Supplemental Appendix

to accompany

"All-receive procurement auctions"

by

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This Supplemental Appendix contains supplemental details and illustrations. Appendix A provides mechanism design foundations for the optimal auction. Appendix B provides supplemental details for some points made in the paper. Appendix C provides supplemental illustrations. Appendix D provides supplemental details for the analysis of collusion.

A Optimal auction design

We characterize here the mechanism that maximizes the buyer's expected payoff. Let $P_i(c_i)$ denote the expected probability that supplier i is selected as supplier and $T_i(c_i)$ denote its expected payment when its cost is c_i. Supplier i's expected gain can then be expressed as

$$G_i(c_i) = T_i(c_i) - c_i P_i(c_i).$$

Standard arguments imply that the mechanism is incentive-compatible if and only if, for every supplier i, $P_i(\cdot)$ is continuous and nonincreasing and, for every $c_i \in \mathcal{C}$,

$$G_i(c_i) = G_i(\bar{c}) + \int_{c_i}^{\bar{c}} P_i(c)\, dc. \tag{A.1}$$

The buyer's expected payoff U can be expressed as the total expected surplus, $\sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i)\, dF(c_i)$, minus the suppliers' total expected gains, $\sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} G_i(c_i)\, dF(c_i)$:

$$\begin{aligned} U &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \left[(v - c_i) P_i(c_i) - G_i(c_i)\right] dF(c_i) \\ &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \left[(v - c_i) P_i(c_i) - \int_{c_i}^{\bar{c}} P_i(c)\, dc\right] dF(c_i) - \sum_{i=1}^{n} G_i(\bar{c}). \end{aligned}$$

Obviously, it is optimal for the buyer to set $G_i(\bar{c}) = 0$ for every $i \in \{1, \ldots, n\}$ to minimize the suppliers' expected gains. Using Fubini's theorem, we can then rewrite the buyer's expected payoff as

$$\begin{aligned} U &= \sum_{i=1}^{n} \Big(\int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i) f(c_i) dc_i - \int_{\underline{c}}^{\bar{c}} \int_{c_i}^{\bar{c}} P_i(x) f(c_i) dx dc_i \Big) \\ &= \sum_{i=1}^{n} \Big(\int_{\underline{c}}^{\bar{c}} (v - c_i) P_i(c_i) f(c_i) dc_i - \int_{\underline{c}}^{\bar{c}} P_i(x)\, F(x)\, dx \Big) \\ &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) P_i(c_i) dF(c_i). \end{aligned}$$

Finally, for $\mathbf{c} \in \mathcal{C}^n$, let $\hat{P}_i(\mathbf{c})$ denote supplier i's probability of being selected, as a function of *all* suppliers' costs, $\mathbf{c} = (\mathbf{c}_{-i}, c_i) = (c_1, ..., c_n)$. We thus have:

$$P_i(c_i) = \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i(\mathbf{c}_{-i}; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i}),$$

and the buyer's expected payoff can be expressed as:

$$\begin{aligned} U &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) P_i(c_i) dF(c_i) \\ &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \left[\int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i(\mathbf{c}_{-i}; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i}) \right] dF(c_i) \\ &= \sum_{i=1}^{n} \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \hat{P}_i(\mathbf{c})\, dF(c_1) \cdots dF(c_n) \\ &= \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \sum_{i=1}^{n} \Big(v - c_i - \frac{F(c_i)}{f(c_i)} \Big) \hat{P}_i(\mathbf{c})\, dF(c_1) \cdots dF(c_n) \\ &= \mathbb{E}_{\mathbf{c}} \Big[\sum_{i=1}^{n} (v - \gamma(c_i)) \hat{P}_i(\mathbf{c}) \Big], \end{aligned}$$

where the final equality uses

$$\gamma(c) \equiv c + \frac{F(c)}{f(c)}.$$

Hence, ideally the auction designer would like to maximize, for every $\mathbf{c} \in \mathcal{C}^n$,

$$\sum_{i=1}^{n} (v - \gamma(c_i)) \hat{P}_i(\mathbf{c}).$$

The probabilities $\hat{P}_i(\cdot)$ must satisfy the feasibility constraints $\hat{P}_i(\cdot) \geq 0$ and $\sum_{i=1}^{n} \hat{P}_i(\cdot) \leq 1$. Ignoring incentive constraints, the buyer would therefore not procure if $v < \min_{i \in \mathcal{N}} \gamma(c_i)$, and otherwise would like to select the supplier with the lowest $\gamma(c_i)$. Let

$$\hat{P}_i^*(\mathbf{c}) = \begin{cases} 1 \text{ if } \gamma(c_i) < \min\{\min_{j \neq i} \gamma(c_j), v\}, \\ 0 \text{ otherwise,} \end{cases}$$

denote this unconstrained solution. Finally, for every supplier i, let

$$P_i^*(c_i) \equiv \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i^*(\mathbf{c}_i; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i})$$

denote the resulting interim expected selection probability.

Related to Corollary 1, building on the analysis above, if $\gamma(\cdot)$ is increasing, the unconstrained solution selects the lowest-cost supplier if its cost lies below $r^* \equiv \min\{\gamma^{-1}(v), \bar{c}\}$, and does not select any supplier otherwise:

$$\hat{P}_i^*(\mathbf{c}) \equiv \begin{cases} 1 \text{ if } c_i < \min\{\min_{j \neq i} c_j, r^*\}, \\ 0 \text{ otherwise.} \end{cases}$$

The resulting expected selection probability is then symmetric and given by

$$P_i^*(c_i) = P^*(c_i) \equiv \begin{cases} P_n(c_i) \text{ if } c_i < r^*, \\ 0 \text{ otherwise.} \end{cases}$$

If instead $\gamma(\cdot)$ is not everywhere increasing, then the unconstrained solution does not have the monotonicity property required for incentive compatibility. The optimal allocation then relies on an "ironed" version of the virtual cost, $\bar{\gamma}(c)$, which is nondecreasing, continuous, and coincides with $\gamma(c)$ whenever $\bar{\gamma}(c)$ is increasing (see Myerson, 1981). Specifically, there exists a sequence of $K = |\mathcal{K}|$ "bunching" ranges (or "ironing intervals") $\mathcal{C}_k \equiv [\underline{c}(k), \bar{c}(k)]$, for $k \in \mathcal{K}$ satisfying $\{\underline{c}_k, \bar{c}_k\}_{k \in \mathcal{K}}$ with $\underline{c} \leq \underline{c}_1 < \bar{c}_1 < \underline{c}_2 < \cdots < \bar{c}_{K-1} < \underline{c}_K < \bar{c}_K \leq \bar{c}$, such that:

- $\bar{\gamma}(c) = \gamma(c)$ for $c \notin \cup_k \{\mathcal{C}_k\}$;
- $\bar{\gamma}(c) = \gamma(\underline{c}_k) = \gamma(\bar{c}_k) \equiv \overline{\gamma}_k$ for $c \in \mathcal{C}_k$; and
- $\int_{\underline{c}_k}^{\bar{c}_k} [\bar{\gamma}(c) - \gamma(c)]\, dF(c) = 0$.

The optimal mechanism is then such that:

$$\hat{P}_i^{**}(\mathbf{c}) = \begin{cases} 1 \text{ if } \bar{\gamma}(c_i) < \min\{\min_{j\neq i}\bar{\gamma}(c_j), v\}. \\ 0 \text{ otherwise.} \end{cases}$$

implying:[1]

$$P_i^{**}(c_i) = P^{**}(c_i) \equiv \int_{\underline{c}}^{\bar{c}} \cdots \int_{\underline{c}}^{\bar{c}} \hat{P}_i^{**}(\mathbf{c}_{-i}; c_i)\, d\mathbf{F}_{-i}(\mathbf{c}_{-i}),$$

and:

$$T_i^{**}(c_i) = T^{**}(c_i) \equiv c_i P^{**}(c_i) + \int_{c_i}^{\bar{c}} P^{**}(c)\, dc,$$

By construction, $\hat{P}_i^{**}(\cdot; c_i)$ and $P_i^{**}(c_i)$ are constant in any range $c_i \in \mathcal{C}_k$; from incentive-compatibility, $T_i^{**}(c_i)$ is thus also constant in any such range. By contrast, outside these ranges, $\hat{P}_i^{**}(\cdot; c_i)$, $P_i^{**}(c_i)$, and $T_i^{**}(c_i)$ are all decreasing in c_i.

