# Time-based Competition with Benchmark Effects

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May 9, 2012

**Abstract**

We consider a duopoly where firms compete on waiting times in the presence of an industry benchmark. The demand captured by a firm depends on the gap between the firm's offer and the benchmark. We refer to the benchmark effect as the impact of this gap on demand. The formation of the benchmark is endogenous and depends on both firms' choices. When the benchmark is equal to the shorter of the two offered delays, we characterize the unique Pareto Optimal Nash equilibrium. Our analysis reveals a *stickiness effect* by which firms equate their delays at the equilibrium when the benchmark effect is strong enough. When the benchmark corresponds to the average of the two offered delays, we show the existence of a pure Nash equilibrium. In this case, we reveal a *reversal effect*, by which the market leader, in terms of offered shorter delay, becomes the follower when the benchmark effect is strong enough. In both cases, we show that customers benefit when an industry benchmark exists because equilibrium waiting times are shorter. Our models also capture customers' loss aversion, which, in our setting, states that demand is more sensitive to the gap when the delay is longer than the benchmark (loss) rather than shorter (gain). We characterize the impact of this loss aversion on the equilibrium in both settings.

**Keywords:** Waiting Time Competition, Benchmark Effect, Loss Aversion, Queues, Game Theory

# 1 Introduction

Service firms often compete on waiting times (see Allon and Federgruen (2007) and references therein). In this classical context, a firm adjusts the expected delay it offers to the market in order to attract additional demands so as to maximize profit. Customers, in turn, derive their utility from the waiting time they experience at the firm they choose to join. However, customer satisfaction may also directly depend on other offers in the competition. For instance, a given level of delay can be more or less dissatisfying depending on whether it is longer or shorter than some industry benchmark. This is because customers are subject to reference effects (Kahneman and Tversky 1979) when evaluating waiting times (Leclerc et al. 1995) or, more generally, service quality (Cadotte et al. 1987). When setting an expected waiting time, a firm also influences the benchmark against which customers evaluate the delays they experience at other firms. The main contribution of this paper is to offer a theoretical analysis on how benchmark effects such as these influence waiting time competition between firms.

To that end, we consider a simple duopoly where firms offer expected waiting times for their service by adjusting their service rates at a capacity cost. Demand is then split between firms according to the different levels of customer satisfaction. More precisely, the customer's satisfaction at a firm is a function of both firms' expected waiting times as in the classical duopoly but also of the gap between the firm's offered delay and the benchmark. We refer to the *benchmark effect* as the impact the gap has on demand. Our model also allows for customers to be loss averse, in the sense that a positive gap, i.e. when the offered delay exceeds the benchmark, has a greater impact on satisfaction than the corresponding negative gap, i.e. when the benchmark exceeds the delay by the same amount. Thus, a firm's decision has a direct effect on demand through its choice of waiting time, and an indirect one through the benchmark effect. Following Cadotte et al. (1987), we consider two different settings depending on how the benchmark is formed. In the first, the benchmark equals the smaller of the two offered waiting times, while in the second, the benchmark corresponds to the average. Without the benchmark effect, our model corresponds to a classical set-up of time-based competition.

This situation gives rise to a game in which each firm strategically chooses a waiting time in order to maximize profit. We show the existence of and characterize the Nash equilibriums for this problem. We find that the presence of a benchmark benefits customers, in the sense that customers experience shorter delays in the equilibrium with benchmark effect compared to the classical duopoly case without benchmark. In fact, the expected waiting times in equilibrium become smaller as the benchmark effect gets stronger.

In our first setting, where the benchmark corresponds to the smaller expected waiting time, our analysis reveals a *stickiness effect* by which firms equate their offers at the equilibrium as long as the ratio of their profit margins belongs to a given interval. By contrast, in the absence of the benchmark, the offered waiting times are equal only when the ratio takes a specific value. We further show that, while the interval expands as the benchmark effect intensifies, it may shrink or expand with the level of loss aversion depending on whether the benchmark effect is strong enough. In other words, the intensity of the benchmark effect changes the direction of the impact of loss aversion on the equilibrium waiting time.

In our second setting, where the benchmark corresponds to the average waiting time between the two firms, we identify a *reversal effect* in the sense that the leader in a duopoly without benchmark effect, in terms of offering shorter waiting time, can become the follower (offering longer waiting time) if the benchmark effect is strong enough. The market leader is determined by a threshold on the profit-margin ratio, which is monotone in both benchmark effect and loss aversion effects. Similar to the minimum benchmark case, the threshold may either decrease or increase with the level of loss aversion depending on the intensity of the benchmark effect.

The presence of a benchmark effect is supported by results of prospect theory (Kahneman and Tversky 1979), which demonstrates that people generally view a final outcome as a gain or a loss with respect to a certain context-dependent reference point. While the original applications of prospect theory mainly studied people's preference towards monetary payoffs (Niedrich et al. 2001, Erdem et al. 2001, Popescu and Wu 2007), empirical study (Leclerc et al. 1995) demonstrates that a similar reference effect also exists in waiting times. In our context, the service benchmark determines the waiting time reference point. The benchmark effect corresponds then to the gap between the benchmark and the actual waiting time. In other words, customers might experience "waiting time gain" if the actual waiting time is shorter than the benchmark, and "waiting time loss" otherwise.

The benchmark effect is also highly consistent with the stream of research on service quality and customer satisfaction. According to Anderson and Sullivan (1993), for instance, customer satisfaction from a service is a function of perceived quality as well as the gap between customers' expectation (reference point) and perceived quality. Moreover, when perceived quality falls short of expectation, the gap has a greater impact on satisfaction than the corresponding gap when quality exceeds expectation, a result consistent to Prospect Theory's loss aversion. Lin et al. (2008) empirically studied the impact of the service quality gap on customers' behavior intentions. They show that a customer's behavior depends on the service quality gap according to a value function that is "kinked" at the reference point, with the loss of service quality influencing the customer's behavior more than does a corresponding service quality gain. This is also consistent with prospect theory.

Empirical studies further suggest how firms' decisions might determine together the benchmark, i.e. the reference point (see Zeithaml et al. (1993)). For instance, Cadotte et al. (1987) show that two types of comparisons better explain customer satisfaction with service quality: "best brand norm" and "product type norm." According to the "best brand norm" case, customers select the best brand in the category as their reference. The "product type norm" case, on the other hand, represents the setting where the average performance is perceived as typical of a group of similar brands. This case is also consistent with the adaptation-level theory where the reference standard is perceived as the mean of the stimuli presented within a contextual set (Helson 1964, Wedell 1995). In our setting, service quality corresponds to offered waiting time and, therefore, the minimum and average delays between the two firms correspond to the best brand and product type norms in the customer satisfaction literature, respectively.[1]

This work is closely related to the literature on competition between service firms

[1]Theoretically, the product type norm should represent the weighted average of the two firms' waiting times, with the weights to be each firm's demand rate. However, such weights are endogenous and will make the analysis much harder. We thus use the simple average to avoid the unnecessary technicality while still capturing the essence.

when customers are sensitive to waiting time, or service quality. Service competition has been a subject of many studies (see Hassin and Haviv (2002) for a comprehensive review in queueing settings). In some of these models (De Vany and Saving 1983, Kalai et al. 1992, So 2000, Cachon and Harker 2002, Ho and Zheng 2004, Allon and Federgruen 2007), firms compete in both waiting times and prices either in an aggregated form (full price) or as separate attributes. Gaur and Park (2007), Liu et al. (2007) and Hall and Porteus (2000), instead, consider the situations in which customers' demand solely depends on waiting time, or service quality. In fact, certain industries experience a higher level of price rigidity compared to their ability to vary service rates. Blinder et al. (1998) provides extensive empirical evidence. For example, half of the businesses in their study change prices no more than once per year. Among all industries under study, service companies adjust prices the most slowly. Therefore, in this paper we focus on waiting time competition, and leave price as exogenous.

Our model is most closely related to Allon and Federgruen (2007), which investigates service competition in both service levels and prices. In the absence of the benchmark effect, our model corresponds to a special scenario of their "price first" case, for which our main findings disappear. Ho and Zheng (2004) also study a similar duopoly competition in waiting time announcements when demand is affected by service quality. The paper studies the case where firms do not need to comply to the waiting times they offer as Allon and Federgruen (2007) and we do. On the other hand, Ho and Zheng (2004) do consider benchmark effects and loss aversion, the main focus of our paper.

Our paper also contributes to the emerging literature on competition between firms with customer reference effects. Heidhues and Koszegi (2008) study price competition when customers base their reference on their recent expectations about the product. They have shown the existence of focal price equilibrium with the presence of the reference effect. Zhou (2011) also examines firms' price competition, yet with customers' reference point based more on the "prominent" firm. In this setting the equilibrium price randomizes between high and low levels. These two papers consider price competition rather than service operations competition. They also consider exogenous reference points while the benchmark is endogenously determined by the firms' strategies in our set-up.

The remainder of the paper is organized as follows. Section 2 presents the minimum benchmark model and the corresponding waiting time competition game. After characterizing the unique (Pareto) Nash equilibrium structure, we analyze the "stickiness" effect of equilibrium waiting times, and how they are affected by the benchmark and loss aversion effects. Section 3 analyzes the average benchmark model, its Nash equilibrium characterization and the impacts of the benchmark and loss aversion effects on the "reversal" phenomenon. We conclude the paper and discuss future research directions in Section 4.

## 2 The Minimum Benchmark Case

Consider two competing firms, each acting as an $M/M/1$ facility. We assume that firm $i$ chooses and commits to an expected waiting time $w_i$. That is, for a given demand arrival rate $\lambda_i$, firm $i$ offers service rate $\mu_i$ to match the waiting time commitment $w_i = 1/(\mu_i - \lambda_i)$. Therefore, firm $i$ has to offer capacity $\mu_i = \lambda_i + 1/w_i$. We use subscript $-i$ to represent the firm competing with firm $i$.

Empirical evidence from Cadotte et al. (1987) indicates that customers sometimes use a "best brand norm" to form a benchmark of service quality, i.e. customers select the best brand in the category as their benchmark. In our setting, this corresponds to the shorter waiting time, or, $r \equiv \min(w_i, w_{-i})$, where $r$ denotes the waiting time benchmark. We study the average case in the following section.

