(a^R) + X\right) - a^R\right], \tag{12}$$

where $\Theta'\left(a^R\right) = 1/\tau$. The first derivative of Π_e^R w.r.t. τ is

$$\begin{aligned} \frac{\partial \Pi_e^R}{\partial \tau} &= \tau \frac{\partial \Theta\left(a^R\right)}{\partial a^R}\frac{\partial a^R}{\partial \tau} + \left(\Theta\left(a^R\right) + E(X)\right) - \frac{\partial a^R}{\partial \tau} \\ &= \tau \frac{1}{\tau}\frac{\partial a^R}{\partial \tau} + \left(\Theta\left(a^R\right) + E(X)\right) - \frac{\partial a^R}{\partial \tau} \\ &= \Theta\left(a^R\right) + E(X) \geq 0. \end{aligned}$$

Therefore, the optimal profit Π_e^R is increasing in τ. □

From the above proposition, we see that since the e-tailer is compensated based on referred demand, its optimal marketing expense level only depends on the referral price τ. When τ is large, the e-tailer has incentive to exert high level of marketing effort.

The retailer, in anticipation of its own and the referred demand, chooses an order quantity Q_r that maximizes (5). Its optimal decision and profit are characterized by the following proposition.

Proposition 3. *The retailer's expected profit Π_r is concave in Q_r. Its optimal order quantity Q_r^R is*

$$Q_r^R = \Theta(a^R) + \Psi^{-1}\left(\frac{p-w}{p}\right), \tag{13}$$

where $\Psi(\cdot)$ is the cumulative density function of $X+Y$. Its optimal profit is

$$\Pi_r^R \quad = \quad (p-w)\Theta(a^R) - \tau\left(\Theta\left(a^R\right) + E(X)\right) + p\Gamma_{X+Y}, \tag{14}$$

where $\Gamma_{X+Y} = \int_0^{\Psi^{-1}(\frac{p-w}{p})} \int_0^{\Psi^{-1}(\frac{p-w}{p})-y} (x+y)f(x)g(y)dxdy$.

Proof. The retailer's expected profit for a fixed price τ can be further written as:

$$\Pi_r = (p-w)Q_r - p\cdot E\left[Q_r - (\Theta(a)+X+Y)\right]^+ - \tau E\left[\Theta(a)+X\right].$$

Since $E\left[Q_r - (\Theta(a)+X+Y)\right]^+$ is convex in Q_r, $-pE\left[Q_r - (\Theta(a)+X+Y)\right]^+$ is concave in Q_r. Therefore, Π_r is concave in Q_r. The optimal solution Q_r^R can then be calculated by solving the first order condition. To derive the first order derivative, rewrite Π_r as

$$\begin{aligned}\Pi_r \quad &= \quad (p-w)Q_r - p\int_0^{Q_r-\Theta(a)}\int_0^{Q_r-\Theta(a)-y}\left[Q_r - \Theta(a) - (x+y)\right]f(x)g(y)dxdy \\ &\quad -\tau E\left[\Theta(a)+X\right].\end{aligned}$$

The first order condition is:

$$\begin{aligned}0 \quad &= \quad \frac{\partial \Pi_r}{\partial Q_r} = (p-w) \\ &\quad -p\int_0^{Q_r-\Theta(a)}\left[\frac{\partial}{\partial Q_r}\left(\int_0^{Q_r-\Theta(a)-y}\left(Q_r-\Theta(a)-(x+y)\right)f(x)g(y)dx\right)\right]dy \\ &= \quad (p-w) - p\int_0^{Q_r-\Theta(a)}\left[\int_0^{Q_r-\Theta(a)-y} f(x)g(y)dx\right. \\ &\qquad \left. + \left(Q_r-\Theta(a)-\left(Q_r-\Theta(a)-y+y\right)\right)f\left(Q_r-\Theta(a)-y\right)g(y)\right]dy \\ &= \quad (p-w) - p\int_0^{Q_r-\Theta(a)}\int_0^{Q_r-\Theta(a)-y} f(x)g(y)dxdy\end{aligned}$$

Solving the above equation, we obtain the following newsvendor type solution for Q_r.

$$\Pr\left[X+Y < Q_r - \Theta(a)\right] = \frac{p-w}{p}$$

Thus, $Q_r = \Theta(a) + \Psi^{-1}\left(\frac{p-w}{p}\right)$.

The optimal marketing expense for the e-tailer and the optimal order quantity for the retailer can be obtained by solving the above equation and (11) simultaneously. Substituting Q_r^R and a^R into the retailer's profit function Π_r, we obtain the optimal profit for the retailer as follows.

$$
\begin{aligned}
\Pi_r^R &= (p-w)Q_r - pE\left[Q_r^R - \left(\Theta(a^R)+X+Y\right)\right]^+ - \tau E\left[\Theta(a^R)+X\right] \\
&= (p-w)\Theta\left(a^R\right) - \tau\left(\Theta\left(a^R\right)+E(X)\right) \\
&\quad + p\int_0^{\Psi^{-1}\left(\frac{p-w}{p}\right)}\int_0^{\Psi^{-1}\left(\frac{p-w}{p}\right)-y}(x+y)f(x)g(y)dxdy. \quad \square
\end{aligned}
$$

The retailer's optimal order quantity is composed of two parts. One part relates to the e-tailer's customer acquisition effort. The other incorporates demand randomness. Upon the agreement of buying all generated demand and thus making a lump sum payment to the e-tailer (we refer to this policy as "lump sum purchase" hereafter), the retailer pools the two streams of demand together. Therefore, the newsvendor part of the optimal order quantity leverages uncertainties of the *aggregated* demand.

Note that under "lump sum purchase" policy, the retailer needs not differentiate the two demand sources. However, if the retailer adopts other fulfillment policies that discriminate demands from different channels, an inevitable question facing the retailer is how to allocation its inventory. A subsequent issue is that how such allocation policies affect the supply chain's performance. We study these questions in the next section.

