Results and Proofs for the Asymmetric Setup for "Merger Review for Markets with Buyer Power"

Simon Loertscher* Leslie M. Marx†

June 27, 2018

In this appendix, we provide the results and proofs for the asymmetric setup. In Section 1, we summarize how the results for the asymmetric setup contrast with those for the symmetric setup. In Section 2, we review the details of the asymmetric setup, and in Section 3, we provide the results for the asymmetric setup with proofs.

1 Comparison between the results for the asymmetric setup and for the symmetric setup

As stated in the paper, Theorem 1 holds for both the symmetric and the asymmetric setups. Propositions 1–11 in the paper are stated for the symmetric setup. With a few exceptions, they or their analogs continue to hold in the asymmetric setup with no or minor additional assumptions. Corollaries 1 and 2 in the paper hold in the asymmetric setup with no adjustments.

Propositions 2 and 4 in the paper are places where a key difference occurs. As discussed in the paper following Proposition 2, the result that $\Delta SS^1 < 0$, which is a component of both Propositions 2 and 4, does not hold in the asymmetric setup: When the pre-merger suppliers are asymmetric, a merger can reduce asymmetries and so reduce the extent to which a powerful buyer discriminates, thereby increasing social surplus.

*Department of Economics, Level 4, FBE Building, 111 Barry Street, University of Melbourne, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

†The Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, USA: Email: marx@duke.edu.

Proposition 3 shows that the Bulow-Klemperer-like result that $BS^0_{pre}(v) > BS^1_{post}(v)$ holds for sufficiently large v, and that the opposite inequality holds for sufficiently small v. In contrast, in the asymmetric setup, the result for sufficiently large v no longer holds. Even for large v, it can be the case that $BS^0_{pre}(v) < BS^1_{post}(v)$.

In addition, a component of Propositions 4 and 9 does not extend to the fully asymmetric setup. The result that $\Delta BS^1 > \Delta BS^0$ is derived in Proposition 4 for the case without cost synergies and in Proposition 9 for the case with cost synergies. As discussed in the paper following Proposition 4, this result rests on symmetry between the two merging suppliers, but continues to hold with asymmetries among other suppliers. When the merging suppliers are not symmetric, a merger imposes an incremental harm on a powerful buyer by removing the buyer's ability to discriminate between the two merging suppliers, opening the possibility that a powerful buyer could experience greater harm from a merger than one without power.

Finally, a component of Proposition 5 relates to a lower bound for ΔBS^1. In the symmetric setup, this lower bound is increasing in n. In the asymmetric setup, we require suppliers 1 and 2 to be symmetric in order to obtain the result that the corresponding lower bound increases as the set of nonmerging suppliers expands in the sense of set inclusion.

The remaining results continue to hold in the asymmetric setup either with no additional assumptions or with the assumption of virtual dominance (defined in Section 6 of the paper and also below). As indicated below, for the parts of Propositions 5 and 6 that provide limiting results or comparisons as the number of nonmerging suppliers increases, the analog for the asymmetric setup involves the replication of a nonempty finite set of nonmerging suppliers or the comparison of nested sets of nonmerging suppliers.

Table 1: Comparison between the symmetric and asymmetric setups

Prop.	**Additional assumptions**	**Results that no longer hold**
1	–	
2	Virtual dominance*	$0 > \Delta SS^1$
3	–	$BS^0_{pre}(v) > BS^1_{post}(v)$ for large v
4	Quantity result: –; Buyer surplus result: $G_1 = G_2$	$0 = \Delta SS^0 > \Delta SS^1$
5**	Without BP: –; With BP: Monotonicity of lower bound result: $G_1 = G_2$; Limit result: –	
6**	Without BP: –; With BP: virtual dominance	
7	Without BP: –; With BP: virtual dominance	
8	Without BP: –; With BP: virtual dominance	
9	Monotonicity result: –; Large synergy result: –; Small synergy result: $G_1 = G_2$	
10	–	
11	–	
12	–	

*In the asymmetric setup, the requirement for $\Delta Q^1 < 0$ becomes: if $n = 2$ or if $n \geq 3$ and $v < \min\{\Gamma_3(\overline{c}), ..., \Gamma_n(\overline{c})\}$.

**For limit results, we replicate a nonempty finite set of nonmerging suppliers. For monotonicity results, we consider market expansion that involves an expansion of the set of nonmerging suppliers to be a superset of the prior set of nonmerging suppliers.

In Table 1, we summarize the additional assumptions required, if any, for results for the symmetric setup to continue to hold in the asymmetric setup. A dash "–" indicates that no additional assumptions are required. We abbreviate buyer power with "BP" in the table to conserve on space.

2 Summary of the asymmetric setup

In the asymmetric setup, there are $n \geq 2$ suppliers, indexed $1, ..., n$. Each supplier $i \in \{1, ..., n\}$ draws a cost c_i independently from a continuously differentiable distribution G_i with support $[\underline{c}, \overline{c}]$ and density g_i that is positive on the interior of the support. Each supplier is privately informed about its type, and so the suppliers' types are unknown to the buyer. The buyer has value $v > \underline{c}$ for one unit of the product. All of this is common knowledge.

Supplier i's virtual cost is denoted

$$\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)}.$$

We assume that for all i, $\Gamma_i(c)$ is increasing.

We denote the distribution for the minimum of the pre-merger costs of suppliers 1 and 2 by $\hat{G}(c) \equiv 1 - (1 - G_1(c))(1 - G_2(c))$, with density $\hat{g}$. We denote the merged entity's virtual cost by

$$\hat{\Gamma}(c) \equiv c + \frac{\hat{G}(c)}{\hat{g}(c)},$$

which we assume is increasing.

We assume that for all i, g_i is finite at $\overline{c}$, and for all i, we define $\Gamma_i(\underline{c}) \equiv \lim_{c \to \underline{c}} \Gamma_i(c) = \underline{c}$, and analogously for $\hat{\Gamma}$. For $x > \Gamma_i(\overline{c})$, define $\Gamma_i^{-1}(x) \equiv \overline{c}$.

We say that *virtual dominance* holds, if for all $c_1, c_2 \in [\underline{c}, \overline{c}]$,

$$\hat{\Gamma}\left(\min\left\{c_1, c_2\right\}\right) \geq \min\{\Gamma_1(c_1), \Gamma_2(c_2)\},$$

with a strict inequality for a positive measure set of costs. Because $\min\{\Gamma(c_1), \Gamma(c_2)\} = \Gamma(\min\{c_1, c_2\})$ and $\hat{\Gamma}(c) \geq \Gamma(c)$, virtual dominance holds if suppliers 1 and 2 are symmetric.

Lemma A.1. *Virtual dominance holds if $G_1 = G_2$.*

In addition, virtual dominance holds in some cases when the merging suppliers are not symmetric. For example, when G_1 is uniform on $[0, 1]$ and $G_2(c) = c$ for $c \in [0, 1/4]$ and $G_2(c) = (1 + 24c^2 - 16c^3)/9$ for $c \in (1/4, 1]$, which is depicted as the solid line in Figure 2(b) in the paper and has continuous density and increasing virtual cost.

3 Results for the asymmetric setup

In this section, we provide the details of the results for the asymmetric setup. The propositions are labelled similarly to those for the symmetric setup, but preceded by "A"so that, for example, Proposition A.1 below corresponds to Proposition 1 for the symmetric case. Additional comments highlight places where the results differ from their counterparts for the symmetric setup.

Proposition A.1. *In the absence of buyer power, a merger results in the same allocation for any realization of costs (implying that $\Delta Q^0 = 0$ and $\Delta SS^0 = 0$), and a higher expected payment by the buyer ($\Delta P^0 > 0$).*

Proof. In the absence of buyer power, distributional assumptions are not relevant, and so the proof is the same as for Proposition 1 in the symmetric setup. ■

Although Proposition 2 shows that in the symmetric setup with buyer power, social surplus is reduced by a merger, this result does not necessarily generalize to the case of asymmetric pre-merger suppliers because then the buyer discriminates among suppliers both before and after the merger. This discrimination can be socially more wasteful before the merger than after the merger. For example, if $n \geq 3$ and for all $i \in \{3, ..., n\}$, $G_i = G$, and for $i \in \{1, 2\}$, $G_i = 1 - \sqrt{1-G}$, then for any $v > \Gamma(\overline{c})$, we get $\Delta SS^1 > 0$ because there is inefficient discrimination before but not after the merger and production occurs with probability 1 both before and after merger. Consequently, the result in Proposition 2 that $\Delta SS^1 < 0$ does not extend to the case of pre-merger asymmetries among suppliers without additional restrictions.

