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# Inducing Environmental Disclosures: A Dynamic Mechanism Design Approach

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This paper studies the design of voluntary disclosure regulations for a firm that faces a stochastic environmental hazard. The occurrence of such a hazard is known only to the firm. The regulator, if finding a hazard, collects a fine and mandates the firm to perform costly remediation that reduces the environmental damage. The regulator may inspect the firm at any time to uncover the hazard. However, because inspections are costly, the regulator also offers a reward to the firm for voluntarily disclosing the hazard. The reward corresponds to either a subsidy or a reduced fine, depending on whether it is positive or negative. Thus, the regulator needs to dynamically determine the reward and inspection policy that minimizes expected societal cost in the long run. We model this problem as a dynamic adverse selection problem with costly state verification in continuous time. Despite the complexity and generality of this setup, we show that the optimal regulation policy follows a very simple cyclic structure, which we fully characterize in closed form. Specifically, the regulator runs scheduled inspections periodically. After each inspection, the reward level decreases over time until a subsequent inspection takes place. If a hazard is not revealed, the reward level is reset to a high level, restarting the cycle. In contrast to the reward level, the mandated remediation level is constant over time. Nonetheless, when subsidies are not allowed in the industry, we show that the regulator should dynamically adjust this remediation level, which then acts as a substitute for a subsidy. Our analysis further reveals that optimal inspection frequency increases not only when the inspection accuracy decreases, but also when the penalty for not disclosing the hazard increases.

*Keywords*: dynamic mechanism design; optimal control; asymmetric information; environmental regulation; voluntary disclosure.
*Subject classifications*: dynamic programming/optimal control; games/group/decisions; environment.
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## 1. Introduction

Environmental inspections (also called *audits*) are indispensable regulatory instruments used to expose environmental noncompliance in private industries (Cohen 1999). However, inspections are costly and sometimes ineffective, especially when hazards result from random events beyond firms' control, such as equipment malfunctions or natural causes (Beavis and Walker 1983, Kim 2015). To improve the efficacy of their investigative tools and to limit the use of inspections, regulatory agencies have instituted voluntary disclosure programs, which reward companies for self-reporting environmental hazards. The Audit Policy of the U.S. Environmental Protection Agency (EPA), for instance, eliminates or significantly reduces penalties for firms that self-report hazards (EPA 2000).

Nonetheless, designing an efficient and effective disclosure program remains a challenging task, as evidenced by the failure of previous voluntary programs. For instance, the EPA had to terminate one of its key initiatives (Shiffman et al. 2009); see also Rivera et al. (2006) or King and Lenox (2000). Indeed, although the notion that incentives for self-reporting can serve as cheaper substitutes for inspections may be well established (Kaplow and Shavell 1994, Malik 1993), how to efficiently operationalize this principle remains poorly understood.

The purpose of this paper, therefore, is to provide theoretical guidelines for designing efficient voluntary disclosure programs that combine reward and inspection instruments to detect an environmental hazard that occurs randomly beyond the firm's control. The regulator is not immediately aware of such a hazard, and the firm can choose to conceal it to evade or delay costly regulatory consequences. This corresponds, for instance, to the leakage of a toxic substance. Because of the nature of this setting in which information is hidden (i.e., adverse selection), we formulate the problem of setting reward levels for self-reporting over time using a dynamic mechanism design framework. We incorporate inspection instruments by enriching the model with costly state verifications.

The main contribution of this paper is to demonstrate that, despite the complex strategic and dynamic interactions between the regulator and the firm, and the generality of the regulation policies under consideration, the optimal policy takes on a simple cyclic structure. The policy is characterized in closed form and is easy to compute and implement.

Specifically, we consider a regulator who seeks to discover whether a random environmental hazard has occurred at a firm. If a hazard that imposes adverse consequences on society is found, the regulator mandates that the firm incurs a remediation (or repair) cost to remove the hazard and pays compensations for environmental damages. These costs, however, encourages the firm to conceal the hazard. The firm's incentives, therefore, are not aligned with those of the regulator.

To determine whether a hazard has occurred, the regulator can decide at any time (possibly randomly) to inspect the firm and inflict a nondisclosure penalty if a hazard is detected. Inspections, however, are costly and may be imperfect.

To limit the use of inspections, the regulator can provide the firm with incentives for voluntarily disclosing the hazard, in the form of a monetary reward or an adjusted remediation level. The reward corresponds to either a subsidy or a reduced fine, depending on whether the reward amount is positive or negative. Varying this amount according to the time the hazard is disclosed adjusts the firm's incentive on when to report the hazard. Similarly, the regulator can also adjust the level of mandated remediation levels and the associated costs over time.

The regulator's objective is to minimize expected societal cost in the long run, that is, to maximize social welfare. Following the regulation economics literature, the reward for voluntary disclosure generates frictions to the economy in the form of a shadow cost of public funds incurred by the regulator (Laffont and Tirole 1993, Assumption 8 on page 38). That is, each dollar spent by the regulator is raised through distortionary taxes, which are costly to society.

In the presence of these frictions, and when inspections are perfect (i.e., inspections always reveal an existing hazard), we show that the optimal policy has a simple deterministic cyclic structure. The regulator starts the cycle by offering a reward and mandates a remediation level when the hazard is self-reported. The reward amount decreases over time throughout the cycle, until an inspection takes place. If that inspection does not reveal a hazard, the reward amount is reset to its initial level, restarting the cycle. The mandated remediation level, by contrast, remains constant over time. Such an optimal policy proceeds as long as no hazard is revealed through disclosure or inspection. (Figures 3 and 4 in §5 visualize this optimal policy.)

When inspections are imperfect (i.e., the probability of finding a hazard is less than one), the optimal policy follows a similar cyclic structure, but differs in two aspects (Figure 6 of §6). First, the first inspection period takes longer time than the subsequent ones, and the policy starts with a reward level that is higher than the starting level of the subsequent cycles. Second, each cycle ends with a reward level that is higher than that in the perfect-inspection case.

Taken together, these results suggest that the regulator of a voluntary disclosure program should not randomly inspect the firm but rather schedule the inspections. This is in contrast with the more common practice where the regulator relies only on inspections to reveal a hazard. Indeed, if the regulator were to induce voluntary disclosure without rewards, we show that optimal inspection time epochs are randomly distributed according to a Poisson process.

Our approach yields additional insights into the frequency of inspections. First, the inspection cycle shrinks as the accuracy of inspections decreases (Figure 7 of §6). Also, and perhaps more surprisingly, the nondisclosure penalty complements inspection frequency: inspections are more frequent for higher nondisclosure penalties (Figure 5 of §5).

Thus far, all our results hold when the shadow cost of public funds is positive. When the shadow cost is null, the use of subsidies is "free of charge" to the regulator, who can then offer a very large subsidy and induce the firm to reveal the hazard at no cost.[1] Nonetheless, offering subsidies in the event of a hazard is sometimes criticized for its perverse effect on the environment (EPA 2001). Therefore, we also consider restricting rewards to nonpositive values.

When the shadow cost of public funds is null, and rewards are nonpositive, the optimal policy retains a cyclic structure. In sharp contrast to the previous case, however, the optimal policy sometimes dynamically adjusts the level of the mandated remediation. Specifically, at the beginning of a cycle, instead of offering a subsidy (positive reward), the regulator lowers the remediation level, which reduces the cost to the firm. Hence, the remediation level is used as a substitute for subsidies. And to emulate a decrease in the subsidy amount, the regulator increases the remediation level accordingly over time. Once the remediation level reaches its maximum, the regulator starts decreasing the reward amount from zero. Therefore, the reward is always nonpositive and corresponds to a reduced fine. As before, the reward amount continues to decrease until the next inspection takes place, which restarts the cycle (Figure 8 of §7).

We derive our results by developing a recursive formulation of the dynamic mechanism design problem, which ultimately is transformed to an optimal control problem of a piecewise deterministic process (Davis 1984). We then develop optimality conditions in the spirit of *quasi-variational inequalities* (Bensoussan and Lions 1982) and verify that the cost-to-go functions of the proposed policies satisfy these optimality conditions. Despite the technical challenges, our approach yields a closed-form characterization of the optimal policy. This allows us to conduct a series of sensitivity analysis and facilitates the policy interpretation of our findings.

Our results speak to the recent literature on disclosure and inspection policies for unintentional environmental hazards, whose occurrence is beyond the firm's control despite its best intentions. Kim (2015), who focuses on a game between the regulator and the firm, is particularly relevant to our work and motivated our model with perfect inspections. In the framework of mechanism design (i.e., design of the game), we consider more general inspection schedules than the deterministic and exponentially distributed interinspection times studied by Kim (2015). Furthermore, we allow the regulator to offer disclosure rewards, while inspections are the only available regulatory instrument in Kim (2015). In our setup, this implies that scheduled inspections are always optimal. However, differences in model assumptions between the two papers limit direct comparisons. In particular, Kim's (2015) model allows hazards to be removed without the regulator's knowledge. By contrast, we assume that the hazard would persist without regulatory intervention.

Our paper is also related to recent studies of firms' strategic decision-making regarding whether to identify/assess or even conceal their environmental impacts. For instance, Kalkanci et al. (2012) show that requiring a firm to disclose these impacts can deter the firm from assessing them. Plambeck and Taylor (Forthcoming) also analyze how an environmentally conscientious firm can encourage suppliers to report and thus limit their social and environmental violations. Even though we do not endogenize the firm's effort to conceal or measure hazards as this stream of work does, we take into account the firm's ability to deceive regulators by assuming that inspections may not be perfectly accurate.

There is a vast literature on the economics of compliance and law enforcement dating back to Becker (1968). Cohen (1999) reviews the literature dedicated to the environmental economics of compliance and distinguishes intentional violations from unintentional ones (see also Cropper and Oates 1992, Beavis and Walker 1983, Malik 1993, Kim 2015). In this line of inquiry, we follow Kim (2015) and others, who focus on random hazards that are beyond the firm's control. This differs from the self-policing economics literature, where the focus is not on self-disclosure but rather on self-regulation (see Toffel and Short 2011, and references therein). This distinction is also reflected in our modeling approach, which belongs to the literature on adverse selection (i.e., hidden information), as opposed to model hazard (i.e., hidden action).

From a methodological perspective, our paper borrows technical apparatus from and contributes to the growing literature on dynamic mechanism design. The recursive representation of repeated strategic interactions between agents is pioneered by Spear and Srivastava (1987) and Abreu et al. (1990), and has been applied to solve several discrete-time long-term contracting problems (e.g., Ljungqvist and Sargent 2004, Ch. 19 & 20, and references therein). Recently, an emerging stream of research in economics and finance (e.g., Sannikov 2008) has extended this approach to a continuous-time framework, albeit primarily for moral hazard rather than for adverse selection problems like ours. In particular, Biais et al. (2010) have examined the dynamic moral hazard problem in the absence of inspections, where an insurance company induces the firm to exert effort to mitigate environmental risks.

Our recursive formulation leads to an optimal stochastic control problem of a *piecewise deterministic process* (PDP) with both intensive and impulsive controls. The notion of PDPs was first formalized by Davis (1984) as a class of stochastic models that cannot be captured by diffusion processes. Several studies in the operations research and management science literature rely on this modeling framework (e.g., Davis et al. 1987). Optimal control problems such as these, however, are often intractable. Our closed-form solutions, therefore, yield a new class of analytically solvable yet nontrivial problems for controlled PDPs.

In the operations management literature, dynamic mechanism design approaches have recently been applied to long-term contracts to induce efforts from competing suppliers (Li et al. 2013) and to efficiently manage inventory systems (Lobel and Xiao 2014). Given our focus on mechanisms involving costly inspections, our paper is more closely related to Ravikumar and Zhang (2012), which incorporates "costly state verification" initiated by Townsend (1979) into a dynamic setting. They consider the problem of a tax authority in designing efficient policies to induce taxpayers to truthfully report an increase in their income and offers a partial characterization of the corresponding optimal policy. Tax regulation problems, however, require different modelling assumptions than ours, especially those relative to auditing accuracy and taxpayer risk aversion. In particular, our context also allows us to obtain a complete analytical characterization of the optimal regulation policy in closed-form.

Finally, our work relates to the vast literature on system reliability (for comprehensive reviews of the research in this area, see Barlow and Proschan 1965, Parmigiani 1993). In particular, Sengupta (1982) considers a single decision maker's problem of using only inaccurate inspections to observe a hazard that occurs according to an exponential distribution. Our paper, therefore, can be regarded as a generalisation of this model to a strategic setting with asymmetric information.

We first present the model in §2 and introduce the recursive formulation of the problem in §3. Section 4 contains the study of two simple benchmark policies. We then study the case where inspections are perfectly accurate in §5. The results are then generalized to imperfect accuracy cases in §6. In §7, we further examine the case where the shadow cost of public funds is null and only fines are allowed. Finally, §8 concludes the paper. The appendix collects the proofs for §5, whereas all other technical details and proofs are relegated to an electronic companion (available as supplemental material at http://www.dx.doi.org/10.1287/opre.2016.1476).

## 2. Model

A regulated firm operating in a continuous infinite time horizon $t \in [0, \infty)$ is subject to an environmental hazard (e.g., a leakage of a toxic substance), which occurs exogenously to the firm's operation at a random time $T$. Time $T$ follows an exponential distribution with rate $\lambda > 0$. As in much of the literature in this line of research (e.g., Kim 2015), we assume that $\lambda$ is known by both the regulator and the firm. This captures the reality that production processes involving environmental risks are often certified by the regulator at the onset of their operations, and therefore, the hazard rate $\lambda$ is publicly observable and regulated (see, for example, EPA 1995 for requirements on U.S.-operated underground storage systems). The occurrence of the hazard, however, is the firm's private information. The time discount rate of the regulator and the firm is denoted by $\theta$, which also measures the benefit of delaying cash outflow.