Define then $s(t, r)$ to be the customer satisfaction when she waits $t$ units of time. Following the marketing literature (see, for instance, Anderson and Sullivan (1993)), we assume that the customer satisfaction with a service depends on service quality and the gap between a customer's expectation and the actual service quality. In our context, service quality corresponds to the experienced waiting time $t$, while customer expectation is set by benchmark $r$. We can then assume that customer satisfaction is given by $s(t, r) = f_1(t) + f_2(r - t)$, where function $f_1$ represents the direct impact of the waiting time (the service quality), and $f_2$ the indirect impact of the benchmark (the gap). For simplicity, we assume that both $f_1$ and $f_2$ are piecewise linear functions, such that

$$s\,(t,r) = -\alpha t + \begin{cases} \beta_r (r-t)\ , & \text{if } t < r\ ; \\ \beta_r \beta_l (r-t)\ , & \text{if } t \geq r\ , \end{cases}$$

where $\alpha$ and $\beta_r > 0$ represent customers' time sensitivity for the direct impact ($f_1$) and the benchmark effect ($f_2$), respectively. On the other hand, $\beta_l > 1$ captures the level of loss aversion (Kahneman and Tversky (1979)), which, in our context, states that customers are more sensitive to waiting times that exceed the benchmark.

We refer to $S_i$ as Firm $i$'s aggregate satisfaction level, which corresponds to the expectation of $s_i(t,r)$ with respect to waiting time $t$. In the $M/M/1$ queueing setting, $t$ follows an exponential distribution with the rate parameter $1/w_i$. And we have,

$$S_i(w_i, r) = E_t[s_i(t,r)] = -\alpha w_i + \beta_r \left[ (r - w_i) - e^{-r/w_i} \left( \beta_\ell - 1 \right) w_i \right]\ .$$

Following Allon and Federgruen (2007), we assume that firm $i$'s demand rate $\lambda_i$ is affected by both customers' aggregate satisfaction level of firm $i$ and of firm $-i$ in a linear fashion:

$$\lambda_i = a_0 + a_i S_i(w_i, r) - a_{-i,i} S_{-i}(w_{-i}, r)\ .$$

Firm $i$'s demand rate increases in the satisfaction level of its own customers, and decreases in firm $-i$'s satisfaction level. Parameters $a_i$ and $a_{-i,i}$ represent the impacts of these two attributes. We assume that $a_i > a_{-i,i} > 0$, so that a firm's own attribute has a larger impact on its demand. Note that without the benchmark effect ($\beta_r = 0$), the demand model reduces to one of the models in Allon and Federgruen (2007). Further following Allon and Federgruen (2007), we assume that the model parameters are such that $\lambda_i$ is guaranteed to be positive to keep the model simple.

Furthermore, Firm $i$ incurs capacity cost $c_i$ per unit of service rate. The price of the service, denoted $p$, is identical across both service firms. Technically, all our results still hold when the two firms' prices are different. Denote quantity $\rho_i = (p - c_i)/c_i$ to represent firm $i$'s profit margin. Since $r = \min(w_i, w_{-i})$, demand rate $\lambda_i$ also depends on $w_i$ and $w_{-i}$ and we can define firm $i$'s profit as:

$$P_i(w_i, w_{-i}) \equiv c_i \left( \rho_i \lambda_i(w_i, w_{-i}) - \frac{1}{w_i} \right)\ . \tag{1}$$

Firm $i$'s objective is then to choose $w_i$ in order to maximize $P_i(\cdot, w_{-i})$.

The following proposition reveals important properties of profit function $P_i$, which will prove useful to analyzing and providing insight into the competition between the two firms.

**Proposition 1.** *Firm i's profit $P_i$, defined in Eq. (1), has the following two structural properties:*

1. *$P_i$ is a quasi-concave function of waiting time $w_i$; and*

2. *$P_i$ is supermodular when $w_i > w_{-i}$, and submodular when $w_i < w_{-i}$.*

*Proof.* See Appendix A ■

The idea of the proof for quasi-concavity is as follows. We first show that firm $i$'s profit $P_i$ is concave in $w_i$ when $w_i \geq w_{-i}$, and quasi-concave in $w_i$ when $w_i < w_{-i}$. Hence $P_i$ can be reduced to the four cases shown in Figure 1. In all cases but case (4), $P_i$ is quasi-concave. Furthermore, we show that if the left derivative of $P_i$ with respect to $w_i$ at $w_i = w_{-i}$ is negative, then the right derivative cannot be positive, so that case (4) is not possible.

As for the supermodularity and submodularity properties, when $w_i > w_{-i}$, increasing $w_{-i}$ lowers customers' expectations, making them easier to satisfy. This provides incentive for firm $i$ to extend its waiting time $w_i$, which leads to the supermodularity of $P_i$. On the other hand, when $w_i < w_{-i}$, the increase of $w_{-i}$ makes firm $-i$ look even worse, and thus increases firm $i$'s demand. This makes it more affordable for firm $i$ to lower its waiting time $w_i$, which leads to the submodularity of $P_i$.

## 2.1 Nash Equilibrium

Given the choice of the other firm, each firm set its waiting time so as to maximize its profit. Because demands and hence profits depend on both firm decisions, the situation gives rise to a game. In the following, we show the existence of and fully characterize the Nash equilibrium for this game.

Using Proposition 1, we first study firm $i$'s best response curve as a function of $w_{-i}$. The quasi-concavity of $P_i$ implies that the first-order condition is a sufficient condition

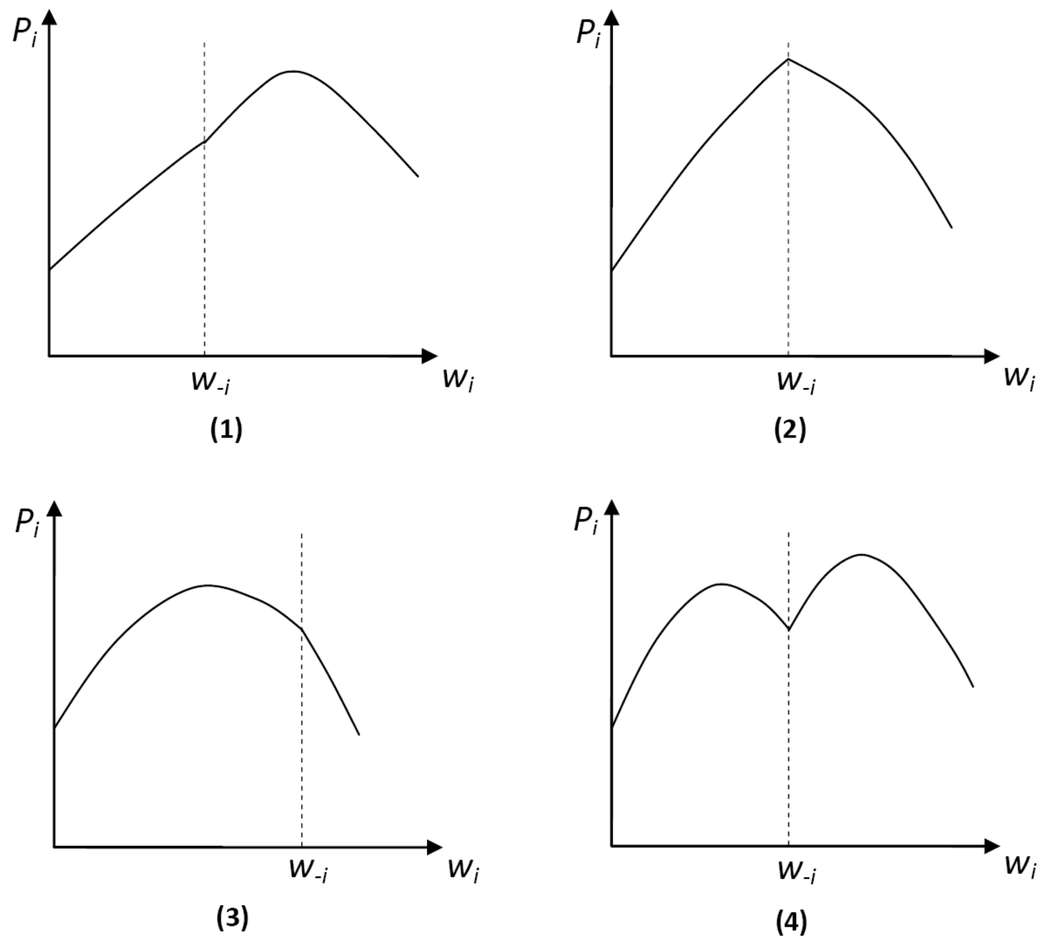


Figure 1: This figure depicts the four possibilities of a function of $w_i$ that is concave when $w_i \leq w_{-i}$ and quasi-concave when $w_i > w_{-i}$. In the proof of Proposition 1, we show that case (4) is not a possible scenario for the profit function $P_i$, which implies that $P_i$ is a quasi-concave function.

for firm $i$'s best response. However, since $r = \min(w_i, w_{-i})$, firm $i$'s demand function $\lambda_i$ is not differentiable at $w_i = w_{-i}$. Nonetheless, we can define the left and right derivatives of $\lambda_i$ with respect to $w_i$ when $w_i = w_{-i}$, respectively, as

$$\underline{\delta}_i \equiv -\left.\frac{\partial \lambda_i}{\partial w_i}\right|_{w_i \uparrow w_{-i}} = \left[a_i\alpha + a_{-i,i}\beta_r + (a_i + a_{-i,i})e^{-1}\beta_r(\beta_l - 1)\right] ,$$

$$\overline{\delta}_i \equiv -\left.\frac{\partial \lambda_i}{\partial w_i}\right|_{w_i \downarrow w_{-i}} = \left[a_i\alpha + a_i\beta_r + 2a_ie^{-1}\beta_r(\beta_l - 1)\right] .$$

Partial derivatives $\underline{\delta}_i$ and $\overline{\delta}_i$ represent the impact of waiting time decision $w_i$ to arrival rate $\lambda_i$, when $w_i$ approaches the competitor's decision $w_{-i}$ from below and above, respectively. Clearly, $\underline{\delta}_i < \overline{\delta}_i$ when $\beta_r > 0$, which means that the marginal demand increase from setting a shorter waiting time is smaller than the marginal demand decrease from setting a longer waiting time. Given these two quantities, firm $i$'s best response curve is determined by the following Proposition:

**Proposition 2.** *Firm $i$'s best response curve, $w_i^\star(w_{-i})$, is a piecewise function on the following three intervals:*

1. *When $w_{-i} \in [0, \underline{w}_{-i})$, $w_i^\star(w_{-i}) > w_{-i}$ and is an increasing function of $w_{-i}$;*
2. *When $w_{-i} \in [\underline{w}_{-i}, \overline{w}_{-i}]$, $w_i^\star = w_{-i}$;*
3. *When $w_{-i} \in (\overline{w}_{-i}, \infty)$, $w_i^\star(w_{-i}) < w_{-i}$ and is a decreasing function of $w_{-i}$; furthermore, $w_i^\star$ converges to a limit $L_i > 0$;*

*where $\underline{w}_{-i} = \left(\rho_i\overline{\delta}_i\right)^{-\frac{1}{2}}$ and $\overline{w}_{-i} = (\rho_i\underline{\delta}_i)^{-\frac{1}{2}}$.*

We provide the proof of Proposition 2 in Appendix B. The three cases of Proposition 2 correspond to cases (1), (2) and (3) in Figure 1, respectively. Firm $i$'s best response is calculated from the first-order condition and is illustrated in Figure 2.