B Supplemental details

B.1 Derivation of $\overline{P}_n(k)$

$$\begin{aligned}
\overline{P}_n(k) &= \sum_{k=0}^{n-1} \binom{n-1}{k} [1 - F(\bar{c}_k)]^{n-1-k} \frac{[F(\bar{c}_k) - F(\underline{c}_k)]^k}{k+1} \\
&= \sum_{k=0}^{n-1} \frac{(n-1)!}{k!\,(n-1-k)!} [1 - F(\bar{c}_k)]^{n-1-k} \frac{[F(\bar{c}_k) - F(\underline{c}_k)]^k}{k+1} \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \sum_{k=0}^{n-1} \frac{n!}{(k+1)!\,[n-(k+1)]!} [1 - F(\bar{c}_k)]^{n-(k+1)} [F(\bar{c}_k) - F(\underline{c}_k)]^{k+1} \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \sum_{k=1}^{n} \frac{n!}{k!\,(n-k)!} [1 - F(\bar{c}_k)]^{n-k} [F(\bar{c}_k) - F(\underline{c}_k)]^k \\
&= \frac{1}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]} \left(\sum_{k=0}^{n} \binom{n}{k} [1 - F(\bar{c}_k)]^{n-k} [F(\bar{c}_k) - F(\underline{c}_k)]^k - [1 - F(\bar{c}_k)]^n \right) \\
&= \frac{[1 - F(\underline{c}_k)]^n - [1 - F(\bar{c}_k)]^n}{n\,[F(\bar{c}_k) - F(\underline{c}_k)]}.
\end{aligned}$$

[1] By construction, $\hat{P}_i^{**}(\mathbf{c})$ is symmetric, implying that $P_i^{**}(c_i)$ is also symmetric: $\hat{P}_i^{**}((\mathbf{c}_{-i,j}, c_j); c_i) = \hat{P}_j^{**}((\mathbf{c}_{-i,j}, c_i); c_j)$ and $P_i^{**}(c) = P_j^{**}(c)$ for any $i \neq j \in \{1, \ldots, n\}$.

Further, we have:

$$\begin{aligned}\mathbb{E}_c\left[P_n\left(c\right) \mid c \in \mathcal{I}_k\right] &= \int_{\underline{c}_k}^{\overline{c}_k} \left[1-F\left(c\right)\right]^{n-1} \frac{f\left(c\right)}{F(\overline{c}_k)-F(\underline{c}_k)} dc \\ &= \frac{\left[1-F\left(\underline{c}_k\right)\right]^n - \left[1-F\left(\overline{c}_k\right)\right]^n}{n\left[F(\overline{c}_k)-F(\underline{c}_k)\right]} \\ &= \overline{P}_n\left(k\right).\end{aligned}$$

B.2 Dominant strategies in the adjusted SPPA

As noted, truthful bidding is a dominant strategy in the SPPA*. The usual reasoning ensures that, compared with bidding at cost, no supplier can benefit from bidding below cost, or from bidding above cost when the second-lowest cost does not lie in $\mathcal{I}(v)$.[2] Hence, to establish dominant strategy incentive compatibility (DIC), we only need to check that a supplier with cost c cannot benefit from bidding $b > c$ when $m > 0$ next-lowest costs lie in $\mathcal{I}_k$, for some $k \in \mathcal{K}_v$. If $c \in \mathcal{I}_k$, then any $b \in \mathcal{I}_k$ yields the same payoff as bidding c, and any $b > \overline{c}_k$ would prevent the supplier from being selected, and is thus dominated by bidding c. Consider now a bidder with cost $c \leq \underline{c}_k$ that is paid p when bidding below $\underline{c}_k$. By bidding in $\mathcal{I}_k$, the supplier would be selected with probability $1/(m+1)$ and paid in that case $\overline{c}_k$. DIC thus requires $p - c \geq \frac{\overline{c}_k - c}{m+1}$. This constraint is tightest at $c = \underline{c}_k$ because the derivative with respect to c of the right-hand side, $-1/(m+1)$, exceeds that of the left-hand side, -1. Setting $p = p_m(k)$ thus satisfies DIC with equality for the cost $\underline{c}_k$ and with a strict inequality for all lower costs.

B.3 Details for multiple-receive auctions

In this appendix, we now briefly elaborate on the scope and challenges for using multiple-receive procurement auctions. For this purpose, we focus on ex post efficiency and assume that $v \geq \overline{c}$.

For there to be an equilibrium in which paying the m lowest or highest bidders means paying the m suppliers with the lowest costs, the equilibrium bid function needs to be strictly monotone. While this is, evidently, the case for $m \in \{1, n\}$ regardless of the distribution, whether it holds for any $m \in \{2, \ldots, n-1\}$ depends on the distribution, as we show next. This suggests that the scope for using multiple-receive procurement auctions to implement the efficient allocation, while striking a different balance between ex post participation constraints and resilience, comes with a caveat.

Even if equilibrium bidding is monotone for all m, because it is increasing for $m = 1$ and

[2] Recall that $\mathcal{I}_k$ is the k-th ironing interval, and we define $\mathcal{K}(v) \equiv \{k \in \mathcal{K} \mid \overline{c}_k \leq r^*\}$ and $\mathcal{I}(v) \equiv \cup_{k \in \mathcal{K}(v)} \mathcal{I}_k$.

decreasing for $m = n$, to determine the rules of the auction, the designer will need to know for which values of m the bid function is increasing (decreasing). The approach we take is, therefore, akin to reverse auction theory—we first assume that a monotone equilibrium exists in which either the m lowest or m highest bidders are paid, and then we verify whether the assumption is correct and, if so, determine the appropriate auction rules. For $m \in \{1, \ldots, n\}$ and $c \in [\underline{c}, \overline{c}]$, let

$$q_{m,n}(c) \equiv \sum_{i=0}^{m-1} \binom{n-1}{i} F(c)^i [1 - F(c)]^{n-1-i}$$

denote the probability that no more than $m - 1$ draws among $n - 1$ are less than c. Notice that $q_{1,n}(c) = [1 - F(c)]^{n-1} = P_n(c)$ and $q_{n,n}(c) = 1$. Moreover, we have $q_{m+1,n}(c) > q_{m,n}(c)$ for any $m < n$ and $c \in (\underline{c}, \overline{c})$.

In a symmetric equilibrium with bid function $\beta_{m,n}(c)$, a supplier with cost c maximizes

$$\beta_{m,n}(\hat{c}) q_{m,n}(\hat{c}) - P_n(\hat{c})c.$$

As usual, this has to be maximized at $\hat{c} = c$, yielding

$$\beta_{m,n}(c) = \frac{cP_n(c) + \int_c^{\overline{c}} P_n(x)dx}{q_{m,n}(c)}.$$

Because $q_{1,n}(c) = P_n(c)$, we have $\beta_{1,n}(c) = c + \frac{\int_c^{\overline{c}} P_n(x)dx}{P_n(c)}$, which is increasing, as it should be, because it is the equilibrium bid function in a FPPA. Similarly, because $q_{n,n}(c) = 1$, $\beta_{n,n}(c) = cP_n(c) + \int_c^{\overline{c}} P_n(x)dx$, which is the (decreasing) ARPA bid function. Moreover, because $q_{m,n}(c)$ decreases in m, $\beta_{m,n}(c)$ decreases in m, in line with the hypothesis that decreasing m reduces the scope that a selected supplier's participation constraint is violated ex post.