Proposition A.2. *With buyer power, assuming virtual dominance holds, a merger results in a weakly lower expected quantity traded ($\Delta Q^1 \leq 0$) (strictly if $n = 2$ or if $n \geq 3$ and $v < \min\{\Gamma_3(\overline{c}), ..., \Gamma_n(\overline{c})\}$).*

Proof. Because $\hat{\Gamma}(\min\{c_1, c_2\}) \geq \min\{\Gamma_1(c_1), \Gamma_2(c_2)\}$ with a strict inequality for a positive measure set of costs (c_1, c_2), with positive probability the buyer discriminates against the merged entity relative to the merging supplier with the lower virtual cost in the pre-merger market. Thus, a merger results in a weakly lower expected quantity traded. Because $v < \hat{\Gamma}(\min\{c_1, c_2\})$ with positive probability, $\min\{\Gamma_1(c_1), \Gamma_2(c_2)\} < v < \hat{\Gamma}(\min\{c_1, c_2\})$, which implies there is trade in the pre-merger market but not in the post-merger market. Thus, $\Delta Q^1 < 0$ if $n = 2$. In addition, if $n \geq 3$ and $v < \min\{\Gamma_3(\overline{c}), ..., \Gamma_n(\overline{c})\}$, then with positive probability

$$\min\{\Gamma_1(c_1), \Gamma_2(c_2)\} < v < \min\{\hat{\Gamma}(\min\{c_1, c_2\}), \Gamma_3(c_3), ..., \Gamma_n(c_n)\},$$

in which case there is trade in the pre-merger market but not in the post-merger market. It follows that $\Delta Q^1 < 0$ if $n \geq 3$ and $v < \min\{\Gamma_3(\overline{c}), ..., \Gamma_n(\overline{c})\}$. ■

In the symmetric setup, Proposition 3 shows that the Bulow-Klemperer-like result that $BS^0_{pre}(v) > BS^1_{post}(v)$ holds for sufficiently large v, and that the opposite inequality holds for sufficiently small v. In the asymmetric setup, the result for sufficiently large v no longer holds. To see this, suppose that $n = 2$ and that distribution G_1 has almost all of its probability weight near $\underline{c}$ and G_2 has almost all of its probability weight near $\overline{c}$. Then for v sufficiently large, the expected value of the second-lowest cost is close to $\overline{c}$, and so $BS^0_{pre}(v)$ is close to $v - \overline{c}$. However, in the post-merger market, if a powerful buyer were to use $\frac{\underline{c}+\overline{c}}{2}$ as its final offer to the merged entity, that offer would be accepted with high probability because $\min\{c_1, c_2\}$ is close to $\underline{c}$ with high probability. In that case, the buyer's expected surplus would be close to $v - \frac{\underline{c}+\overline{c}}{2}$, which is greater than $BS^0_{pre}(v)$. Because the optimal final offer by a powerful buyer in the post-merger market does at least as well, we have $BS^0_{pre}(v) < BS^1_{post}(v)$.

Proposition A.3. *There exists $v' > \underline{c}$ such that for all $v \in (\underline{c}, v')$, $BS^0_{pre}(v) < BS^1_{post}(v)$.*

Proof. It is useful to define notation for the lowest derivative of one of $g_1, ..., g_n$ that is nonzero at $\underline{c}$. Specifically, define

$$k \equiv \min\left\{i \in \{0, 1, 2, ...\} \mid \exists j \in \{1, ..., n\} \text{ s.t. } g_j^{(i)}(\underline{c}) > 0\right\}.$$

That is, k is such that for all $j \in \{1, ..., n\}$, $g_j^{(0)}(\underline{c}) = ... = g_j^{(k-1)}(\underline{c}) = 0$ and for some $j \in \{1, ..., n\}$, $g_j^{(k)}(\underline{c}) > 0$ (the value of k is well defined because the densities $g_1, ..., g_n$ are assumed positive on the interior of the support). The case with $g_j(\underline{c}) > 0$ for some $j \in \{1, ..., n\}$ corresponds to $k = 0$.

Let H be the distribution of the second-lowest of the n cost draws, with density h, and note that

$$h(y) = \sum_{i=1}^{n} g_i(y) \sum_{j\in\{1,...,n\}\setminus\{i\}} G_j(y) \prod_{\ell\in\{1,...,n\}\setminus\{i,j\}} (1 - G_\ell(y))$$

and $h(\underline{c}) = 0$. Before the merger with no buyer power, the buyer pays $\min\left\{c_{(2)}, v\right\}$, where $c_{(2)}$ is the second-lowest cost, so

$$BS^0_{pre}(v) = E\left[v - c_{(2)} \mid c_{(2)} < v\right] \Pr\left(c_{(2)} < v\right) = \int_{\underline{c}}^{\min\{v,\overline{c}\}} (v - y) dH(y).$$

Thus, for $v < \overline{c}$, $BS^{0\prime}_{pre}(v) = \int_{\underline{c}}^{v} dH(y) = H(v)$, which is zero at $v = \underline{c}$. Differentiating again, we get, for $v < \overline{c}$, $BS^{0\prime\prime}_{pre}(v) = h(v)$, which is also zero at $v = \underline{c}$. More generally, letting $BS^{0(j)}_{pre}$ denote the j-th derivative of BS^0_{pre},

$$BS^{0(k+2)}_{pre}(\underline{c}) = h^{(k)}(\underline{c}) = 0.$$

Turning to the post-merger market with buyer power, it will be useful to consider the post-merger mechanism that evaluates each of the nonmerging suppliers using the virtual cost function of supplier 3. Denote the expected buyer surplus from such a potentially nonoptimal mechanism as $\widetilde{BS}(v)$ and note that for all v, $\widetilde{BS}(v) \leq BS^1_{post}(v)$. Let $\hat{H}$ be the distribution of the lowest cost among $c_3, ..., c_n$, with density $\hat{h}$. Thus, $\hat{H}(c) = 1-(1-G_3(c))\cdots(1-G_n(c))$ and $\hat{h}(c) = \sum_{i=3}^n g_i(c) \prod_{j\in\{3,...,n\}\setminus\{i\}} (1-G_j(c))$. Let $\hat{G}$ be the distribution of the minimum of c_1 and c_2, with density $\hat{g}$. Thus, $\hat{G}(c) = 1-(1-G_1(c))(1-G_2(c))$ and $\hat{g}(c) = g_1(c)(1-G_2(c)) + g_2(c)(1-G_1(c))$. Letting $\hat{c}_3$ denote the lowest cost among $c_3, ..., c_n$ and $\hat{c}$ denote $\min\{c_1, c_2\}$,

$$\begin{aligned}
\widetilde{BS}(v) &= E\left[v - \min\left\{\Gamma_3^{-1}(\hat{\Gamma}(\hat{c})), \Gamma_3^{-1}(v)\right\} \mid \Gamma_3(\hat{c}_3) \leq \min\left\{\hat{\Gamma}(\hat{c}), v\right\}\right] \\
&\quad \cdot \Pr\left(\Gamma_3(\hat{c}_3) \leq \min\left\{\hat{\Gamma}(\hat{c}), v\right\}\right) \\
&\quad + E\left[v - \min\left\{\hat{\Gamma}^{-1}(\Gamma_3(\hat{c}_3)), \hat{\Gamma}^{-1}(v)\right\} \mid \hat{\Gamma}(\hat{c}) \leq \min\left\{\Gamma_3(\hat{c}_3), v\right\}\right] \\
&\quad \cdot \Pr\left(\hat{\Gamma}(\hat{c}) \leq \min\left\{\Gamma_3(\hat{c}_3), v\right\}\right),
\end{aligned}$$

which we can write as

$$\begin{aligned}
\widetilde{BS}(v) &= \int_{\underline{c}}^{\hat{\Gamma}^{-1}(v)} \int_{\underline{c}}^{\Gamma_3^{-1}(\hat{\Gamma}(\hat{c}))} \left(v - \Gamma_3^{-1}(\hat{\Gamma}(\hat{c}))\right) d\hat{H}(c_3) d\hat{G}(\hat{c}) \\
&\quad + \int_{\hat{\Gamma}^{-1}(v)}^{\overline{c}} \int_{\underline{c}}^{\Gamma_3^{-1}(v)} \left(v - \Gamma_3^{-1}(v)\right) d\hat{H}(c_3) d\hat{G}(\hat{c}) \\
&\quad + \int_{\underline{c}}^{\Gamma_3^{-1}(v)} \int_{\underline{c}}^{\hat{\Gamma}^{-1}(\Gamma_3(c_3))} \left(v - \hat{\Gamma}^{-1}(\Gamma_3(c_3))\right) d\hat{G}(\hat{c}) d\hat{H}(c_3) \\
&\quad + \int_{\Gamma_3^{-1}(v)}^{\overline{c}} \int_{\underline{c}}^{\hat{\Gamma}^{-1}(v)} \left(v - \hat{\Gamma}^{-1}(v)\right) d\hat{G}(\hat{c}) d\hat{H}(c_3).
\end{aligned}$$

Taking the derivative with respect to v, one can show that $\widetilde{BS}'(\underline{c}) = 0$. Differentiating again and using $\Gamma_3^{-1\prime}(\underline{c}) = \hat{\Gamma}^{-1\prime}(\underline{c}) = \frac{1}{2}$, one can show that

$$\begin{aligned}
\widetilde{BS}''(\underline{c}) &= 2\Gamma_3^{-1\prime}(\underline{c})\left(1 - \Gamma_3^{-1\prime}(\underline{c})\right)\hat{h}(\Gamma_3^{-1}(\underline{c})) + 2\hat{\Gamma}^{-1\prime}(\underline{c})\left(1 - \hat{\Gamma}^{-1\prime}(\underline{c})\right)\hat{g}(\hat{\Gamma}^{-1}(\underline{c})) \\
&= \frac{1}{2}\left(\hat{h}(\underline{c}) + \hat{g}(\underline{c})\right) \\
&= \frac{1}{2}\left(\sum_{i=3}^n g_i(\underline{c}) + g_1(\underline{c}) + g_2(\underline{c})\right) \\
&= \frac{1}{2}\sum_{i=1}^n g_i(\underline{c}),
\end{aligned}$$

which is positive if for some i, $g_i(\underline{c}) > 0$. More generally, for $k \in \{0, 1, ..\}$, one can show that $\widetilde{BS}^{(k+2)}(v)$ contains the following terms that involve either the k-th or $k+1$-st derivatives of $\hat{g}$ and $\hat{h}$:

$$\begin{aligned}&(k+2)\Gamma_3^{-1\prime}(v)\left(1-\Gamma_3^{-1\prime}(v)\right)\hat{h}^{(k)}(\Gamma_3^{-1}(v))(\Gamma_3^{-1\prime}(v))^k(1-\hat{G}(\hat{\Gamma}^{-1}(v)))\\&+\Gamma_3^{-1\prime}(v)\left(v-\Gamma_3^{-1}(v)\right)\hat{h}^{(k+1)}(\Gamma_3^{-1}(v))\left(\Gamma_3^{-1\prime}(v)\right)^{k+1}(1-\hat{G}(\hat{\Gamma}^{-1}(v)))\\&+(k+2)\hat{\Gamma}^{-1\prime}(v)(1-\hat{\Gamma}^{-1\prime}(v))\hat{g}^{(k)}(\hat{\Gamma}^{-1}(v))(\hat{\Gamma}^{-1\prime}(v))^k(1-\hat{H}(\Gamma_3^{-1}(v)))\\&+\hat{\Gamma}^{-1\prime}(v)(v-\hat{\Gamma}^{-1}(v))\hat{g}^{(k+1)}(\hat{\Gamma}^{-1}(v))(\hat{\Gamma}^{-1\prime}(v))^{k+1}(1-\hat{H}(\Gamma_3^{-1}(v))).\end{aligned} \tag{1}$$