Case (1) in Figure 1 illustrates the scenario in which $w_{-i}$ is below $\underline{w}_{-i}$. In this case, the competitor's waiting time is so short that decreasing $w_i$ to below $w_{-i}$ costs firm $i$ too much to be offset by the gain in revenue. Hence, firm $i$ chooses a waiting time $w_i^\star$ longer than $w_{-i}$. Furthermore, $w_i^\star$ increases in $w_{-i}$ due to the supermodularity of profit function $P_i$.

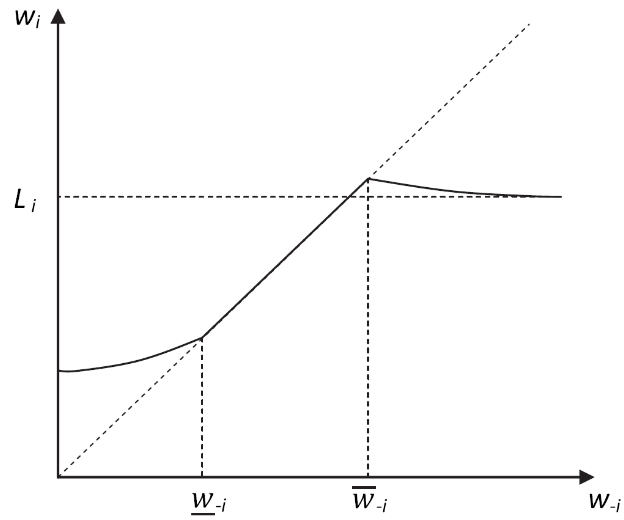


Figure 2: This figure depicts the properties of firm $i$'s best response curve following Proposition 2.

Case (3) in Figure 1, on the other hand, illustrates the scenario in which $w_{-i}$ is longer than $\overline{w}_{-i}$. In this case the competitor's waiting time $w_{-i}$ is so long that the marginal cost increase for firm $i$ to set $w_i$ to be below $w_{-i}$ can be offset by the revenue increase. It follows that $w_i^\star < w_{-i}$. Furthermore, $w_i^\star$ is decreasing in $w_{-i}$ since $P_i$ is submodular.

Case (2) in Figure 1 illustrates the scenario in which $w_{-i}$ is between $\underline{w}_{-i}$ and $\overline{w}_{-i}$. In this case firm $i$ maximizes profit when matching firm $-i$'s waiting time.

In other words, bound $\underline{w}_{-i}$ can be interpreted as the shortest waiting time that firm $i$ is able to match, while $\overline{w}_{-i}$ corresponds to the longest waiting time that firm $i$ is willing to match.

We are now ready to state one of our main results, which characterizes the equilibrium of the game.

**Theorem 1.** *The game has the following three equilibrium scenarios:*

1. $\underline{w}_i > \overline{w}_{-i}$. *There exists a unique Nash equilibrium* $(w_i^*, w_{-i}^*)$ *with* $w_i^* < w_{-i}^*$*;*

2. $\underline{w}_i \leq \overline{w}_{-i}$ *and* $\overline{w}_i \geq \underline{w}_{-i}$. *Any strategy profile* $(w_i^*, w_{-i}^*)$ *such that*

$$w_i^* = w_{-i}^* \in [\max(\underline{w}_i, \underline{w}_{-i}), \min(\overline{w}_i, \overline{w}_{-i})]$$

*is a Nash equilibrium. In particular,*

$$w_i^* = w_{-i}^* = \min(\overline{w}_i, \overline{w}_{-i})$$

*is the Pareto Nash equilibrium;*

*3.* $\overline{w}_i < \underline{w}_{-i}$*. There exists a unique Nash equilibrium* $(w_i^*, w_{-i}^*)$ *with* $w_i^* > w_{-i}^*$*.*

*Proof.* The quasi-concavity of $P_i$ implies that there exists a pure strategy Nash equilibrium (Fudenberg and Tirole 1991). The first two scenarios presented in Theorem 1 are illustrated in Figures 3 and 4.

The scenario with $\underline{w}_i > \overline{w}_{-i}$ corresponds to Figure 3. Given that $w_i^*(w_{-i})$ decreases in $w_{-i}$ when $w_{-i} > \overline{w}_{-i}$ and is bounded below by a positive lower bound, while $w_{-i}^*(w_i)$ increases in $w_i$ when $w_i < \underline{w}_i$, there exists a unique Nash equilibrium $(w_i^*, w_{-i}^*)$ with strategy profile $w_i^* < w_{-i}^*$, which can be characterized by the following first order conditions:

$$\begin{cases} (p-c_i)\left[a_i\alpha + a_i e^{-1}\beta_r(\beta_\ell - 1) + a_{-i,i}\beta_r\left(1 + e^{-w_i^*/w_{-i}^*}(\beta_\ell - 1)\right)\right] = c_i/w_i^{*2}\ , \\ (p-c_{-i})\left[a_{-i}\alpha + a_{-i}\beta_r + a_{-i}e^{-w_i^*/w_{-i}^*}\beta_r(\beta_\ell - 1)\left(1 + w_i^*/w_{-i}^*\right)\right] = c_{-i}/w_{-i}^{*2}\ . \end{cases}$$

The scenario with $\underline{w}_i \le \overline{w}_{-i}$ and $\overline{w}_i \ge \underline{w}_{-i}$ corresponds to Figure 4. Any point in the interval $\left[\max(\underline{w}_i, \underline{w}_{-i}), \min(\overline{w}_{-i}, \overline{w}_i)\right]$ is a Nash equilibrium. We next show that a unique Pareto optimal Nash equilibrium exists. Let $P_i(w_i, w_{-i})$ be firm $i$'s profit function and $w^* = \min(\overline{w}_{-i}, \overline{w}_i)$, then

$$P_i(w^*, w^*) > P_i(w, w^*) > P_i(w, w)$$

for any $w < w^*$. The first inequality is based on the fact that $(w^*, w^*)$ is an equilibrium, while the second inequality occurs because it is always better for firm $i$ to have firm $-i$ choosing longer waiting times since

$$\frac{\partial P_i(w_i, w_{-i})}{\partial w_{-i}} = (p-c_i)\left[a_{-i,i}\alpha + a_{-i,i}\left(\beta_r + e^{-w_i/w_{-i}}(\beta_l - 1)\left(1 + \frac{w_i}{w_{-i}}\right)\right)\right] > 0$$

for any $w_i \le w_{-i}$. The same inequalities apply to firm $-i$'s profit function. Therefore,

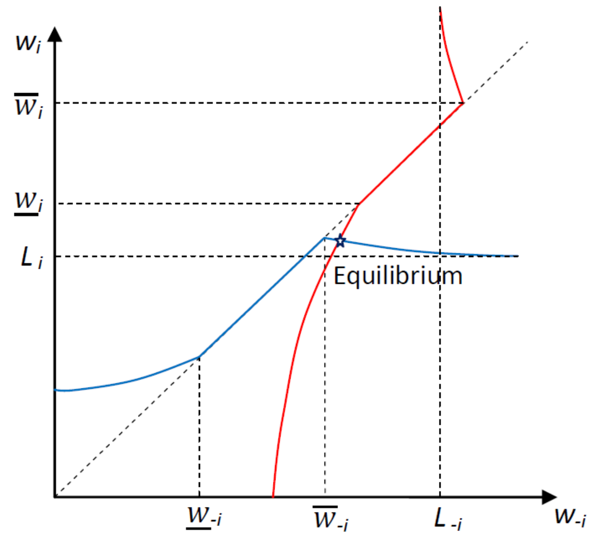


Figure 3: This figure depicts the first scenario of the pure strategy Nash equilibrium in Theorem 1. The blue curve is firm $i$'s response curve, while the red curve is firm $-i$'s response curve. They have one intersection below the forty five degree line, which is represented by the blue star.

$w_i^* = w_{-i}^* = \min(\overline{w}_i, \overline{w}_{-i})$ is the unique Pareto optimal Nash equilibrium.

The third scenario is a symmetric mirror reflection of Figure 3. ■

Note that in scenario 2 of Theorem 1, when there are multiple equilibria, the one with the longest waiting time is Pareto optimal. In this scenario, the two firms match each other's waiting times, and they both set the benchmark. But with longer waiting times, both firms can reduce their operation costs. Therefore, both firms prefer the longest possible benchmark.