Lemma B.1. *For $n \geq 2$ and $m \in \{1, \ldots, n\}$,*

$$\beta'_{2,n}(c) = \frac{(n-1)(1-F(c))^{n-3} f(c)}{(q_{2,n}(c))^2} \left(-c(1-F(c))^{n-1} + (n-2)F(c) \int_c^{\overline{c}} (1-F(x))^{n-1} dx \right).$$

Proof. We have

$$\frac{P_n(c)}{q_{m,n}(c)} = \frac{1}{\sum_{i=0}^{m-1} \binom{n-1}{i} \left[\frac{F(c)}{1-F(c)} \right]^i},$$

which is decreasing in c for $m > 1$. The derivative of $\beta_{m,n}$ is

$$\beta'_{m,n}(c) = c\left[\frac{P_n(c)}{q_{m,n}(c)}\right]' - \frac{q'_{m,n}(c)}{q_{m,n}(c)}\frac{\int_c^{\bar{c}} P_n(x)dx}{q_{m,n}(c)}.$$

The first term is negative and the second is negative, which with the negative sign makes it positive. Focusing on the case of $m = 2$, $q_{2,n}(c) = (1 - F(c))^{n-2}(1 - F(c) + (n-1)F(c))$, $q'_{2,n}(c) = -(n-1)(n-2)F(c)(1-F(c))^{n-3}f(c)$, and $\left[\frac{P_n(c)}{q_{2,n}(c)}\right]' = \frac{-(n-1)(1-F(c))^{2n-4}f(c)}{(q_{2,n}(c))^2}$, so

$$\begin{aligned}\beta'_{2,n}(c) &= c\frac{-(n-1)(1-F(c))^{2n-4}f(c)}{(q_{2,n}(c))^2} - \frac{q'_{2,n}(c)}{q_{2,n}(c)}\frac{\int_c^{\bar{c}} P_n(x)dx}{q_{2,n}(c)} \\ &= \frac{(n-1)(1-F(c))^{n-3}f(c)}{(q_{2,n}(c))^2}\left(-c(1-F(c))^{n-1} + (n-2)F(c)\int_c^{\bar{c}}(1-F(x))^{n-1}dx\right),\end{aligned}$$

which completes the proof. ■

Using Lemma B.1, for uniformly distributed costs, $\beta_{2,n}(c)$ is decreasing. This means that for uniformly distributed costs, the multiple-receive procurement auction that pays the 2 highest bidders and selects the highest bidder to produce has a monotone equilibrium.

However, there are distributions such that this is not the case. For example, for $F(c) = \sqrt{c}$ with support $[0,1]$, $\beta_{2,n}(c)$ is nonmonotone for $n \geq 3$. To see this, note that using Lemma B.1, for the case of $F(c) = \sqrt{c}$, we have

$$\beta'_{2,n}(c) = -\frac{(n-1)\left(-\sqrt{c}\,(n^2-7n+4) + 2c(n-2)n - 2n + 4\right)}{2n(n+1)\left(\sqrt{c}(n-2)+1\right)^2},$$

where $\beta'_{2,n}(0) = \frac{2-3n+n^2}{n+n^2}$, which is positive for $n > 2$, and $\beta'_{2,n}(1) = \frac{-1}{2(n-1)}$, which is negative for $n \geq 2$, establishing that $\beta_{2,n}(c)$ is not monotone for this case. This is illustrated in Figure B.1.

(a) $\beta_{1,4}$ with $F(c) = \sqrt{c}$ (b) $\beta_{m,4}$ with $F(c) = \sqrt{c}$ for $m \in \{2, 3, 4\}$

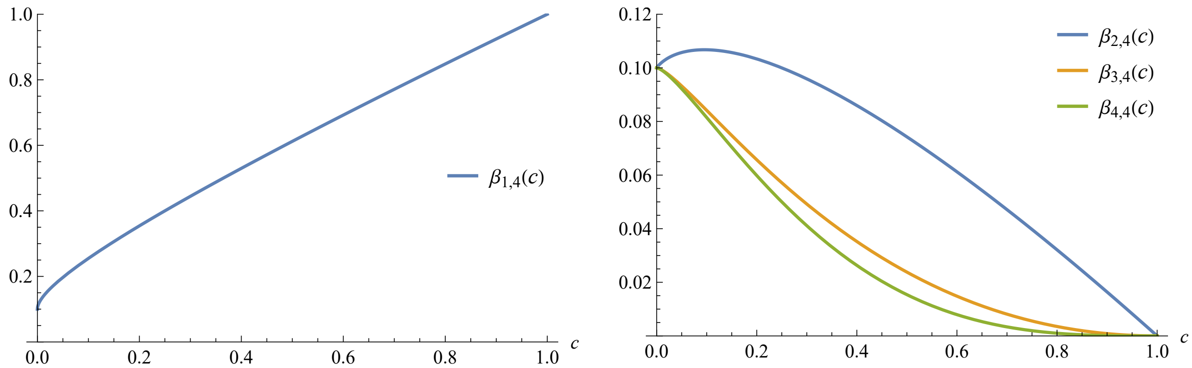

Figure B.1: Bid functions $\beta_{m,n}(c)$ for $n = 4$ and $m \in \{1, 2, 3, 4\}$ assuming $F(c) = \sqrt{c}$ with support $[0, 1]$.

One can also show that the expression for $\beta'_{2,n}(c)$ in Lemma B.1 is positive for $F(c) = \sqrt{c}$ with support $[0, 1]$ and $n \in \{3, \dots\}$ if $c = 0$, but negative if $c = 1$. Thus, for $F(c) = \sqrt{c}$ and $n = 4$, we require m of at least 3 to have a monotone decreasing bid function.

C Supplemental illustrations

C.1 Illustration of the ARPA bid function and reserve.

In Figure C.1, will illustrate the ARPA bid function and reserve.

(a) ARPA bid function ($r = 0.75$) (b) ARPA reserve

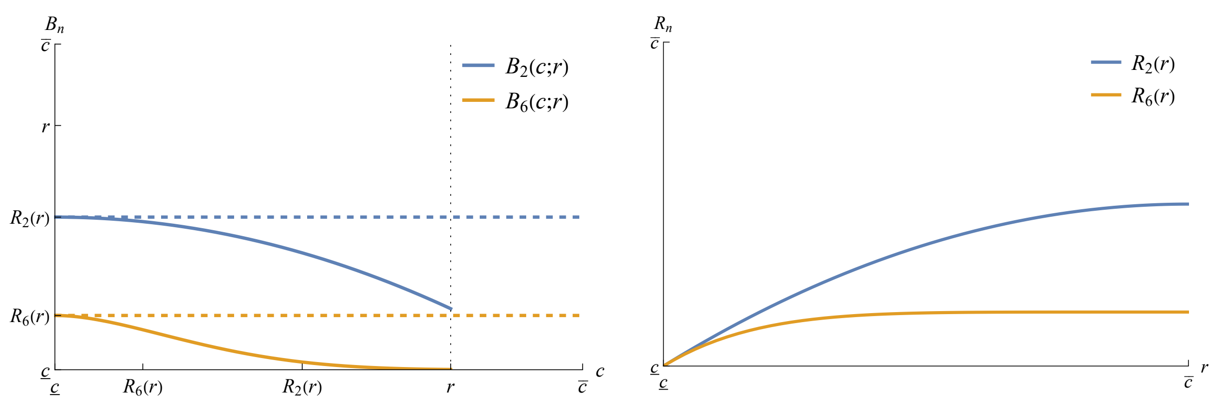

Figure C.1: Illustration of the ARPA bid function and reserve. Panel (a) shows the ARPA equilibrum bid strategy for $r = 0.75$ and $n \in \{2, 6\}$. Panel (b) shows the ARPA reserve corresponding to a second-price or first-price procurement reserve of r. Assumes uniformly distributed costs on $[0, 1]$.

C.2 Illustration of the comparison between FPPA and ARPA bid functions

A supplier with cost $\underline{c}$ bids the same amount in the ARPA and in the FPPA. In the ARPA, bids are decreasing in c, but in the FPPA are increasing in c. As a result, all bids in the FPPA weakly exceed all bids in the ARPA. This is illustrated in Figure C.2.