All other terms are zero when evaluated at $v=\underline{c}$. Thus, setting $v=\underline{c}$ and using $\hat{\Gamma}^{-1\prime}(v)=\Gamma_3^{-1\prime}(v)=\frac{1}{2}$, we are left with the following:[1]

$$\widetilde{BS}^{(k+2)}(\underline{c})=\frac{k+2}{2^{k+2}}(\hat{h}^{(k)}(\underline{c})+\hat{g}^{(k)}(\underline{c}))=\frac{k+2}{2^{k+2}}\sum_{i=1}^{n}g_i^{(k)}(\underline{c}),$$

which is positive by the definition of k. Thus, all derivatives of $BS^0_{pre}(v)$ at $v=\underline{c}$ up to and including the $k+2$-nd derivative are zero, and all derivatives of $\widetilde{BS}(v)$ at $v=\underline{c}$ up to the $k+2$-nd derivative are zero, but the $k+2$-nd derivative is positive. Because $\widetilde{BS}(v)$ increases faster than $BS^0_{pre}(v)$ at $v=\underline{c}$, it follows that there exists $v'>\underline{c}$ such that for all $v\in(\underline{c},v')$, $BS^1_{post}(v)\geq\widetilde{BS}(v)>BS^0_{pre}(v)$. ■

As indicated in Proposition A.4, we require symmetry between the merging suppliers (and only between the merging suppliers) to get the result that buyer power mitigates the harm to the buyer from the transaction. When the merging suppliers are asymmetric, a buyer with power benefits from the ability to price discriminate between suppliers 1 and 2. The loss of this ability as a result of a merger negatively affects a powerful buyer in the post-merger market. This results in an incremental source of harm to the buyer that only affects powerful buyers.

Proposition A.4. $0=\Delta Q^0\geq\Delta Q^1$ *and, if suppliers* 1 *and* 2 *are symmetric, then* $0>\Delta BS^1>\Delta BS^0$.

Proof. The result that $0=\Delta Q^0\geq\Delta Q^1$ follows from Propositions A.1 and A.2. The result that ΔBS^1 and ΔBS^0 are negative follows from Theorem 1, which applies to both the symmetric and asymmetric setup. We show that if $G_1=G_2$, then $\Delta BS^1>\Delta BS^0$. To

[1] To see the induction step, note that $\widetilde{BS}^{(k+3)}(\underline{c})$ can be obtained by taking the derivative of (1) once again with respect to v and evaluating at $v=\underline{c}$, which delivers terms involving $\hat{h}^{(k+1)}$ and $\hat{g}^{(k+1)}$ that have coefficient $(k+3)/2^{k+3}$.

do so, it is useful to consider a range of buyer power $\beta \in [0,1]$ and to define the weighted virtual cost function for supplier i by

$$\Gamma_{i,\beta}(c) \equiv (1-\beta)c + \beta\Gamma_i(c) \quad \text{and} \quad \hat{\Gamma}_\beta(c) \equiv (1-\beta)c + \beta\hat{\Gamma}(c).$$

Suppose temporarily that a buyer with buyer power β uses virtual cost function $\Gamma_{i,\beta}$ to evaluate supplier i in both the pre-merger and post-merger market, with virtual cost function $\Gamma_{1,\beta}(c) = \Gamma_{2,\beta}(c)$ used to evaluate the merged entity in the post-merger market. Let $x \equiv \min\{\Gamma_3(c_3), ..., \Gamma_n(c_n), \Gamma_1(\overline{c})\}$ and let F be the distribution of x, with support $[\underline{c}, \overline{\gamma}]$ with $\overline{\gamma} \equiv \min\{\Gamma_3(\overline{c}), ..., \Gamma_n(\overline{c}), \Gamma_1(\overline{c})\}$. For such a buyer, the probability of trade is not affected by the merger, and the payment is affected only when $\max\{c_1, c_2\} < \min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}$, in which case the buyer pays $\min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}$ instead of $\max\{c_1, c_2\}$. Thus, the expected change in buyer surplus as a result of a merger is

$$\begin{aligned} 2E_{\mathbf{c}}[\max\{c_1,c_2\} - \min\{\Gamma_{1,\beta}^{-1}(v), x\} \quad | \quad & \max\{c_1,c_2\} < \min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}] \\ & \cdot \Pr\left(\max\{c_1,c_2\} < \min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}\right), \end{aligned}$$

which we can write as

$$\begin{aligned} &\int_{\underline{c}}^{\overline{\gamma}} \int_{\underline{c}}^{\min\left\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\right\}} \int_{\underline{c}}^{c_2} (c_2 - \min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}) dG_1(c_1) dG_2(c_2) dF(x) \\ &+ \int_{\underline{c}}^{\overline{\gamma}} \int_{\underline{c}}^{\min\left\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\right\}} \int_{\underline{c}}^{c_1} (c_1 - \min\{\Gamma_{1,\beta}^{-1}(v), \Gamma_{1,\beta}^{-1}(x)\}) dG_2(c_2) dG_1(c_1) dF(x). \end{aligned}$$

Differentiating with respect to β, we get:

$$\begin{aligned} &-\frac{\partial \Gamma_{1,\beta}^{-1}(v)}{\partial \beta} \int_{v}^{\overline{\gamma}} \int_{\underline{c}}^{\Gamma_{1,\beta}^{-1}(v)} \int_{\underline{c}}^{c_2} dG_1(c_1) dG_2(c_2) dF(x) \\ &-\frac{\partial \Gamma_{1,\beta}^{-1}(v)}{\partial \beta} \int_{v}^{\overline{\gamma}} \int_{\underline{c}}^{\Gamma_{1,\beta}^{-1}(v)} \int_{\underline{c}}^{c_1} dG_2(c_2) dG_1(c_1) dF(x) \geq 0, \end{aligned}$$

where the inequality follows because $\frac{\partial \Gamma_{1,\beta}^{-1}(v)}{\partial \beta} \leq 0$. Thus, for such a buyer, the change is surplus as a result of a merger is weakly greater (closer to zero) when $\beta = 1$ than when $\beta = 0$. This implies that for a buyer who uses virtual cost $\hat{\Gamma}$ for the merged entity, $\Delta BS^1 > \Delta BS^0$. ∎

We now extend Proposition 5 to the asymmetric setup. In addition to the two merging suppliers, we posit the existence of an infinite set of potential nonmerging suppliers with cost distributions $\{G_i\}_{i=3}^{\infty}$ on $[\underline{c}, \overline{c}]$, where each G_i satisfies our assumptions (continuously differentiable with density g_i that is positive on the interior of the support and has increasing virtual cost function Γ_i). In order to consider effects as the market grows large in the asymmetric setup, we consider η-fold replicas of a finite set of nonmerging suppliers. In an η-fold replication of suppliers in finite set $K \subset \{3, 4, ...\}$, the set of nonmerging suppliers consists of η suppliers drawing cost types independently from G_i for each $i \in K$. (A straightforward extension allows there to be an additional finite set of nonmerging suppliers that are not replicated.) Given a nonempty finite set of nonmerging suppliers $K \subset \{3, 4, ...\}$ and given the number of replicas $\eta \in \{1, 2, ...\}$, we let L_K^η be the distribution of the lowest cost among the nonmerging suppliers when the set of nonmerging suppliers consists of η replicas of the suppliers in K, i.e.,

$$L_K^\eta(c) \equiv 1 - \Pi_{i \in K}(1 - G_i(c))^\eta.$$

In addition, we let $\hat{L}_K^\eta$ be the distribution of the lowest *virtual* cost among the η replicas of the suppliers in K, $\hat{L}_K^\eta(z) \equiv 1 - \Pi_{i \in K}(1 - G_i(\Gamma_i^{-1}(z)))^\eta$.