The environmental damage costs the society an expected present value $\bar{c}$ at the time of the hazard. This damage can be mitigated by the firm through a costly remediation (or repair) process such as a cleanup (see, for instance, Innes 1999a, b or the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) and the Resource Conservation and Recovery Act (RCRA)[2]). Specifically, the firm can incur a cost $R \in [0, \bar{r}]$ to reduce the environmental damage to

$$\bar{c} - cR/\bar{r}. \tag{1}$$

Thus, a *full remediation* reduces the environmental damage to $\bar{c} - c \geqslant 0$ at a maximum cost $\bar{r}$ to the firm.[3]

Because the regulator bears the environmental costs, the regulator seeks to uncover the occurrence of the hazard so as to mandate remediation levels. Facing the possibility to repair or compensate for the environmental damage, the firm has an incentive to conceal the hazard's occurrence. Nonetheless, the total compensation and remediation costs mandated by the regulator cannot exceed the firm's limited financial liability, detonated by $F$ (e.g., EPA 1986, 2015; Cohen 1987).

The regulator can only discover the occurrence of a hazard through either (i) inspections, or (ii) a voluntary disclosure by the firm. Following the standard assumption of the disclosure literature (e.g., Verrecchia 2001, Kalkanci et al. 2012, Toffel and Short 2011), the regulator is able to verify the firm's claim that a hazard has occurred.[4] Depending on which of the two discovery channels is used, the following regulatory responses occur.

(i) Upon the detection of a hazard by an inspection, the regulator simply penalizes the firm with the highest possible fine, which equals its liability $F$.[5]

(ii) If the hazard is disclosed by the firm at time $t$, the regulator mandates a remediation level $R_t$ and offers a monetary *reward* $S_t$ to the firm, such that $R_t - S_t \leqslant F$.

Reward $S_t$ may represent either a subsidy or a fine, depending on whether it is positive or negative. Because $S_t \geqslant -F$, a negative reward for disclosing the hazard corresponds to a *reduced fine* relative to the highest possible fine for getting caught by an inspection.

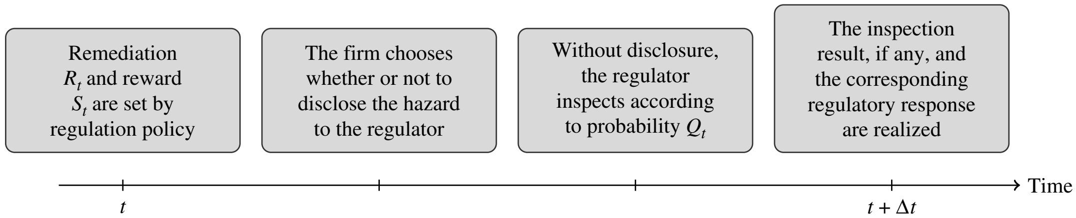
Each inspection costs the regulator $K$. Inspections may be imperfect, in the sense that the probability that an inspection reveals a hazard which has already occurred is $p > 0$, independent from previous inspections. We refer to probability $p$ as the *accuracy* of inspection. We also assume that inspections never erroneously reveal a hazard that has not occurred. Furthermore, inspections are conditionally independent given the true state of the firm (hazardous or not).

We allow the regulator to use very general inspection policies that combine "impulsive" and/or "intensive" inspections. An impulsive inspection is an inspection that takes place at an exact time epoch $t$ with probability $q_t^m \in [0, 1]$, where we require only finitely many impulsive inspection time epochs with $q_t^m > 0$ within any finite time interval. By contrast, an intensive inspection occurs in time interval $[t, t + \Delta t)$ with probability $q_t^n \Delta t + o(\Delta t)$ where the inspection rate $q_t^n \geqslant 0$. Denote $Q_t := (q_t^m, q_t^n)$ to represent an *inspection policy*, which specifies the chance of inspection contingent upon realizations of earlier uncertainties. (A rigorous definition is provided in Appendix A.) For instance, $Q_t = (0, q)$ for $t \geqslant 0$ corresponds to a *random* inspection policy in which inspections follow a Poisson process with rate $q \geqslant 0$. A *scheduled* inspection policy that deterministically inspects at time epochs $t_1, t_2, \ldots$ can be expressed as $q_t^n = 0$ for all $t$ while $q_t^m = 1$ when $t = t_i$ for $i = 1, 2, \ldots,$ and $q_t^m = 0$ otherwise. More generally, we allow inspection policies to take any contingency forms. For example, under an admissible inspection policy, the regulator may run a random impulsive inspection at time $t$ (with probability $q_t^m \in (0, 1)$), followed by intensive inspections, the rate of which depends on whether the impulsive inspection actually occurred at time $t$.

At the beginning of the time horizon ($t = 0$), the regulator announces a *regulation policy*, which includes a remediation policy $R_t$, a reward policy $S_t$, and an inspection policy $Q_t$ for all $t$, and then commits to this policy throughout the entire time horizon. At arbitrary time $t$, the following sequence of events takes place, illustrated in Figure 1. First, given remediation level $R_t$ and reward level $S_t$, the firm, if having observed the occurrence of the hazard, decides whether or not to report the hazard. As long as the hazard has not been disclosed, the regulator inspects the firm with probability $q_t^m + q_t^n \Delta t + o(\Delta t)$. Once the hazard has been disclosed or detected, strategic interactions between the firm and the regulator end with the regulatory responses specified above. The corresponding relevant history, or state of the system, is the set of inspection epochs during time period $[0, t)$,[6] which is denoted by $\mathcal{I}_t$, with convention $\mathcal{I}_0 = \varnothing$. We denote $\mathcal{R} := (R_t, S_t, Q_t)_{t \in [0,\infty)}$ to represent the regulator's overall regulation policy adaptive to $\mathcal{I}_t$.[7]

The strategic interaction between the regulator and the firm described above naturally gives rise to a game, where the regulator commits to a prespecified regulation policy, and, in response, the firm decides when to report an existing hazard. Any reporting strategy that postpones hazard disclosure to

**Figure 1.** Sequence of events at any moment in time $t$ ($\Delta t \approx 0$).




a time after its occurrence is feasible for the firm. In this paper, we take a mechanism design approach to formulate and solve the regulator's problem of designing its regulation policy. In Lemma A.1 of Appendix A, we establish a version of the Revelation Principle (Dasgupta et al. 1979, Myerson 1979, Townsend 1988) pertaining to our setting. Specifically, Lemma A.1 states that for any regulation policy under which the firm chooses to delay the hazard disclosure, there exists an equivalent policy that induces the firm to reveal the hazard immediately without increasing the regulator's cost. Hence, the regulator can restrict the search for the optimal regulation policy within the set of *direct revelation policies*—regulation policies that induce the firm to disclose the hazard without delay.

A regulatory policy $\mathcal{R}$ induces the firm to immediately report the occurrence of a hazard at any time $t$ if and only if the firm's cost upon immediate disclosure $R_t - S_t$ is no greater than the firm's expected total discounted cost of postponing the report to some later time $t' > t$. Namely, we focus on *direct regulatory policies* $\mathcal{R}$ that satisfy the following *incentive compatibility* (IC) constraint:

$$R_t - S_t \leqslant \mathbb{E}[D_t^{\mathcal{R}}(t') \mid \mathcal{F}_t], \quad \text{for any } \mathcal{F}_t \text{ and } t' \geqslant t, \tag{2}$$

where random variable $D_t^{\mathcal{R}}(t')$ represents the firm's expected discounted cost of delaying the disclosure of a hazard occurring at time $t$ to $t'$, subject to regulation policy $\mathcal{R}$. The explicit formula for $D_t^{\mathcal{R}}(t')$ is provided in Lemma A.1 of Appendix A.

Under a direct regulatory policy $\mathcal{R}$, the firm promptly discloses the hazard at time $T$, and incurs an expected discounted cost of

$$\mathbb{E}[e^{-\theta T}(R_T - S_T)]. \tag{3}$$

By (1), the regulator bears an expected discounted environmental cost of

$$\mathbb{E}[e^{-\theta T}(\bar{c} - cR_T/\bar{r})] = \frac{\lambda}{\lambda+\theta}\bar{c} - \mathbb{E}[e^{-\theta T} cR_T/\bar{r}]. \tag{4}$$

In addition, the regulator incurs regulatory costs because of reward payments ($S_T$) and inspections ($K$ per inspection). Following a large body of literature on regulation economics (e.g., Baron and Myerson 1982, Laffont and Tirole 1993), public economics (e.g., Dahlby 2008), and also environmental regulations (e.g., Boyer and Laffont 1999, Lyon and Maxwell 2003), we assume the regulatory cost is $1+\alpha$ times the regulatory payment using public funds, with $\alpha \geqslant 0$.[8] This corresponds to a shadow cost of applying public funds, and captures economic inefficiencies that the regulation creates, for instance, by raising distortionary taxes. Therefore, the society at large (excluding the firm) incurs a total expected discounted regulatory cost of

$$(1+\alpha)\,\mathbb{E}\Bigg[e^{-\theta T} S_T + K \sum_{\tau_i \in \mathcal{F}_T^Q} e^{-\theta \tau_i}\Bigg], \tag{5}$$

where $\mathcal{F}_T^Q$ is the set of inspection time epochs by time $T$ following inspection policy $Q$ as part of the regulation policy $\mathcal{R}$.

As a social welfare maximizer (or cost minimizer), the regulator accounts for the firm's cost (3), the environmental cost (4), and the society's regulatory cost (5). Thus, the regulator solves the following minimization problem

$$C^\star := \frac{\lambda}{\lambda+\theta}\bar{c} + \min_{\mathcal{R}} \mathbb{E}\Bigg[e^{-\theta T}[(1-c/\bar{r})R_T + \alpha S_T] + k \sum_{\tau_i \in \mathcal{F}_T^Q} e^{-\theta \tau_i}\Bigg], \quad \text{subject to (2)}, \tag{6}$$

where $k := (1+\alpha)K$. We denote the optimal regulation policy as $\mathcal{R}^\star = (R_t^\star, S_t^\star, Q_t^\star)_{t \in [0,\infty)}$.

The first fixed term in $C^\star$ corresponds to the discounted environmental cost to society, when the firm is not regulated. Because the hazard occurs at time $T$, the total environmental cost $\bar{c}$ is discounted by the discount factor $\lambda/(\lambda+\theta)$. The second term of $C^\star$ is then the benefit that the regulation brings about, which needs to be nonpositive. In particular, the optimal regulation does not depend on cost $\bar{c}$, but rather on the maximum possible environmental damage reduction $c$.

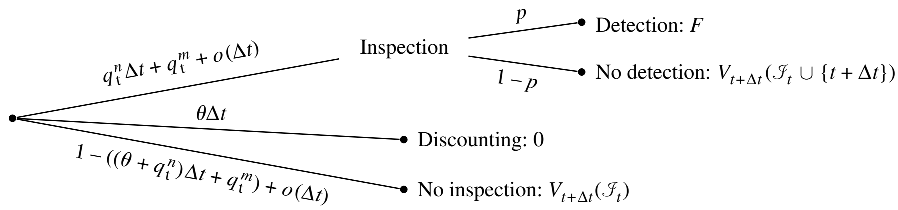
## 3. Recursive Formulation

In this section, we present recursive formulations of the firm's problem (the IC constraint (2)), as well as the regulator's problem (6), which yields the optimality condition.

### 3.1. Recursive Formulation of IC Constraint

The IC constraint (2), in its original form, is an object of exploding complexity and lacks tractable structure. To proceed with our analysis, we establish its recursive representation, which explicitly captures the firm's dynamic

**Figure 2.** Probability tree for the firm at $t+\Delta t$, after the hazard has occurred.




disclosure strategies. We first provide a heuristic argument based on a discrete time dynamic programming approximation to build intuition, and conclude the subsection with the formal result.