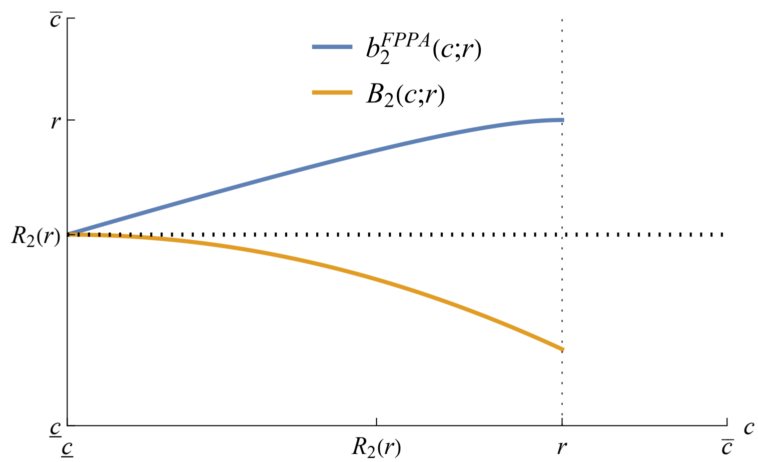

Figure C.2: Comparison of the ARPA and FPPA bid functions. Assumes $r = 0.75$ and uniformly distributed costs on $[0, 1]$.

C.3 Illustration of the interim expected payoffs and threshold type in the ARPA

As discussed in Section 3, there exists a unique $\tilde{c}_n(r) \in (\underline{c}, r)$ such that $B_n(c; r) \gtreqless c$ if and only if $c \lesseqgtr \tilde{c}_n(r)$. This means that suppliers with costs below $\tilde{c}_n(r)$ make a profit even if called upon to produce, whereas suppliers with costs above $\tilde{c}_n(r)$ make a profit only if they do not have to produce (and make a loss otherwise). Because $B_n(c; r)$ is decreasing in n and increasing in r, $\tilde{c}_n(r)$ is also decreasing in n and increasing in r. This is illustrated in Figure C.3.

(a) ARPA interim expected payoffs ($r = 0.75$)

(b) Threshold type for payoff sign

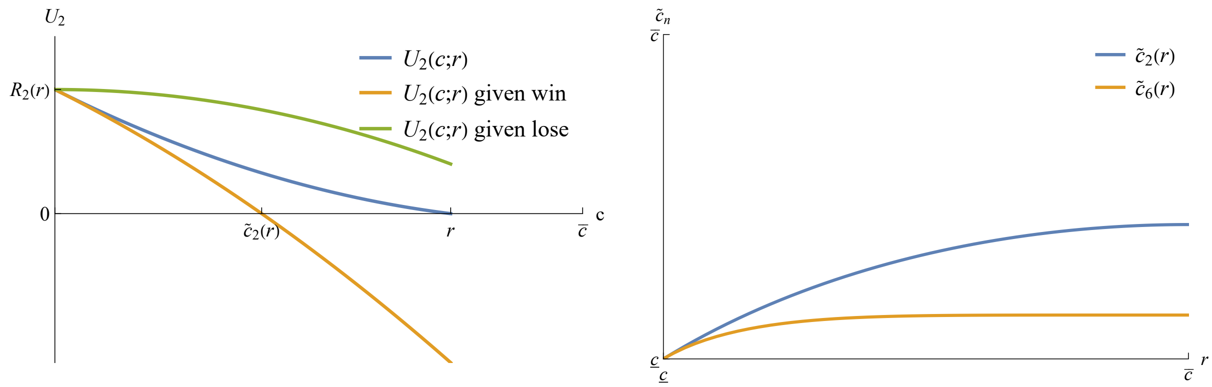

Figure C.3: Panel (a): interim expected payoffs in the ARPA for $r = 0.75$ and $n = 2$; Panel (b): threshold type $\tilde{c}_n(r)$ for $n \in \{2, 6\}$ such that, conditional on producing the good, interim expected payoffs are positive for lower types and negative for higher types. Assumes uniformly distributed costs on $[0, 1]$.

C.4 Illustration of the selection probability in the optimal mechanism

As defined in equation (5), $P_n^*(c)$ is the selection probability for a supplier with cost c in the optimal mechanism with uniform random tie-breaking. It is illustrated in Figure C.4.

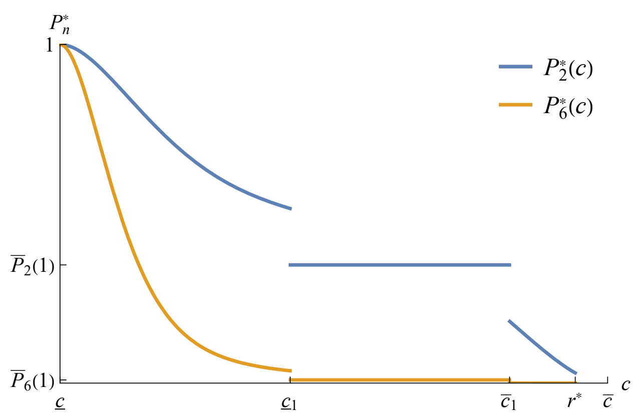

Figure C.4: Selection probability $P_n^*(c)$ in the optimal mechanism for a nonregular cost distribution. Assumes $f(c) = \text{Beta}(c; 2, 8)/2 + \text{Beta}(c; 6, 2)/2$, which is shown in Figure 1(a), and $r^* = 0.9409$, which is optimal for $v = 2$.

C.5 Illustration of the survival rate in the ARPA

In Figure C.5, we illustrate the survival rate in the ARPA, which is discussed in Section 5.1.

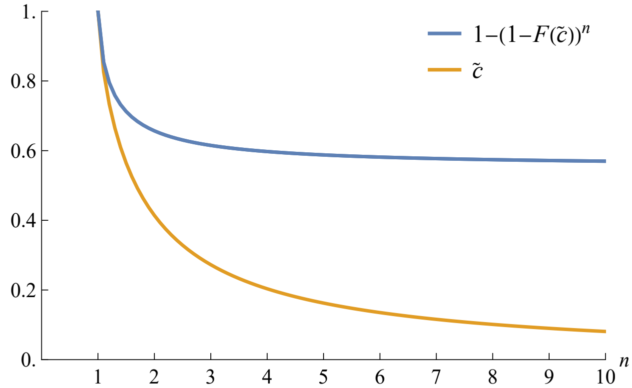

Figure C.5: Threshold type $\tilde{c}$ such that $B_n(\tilde{c};\overline{c}) - \tilde{c} = 0$ and the probability, $1-(1-F(\tilde{c}))^n$, that the producing supplier survives a small liquidity shock in the ARPA. Assumes uniformly distributed costs on $[0,1]$ and $r = \overline{c}$.

C.6 Illustration of the adjusted FPPA

In Figure C.6, we illustrate the equilibrium bid functions in the adjusted FPPA, which is defined in Section 5.2.

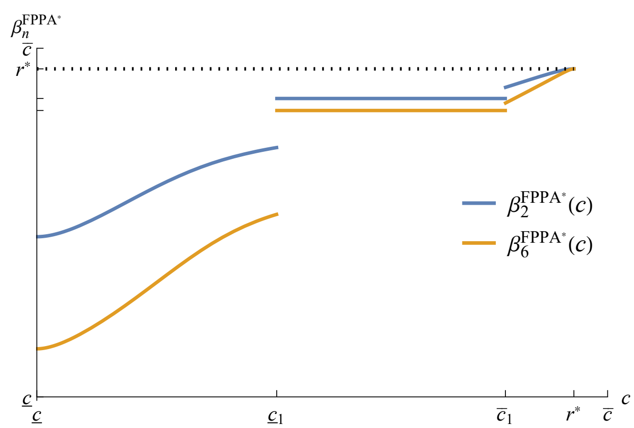

Figure C.6: FPPA* bid function assuming $f(c) = \text{Beta}(c;2,8)/2 + \text{Beta}(c;6,2)/2$, which is shown in Figure 1(a), and $r^* = 0.9409$, which is optimal for $v = 2$.

D Details for analysis of collusion

D.1 Framework

We consider the framework of Iossa et al. (2024). There are two identical markets. A buyer operates in both markets and has value v for one unit of the good in each market in each period over time. There are two suppliers 1 and 2, with costs drawn from distribution F with density f over the support $[\underline{c},\overline{c}]$ and increasing reversed hazard rate $F(c)/f(c)$. Cost draws are independent across suppliers, markets, and time. All agents are risk neutral with quasi-linear utility, and discount the future according to the common discount factor $\delta \in [0,1)$. Bids are observed at the end of each period.