Analogous to our analysis of the symmetric case, we now develop a lower bound for ΔBS^1. With buyer power, the buyer's expected surplus before a merger when the set of nonmerging consists of η replicas of the suppliers in K is

$$\begin{aligned} BS_{pre}^1(K, \eta) &= \int_{\underline{c}}^{\Gamma_1^{-1}(v)} (v - \Gamma_1(c))(1 - \hat{L}_K^\eta(\Gamma_1(c)))(1 - G_2(\Gamma_2^{-1}(\Gamma_1(c))))dG_1(c) \\ &\quad + \int_{\underline{c}}^{\Gamma_2^{-1}(v)} (v - \Gamma_2(c))(1 - \hat{L}_K^\eta(\Gamma_2(c)))(1 - G_1(\Gamma_1^{-1}(\Gamma_2(c))))dG_2(c) \\ &\quad\quad + Y_K^\eta, \end{aligned}$$

where

$$\begin{aligned} Y_K^\eta \equiv\ & \eta \sum_{i \in K} \int_{\underline{c}}^{\Gamma_i^{-1}(v)} (v - \Gamma_i(c)) \left[\Pi_{\ell \in K \setminus \{i\}}(1 - G_\ell(\Gamma_\ell^{-1}(\Gamma_i(c)))^\eta\right] (1 - G_i(c))^{\eta - 1} \\ & \cdot (1 - G_2(\Gamma_2^{-1}(\Gamma_i(c))))(1 - G_1(\Gamma_1^{-1}(\Gamma_i(c))))dG_i(c). \end{aligned}$$

We construct a lower bound for the post-merger expected buyer surplus by assuming that the buyer evaluates the merged entity using the (suboptimal) virtual cost function

$\tilde{\Gamma}$ defined so that

$$(1 - G_2(\Gamma_2^{-1}(z)))(1 - G_1(\Gamma_1^{-1}(z))) = 1 - \hat{G}(\tilde{\Gamma}^{-1}(z)),$$

i.e., for $z \in [\underline{c}, \infty)$,

$$\tilde{\Gamma}^{-1}(z) = \hat{G}^{-1}(1 - (1 - G_2(\Gamma_2^{-1}(z)))(1 - G_1(\Gamma_1^{-1}(z)))).$$

Given our assumptions, $\tilde{\Gamma}$ is increasing on $[\underline{c}, \overline{c}]$. Then we have a lower bound on the post-merger expected buyer surplus of

$$\begin{aligned} \underline{BS}^1_{post}(K, \eta) &= \int_{\underline{c}}^{\tilde{\Gamma}^{-1}(v)} (v - \hat{\Gamma}(c))(1 - \hat{L}_K^\eta(\tilde{\Gamma}(c)))d\hat{G}(c) \\ &\quad + Y_K^\eta \frac{1 - \hat{G}(\tilde{\Gamma}^{-1}(\Gamma_i(c)))}{(1 - G_2(\Gamma_2^{-1}(\Gamma_i(c))))(1 - G_1(\Gamma_1^{-1}(\Gamma_i(c))))} \\ &= \int_{\underline{c}}^{\tilde{\Gamma}^{-1}(v)} (v - \hat{\Gamma}(c))(1 - \hat{L}_K^\eta(\tilde{\Gamma}(c)))d\hat{G}(c) + Y_K^\eta, \end{aligned}$$

where the final equality uses the definition of $\tilde{\Gamma}$. This implies that a lower bound for the change in expected buyer surplus is

$$\begin{aligned} \underline{\Delta BS}^1(K, \eta) &= \int_{\underline{c}}^{\tilde{\Gamma}^{-1}(v)} (v - \hat{\Gamma}(c))(1 - \hat{L}_K^\eta(\tilde{\Gamma}(c)))d\hat{G}(c) \\ &\quad - \int_{\underline{c}}^{\Gamma_1^{-1}(v)} (v - \Gamma_1(c))(1 - \hat{L}_K^\eta(\Gamma_1(c)))(1 - G_2(\Gamma_2^{-1}(\Gamma_1(c))))dG_1(c) \\ &\quad - \int_{\underline{c}}^{\Gamma_2^{-1}(v)} (v - \Gamma_2(c))(1 - \hat{L}_K^\eta(\Gamma_2(c)))(1 - G_1(\Gamma_1^{-1}(\Gamma_2(c))))dG_2(c). \end{aligned}$$

For a given K, for all $c \in (\underline{c}, \overline{c}]$, $1-\hat{L}_K^\eta(\tilde{\Gamma}(c))$, $1-\hat{L}_K^\eta(\Gamma_1(c))$, and $1-\hat{L}_K^\eta(\Gamma_2(c))$ converge uniformly to zero, which implies that for all finite sets $K \subset \{3, 4, ...\}$, $\lim_{\eta \to \infty} \underline{\Delta BS}^1(K, \eta) = 0$.

If suppliers 1 and 2 are symmetric, i.e., $G_1 = G_2 = G$, then $\tilde{\Gamma} = \Gamma$ and

$$\begin{aligned} \underline{\Delta BS}^1(K, \eta) &= \int_{\underline{c}}^{\Gamma^{-1}(v)} \left[2\Gamma(c)(1 - G(c))g(c) - \hat{\Gamma}(c)\hat{g}(c)\right] (1 - \hat{L}_K^\eta(\Gamma(c)))d\hat{G}(c) \\ &= \int_{\underline{c}}^{\Gamma^{-1}(v)} \left(\Gamma(c) - \hat{\Gamma}(c)\right) 2(1 - G(c))g(c)(1 - \hat{L}_K^\eta(\Gamma(c)))d\hat{G}(c). \end{aligned}$$

It follows that when suppliers 1 and 2 are symmetric, given nonempty finite sets $K, K' \subset \{3, 4, ...\}$, if K is a strict subset of K'—in which case we refer to the change from K to K' as a"market expansion"—then $\underline{\Delta BS}^1(K, 1) < \underline{\Delta BS}^1(K', 1)$. Thus, when the merging suppliers are symmetric, the change in expected buyer surplus increases (towards zero) as the market expands.

Proposition A.5. *With no buyer power, buyer harm decreases as the market expands, and it goes to zero as any nonempty finite subset of nonmerging suppliers is replicated infinitely often. With buyer power, for any nonempty finite subset of nonmerging suppliers K and positive number of replicas η, $\underline{\Delta BS}^1(K, \eta) \leq \Delta BS^1(K, \eta) \leq 0$. Moreover, if $G_1 = G_2$, then $\underline{\Delta BS}^1(K, \eta)$ increases as the market expands, and for any G_1 and G_2, $\underline{\Delta BS}^1(K, \eta)$ goes to zero as the number of replicas goes to infinity. Consequently, buyer harm goes to zero as the number of replicas goes to infinity. Likewise, harm to social surplus goes to zero as the number of replicas goes to infinity. That is, given a nonempty finite set of nonmerging suppliers K, $\lim_{\eta \to \infty} \Delta BS^1(K, \eta) = 0$ and $\lim_{\eta \to \infty} \Delta SS^1(K, \eta) = 0$.*

Proof. Let a nonempty finite set $K \subset \{3, 4, ...\}$ be given. Without buyer power, the buyer's expected surplus before the merger when facing η replicas of the nonmerging suppliers in K is

$$\begin{aligned} BS^0_{pre}(K, \eta) &= \int_{\underline{c}}^{\min\{v, \overline{c}\}} (v - \Gamma_1(c))(1 - L_K(c))^\eta (1 - G_2(c)) dG_1(c) \\ &\quad + \int_{\underline{c}}^{\min\{v, \overline{c}\}} (v - \Gamma_2(c))(1 - L_K(c))^\eta (1 - G_1(c)) dG_2(c) + Z^\eta_K, \end{aligned}$$

where

$$Z^\eta_K \equiv \sum_{i \in K} \int_{\underline{c}}^{\min\{v, \overline{c}\}} (v - \Gamma_i(c))(1 - \hat{G}(c)) \left[\Pi_{j \in K \setminus \{i\}} (1 - G_j(c))^\eta\right] (1 - G_i(c))^{\eta - 1} dG_i(c).$$

After the merger, the buyer's expected surplus is

$$BS^0_{post}(K, \eta) = \int_{\underline{c}}^{\min\{v, \overline{c}\}} (v - \hat{\Gamma}(c))(1 - L_K(c))^\eta d\hat{G}(c) + Z^\eta_K.$$

Taking the difference, we obtain

$$\begin{aligned}
& \Delta BS^0(K,\eta) \\
= \; & BS^0_{post}(K,\eta) - BS^0_{pre}(K,\eta) \\
= \; & \int_{\underline{c}}^{\min\{v,\overline{c}\}} (v-\hat{\Gamma}(c))(1-L_K(c))^{\eta} d\hat{G}(c) - \int_{\underline{c}}^{\min\{v,\overline{c}\}} (v-\Gamma_1(c))(1-L_K(c))^{\eta}(1-G_2(c))dG_1(c) \\
& - \int_{\underline{c}}^{\min\{v,\overline{c}\}} (v-\Gamma_2(c))(1-L_K(c))^{\eta}(1-G_1(c))dG_2(c) \\
= \; & \int_{\underline{c}}^{\min\{v,\overline{c}\}} \Big[(1-G_2(c))(\Gamma_1(c)-\hat{\Gamma}(c))g_1(c) + (1-G_1(c))(\Gamma_2(c)-\hat{\Gamma}(c))g_2(c)\Big](1-L_K(c))^{\eta} dc \\
= \; & -\int_{\underline{c}}^{\min\{v,\overline{c}\}} (1-L_K(c))^{\eta} G_1(c)G_2(c)dc,
\end{aligned}$$

where the final equality uses the fact that the expression in square brackets is equal to $-G_1(c)G_2(c)$. Given nonempty finite set $K' \subset \{3,4,...\}$ such that $K \subset K'$, for $c \in (\underline{c},\overline{c})$, $1-L_{K'}(c) < 1-L_K(c)$, implying that $\Delta BS^0(K') > \Delta BS^0(K)$. Thus, buyer harm decreases as the market expands. In addition, because for $c \in (\underline{c},\overline{c}]$, $(1-L_K(c))^{\eta}$ converges uniformly to zero as η goes to infinity, $\lim_{\eta\to\infty} \Delta BS^0(K,\eta) = 0$.