At time $t$, assuming that a hazard has already occurred, the firm faces an optimal stopping problem. Specifically, given state $\mathcal{I}_t$ (the set of previous inspection times), denote $V_t(\mathcal{I}_t)$ to represent the firm's optimal cost-to-go function (in current value). The firm chooses between postponing the decision to $t+\Delta t$ and disclosing now, which implies a cost:

$$W_t := R_t - S_t. \qquad (7)$$

The firm faces several uncertainties by time $t+\Delta t$, as illustrated in Figure 2. First, an inspection is conducted with probability $q_t^n \Delta t + q_t^m + o(\Delta t)$. If the inspection takes place, the hazard is detected with probability $p$, which inflicts a cost $F$ on the firm. The firm eludes detection with probability $1-p$ and moves to time $t+\Delta t$. This yields cost-to-go function $V_{t+\Delta t}(\mathcal{I}_t \cup \{t+\Delta t\})$, where the state (the set of inspection epochs) is updated to include the new inspection time $t+\Delta t$. This process is terminated by discounting with probability $\theta\Delta t$, resulting in a cost of 0. If neither inspection nor discounting occurs, which happens with probability $1-((\theta+q_t^n)\Delta t+q_t^m)+o(\Delta t)$, the firm's cost-to-go function becomes $V_{t+\Delta t}(\mathcal{I}_t)$, and the inspection set remains unchanged with $\mathcal{I}_{t+\Delta t}=\mathcal{I}_t$. Overall, the firm's Bellman equation is

$$\begin{aligned} V_t(\mathcal{I}_t) = \min\{ & W_t, (q_t^n \Delta t + q_t^m + o(\Delta t)) \\ & \cdot (pF + (1-p)V_{t+\Delta t}(\mathcal{I}_t \cup \{t+\Delta t\})) \\ & + (1-((\theta+q_t^n)\Delta t + q_t^m) + o(\Delta t))V_{t+\Delta t}(\mathcal{I}_t)\}. \end{aligned}$$

Incentive compatibility requires that the firm is better off disclosing at time $t$ than postponing the decision to $t+\Delta t$. This means that $V_t(\mathcal{I}_t)=W_t$ for any time $t$ and inspection set $\mathcal{I}_t$, which further implies that $V_{t+\Delta t}(\mathcal{I}_t)=W_{t+\Delta t}$ if no additional inspection takes place. Should an inspection occur, on the other hand, the state space changes from $\mathcal{I}_t$ to $\mathcal{I}_t \cup \{t+\Delta t\}$, and we introduce notation $W_{t+\Delta t}^I$ to represent the firm's expected total cost immediately after the inspection. We also have $V_{t+\Delta t}(\mathcal{I}_t \cup \{t+\Delta t\}) = W_{t+\Delta t}^I$ because of the IC constraint. Overall, incentive compatibility implies that the second term in the previous Bellman equation dominates the first term, which, after some straightforward algebraic manipulation, yields

$$\begin{aligned} W_t \leqslant\ & W_{t+\Delta t} + \Delta t[q_t^n(pF+(1-p)W_{t+\Delta t}^I - W_{t+\Delta t}) - \theta W_{t+\Delta t}] \\ & + q_t^m(pF+(1-p)W_{t+\Delta}^I - W_{t+\Delta t}) + o(\Delta t). \end{aligned} \qquad (8)$$

The limit of (8) as $\Delta t$ approaches zero depends on whether $q_t^m$ is positive or null. When $q_t^m > 0$, condition (8) reduces to

$$W_t \leqslant (1-q_t^m)W_{t+} + q_t^m[pF+(1-p)W_{t+}^I], \quad \text{if } q_t^m > 0. \qquad (9)$$

If, otherwise, $q_t^m = 0$, condition (8) is satisfied either when $W_{t+}$ is strictly larger than $W_t$ (an upward jump of $W_t$ at time $t$), or when the derivative of $W_t$ is bounded from below, which leads to

$$W_t < W_{t+}, \quad \text{or} \quad \frac{dW_t}{dt} \geqslant \theta W_t - q_t^n[pF+(1-p)W_{t+}^I - W_t], \quad \text{if } q_t^m = 0. \qquad (10)$$

(To see this, first subtract $W_{t+\Delta t}$ and then divide by $\Delta t$ on both sides of (8), and finally take limit $\Delta t \to 0$.)

The following formal result, which is based on Dynkin's formula for piecewise deterministic processes (e.g., Davis 1993), establishes the equivalence between the instantaneous IC constraints (9), and (10) and the original IC constraint (2).

Lemma 1. *The IC constraint (2) holds, if and only if $W_t \leqslant F$ is a left-continuous piecewise deterministic process whose evolution between any two consecutive inspections satisfies (9), and (10), and $W_t$ is reset to $W_{t+}^I$ immediately after an inspection at time $t$.*

It is worth pointing out that the regulator can always introduce instantaneous upward jumps in the firm's disclosure cost trajectory, $W_t$, without violating the IC constraint or incurring any cost. Intuitively, this is because an upward jump of $W_t$ in the next moment only strengthens the incentive for the firm to report in the current moment. By contrast, disclosure cost $W_t$ can only jump downward through an impulsive inspection.

## 3.2. Optimality Condition

To ensure its applicability to different extensions of our base model, we develop the optimality condition for a general set of feasible regulatory responses $\mathcal{L} := \{(R,S)\colon R \in [0,\bar{r}], S \leqslant \bar{B}, R-S \leqslant F\}$ with $0 \leqslant \bar{B} \leqslant \infty$. Our base model corresponds to $\bar{B}=\infty$, whereas our extension in §7 corresponds to $\bar{B}=0$. We first claim that the regulator's dynamic optimization uses $W_t$ as the state variable that summarizes the entire history $\mathcal{F}_t$. This way of reducing the dimension of the state space in the dynamic mechanism design problem is pioneered by Spear and Srivastava (1987) and Abreu et al. (1990).

Lemma 2. *The regulator's optimal cost-to-go in time $t$ value starting from $W_t = w$ is given by the following time-homogeneous function.*

$$C^\star(w) := \min_{(W_t, R_t, Q_t)_{t\geqslant 0}} \mathbb{E}\left[\int_0^\infty e^{-(\lambda+\theta)t}(\lambda(\beta R_t - \alpha W_t) + kq_t^n)\,dt + k\sum_{q_t^m>0} e^{-(\lambda+\theta)t} q_t^m \mid W_0 = w\right] \quad (11)$$

$$\text{subject to} \quad (9), (10), \text{ and } (R_t, R_t - W_t) \in \mathcal{L}, \quad \text{for all } t \geqslant 0,$$

*where $\beta := 1+\alpha - c/\bar{r}$. In particular, the regulator's optimal cost is $C^\star := (\lambda/(\lambda+\theta))\bar{c} + \min_{w\leqslant F} C^\star(w)$.*

As in the previous section, we first provide a heuristic argument for the recursive formulation of the regulator's problem before establishing a formal result on the optimality conditions.

At time $t$ when $W_t = w$, the regulator first decides whether to conduct an impulsive inspection or an upward jump of $W_t$. If neither is used (i.e., $q^m = 0$ and $W_{t+} = W_t$), the regulator then chooses the remediation level $r$ and inspection intensity $q^n$ during the next $\Delta t$, as well as the firm's disclosure cost $w_+^I$ right after an inspection during $\Delta t$.[9] The regulator's optimization problem (11) during this $\Delta t$ becomes

$$\min_{\substack{(r,r-w)\in\mathcal{L},\\ w_+^I \leqslant F,\, q^n\geqslant 0}} \{\lambda\Delta t(\beta r - \alpha w) + q^n \Delta t(k + C^\star(s_+^I)) + (1-(\theta+\lambda+q^n)\Delta t)C^\star(W_{t+\Delta t}) + o(\Delta t)\}$$
$$= C^\star(w) + [\mathcal{N}C^\star(w) - \lambda\alpha w - (\theta+\lambda)C^\star(w) +]\Delta t + o(\Delta t),$$

where operator $\mathcal{N}$ is defined as

$$\mathcal{N}C^\star(w) := \min_{\substack{(r,r-w)\in\mathcal{L},\\ w_+^I\leqslant F,\, q^n,\, z\geqslant 0}} \lambda\beta r + q^n[k + C^\star(w_+^I) - C^\star(w)] + \{\theta w - q^n[pF + (1-p)w_+^I - w] + z\}\frac{dC^\star(w)}{dw}. \quad (12)$$

(The equality follows from the first-order Taylor expansion of $C^\star(W_{t+\Delta t})$ at state $W_t = w$, assuming $W_{t+\Delta t} - W_t = O(\Delta t)$, and IC constraint (10), with slack $z = dW_t/dt - \theta w + q^n[pF + (1-p)w_+^I - w] \geqslant 0$.)

If impulsive inspection is used ($q^m > 0$), the regulator's optimization over decisions $q^m$, $w_+^I$ and $w_+ := W_{t+}$ can be expressed as the following recursive formulation:

$$\mathcal{M}C^\star(w) := \min_{\substack{w_+,\, w_+^I\leqslant F,\\ q^m\in[0,1]}} \{q^m(k + C^\star(w_+^I)) + (1-q^m)C^\star(w_+)\} \quad (13)$$

$$\text{subject to} \quad w \leqslant (1-q^m)w_+ + q^m[pF + (1-p)w_+^I],$$

where IC constraint (9) is enforced in the above constraint. When impulsive inspection $q^m > 0$, it dominates any bounded intensive control $q^n$. Therefore, $q^n$ is not a decision variable in this case. Also note that we still allow the feasible set of $q^m$ to start from 0. When $q^m = 0$, the functional operator $\mathcal{M}$ reduces to a simple upward adjustment of the firm's disclosure cost level $s_+ \geqslant s$.

Overall, the Bellman equation of the regulator's problem is

$$C^\star(w) = \min\{\mathcal{M}C^\star(w), C^\star(w) + [\mathcal{N}C^\star(w) - \lambda\alpha w - (\theta+\lambda)C^\star(w)]\Delta t + o(\Delta t)\},$$

where the first and second terms correspond to $q^m > 0$ and $q^m = 0$, respectively. By rearranging terms and letting $\Delta t$ go to zero, this further leads to the following optimality condition in the form of *quasi-variational inequalities* (Bensoussan and Lions 1982):

$$\min\{\mathcal{M}C^\star(w) - C^\star(w), \mathcal{N}C^\star(w) - \lambda\alpha w - (\theta+\lambda)C^\star(w)\} = 0. \quad (14)$$

The following Verification Theorem rigorously establishes the connection between the previous recursive formulation and the original problem.

Theorem 1 (Verification Theorem). *Given the feasible set of regulatory responses $\mathcal{L}$, if $C(w)$ is a bounded, nondecreasing, continuous, and differentiable function on $(-\infty, F]$ that satisfies*

$$\begin{aligned} &\mathcal{M}C(w) - C(w) \geqslant 0,\\ &\mathcal{N}C(w) - \lambda\alpha w - (\theta+\lambda)C(w) \geqslant 0, \quad \text{for } w \leqslant F, \end{aligned} \quad (15)$$

*where functional operators $\mathcal{N}$ and $\mathcal{M}$ are defined in (12) and (13), respectively, then $C(w)$ is a lower bound of the optimal value-to-go function $C^\star(w)$ defined in (11).*

Note that Theorem 1 only presents conditions for a lower bound of the optimal value function $C^\star(w)$. This is because condition (15) does not require that at least one of the two inequalities holds at equality, as in (14). Our solution strategy in the subsequent sections is to propose an admissible policy and compute its cost-to-go function, which bounds the optimal cost-to-go from above. We then demonstrate that the cost-to-go function of our proposed policy also satisfies (15), thereby also bounds the optimal cost-to-go from below.

## 4. Benchmark Policies and Preliminary Insights

Before presenting the optimal regulation policy in §5, we first introduce two simple benchmark policies that help reveal important insights into the optimal policy. These two simple benchmark policies rely on using only one regulatory instrument, either the rewards or the inspections, to uncover the hazard.

First, note that because of time discounting, the firm has an economic incentive to postpone the hazard's disclosure as long as the corresponding cost is positive and the regulator does not inspect the firm. To eliminate such incentive, the regulator needs to dynamically adjust the regulatory responses so that the firm's total cost for disclosing the hazard immediately, $R_t - S_t$, is no greater than the present value of the firm's cost from delaying the disclosure by $\Delta t$, that is,

$$R_t - S_t \leqslant e^{-\theta\Delta t}(R_{t+\Delta t} - S_{t+\Delta t}). \quad (16)$$

In particular, when the regulator always fully covers the firm's remediation cost ($S_t \equiv R_t$), the firm does not incur any cost for disclosing the hazard and hence reports the hazard immediately. Thus, the regulator should never offer a subsidy amount which is higher than the remediation cost ($S_t > R_t$). More crucially, when the regulator does not fully cover the remediation costs ($S_t < R_t$), the firm's total payment $R_t - S_t$ needs to increase over time so as to offset any incentive of delaying the hazard's disclosure (i.e., condition (16) implies that $R_{t+\Delta t} - S_{t+\Delta t} > R_t - S_t$ when $R_t - S_t > 0$, which can also be seen from (10) with $q_t^n = 0$). However, the total cost that the firm can incur is bounded from above by its limited liability $F$, so $R_t - S_t$ cannot increase infinitely. Thus, when inspections are not possible, the regulator can only incentivize the firm by covering all remediation costs at all times ($S_t \equiv R_t$). This policy provides our first benchmark.

Proposition 1 (No-Inspection Policy). *In the absence of inspection (i.e., $q_t^m = q_t^n = 0$ for all $t > 0$), if $c > (1+\alpha)\bar{r}$, the optimal regulation policy for (6) always subsidizes a full remediation: $S_t = R_t = \bar{r}$ for all $t \geqslant 0$. Otherwise, the firm is not regulated and $S_t = R_t = 0$ for all $t \geqslant 0$. Furthermore, the regulator's cost is equal to $\lambda[\bar{c} - (c - (1+\alpha)\bar{r})^+]/(\lambda+\theta)$.*

Thus, without inspections, the regulator either fully subsidizes a full remediation or does not regulate the firm. Indeed, the total discounted damage reduction that the regulation brings about is proportional to $(c - (1+\alpha)\bar{r})^+$. This benefit, however, is only positive when $c$ is larger than $(1+\alpha)\bar{r}$. If $c$ is too small, regulating the firm is too expensive.

By contrast, our second benchmark policy consists of relying on inspections only. Specifically, the reward $S_t$ is kept at 0, whereas the mandated remediation cost $R_t$ is set at the highest level $\bar{r}$, which is feasible only if $\bar{r} \leqslant F$. As discussed above, this always incentivizes the firm to delay the disclosure of the hazard to a later date. Thus, the regulator can only use the threat of an inspection to uncover the hazard. In this case, the regulator should not schedule the inspections but run them randomly, as stated in the following result.