We consider three auction formats: FPPA or SPPA with reserve $r \in (\underline{c}, \overline{c}]$, and ARPA with reserve $R(r)$, where $R(r)$ is defined to be the same as $R_2(r)$ as defined in equation (2). Thus, we have

$$R(r) \equiv \underline{c} + \int_{\underline{c}}^{r} [1 - F(c)]\, dc = r - \Phi(r), \tag{D.2}$$

where

$$\Phi(r) \equiv \int_{\underline{c}}^{r} F(c)\, dc$$

is the primitive of $F(\cdot)$ satisfying $\Phi(\underline{c}) = 0$. By construction, $R(r) \in (\underline{c}, r)$ and is strictly increasing in r in the range $r \in (\underline{c}, \overline{c}]$. Note that:[3]

$$R(r) > [1 - F(r)]\, r. \tag{D.3}$$

We begin by defining the competitive profit. Under a SPPA, a supplier with cost c obtains an expected profit equal to:

$$\pi^c(c;r) \equiv \mathbb{E}_{\tilde{c}}\left[\max\{0, \hat{\pi}^c(c;r) \min\{r, \tilde{c}\} - c\}\right] = \mathbf{1}_{c<r} \times \hat{\pi}^c(c;r),$$

where (using $n = 2$)

$$\begin{aligned} \hat{\pi}^c(c;r) &\equiv \int_{c}^{r} (\tilde{c} - c)\, dF(\tilde{c}) + [1 - F(r)](r - c) \\ &= [(\tilde{c} - c) F(\tilde{c})]_{c}^{r} - \int_{c}^{r} F(\tilde{c})\, d\tilde{c} + [1 - F(r)](r - c) \\ &= (r - c) F(r) - [\Phi(r) - \Phi(c)] + [1 - F(r)](r - c) \\ &= r - c - [\Phi(r) - \Phi(c)] \\ &= R(r) - R(c). \end{aligned} \tag{D.4}$$

Using the revenue equivalence theorem, under all three formats, the expected competitive profit is:

$$\overline{\pi}^c(r) \equiv \mathbb{E}_c[\pi^c(c;r)] = \int_{\underline{c}}^{r} F(c)[1 - F(c)]\, dc = \Phi(r) - \Phi_2(r),$$

[3] Indeed, for $r = \underline{c}$ we have $R(\underline{c}) = [1 - F(\underline{c})]\, \underline{c}\, (= \underline{c})$ and, for $r > \underline{c}$:

$$\frac{d}{dr}\left(R(r) - [1 - F(r)]\, r\right) = [1 - F(r)] - ([1 - F(r)] - f(r)\, r) = f(r)\, r > 0.$$

where

$$\Phi_2(r) \equiv \int_{\underline{c}}^{r} F^2(c)dc = [F(c)\Phi(c)]_{\underline{c}}^{r} - \int_{\underline{c}}^{r} \Phi(c)\,dF(c) = F(r)(\Phi(r) - \mathbb{E}_c[\Phi(c) \mid c \leq r]).$$

Total industry profit is then

$$\overline{\Pi}_C(r) \equiv 2\pi^c(r) = 2[\Phi(r) - \Phi_2(r)].$$

D.2 Market allocation

We consider collusion that takes the form of a market allocation. The general idea is that in each market a designated supplier is selected as provider; the other bids slightly less aggressively. In a FPPA with reserve r, the designated supplier bids slightly below the reserve if its cost lies below it, and at cost otherwise; nondesignated supplier bids the reserve if its cost lies below it, and at cost otherwise. In an SPPA with reserve r, the designated supplier bids its cost; the nondesignated supplier bids the reserve if its cost lies below it, and at cost otherwise.

We now turn to the ARPA with reserve $R(r)$. We look for a collusive mechanism that has the following features:

- each supplier is either active (bids weakly below the reserve) or inactive (bids strictly above the reserve);
- active suppliers obtain (almost) $R(r)$;
- when both suppliers are active, one of them (the designated supplier, hereafter) is selected to be the provider;
- this is achieved by requiring the designated supplier to bid $R(r)$ whenever active, and the nondesignated supplier to bid slightly below $R(r)$ whenever active.

We can distinguish two variants, depending on whether the designated supplier is always active. If the designated supplier is always active, it obtains

$$R(r) - c.$$

The nondesignated supplier then obtains $R(r)$ whenever active; hence, the most profitable scheme has the nondesignated supplier also being always active (Variant 1 hereafter). If instead the designated supplier is not always active, then the on-path incentive constraint (namely deviating from active to inactive) implies that it cannot be active when its cost strictly exceeds $R(r)$; conversely, whenever its cost lies below $R(r)$, it is profitable for it to be active. Hence, the most profitable scheme has the designated supplier being active if and only if its cost lies below $R(r)$. If active, the nondesignated supplier with cost c then obtains

$$R(r) - [1 - F(R(r))]\, c = [1 - F(R(r))]\, [\hat{c}(r) - c]\,,$$

where

$$\hat{c}(r) \equiv \frac{R(r)}{1 - F(R(r))}.$$

Two subcases can therefore be distinguished:

- If $\hat{c}(r) \geq \bar{c}$, then the the most profitable scheme has the nondesignated supplier being always active (Variant $2a$ hereafter).

- If instead $\hat{c}(r) < \bar{c}$, then in addition to Variant $2a$, there exists an alternative variant in which the nondesignated supplier is not always active. The on-path incentive constraint (namely deviating from active to inactive) then implies that it cannot be active when its cost strictly exceeds $\hat{c}(r)$; conversely, whenever its cost lies below $\hat{c}(r)$, it is profitable for it to be active. Hence, the most profitable scheme has the designated supplier being active if and only if its cost lies below $\hat{c}(r)$ (Variant $2b$ hereafter).

Summing-up, there are two relevant variants, and sometimes a third one. In Variant 1, the designated supplier bids $R(r)$ regardless of its cost, and obtains $R(r) - c$; the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r)$. In Variant $2a$, the designated supplier bids $R(r)$ if its cost lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0; the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r) - [1 - F(R(r))]\, c$. In Variant $2b$ (if $\hat{c}(r) < \bar{c}$), the designated supplier acts as in Variant $2a$: it bids $R(r)$ if its cost c lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0; the nondesignated supplier bids slightly below $R(r)$ only if its cost lies below $\hat{c}(r)$, in which case it obtains $R(r) - [1 - F(R(r))]\, c$, and at cost otherwise, in which case it obtains 0.

D.3 Costs and benefits of collusion

FPPA and SPPA with reserve r

In the FPPA and SPPA, the designated supplier has expected payoff (with subscript P standing for first/second-Price procurement auction)

$$\pi_P^d(c;r) \equiv \max\{0, r-c\} = \mathbf{1}_{c<r} \times \hat{\pi}_P^d(c;r),$$

where

$$\hat{\pi}_P^d(c;r) \equiv r - c > \hat{\pi}^c(c;r),$$

where the inequality stems from (D.4) and $R'(c) = 1 - F(c) < 1$,[4] and

$$\overline{\pi}_P^d(r) \equiv \mathbb{E}_c\left[\pi_P^d(c)\right] = \Phi(r)\,(> \overline{\pi}^c(r)).$$

The nondesignated supplier has expected payoff

$$\pi_P^n(c;r) \equiv [1 - F(r)]\,\pi_P^d(c;r) = \mathbf{1}_{c<r} \times \hat{\pi}_P^n(c;r),$$

where

$$\hat{\pi}_P^n(c;r) \equiv [1 - F(r)](r - c) < \hat{\pi}^c(c;r),$$

where the inequality stems from

$$\hat{\pi}^c(c;r) - \hat{\pi}_P^n(c;r) = \int_c^r (\tilde{c} - c)\,dF(\tilde{c}) > 0,$$

and

$$\overline{\pi}_P^n(r) \equiv \mathbb{E}_c\left[\pi_P^n(c;r)\right] = [1 - F(r)]\,\overline{\pi}_P^d(r)\,(< \overline{\pi}^c(r)).$$

The net benefit from collusion is constructed as:

$$\begin{aligned} \overline{\Pi}_P(r) &\equiv \overline{\pi}_P^d(r) + \overline{\pi}_P^n(r) = [2 - F(r)]\,\Phi(r), \\ \overline{\Pi}_C(r) &\equiv 2\overline{\pi}^c(r) = 2\left[\Phi(r) - \Phi_2(r)\right], \\ \Delta_P(r) &\equiv \overline{\Pi}_P(r) - \overline{\Pi}_C(r) = 2\Phi_2(r) - F(r)\,\Phi(r) > 0, \end{aligned} \tag{D.5}$$

where the inequality stems from the monotonicity of the hazard rate (Iossa et al., 2024,

[4] Using (D.4), we have: $\partial(r - c - \hat{\pi}^c(c;r))/\partial c = R'(c) - 1 < 0$; the conclusion then follows from $r - c = \hat{\pi}^c(c;r) = 0$ for $c = r$.