4.2 Effect of Retailer's Inventory Allocation Policies

Previously, online order fulfillment follows a simple form: the retailer pays for all demand referred by the e-tailer. However, it is possible that the retailer only agrees to compensate the e-tailer on *fulfilled* demand. Under this circumstance, the retailer needs to allocate its inventory. One possibility is that the retailer favors and thus satisfies its own demand in the traditional channel first. We refer to this as "priority policy." Another possibility is that the retailer rations the inventory between the traditional and e-demand, which is referred to as "proportional policy." Next, we investigate the impact of these fulfillment agreements.

Priority Policy

With priority policy, the retailer fulfills its own demand first and only uses leftover inventory to satisfy the e-tailer's demand. Accordingly, the e-tailer is compensated on fulfilled e-demand. Then, the retailer's problem is to choose an order quantity Q_r to maximize

$$\Pi_r = E\left[p\min(Y,Q_r) + (p-\tau)\min\left(\Theta(a)+X,(Q_r-Y)^+\right) - wQ_r\right],$$

and the e-tailer's objective is to choose an effort level a that maximizes

$$\Pi_e = E\left[\tau\min\left(\Theta(a)+X,(Q_r-Y)^+\right)\right] - a.$$

The optimal solution is characterized as follows.

Proposition 4. *The retailer and the e-tailer simultaneously choose a stocking level and marketing expense such that*

$$\begin{aligned} \tau \Pr\left(Y \le Q_r^{PI}\right) + (p-\tau)\Pr\left(\Theta\left(a^{PI}\right) + X + Y \le Q_r^{PI}\right) &= p - w \qquad (15) \\ \tau\Theta'\left(a^{PI}\right)\Pr\left(\Theta\left(a^{PI}\right) + X + Y \le Q_r^{PI}\right) - 1 &= 0, \qquad (16) \end{aligned}$$

where the superscript PI denotes "priority".

Proof. The retailer's expected demand can be rewritten as

$$\Pi_r = E\left[(p-w)Q_r - \tau\left(Q_r - Y\right)^+ - (p-\tau)\left(\left(Q_r - Y\right)^+ - \Theta(a) - X\right)^+\right].$$

It follows that the above function is concave in Q_r and the first order condition leads to

$$\frac{\partial \Pi_r}{\partial Q_r} = p - w - \tau \Pr\left(Y \le Q_r\right) - (p-\tau)\Pr\left(\Theta(a) + X + Y \le Q_r\right) = 0.$$

The e-tailer's objective function is

$$\begin{aligned} \Pi_e &= \tau E\left[\left(Q_r - Y\right)^+ - \left(\left(Q_r - Y\right)^+ - \Theta(a) - X\right)^+\right] - a \\ &= \tau \int_0^{Q_r} (Q_r - y) g(y) dy \\ &\quad -\tau \int_0^{Q_r - \Theta(a)} g(y) \int_0^{Q_r - \Theta(a) - y} \left(Q_r - \Theta(a) - y - x\right) f(x) dx dy - a \end{aligned}$$

The second derivative of the e-tailer's profit function is

$$\frac{\partial^2 \Pi_e}{\partial a^2} = \tau \int_0^{Q_r - \Theta(a)} \left[\int_0^{Q_r - \Theta(a) - y} \Theta''(a) f(x) dx - \left(\Theta'(a)\right)^2 f\left(Q_r - \Theta(a) - y\right)\right] g(y) dy,$$

which is less than zero. Therefore, the e-tailer's objective function is concave in a and the optimal marketing decision a can be derived from the following first order condition.

$$\tau\Theta'(a)\Pr\left(\Theta(a) + X + Y \le Q_r\right) - 1 = 0.$$

□

Rewrite (15) as

$$\Pr\left(\Theta\left(a^{PI}\right) + X + Y \le Q_r^{PI}\right) = \left[p - w - \tau Pr\left(Q_r^{PI} - \Theta\left(a^{PI}\right) - X \le Y \le Q_r^{PI}\right)\right]/p. \qquad (17)$$

This is a revised newsvendor quantity. Recall that the newsvendor problem is to leverage overstocking and understocking cost. In the typical case, the retailer faces one (aggregated) demand stream, e.g., under lump sum purchase agreement. The newsvendor solution satisfies

$$\Pr\left(\Theta\left(a^R\right) + X + Y \le Q_r^R\right) = (p-w)/p,$$

where the L.H.S. represents the probability of overstocking, and the R.H.S. represents the relative understocking cost. In (17), the numerator of the R.H.S. is related to understocking cost. Note that when Y is greater than Q_r^{PI}, unit understocking cost is $p - w$. When Y is less than Q_r^{PI} and the sum of demand, $\Theta\left(a^{PI}\right) + X + Y$, is greater than Q_r^{PI}, unit understocking cost is $p - w - \tau$. Therefore, (17) represents a revised newsvendor solution in the case when a retailer satisfies its own demand first and uses leftover inventory to satisfy another demand stream.

Comparing the two fulfillment policies (priority and lump sum purchase), we observe the following.

Proposition 5. *System-wide, priority policy performs worse than lump sum purchase policy. In other words,* $a^{PI} \leq a^R$, $Q_r^{PI} \leq Q_r^R$.

Proof. Compare the following solutions

$$\begin{aligned}
\Theta'(a^R) &= \frac{1}{\tau} \\
\Theta'(a^{PI}) &= \frac{1}{\tau \Pr\left(\Theta(a) + X + Y \leq Q_r^{PI}\right)}.
\end{aligned}$$

Since $\Pr\left(\Theta(a) + X + Y \leq Q_r^{PI}\right) \leq 1$, as $\Theta(a)$ is concave, we have $a^{PI} \leq a^R$.