Proposition 2 (Inspection-Only Policy). *When $R_t = \bar{r} < F$ and $S_t = 0$ for all $t \geqslant 0$, the optimal regulation policy for (6) conducts random inspections according to a Poisson process with constant rate $q^n = \theta\bar{r}/(p(F-\bar{r}))$. Furthermore, the regulator's cost is equal to $(\lambda(\bar{c} - c + \bar{r}) + kq^n)/(\lambda+\theta)$.*

Under an inspection-only policy, the regulator prefers random intensive inspections because they are cheaper than impulsive inspection if $R_t - S_t$ is maintained constant over time. To see this, consider any time interval $[t, t+\Delta t)$. If an impulsive inspection with probability $q_t^m > 0$ is used at time $t$, the inspection cost over this time period (with or without intensive inspection) is $kq_t^m + O(\Delta t)$. By contrast, using intensive inspection with rate $q_t^n$ only costs $kq_t^n\Delta t$, which is on the order of $O(\Delta t)$.

Furthermore, the optimal intensity $q_t^n$ remains constant over time for the inspection-only policy. Indeed, intensity $q_t^n$ needs to ensure that the benefit from delaying the disclosure (and hence the remediation payment) for a small time interval $\Delta t$, which is $\theta\Delta t\bar{r} + o(\Delta t)$, is offset by the expected penalty of being caught, which is $q_t^n\Delta t \times p(F-\bar{r}) + o(\Delta t)$. That is, inspection intensity $q_t^n$ is set such that $\theta\bar{r}\Delta t = q_t^n\Delta t p(F-\bar{r})$, which corresponds to the binding IC constraint (10).

## 5. Optimal Regulation Policy with Accurate Inspections ($p = 1$)

We now present our main result, which characterizes the optimal regulation policy of (6) when inspections are accurate ($p = 1$). As described in the following theorem, the optimal policy features a simple cyclic deterministic structure. This is in sharp contrast with the previous two benchmark policies.

Theorem 2. *When inspection accuracy is perfect ($p = 1$), the optimal regulation policy is determined by time $t^\star$, the unique positive solution of the following equation:*

$$\alpha\lambda F(1 - e^{-(\lambda+\theta)t^\star}) = (\lambda+\theta)[(k+\alpha F)(1 - e^{-\lambda t^\star}) - k]. \quad (17)$$

*The policy inspects the firm periodically at deterministic time epochs $\tau_i = t^\star \times i$, for $i = 1, 2, \ldots$ with $q_{\tau_i}^{m\star} = 1$, $q_t^{m\star} = 0$ for $t \neq \tau_i$ and $q_t^{n\star} = 0$ for $t \geqslant 0$. Let $\tau_0 = 0$.*

- *If $c > (1+\alpha)\bar{r}$, a full remediation is mandated with $R_t^\star = \bar{r}$ for $t \geqslant 0$, and the reward follows*

$$S_t^\star = \bar{r} - Fe^{\theta(t-\tau_i)}, \quad \text{for } t \in (\tau_{i-1}, \tau_i], \quad \text{with } S_{\tau_{i-1}+}^\star = \bar{s}^\star := \bar{r} - Fe^{-\theta t^\star} \quad \text{and} \quad S_{\tau_i}^\star = \bar{r} - F. \quad (18)$$

- *If $c \leqslant (1+\alpha)\bar{r}$, no remediation is mandated with $R_t^\star = 0$ for $t \geqslant 0$, and the reward follows*

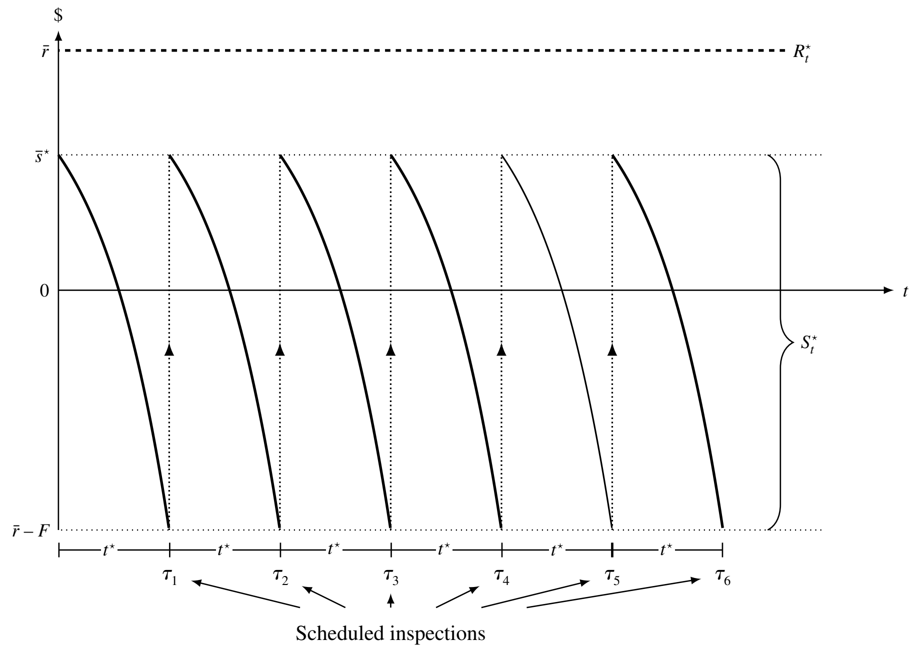
$$S_t^\star = -Fe^{\theta(t-\tau_i)}, \quad \text{for } t \in (\tau_{i-1}, \tau_i], \quad \text{with } S_{\tau_{i-1}+}^\star = \bar{s}^\star := -Fe^{-\theta t^\star} \quad \text{and} \quad S_{\tau_i}^\star = -F. \quad (19)$$

*Finally, the regulator's optimal cost at time 0 is*

$$C^\star = \frac{\lambda}{\lambda+\theta}[\bar{c} - (c - (1+\alpha)\bar{r})^+ - \alpha Fe^{-\theta t^\star}]. \quad (20)$$

Figure 3 depicts the cyclic structure of the optimal policy when the damage reduction from a full remediation is large enough ($c > (1+\alpha)\bar{r}$). The remediation level $R_t^\star$ is set at

**Figure 3.** The optimal regulation policy with perfect inspection ($p = 1$) and $c > (1+\alpha)\bar{r}$ for $c = 10$, $\alpha = 0.3$, $\bar{r} = 6$, $F = 12$, $k = 0.6$, $\lambda = 0.2$, and $\theta = 1$.




its upper bound $\bar{r}$, and the reward level $S_t^\star$ decreases from $\bar{s}^\star \leqslant \bar{r}$ to $\bar{r} - F$ between two consecutive inspections.

Thus, the optimal regulation policy always mandates a full remediation as in the noninspection policy of Proposition 1. (In fact, the noninspection policy is essentially the limiting case of the optimal policy for infinitely large inspection cost, since $t^\star$ approaches infinity as $k$ increases to infinity, according to (17).) However, although the noninspection policy fully subsidizes the firm's remediation cost (with $S_t = \bar{r}$ for all $t$), the optimal policy in Theorem 2 decreases the reward level $S_t$ below $\bar{r}$. This dynamic adjustment reduces the regulator's cost and exactly offsets the firm's incentive to delay the hazard's disclosure, as explained in §4 (in particular, trajectory (18) guarantees that (16) holds at equality). The reward amount continues to decrease until the firm's total disclosure cost reaches the firm's liability, i.e., $R_t - S_t = \bar{r} - S_t = F$. At this point, the regulator can no longer increase the firm's cost. Instead, the regulator runs an inspection that resets $S_t$ to $\bar{s}^\star$, which also starts the next cycle. This is in contrast with the noninspection policy, under which the regulator cannot circumvent the firm's liability constraint with an inspection. And in contrast with the inspection-only policy of Proposition 2, the regulator does not run any random inspections but instead follows a periodic inspection schedule.

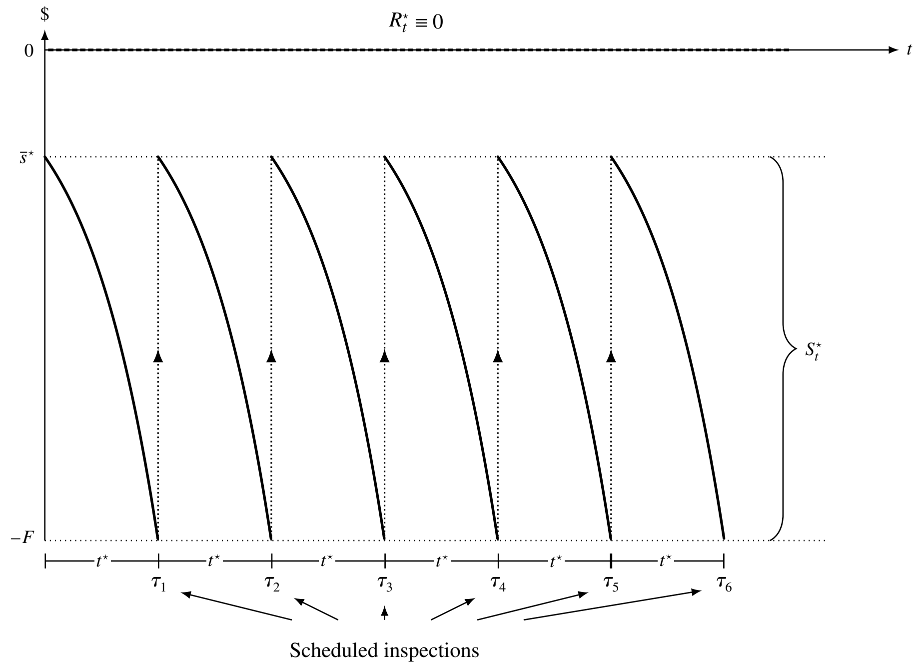
Figure 4 depicts the optimal policy when the damage reduction of a full remediation is small ($c \leqslant (1+\alpha)\bar{r}$). The optimal policy retains the same structure, except that both $R_t$ and $S_t$ are shifted downward. Specifically, the regulator never mandates a repair with $R_t = 0$; and the reward amount $S_t$ decreases from a negative value $\bar{s}^\star$ all the way to $-F$, at which point an inspection takes place.

To understand this difference, note first that the reduced environmental damage $c$ does not directly affect the firm's incentive to disclose the hazard ($c$ does not appear in the (IC) constraint (2)). What matters to the firm is the total cost $R_t - S_t$ of disclosing the hazard at time $t$. How this total cost is split between the mandated remediation level or the reward is irrelevant to the firm. This is not the case, however, for the regulator. As in Proposition 1, a remediation is too expensive to society and $R_t = 0$ if $c < (1+\alpha)\bar{r}$. Since $S_t$ needs to be lower than $R_t$ (as discussed in §4), the reward amount is always negative in this case. Therefore, the reward corresponds to a reduced fine, with $0 \geqslant S_t \geqslant -F$. By contrast, mandating a full remediation is always optimal if $c > (1+\alpha)\bar{r}$. In this case, and if $\bar{s}^\star > 0$, the reward amount satisfies $\bar{r} \geqslant S_t \geqslant 0$ during the early phase of a policy cycle. The reward corresponds then to a (partial) subsidy for a full remediation, which results in net cost $\bar{r} - S_t \geqslant 0$ for the firm.

Overall, Theorem 2 demonstrates how the use of inspections enables the regulator to incentivize the firm with a decreasing reward level. In fact, the theorem also reveals the impact of these inspections on social cost. Specifically, the optimal cost $C^\star$ is exactly equal to the cost of the no-inspection policy in Proposition 1, minus a term proportional to $\alpha F e^{-\theta t^\star}$. Thus, this last term captures the reduction in social cost that inspections bring about.

Furthermore, Theorem 2 reveals how the size of the reward level is related to the frequency of inspection. Specifically, reward threshold $\bar{s}^\star$ given in (18) and (19) is increasing in $t^\star$. Thus, to reduce the inspection costs, the regulator needs to trade off less frequent inspections (i.e., longer $t^\star$) with higher rewards (i.e., higher values of $\bar{s}^\star$). The optimal

**Figure 4.** The optimal regulation policy with perfect inspection ($p=1$) and $c \leqslant (1+\alpha)\bar{r}$ for $c=5$, $\alpha=0.3$, $\bar{r}=6$, $F=12$, $k=0.6$, $\lambda=0.2$, and $\theta=1$.




trade-off is reflected in (17), which can be thought of as a first order optimality condition for the cycle length $t^\star$.

Finally, Theorem 2 points out the importance of the firm's limited liability in the optimal regulation policy. In fact, the next result shows that the key parameters of the optimal policy are monotone in $F$.

Corollary 1. *Ceteris paribus, both $\bar{s}^\star$ and $t^\star$ are decreasing in $F$.*

In particular, since $t^\star$ is decreasing in $F$, the cost reduction $\alpha F e^{-\theta t^\star}$ increases in $F$. In other words, the social benefit of using inspections increases with the firm's liability, as should be expected.

Figure 5 illustrates this points numerically and depicts the effect of the firm's limited liability $F$ on the key parameters of the reward and inspection policies specified in Theorem 2. The corollary shows that higher liability $F$ allows the regulator to apply more stringent reward policies and thus reduces $\bar{s}^\star$. In essence, as the firm's ability to pay for environmental hazard increases, the regulator is able to offer lower rewards.

