Madoff Finance Fraud Detection with Central Limit Theorem

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Introduction

On December 11, 2008, the financial world was shaken by the arrest of Bernard Madoff, the master mind of a multi-billion dollar Ponzi scheme that started in 1990[1]. In his testimony, Mr. Madoff reported that he fraudulently informed investors that he utilized a "split-strike conversion strategy" to achieve his reported returns. Disturbingly, the US Securities and Exchange Commission (SEC) was informed of Madoff's fraud in May of 1999 by Harry Markopolos[2], who sought to have the SEC take action. Unfortunately, the information he presented did not sufficiently convey the significance of his discovery. The SEC's inaction highlights the fact that regulators and the investing public lack standard, commonly recognized methods for detecting such frauds.

Objective

Our goal is to develop an innovative method for calculating the probability distribution function of a randomly selected portfolio of known size from a large population of potential investments. Given a known universe of potential investments and a known portfolio size and holding period, this method gives the probability that a portfolio selected at random would achieve the reported return. Highly improbable results are also highly suspicious, because it is well-established that even high-performing US public equity investors exceed a randomly-generated portfolio by only a small margin[3].

Applying our technique to the reported Madoff fund returns demonstrates not only the extreme improbability that the Madoff results could have been real, but how simple it is to establish this improbability.

Madoff Strategy

Madoff's "split-strike" strategy supposedly involved purchasing and selling a basket of 35 stocks drawn from the Standard and Poor's (S&P) 100 Index[4], which is a collection of the 100 largest publicly traded US stocks by market capitalization and hedging his basket of stocks by buying and selling option contracts on the index to limit the potential losses to his clients[2]. He further claimed to at times forgo buying a basket of stocks, ideally during adverse market conditions, and instead held short-term US Government debt.

Hypothesis

Due to constraints regarding available information on Madoff's strategy, we make the following simplifying assumptions to develop a feasible model:

- Basket size limited to 35 equally-weighted stocks from S&P 100 Index based on statements[2] by feeder funds that no stock was weighted more than 5%
- Basket of 35 stocks are traded once monthly
- Madoff's split-strike strategy accomplished by writing OEX calls and buying OEX puts each month "at the money," with one month time to expiry, call price equal to put price, transaction costs zero, all options held to expiry
- The required absolute return needed to produce Madoff's reported return is the return of the S&P 100 plus Madoff's reported return

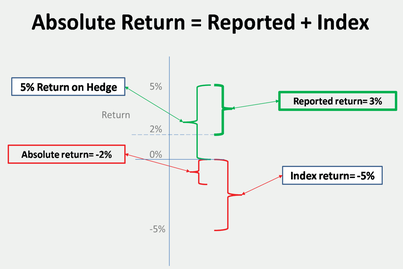

Figure 1 Depiction of Madoff's Reported Absolute Return

Methodology

- Gather S&P 100 Data from 1991 to 2003 totaling 156 months
- Acquire monthly reported returns of Madoff for the same time period
- Compute the probability, using **Central Limit Theorem**, for any basket of 35 stocks to achieve Madoff's reported returns for each month
- Compute the rolling probability of Madoff achieving a return equal to or greater than his reported return for each month

Central Limit Theorem

The Central Limit Theorem (CLT) states that the mean of a sufficiently large number of independent random variables, each with finite mean and variance, will be approximately normally distributed[5].

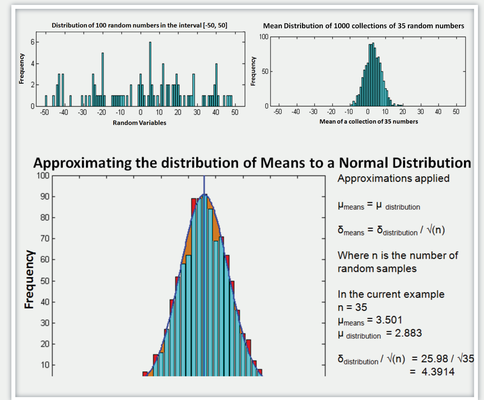

Figure 2 Visual Display of Central Limit Theorem

The counter-intuitive thing about the Central Limit Theorem is that no matter what the shape of the original distribution, the sampling distribution of the mean approaches a normal distribution. For most distributions, a normal distribution is approached very quickly as N increases, where N is the sample size for each mean and not the number of samples. In a sampling distribution, the number of samples is assumed to be infinite. and since our distribution has a finite number of samples, we needed to apply an approximation as shown in figure 2.