To compare Q_r^{PI} with Q_r^R, we have

$$\begin{aligned}
0 &= p - w - \tau \Pr\left(Y \leq Q_r^{PI}\right) - (p - \tau) \Pr\left(\Theta(a^{PI}) + X + Y \leq Q_r^{PI}\right) \\
&\leq p - w - \tau \Pr\left(\Theta(a^{PI}) + X + Y \leq Q_r^{PI}\right) - (p - \tau) \Pr\left(\Theta(a^{PI}) + X + Y \leq Q_r^{PI}\right) \\
&= p - w - p \Pr\left(\Theta(a^{PI}) + X + Y \leq Q_r^{PI}\right).
\end{aligned}$$

The inequality follows because $\Pr\left(Y \leq Q_r^{PI}\right) \geq \Pr\left(Y \leq Q_r^{PI} - \Theta(a^{PI}) - X\right)$. Therefore,

$$\Pr\left(X + Y \leq Q_r^{PI} - \Theta(a^{PI})\right) \leq \frac{p - w}{p} = \Pr\left(X + Y \leq Q_r^R - \Theta(a^R)\right)$$

which implies that $Q_r^{PI} - \Theta(a^{PI}) \leq Q_r^R - \Theta(a^R)$. Furthermore, since $a^{PI} \leq a^R$, we have $Q_r^{PI} \leq Q_r^R$. □

From this result, we see that under lump sum purchase policy, the e-tailer can concentrate and thus invest more on marketing, since it is paid on all generated demand. On the other hand, the retailer treats both demands as its own and thus has incentive to stock more. Therefore, from the supply chain's perspective, loss of supply chain efficiency, i.e., system-wise invest in inventory and customer acquisition, is less with lump sum purchase agreement.

Proportional Allocation Policy

Under a proportional policy, the retailer agrees to assign a fixed proportion of its inventory to the e-tailer's referred demand. At the end of the season, unfulfilled demand is lost and there is no inventory substitution. In other words, the retailer does not use leftover inventory that was originally assigned to one demand stream to satisfy excess demand from another stream. Denote the proportion of inventory for the retailer's demand by η, the proportion for the e-tailer's demand by $1-\eta$. Then, the retailer chooses an order quantity Q_r and the e-tailer chooses customer acquisition effort a, respectively, to maximize their expected profit functions

$$\begin{aligned}\Pi_r &= E\left[(p-\tau)\min\left(\Theta(a)+X,(1-\eta)Q_r\right)+p\min\left(Y,\eta Q_r\right)-wQ_r\right],\\ \Pi_e &= E\left[\tau\min\left(\Theta(a)+X,(1-\eta)Q_r\right)-a\right].\end{aligned}$$

The optimal solution is characterized by the following proposition.

Proposition 6. *The retailer's optimal order quantity and the e-tailer's optimal marketing costs are such that*

$$\begin{aligned}(p-\tau)(1-\eta)\Pr\left(X+\Theta(a^{PO})\le(1-\eta)Q_r^{PO}\right)+p\eta\Pr\left(Y\le\eta Q_r^{PO}\right) &=\\ p-w-\tau(1-\eta)&\\ \tau\Theta'(a^{PO})\Pr\left(X+\Theta(a^{PO})\le(1-\eta)Q_r^{PO}\right)-1=0,&\end{aligned}$$

where the superscript PO represents "proportional".

Proof. The retailer's expected profit is

$$\begin{aligned}\Pi_r &= (p-\tau)E\left[(1-\eta)Q_r-((1-\eta)Q_r-\Theta(a)-X)^+\right]\\ &\quad +pE\left[\eta Q_r-(\eta Q_r-Y)^+\right]-wQ_r,\end{aligned}$$

which is concave in Q_r. Therefore, the optimal quantity Q_r can be calculated from the first order condition

$$\frac{\partial\Pi_r}{\partial Q_r}=p-w-\tau(1-\eta)-(p-\tau)(1-\eta)\Pr\left(X+\Theta(a)\le(1-\eta)Q_r\right)-p\eta\Pr\left(Y\le\eta Q_r\right)=0.$$

Rewrite the e-tailer's expected profit function as

$$\begin{aligned}\Pi_e &= \tau E\left[(1-\eta)Q_r-((1-\eta)Q_r-\Theta(a)-X)^+\right]-a\\ &= \tau(1-\eta)Q_r-\tau\int_0^{(1-\eta)Q_r-\Theta(a)}\left((1-\eta)Q_r-\Theta(a)-x\right)f(x)dx.\end{aligned}$$

The second order derivative is

$$\frac{\partial^2\Pi_E}{\partial a^2}=\tau\int_0^{(1-\eta)Q_r-\Theta(a)}\Theta''(a)f(x)dx-r\left(\Theta'(a)\right)^2f\left((1-\eta)Q_r-\Theta(a)\right)<0,$$

which implies that Π_e is concave in a. Thus, the optimal a can be obtained from the following first order condition.

$$\tau\Theta'(a)\Pr(X+\Theta(a)\leq(1-\eta)Q_r)-1=0.$$

$\square$

Following a similar argument, we can prove that $a^{PO}\leq a^R$. However, it is not clear how Q_r^{PO} is compared to Q_r^R. Numerical comparison is made in Section 5.2.

From the above discussions on fulfillment policies, we see that system-wide, "lump sum purchase" policy performs well. The reason is that when the retailer promises to compensate the e-tailer on every unit of generated e-demand, the e-tailer has incentive to acquire more customers. While with other fulfillment policies, the allocation mechanism is a new source of inefficiency because uncertainty of getting adequate fulfillment capacity hurts the e-tailer's incentive to exert effort. In the next section, we investigate when the firms prefer one type of cooperation to the other.

5 Discussions

To better understand the differences between the three inventory allocation policies, we conducted an extensive numerical study in this section. In the numerical examples, let $\Theta(a)=b\sqrt{a}$, where b can be interpreted as the efficiency index of the e-tailer's marketing function.