Perhaps more surprising, however, is the effect of the firm's liability on the regulator's inspection policy. Corollary 1 suggests that the inspection frequency, $1/t^\star$, *increases* with $F$. It appears that Kim (2015) is the first and only study to demonstrate the potential complementarity of the firm's limited liability and inspections. Nonetheless, the rationale behind our result differs from Kim's (2015), whereby the complementarity emerges when the firm has incentives to cheat. In his setting, a higher firm's ability-to-pay induces the regulator to increase the inspection frequency to detect more hazards and hence generate more revenues from penalties. In our setting, however, a rational firm never cheats under the optimal policy and, therefore, the maximum penalties (equal to the firm's liability) are never collected in equilibrium (or only collected when the hazard happens to occur at the scheduled inspection epochs, which are zero-probability events). Nevertheless, higher firm liability strengthens the power of the regulator's threat and hence improves the effectiveness of inspections in inducing voluntary disclosures. This allows the regulator to use inspections more frequently (i.e., to lower $t^\star$) so as to decrease the reward levels (i.e., to lower $\bar{s}^\star$ and hence $S_t$).

## Proof of Theorem 2

We conclude this section by outlining the main steps to prove Theorem 2, which also provide the logic of the more intricate proofs for the extensions of the base model in the next two sections. The proof derivations for the major steps are presented in Appendix B.

First, we verify that $t^\star$ is well-defined according to (17).

Lemma 3. *There exists a unique solution $t^\star > 0$ to (17).*

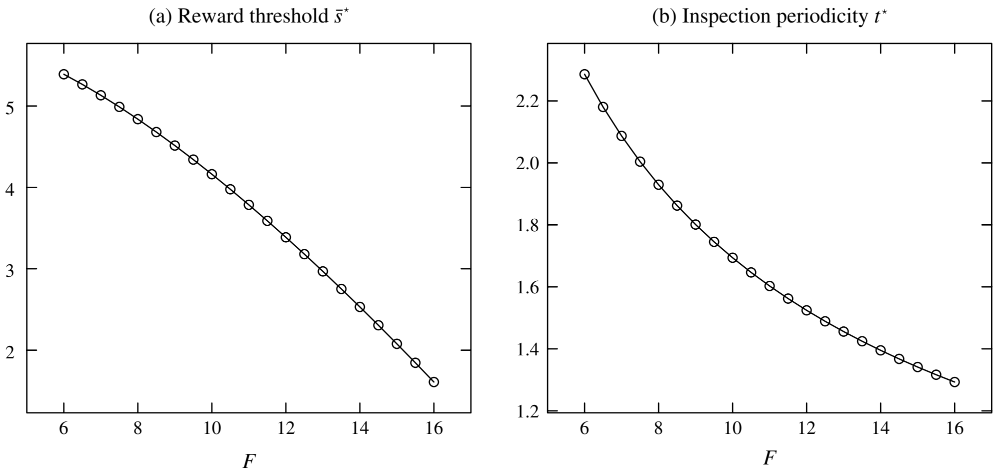
We then verify that the policy prescribed in Theorem 2 is indeed incentive compatible.

Proposition 3. *Under the policy prescribed in Theorem 2, the firm's total cost upon disclosure, is given by*

$$W_t^\star = R_t^\star - S_t^\star = F e^{\theta(t-\tau_i)}, \quad \text{for } t \in (\tau_{i-1}, \tau_i],\ i = 1, 2, \ldots, \tag{21}$$

*which satisfies (9), and (10) with $p=1$.*

**Figure 5.** The effect of the firm's limited liability ($F$) on the optimal regulation policy with $p=1$, $c=10$, $\alpha=0.3$, $\bar{r}=6$, $k=0.6$, $\lambda=0.2$, and $\theta=1$.




Next, we derive the cost-to-go function under the policy prescribed in Theorem 2.

Proposition 4. *The regulator's cost-to-go function under the policy specified in Theorem 2 is given by*

$$C(w) := -\frac{\lambda}{\lambda+\theta}[c-(1+\alpha)\bar{r}]^+ + \frac{k+\alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})} \cdot w^{(\lambda+\theta)/\theta} - \alpha w, \quad \text{for } w \in [Fe^{-\theta t^\star}, F], \tag{22}$$

*which is strictly, convex increasing with*

$$C(Fe^{-\theta t^\star}) = -\frac{\lambda}{\lambda+\theta}\{[c-(1+\alpha)\bar{r}]^+ + \alpha F e^{-\theta t^\star}\}, \quad \text{and} \quad \left.\frac{dC(w)}{dw}\right|_{w=Fe^{-\theta t^\star}} = 0, \tag{23}$$

Since the policy specified in Theorem 2 is admissible, cost-to-go function $C(w)$ is an upper bound of the optimal cost-to-go function, $C^\star(w)$. The next result demonstrates that cost-to-go function $C(w)$ satisfies the sufficient condition of Theorem 1, which is, therefore, also a lower bound of $C^\star(w)$.

Proposition 5. *Extend function $C(w)$ defined in (22) by defining $C(w) \equiv C(Fe^{-\theta t^\star})$ for $w \leqslant Fe^{-\theta t^\star}$. Then $C(w)$ satisfies (15) with equalities for $w \in [Fe^{-\theta t^\star}, F]$. Therefore $C^\star(w) = C(w)$ for $w \in [Fe^{-\theta t^\star}, F]$.*

Thus, $C(w) = C^\star(w)$, and the policy is optimal, completing the proof of the optimality of the policy in Theorem 2. In particular, since $W_0^\star = Fe^{-\theta t^\star}$ minimizes $C(w)$, Lemma 2 suggests that the regulator's expected cost at time 0 is $C^\star = \bar{c}\lambda/(\lambda+\theta) + C(Fe^{-\theta t^\star})$, which is essentially (20).

## 6. Optimal Regulation Policy with Inaccurate Inspections ($p \leqslant 1$)

When inspections are inaccurate ($p<1$), the optimal policy retains the same cyclic structure as with accurate inspection ($p=1$), except for the very first cycle. Indeed, the following theorem shows that when $p<1$, the first inspection takes place after a longer time period than the subsequent ones. The reward amount is then set at its highest possible level at the beginning of the time horizon ($t=0$). Furthermore, the reward amount is set such that the firm's total cost never reaches liability $F$.

Theorem 3. *The optimal regulation policy is determined by a single parameter, $t^\star$, which is the unique positive solution to the following equation:*

$$\alpha\lambda p F(1-e^{-(\lambda+\theta)t^\star}) = (\lambda+\theta)(1-(1-p)e^{-\theta t^\star}) \cdot \left[\left(\frac{k}{p}+\alpha F\right)(1-e^{-\lambda t^\star})-k\right]. \tag{24}$$

*Then the regulator inspects only at time epochs $\tau_i = t_0^\star + (i-1)t^\star$, for $i=1,2,\ldots,$ where*

$$t_0^\star := -\frac{1}{\lambda}\ln\left(1-\frac{1-e^{-\lambda t^\star}}{p}\right) \geqslant t^\star. \tag{25}$$

*Namely, $q_{\tau_i}^{m\star}=1$, $q_t^{m\star}=0$ for $t\neq\tau_i$ and $q_t^{n\star}=0$ for $t\geqslant 0$. Let $\hat{F} := pF/[1-(1-p)e^{-\theta t^\star}]$ and $\tau_0 = 0$.*

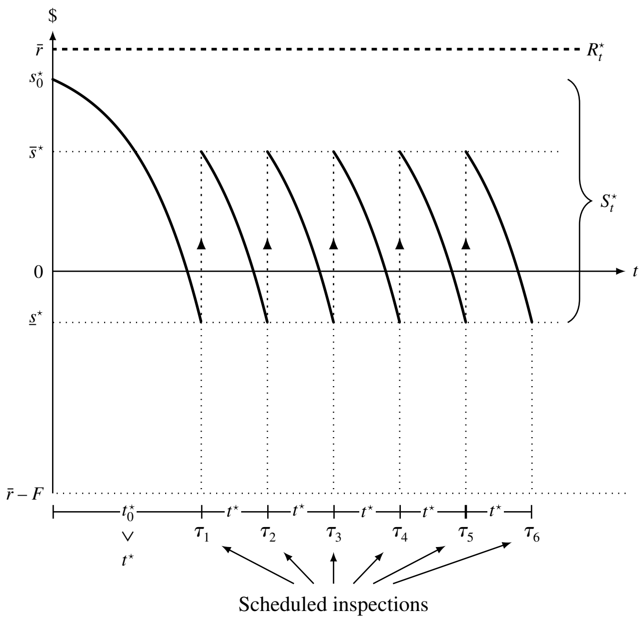
- *If $c > (1+\alpha)\bar{r}$, we have $R_t^\star = \bar{r}$ for all $t \geqslant 0$, and $S_t^\star = \bar{r} - \hat{F}e^{\theta(t-\tau_i)}$ for $t\in[\tau_{i-1},\tau_i]$, with*

$$S_0^\star = s_0^\star := \bar{r} - \hat{F}e^{-\theta t_0^\star} \geqslant S_{\tau_i+}^\star = \bar{s}^\star := \bar{r} - \hat{F}e^{-\theta t^\star} > S_{\tau_i}^\star = \underline{s}^\star := \bar{r} - \hat{F} \geqslant \bar{r} - F. \tag{26}$$

- *If $c \leqslant (1+\alpha)\bar{r}$, we have $R_t^\star = 0$ for all $t\geqslant 0$, and $S_t^\star = -\hat{F}e^{\theta(t-\tau_i)}$ for $t\in[\tau_{i-1},\tau_i]$, with*

$$S_0^\star = s_0^\star := -\hat{F}e^{-\theta t_0^\star} \geqslant S_{\tau_i+}^\star = \bar{s}^\star := -\hat{F}e^{-\theta t^\star} > S_{\tau_i}^\star = \underline{s}^\star := -\hat{F} \geqslant -F. \tag{27}$$

**Figure 6.** The optimal regulation policy with imperfect inspection ($p = 0.5$) and $c > (1+\alpha)\bar{r}$ for $c = 10$, $\alpha = 0.3$, $\bar{r} = 6$, $F = 12$, $k = 0.6$, $\lambda = 0.2$, and $\theta = 1$.




*All nonstrict inequalities in* (25)–(27) *hold as equalities if and only if* $p = 1$. *Finally, the regulator's optimal cost is*

$$C^\star = \frac{\lambda}{\lambda+\theta}\{\bar{c} - (c - (1+\alpha)\bar{r})^+ - \alpha\hat{F}e^{-\theta t_0^\star}\}. \tag{28}$$

Figure 6 depicts an example of the optimal policy for inaccurate inspection ($p < 1$) when the damage reduction from a full remediation is sufficiently large ($c > (1+\alpha)\bar{r}$). For each cycle, the lower reward threshold $\underline{s}^\star$ is larger than $\bar{r} - F$, with the first cycle starting from a higher reward level $s_0^\star > \bar{s}^\star$ and taking a longer time $t_0^\star > t^\star$ before the first inspection.

The choices of reward thresholds $\bar{s}^\star$ and $\underline{s}^\star$ reflect that the IC constraint (9) associated with impulsive inspections is binding at the optimum. Specifically, consider the firm's trade-off between immediately reporting a hazard that occurs right before an inspection, and delaying the disclosure of the hazard until right after the inspection. The expected benefit of waiting until after the inspection is $(1-p)\bar{s}^\star$, whereas the expected penalty cost is $pF$. Thus, the reward for disclosing right before an inspection has to satisfy $\underline{s}^\star \leqslant (1-p)\bar{s}^\star - pF$, which corresponds to the IC constraint (9) for $q^{m\star} = 1$. This inequality holds at equality for $\bar{s}^\star$ and $\underline{s}^\star$ specified in (26). By contrast, when inspections are perfectly accurate, the firm cannot expect a benefit for disclosing the hazard right after the inspection. Indeed, the inspection always reveals the hazard in this case and $(1-p)\bar{s}^\star = 0$ when $p = 1$ so that the lower threshold $\underline{s}^\star$ is set at the lowest possible bound allowed by the firm's liability (i.e., $\bar{r} - F$ if $c > (1+\alpha)\bar{r}$ and $-F$ otherwise).

This also explains why the first cycle is longer than the subsequent ones. Indeed, as discussed above, the regulator should not set $\bar{s}^\star$ to too high a level so as to offset the firm's incentive to take the chance of slipping through an inaccurate inspection. But such a concern is not present when $t = 0$, which leads to a higher $\bar{s}_0^\star$ and hence a longer first cycle time $t_0^\star$.

Furthermore, Theorem 3 reveals an important connection between the regulator's inspection accuracy and the firm's limited liability. In short, inaccurate inspections weaken the regulator's power but strengthen the firm's position. This is reflected by the maximum possible cost that the regulator actually imposes on the firm. Indeed, the optimal policy in Theorem 3 for $p < 1$ resembles that of Theorem 2 for $p = 1$, if $\hat{F}$ is replaced with $F$. (This can be readily seen by comparing (18) and (19) with (26) and (27), respectively.) Since $\hat{F} \leqslant F$ with the equality holding only when $p = 1$, the effect of inaccurate inspections acts as if the firm's liability is decreased and, as a result, the firm's position is strengthened. As such, the regulator's total cost given by (28) becomes higher than that given by (20).

Overall, Theorem 3 points to the importance of inspection accuracy in optimal regulation policy. As shown in the next result, the key parameters of the optimal policy are all monotone in $p$.

Corollary 2. *Ceteris paribus,* $s_0^\star$, $\underline{s}^\star$ *and* $t_0^\star$ *decreases in* $p$, *while* $t^\star$ *increases in* $p$.