## 2.2 The Stickiness Effect

One important insight from Theorem 1 is that for a range of parameters, both firms equate their waiting times. In this section, we explore further this *stickiness effect*. More generally we study the impact of the benchmark and loss aversion on the equilibrium. To that end, we consider first the situation without the benchmark effect, that is, when $\beta_r = 0$. In this setting, the demand function becomes linear and the model reduces to a special case of Allon and Federgruen (2007). Each firm's best response is then constant over the other firm's decision and the game is effectively degenerate, as stated in the

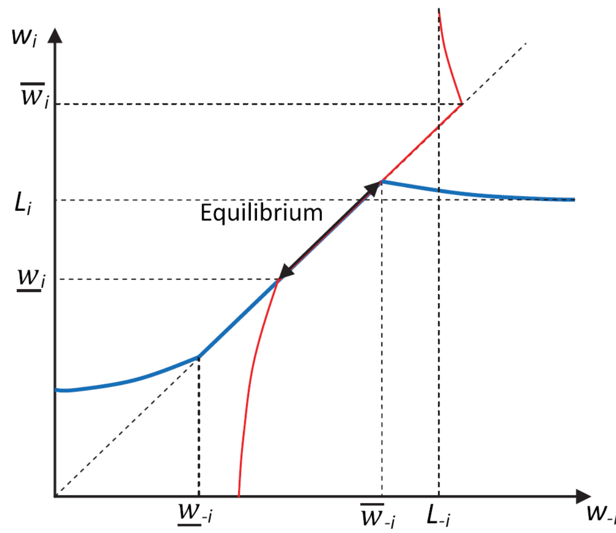


Figure 4: This figure depicts the second scenario of the pure strategy Nash equilibrium in Theorem 1. The blue curve is firm $i$'s response curve, while the red curve is firm $-i$'s response curve. The double-sided arrow represents the intersection of the two responses curves.

following proposition,

**Proposition 3.** *When $\beta_r = 0$, the game is degenerate and the firms' optimal strategies are*

$$w_i^0 = (\rho_i a_i \alpha)^{-\frac{1}{2}} \quad , \quad w_{-i}^0 = (\rho_{-i} a_{-i} \alpha)^{-\frac{1}{2}} \quad .$$

According to Proposition 3, in the absence of a benchmark, firms $i$ and $-i$ choose different waiting times unless $\rho_{-i}/\rho_i = a_i/a_{-i}$. To highlight the impact of model parameters on equilibrium waiting times, we restate Theorem 1 as the following corollary:

**Corollary 1.** *There exist two thresholds, $\underline{R}(\beta_r, \beta_l)$ and $\overline{R}(\beta_r, \beta_l)$, such that the structure of the equilibrium strategies can be characterized by the following three intervals:*

1. *If $\rho_{-i}/\rho_i < \underline{R}(\beta_r, \beta_l)$, then $w_i^* < w_{-i}^*$;*
2. *If $\underline{R}(\beta_r, \beta_l) \leq \rho_{-i}/\rho_i \leq \overline{R}(\beta_r, \beta_l)$, then $w_i^* = w_{-i}^*$; and*
3. *If $\rho_{-i}/\rho_i > \overline{R}(\beta_r, \beta_l)$, then $w_i^* > w_{-i}^*$;*

*where*

$$\underline{R} = \frac{\theta_3 \alpha + (\theta_1 + \theta_3) e^{-1} \beta_r (\beta_l - 1) + \theta_1 \beta_r}{\alpha + 2e^{-1} \beta_r (\beta_l - 1) + \beta_r} \; < \; \overline{R} = \frac{\theta_3 \left[\alpha + 2e^{-1} \beta_r (\beta_l - 1) + \beta_r\right]}{\alpha + (1 + \theta_2) e^{-1} \beta_r (\beta_l - 1) + \theta_2 \beta_r} \; ,$$

*and*

$$\theta_1 = a_{-i,i}/a_{-i}\ ,\ \theta_2 = a_{i,-i}/a_{-i}\ <\ \theta_3 = a_i/a_{-i}\ . \tag{2}$$

The complete derivation from Theorem 1 is in Appendix C. Corollary 1 shows that, with the benchmark effect, the two firms will match each other's waiting time at the equilibrium as long as their profit margin ratio is within a certain range. We call this phenomenon the *stickiness effect*. It is similar to the "focal price equilibrium" in a different setting involving price competition with reference effect, as shown in Heidhues and Koszegi (2008). We denote $\left[\underline{R}, \overline{R}\right]$ as the stickiness interval. The following result determines the impact of the benchmark effect $\beta_r$ and the loss aversion effect $\beta_l$ on the stickiness interval.

**Proposition 4.** *$\underline{R}(\beta_r, \beta_l)$ and $\overline{R}(\beta_r, \beta_l)$ have the following structural properties:*

1. *When $\beta_r = 0$, $\underline{R} = \theta_3 = \overline{R}$; when $\beta_r > 0$, $\underline{R} < \theta_3 < \overline{R}$;*

2. *For fixed $\beta_l$, $\underline{R}$ is decreasing in $\beta_r$; $\overline{R}$ is increasing in $\beta_r$;*

3. *For fixed $\beta_r$,*

   - *If $\beta_r < \alpha$, $\underline{R}$ is decreasing in $\beta_l$; $\overline{R}$ is increasing in $\beta_l$;*
   - *If $\beta_r = \alpha$, $\underline{R} = (\theta_1 + \theta_3)/2$ and $\overline{R} = 2\theta_3/(1 + \theta_2)$, which does not change with $\beta_l$;*
   - *If $\beta_r > \alpha$, $\underline{R}$ is increasing in $\beta_l$; $\overline{R}$ is decreasing in $\beta_l$.*

*Proof.* See Appendix D ■

Proposition 4 essentially states that the stickiness interval expands with $\beta_r$, but may either expand or shrink with $\beta_l$ depending on the relative strength of the benchmark effect ($\beta_r$) compared to the direct impact of waiting time ($\alpha$).

Figure 5 depicts the effect of $\beta_r$ on the equilibrium for a particular example. Without the benchmark effect, i.e. when $\beta_r = 0$, the firms equate their waiting times if and only if $\rho_{-i}/\rho_i = \theta_3 = 1$. As the benchmark effect gets stronger, i.e. as $\beta_r$ increases, both firms are increasingly incentivized to influence the benchmark. Since a firm cannot influence the benchmark as long as its waiting time is longer than the other firm's, both firms match each other's offers and the stickiness interval expands.

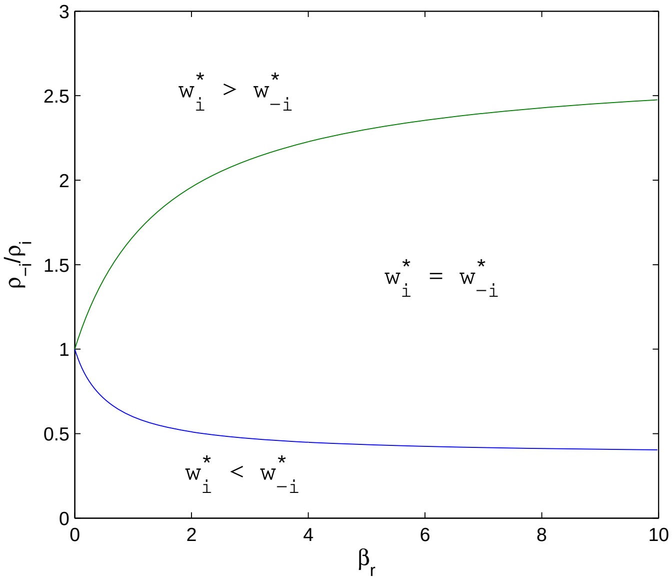


Figure 5: This figure depicts the combinations of $\rho_{-i}/\rho_i$ and $\beta_r$ such that $w_i^* > w_{-i}^*$, $w_i^* = w_{-i}^*$ and $w_i^* < w_{-i}^*$. The upper curve denotes how $\overline{R}$ changes with $\beta_r$, while the lower curve denotes how $\underline{R}$ changes with $\beta_r$ ($a_i = a_{-i} = 1, a_{-i,i} = a_{i,-i} = 0.2, \alpha = 1, \beta_l = 2$).

Similarly, Figure 6 describes the impact of $\beta_l$ on the equilibrium. The effect of $\beta_l$ is also monotone but depends on whether $\beta_r > \alpha$ or not. In particular, the stickiness interval increases for $\beta_r = .3 < 1 = \alpha$ and decreases when $\beta_r = 3 > \alpha$. When $\beta_r = \alpha = 1$, $\beta_l$ has no effect on the interval. In other words, the intensity of the benchmark effect can reverse the impact of loss aversion on the equilibrium.

Thus far, we have studied how the benchmark affects how waiting times compare to each other at the equilibrium. We conclude this section by analyzing the impact of $\beta_r$ and $\beta_l$ on $w_i^*(\beta_r, \beta_l)$ and $w_{-i}^*(\beta_r, \beta_l)$, the unique Pareto equilibrium of the game.

**Proposition 5.** *Unique (Pareto) equilibriums* $w_i^*(\beta_r, \beta_l), w_{-i}^*(\beta_r, \beta_l)$ *are non-increasing in both* $\beta_r$ *and* $\beta_l$.

*Proof.* See Appendix E. ■

Proposition 5 implies that both waiting times at the equilibrium with the benchmark effect are shorter than the corresponding waiting times without benchmark ($\beta_r = 0$). The benchmark effect, but also loss aversion, make customers more sensitive to delays, and thus intensifies the competition. This leads to shorter waiting times for customers.