As shown above prior to the statement of the proposition, with buyer power, for any nonempty finite subset of nonmerging suppliers K and number of replicas η, $\underline{\Delta BS}^1(K,\eta) \leq \Delta BS^1(K,\eta) \leq 0$. Further, when $G_1 = G_2$, $\underline{\Delta BS}^1(K,\eta)$ increases as the market expands, and for general G_1 and G_2, $\underline{\Delta BS}^1(K,\eta)$ goes to zero as the number of replicas goes to infinity.

It remains to show that $\lim_{\eta\to\infty} \Delta SS^1(K,\eta) = 0$. Before the merger, we have

$$\begin{aligned}
SS^1_{pre}(K,\eta) \;=\; & \int_{\underline{c}}^{\Gamma_1^{-1}(v)} (v-c)(1-\hat{L}^{\eta}_K(\Gamma_1(c)))(1-G_2(\Gamma_2^{-1}(\Gamma_1(c))))dG_1(c) \\
& + \int_{\underline{c}}^{\Gamma_2^{-1}(v)} (v-c)(1-\hat{L}^{\eta}_K(\Gamma_2(c)))(1-G_1(\Gamma_1^{-1}(\Gamma_2(c))))dG_2(c) \\
& + \eta \sum_{i\in K} \int_{\underline{c}}^{\Gamma_i^{-1}(v)} (v-c)\left[\Pi_{\ell\in K\setminus\{i\}}(1-G_\ell(\Gamma_\ell^{-1}(\Gamma_i(c)))^{\eta}\right](1-G_i(c))^{\eta-1} \\
& \cdot(1-G_2(\Gamma_2^{-1}(\Gamma_i(c))))(1-G_1(\Gamma_1^{-1}(\Gamma_i(c))))dG_i(c).
\end{aligned}$$

After the merger, we have

$$
\begin{aligned}
SS^1_{post}(K,\eta) &= \int_{\underline{c}}^{\hat{\Gamma}^{-1}(v)} (v-c)(1-\hat{L}^{\eta}_K(\hat{\Gamma}(c)))d\hat{G}(c) \\
&+\eta \sum_{i\in K} \int_{\underline{c}}^{\Gamma_i^{-1}(v)} (v-c)\left[\Pi_{\ell\in K\setminus\{i\}}(1-G_\ell(\Gamma_\ell^{-1}(\Gamma_i(c)))^{\eta}\right](1-G_i(c))^{\eta-1} \\
&\cdot(1-\hat{G}(\hat{\Gamma}^{-1}(\Gamma_i(c))))dG_i(c).
\end{aligned}
$$

Because for all $c \in (\underline{c}, \overline{c}]$, $1-\hat{L}^{\eta}_K(\Gamma_1(c))$, $1-\hat{L}^{\eta}_K(\Gamma_2(c))$, $1-\hat{L}^{\eta}_K(\hat{\Gamma}(c))$, and $(1-G_i(c))^{\eta-1}$ converge uniformly to zero as η goes to infinity, it follows that $\lim_{\eta\to\infty} \Delta SS^1(K,\eta) = 0$, which completes the proof. ■

Proposition A.6. *With no buyer power, a merger is profitable for the merging suppliers and neutral for nonmerging suppliers. The remainder of the proposition assumes buyer power and that virtual dominance holds. Any merger, whether it is profitable or not, benefits the nonmerging suppliers. For v sufficiently large, a merger to monopoly is profitable, and for v sufficiently small, a merger to monopoly is not profitable. Likewise, considering η-fold replicas of the nonmerging suppliers, for η sufficiently large, a merger is not profitable.*

Proof. The results with no buyer power do not rely on distributional assumptions and so the proof is the same as for Proposition 6 for the symmetric setup.
With buyer power a merger benefits the nonmerging suppliers: With buyer power, the expected surplus of a nonmerging supplier i is

$$
E\left[\Gamma_i(c_i) - c_i \mid \Gamma_i(c_i) < \min_{j\neq i}\{\Gamma_j(c_j), v\}\right] \Pr\left(\Gamma_i(c_i) < \min_{j\neq i}\{\Gamma_j(c_j), v\}\right)
$$

before the merger and

$$
\begin{aligned}
&E\left[\Gamma_i(c_i) - c_i \mid \Gamma_i(c_i) < \min_{j\in\{3,\dots,n\}\setminus\{i\}}\{\hat{\Gamma}(\min\{c_1,c_2\}), \Gamma_j(c_j), v\}\right] \\
&\cdot \Pr\left(\Gamma_i(c_i) < \min_{j\in\{3,\dots,n\}\setminus\{i\}}\{\hat{\Gamma}(\min\{c_1,c_2\}), \Gamma_j(c_j), v\}\right)
\end{aligned}
$$

after the merger. Because $\Gamma_i(c_i) - c_i > 0$ for all $c_i > \underline{c}$ and because the conditioning event is less strict in the post-merger market (because $\hat{\Gamma}(\min\{c_1, c_2\}) > \min\{\Gamma(c_1), \Gamma(c_2)\}$), it follows that a merger benefits the nonmerging suppliers.

With buyer power and v large, a merger to monopoly is profitable: When $n = 2$, the merging suppliers' joint pre-merger expected surplus is

$$E\left[\Gamma_1(c_1) - c_1 \mid \Gamma_1(c_1) < \min\{\Gamma_2(c_2), v\}\right] \Pr\left(\Gamma_1(c_1) < \min\{\Gamma_2(c_2), v\}\right)$$
$$+E\left[\Gamma_2(c_2) - c_2 \mid \Gamma_2(c_2) < \min\{\Gamma_1(c_1), v\}\right] \Pr\left(\Gamma_2(c_2) < \min\{\Gamma_1(c_1), v\}\right)$$

and after the merger it is

$$E\left[\hat{\Gamma}(\min\{c_1, c_2\}) - \min\{c_1, c_2\} \mid \hat{\Gamma}(\min\{c_1, c_2\}) < v\right] \Pr\left(\hat{\Gamma}(\min\{c_1, c_2\}) < v\right).$$

In the limit as v goes to infinity, the probability of trade in the post-merger market goes to 1. Thus, in the limit as v goes to infinity, the joint expected surplus of the merging suppliers after the merger is

$$E\left[\hat{\Gamma}(\min\{c_1, c_2\}) - \min\{c_1, c_2\}\right]$$

and their joint expected surplus before the merger is

$$\begin{aligned}
& E\left[\Gamma_1(c_1) - c_1 \mid \Gamma_1(c_1) < \Gamma_2(c_2)\right] \Pr\left(\Gamma_1(c_1) < \Gamma_2(c_2)\right) \\
& +E\left[\Gamma_2(c_2) - c_2 \mid \Gamma_2(c_2) < \Gamma_1(c_1)\right] \Pr\left(\Gamma_2(c_2) < \Gamma_1(c_1)\right) \\
\leq\ & E[\min\left\{\Gamma_1(c_1), \Gamma_2(c_2)\right\} - \min\{c_1, c_2\}] \\
<\ & E\left[\hat{\Gamma}(\min\{c_1, c_2\}) - \min\{c_1, c_2\}\right],
\end{aligned}$$

where the final inequality uses virtual dominance and establishes the result.

With buyer power and v small, a merger to monopoly is not profitable: The proof that with buyer power and v small, a merger to monopoly is not profitable follows in similar fashion to the proof for the symmetric setup. Suppose a merger to monopoly, i.e., $n = 2$. We define $p_1(v)$ and $p_2(v)$ to be the buyer's optimal final offers to suppliers 1 and 2, respectively, as a function of v in the pre-merger market. Define $\hat{p}(v)$ to be the buyer's optimal final offer to the merged entity in the post-merger market. Thus, for $i \in \{1, 2\}$, $\Gamma_i(p_i(v)) = v$ and $\hat{\Gamma}(\hat{p}(v)) = v$, implying that $p_1(\underline{c}) = p_2(\underline{c}) = \hat{p}(\underline{c}) = \underline{c}$. We define $\Pi(p_1, p_2)$ and $\hat{\Pi}(p)$ to be the joint expected profit of the two merging suppliers as a function of the final offers. Before the merger, given final offers p_1 and p_2, the joint expected profit of the two merging suppliers is

$$\begin{aligned}\Pi(p_1,p_2) &= \int_{\underline{c}}^{p_1}(p_1-c_1)(1-G_2(p_2))dG_1(c_1)+\int_{\underline{c}}^{p_2}(p_2-c_2)(1-G_1(p_1))dG_2(c_2)\\ &\quad+\int_{\underline{c}}^{p_1}\int_{\underline{c}}^{\Gamma_2^{-1}(\Gamma_1(c_1))}(\Gamma_2^{-1}(\Gamma_1(c_1))-c_2)dG_2(c_2)dG_1(c_1)\\ &\quad+\int_{\underline{c}}^{p_2}\int_{\underline{c}}^{\Gamma_1^{-1}(\Gamma_2(c_2))}(\Gamma_1^{-1}(\Gamma_2(c_2))-c_1)dG_1(c_1)dG_2(c_2),\end{aligned}$$

where the first two terms are the joint profit of the two suppliers if they do not compete against each other and the lowest-cost supplier is paid the final offer. The other two terms reflect the low-virtual-cost supplier being paid its threshold payment as determined by the other supplier's report. Given a take-it-or-leave-it offer p, the post-merger expected profit is

$$\hat{\Pi}(p)=\int_{\underline{c}}^{p}(p-c)d\hat{G}(c)=\int_{\underline{c}}^{p}G(y)(2-G(y))dy.$$

Note that $\Pi(p_1(\underline{c}),p_2(\underline{c}))=\hat{\Pi}(\hat{p}(\underline{c}))=0$. We show that $\Pi(p_1(v),p_2(v))$ increases faster than $\hat{\Pi}(\hat{p}(v))$ at $v=\underline{c}$, implying by continuity that $\Pi(p_1(v),p_2(v))>\hat{\Pi}(\hat{p}(v))$ for v in a neighborhood to the right of $\underline{c}$. Specifically, letting $f(v)\equiv\Pi(p_1(v),p_2(v))$ and $\hat{f}(v)\equiv\hat{\Pi}(\hat{p}(v))$, we look at the derivatives of $f(v)-\hat{f}(v)$, evaluated at $v=\underline{c}$, and show that the "first time" the derivatives differ, the derivative is positive, i.e., if $j=\min\{i\in\{1,2,...\}\mid f^{(i)}(\underline{c})\neq\hat{f}^{(i)}(\underline{c})\}$, then $f^{(j)}(\underline{c})-\hat{f}^{(j)}(\underline{c})>0$, where $f^{(j)}$ denotes the j-th derivative of f. The index j is well defined because $f(\underline{c})=\hat{f}(\underline{c})$ and for v sufficiently large, $f(v)\neq\hat{f}(v)$.