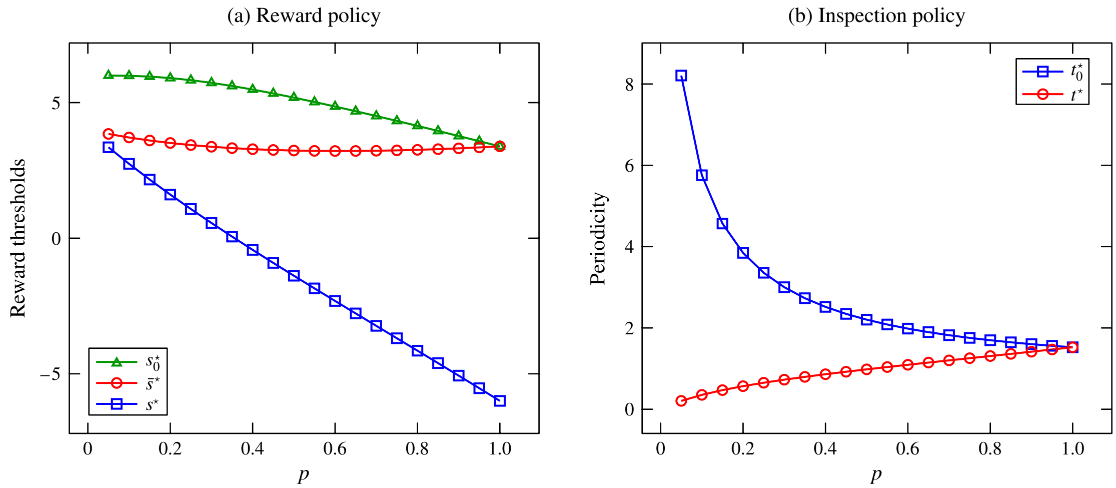
Figure 7 illustrates Corollary 2 and depicts the effect of accuracy $p$ on the parameters of the reward and inspection policies as described in Theorem 3. In particular, the inspection frequency, $1/t^\star$, increases as inspections become less accurate (i.e., as $p$ decreases), compensating for the lack of accuracy. A lower accuracy level $p$ is also compensated for by a higher initial reward $s_0^\star$ and thus a longer first cycle $t_0^\star$. Indeed, because a lower accuracy level requires more frequent, and hence more expensive inspections, the regulator seeks to postpone the use of inspections altogether by increasing $t_0^\star$. Similarly, the regulator also offers a higher reward threshold $\underline{s}^\star$ for a lower level of accuracy. However, in general, $\bar{s}^\star$ is not monotonic for $p < p^\star$, as illustrated in Figure 7(a). (Our numerical experiments (not reported) show that $\bar{s}^\star$ is either monotonic or first decreasing and then increasing in $p$.)

Note finally that the effect of the firm's liability $F$ for $p = 1$ in Corollary 1 also holds when $p < 1$ (see Proposition EC.5 of the e-companion).

## 7. Optimal Regulation Policy with Frictionless Fine ($\alpha = 0, S_t \leqslant 0$)

Thus far, we have assumed that each dollar spent by the regulator is raised through distortionary taxes that are costly to society. This corresponds to a positive shadow cost of public funds with $\alpha > 0$ for the regulator. When the application of public funds is frictionless ($\alpha = 0$), the use

**Figure 7.** (Color online) The effect of the inspection accuracy ($p$) on the optimal regulation policy with $c = 10$, $\alpha = 0.3$, $\bar{r} = 6$, $F = 12$, $k = 0.6$, $\lambda = 0.2$, and $\theta = 1$.




of subsidies is "free of charge" to the regulator, who can then offer an infinitely large subsidy to reward voluntary disclosure. The regulator can induce the firm to promptly reveal the hazard at no cost, and the design of the regulation policy becomes trivial, as stated in the next result.

Corollary 3. *If $\alpha = 0$, the optimal regulation policy conducts no inspections at all and fully reimburses the firm's remediation cost, if any (i.e., $S_t^\star = R_t^\star$ for all $t \geqslant 0$), upon disclosure. In particular, if $c > \bar{r}$, a full remediation is mandated upon disclosure, i.e., $R_t^\star = \bar{r}$ for all $t \geqslant 0$; otherwise, no remediation is mandated upon disclosure, i.e., $R_t^\star = 0$ for all $t \geqslant 0$. Furthermore, the regulator's optimal cost is equal to $C^\star = \lambda[\bar{c} - (c - \bar{r})^+]/(\lambda + \theta)$.*

Thus, if the damage reduction is worth its cost ($c > \bar{r}$), the regulator fully subsidizes a full remediation. Otherwise, the firm is not regulated.

However, subsidies may not be allowed in practice (EPA 2001). As such, the regulator cannot offer any positive reward to the firm and can only fine the firm when the hazard is disclosed. In our setup, this corresponds to restricting the reward amount to nonpositive values, i.e., $S_t \leqslant 0$. In other words, when $\alpha = 0$ and $S_t \leqslant 0$, the frictions associated with the application of public funds takes the form of an upper bound at 0 on the reward amount. In this case, the regulator's problem becomes

$$C^\star := \frac{\lambda}{\lambda + \theta}\bar{c} + \min_{\mathscr{R}} \mathbb{E}\left[e^{-\theta T}(1 - c/\bar{r})R_T + K \sum_{\tau_i \in \mathcal{T}_T^Q} e^{-\theta \tau_i}\right],$$

$$\text{subject to (2) and } S_t \leqslant 0. \tag{29}$$

When positive rewards are not allowed, the next theorem shows that the optimal policy retains a cyclic structure, in which the dynamic adjustment of mandated remediation level sometimes acts as a substitute for the subsidy.

Theorem 4. *Suppose $\alpha = 0$, $S_t \leqslant 0$ for $t \geqslant 0$ and $p = 1$. If $c \leqslant \bar{r}$, the optimal regulation policy does not regulate the firm, i.e., $R_t = S_t = q_t^m = q_t^n = 0$ for all $t$. Otherwise, the optimal regulation policy is characterized by time $t^\star$, which is the unique solution to the following equation,*

$$\lambda e^{-(\lambda+\theta)t^\star} - \left[\theta\left(\min\left\{\frac{\bar{r}}{F}, 1\right\}\right)^{-\lambda/\theta} + \lambda\left(\min\left\{\frac{\bar{r}}{F}, 1\right\}\right) + (\lambda + \theta)\frac{\bar{r}}{F}\frac{K}{c - \bar{r}}\right]e^{-\lambda t^\star} + \theta = 0, \tag{30}$$

*such that $t^\star > \bar{t} := [(1/\theta)\ln(F/\bar{r})]^+$. The regulator inspects the firm periodically at deterministic time epochs $\tau_i = t^\star \times i$ for $i = 1, 2, \ldots$, with $q_{\tau_i}^{m\star} = 1$, $q_t^{m\star} = 0$ for $t \neq \tau_i$ and $q_t^{n\star} = 0$ for $t \geqslant 0$. Then,*

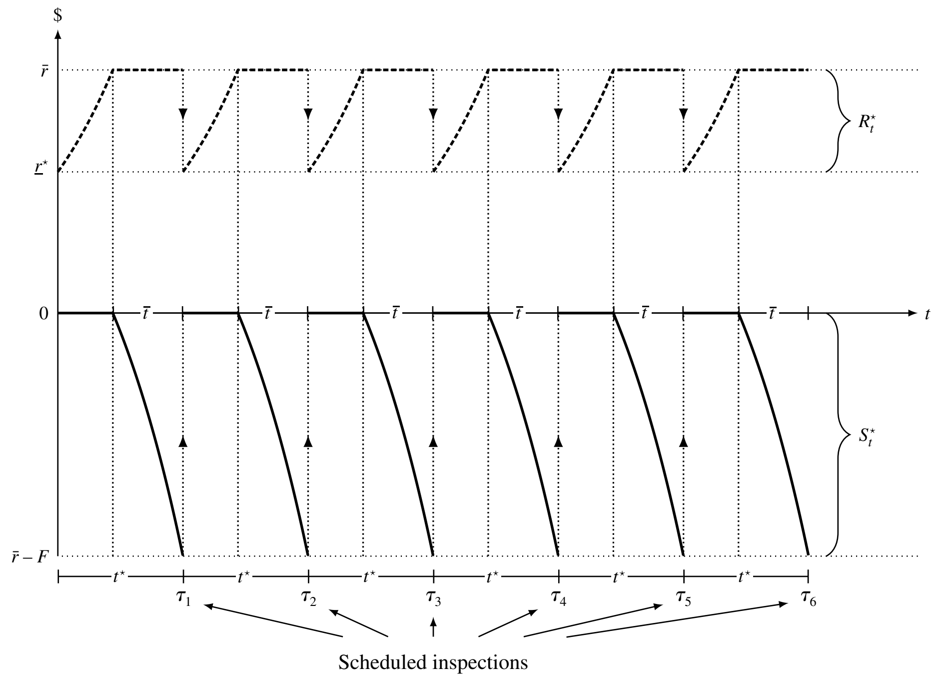
$$\begin{aligned} &R_t^\star = Fe^{-\theta(\tau_i - t)}, \quad S_t^\star = 0, \quad \text{for } t \in (\tau_{i-1}, \tau_i - \bar{t}\,];\\ &R_t^\star = \bar{r}, \quad S_t^\star = \bar{r} - Fe^{-\theta(\tau_i - t)}, \quad \text{for } t \in (\tau_i - \bar{t}, \tau_i], \end{aligned} \tag{31}$$

*with $R^\star_{\tau_{i-1}+} = \underline{r}^\star := Fe^{-\theta t^\star}$ and $S^\star_{\tau_i} = -(F - \bar{r})^+$. Finally, the regulator's optimal cost at time 0 is*

$$C^\star = \frac{\lambda}{\lambda + \theta}\{\bar{c} - (c/\bar{r} - 1)^+ Fe^{-\theta t^\star}\}. \tag{32}$$

Figure 8 illustrates the optimal policy described in Theorem 4. The policy retains a deterministic cyclic structure, where scheduled inspections are performed periodically. In contrast to the previous cases, however, the optimal policy dynamically adjusts the remediation level upon the disclosure. Specifically, at the beginning of each cycle, the regulator wishes to dynamically adjust a subsidy (i.e., a positive reward) to the firm for self-reporting the hazard. But because subsidies are not allowed, the regulator instead resorts to adjusting the mandated remediation level (up to the full repair $\bar{r}$). To emulate a decrease in the subsidy amount,

**Figure 8.** The optimal regulation policy with $\alpha = 0$, $S_t \leqslant 0$, $c > \bar{r}$ and $\bar{r} \leqslant F$ for $p = 1$, $\bar{r} = 6$, $F = 12$, $k = 0.6$, $\lambda = 0.2$, and $\theta = 1$.




the regulator needs to increase the remediation level. In this sense, the regulator uses the remediation level as a substitute for subsidies. This allows the regulator to gradually decrease the firm's total cost $R_t - S_t$ so as to incentivize the firm to disclose the hazard immediately, as discussed in §4.

When the remediation level acts as a substitute for a subsidy, the regulator faces a new tradeoff between reducing regulatory costs and limiting the reduction of environmental damage. That is, the level of $c$ also affects the frequency of inspection $1/t^\star$, which is in contrast with the case $\alpha > 0$ discussed in §5. The following corollary formally examines the effect of maximum damage reduction $c$. Recall from Theorem 4 that the firm is not regulated when $c \leqslant \bar{r}$ .

Corollary 4. *Ceteris paribus, $t^\star$ is decreasing in $c > \bar{r}$ while $\underline{r}^\star$ is increasing in $c > \bar{r}$.*

Figure 9 illustrates the result in Corollary 4. Fixing $\bar{r}$, the environmental cost associated with a low remediation level ($R_t < \bar{r}$) increases with $c$. The regulator, therefore, sets the minimum level of mandated repairs $\underline{r}^\star$ to a higher level for higher $c$. This reduces the regulator's ability to use the remediation level as a substitute for a subsidy. Thus, the regulator resorts instead to more frequent inspections with shorter cycle length $t^\star$.

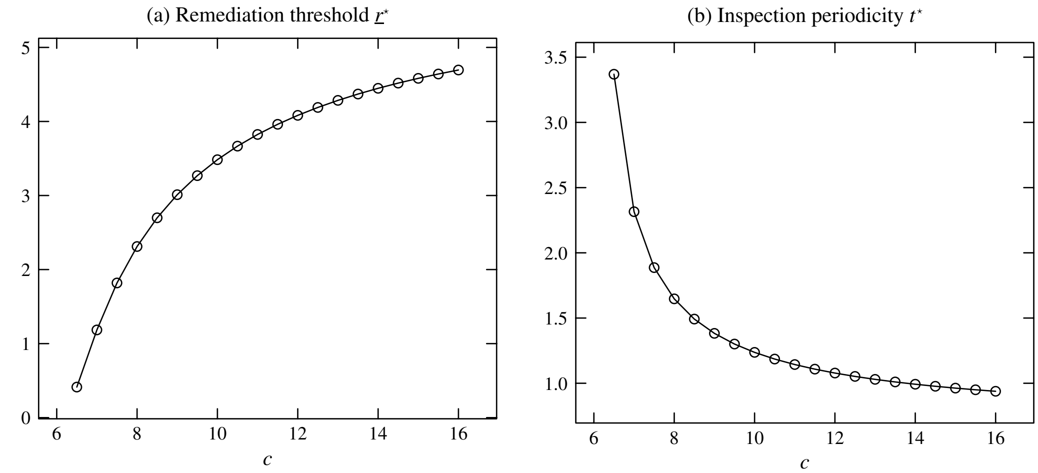
## 8. Discussions and Conclusion

This paper proposes a dynamic mechanism design framework to study voluntary disclosure of environmental hazards. This framework is adequate when the occurrence of the hazard is beyond the firm's control and privately known only to the firm, and when the regulatory responses, costly to the firm, discourage it from revealing on this information. The approach enables us to explicitly account for the timing of the uncertain event, without a priori restricting the possible set of admissible inspection and reward policies. Despite the generality and complexity of our framework, we obtain simple and easy-to-implement optimal regulation policies in closed-form. The optimal structure reveals that the regulator should rely on offering a reward for a voluntary disclosure for as long as possible before resorting to an inspection.

