

Online Appendix for Merger Review for Markets with Buyer Power

Simon Loertscher* Leslie M. Marx[†]

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Introduction

In this appendix we extend the framework of Loertscher and Marx (forthcoming) to allow two products that are perfect complements for the buyer. We allow there to be some suppliers who produce both of the products demanded by the buyer and others who produce only one. We can accommodate the possibility that the buyer has a preference for purchasing both products from the same supplier.

This extension is motivated in part by the proposed merger of oilfield services firms Halliburton and Baker Hughes, which posed challenges for competition authorities because of the multi-product nature of the merging firms.¹ In particular, existing methodologies, which typically focus on individual relevant antitrust markets, have limited ability to account for cost synergies associated with multi-product suppliers, demand side complementarities that cause buyers to prefer a single source for multiple products, or the interaction of these with buyer power.² The extension of Loertscher and Marx (forthcom-

*Department of Economics, Level 4, FBE Building, 111 Barry Street, University of Melbourne, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

[†]The Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, USA: Email: marx@duke.edu.

¹The proposed merger was announced in 2014, but the parties ultimately abandoned attempts to merge in the face of opposition from the DOJ: United States of America v. Halliburton Co. and Baker Hughes, Inc., *Complaint*, April 6, 2016, Case 1:16-cv-00233-UNA, hereafter “DOJ Complaint.”

²The argument that the DOJ put forward in its opposition to the Halliburton-Baker Hughes transaction focused on effects within individual product markets, ignoring possible synergies and complementarities, but then returned to these issues as overarching concerns that amplified competitive concerns created by the transaction, noting that the merging parties “offer similar types of integrated solutions, bundled services, and other multiple-product and service combinations.” (DOJ Complaint, p. 30)

ing) provided here allows one to address these issues.

Extension to multi-product suppliers

In order to address cross-market issues such as demand-side complementarities, we consider a setup with two products, A and B . The buyer has value zero for product A or B individually, value v for the pair of products A and B if purchased from two different suppliers, and value $V \geq v$ for the pair of products A and B if purchased from the same supplier. Thus, the difference between $V - v \geq 0$ captures demand-side complementarities, which are sometimes referred to as the value of “one-stop shopping.”

This multi-product extension accommodates the case in which suppliers produce multiple complementary products, as well as the case in which products are location specific, with one product supplied in location A and the other in location B , where the buyer demands coverage that spans both locations. Further, this extension can be interpreted in terms of vertically related products. For example, product A might be the transportation or marketing of product B , both of which are demanded by the buyer. In that case, a merger of a supplier of A with a supplier of B is a vertical merger.

Let \mathbb{M} be the set of multi-product suppliers, \mathbb{A} be the set of suppliers of only A , and \mathbb{B} be the set of suppliers of only B , with $|\mathbb{M}| + |\mathbb{A}| + |\mathbb{B}| = n$. The cost type of a multi-product supplier is the cost to that supplier of producing both products, whereas the cost type of a single-product supplier is the cost to that supplier of producing only one product. Thus, each supplier has a single-dimensional type. Multi-product suppliers can supply individual products to the buyer for a commonly known proportion of their joint production cost, where we allow for the possibility of cost synergies in production. Specifically, the cost to multi-product supplier $i \in \mathbb{M}$ of supplying just product A is $\gamma^A c_i$ and of supplying just product B is $\gamma^B c_i$, where $\gamma^A, \gamma^B < 1$ and where γ^A and γ^B are known by the buyer. As a matter of notation, we set $\gamma_i^A = \gamma^A$ and $\gamma_i^B = \gamma^B$ for $i \in \mathbb{M}$ and $\gamma_i^A = \gamma_i^B \equiv 1$ for $i \in \mathbb{A} \cup \mathbb{B}$. Cost synergies are accommodated by letting $\gamma^A + \gamma^B > 1$, which implies that multi-product suppliers can produce both products at a cost that is less than the sum of the costs of producing each separately.