In order to illustrate the logic of the proof, we begin by considering the case with $g_1(\underline{c})>0$ and $g_2(\underline{c})>0$. First, consider the derivatives of Γ_i and $\hat{\Gamma}$. As always, $\Gamma_i(\underline{c})=\hat{\Gamma}(\underline{c})=\underline{c}$. When $g_1(\underline{c})>0$ and $g_2(\underline{c})>0$, $\Gamma_1'(\underline{c})=\Gamma_2'(\underline{c})=\hat{\Gamma}'(\underline{c})=2$, $\Gamma_1''(\underline{c})=-\frac{g_1'(\underline{c})}{g_1(\underline{c})}$, $\Gamma_2''(\underline{c})=-\frac{g_2'(\underline{c})}{g_2(\underline{c})}$, and

$$\hat{\Gamma}''(\underline{c})=-\frac{g_1'(\underline{c})+g_2'(\underline{c})}{g_1(\underline{c})+g_2(\underline{c})}+\frac{2g_1(\underline{c})g_2(\underline{c})}{g_1(\underline{c})+g_2(\underline{c})},$$

implying that

$$\begin{aligned}&(g_1(\underline{c})+g_2(\underline{c}))\,\hat{\Gamma}''(\underline{c})-g_1(\underline{c})\Gamma_1''(\underline{c})-g_2(\underline{c})\Gamma_2''(\underline{c})\\ =\;&-g_1'(\underline{c})-g_2'(\underline{c})+2g_1(\underline{c})g_2(\underline{c})+g_1'(\underline{c})+g_2'(\underline{c}) \qquad (2)\\ =\;&2g_1(\underline{c})g_2(\underline{c}).\end{aligned}$$

Using the definitions of p and $\hat{p}$ and the equality of Γ_i and $\hat{\Gamma}$ and their first derivatives at $\underline{c}$, we have

$$p_1'(\underline{c}) = p_2'(\underline{c}) = \hat{p}'(\underline{c}) = 1/\Gamma_1'(\underline{c}) > 0 \tag{3}$$

and for $i \in \{1, 2\}$,

$$p_i''(\underline{c}) = -\frac{(p_i'(\underline{c}))^2}{\Gamma_i'(\underline{c})}\Gamma_i''(\underline{c}) \ \text{ and } \hat{p}''(\underline{c}) = -\frac{(\hat{p}')^2}{\hat{\Gamma}'(\underline{c})}\hat{\Gamma}''(\underline{c}).$$

We now turn to the functions Π and $\hat{\Pi}$. We let $\hat{\Pi}^{(i)}$ denote the i-th derivative of $\hat{\Pi}$, and we let $\Pi^{(i,j)} \equiv \frac{\partial^{i+j}}{\partial p_1^i \partial p_2^j}\Pi$. Note that $\Pi^{(1,0)}(\underline{c}, \underline{c}) = \Pi^{(0,1)}(\underline{c}, \underline{c}) = \hat{\Pi}'(\underline{c}) = 0$. In addition, when $g_1(\underline{c}) > 0$ and $g_2(\underline{c}) > 0$, then

$$\Pi^{(1,1)}(\underline{c}, \underline{c}) = 0, \ \Pi^{(2,0)}(\underline{c}, \underline{c}) = g_1(\underline{c}), \ \Pi^{(0,2)}(\underline{c}, \underline{c}) = g_2(\underline{c}),$$

$$\Pi^{(2,1)}(\underline{c}, \underline{c}) = \Pi^{(1,2)}(\underline{c}, \underline{c}) = -g_1(\underline{c})g_2(\underline{c}),$$

$$\Pi^{(3,0)}(\underline{c}, \underline{c}) = g_1'(\underline{c}) + g_1(\underline{c})g_2(\underline{c}) \ \text{ and } \ \Pi^{(0,3)}(\underline{c}, \underline{c}) = g_2'(\underline{c}) + g_1(\underline{c})g_2(\underline{c}).$$

For the post-merger market,

$$\hat{\Pi}^{(2)}(\underline{c}) = \hat{g}(\underline{c}) = g_1(\underline{c}) + g_2(\underline{c})$$

and

$$\hat{\Pi}^{(3)}(\underline{c}) = \hat{g}'(\underline{c}) = g_1'(\underline{c}) + g_2'(\underline{c}) - 2g_1(\underline{c})g_2(\underline{c}).$$

Thus,

$$\Pi^{(3,0)}(\underline{c}, \underline{c}) + \Pi^{(0,3)}(\underline{c}, \underline{c}) + 3\left(\Pi^{(2,1)}(\underline{c}, \underline{c}) + \Pi^{(1,2)}(\underline{c}, \underline{c})\right) - \hat{\Pi}^{(3)}(\underline{c}) = -2g_1(\underline{c})g_2(\underline{c}) \tag{4}$$

and (dropping the argument(s) $\underline{c}$ for readability)

$$\begin{aligned}
\Pi^{(2,0)}p_1'' + \Pi^{(0,2)}p_2'' - \hat{\Pi}^{(2)}\hat{p}'' &= -g_1\frac{(p_1')^2}{\Gamma_1'}\Gamma_1'' - g_2\frac{(p_2')^2}{\Gamma_2'}\Gamma_2'' + (g_1 + g_2)\frac{(\hat{p}')^2}{\hat{\Gamma}'}\hat{\Gamma}'' \\
&= \left(-g_1\Gamma_1'' - g_2\Gamma_2'' + (g_1 + g_2)\hat{\Gamma}''\right)\frac{(p_1')^2}{\Gamma_1'} \\
&= 2g_1g_2\frac{(p_1')^2}{\Gamma_1'} \\
&= g_1g_2(p_1')^2,
\end{aligned} \tag{5}$$

where the second equality uses $\Gamma_1' = \Gamma_2' = \hat{\Gamma}'$ and $p_1' = p_2' = \hat{p}'$, the third equality uses (2), and the fourth equality uses $\Gamma_1' = 2$.

Moving beyond the case in which $g_1(\underline{c}) > 0$ and $g_2(\underline{c}) > 0$, under virtual dominance, there exists $k \in \{0, 1, ...\}$ such that for $i \in \{1, 2\}$, $g_i(\underline{c}) = g_i'(\underline{c}) = ... = g_i^{(k-1)}(\underline{c}) = 0$ and for $i \in \{1, 2\}$, $g_1^{(k)}(\underline{c}) > 0$ and $g_2^{(k)}(\underline{c}) > 0$. In this case, (4) and (5) can be written in terms of k as

$$\begin{aligned} & \Pi^{(2k+3,0)}(\underline{c}, \underline{c}) + \Pi^{(0,2k+3)}(\underline{c}, \underline{c}) \\ & + \binom{2k+3}{k+2} \left(\Pi^{(k+1,k+2)}(\underline{c}, \underline{c}) + \Pi^{(k+2,k+1)}(\underline{c}, \underline{c}) \right) - \hat{\Pi}^{(k)}(\underline{c}) \\ = & -(k+1)(k+2) g_1^{(k)}(\underline{c}) g_2^{(k)}(\underline{c}), \end{aligned} \tag{6}$$

where $\binom{a}{b}$ denotes the binomial coefficient $a!/(b!(a-b)!)$, and

$$\begin{aligned} & \Pi^{(k+2,0)}(\underline{c}, \underline{c}) p_1^{(k+2)}(\underline{c}) + \Pi^{(0,k+2)}(\underline{c}, \underline{c}) p_2^{(k+2)}(\underline{c}) - \hat{\Pi}^{(k+2)} \hat{p}^{(k+2)}(\underline{c}) \\ = & \; g_1^{(k)}(\underline{c}) g_2^{(k)}(\underline{c}) (p_1'(\underline{c}))^{k+2}. \end{aligned} \tag{7}$$

Now, for the case of $g_1(\underline{c}) > 0$ and $g_2(\underline{c}) > 0$, consider the derivatives of $f(v) - \hat{f}(v)$, evaluated at $v = \underline{c}$. The first derivative is zero. Evaluating the second derivative at $\underline{c}$ and dropping the argument $\underline{c}$ for readability, we have:

$$\begin{aligned} f'' - \hat{f}'' &= \Pi^{(2,0)} p_1'^2 + \Pi^{(0,2)} p_2'^2 + 2\Pi^{(1,1)} p_1' p_2' + \Pi^{(1,0)} p_1'' + \Pi^{(0,1)} p_2'' - \left(\hat{\Pi}'' \hat{p}'^2 + \hat{\Pi}' \hat{p}'' \right) \\ &= g_1 p_1'^2 + g_2 p_2'^2 - (g_1 + g_2) \hat{p}'^2 \\ &= 0. \end{aligned}$$

At the third derivative, we have a difference:

$$\begin{aligned} f''' - \hat{f}''' &= \left(\Pi^{(3,0)} + \Pi^{(0,3)} + 3 \left(\Pi^{(2,1)} + \Pi^{(1,2)} \right) - \hat{\Pi}^{(3)} \right) (p_1')^3 \\ &\quad + 3 \left(\Pi^{(2,0)} p_1'' + \Pi^{(0,2)} p_2'' - \hat{\Pi}^{(2)} \hat{p}'' \right) p_1' \\ &= -2 g_1 g_2 (p_1')^3 + 3 \left(g_1 g_2 (p_1')^2 \right) p_1' \\ &= (-2 + 3) \, g_1 g_2 (p_1')^3 \\ &> 0, \end{aligned}$$

where the second equality uses (4) and (5), the third equality rearranges, and the inequality uses the assumptions that $g_1(\underline{c}) > 0$ and $g_2(\underline{c}) > 0$ and the result that $p_1' > 0$, which is stated in (3). This completes the proof for the case in which $g_i(\underline{c}) > 0$.