Our results critically depend on three "first principles" in our model: private information, limited liability, and frictions of applying public funds. Without any one of these three principles, the misalignment of incentives, which constitutes the core of our problem, does not exist in the first place. In particular, to the extent that the private information in our setting is the timing of an event, our problem is, in essence, dynamic.

Our model can be generalized in several meaningful directions. First, our model can easily capture the situation under which the hazard cannot be concealed forever. Instead, if the hazard is inevitably observable by the public some time after its occurrence—say, after an exponentially distributed time—then we only need to generalize the model to allow the firm's time discount factor to be different from the regulator's. The solution procedures in our current model remain applicable, and the corresponding optimal regulation policy, although more complex, retains the same qualitative structure as demonstrated in this paper. Along this line of thought, our model can be easily extended to study regulations that prevent environmental disasters, which may

**Figure 9.** The effect of the environmental damage reduction from full remediation ($c > \bar{r}$) on the optimal regulation policy when only fines ($S_t \leqslant 0$) are allowed with $\alpha = 0$, $\bar{r} = 6$, $F = 12$, $k = 0.6$, $\lambda = 0.2$, and $\theta = 1$.




occur some time after a certain risk indicator becomes noticeable only to the firm.

Furthermore, the main driver of our deterministic cyclic structure is that the cost to the firm for hiding the hazard—and hence for being caught by an inspection—is equal to the firm's limited liability. In some situations, a hazard exposed by an inspection incurs additional intangible costs to the firm, such as a loss of reputation or public goodwill. In this case, the total cost for being caught by an inspection is effectively higher than the firm's financial liability. These cases constitute interesting extensions of our work, where the optimal inspections may, in fact, be random.

To capture the essential tradeoff between environmental and remediation costs, we make the simplified technical assumption that the reduced damage $c$ is linear in the remediation cost $R$. If this relationship is nonlinear, the optimal remediation level may not be set at an extreme value (either the lower bound 0 or the upper bound $\bar{r}$), but somewhere in between. Furthermore, the optimal solution may involve dynamically adjusting the remediation level as we find in our extension, where subsidies are infeasible.

Finally, we focus in this paper on adverse selection problems and thus consider unintentional hazards. In some settings, the firm can take additional actions to reduce the chance of the hazard, or make costly efforts to elicit information about its occurrence. These issues give rise to moral hazard problems. Combining dynamic adverse selection and moral hazard issues is, however, notoriously difficult and often intractable. Nonetheless, the closed form solutions we derive in this paper provide a promising starting point for future investigations in this direction.

## Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2016.1476.

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## Appendix A. Definition of Inspections and Revelation Principle

In this section, we use $X_t$ to denote the corresponding environmental state: the state is *normal*, i.e., $X_t = 1$, if and only if $t < T$ and the state is *hazardous*, i.e., $X_t = 0$, otherwise. We use $Y_t \in \{0, 1\}$ to denote the result of an inspection at time $t$. According to our model setup, we have

$$\mathbb{P}[Y_t = 0 \mid X_t = 1] = 0, \quad \mathbb{P}[Y_t = 0 \mid X_t = 0] = p. \tag{A.1}$$

We notice that the more precise way of writing the right-hand side of the IC constraint $\mathbb{E}[D_t^{\mathcal{R}}(t') \mid \mathcal{I}_t]$ is $\mathbb{E}[D_t^{\mathcal{R}}(t') \mid \mathcal{I}_t, X_t = 0]$.

We use $\mathcal{I}^Q_{[t_1, t_2]}$ to denote the set of inspection time epochs during $[t_1, t_2]$ under the inspection policy $Q$. To be consistent with the notation in the main text, we denote $\mathcal{I}^Q_t := \mathcal{I}^Q_{[0, t]}$ and $\mathcal{I}^Q := \mathcal{I}^Q_{[0, \infty)}$. We define the corresponding counting process as $N^Q_t := \sum \mathbb{1}_{[\tau_i \in \mathcal{I}^Q_t]}$ and the filtration generated by $N^Q_t$ as $\mathcal{F}_t$. Let $Z^Q_{[t, \zeta)} := \prod_{\tau_i \in \mathcal{I}^Q_{[t, \zeta)}} Y_{\tau_i}$ for any $\zeta \geqslant t$ be the indicator of whether the firm has survived inspections during time $[t, \zeta)$. That is, $Z^Q_{[t, \zeta)} = 1$ if no inspection takes place ($Z^Q_{[t, \zeta)}$ is an empty product) or any possible inspections during time $[t, \zeta)$ produces "no hazard" results (because of imperfection of the inspection technology), and $Z^Q_{[t, \zeta)} = 0$ if an inspection during $[t, \zeta)$ correctly identified the hazard. If a hazard is detected at time $\zeta$, we must have $\Delta Z^Q_{[t, \zeta]} := Z^Q_{[t, \zeta]} - Z^Q_{[t, \zeta)} = -1$.

Definition A.1 (Inspection Policy). Let $dt$ and $\delta_t$ denote the usual Lebesgue measure and Dirac measure on time horizon $t \in [0, \infty)$, respectively. We call $\{q^n_t \in [0, \infty)\colon t \geqslant 0\}$ and $\{q^m_t \in [0, 1]\colon t \geqslant 0\}$ an *intensity inspection policy* and an *impulsive inspection policy*, respectively, if

1. the process $q^n_t$ and $q^m_t$ are $\mathcal{F}_t$-predictable;
2. the measure $\mu(dt) := q^n_t dt + q^m_t \delta_t$ satisfies

$$\int_0^t \mu(ds) < \infty, \quad t \geqslant 0; \text{ and} \tag{A.2}$$

3. the measure $\mu(dt)$ consists of an $\mathcal{F}_t$-predictable compensator (c.f. Brémaud 1981, Liptser and Shiryaev 2010) for the counting process $N_t^Q$, i.e.,

$$\mathbb{E}\left[\sum_{t\in[0,\infty)} A_t \Delta N_t^Q\right] = \mathbb{E}\left[\int_0^\infty A_t \mu(dt)\right] \tag{A.3}$$

for any bounded $\mathcal{F}_t$-predictable process $A_t$. □

To establish the most general version of the Revelation Principle that is applicable to all our settings, we consider a general feasible set of regulatory responses $\mathcal{L} := \{(R,S)\colon R\in[0,\bar{r}], S\leqslant \bar{B}, R-S\leqslant F\}$ with $0\leqslant \bar{B}\leqslant\infty$.

Lemma A.1 (Revelation Principle). *It is without loss of generality to restrict the search for the optimal regulation policy within the class of direct revelation policies. Namely, for an arbitrary regulation policy $\hat{\mathcal{R}}$, there exists an alternative regulation policy $\mathcal{R}$, which is payoff-equivalent to the firm without increasing the regulator's expected cost. In particular, one can choose policy $\mathcal{R}$, under which it is optimal for the firm to disclose the hazard once it occurs. Moreover, such policy $\mathcal{R} := (R_t, S_t, Q_t)_{t\in[0,\infty)}$ satisfies the IC constraint (2) with*

$$D_t^{\mathcal{R}}(t') = -F\sum_{\zeta\in[t,t')} e^{-\theta(\zeta-t)}\Delta Z^Q_{[t,\zeta]} + e^{-\theta(t'-t)} Z^Q_{[t,t')}(R_{t'} - S_{t'}), \quad \forall\, t' > t. \tag{A.4}$$

Proof. See Appendix EC.1. □

## Appendix B. Proofs in §5

Proof of Lemma 3. Let $f(t) := \alpha\theta F + \alpha\lambda F e^{-(\lambda+\theta)t} - (\theta+\lambda)(\alpha F + k)e^{-\lambda t}$. Then, (17) is equivalent to $f(t^\star)=0$. The existence and uniqueness of $t^\star$ thus follow from the straightforward verification that $f(0) = -(\lambda+\theta)k < 0$, $f(\infty) = \alpha\theta F > 0$, and

$$f'(t) = \lambda(\lambda+\theta)e^{-\lambda t}(k + \alpha F - \alpha F e^{-\theta t}) \geqslant \lambda(\lambda+\theta)k e^{-\lambda t} > 0,$$

because $e^{-\theta t}\leqslant 1$. □

Proof of Proposition 3. By definition, $W_t^\star = R_t^\star - S_t^\star$ for $t\geqslant 0$. Thus, (21) follows immediately from (18) and (19). In particular, we have $W^\star_{\tau_i} = F$ and $W^\star_{\tau_i+} = Fe^{-\theta t^\star}$ for $i = 1, 2, \ldots$.

According to the policy prescribed in Theorem 2, impulsive inspections, $q^{m\star}_{\tau_i} = 1$, are used only at $\tau_i = t^\star\times i$. It is straightforward to verify that,

$$\begin{aligned} W^\star_{\tau_i} &= F = (1-1)\cdot Fe^{-\theta t^\star} + 1\cdot F \\ &= (1 - q^{m\star}_{\tau_i})W^\star_{\tau_i+} + q^{m\star}_{\tau_i}[1\cdot F + (1-1)\cdot W^{I\star}_{\tau_i+}]. \end{aligned}$$

Namely, (9) holds with equality at those inspection epochs.

For any other time moments, no impulsive inspection is used in the policy prescribed in Theorem 2. Thus, we just need to check (10). Indeed, for $t\in(\tau_{i-1},\tau_i]$ and any $i$, (21) implies that

$$\frac{dW_t^\star}{dt} = \theta W_t^\star,$$

which is essentially the binding constraint (10) because $q_t^{n\star}\equiv 0$ for all $t\geqslant 0$. □

Proof of Proposition 4. By (21), $W_t^\star$, starting from $W_0^\star = w\leqslant \bar{w}^\star$, takes $\tau_1(w) := (1/\theta)\ln(F/w)$ to reach the upper threshold $F$ when the first inspection takes place. Subsequent inspections take place at $\tau_i(w) = \tau_1(w) + (i-1)t^\star$ for $i = 2, 3, \ldots$, where $t^\star$ is given by the solution to (17).

According to the policy specified in Theorem 2, $R_t^\star \equiv \bar{r}\mathbb{1}[c\geqslant(1+\alpha)\bar{r}]$ and $q_t^{n\star}\equiv 0$ for $t\geqslant 0$, and $q_t^{m\star}=1$ if $t=\tau_i(w)$ for $i = 1, 2, \ldots$ but $q_t^{m\star}=0$ otherwise, implying the corresponding regulator's cost-to-go function to be, by (11),

$$C(w) = -\frac{\lambda}{\lambda+\theta}[c-(1+\alpha)\bar{r}]^+ + \underbrace{\mathbb{E}\left[\int_0^\infty e^{-(\lambda+\theta)t}(-\alpha\lambda W_t^\star)\,dt + k\sum_{i=1}^\infty e^{-(\lambda+\theta)\tau_i(w)} \,\middle|\, W_0^\star = w\right]}_{:=\,\tilde{C}(w)}.$$

Taking advantage of the cyclical structure of the policy specified in Theorem 2, we compute

$$\begin{aligned} \tilde{C}(w) &= \int_0^{\tau_1(w)} e^{-(\lambda+\theta)t}(-\alpha\lambda w e^{\theta t})\,dt + e^{-(\lambda+\theta)\tau_1(w)}\tilde{C}(F) \\ &= \alpha(e^{-\lambda\tau_1(w)} - 1) + \left(\frac{w}{F}\right)^{(\lambda+\theta)/\theta}\tilde{C}(F) \\ &= \frac{\tilde{C}(F)+\alpha F}{F^{(\lambda+\theta)/\theta}} w^{(\lambda+\theta)/\theta} - \alpha w. \end{aligned} \tag{B.1}$$

Since a deterministic inspection is conducted once $W_t^\star = F$ and $W_t^\star$ is instantaneously adjusted to $Fe^{-\theta t^\star}$, we must have

$$\begin{aligned} \tilde{C}(F) &= k + \tilde{C}(Fe^{-\theta t^\star}) \\ &= k + e^{-(\lambda+\theta)t^\star}[\tilde{C}(F) + \alpha F] - \alpha F e^{-\theta t^\star}, \end{aligned}$$

where we use (B.1) to compute $\tilde{C}(Fe^{-\theta t^\star})$ and obtain the last equality above. We solve from the above equality for

$$\frac{\tilde{C}(F)+\alpha F}{F^{(\lambda+\theta)/\theta}} = \frac{k + \alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})},$$

which implies that

$$\tilde{C}(w) = \frac{k+\alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})} w^{(\lambda+\theta)/\theta} - \alpha w,$$

and subsequently (22).

It is easy to see that $(k+\alpha F(1-e^{-\theta t^\star}))/(F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})) > 0$. By noting that $(\lambda+\theta)/\theta > 1$, we immediately have $C(w)$ defined in (22) is strictly convex in $w\in[0,F]$. Straightforward calculation yields

$$\begin{aligned} &\frac{d}{dw}C(Fe^{-\theta t^\star}) \\ &\quad = \frac{\lambda+\theta}{\theta}\frac{k+\alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})}F^{\lambda/\theta}e^{-\lambda t^\star} - \alpha \\ &\quad = \frac{(\lambda+\theta)(k+\alpha F)e^{-\lambda t^\star} - \alpha\lambda F e^{-(\lambda+\theta)t^\star} - \alpha\theta F}{\theta F(1-e^{-(\lambda+\theta)t^\star})} = 0, \end{aligned}$$

which follows from (17). Therefore, we obtain the second equality in (23) and that $C(w)$ is strictly increasing in $w\in[Fe^{-\theta t^\star}, F]$.

