1. Use L’Hospital’s rule to find the limit of \( \frac{\sin(x)}{x} \) as \( x \to 0 \).

2. Derive the expression of the 1st and 2nd order Taylor expansion of the function \( f(x) = c \exp(-dx^2) \) around the point \( x_0 \). Find an expression for the \( k \)'th order Taylor expression that works for all \( k \).

3. Consider modeling a dataset \( D = \{(x_i, y_i), i=1,...,n\} \) with a line through the origin \( y = mx \), where \( m \) is a parameter. The error \( E_i(m) \) between the line and the \( i \)'th datapoint is \( E_i(m) = |mx_i - y_i| \). Let the objective function \( f(m) \) be the \textit{sum of squared errors} \( f(m) = \sum_{i=1}^{n} E_i^2(m) \). Find the critical points of \( f \). Show that there is one unique point and that it is indeed a local minimum.

4. Prove that if a function \( f(x) \) is differentiable, and its derivative \( f'(x) \) satisfies \( |f'(x)| \leq K \) for all \( x \), then \( f \) satisfies the Lipschitz condition with constant \( K \). Use Rolle’s theorem, which states that if a differentiable function \( g(x) \) attains the same values at points \( a \) and \( b \), then \( g'(x) = 0 \) at some point \( x \in (a,b) \). Give an example of a function that is Lipschitz but is not differentiable.

5. Give pseudocode for a constrained Newton's method that limits the step \( \Delta x_t = -\frac{g(x_t)}{g'(x_t)} \) in order to prevent divergence. The iterates should satisfy \( |g(x_{t+1})| < |g(x_t)| \).