As in the single-product case, we model the merger of two suppliers of substitute products (both producing A , both producing B , or both producing A and B) by assuming that the merged entity has cost type equal to the minimum of the cost types of the merging suppliers. We model a merged entity that combines a supplier of product A with a supplier of product B as drawing its cost type from the distribution for the sum of the cost types

of the two merging suppliers.³

Analogously to the single-product case, we assume that market outcomes correspond to the allocation and payments of the optimal mechanism with the objective that is a weighted average of buyer surplus and social surplus, with weight $\beta \in \{0, 1\}$ on buyer surplus. Thus, focusing on the case with buyer power and using the convention that subscripts on virtual type functions index suppliers, when the type vector is \mathbf{c} ,⁴ quantities traded in the pre-merger market (and analogously for the post-merger market) are determined by the maximizer of the set

$$\{0\} \cup \{V - \Gamma_i(c_i)\}_{i \in \mathbb{M}} \cup \{v - \gamma_i^A \Gamma_i(c_i) - \gamma_j^B \Gamma_j(c_j)\}_{i \in \mathbb{M} \cup \mathbb{A}, j \in \mathbb{M} \cup \mathbb{B}, i \neq j}$$

as follows: If there exists $i \in \mathbb{M}$ such that $V - \Gamma_i(c_i)$ is a maximizer, then $q_i^A(\mathbf{c}) = q_i^B(\mathbf{c}) = 1$; otherwise, if there exist $i \in \mathbb{M} \cup \mathbb{A}$ and $j \in \mathbb{M} \cup \mathbb{B}$ with $i \neq j$ such that $v - \gamma_i^A \Gamma_i(c_i) - \gamma_j^B \Gamma_j(c_j)$ is a maximizer, then $q_i^A(\mathbf{c}) = q_j^B(\mathbf{c}) = 1$; otherwise, zero is a maximizer and there is no trade. The allocation rule is monotone. For the case without buyer power, virtual type functions are replaced by the identity function.

Given the allocation rule, expected buyer surplus is

$$E_{\mathbf{c}} \left[\begin{aligned} & \sum_{i \in \mathbb{M}} q_i^A(\mathbf{c}) q_i^B(\mathbf{c}) (V - \Gamma_i(c_i)) \\ & + \sum_{i \in \mathbb{M} \cup \mathbb{A}, j \in \mathbb{M} \cup \mathbb{B}, i \neq j} q_i^A(\mathbf{c}) q_j^B(\mathbf{c}) (v - \gamma_i^A \Gamma_i(c_i) - \gamma_j^B \Gamma_j(c_j)) \end{aligned} \right].$$

In the dominant strategy implementation for the multi-product setup, payments to suppliers are defined by multiple threshold cost types as described in Lemma 1.

Lemma 1. *There exist threshold cost types that define the dominant-strategy implementation of the optimal direct mechanism with objective*

$$\beta(\text{buyer surplus}) + (1 - \beta)(\text{social surplus}), \quad (1)$$

subject to incentive compatibility and individual rationality.

Proof of Lemma 1. Given \mathbf{c}_{-i} , possible threshold types for supplier i are $c_i^{AB,-}$, $c_i^{A,-}$, $c_i^{B,-}$, $c_i^{AB,A}$, $c_i^{AB,B}$, $c_i^{B,A}$, and $c_i^{A,B}$, where $c^{X,Y}$ is defined so that for cost types below $c_i^{X,Y}$

³For a merger that combines a single-product supplier with a multi-product supplier, various approaches can be considered, but we do not deal with this case.

⁴In the direct implementation, each agent i reports a type $c_i \in [\underline{c}, \bar{c}]$ to the buyer. As described in Lemma 1 below, one can structure transfers in the direct implementation so that reporting truthfully is a dominant strategy for every agent i of every possible cost type. Whether the optimal direct mechanism with multi-product firms permits a descending clock-auction implementation is left for future research.

and above the next lower threshold, supplier i supplies product X , and for cost types above $c_i^{X,Y}$ and below the next higher threshold, supplier i supplies product Y , where “ $-$ ” denotes the empty set.