Finally, to see the first equality in (23), we note that

$$C(w)=-\frac{\lambda}{\lambda+\theta}[c-(1+\alpha)\bar{r}]^{+}+\frac{k+\alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})}w^{(\lambda+\theta)/\theta}-\alpha w$$
$$=-\frac{\lambda}{\lambda+\theta}\{[c-(1+\alpha)\bar{r}]^{+}+\alpha w\}+\frac{\theta}{\lambda+\theta}w\frac{d}{dw}C(w). \quad \square$$

Proof of Proposition 5. In this proof, we denote $\underline{w}^\star=Fe^{-\theta t^\star}<F$ and

$$K^\star:=\frac{k+\alpha F(1-e^{-\theta t^\star})}{F^{(\lambda+\theta)/\theta}(1-e^{-(\lambda+\theta)t^\star})}. \tag{B.2}$$

Then, the extended function $C(w)$ defined in the proposition can be rewritten as

$$C(w)=\begin{cases}-\dfrac{\lambda}{\lambda+\theta}[c-(1+\alpha)\bar{r}]^{+} \\ \quad +K^\star w^{(\lambda+\theta)/\theta}-\alpha w, \quad \text{for } w\in[\underline{w}^\star,F], \\ C(\underline{w}^\star), \quad \text{for } w\leqslant \underline{w}^\star,\end{cases} \tag{B.3}$$

where $C(\underline{s}^\star)$ is given by (23).

By Proposition 4, $C(w)$ defined above is bounded, nondecreasing and continuous-differentiable. By Theorem 1, therefore, $C^\star(w)=C(w)$ for $w\leqslant F$ once we demonstrate that $C(w)$ satisfies (15) with $p=1$.

*We first show* $\mathcal{N}C(w)-\lambda\alpha w-(\theta+\lambda)C(w)\geqslant 0$, where the functional operator $\mathcal{N}$ defined by (12) specializes to

$$\mathcal{N}C(w)=\min_{\substack{0\leqslant r\leqslant\bar{r},\\ w_+^l\leqslant F,\, q,\, z\geqslant 0}}\Big\{\lambda\beta r+q[k+C(w_+^l)-C(w)]+\{\theta w-q[F-w]+z\}\frac{dC(w)}{dw}\Big\}$$
$$=-\lambda[c-(1+\alpha)\bar{r}]^{+}+\min_{q,\,z\geqslant 0}\Big\{q[k+C(\underline{w}^\star)-C(w)]+\{\theta w-q[F-w]+z\}\frac{dC(w)}{dw}\Big\}, \tag{B.4}$$

where we use the fact that $C(w)$ reaches its minimal value $C(\underline{w}^\star)$ by Proposition 4 to eliminate $w_+^l$.

- For $w\leqslant \underline{w}^\star$, $C(w)\equiv C(\underline{w}^\star)$ is a constant that equals the minimal value of $C(w)$ by Proposition 4. Hence, (B.4) suggests that

$$\mathcal{N}C(w)-\lambda\alpha w-(\theta+\lambda)C(w)$$
$$=-\lambda[c-(1+\alpha)\bar{r}]^{+}+\min_{q\geqslant 0}\{kq-\lambda\alpha w-(\theta+\lambda)C(\underline{w}^\star)\}$$
$$=\lambda\alpha(\underline{w}^\star-w)\geqslant 0,$$

where we used the expression of $C(\underline{w}^\star)$ given by (23) to obtain the second equality.

- For $w\in(\underline{w}^\star,F]$, again by (B.4),

$$\mathcal{N}C(w)-\lambda\alpha w-(\theta+\lambda)C(w)$$
$$=-\lambda[c-(1+\alpha)\bar{r}]^{+}-\lambda\alpha w-(\theta+\lambda)C(w)+\theta w\frac{dC(w)}{dw}+\min_{q,\,z\geqslant 0}q\left[k+C(\underline{w}^\star)-C(w)-(F-w)\frac{dC(w)}{dw}\right], \tag{B.5}$$

which follows from the fact that $C(w)$ is increasing for $w\in(\underline{w}^\star,F]$ according to Proposition 4, implying that $z$ must be 0 at the optimum.

We now claim

$$k+C(\underline{w}^\star)-C(w)-(F-w)\frac{dC(w)}{dw}\geqslant 0, \quad \forall\, w\in(\underline{w}^\star,F]. \tag{B.6}$$

Indeed, straightforward calculation reveals

$$\frac{d}{dw}\left\{k+C(\underline{w}^\star)-C(w)-(F-w)\frac{dC(w)}{dw}\right\}=-(F-w)\frac{d^2C(w)}{dw^2}\leqslant 0,$$

because $C(w)$ is convex in $w$. Therefore, it suffices to show that (B.6) holds at $w=F$. Indeed, by (22) and (23),

$$k+C(\underline{w}^\star)-C(F)$$
$$=k-\frac{\lambda}{\lambda+\theta}\alpha Fe^{-\theta t^\star}-\frac{k+\alpha F(1-e^{-\theta t^\star})}{1-e^{-(\lambda+\theta)t^\star}}+\alpha F$$
$$=\frac{e^{-\theta t^\star}}{(\lambda+\theta)(1-e^{-(\lambda+\theta)t^\star})}\cdot[\alpha\theta F+\alpha\lambda Fe^{-(\lambda+\theta)t^\star}-(\lambda+\theta)(k+\alpha F)e^{-\lambda t^\star}]=0,$$

which follows from (17).

Now that (B.6) implies that

$$\min_{q,\,z\geqslant 0}q\left[k+C(\underline{w}^\star)-C(w)-(F-w)\frac{dC(w)}{dw}\right]=0,$$

which, in turn, reduces (B.5) to

$$\mathcal{N}C(w)-\lambda\alpha w-(\theta+\lambda)C(w)$$
$$=-\lambda[c-(1+\alpha)\bar{r}]^{+}-\lambda\alpha w-(\theta+\lambda)C(w)+\theta w\frac{dC(w)}{dw}$$
$$=-\lambda[c-(1+\alpha)\bar{r}]^{+}-\lambda\alpha w+\lambda[c-(1+\alpha)\bar{r}]^{+}-(\theta+\lambda)K^\star w^{(\lambda+\theta)/\theta}+\alpha(\theta+\lambda)w+\theta w\left[\frac{\lambda+\theta}{\theta}K^\star w^{\lambda/\theta}-\alpha\right]$$
$$=0.$$

That is, $\mathcal{N}C(w)-\lambda\alpha w-(\theta+\lambda)C(w)\geqslant 0$ holds with equality for $w\in[\underline{w}^\star,F]$.

*We then show* $\mathcal{M}C(w)-C(w)\geqslant 0$, where the functional operator $\mathcal{M}$ is defined by (13), which, in the case of $p=1$, specializes to

$$\mathcal{M}C(w):=\min_{w_+,\,q^m\in[0,1]}\{q^m(k+C(\underline{w}^\star))+(1-q^m)C(w_+)\} \quad \text{subject to} \quad (1-q^m)w_++q^mF\geqslant w. \tag{B.7}$$

- For $w \leqslant \underline{w}^\star$, we must have $(1-q^m)\underline{w}^\star + q^m F \geqslant w$. Thus, the fact that $\underline{w}^\star$ minimizes $C(w)$ suggests that

$$C(w) \equiv C(\underline{w}^\star) \leqslant q[k + C(\underline{w}^\star)] + (1-q)C(\underline{w}^\star), \quad \forall q \in [0,1].$$

Therefore, by definition (B.7), we must have $\mathscr{M}C(w) - C(w) \geqslant 0$.

- For $w \in [\underline{w}^\star, F]$, we define, for $q \in [0,1]$,

$$\Upsilon(q \mid w) := \min_{w_+ \leqslant F} q(k + C(\underline{w}^\star)) + (1-q)C(w_+) \tag{B.8}$$

$$\text{subject to} \quad (1-q)w_+ + qF \geqslant w. \tag{B.9}$$

It is straightforward to see that $\mathscr{M}C(w) = \min_{q\in[0,1]} \Upsilon(q \mid w)$. Since $C(\cdot)$ is convex increasing and reaches its minimum at $\underline{w}^\star$, we consider the following two cases.

—If $q \geqslant (w - \underline{w}^\star)/(F - \underline{w}^\star)$, then $(1-q)\underline{w}^\star + qF \geqslant w$, i.e., the constraint (B.9) will not bind at $w_+ = \underline{w}^\star$. Therefore, $\Upsilon(q \mid w) = qk + C(\underline{w}^\star)$, which is increasing in $q$.

—If $q \leqslant (w - \underline{w}^\star)/(F - \underline{w}^\star)$, then $(1-q)\underline{w}^\star + qF < w$, then (B.9) must be binding and

$$\Upsilon(q \mid w) = q(k + C(\underline{w}^\star)) + (1-q)C\left(\frac{w - Fq}{1-q}\right). \tag{B.10}$$

Direct calculation reveals that

$$\frac{d}{dq}\Upsilon(q \mid w) = k + C(\underline{w}^\star(q,w)) - C\left(\frac{w-Fq}{1-q}\right) - \left[F - \frac{w-Fq}{1-q}\right]\frac{d}{dw}C\left(\frac{w-Fq}{1-q}\right) \geqslant 0,$$

which follows from the fact that $(w - Fq)/(1-q) \geqslant \underline{w}^\star$ and (B.6). That is, $\Upsilon(q \mid w)$ is again nondecreasing in $q$. Therefore, $\mathscr{M}C(w) = \min_{q\in[0,1]} \Upsilon(q \mid w) = \Upsilon(0 \mid w) = C(w)$ and hence $\mathscr{M}C(w) = C(w)$ for $w \in [\underline{w}^\star, F]$. □

Proof of Corollary 1. This corollary is a special case of Proposition EC.5 when $p = 1$. □

## Endnotes

1. This result corresponds to transferring a reward equal to the consumer surplus in the economic literature on regulation, which directly achieves first best in the absence of frictions (Laffont and Tirole 1993).
2. To implement CERCLA and RCRA, the U.S. EPA has established comprehensive cleanup standards for various types of hazards (see http://www.epa.gov/superfund/policy/index.htm for more details). For example, EPA (2004) provides cleanup standards for groundwater, and EPA (1998) provides remediation goals for radioactively contaminated soils. In particular, EPA (1996) describes the role of cost in adjusting remediation levels. As documented on http://www.cleanuplevels.com/, different states have also established comparable cleanup standards for various environmental hazards.
3. For example, when the hazard's effect is deterministic, environmental cost $\bar{c}$ can be expressed as $\bar{c} = \int_0^\infty e^{-\theta s}\bar{\delta}\,ds$, where $\bar{\delta}$ is the instantaneous damage at time $s$ in the absence of any repair, after the hazard has already lasted for $s$ units of time. Similarly, if a full remediation with $R = \bar{r}$ decreases the damage rate to $\underline{\delta} < \bar{\delta}$, the resulting minimum environmental cost to society is equal to $\bar{c} - c = \int_0^\infty e^{-\theta s}\underline{\delta}\,ds$. For a partial remediation $(R < \bar{r})$, the environmental cost to society is then $\bar{c} - cR/\bar{r} = \int_0^\infty e^{-\theta s}[(1 - R/\bar{r})\bar{\delta} + R/\bar{r}\underline{\delta}]\,ds$. (Similar expressions can be obtained when the hazard's effect is random.)
4. Note, also, that in most practical situations, even though the regulator is able to verify whether the hazard has occurred, it cannot verify when it occurred, as in our setup.
5. As demonstrated in Lemma A.1, the mechanism design framework allows us to restrict our search for the optimal regulation policy within the set of direct regulatory policies that always induce prompt disclosure of a hazard from the firm. As such, the detection of a hazard by an inspection is an off-equilibrium outcome, and, hence, the regulator, even given the option to set the remediation level and the reward to the firm, will always impose on the firm the highest possible cost, equal to the firm's limited liability $F$, if an inspection detects a hazard (see the proof of Lemma A.1 for a formal argument).
6. Note that all previous inspection results must indicate no hazard for $\tau_i \in \mathcal{I}_t$. It is worth pointing out that $\mathcal{I}_t$ can be an off-equilibrium history and be different from the inspection epochs under policy $\mathcal{R}$.
7. Since $R_t$, $S_t$, $Q_t$ are the regulator's decision, they need to be predictable with respect to the natural filtration. In this paper, therefore, we assume that they are left-continuous in time $t$.
8. If the regulator generates revenue, i.e., she incurs negative regulatory cost, the interpretation of $\alpha$ still holds as long as it is negligible compared to the overall economy. More specifically, negative regulatory cost means a reduction of taxes that would otherwise be collected elsewhere in the economy. A reasonable estimate for $\alpha$ is significantly positive in the magnitude of 0.3 for the U.S. economy (see, e.g., Ballard et al. 1985, Jones et al. 1990) for empirical estimations of $\alpha$.
9. Following control theory convention, we use upper case letters $W_t$, $W_{t+}$, and $W_{t+}^I$ to represent the PDP state variables, while using lower case letters $w$, $w_+^I$, and $w_{t+\Delta t}$ to represent their values at a specific time.

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