5.1 Under Priority Policy

It is shown analytically in Section 4.2 that for a fixed τ, priority policy leads to understocking in inventory and underinvestment in marketing expense. It is intuitive that when the retailer prioritizes demands, the e-tailer may not benefit from engaging in horizontal cooperation, which is confirmed from our numerical test as well. Among the numerical examples, we list two tables for illustration. The parameters for Table 1 are as follows: $X\sim N(200,20^2)$, $Y\sim N(500,50^2)$, $p=10$, $w=8$, $c=7$ and $s=0.3$. The e-tailer's marketing function is $\Theta(a)=30\sqrt{a}(b=30)$. Those for Table 2 are the same except that $s=1.2$.

From the numerical study, we make the following observation.

(i) For a given τ, stocking quantity and marketing expense are lower under priority policy.

(ii) When shipping saving is trivial, priority policy does not warrant horizontal cooperation. For example in Table 1, there does not exist a transfer price under priority policy that makes the e-tailer better off in the horizontal structure. However, under lump sum policy, both parties

τ	Q^{PI}	$\theta(a^{PI})$	π_r^{PI}	π_e^{PI}	Q^{R}	$\theta(a^{R})$	π_r^{R}	π_e^{R}	π_r^{T}	π_e^{D}
0.2	668.0	16.5	1249.5	32.2	744.7	90	1371.2	49.0	860	451
0.3	673.4	23.7	1245.4	49.9	789.7	135	1418.7	80.3	860	451
0.4	677.9	30.0	1238.8	68.3	834.7	180	1457.2	116.0	860	451
0.5	681.5	35.5	1229.9	87.1	879.7	225	1486.7	156.3	860	451
0.6	684.2	40.2	1218.8	106.2	924.7	270	1507.2	201.0	860	451
0.7	685.8	44.0	1205.9	125.2	969.7	315	1518.7	250.3	860	451
0.8	686.4	47.0	1191.3	143.9	1014.7	360	1521.2	304.0	860	451
0.9	685.9	49.0	1175.3	161.8	1059.7	405	1514.7	362.3	860	451
1.0	684.3	50.0	1158.2	178.6	1104.7	450	1499.2	425.0	860	451
1.1	681.4	50.1	1140.3	193.9	1149.7	495	1474.7	492.3	860	451
1.2	677.2	49.1	1121.7	207.2	1194.7	540	1441.2	564.0	860	451
1.3	671.6	47.1	1103.0	218.1	1239.7	585	1398.7	640.3	860	451
1.4	664.5	44.0	1084.4	225.8	1284.7	630	1347.2	721.0	860	451
1.5	655.6	39.8	1066.2	229.7	1329.7	675	1286.7	806.3	860	451
1.6	644.9	34.5	1048.9	228.8	1374.7	720	1217.2	896.0	860	451
1.7	632.1	28.3	1032.9	222.4	1419.7	765	1138.7	990.3	860	451
1.8	617.2	21.4	1018.7	209.8	1464.7	810	1051.2	1089.0	860	451
1.9	601.2	14.9	1006.6	192.1	1509.7	855	954.7	1192.3	860	451
2.0	585.9	9.7	996.7	173.0	1554.7	900	849.2	1300.0	860	451

Table 1: When Shipping Saving is Trivial Under Priority Policy

are better off when τ ranges from 1.1 to 1.9. When shipping saving is significant, priority policy leads to horizontal cooperation. For instance in table 2, under lump sum policy when τ ranges from 0.8 to 1.9, and under priority policy when τ is from 1 to 1.9, both demand transfer mechanisms make the firms better off compared to the vertical structure. Nonetheless, both firms' profits are lower when the retailer gives priority to its own demand.

(iii) While under lump sum policy, Q and a are monotone in τ, the property does not hold under priority policy. In the latter case, the impact of τ on Q and a is more complex. On the one hand, increase in τ tends to increase a, which may lead to larger Q. On the other hand, increase in τ implies lower underage cost and thus lower Q for the retailer. This means possibly less leftover inventory for the e-tailer, which in turn depresses the level of a. Therefore, we observe the non-monotone relationship between τ, and Q and a. For observation (iii), if we differentiate (15) and (16) w.r.t. τ, we obtain the following.

$$\begin{aligned}
\tau^2\Theta' g(Q)\frac{\partial Q}{\partial \tau} &= (p\Theta' - p\tau\Theta'' + \tau^2\Theta'')\Psi(Q-\Theta(a)) - \tau\Theta' G(Q) \\
\tau\Theta'^2\psi(Q-\Theta(a))\frac{\partial a}{\partial \tau} &= \tau\Theta'\psi(Q-\Theta(a))\frac{\partial Q}{\partial \tau} + (\Theta' - \tau\Theta'')\Psi(Q-\Theta(a)).
\end{aligned}$$

These derivatives also suggest non-monotonicity. Moreover, our numerical study shows that Q^{PI}, a^{PI}, Π_e^{PI} are unimodal in τ. Recall that under lump sum policy, the e-tailer's profit only depends on τ. This simple relationship may increase efficiency. While under priority policy, one firm's profit depends not only on its own "investment," but on its partner's "investment." Therefore, if the e-tailer negotiates the transfer price, it is no longer the larger the better,