In the general case in which for $i \in \{1,2\}$, $g_i(\underline{c}) = g_i'(\underline{c}) = ... = g_i^{(k-1)}(\underline{c}) = 0$ and for $i \in \{1,2\}$, $g_1^{(k)}(\underline{c}) > 0$ and $g_2^{(k)}(\underline{c}) > 0$, we have:

$$\begin{aligned}
f^{(2k+3)} - \hat{f}^{(2k+3)} &= \left(\Pi^{(2k+3,0)} + \Pi^{(0,2k+3)} + 3\left(\Pi^{(k+2,k+1)} + \Pi^{(k+1,k+2)}\right) - \hat{\Pi}^{(2k+3)}\right)(p_1')^{2k+3} \\
&\quad + \binom{2k+3}{k+2}\left(\Pi^{(k+2,0)}p_1^{(k+2)} + \Pi^{(0,k+2)}p_2^{(k+2)} - \hat{\Pi}^{(k+2)}\hat{p}^{(k+2)}\right)(p_1')^{k+1} \\
&= -(k+1)(k+2)g_1^{(k)}g_2^{(k)}(p_1')^{2k+3} + \binom{2k+3}{k+2}\left(g_1^{(k)}g_2^{(k)}(p_1')^{k+2}\right)(p_1')^{k+1} \\
&= \left(-(k+1)(k+2) + \binom{2k+3}{k+2}\right)g_1^{(k)}g_2^{(k)}(\hat{p}')^{2k+3} \\
&> 0,
\end{aligned}$$

where the second equality uses (6) and (7), the third inequality rearranges, and the inequality uses the definition of k, the positivity of $p'(\underline{c})$ as stated in (3), and the result that for all $k \in \{0,1,...\}$,

$$0 < -(k+1)(k+2) + \frac{(2k+3)!}{(k+2)!\,(k+1)!}.$$

This completes the proof that with buyer power and v small, a merger to monopoly is not profitable.

With buyer power and considering η-fold replicas of the nonmerging suppliers, for η sufficiently large, a merger is not profitable: Let $n > 2$ and consider a market that includes suppliers 1 and 2 plus η-fold replication of the nonmerging suppliers $\{3,...,n\}$. Denote by $\tilde{c}_{(1)}$ the lowest *virtual* cost for any of the $\eta(n-2)$ nonmerging suppliers, i.e., $\tilde{c}_{(1)} \equiv \min_{j\in\{3,...,n\},\kappa\in\{1,...,\eta\}} \Gamma_j(c_j^\kappa)$. Before the merger, in the limit as η goes to infinity, with probability 1, the reserve based on the buyer's value does not bind and a winning supplier $i \in \{1,2\}$ is paid $\Gamma_i^{-1}(\tilde{c}_{(1)})$. Thus, in the limit, the joint surplus of the merging suppliers, conditional on one of them winning, is $\Gamma_1^{-1}(\tilde{c}_{(1)}) - c_1$ if $\Gamma_1(c_1) < \Gamma_2(c_2)$ and $\Gamma_2^{-1}(\tilde{c}_{(1)}) - c_2$ if $\Gamma_2(c_2) < \Gamma_1(c_1)$. As η goes to infinity, $\tilde{c}_{(1)} \to \underline{c}$ almost surely and for $i \in \{1,2\}$, $\Gamma_i^{-1}(\tilde{c}_{(1)}) \to \underline{c}$ almost surely. Because Γ_1 and Γ_2 are equal and have the same slope at $\underline{c}$ (recall that $\Gamma_1'(\underline{c}) = \Gamma_2'(\underline{c}) = 2$), in the limit as η goes to infinity, conditional on supplier 1 or supplier 2 winning, with probability 1 the supplier with the lower virtual cost also has the lower cost. Thus, in the limit as η goes to infinity, in the premerger market, conditional on one of the merging supplier's winning, their joint surplus is $\min\left\{\Gamma_1^{-1}(\tilde{c}_{(1)}), \Gamma_2^{-1}(\tilde{c}_{(1)})\right\} - \min\{c_1, c_2\}$. After the merger, in the limit as η goes to infinity, with probability 1, a winning merged entity has surplus $\hat{\Gamma}^{-1}(\tilde{c}_{(1)}) - \min\{c_1, c_2\}$. As η goes to infinity, $\hat{\Gamma}^{-1}(\tilde{c}_{(1)}) \to \underline{c}$ almost surely. By virtual dominance, for all $\tilde{c}_{(1)} > \underline{c}$,

$\hat{\Gamma}^{-1}(\tilde{c}_{(1)}) < \min\left\{\Gamma_1^{-1}(\tilde{c}_{(1)}), \Gamma_2^{-1}(\tilde{c}_{(1)})\right\}$, which implies that for η sufficiently large, the expected joint surplus of the merging suppliers is greater before the merger than after the merger. ■

Proposition A.7. *In the absence of buyer power, a merger is neutral for nonmerging suppliers and so does not induce entry, but with buyer power and assuming virtual dominance holds, a merger increases the expected payoff from entry and so potentially induces entry.*

Proof. In the absence of buyer power, distributional assumptions are not relevant, and so the proof is the same as for Proposition 7 for the symmetric setup. With buyer power and assuming virtual dominance holds, a merger induces the buyer to discriminate against the merging suppliers, which shifts market shares to the nonmerging suppliers and therefore increases the profitability of entry. ■

Proposition A.8. *Without buyer power, a merger is neutral for nonmerging suppliers' incentives to invest and increases the merging suppliers' incentives to invest. With buyer power and assuming virtual dominance holds, a merger increases incentives to invest for nonmerging suppliers but can either increase or decrease incentives for the merging suppliers.*

Proof. Let $\pi_i^\beta(c)$ be supplier i's expected profit before the merger when its cost is c and the buyer's power is β, and let $\hat{\pi}_i^\beta(c)$ be the expected profit after the merger of supplier i in the same contingency, where for $i \in \{1,2\}$, this is expected profit per plant. Letting $G_{i,I}$ denote supplier i's cost distribution after investment, we assume $G_{i,I}(c) \geq G_i(c)$ for all $c \in [\underline{c}, \overline{c}]$. We say that for a given β, supplier i's incentives to invest increase with the merger if

$$\int_{\underline{c}}^{\overline{c}} \hat{\pi}_i^\beta(c)[dG_{i,I}(c) - dG_i(c)] > \int_{\underline{c}}^{\overline{c}} \pi_i^\beta(c)[dG_{i,I}(c) - dG_i(c)], \tag{8}$$

and we say that a merger is neutral (decreases incentives to invest) for supplier i if $\int_{\underline{c}}^{\overline{c}} \hat{\pi}_i^\beta(c)[dG_{i,I}(c) - dG_i(c)] = (<) \int_{\underline{c}}^{\overline{c}} \pi_i^\beta(c)[dG_{i,I}(c) - dG_i(c)]$. Evidently, (8) is equivalent to

$$\int_{\underline{c}}^{\overline{c}} [\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)]dG_{i,I}(c) > \int_{\underline{c}}^{\overline{c}} [\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)]dG_i(c). \tag{9}$$

Because we assume that G_i first-order stochastically dominates $G_{i,I}$, it follows that a merger increases (decreases) incentives to invest for i if $\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)$ decreases (increases) in c and is neutral for i if $\hat{\pi}_i^{\beta}(c) - \pi_i^{\beta}(c)$ is constant. In what follows, our focus is thus naturally on the sign of the derivative of the expression $\hat{\pi}_i^{\beta}(c) - \pi_i^{\beta}(c)$.

Rival's' incentives: Let us first consider the incentive effects of the rivals of the merging suppliers, that is, suppliers $i = 3, .., n$, beginning with the case without buyer power. By the revenue (or payoff) equivalence theorem, we know that in any incentive compatible mechanism the interim expected payoff of supplier i when his cost is $c \in [\underline{c}, \overline{c}]$ is $\int_c^{\overline{c}} q_i(x)dx$ plus a constant (which under the assumptions we impose is 0), where $q_i(x)$ is the probability that i produces and is determined by the allocation rule. Before the merger, the probability that i produces is $\Pi_{j \neq i}(1 - G_j(x))$. Thus, for all $c \leq \min\{v, \overline{c}\}$,

$$\pi_i^0(c) = \int_c^{\min\{v,\overline{c}\}} \Pi_{j \neq i}(1 - G_j(x))dx, \tag{10}$$

and $\pi_i^0(c) = 0$ for all c larger than $\Gamma_i^{-1}(v)$. Without buyer power, $\Pi_{j \neq i}(1 - G_j(x))$ is also the probability that i produces after the merger, so we have $\hat{\pi}_i^0(c) = \pi_i^0(c)$ for all c and all $i \in \{3, ..., n\}$. Thus, without buyer power, a merger is neutral for rivals' incentives to invest.