Data

The time period covered by our data is Jan 1991 to Dec 2003. The bulk of our return data is obtained from the Center for Research in Security Prices (CRSP)[6]. For Data not found on CRSP, Yahoo Finance was used to fill in for the missing data.

Computation for Monthly Probability of Success for Reported Return

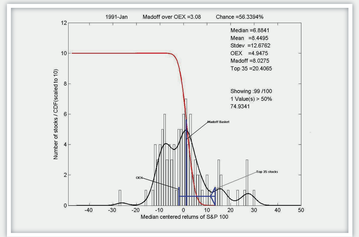

Figure 3 Strong Success Probability

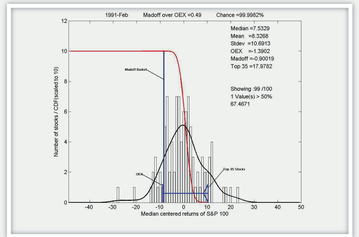

Figure 4 Weak Success Probability

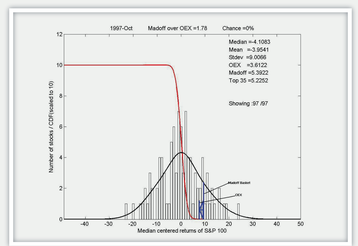

Figure 5 Success Impossible

Results

Out of 156 months, 25 or 16% of the months had returns that were impossible to achieve for any basket of 35 stocks while 34 or 22% were extremely unlikely. Madoff could have opted to not buy stocks for all the months that his probability of achieving his stated return was less that 5%.

To model this possibility, we calculate a rolling probability using 50% (essentially a coin flip) to represent the probability that Madoff stayed out of the market and held short-term US Government debt instead.

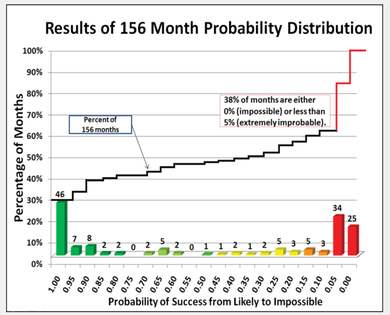

Figure 6 Dispersion of Returns

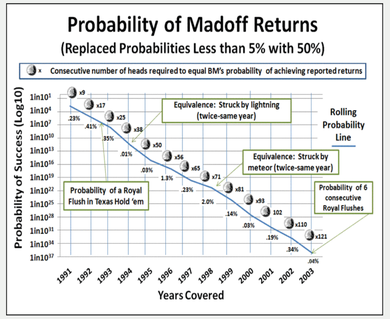

Figure 7 Rolling Annual Probability of Success

Conclusion

In 25 of 156 months when the Madoff Fund was allegedly operating, it could not have achieved its reported results with any combination of stocks. Assuming that Madoff chose to stay out of the market for those 25 months – and for the 34 months when the probability of obtaining his reported results is less than or equal to 5% - and further assuming that Madoff had a 50% chance each month of guessing correctly whether to stay out of the market, then his reported returns for the full 156 months would be an astronomically small 1 in 10^{37}

Literature Cited

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[4] Standard & Poor's Index Committee. S&P 100 Fact Sheet [http://www.standardandpoors.com/indices/main/en/us]

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Acknowledgements

We are indebted to Daniel Egger for his input on how to best model Bernard Madoff's Split-Strike scheme, to the Standard & Poor's Index & Portfolio Services for providing us S&P 100 Index component lists, and to Jai Jacobs for his assistance in finding elusive stock information

November 2009