Lemma 1).[5]

ARPA with reserve $R(r)$

- Variant 1:
  - the designated supplier bids $R(r)$ regardless of its cost, and obtains $R(r) - c$;
  - the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r)$.

- Variant $2a$:
  - the designated supplier bids $R(r)$ if its cost lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0;
  - the nondesignated supplier bids slightly below $R(r)$ regardless of its cost, and obtains $R(r) - [1 - F(R(r))]\,c$.

- Variant $2b$ (if $\hat{c}(r) < \bar{c}$):
  - the designated supplier acts as in Variant $2a$: it bids $R(r)$ if its cost c lies below $R(r)$, in which case it obtains $R(r) - c$, and at cost otherwise, in which case it obtains 0;
  - the nondesignated supplier bids slightly below $R(r)$ only if its cost lies below $\hat{c}(r)$, in which case it obtains $R(r) - [1 - F(R(r))]\,c$, and at cost otherwise, in which case it obtains 0. [It may therefore still be active even when its cost exceeds the reserve (namely, when its cost lies in $(R(r), \hat{c}(r)]$), as the cost is incurred only if the designated supplier's cost exceeds the reserve; but contrary to Variant 2, the nondesignated supplier is inactive when its cost lies in $(\hat{c}(r), \bar{c}]$.]

In Variant 1, the designated supplier's expected payoff is

$$\pi^d_{R1}(c; r) \equiv R(r) - c < \hat{\pi}^c(c; r),$$

[5] To see this, note that $\Delta_P(\underline{c}) = 0$ and

$$\Delta'_P(r) = F^2(r) - f(r)\Phi(r) = F(r)\int_{\underline{c}}^{r} f(c)dc - \int_{\underline{c}}^{r} f(r)F(c)dc = \int_{\underline{c}}^{r} F(r)F(c)\Big(\frac{f(c)}{F(c)} - \frac{f(r)}{F(r)}\Big)dc > 0.$$

where the inequality stems from $(D.4)$ and $c > R(c)$, and:[6]

$$\overline{\pi}_{R1}^{d}(r) \equiv \mathbb{E}_c\left[\pi_{R1}^{d}(c;r)\right] = R(r) - \int_{\underline{c}}^{\overline{c}} c dF(c) = R(r) - R(\overline{c})\,(\leq 0)\,.$$

The nondesignated supplier's expected payoff is

$$\pi_{R1}^{n}(c;r) \equiv R(r)\,.$$

Thus, the net benefit from collusion is constructed as:

$$\begin{aligned}\overline{\Pi}_{R1}(r) &\equiv \overline{\pi}_{R1}^{d}(r) + \overline{\pi}_{R1}^{n}(r) = 2R(r) - R(\overline{c})\,,\\ \Delta_{R1}(r) &\equiv \overline{\Pi}_{R1}(r) - \overline{\Pi}_{C}(r) = 2R(r) - R(\overline{c}) - 2\left[\Phi(r) - \Phi_2(r)\right]. \qquad \text{(D.6)}\end{aligned}$$

It follows that collusion in Variant 1 is profitable if and only if $\Delta_{R1}(r) > 0$. This is not always the case. For example, for $F(c) = c^s$ and $s \in (0, 0.5)$, $\Delta_{R1}(\overline{c}) < 0$, in which case collusion is not profitable under the ARPA, although it is profitable under the FPPA and under the SPPA.

In Variant $2a$, the designated supplier has expected payoff

$$\pi_{Ra}^{d}(c;r) \equiv \max\{0, R(r) - c\} = \mathbf{1}_{c<R(r)} \times \pi_{R1}^{d}(c;r),$$

and

$$\overline{\pi}_{Ra}^{d}(r) \equiv \mathbb{E}_c\left[\pi_{Ra}^{d}(c;r)\right] = \int_{\underline{c}}^{R(r)} \left[R(r) - c\right] dF(c) = \Phi(R(r))\,(< \overline{\pi}^{c}(r))\,.$$

The nondesignated supplier has expected payoff

$$\pi_{Ra}^{n}(c;r) \equiv R(r) - \left[1 - F(R(r))\right] c = \left[1 - F(R(r))\right]\left[\hat{c}(r) - c\right],$$

and:

$$\begin{aligned}\overline{\pi}_{Ra}^{n}(r) &\equiv \mathbb{E}_c\left[\pi_{Ra}^{n}(c;r)\right]\\ &= \left[1 - F(R(r))\right] \int_{\underline{c}}^{\overline{c}} \left[\hat{c}(r) - c\right] dF(c)\\ &= \left[1 - F(R(r))\right]\left[\hat{c}(r) - R(\overline{c})\right]\\ &= R(r) - \left[1 - F(R(r))\right] R(\overline{c})\,.\end{aligned}$$

[6] Note that $R(\overline{c}) = \mathbb{E}_c[c]$.

The net benefit from collusion is constructed as:

$$\overline{\Pi}_{Ra}(r) \equiv \overline{\pi}^{d}_{R2}(r) + \overline{\pi}^{n}_{Ra}(r) = \Phi(R(r)) + R(r) - [1 - F(R(r))] R(\overline{c}),$$

$$\begin{aligned} \Delta_{Ra}(r) &\equiv \overline{\Pi}_{Ra}(r) - \overline{\Pi}_{C}(r) \\ &= \Phi(R(r)) + R(r) - [1 - F(R(r))] R(\overline{c}) - 2[\Phi(r) - \Phi_2(r)]. \end{aligned} \tag{D.7}$$

Finally, in Variant $2b$ (if $\hat{c}(r) < \overline{c}$), the designated supplier has the same expected payoff as in Variant $2a$. The nondesignated supplier's expected payoff is

$$\pi^{n}_{Rb}(c; r) \equiv \max\{0, R(r) - [1 - F(R(r))] c\} = \mathbf{1}_{c<\hat{c}(r)} \times \pi^{n}_{Ra}(c; r),$$

and:

$$\overline{\pi}^{n}_{Rb}(r) \equiv \mathbb{E}_c[\pi^{n}_{Rb}(c; r)] = [1 - F(R(r))] \int_{\underline{c}}^{\hat{c}(r)} [\hat{c}(r) - c] \, dF(c) = [1 - F(R(r))] \Phi(\hat{c}(r)).$$

The net benefit from collusion is then constructed as:

$$\overline{\Pi}_{Rb}(r) \equiv \overline{\pi}^{d}_{Rb}(r) + \overline{\pi}^{n}_{Rb}(r) = \Phi(R(r)) + [1 - F(R(r))] \Phi(\hat{c}(r)),$$

$$\begin{aligned} \Delta_{Rb}(r) &\equiv \overline{\Pi}_{Rb}(r) - \overline{\Pi}_{C}(r) \\ &= \Phi(R(r)) + [1 - F(R(r))] \Phi(\hat{c}(r)) - 2[\Phi(r) - \Phi_2(r)]. \end{aligned} \tag{D.8}$$

D.4 Sustainability

We now consider the sustainability of collusion in the different formats. A market allocation is sustainable in a procurement auction of type $\tau \in \mathcal{T} \equiv \{P, F, S, R1, Ra, Rb\}$ (with F standing for First-price procurement auction, S standing for Second-price procurement auction, and R standing for all-Receive procurement auction), if and only if

$$\frac{\delta}{1-\delta} \geq \lambda_\tau(r) \equiv \frac{SG_\tau(r)}{LL_\tau(r)}, \tag{D.9}$$

where $SG(r)$ denotes the *short-term gain* from a deviation (see below), whereas $LL_\tau(r)$ denotes the (per-period) *long-term loss* of giving up collusion in the future. This condition can equivalently be expressed as:

$$\delta \geq \hat{\delta}_\tau (r) \equiv \frac{\lambda_\tau (r)}{1 + \lambda_\tau (r)}$$

In the FPPA and SPPA, the long-term loss is equal to:

$$LL_F (r) = LL_S (r) = \Delta_P (r) ,$$

where $\Delta_P (r)$ is given by (D.5). Furthermore, the nondesignated supplier is the only one that may be tempted to deviate, and its gain from a deviation is maximal when it has the lowest possible cost, $\underline{c}$.