τ	Q^{PI}	$\Theta(a^{PI})$	π_r^{PI}	π_e^{PI}	Q^R	$\Theta(a^R)$	π_r^R	π_e^R	π_r^T	π_e^D
0.2	668.0	16.5	1249.5	32.2	744.7	90	1371.2	49.0	860	174
0.3	673.4	23.7	1245.4	49.9	789.7	135	1418.7	80.3	860	174
0.4	677.9	30.0	1238.8	68.3	834.7	180	1457.2	116.0	860	174
0.5	681.5	35.5	1229.9	87.1	879.7	225	1486.7	156.3	860	174
0.6	684.2	40.2	1218.8	106.2	924.7	270	1507.2	201.0	860	174
0.7	685.8	44.0	1205.9	125.2	969.7	315	1518.7	250.3	860	174
0.8	686.4	47.0	1191.3	143.9	1014.7	360	1521.2	304.0	860	174
0.9	685.9	49.0	1175.3	161.8	1059.7	405	1514.7	362.3	860	174
1.0	684.3	50.0	1158.2	178.6	1104.7	450	1499.2	425.0	860	174
1.1	681.4	50.1	1140.3	193.9	1149.7	495	1474.7	492.3	860	174
1.2	677.2	49.1	1121.7	207.2	1194.7	540	1441.2	564.0	860	174
1.3	671.6	47.1	1103.0	218.1	1239.7	585	1398.7	640.3	860	174
1.4	664.5	44.0	1084.4	225.8	1284.7	630	1347.2	721.0	860	174
1.5	655.6	39.8	1066.2	229.7	1329.7	675	1286.7	806.3	860	174
1.6	644.9	34.5	1048.9	228.8	1374.7	720	1217.2	896.0	860	174
1.7	632.1	28.3	1032.9	222.4	1419.7	765	1138.7	990.3	860	174
1.8	617.2	21.4	1018.7	209.8	1464.7	810	1051.2	1089.0	860	174
1.9	601.2	14.9	1006.6	192.1	1509.7	855	954.7	1192.3	860	174
2.0	585.9	9.7	996.7	173.0	1554.7	900	849.2	1300.0	860	174

Table 2: When Shipping Saving is Significant Under Priority Policy

because low inventory level might hurt the e-tailer.

5.2 Under Proportional Policy

In the numerical example shown in this section, $X \sim N(200, 20^2)$, $Y \sim N(500, 50^2)$, $p = 10$, $w = 8$, $c = 7$,$s = 0.3$, $b = 30$. In Table 3 ($\eta = 0.5$, i.e., the retailer splits its inventory equally), we illustrate a comparison between lump sum and proportional policy. In Table 4 ($\eta = 0.3, 0.5, 0.7$, i.e., the retailer increases the proportion of inventory assigned to its own demand), we show the effect of η on the cooperation structure and on firms' profit. In Table 5, the parameters are the same as those in Table 4 except that $b = 90$.

From the numerical examples, we observe the following.

(i) For a given τ, stocking quantity and marketing expense are lower under proportional policy, compared to lump sum policy.

(ii) Horizontal cooperation may be preferred under either the lump sum or the proportional policy. For example in Table 3, under lump sum policy, τ that makes horizontal structure favorable ranges from 1.1 to 1.9; under proportional policy, such τ ranges from 1.4 to 1.7. However, proportional policy performs worse in that it leads to lower profits for both firms.

(iii) Similarly to priority policy, Q and a are not necessarily monotone in τ.

(iv) The impact of η is not straightforward. For instance, when η is small, i.e., the assigned fractional inventory to the e-tailer is large, the e-tailer has incentive to exerts higher marketing

τ	Q^{PO}	$\Theta(a^{PO})$	π_r^{PO}	π_e^{PO}	Q^R	$\Theta(a^R)$	π_r^R	π_e^R	π_r^T	π_e^D
0.2	458.4	34.9	817.5	43.4	744.7	90.0	1371.2	49.0	860	451
0.3	490.9	51.5	856.6	69.1	789.7	135.0	1418.7	80.3	860	451
0.4	522.3	67.5	890.2	97.3	834.7	180.0	1457.2	116.0	860	451
0.5	552.3	82.9	918.5	127.9	879.7	225.0	1486.7	156.3	860	451
0.6	581.2	97.7	941.6	160.8	924.7	270.0	1507.2	201.0	860	451
0.7	608.6	111.8	959.6	195.8	969.7	315.0	1518.7	250.3	860	451
0.8	634.7	125.2	972.5	232.7	1014.7	360.0	1521.2	304.0	860	451
0.9	659.2	137.8	980.5	271.4	1059.7	405.0	1514.7	362.3	860	451
1.0	681.9	149.6	983.3	311.7	1104.7	450.0	1499.2	425.0	860	451
1.1	702.7	160.5	981.0	353.2	1149.7	495.0	1474.7	492.3	860	451
1.2	721.1	170.2	973.4	395.6	1194.7	540.0	1441.2	564.0	860	451
1.3	737.0	178.7	960.4	438.5	1239.7	585.0	1398.7	640.3	860	451
1.4	750.2	185.9	942.3	481.6	1284.7	630.0	1347.2	721.0	860	451
1.5	760.9	191.9	919.6	524.5	1329.7	675.0	1286.7	806.3	860	451
1.6	769.3	196.7	893.1	567.1	1374.7	720.0	1217.2	896.0	860	451
1.7	775.6	200.5	863.4	609.2	1419.7	765.0	1138.7	990.3	860	451
1.8	780.2	203.5	831.1	650.8	1464.7	810.0	1051.2	1089.0	860	451
1.9	783.3	205.7	796.8	691.7	1509.7	855.0	954.7	1192.3	860	451
2.0	785.0	207.3	760.7	731.9	1554.7	900.0	849.2	1300.0	860	451

Table 3: An Example of Proportional Policy

effort. But on the other hand, total stocking level is low due to the retailer's lack of incentive, which may in turn hurts the e-tailer's incentive. Therefore, we see from Table 4, that η of 0.3 results to lower inventory *and* lower marketing expense compared with η equal to 0.5. Also, η equal 0.3 does not lead to the horizontal structure (recall that in this case $\Pi_r^T = 860$ and $\Pi_e^D = 451$). While when η equals 0.7, even the retailer has incentive to stock more (when $\tau = 0.2 - 0.9$) compared to $\eta = 0.5$, the low assigned proportion significantly hurts the e-tailer's incentive. Thus, we observe lower marketing effort. Due to the e-tailer's lack of effort, the retailer's stocking quantity drops and is instead lower than when η equals 0.5 as τ further increases from 0.9. Moreover, when $\eta = 0.7$, although the retailer is mostly better off relative to the vertical structure, the allocation mechanism cannot induce the e-tailer to participate in horizontal cooperation. Therefore, in negotiating the percentage to allocation inventory, the retailer should choose a moderate value. A percentage that is too small (smaller proportion to its own demand) hurts its own incentive and benefit. A percentage that is too large hurts the e-tailer's incentive and makes the horizontal structure unraveled.