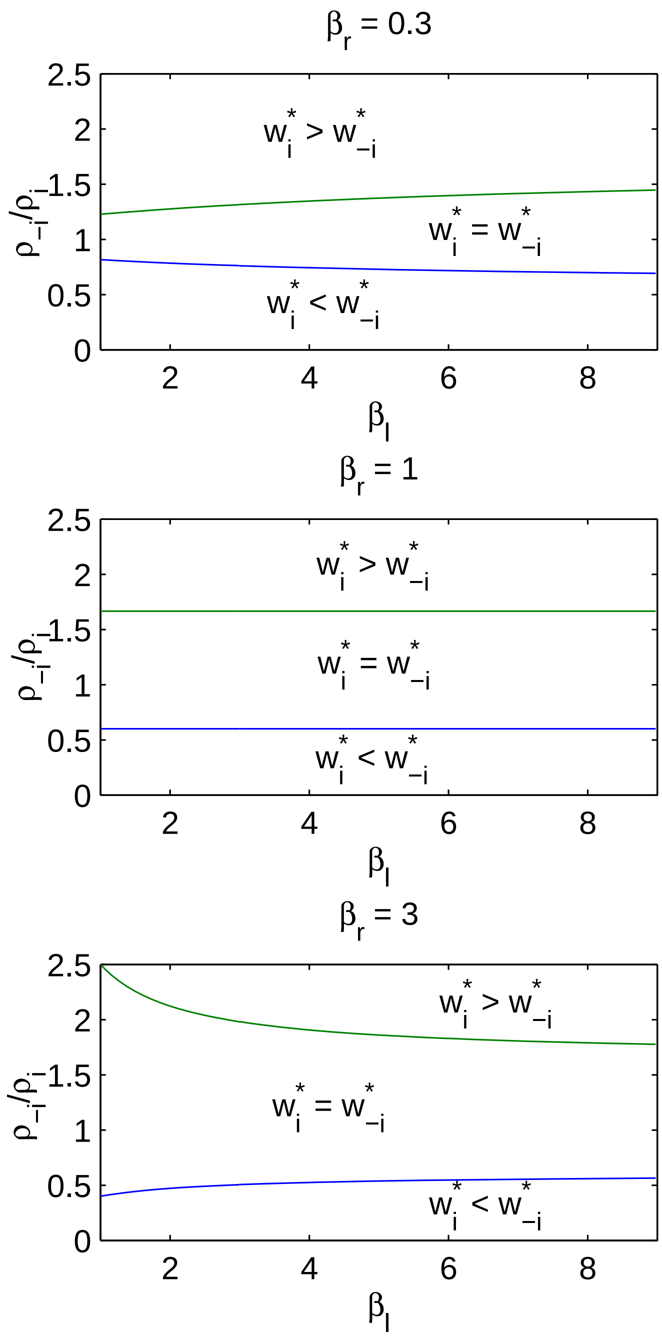


Figure 6: This figure depicts the combinations of $\rho_{-i}/\rho_i$ and $\beta_l$ such that $w_i^* > w_{-i}^*$, $w_i^* = w_{-i}^*$ and $w_i^* < w_{-i}^*$, when $\beta_r = 0.3$, 1 and 3. In each figure, the upper curve denotes the $\overline{R}$ function, while the lower curve denotes the $\underline{R}$ ($a_i = a_{-i} = 1, a_{-i,i} = a_{i,-i} = 0.2, \alpha = 1$).

# 3 Average Benchmark and Reversal Effect

In this section we study the case where $r = (w_i + w_{-i})/2$, which corresponds to the so-called "product type norm" (Cadotte et al. 1987). We follow the approach of the previous section, first characterizing the equilibrium of the game and then studying the impact of the benchmark and loss aversion on the equilibrium.

## 3.1 Nash Equilibrium

We start by showing that firm $i$'s profit $P_i$ is still a quasi-concave function in the average benchmark case.

**Proposition 6.** *Firm i's profit $P_i$ is a quasi-concave function of the waiting time $w_i$.*

*Proof.* See Appendix F ■

Given the quasi-concavity of $P_i$, we can study firm $i$'s best response curve $w_i^*$ as a function of $w_{-i}$. Proposition 6 implies that the first-order condition is sufficient to determine firm $i$'s best response. Since firm $i$'s demand function $\lambda_i$ is now everywhere differentiable, we can define the derivative of $\lambda_i$ with respect to $w_i$ at $w_i = w_{-i}$ as

$$\widehat{\delta}_i \equiv - \left. \frac{\partial \lambda_i}{\partial w_i} \right|_{w_i = w_{-i}} = a_i \alpha + \frac{a_i + a_{-i,i}}{2} \beta_r + \frac{3a_i + a_{-i,i}}{2} e^{-1} (\beta_l - 1) \beta_r \ ,$$

which leads to the following characterization of the response curve:

**Proposition 7.** *Firm i's best response curve, $w_i^*(w_{-i})$, is a piecewise function on the following three intervals:*

1. *When $w_{-i} \in [0, \widehat{w}_{-i})$, we have $w_i^* > w_{-i}$ and $w_i^*$ is quasi-convex in $w_{-i}$;*
2. *When $w_{-i} = \widehat{w}_{-i}$, we have $w_i^* = w_{-i}$; and*
3. *When $w_{-i} \in (\widehat{w}_{-i}, \infty)$, we have $w_i^* < w_{-i}$ and $w_i^*$ is quasi-concave in $w_{-i}$,*

*where* $\widehat{w}_{-i} = \left(\rho_i \widehat{\delta}_i\right)^{-1/2}$.

*Proof.* See Appendix G ■

Proposition 7 is very similar to Proposition 2 for the minimum benchmark case. Specifically, when firm $-i$ chooses a short waiting time ($w_{-i} < \widehat{w}_{-i}$), it is too costly for firm $i$ to match this delay. Thus firm $i$ sets a waiting time $w_i^*$ longer than $w_{-i}$. On the other hand, when firm $-i$'s waiting time is not too short ($w_{-i} > \widehat{w}_{-i}$), it does not cost much for firm $i$ to set a higher benchmark. So firm $i$ will choose a waiting time shorter than $w_{-i}$.

On the other hand, Proposition 7 differs from Proposition 2, which states that for the minimum benchmark case when $w_{-i}$ is within a certain range, firm $i$'s optimal waiting time $w_i^*$ equals $w_{-i}$. According to Proposition 7, $w_i^* = w_{-i}$ only when $w_{-i} = \widehat{w}_{-i}$. This is because in the average form case, both firms can always influence the benchmark. By contrast, in the minimum case, the firm with the longer waiting time has no impact on the benchmark.

Following Proposition 7, we now provide the characterization of a pure strategy Nash equilibrium, the existence of which is guaranteed by Proposition 6.

**Theorem 2.** *A pure strategy Nash Equilibrium* $(w_i^*, w_{-i}^*)$ *exists. Furthermore,*

1. *If* $\widehat{w}_i > \widehat{w}_{-i}$, *then* $w_i^* < w_{-i}^*$.

2. *If* $\widehat{w}_i < \widehat{w}_{-i}$, *then* $w_i^* > w_{-i}^*$.

3. *If* $\widehat{w}_i = \widehat{w}_{-i}$, *then* $w_i^* = w_{-i}^*$.

*Proof.* See Appendix H ■

Note that the structure described by Theorem 2 is similar to the minimum benchmark case. Although we are not able to prove the uniqueness of the pure strategy Nash Equilibrium analytically, numerical tests in Appendix I show that under all the 625 choices of parameter settings, the equilibrium is unique. Our numerical study shows further that both waiting times at the equilibrium decrease with parameters $\beta_r$ and $\beta_l$, which is also consistent with our findings for the minimum benchmark case.

## 3.2 The Reversal Effect

In this section we study the impact of the benchmark effect on the equilibrium. To that end, we present the following Corollary from Theorem 2, which shows that the equilibrium is characterized by a threshold on $\rho_{-i}/\rho_i$.

**Corollary 2.** *There exists a threshold $\widehat{R}(\beta_r, \beta_l)$ such that the structure of the equilibrium strategies can be characterized by the following three cases:*

1. *If $\rho_{-i}/\rho_i < \widehat{R}(\beta_r, \beta_l)$, then $w_i^* < w_{-i}^*$;*
2. *If $\rho_{-i}/\rho_i = \widehat{R}(\beta_r, \beta_l)$, then $w_i^* = w_{-i}^*$; and*
3. *If $\rho_{-i}/\rho_i > \widehat{R}(\beta_r, \beta_l)$, then $w_i^* > w_{-i}^*$;*

*where*

$$\widehat{R} = \frac{2\theta_3\alpha + (\theta_1 + \theta_3)\beta_r + (\theta_1 + 3\theta_3)e^{-1}(\beta_l - 1)\beta_r}{2\alpha + (1 + \theta_2)\beta_r + (3 + \theta_2)e^{-1}(\beta_l - 1)\beta_r} ,$$

*and $\theta_1$, $\theta_2$ and $\theta_3$ are defined in Equation (2).*

*Proof.* The proof is provided in Appendix J. ■

Similar to Proposition 4, we can further study how the threshold $\widehat{R}$ varies with $\beta_r$ and $\beta_l$.

**Proposition 8.** *Without loss of generality, assume that $a_{-i,i}/a_{i,-i} < a_i/a_{-i}$. We have,*

1. *When $\beta_r = 0$, $\widehat{R} = \theta_3$; when $\beta_r > 0$, $\widehat{R} < \theta_3$;*
2. *For fixed $\beta_l$, $\widehat{R}$ is decreasing in $\beta_r$;*
3. *For fixed $\beta_r$,*
   - *If $\beta_r < \alpha$, $\widehat{R}$ is decreasing in $\beta_l$;*
   - *If $\beta_r = \alpha$, $\widehat{R} = (\theta_1 + 3\theta_3)/(3 + \theta_2)$, which does not change with $\beta_l$;*
   - *If $\beta_r > \alpha$, $\widehat{R}$ is increasing in $\beta_l$.*

*Proof.* See Appendix K ■

Figures 7 and 8 illustrate the proposition and depict the impact of $\beta_r$ and $\beta_l$, respectively. The overall structure is akin to the minimum benchmark case of the previous section. In particular, the threshold is increasing when the benchmark effect intensifies and either increases or decreases with loss aversion $\beta_l$ depending on whether $\beta_r$ is larger or smaller than $\alpha$.

The previous analysis reveals that a market leader in terms of offering shorter waiting time without the benchmark effect can sometimes become the follower (offering longer waiting time) when the benchmark effect is strong enough. More specifically, when $\beta_r = 0$, the firms' equilibrium strategies are equal to $w_i^0$ and $w_{-i}^0$ from Proposition 3. Assume, for instance, that firm $i$ is the leader such that $w_i^0 < w_{-i}^0$. Following Corollary 2, firm $i$ becomes the follower with $w_i^* > w_{-i}^*$ if $\rho_{-i}/\rho_i > \widehat{R}(\beta_r, \beta_l)$. Figure 7 illustrates this reversal effect: When $\rho_{-i}/\rho_i$ is equal to 0.9, we have $w_i^* = w_i^0 < w_{-i}^* = w_{-i}^0$ for $\beta_r = 0$ (circle); but when the benchmark effect is strong enough ($\beta_r > 2$ for instance), the order is reversed with $w_{-i}^* < w_i^*$ (star). We conclude this section by providing necessary and sufficient conditions for this reversal effect to occur, as stated by the following result which is a direct consequence of Corollary 2 and Proposition 8:

**Proposition 9.** *Without loss of generality, assume that $a_{-i,i}/a_{i,-i} < a_i/a_{-i}$. There exist values of $\beta_r$ and $\beta_l$ such that the leader of the duopoly without benchmark effect becomes the follower with benchmark effect if and only if*

$$\frac{\theta_1 + \theta_3}{1 + \theta_2} < \frac{\rho_{-i}}{\rho_i} < \theta_3.$$

Thus, according to the proposition, the reversal can only occur when the ratio of profit margins is within a certain range. The phenomena never occurs otherwise.