For example, for the case with buyer power, if for a given \mathbf{c} , supplier i supplies both A and B , then

$$V - \Gamma_i(c_i) \geq \max \left\{ 0, \max_{\substack{j \in \mathbb{M} \\ j \neq i}} (V - \Gamma_j(c_j)), \max_{\substack{k \in \mathbb{M} \cup \mathbb{A} \\ j \in \mathbb{M} \cup \mathbb{B} \\ j \neq k}} (v - \gamma_k^A \Gamma_k(c_k) - \gamma_j^B \Gamma_j(c_j)) \right\}. \quad (2)$$

If the highest element in curly brackets on the right side of (2) does not involve supplier i , then there exists a cost type for supplier i , $c_i^{AB,-}$, such that if supplier i reports a cost less than $c_i^{AB,-}$, supplier i supplies A and B , but if supplier i reports a cost greater than $c_i^{AB,-}$, supplier i supplies nothing. The cost type $c_i^{AB,-}$ is the threshold type for supplier i between supplying both A and B and supplying nothing. However, if the highest element in curly brackets on the right side of (2) involves supplier i , then the threshold types must account for that. Fixing the cost types of suppliers other than i , as the cost type of supplier i varies, the set of products that supplier i supplies also varies. For example, it may be that for low cost types, supplier i supplies A and B , but for intermediate cost types, supplier i supplies only A , and for higher cost types, supplier i supplies nothing. In this case, the cost types defining the cutoffs between the regions in type space would be denoted $c_i^{AB,A}$ and $c_i^{A,-}$. Other threshold types are defined analogously. In all cases, the threshold types for supplier i depend only on the cost types of the other suppliers.

Given the cost vector for the suppliers, the identities of the trading suppliers, and the threshold types for the trading suppliers, payments in the dominant strategy implementation are as shown in Figure 1. As we show, the payments defined in Figure 1 correspond to the dominant-strategy implementation of the optimal mechanism for the multi-product setup.

To see that dominant strategy incentive compatibility is satisfied, suppose that suppliers other than i report truthfully. If supplier i does not trade when it reports truthfully, then all threshold types for supplier i are less than c_i , and so any report that results in supplier i trading gives supplier i a payment that is less than c_i , and so no deviation is profitable. If supplier i does trade when it reports truthfully, then a downward deviation $r_i < c_i$ only changes supplier i 's payoff if r_i is less than a type threshold that is less than c_i . For example, if supplier i is a multi-product supplier and if $r_i < c_i^{AB,A} < c_i < c_i^{A,-}$,

Threshold types	Supplier i 's type	Products i supplies	Payment to supplier i
$c_i^{AB,-}$	$c_i < c_i^{AB,-}$	A and B	$c_i^{AB,-}$
$c_i^{AB,A} < c_i^{A,-}$	$c_i < c_i^{AB,A}$	A and B	$c_i^{AB,A} + \gamma_i^A(c_i^{A,-} - c_i^{AB,A})$
	$c_i \in (c_i^{AB,A}, c_i^{A,-})$	A only	$\gamma_i^A c_i^{A,-}$
$c_i^{AB,B} < c_i^{B,-}$	$c_i < c_i^{AB,B}$	A and B	$c_i^{AB,A} + \gamma_i^B(c_i^{B,-} - c_i^{AB,B})$
	$c_i \in (c_i^{AB,B}, c_i^{B,-})$	B only	$\gamma_i^B c_i^{B,-}$
$c_i^{AB,A} < c_i^{A,B} < c_i^{B,-}$	$c_i < c_i^{AB,A}$	A and B	$c_i^{AB,A} + \gamma_i^A(c_i^{A,B} - c_i^{AB,A})$ $+ \gamma_i^B(c_i^{B,-} - c_i^{A,B})$
	$c_i \in (c_i^{AB,A}, c_i^{A,B})$	A only	$\gamma_i^A c_i^{A,B} + \gamma_i^B(c_i^{B,-} - c_i^{A,B})$
	$c_i \in (c_i^{A,B}, c_i^{B,-})$	B only	$\gamma_i^B c_i^{B,-}$
$c_i^{AB,B} < c_i^{B,A} < c_i^{A,-}$	$c_i < c_i^{AB,B}$	A and B	$c_i^{AB,B} + \gamma_i^B(c_i^{B,A} - c_i^{AB,B})$ $+ \gamma_i^A(c_i^{A,-} - c_i^{B,A})$
	$c_i \in (c_i^{AB,B}, c_i^{B,A})$	B only	$\gamma_i^B c_i^{B,A} + \gamma_i^A(c_i^{A,-} - c_i^{B,A})$
	$c_i \in (c_i^{B,A}, c_i^{A,-})$	A only	$\gamma_i^A c_i^{A,-}$