(v) Compare Table 4 and 5. When $b = 30$, the retailer's profit increases as η increases when τ is given. That is, the retailer is better off by assigning more inventory to its own demand. However, this is not the case when $b = 90$ (Table 5). The implication is that when the e-tailer is very efficient at customer acquisition, the retailer may want to allocate more inventory to the e-demand to induce higher marketing effort.

	$\eta=0.3$				$\eta=0.5$				$\eta=0.7$			
τ	Q^{PO}	$\Theta(a^{PO})$	π_r^{PO}	π_e^{PO}	Q^{PO}	$\Theta(a^{PO})$	π_r^{PC}	π_e^{PO}	Q^{PO}	$\Theta(a^{PO})$	π_r^{PO}	π_e^{PO}
0.2	303.2	24.4	531.3	41.1	458.4	34.9	817.5	43.4	655.4	15.5	1195.8	38.7
0.3	318.5	35.6	539.1	64.5	490.9	51.5	856.6	69.1	657.8	18.9	1181.5	58.4
0.4	332.8	46.1	542.9	89.6	522.3	67.5	890.2	97.3	659.1	21.4	1164.9	78.1
0.5	346.0	55.8	542.9	116.2	552.3	82.9	918.5	127.9	659.7	23.2	1147.1	97.9
0.6	358.2	64.8	539.3	144.1	581.2	97.7	941.6	160.8	659.9	24.6	1128.7	117.6
0.7	369.2	73.1	532.3	173.1	608.6	111.8	959.6	195.8	659.9	25.8	1110.0	137.3
0.8	379.0	80.5	522.0	203.0	634.7	125.2	972.5	232.7	659.6	26.8	1091.0	157.0
0.9	387.6	87.1	508.7	233.6	659.2	137.8	980.5	271.4	659.3	27.6	1071.8	176.6
1.0	395.0	92.9	492.5	264.6	681.9	149.6	983.3	311.7	658.8	28.3	1052.5	196.2
1.1	401.0	97.7	473.7	295.8	702.7	160.5	981.0	353.2	658.2	28.9	1033.1	215.7
1.2	405.7	101.7	452.4	326.8	721.1	170.2	973.4	395.6	657.6	29.4	1013.6	235.2
1.3	409.0	104.7	429.0	357.5	737.0	178.7	960.4	438.5	656.9	29.8	994.2	254.7
1.4	410.9	106.7	403.5	387.5	750.2	185.9	942.3	481.6	656.2	30.2	974.7	274.0
1.5	411.2	107.8	376.4	416.4	760.9	191.9	919.6	524.5	655.4	30.5	955.2	293.4
1.6	410.0	107.8	347.8	443.8	769.3	196.7	893.1	567.1	654.6	30.8	935.6	312.6
1.7	407.1	106.7	318.0	469.4	775.6	200.5	863.4	609.2	653.8	31.0	916.1	331.8
1.8	402.6	104.4	287.3	492.8	780.2	203.5	831.1	650.8	652.9	31.2	896.6	350.9
1.9	396.2	101.0	256.1	513.4	783.3	205.7	796.8	691.7	652.0	31.4	877.1	370.0
2.0	388.0	96.4	224.6	530.8	785.0	207.3	760.7	731.9	651.0	31.5	857.7	389.0

Table 4: Effect of η Under Proportional Policy (1): When b is Small

5.3 Under Lump Sum Purchase

For each of the three inventory allocation policies, benefit of horizontal cooperation comes from inventory risk pooling and shipping cost savings. Moreover, such savings and an appropriate margin can induce the e-tailer to exert higher level of marketing effort, which further benefit both firms.

As suggested in all the numerical examples, e.g. in Tables 1, 2 and 3, lump sum purchase policy outperforms the other two inventory allocation policies. As compared to vertical cooperation, we use Table 1 as an illustration (see the last four columns). For τ between 1.1 and 1.9, both the retailer and the e-tailer are better off under lump sum policy in the horizontal structure, compared to the vertical structure. This is due to the above mentioned inventory and shipping cost factors.

6 Supply Chain Coordination

As mentioned in the Introduction, separation of the operations and marketing functions may result in inefficiencies. In this section, we study the supply chain coordination issue. For drop-shipping supply chains, Netessine and Rudi (2004) show that none of the contracts in the literature can achieve coordination. They propose a coordination scheme: the wholesaler sponsors a part of the retailer's marketing expense, and at the same time, the retailer compensates the wholesaler for each unit of leftover inventory. Following a similar approach, in our setting, the problem for the