# 4 Conclusions

Empirical studies from both the Marketing and Decision Making literatures suggest that customers can be influenced by the gap between their expected delays and a market benchmark. We posit that this, in turn, should influence the way firms compete. The key feature of our approach is that the benchmark is endogenous. This means that companies can affect demand directly through their choices of waiting times, and indirectly by manipulating the benchmark and making the competitor's offer look worse.

Using an analytical approach, our study reveals several new findings. First the presence of a benchmark decreases waiting times at the equilibrium. Second, depending on how the benchmark is formed, either a stickiness or a reversal effect can occur. We

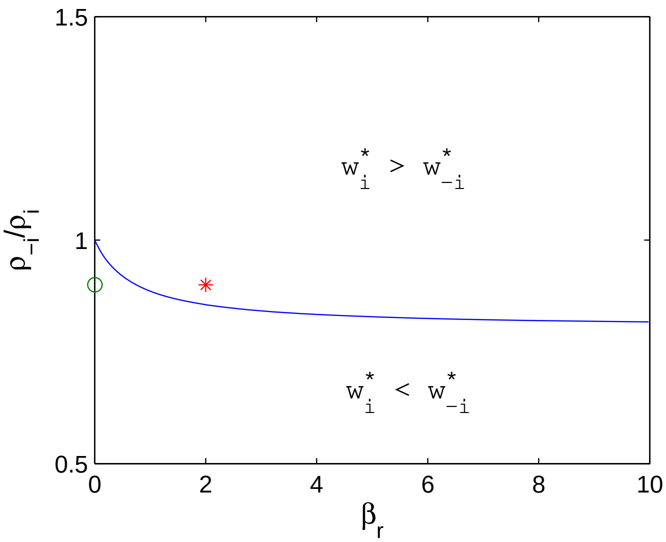


Figure 7: This figure depicts the combinations of $\rho_{-i}/\rho_i$ and $\beta_r$ such that $w_i^* > w_{-i}^*$ and $w_i^* < w_{-i}^*$. The decreasing curve denotes the function $\widehat{R}$ ($a_i = a_{-i} = 1, a_{-i,i} = 0.1, a_{i,-i} = 0.5, \alpha = 1, \beta_l = 2$).

have also disentangled the impacts that loss aversion and the benchmark effect have on the equilibrium. We show in particular that in both cases, the direction of the impact of loss aversion changes with the strength of the benchmark effect.

Our paper appears to be the first to study benchmark effects between service firms competing on waiting times. Interesting extensions include investigating other forms of benchmark, for example, the weighted average form. The equilibrium structure of the weighted average form will be similar to the average case, and the *reversal effect* result may hold as well. One can also generalize our model to other queuing systems. Taking the $M/G/1$ queue, for example, the customers' aggregate satisfaction with one firm's service will depend not only on the waiting time expectation, but also the higher moments (e.g. variance), which will remarkably complicate the model and the analysis. We could also consider the segmentation of customers with respect to their degree of benchmark dependence and loss aversion or the presence of more than two firms. Equilibrium may not be unique any more for these extensions. However, we suspect that effects akin to the stickiness or reverse effects we identify in this paper should continue to exist.

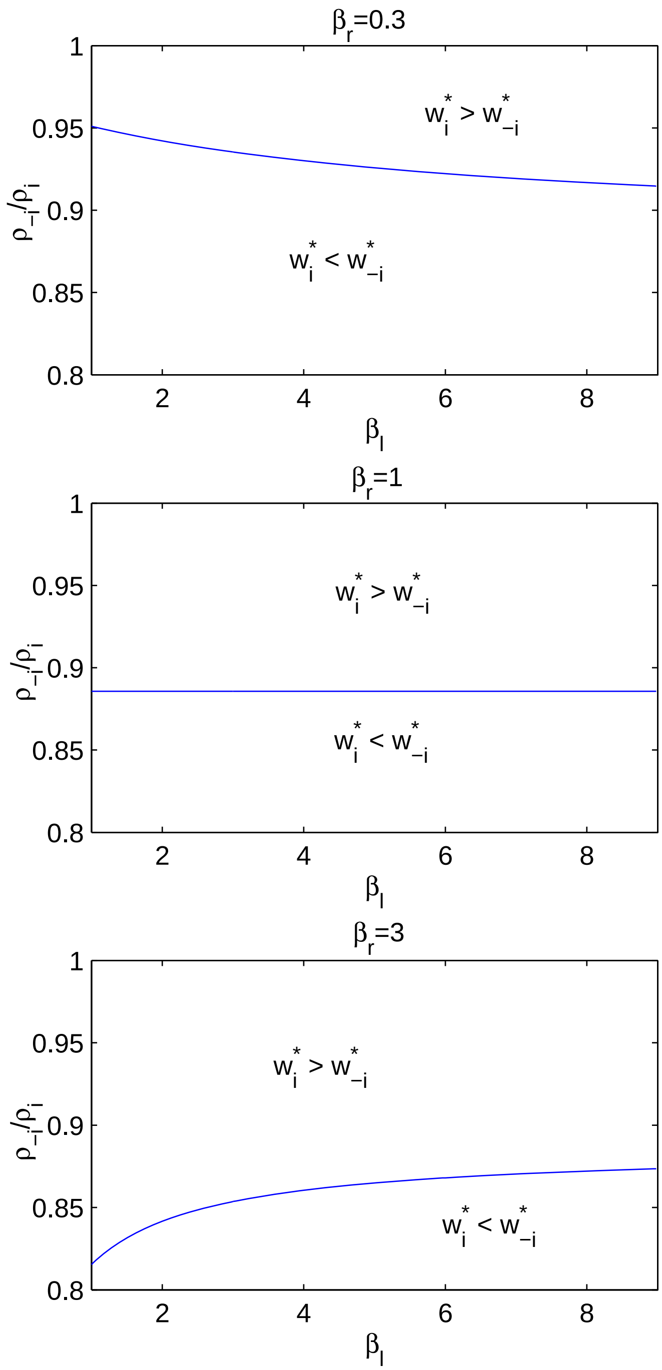


Figure 8: This figure depicts the combinations of $\rho_{-i}/\rho_i$ and $\beta_l$ such that $w_i^* > w_{-i}^*$ and $w_i^* < w_{-i}^*$, when $\beta_r = 0.3$, 1 and 3. The curve in each figure denotes the function $\widehat{R}$ ($a_i = a_{-i} = 1, a_{-i,i} = 0.1, a_{i,-i} = 0.5, \alpha = 1$).

# References


Allon, G. and Federgruen, A. (2007). Competition in service industries. *Operations Research*, 55(1):37–55.

Anderson, E. W. and Sullivan, M. W. (1993). The antecedents and consequences of customer satisfaction for firms. *Marketing Science*, 12(2):125–143.

Blinder, A. S., Canetti, E. R., and Lebow, D. E. (1998). *Asking About Prices: A New Approach to Understanding Pricing Stickiness*. Russell Sage Foundation, New York.

Cachon, G. P. and Harker, P. T. (2002). Competition and outsourcing with scale economies. *Management Science*, 48(10):1314–1333.

Cadotte, E. R., Woodruff, R. B., and Jenkins, R. L. (1987). Expectations and norms in models of consumer satisfaction. *Journal of Marketing Research*, 24(3):305–314.

Crouzeix, J. P. (1980). On second order conditions for quasiconvexity. *Mathematical Programming*, 18:349–352.

De Vany, A. S. and Saving, T. R. (1983). The economics of quality. *Journal of Political Economy*, 91(6):979–1000.

Erdem, T., Mayhew, G., and Sun, B. (2001). Understanding reference-price shoppers: A within- and cross- category analysis. *Journal of Marketing Research*, 38(4):445–457.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. MIT Press.

Gaur, V. and Park, Y. H. (2007). Asymmetric consumer learning and inventory competition. *Management Science*, 53(2):227–240.

Hall, J. and Porteus, E. (2000). Customer service competition in capacitated systems. *Manufacturing and Service Operations Management*, 2(2):144–165.

Hassin, R. and Haviv, M. (2002). *To Queue or Not to Queue: Equilibrium Behavior in Queueing Systems*. Springer.

Heidhues, P. and Koszegi, B. (2008). Competition and price variation when consumers are loss averse. *American Economic Review*, 98(4):1245C1268.

Helson, H. (1964). *Adaptation-Level Theory*. Harper and Row, New York.

Ho, T. H. and Zheng, Y. S. (2004). Setting customer expectation in service delivery: An integrated marketing-operations perspective. *Management Science*, 50(4):479–488.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–292.

Kalai, E., Kamien, M. I., and Rubinovitch, M. (1992). Optimal service speeds in a competitive environment. *Management Science*, 38(8):1154–1163.

Leclerc, F., Schmitt, B. H., and L., D. (1995). Waiting time and decision making: Is time like money? *Journal of Consumer Research*, 22(1):110–119.

Lin, J. H., Lee, T. R., and Jen, W. (2008). Assessing asymmetric response effect of behavioral intention to service quality in an integrated psychological decision-making process model of intercity bus passengers: A case of taiwan. *Transportation*, 35:129–144.

Liu, B. S., Petruzzi, N. C., and Sudharshan, D. (2007). A service effort allocation model for assessing customer lifetime value in service marketing. *Journal of Services Marketing*, 21(1):24–35.

Niedrich, R. W., Sharma, S., and Wedell, D. (2001). Reference price and price perceptions: A comparison of alternative models. *Journal of Consumer Research*, 28(3):339–354.

Popescu, I. and Wu, Y. (2007). Dynamic pricing strategies with reference effects. *Operations Research*, 55(3):413–429.

So, K. C. (2000). Price and time competition for service delivery. *Manufacturing and Service Operations Management*, 2(4):392–409.

Wedell, D. H. (1995). Contrast effects in paired comparisons: Evidence for both stimulus-based and response-based processes. *Journal of Experimental Psychology: Human Perception and Performance*, 21(5):1158–1173.