Figure 1: Threshold types in the multi-product setup with associated payments

then under truthful reporting supplier i supplies A , for a payoff of $\gamma_i^A c_i^{A,-} - \gamma_i^A c_i$, but under report r_i , supplier i supplies A and B , for a payoff of

$$\begin{aligned}
c_i^{AB,A} + \gamma_i^A(c_i^{A,-} - c_i^{AB,A}) - c_i &= (1 - \gamma_i^A)(c_i^{AB,A} - c_i) + \gamma_i^A c_i^{A,-} - \gamma_i^A c_i \\
&< \gamma_i^A c_i^{A,-} - \gamma_i^A c_i,
\end{aligned}$$

where the inequality uses $c_i^{AB,A} - c_i < 0$ and $\gamma_i^A < 1$ for a multi-product supplier, and so the deviation is not profitable. A similar analysis shows that no other downward deviation is profitable.

If supplier i reports $r_i > c_i$, then its payoff is only affected if r_i is greater than a type threshold that is greater than c_i . For example, consider a multi-product supplier i , with $c_i < c_i^{AB,A} < r_i < c_i^{A,-}$. Then under truthful reporting, supplier i supplies A and B for a payoff of $c_i^{AB,A} + \gamma_i^A(c_i^{A,-} - c_i^{AB,A}) - c_i$, but under report r_i , supplier i supplies only A for a payoff of $\gamma_i^A c_i^{A,-} - \gamma_i^A c_i$, which is less for any $c_i < c_i^{AB,A}$, and so the deviation is not profitable. Similarly, no other upward deviation is profitable. ■

Using the setup described above, one can analyze a merger in the multi-product environment.

Merger of suppliers of complements

Our framework also allows us to consider the merger of supplier 1 producing only product A with supplier 2 producing only product B , which is a merger of suppliers of complements. As mentioned, these complementary products could be two inputs used together by the buyer, or inputs in different geographic locations for a buyer that demands coverage for both locations, or vertically related products, such as a product and its distribution.

To illustrate effects, consider the case of $n = 2$. Let $\bar{\Gamma}$ be the virtual cost function for a supplier who draws its cost type from the distribution that is the convolution of G_1 and G_2 . Even if the virtual cost functions for suppliers 1 and 2 are bounded on $[\underline{c}, \bar{c}]$, $\bar{\Gamma}$ is necessarily unbounded on $[2\underline{c}, 2\bar{c}]$.⁵

If $v \geq 2\bar{c}$, then a buyer without power purchases both before and after the merger, paying $2\bar{c}$ in both cases. Thus, the buyer's quantity and payment are not affected by the merger, but it benefits from one-stop shopping if $V > v$.

Considering a buyer with power, if $v \geq \Gamma_1(\bar{c}) + \Gamma_2(\bar{c}) \geq 2\bar{c}$, then the buyer purchases before the merger and pays $2\bar{c}$. After the merger, the buyer only purchases if $\bar{\Gamma}(c_1 + c_2) \leq V$, in which case the buyer pays $\bar{\Gamma}^{-1}(V)$, which is less than $2\bar{c}$ because $V < \bar{\Gamma}(2\bar{c}) = \infty$. By revealed preference, the buyer's expected surplus following the merger is greater than if the buyer committed to always purchase from the merged entity at price $2\bar{c}$, which would be incentive compatible and generate the same surplus as in the case of no merger. Thus, with buyer power, the buyer's expected surplus increases as a result of a merger, and more so the greater is the buyer's value for one-stop shopping.

Thus, we have the following result:

Proposition 1. *Assuming $v \geq \Gamma_1(\bar{c}) + \Gamma_2(\bar{c})$, a merger of monopoly suppliers of complementary products increases a buyer's expected surplus if the buyer has buyer power or has a positive value for one-stop shopping, and the effect on expected buyer surplus is increasing in the value of one-stop shopping.*

This result is consistent with the usual intuition that the merger of suppliers of complementary products typically produces benefits for the buyer.

⁵The density of the convolution is $g_m(c) = \int_{\underline{c}}^{\bar{c}} g_1(x)g_2(c-x)dx$, and so $g_m(2\bar{c}) = \int_{\bar{c}}^{\bar{c}} g_1(x)g_2(2\bar{c}-x)dx = 0$, which follows because $g_2(2\bar{c} - x)$ is zero for all $x \in [\underline{c}, \bar{c}]$.

References

LOERTSCHER, S. AND L. M. MARX (forthcoming): “Merger Review for Markets with Buyer Power,” *Journal of Political Economy*.