		η=0.3				η=0.5					η=0.7	
τ	Q^{PO}	$\theta(a^{PO})$	π_r^{PO}	π_e^{PO}	Q^{PO}	$\theta(a^{PO})$	π_r^{PO}	π_e^{PO}	Q^{PO}	$\theta(a^{PO})$	π_r^{PO}	π_e^{PO}
0.2	582.0	219.6	1050.1	74.9	863.8	242.4	1584.2	78.4	668.1	34.8	1214.6	39.9
0.3	725.2	320.3	1267.1	138.7	899.8	265.4	1598.8	125.5	668.3	37.8	1195.6	59.9
0.4	859.3	414.6	1448.5	218.2	914.5	276.3	1574.1	172.7	668.0	39.8	1176.1	79.9
0.5	984.0	502.4	1595.6	311.8	921.9	282.6	1537.7	219.9	667.4	41.2	1156.4	99.8
0.6	1098.6	583.1	1709.1	417.7	925.8	286.6	1496.5	266.9	666.8	42.3	1136.6	119.7
0.7	1199.4	654.3	1785.7	533.0	927.8	289.3	1453.0	313.7	666.0	43.1	1116.7	139.6
0.8	1277.5	709.7	1814.8	651.2	928.5	291.1	1408.3	360.3	665.2	43.8	1096.9	159.3
0.9	1329.4	747.0	1796.3	766.5	928.4	292.3	1362.9	406.6	664.4	44.4	1077.0	179.1
1.0	1361.8	770.8	1744.8	877.8	927.7	293.0	1317.1	452.6	663.5	44.8	1057.1	198.7
1.1	1381.8	785.8	1673.7	985.6	926.6	293.4	1271.0	498.3	662.6	45.2	1037.3	218.3
1.2	1393.7	795.2	1590.8	1090.5	925.1	293.6	1224.9	543.8	661.7	45.5	1017.5	237.9
1.3	1399.9	800.5	1500.5	1192.7	923.3	293.5	1178.6	588.9	660.7	45.8	997.7	257.4
1.4	1401.7	802.7	1405.4	1292.1	921.3	293.2	1132.5	633.7	659.8	46.0	977.9	276.8
1.5	1399.9	802.3	1307.0	1388.5	919.0	292.8	1086.3	678.1	658.8	46.2	958.1	296.1
1.6	1394.9	799.6	1206.4	1481.4	916.6	292.2	1040.2	722.1	657.8	46.3	938.4	315.4
1.7	1386.7	794.6	1104.3	1570.3	914.0	291.5	994.2	765.8	656.8	46.4	918.7	334.6
1.8	1375.0	787.2	1000.9	1654.1	911.2	290.7	948.4	809.1	655.8	46.5	899.0	353.8
1.9	1359.1	776.9	896.5	1731.3	908.3	289.7	902.6	851.9	654.7	46.6	879.3	372.9
2.0	1337.8	762.7	791.1	1799.3	905.2	288.7	857.0	894.3	653.7	46.6	859.7	391.9

Table 5: Effect of η Under Proportional Policy (2): When b is Large

centralized drop-shipping supply chain is to maximize

$$\begin{aligned}\Pi_s + \Pi_e &= E\left[(p_e - s)\min\left(\Theta(a) + X, Q_e\right) - cQ_e - a\right] \\ &= (p_e - c - s)Q_e - (p_e - s)E(Q_e - \Theta(a) - X)^+ - a.\end{aligned}$$

We denote the marketing sponsorship to the retailer by ϑ and the inventory compensation to the supplier υ per unit. Then, the e-tailer and the supplier's objectives are to maximize

$$\begin{aligned}\Pi_e &= E\left[(p_e - w - s)\min\left(\Theta(a) + X, Q_e\right) - \upsilon(Q_e - \Theta(a) - X)^+ - (1-\vartheta)a\right] \\ \Pi_s &= E\left[w\min\left(\Theta(a) + X, Q_e\right) - cQ_e + \upsilon(Q_e - \Theta(a) - X)^+ - \vartheta a\right].\end{aligned}$$

It can be verified that when $\vartheta = (w-c)/(p_e - c - s)$ and $\upsilon = c(1-\vartheta)$,

$$\begin{aligned}\Pi_e &= \frac{p_e - w - s}{p_e - c - s}\left[(p_e - c - s)Q_e - (p_e - s)E(Q_e - \Theta(a) - X)^+ - a\right] = (1-\vartheta)\left(\Pi_s + \Pi_e\right) \\ \Pi_s &= \frac{w - c}{p_e - c - s}\left[(p_e - c - s)Q_e - (p_e - s)E(Q_e - \Theta(a) - X)^+ - a\right] = \vartheta\left(\Pi_s + \Pi_e\right).\end{aligned}$$

Therefore, this contract coordinates the supply chain. Furthermore, if the supplier can choose the wholesale price, a proportion of marketing expense to subsidize, and the return value, supply chain profits can be split arbitrarily.

6.1 Coordination Under Horizontal Structure

Under horizontal cooperation, separation of the marketing and operations functions still results in inefficiencies. In other words, the decentralized system only achieves suboptimal performance. In this section, we investigate how coordination can be achieved. Recall that we examine three kinds of inventory allocation policies. Hence, we explore the coordination mechanism under each policy. All proofs in this section are similar and omitted, which are available upon request.

6.1.1 Coordination Under Lump Sum Policy

Under lump sum policy, were the e-tailer and the retailer controlled by a central decision maker, the objective is to maximize

$$\Pi = E\left[p \min\left(Q, \Theta(a) + X + Y\right) - wQ - a\right].$$

The optimal customer acquisition expense and order quantity are then characterized by

$$\Theta'(a^{R^*}) = \frac{1}{p-w} \tag{18}$$

$$Q^{R^*} = \Theta(a^{R^*}) + \Psi^{-1}(\frac{p-w}{p}). \tag{19}$$

Coordination issue of such a supply chain is characterized by the following proposition.

Proposition 7. *The supply chain in which an e-tailer sells its demand to a retailer can be coordinated by the following contract: the retailer pays the e-tailer a referral price τ per unit of demand that the e-tailer refers, and at the same time the retailer reimburses a portion of the e-tailer's customer acquisition expenses $\varsigma = 1 - \frac{\tau}{p-w}$.*

6.1.2 Coordination Under Priority Policy

Under priority policy in a centralized supply chain, a single decision maker chooses the marketing expense level and order quantity to optimize the following profit.

$$\Pi_r + \Pi_e = E\left[p \min\left(Y, Q\right) + p \min\left(\Theta(a) + X, (Q-Y)^+\right) - wQ - a\right].$$

The optimal solution (a^{PI^*}, Q^{PI^*}) is characterized by

$$\Theta'\left(a^{PI^*}\right) = \frac{1}{p-w}$$

$$Q^{PI^*} = \Theta(a^{PI^*}) + \Psi^{-1}(\frac{p-w}{p}).$$

We show that the following contract coordinates the decentralized supply chain.