Zeithaml, V. A., Berry, L. L., and Parasuraman, A. (1993). The nature and determinants of customer expectations of service. *Journal of the Academy of Marketing Science*, 21(1):1–12.

Zhou, J. (2011). Reference dependence and market competition. *Journal of Economics and Management Strategy*, 20(4):1073–1097.

## A Proof of Proposition 1

*Proof.* We first prove that $P_i$ is a quasi-concave function of $w_i$. We first show that firm $i$'s profit function $P_i(w_i, w_{-i})$ is concave in $w_i$ when $w_i \geq w_{-i}$. When $w_i \geq w_{-i}$, $r = w_{-i}$ and

$$P_i(w_i, w_{-i}) = (p - c_i)\left[a_0 + a_i S_i(w_i, w_{-i}) - a_{-i,i} S_{-i}(w_{-i}, w_{-i})\right] - \frac{c_i}{w_i} \ ,$$

where

$$S_i(w_i, w_{-i}) = -\alpha w_i + \beta_r \left[(w_{-i} - w_i) - e^{-\frac{w_{-i}}{w_i}}\left(\beta_\ell - 1\right) w_i\right]$$

and

$$S_{-i}(w_{-i}, w_{-i}) = -\alpha w_{-i} - \beta_r e^{-1}\left(\beta_\ell - 1\right) w_{-i}.$$

Take the first and second order derivatives of $P_i$ with respect to $w_i$:

$$\frac{\partial P_i}{\partial w_i} = -(p - c_i) a_i \left\{\alpha + \beta_r \left[1 + e^{-\frac{w_{-i}}{w_i}}\left(\beta_\ell - 1\right)\left(1 + \frac{w_{-i}}{w_i}\right)\right]\right\} + \frac{c_i}{w_i^2}$$

$$\frac{\partial^2 P_i}{\partial w_i^2} = -(p - c_i) a_i \beta_r e^{-\frac{w_{-i}}{w_i}}\left(\beta_\ell - 1\right)\frac{w_{-i}^2}{w_i^3} - \frac{2c_i}{w_i^3} < 0 \ .$$

Therefore $P_i$ is concave in $w_i$ when $w_i \geq w_{-i}$.

We next show that $P_i$ is quasi-concave in $w_i$ when $w_i \leq w_{-i}$. When $w_i \leq w_{-i}$, $r = w_i$ and

$$P_i(w_i, w_{-i}) = (p - c_i)\left[a_0 + a_i S_i(w_i, w_i) - a_{-i,i} S_{-i}(w_{-i}, w_i)\right] - \frac{c_i}{w_i} \ ,$$

where

$$S_i(w_i, w_i) = -\alpha w_i - \beta_r e^{-1}\left(\beta_\ell - 1\right) w_i$$

and

$$S_{-i}(w_{-i}, w_i) = -\alpha w_{-i} + \beta_r \left[(w_i - w_{-i}) - e^{-\frac{w_i}{w_{-i}}}\left(\beta_\ell - 1\right) w_{-i}\right].$$

For simplicity, we let $P(w_i) = P_i(w_i, w_{-i})$. By definition, given any $w_1 < w_2 < w_3 \leq w_{-i}$, $P(w_i)$ is quasi-concave in $w_i$ iff $P(w_2) \geq \min\{P(w_1), P(w_3)\}$.

First, assume that $P(w_1) \leq P(w_3)$, we then prove $P(w_2) \geq P(w_1)$. Let $A = a_i\alpha +$

$a_i\beta_r e^{-1}(\beta_\ell - 1) + a_{-i,i}\beta_r$ and $B = a_{-i,i}\beta_r(\beta_\ell - 1) w_{-i}$, we have $A, B > 0$, then

$$P(w_3) - P(w_1) = (p - c_i)\left[A(w_1 - w_3) + B\left(e^{-\frac{w_3}{w_{-i}}} - e^{-\frac{w_1}{w_{-i}}}\right)\right] - c_i\frac{w_1 - w_3}{w_1 w_3} \geq 0 \ .$$

Therefore,

$$\frac{P(w_3) - P(w_1)}{w_1 - w_3} w_3 = (p - c_i)\left[Aw_3 + Bg(w_3)\right] - \frac{c_i}{w_1} \leq 0 \ ,$$

where $g(w_3) = \frac{e^{-\frac{w_3}{w_{-i}}} - e^{-\frac{w_1}{w_{-i}}}}{w_1 - w_3} w_3$. Since

$$\frac{\partial g}{\partial w_3} = \frac{e^{-\frac{w_3}{w_{-i}}}\left(w_1 + \frac{w_3}{w_{-i}}(w_3 - w_1)\right) - e^{-\frac{w_1}{w_{-i}}} w_1}{(w_1 - w_3)^2} > 0 \ ,$$

we have $g(w_2) < g(w_3)$. Therefore,

$$\begin{aligned}
\frac{P(w_2) - P(w_1)}{w_1 - w_2} w_2 &= (p - c_i)\left[Aw_2 + Bg(w_2)\right] - \frac{c_i}{w_1} \\
&< (p - c_i)\left[Aw_3 + Bg(w_3)\right] - \frac{c_i}{w_1} \\
&= \frac{P(w_3) - P(w_1)}{w_1 - w_3} w_3 \leq 0 \ .
\end{aligned}$$

Given that $w_1 < w_2$, we have $P(w_2) > P(w_1)$.

Assume that $P(w_1) \geq P(w_3)$; we then prove $P(w_2) \geq P(w_3)$. Using the same method, we have:

$$\frac{P(w_1) - P(w_3)}{w_3 - w_1} w_1 = (p - c_i)\left[Aw_1 + Bh(w_1)\right] - \frac{c_i}{w_3} \geq 0 \ ,$$

where $h(w_1) = \frac{e^{-\frac{w_1}{w_{-i}}} - e^{-\frac{w_3}{w_{-i}}}}{w_3 - w_1} w_1$. Since

$$\frac{\partial h}{\partial w_1} = \frac{e^{-\frac{w_1}{w_{-i}}}\left(w_3 - \frac{w_1}{w_{-i}}(w_3 - w_1)\right) - e^{-\frac{w_3}{w_{-i}}} w_3}{(w_3 - w_1)^2} > 0 \ ,$$

we have $h(w_2) > h(w_1)$. Therefore,

$$\begin{aligned}
\frac{P(w_2) - P(w_3)}{w_3 - w_2} w_2 &= (p - c_i) \left[ A w_2 + B h(w_2) \right] - \frac{c_i}{w_3} \\
&> (p - c_i) \left[ A w_1 + B h(w_1) \right] - \frac{c_i}{w_3} \\
&= \frac{P(w_1) - P(w_3)}{w_3 - w_1} w_1 \geq 0 \ .
\end{aligned}$$

Given that $w_3 > w_2$, we have $P(w_2) > P(w_3)$. It follows that

$$P(w_2) > \min\{P(w_1), P(w_3)\}.$$

Given that firm $i$'s profit function $P_i(w_i, w_{-i})$ is concave in $w_i$ when $w_i \geq w_{-i}$ and is quasi-concave in $w_i$ when $w_i \leq w_{-i}$, $P_i$ can only be one of the four cases shown in Figure 1. To prove that $P_i$ is quasi-concave in $w_i$, we need only rule out case (4). Assuming that

$$\left. \frac{\partial P_i}{\partial w_i} \right|_{w_i \uparrow w_{-i}} = -(p - c_i) \left[ a_i \alpha + a_{-i,i} \beta_r + (a_i + a_{-i,i}) \beta_r e^{-1} \left( \beta_\ell - 1 \right) \right] + \frac{c_i}{w_{-i}^2} \ < \ 0 \ ,$$

then we have

$$\begin{aligned}
\left. \frac{\partial P_i}{\partial w_i} \right|_{w_i \downarrow w_{-i}} &= -(p - c_i) \left[ a_i \alpha + a_i \beta_r + 2 a_i \beta_r e^{-1} \left( \beta_\ell - 1 \right) \right] + \frac{c_i}{w_{-i}^2} \\
&< -(p - c_i) \left[ a_i \alpha + a_{-i,i} \beta_r + (a_i + a_{-i,i}) \beta_r e^{-1} \left( \beta_\ell - 1 \right) \right] + \frac{c_i}{w_{-i}^2} \\
&= \left. \frac{\partial P_i}{\partial w_i} \right|_{w_i \uparrow w_{-i}} \ < \ 0 \ .
\end{aligned}$$

So if the derivative of $P_i$ with respect to $w_i$ is negative on the left of $w_{-i}$, it can not be positive on the right of $w_{-i}$. Case (4) in Figure 1 is not possible. The result holds.

We next show that $P_i$ is supermodular when $w_i > w_{-i}$, and submodular when $w_i < w_{-i}$. When $w_i > w_{-i}$,

$$\left. \frac{\partial P_i}{\partial w_i} \right|_{w_i > w_{-i}} = -(p - c_i) \left[ a_i \alpha + a_i \beta_r + a_i \beta_r e^{-\frac{w_{-i}}{w_i}} \left( \beta_\ell - 1 \right) \left( 1 + \frac{w_{-i}}{w_i} \right) \right] + \frac{c_i}{w_i^2} \ ,$$

so

$$\left.\frac{\partial^2 P_i}{\partial w_i \partial w_{-i}}\right|_{w_i > w_{-i}} = (p - c_i) a_i \beta_r (\beta_l - 1) e^{-\frac{w_{-i}}{w_i}} \frac{w_{-i}}{w_i^2}.$$

Therefore, $P_i$ is supermodular when $w_i > w_{-i}$.

When $w_i < w_{-i}$,

$$\left.\frac{\partial P_i}{\partial w_i}\right|_{w_i < w_{-i}} = -(p - c_i) \left[ a_i \alpha + a_i \beta_r e^{-1} (\beta_\ell - 1) + a_{-i,i} \beta_r \left( 1 + e^{-\frac{w_i}{w_{-i}}} (\beta_\ell - 1) \right) \right] + \frac{c_i}{w_i^2} \ ,$$

so

$$\left.\frac{\partial^2 P_i}{\partial w_i \partial w_{-i}}\right|_{w_i < w_{-i}} = -(p - c_i) a_{-i,i} \beta_r (\beta_l - 1) e^{-\frac{w_i}{w_{-i}}} \frac{w_i}{w_{-i}^2}.$$

Therefore, $P_i$ is submodular when $w_i < w_{-i}$. ∎

# B Proof of Proposition 2

*Proof.* The quasi-concavity of $P_i$ implies that for any given $w_{-i}$, the first-order condition is a sufficient condition for firm $i$'s best response. $P_i$ can be any of the three cases (1, 2, 3) as shown in Figure 1.