Turning to the short-term gain, in a FPPA, the best deviation for the nondesignated supplier consists in slightly undercutting the designated supplier's collusive bid, which yields a profit arbitrarily close to $\pi_P^d (\underline{c}, r)$; hence:

$$SG_F(r) \equiv \pi_P^d (\underline{c}, r) - \pi_P^n (\underline{c}, r_j) = F (r) (r - \underline{c}) . \tag{D.10}$$

In an SPPA, the best deviation for the nondesignated supplier consists instead in bidding at cost, which yields the competitive profit $\pi^c (\underline{c}, r)$; hence:

$$SG_P(r) \equiv \pi^c (\underline{c}, r) - \pi_P^n (\underline{c}, r_j) = F(r) (r - \underline{c}) - \Phi (r) . \tag{D.11}$$

To consider sustainability in the ARPA, we must consider the different variants. Under Variant 1, the long-term loss is equal to:

$$LL_{R1} (r) = \Delta_{R1} (r) ,$$

where $\Delta_{R1} (r)$ is given by (D.6). By construction, the nondesignated supplier best-responds to the designated supplier's collusive strategy; hence, the designated supplier is the only one that may be tempted to deviate. Furthermore, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$ but let the nondesignated supplier be selected as provider. This gain is maximal when its cost is equal to $\overline{c}$; hence:

$$SG_{R1} (r) \equiv \overline{c}.$$

Under Variant $2a$, the long-term loss is equal to:

$$LL_{Ra} (r) = \Delta_{Ra} (r) ,$$

where $\Delta_{Ra}(r)$ is given by (D.7). For the designated supplier, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$, but let the nondesignated supplier be selected as provider; the resulting gain from the deviation is as follows:

- for $c \leq R(r)$, the designated supplier was and remains active, but is no longer the provider; the gain from the deviation is therefore equal to c;
- for $c > R(r)$, in the absence of a deviation the designated supplier is inactive and, thus, obtain zero payoff; by deviating, it gets paid (almost) $R(r)$; the gain from the deviation is therefore equal to $R(r)$.

It follows that this short-term gain is maximal for $c = R(r)$, where it is equal to:

$$SG^d_{Ra}(r) \equiv R(r).$$

For the nondesignated supplier, the best deviation consists in bidding above the reserve (e.g., at cost) if its cost c lies above $\hat{c}(r)$, so as to avoid the expected loss $[1 - F(R(r))](\hat{c}(r) - c)$; it follows that the short-term gain is maximal for $c = \overline{c}$, where it is equal to:

$$SG^n_{Ra}(r) \equiv \max\{[1 - F(R(r))](\overline{c} - \hat{c}(r)), 0\}.$$

As $R(r) > (\underline{c} \geq) 0$, it follows that the maximal short-term gain is equal to:

$$SG_{Ra}(r) \equiv \max\{R(r), [1 - F(R(r))](\overline{c} - \hat{c}(r))\}.$$

Under Variant 2b, the long-term loss is equal to:

$$LL_{Rb}(r) = \Delta_{Rb}(r),$$

where $\Delta_{Rb}(r)$ is given by (D.8). By construction, the nondesignated supplier best-responds to the designated supplier's collusive strategy; hence, the designated supplier is the only one that may be tempted to deviate. Furthermore, the best deviation consists in slightly undercutting the nondesignated supplier's bid, so as to get paid (almost) $R(r)$, but let the nondesignated supplier be selected as provider whenever active; the resulting gain from the deviation is as follows:

- for $c \leq R(r)$, the designated supplier was and remains active, but with probability $F(\hat{c}(r))$, it is no longer the provider; the gain from the deviation is therefore equal to $F(\hat{c}(r))\, c$, which is increasing in c;

- for $c > R(r)$, in the absence of a deviation the designated supplier is inactive and, thus, obtain zero payoff; by deviating, it gets paid (almost) $R(r)$ and is selected as provider with probability $1 - F(\hat{c}(r))$; the gain from the deviation is therefore $R(r) - [1 - F(\hat{c}(r))]\, c$, which is positive for $c < \hat{c}(r)$ but decreasing in c.

It follows that the short-term gain is again maximal for $c = R(r)$, where it is now equal to:

$$SG_{Rb}(r) \equiv F(\hat{c}(r))\, R(r)\,.$$

To summarize, the question of sustainability depends on a comparison of $\frac{SG_\tau(r)}{LL_\tau(r)}$ to $\frac{\delta}{1-\delta}$, where the long-term loss and short-term gains are:

	LL	SG
FPPA	$2\Phi_2(r) - F(r)\Phi(r)$	$F(r)(r - \underline{c})$
SPPA	$2\Phi_2(r) - F(r)\Phi(r)$	$F(r)(r - \underline{c}) - \Phi(r)$
ARPA-V1	$2R(r) - R(\overline{c}) - 2[\Phi(r) - \Phi_2(r)]$	$\overline{c}$
ARPA-V2a	$\Phi(R(r)) + R(r) - [1 - F(R(r))]\, R(\overline{c}) - 2[\Phi(r) - \Phi_2(r)]$	$\max\{R(r), [1 - F(R(r))]\,[\overline{c} - \hat{c}(r)]\}$
ARPA-V2b	$\Phi(R(r)) + [1 - F(R(r))]\, \Phi(\hat{c}(r)) - 2[\Phi(r) - \Phi_2(r)]$	$F(\hat{c}(r))\, R(r)$

We illustrate in Figure D.7 differences among the ARPA variants in terms of profitability and threshold discount factors. Panel (a) shows that ARPA-V2b is profitable for all r but that the other forms of ARPA collusion require r sufficiently large. For example, ARPA-V1 is profitable only for $r > 0.3700$. Panel (b) shows that the threshold discount factor for sustainability of collusion is lowest for ARPA-V2b for $r < 0.4730$ and lowest for ARPA-V1 for higher r. For sufficiently low r, even with $\delta = 1$, collusion is only sustainable for ARPA-V2b. For $r > 0.3700$, collusion is also sustainable for ARPA-V1 for sufficiently high δ (collusion under format ARPA-V2a is dominated in terms of profitability and sustainability by the other formats when costs are uniformly distributed on $[0, 1]$). As shown in Panel (c), for the case of $F(c) = c^s$, ARPA-V2a is no longer always dominated by ARPA-V2b, but both ARPA-V2a and ARPA-V2b are dominated by ARPA-V1. For $s < 0.5$, collusion is not profitable under the ARPA for any variant.