Proposition 8. *Under the priority allocation agreement, consider a contract as follows. In addition to the transfer payment τ, the e-tailer subsidizes the retailer on leftover inventory at α per unit.*

At the same time, the retailer sponsors part (β percent) of the e-tailer's marketing expense. When (α, β) is chosen to satisfy

$$\begin{aligned} \frac{1-\beta}{\tau+\alpha} &= \frac{1}{p} \\ (\tau+\alpha)\Pr\left(\Theta(a^{PI^*}) + X + Y \leq Q^{PI^*}\right) &= \tau \Pr\left(Y \leq Q^{PI^*}\right), \end{aligned}$$

the supply chain is coordinated.

6.1.3 Coordination Under Proportional Policy

Note that under proportional policy, even in the centralized supply chain, the decision maker will pre-specify the inventory allocation to demand from different channels. Therefore, the problem is to maximize the following total profit.

$$\Pi_r + \Pi_e = E\left[p\min\left(\Theta(a) + X, (1-\eta)Q\right) + p\min\left(Y, \eta Q\right) - wQ - a\right].$$

The optimal solution (a^{PO^*}, Q^{PO^*}) is such that

$$\begin{aligned} p\Theta'\left(a^{PO^*}\right)\Pr\left(\Theta(a) + X \leq (1-\eta)Q^{PO^*}\right) - 1 &= 0 \\ p(1-\eta)Pr\left(\Theta(a) + X \leq (1-\eta)Q^{PO^*}\right) + p\eta\Pr\left(Y \leq \eta Q^{PO^*}\right) &= p - w. \end{aligned}$$

Then it can be shown that the following contract coordinates the decentralized supply chain.

Proposition 9. *Under the proportional allocation mechanism, consider the following contract. In addition to the transfer payment τ, the e-tailer subsidizes the retailer on leftover of the* **assigned** *inventory at α per unit. At the same time, the retailer reimburses part (β percent) of the e-tailer's marketing expense. When (α, β) satisfies*

$$\begin{aligned} \frac{1-\beta}{\tau+\alpha} &= \frac{1}{p} \\ (\tau+\alpha)\Pr\left(\Theta(a^{PO^*}) + X \leq (1-\eta)Q^{PO^*}\right) &= \tau, \end{aligned}$$

the supply chain is coordinated.

7 Extension and Conclusions

7.1 Extension

We can extend the single period model to a multi-period setting. In particular, we assume lost sales but a holding cost h_r for left-over inventory carried over to the next period at the retailer's and h_s at the supplier's. We argue that the results for the single period model hold. Take the traditional supply chain as an example. (Proof for the other settings are available upon request.) The retailer's objective is to maximize the follow profit function

$$\Pi_r = \sum_{i=1}^{\infty} \rho^{i-1} E\left[p_r \min\left(Y^i, Q_r^i\right) - h\left(Q_r^i - Y^i\right)^+ - w\left(Q_r^i - q^i\right)\right],$$

where q^i is the beginning inventory before placing an order in period i and ρ is the discount factor. This profit function can be rewritten as

$$\Pi_r = wq^1 + \sum_{i=1}^{\infty} \rho^{i-1} \Pi_r^i\left(Q_r^i, q^i\right),$$

where Π_r^i is the expectation of the single period objective function

$$\Pi_r^i\left(Q_r^i, q^i\right) = E\left[(p_r + h - \rho w)\min\left(Y^i, Q_r^i\right) - (h + w(1-\rho))\, Q_r^i\right].$$

The optimal solution to the single period objective function without starting inventory is

$$\Pr\left(Y \le Q_r\right) = \frac{p_r - w}{p_r + h - \rho w}. \tag{20}$$

Then this solution is a myopic policy (Heyman and Sobel 1984). When the initial inventory is sufficiently low, it is a stationary solution to the infinite-horizon problem. Therefore, our previous results based on the single period model carry through.

7.2 Concluding Remarks

Both drop-shipping and Internet referral have been exercised in practice. As information technologies make online purchases more popular, these business models become more attractive in that both allow small e-tailers to avoid expensive inventory investment. To the retailers, Internet referral allow them to serve the online buyers without having to invest aggressively in customer acquisition. On the other hand, since in both business models inventory management and customer acquisition are separated, inefficiencies such as understocking and underinvestment in marketing may arise. Therefore, it is worth investigating how to improve efficiencies in such Internet related supply chains.

In this paper, we model horizontal cooperation between an e-tailer and a traditional retailer, Internet referral being an example. Particularly, we examine when the e-tailer is better off referring demand to a horizontal retailer and when it would rather satisfy demand via drop-shipping. We also investigate in which settings both the e-tailer and the retailer are better off under horizontal cooperation. While many studies in marketing focus on price competition between the online and traditional sellers, operational issues such as shift of inventory ownership have not been extensively studied. Since inventory ownership, ways of customer acquisition and transportation costs are among the unique features of supply chains that involve Internet retailers, we investigate the issue from both operational and marketing perspectives.

We find that demand referral may be beneficial to both the retailer and the e-tailer under various inventory allocation policies, when (1) the e-tailer can generate more demand per unit of marketing expense; or (2) the e-tailer's original shipping costs are relatively high. In the first case, the e-tailer can bring more customers to the retailer per unit of marketing expense, if the transfer price is adequate. In the second case, the e-tailer can afford getting a lower margin since compared with drop-shipping, it benefits from significant shipping cost reductions. These findings suggest that an e-tailer with the above mentioned features may expect more profit from Internet referral services than from drop-shipping. Nonetheless, in any supply chain structure studied in this paper, due to separation of the operations and marketing functions, only system-wide suboptimal performance can be achieved. We therefore propose coordination contracts for various supply chains.

Finally, we examine, without coordination, the impact of different inventory allocation mechanisms. We find that from the supply chain's perspective, the policy in which the retailer does not differentiate the retail and online demands performs the best.

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