With buyer power, for all $c \leq \Gamma_i^{-1}(v)$ (recall that we define $\Gamma_i^{-1}(v)$ so that $\Gamma_i^{-1}(v) \leq \overline{c}$) and all $i \in \{3, ..., n\}$,

$$\pi_i^1(c) = \int_c^{\Gamma_i^{-1}(v)} \Pi_{j \neq i}(1 - G_j(\Gamma_j^{-1}(\Gamma_i(x))))dx,$$

and

$$\hat{\pi}_i^1(c) = \int_c^{\Gamma_i^{-1}(v)} \Pi_{j \in \{1,2\}}(1 - G_j(\hat{\Gamma}^{-1}(\Gamma_i(x))))\Pi_{j \in \{3,...,n\} \backslash \{i\}}(1 - G_j(\Gamma_j^{-1}(\Gamma_i(x))))dx,$$

and, of course, $\pi_i^1(c) = 0 = \hat{\pi}_i^1(c)$ for all c greater than $\Gamma_i^{-1}(v)$. Thus, for all $c \leq \Gamma_i^{-1}(v)$,

$$\begin{aligned} \hat{\pi}_i^{1\prime}(c) - \pi_i^{1\prime}(c) &= -\left[\Pi_{j \in \{1,2\}}(1 - G_j(\hat{\Gamma}^{-1}(\Gamma_i(c)))) - \Pi_{j \in \{1,2\}}(1 - G_j(\Gamma_j^{-1}(\Gamma_i(c))))\right] \\ &\quad \cdot \Pi_{j \in \{3,...,n\} \backslash \{i\}}(1 - G_j(\Gamma_j^{-1}(\Gamma_i(c)))) \\ &\leq 0, \end{aligned}$$

where the inequality is strict for all $c \in (\underline{c}, \Gamma_i^{-1}(v))$ because by virtual dominance $\hat{\Gamma}^{-1}(\Gamma_i(c)) < \Gamma_j^{-1}(\Gamma_i(c))$ for all such c. Thus, with buyer power and assuming virtual dominance holds, a merger increases rivals' incentives to invest.

Merging suppliers' incentives: Consider now the merging suppliers' incentives to invest.

Without buyer power, for $i \in \{1,2\}$, $\pi_i^0(c)$ is as defined above. After the merger, for $c \leq \min\{v, \overline{c}\}$,

$$\hat{\pi}_1^0(c) = \int_c^{\min\{v,\overline{c}\}} (1 - G_2(c))\Pi_{j=3}^n (1 - G_j(x))dx$$

and

$$\hat{\pi}_2^0(c) = \int_c^{\min\{v,\overline{c}\}} (1 - G_1(c))\Pi_{j=3}^n (1 - G_j(x))dx,$$

and $\pi_1^0(c) = \pi_2^0(c) = 0$ for all larger c. Observing that, for $i, j \in \{1,2\}$ with $i \neq j$ and $c \leq \min\{v, \overline{c}\}$,

$$\hat{\pi}_i^{0\prime}(c) - \pi_i^{0\prime}(c) = -\frac{g_j(c)}{1 - G_j(c)}\hat{\pi}_i^0(c) \leq 0,$$

with a strict inequality for all $c \in [\underline{c}, \min\{v, \overline{c}\})$, unless $g_j(\underline{c}) = 0$, and $\hat{\pi}_i^{0\prime}(c) - \pi_i^{0\prime}(c) = 0$ otherwise, it follows that without buyer power a merger increases the merging suppliers' incentives to invest per plant.

Finally, we address the merging suppliers' incentives to invest in the presence of buyer power, which is the only case for which the merger-related change in incentives cannot be signed. Before the merger, for $i \in \{1,2\}$, $\pi_i^1(c)$ is as defined above. After the merger, for $c \leq \hat{\Gamma}^{-1}(v)$ and $i, j \in \{1,2\}$ with $i \neq j$,

$$\hat{\pi}_i^1(c) = \int_c^{\hat{\Gamma}^{-1}(v)} (1 - G_j(c))\Pi_{k\in\{3,\ldots,n\}}(1 - G_k(\Gamma_k^{-1}(\hat{\Gamma}(x))))dx,$$

implying, for $c \leq \hat{\Gamma}_i^{-1}(v)$,

$$\begin{aligned} & \hat{\pi}_i^{1\prime}(c) - \pi_i^{1\prime}(c) \\ = \; & -(1 - G_j(c))\underbrace{[\Pi_{k\in\{3,\ldots,n\}}(1 - G_k(\Gamma_k^{-1}(\hat{\Gamma}(x)))) - \Pi_{k\in\{3,\ldots,n\}}(1 - G_k(\Gamma_k^{-1}(\Gamma_i(c))))]}_{<0} \\ & -\frac{g_j(c)}{1 - G_j(c)}\underbrace{\hat{\pi}_i^1(c)}_{>0}, \end{aligned}$$

while for $c \in (\hat{\Gamma}^{-1}(v), \Gamma_i^{-1}(v))$,

$$\hat{\pi}_i^{1\prime}(c) - \pi_i^{1\prime}(c) = -\pi_i^{1\prime}(c) > 0.$$

Thus, with buyer power, the effects of the merger on the merging suppliers' incentives to invest cannot be signed in general. ■

To extend Proposition 9 to the asymmetric setup, as in Proposition A.4, we require symmetry between suppliers 1 and 2 for the comparison between $\Delta BS^1(s)$ and $\Delta BS^0(s)$ for small synergies:

Proposition A.9. *With cost synergies, the buyer's harm from a merger decreases with cost synergies, i.e., $\Delta BS^0(s)$ and $\Delta BS^1(s)$ are both increasing in s. For s sufficiently close to one, $\Delta BS^0(s) > \Delta BS^1(s) > 0$, and assuming $G_1 = G_2$, for s sufficiently close to zero, $0 > \Delta BS^1(s) > \Delta BS^0(s)$.*

Proof. Recall that in the setup with cost synergies, we assume that $\underline{c} = 0$. Given $s \in [0,1]$, c_1, and c_2, the merged entity's cost is $(1-s)\min\{c_1, c_2\}$. The distribution of costs after the merger for the merged entity is, for $c \in [0,(1-s)\overline{c}]$, $\overline{G}(c) \equiv \hat{G}(c/(1-s))$, with density $\overline{g}(c) \equiv \hat{g}(c/(1-s))/(1-s)$. Because cost synergies improve the merged entity's distribution, cost synergies monotonically increase the set of trades that are beneficial to the buyer. Thus, the buyer would benefit monotonically from cost synergies if it employed the same mechanism as without cost synergies. When the buyer optimally adjusts its mechanism to account for synergies, the monotonicity is preserved, where for $\beta = 0$ the adjustment is the set the reserve equal to $\min\{v,(1-s)\overline{c}\}$.

When $s = 1$, the post-merger buyer has payoff v regardless of buyer power, while the pre-merger buyer has higher expected surplus with buyer power than without, implying that $\Delta BS^0(1) > \Delta BS^1(1) > 0$. Assuming $G_1 = G_2$, the result that $\Delta BS^1(s) > \Delta BS^0(s)$ for s sufficiently close to zero follows from Proposition A.4 and continuity. ■

Corollary 1. *In the absence of buyer power, a competition authority using a social surplus standard is more permissive than one using a buyer surplus standard. Formally, $\Delta SS^0(s) \geq 0$ for all $s \in [0,1]$, while $\Delta BS^0(s) \geq 0$ if and only if $s \in [s^*,1]$ with $s^* > 0$.*

Proof. In the absence of buyer power, distributional assumptions are not relevant, and so the proof is the same as for Corollary 1 for the symmetric setup. ■

Proposition A.10. *With cost synergies, no buyer power, and two suppliers, the following holds:*

(i) if $v < \overline{c}$, then the merged entity's expected surplus increases and then decreases as cost synergies increase; that is, there exists $\hat{s} \in (0,1)$ such that $\hat{\Pi}(s)$ increases in s for $s < \hat{s}$ and decreases in s for $s > \hat{s}$),

(ii) if $v \geq \overline{c}$, then the merged entity's expected surplus decreases as cost synergies increase; that is, $\hat{\Pi}(s)$ decreases in s for all $s \in [0, 1]$;

(iii) the merged entity's expected surplus is zero when $s = 1$; that is, $\hat{\Pi}(1) = 0$.

Proof. In the absence of buyer power, distributional assumptions are not relevant, and so the proof is the same as for Proposition 10 for the symmetric setup. ■

Proposition A.11. *With cost synergies, buyer power, and two suppliers, the price faced by the merged entity is 0 if $s = 1$ and otherwise decreases in s.*

Proof. The proof relies only on the properties of the post-merger reserve for the merged entity, $p^1(s) \equiv (1-s)\hat{\Gamma}^{-1}(v/(1-s))$, which is not affected by distributional asymmetries. Thus, the proof is the same as for Proposition 11 for the symmetric setup. ■

Corollary 2. *With cost synergies, the expected surplus of the merged entity is maximized at a level of cost synergies strictly less than one.*

Proof. Without buyer power, Proposition A.10(iii) implies that $\hat{\Pi}(1) = 0$. The same intuition carries over to the case with buyer power—as s goes to 1, the merged entity is left with no private information, so $\hat{\Pi}(1) = 0$. Because, both with and without buyer power, the expected surplus of the merged entity is positive when $s = 0$, the result follows. ■