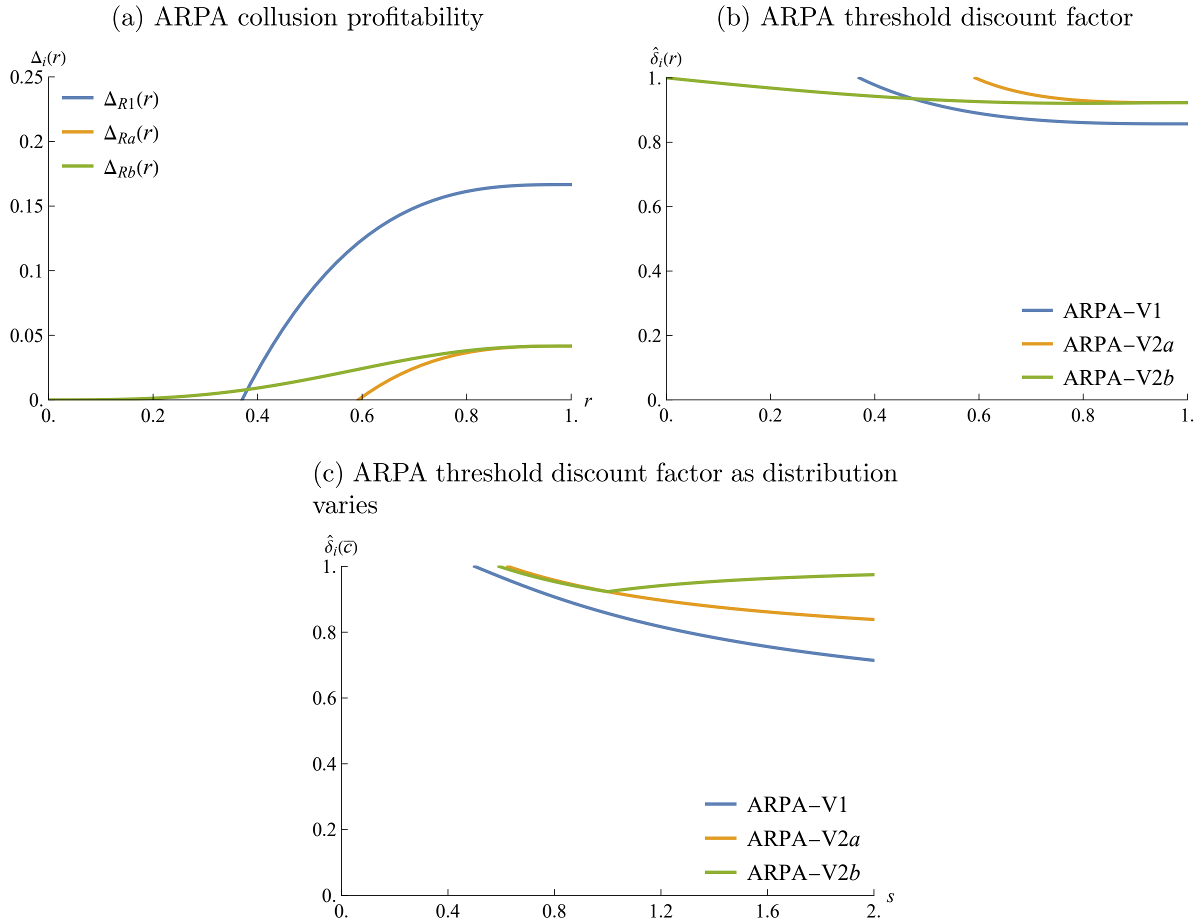

Figure D.7: Panels (a) and (b) assume uniformly distributed costs on $[0, 1]$. Panel (c) assumes that $F(c) = c^s$ on $[0, 1]$ for $s \in (0, 2]$ and $r = 1$.

D.5 Comparisons across auction formats

As mentioned in Section 6.2, the ARPA can be more or less susceptible to collusion than the FPPA and SPPA depending on the setup.

As an example, consider the case of uniformly distributed costs on $[0, 1]$. The long-term losses for the ARPA under Variants 2a and 2b are always below that of the FPPA and SPPA, and the long-term loss for the ARPA under Variant 1 is below that of the FPPA and SPPA for $r < 0.3820$ (Figure D.8(a)). The short-term gain for the ARPA under Variant 1 is always above that of the FPPA and SPPA, and the short-term gain for ARPA under Variants 2a and 2b are greater than that of the SPPA (Figure D.8(b)). The result is that under Variants 2a and 2b, the ARPA is always less susceptible to collusion (higher threshold discount factor) than the FPPA and SPPA; and under Variant 1 is always less susceptible to collusion than the SPPA and, for $r < 0.5$, is less susceptible than the FPPA (Figure D.8(c)). Thus, regardless of the variant of ARPA collusion, the ARPA is less susceptible to collusion than the SPPA for all r and less susceptible than the FPPA for $r < 0.5$.

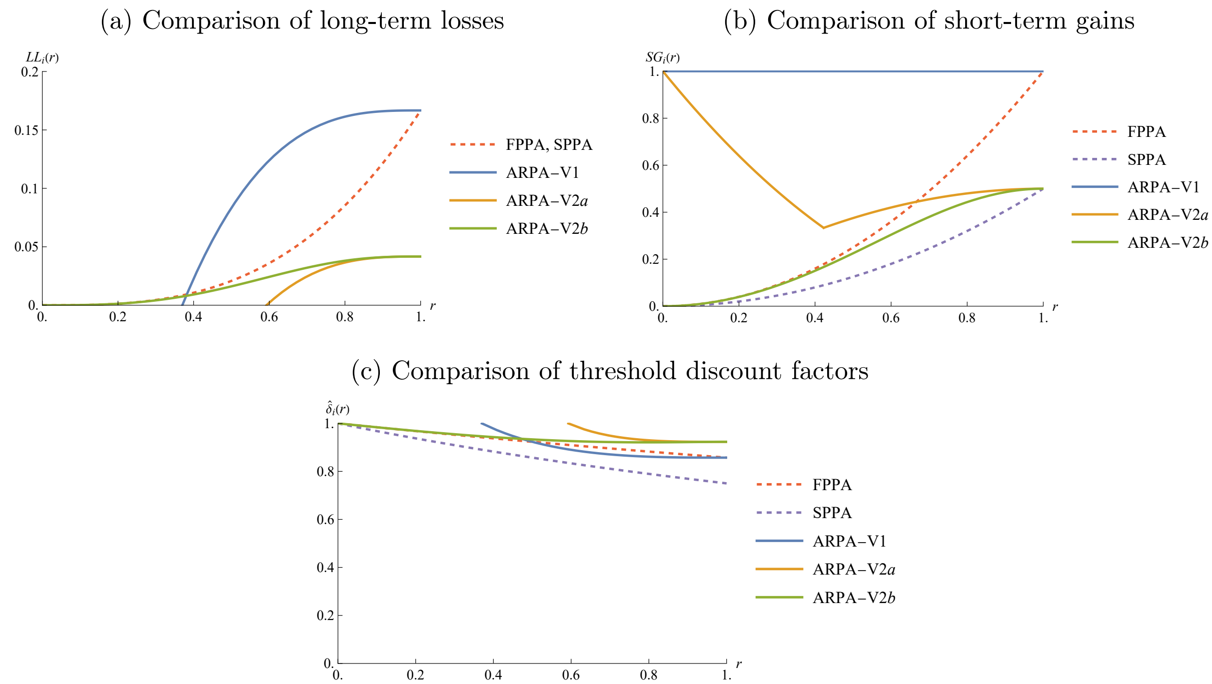

Figure D.8: All panels assume uniformly distributed costs on $[0,1]$.

As another example, consider $r = \overline{c}$ and $F(c) = c^s$ on $[0,1]$. In this case, as illustrated in Figure D.9, ARPA collusion Variants 1 and 2 are less susceptible than the other formats for $s \in (0,1)$, but under Variant 1, the ARPA is even more susceptible to collusion than the SPPA for $s > 1.234$.

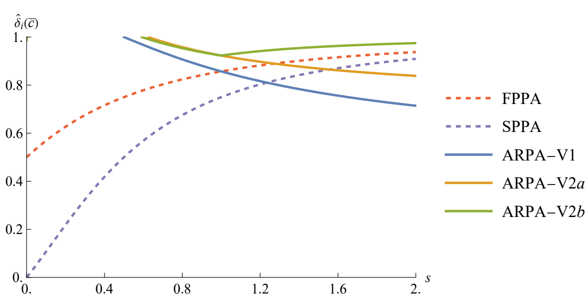

Figure D.9: Comparison of threshold discount factors. Assumes that $F(c) = c^s$ on $[0,1]$ for $s \in (0,2]$ and $r = 1$.

References for the Supplemental Appendix

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