If $\partial P_i / \partial w_i|_{w_i \downarrow w_{-i}} > 0$, then it corresponds to case (1). Since

$$\left.\frac{\partial P_i}{\partial w_i}\right|_{w_i \downarrow w_{-i}} = -(p - c_i) \left[ a_i \alpha + a_i \beta_r + 2 a_i \beta_r e^{-1} (\beta_\ell - 1) \right] + \frac{c_i}{w_{-i}^2} \ ,$$

$\partial P_i / \partial w_i|_{w_i \downarrow w_{-i}} > 0$ is equivalent to $w_{-i} < \left(\rho_i \overline{\delta}_i\right)^{-1/2}$. In this case, $w_i^*$ will be on the right of $w_{-i}$. Since $P_i$ is supermodular when $w_i > w_{-i}$, firm $i$'s best response $w_i^*$ is increasing in $w_{-i}$ in this scenario.

If $\partial P_i / \partial w_i|_{w_i \uparrow w_{-i}} \geq 0$ and $\partial P_i / \partial w_i|_{w_i \downarrow w_{-i}} \leq 0$, which is equivalent to $\left(\rho_i \overline{\delta}_i\right)^{-1/2} \leq w_{-i} \leq (\rho_i \underline{\delta}_i)^{-1/2}$, it corresponds to case (2). In this case, $w_i^* = w_{-i}$.

If $\partial P_i / \partial w_i|_{w_i \uparrow w_{-i}} < 0$, then it corresponds to case (3). Since

$$\left.\frac{\partial P_i}{\partial w_i}\right|_{w_i \uparrow w_{-i}} = -(p - c_i) \left[ a_i \alpha + a_{-i,i} \beta_r + (a_i + a_{-i,i}) \beta_r e^{-1} (\beta_\ell - 1) \right] + \frac{c_i}{w_{-i}^2} \ ,$$

$\partial P_i/\partial w_i|_{w_i \uparrow w_{-i}} < 0$ is equivalent to $w_{-i} > (\rho_i \underline{\delta}_i)^{-1/2}$. In this case, $w_i^*$ will be on the left of $w_{-i}$. Given that $P_i$ is submodular when $w_i < w_{-i}$, we have firm $i$'s best response $w_i^*$ decreasing in $w_{-i}$ in this scenario. Therefore, $w_i^* \geq w_i^*(\infty) = L_i$ where $L_i = \left(\rho_i \left[a_i \alpha + a_i e^{-1} \beta_r \left(\beta_\ell - 1\right) + a_{-i,i} \beta_r \beta_\ell\right]\right)^{-1/2}$. ∎

# C Proof of Corollary 1

*Proof.* According to Theorem 1, if $\underline{w}_i > \overline{w}_{-i}$, then the equilibrium strategies $w_i^* < w_{-i}^*$. The condition corresponds to

$$\frac{\underline{w}_i}{\overline{w}_{-i}} = \left( \frac{\rho_{-i}}{\rho_i} \cdot \frac{\alpha + 2e^{-1}\beta_r(\beta_l - 1) + \beta_r}{\theta_3 \alpha + (\theta_1 + \theta_3) e^{-1} \beta_r (\beta_l - 1) + \theta_1 \beta_r} \right)^{-\frac{1}{2}} > 1 \ ,$$

which leads to $\rho_{-i}/\rho_i < \underline{R}$, where

$$\underline{R} = \frac{\theta_3 \alpha + (\theta_1 + \theta_3) e^{-1} \beta_r (\beta_l - 1) + \theta_1 \beta_r}{\alpha + 2e^{-1}\beta_r(\beta_l - 1) + \beta_r} \ .$$

If $\underline{w}_i \leq \overline{w}_{-i}$ and $\overline{w}_i \geq \underline{w}_{-i}$, then the equilibrium strategies $w_i^* = w_{-i}^*$. The condition corresponds to $\rho_{-i}/\rho_i \geq \underline{R}$ and

$$\frac{\overline{w}_i}{\underline{w}_{-i}} = \left( \frac{\rho_{-i}}{\rho_i} \cdot \frac{\alpha + (1 + \theta_2) e^{-1} \beta_r (\beta_l - 1) + \theta_2 \beta_r}{\theta_3 \left[\alpha + 2e^{-1}\beta_r(\beta_l - 1) + \beta_r\right]} \right)^{-\frac{1}{2}} \geq 1 \ ,$$

which is equivalent to $\underline{R} \leq \rho_{-i}/\rho_i \leq \overline{R}$, where

$$\overline{R} = \frac{\theta_3 \left[\alpha + 2e^{-1}\beta_r(\beta_l - 1) + \beta_r\right]}{\alpha + (1 + \theta_2) e^{-1} \beta_r (\beta_l - 1) + \theta_2 \beta_r} \ .$$

If $\overline{w}_i < \underline{w}_{-i}$, then the equilibrium strategies $w_i^* > w_{-i}^*$. The condition corresponds to $\rho_{-i}/\rho_i > \overline{R}$. ∎

# D Proof of Proposition 4

When we fix $\beta_l$, since $\theta_3 > (\theta_1 + \theta_3)/2$ and $\theta_3 > \theta_1$, we have $\underline{R}$ decreasing in $\beta_r$ and bounded from below by $\left[(\theta_1 + \theta_3) e^{-1} (\beta_l - 1) + \theta_1\right] / \left[2e^{-1}(\beta_l - 1) + 1\right]$. On the other hand, since $\theta_3 < 2\theta_3/(1+\theta_2)$ and $\theta_3 < \theta_3/\theta_2$, we have $\overline{R}$ increasing in $\beta_r$ and bounded from above by $[2\theta_3 e^{-1}(\beta_l - 1) + \theta_3] \,/\, [(1+\theta_2)e^{-1}(\beta_l - 1) + \theta_2]$.

When we fix $\beta_r$, to analyze $\underline{R}$, we need to compare $A = (\theta_3\alpha + \theta_1\beta_r)/(\alpha + \beta_r)$ and $B = (\theta_1 + \theta_3)/2$. If $\beta_r < \alpha$, we have $A > B$, so $\underline{R}$ is decreasing in $\beta_l$ and bounded from below by $B$; if $\beta_r = \alpha$, we have $A = B$, so $\underline{R} = B$; if $\beta_r > \alpha$, we have $A < B$, so $\underline{R}$ is increasing in $\beta_l$ and bounded from above by $B$.

To analyze $\overline{R}$, we need to compare $C = (\theta_3\alpha + \theta_3\beta_r)/(\alpha + \theta_2\beta_r)$ and $D = 2\theta_3/(1+\theta_2)$. If $\beta_r < \alpha$, we have $C < D$, so $\overline{R}$ is increasing in $\beta_l$ and bounded from above by $D$; if $\beta_r = \alpha$, we have $C = D$, so $R_2 = D$; if $\beta_r > \alpha$, we have $C > D$, so $\overline{R}$ is decreasing in $\beta_l$ and bounded from below by $D$. ■

# E Proof of Proposition 5

*Proof.* We first show that the unique (Pareto) equilibrium strategies $\left(w_i^*, w_{-i}^*\right)$ are decreasing with $\beta_r$ when $\underline{w}_i > \overline{w}_{-i}$, $(\underline{w}_i \leq \overline{w}_{-i}) \cup (\underline{w}_{-i} \leq \overline{w}_i)$, $\underline{w}_{-i} \geq \overline{w}_i$ respectively.

We first show that when $\underline{w}_i > \overline{w}_{-i}$, the unique equilibrium strategies $\left(w_i^*, w_{-i}^*\right)$ are decreasing with $\beta_r$. $\left(w_i^*, w_{-i}^*\right)$ can be determined from

$$\begin{cases} (p - c_i)\left[a_i\alpha + a_i e^{-1}\beta_r\left(\beta_\ell - 1\right) + a_{-i,i}\beta_r\left(1 + e^{-\frac{w_i^*}{w_{-i}^*}}\left(\beta_\ell - 1\right)\right)\right] = \frac{c_i}{w_i^{*2}} \ , \\ (p - c_{-i})\left[a_{-i}\alpha + a_{-i}\beta_r + a_{-i}e^{-\frac{w_i^*}{w_{-i}^*}}\beta_r\left(\beta_\ell - 1\right)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\right] = \frac{c_{-i}}{w_{-i}^{*2}} \ . \end{cases}$$

According to implicit function theorem, we can derive the first-order derivative of $w_i^*$ and $w_{-i}^*$ with respect to $\beta_r$:

$$\begin{pmatrix} \frac{dw_i^*}{d\beta_r} \\ \frac{dw_{-i}^*}{d\beta_r} \end{pmatrix} = \begin{pmatrix} -(p - c_{-i})a_{-i}\beta_r e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^{*2}}{w_{-i}^{*3}} - \frac{2c_{-i}}{w_{-i}^{*3}} & (p - c_i)\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^*}{w_{-i}^{*2}} \\ -(p - c_{-i})a_{-i}\beta_r e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^*}{w_{-i}^{*2}} & (p - c_i)\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{1}{w_{-i}^*} - \frac{2c_i}{w_i^{*3}} \end{pmatrix}$$

$$\bullet \frac{1}{ad - bc}\begin{pmatrix} (p - c_i)\left[a_i e^{-1}(\beta_\ell - 1) + a_{-i,i} + a_{-i,i}e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\right] \\ (p - c_{-i})\left[a_{-i} + a_{-i}e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\right] \end{pmatrix},$$

where

$$ad - bc > (p - c_i)(p - c_{-i})a_{-i}\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}} (\beta_\ell - 1)(a_{-i}\alpha + \beta_r)\frac{2}{w_{-i}^*}\left(\frac{2}{w_i^*} - \frac{1}{w_{-i}^*}\right) > 0$$

since $w_i^* < w_{-i}^*$. So we have

$$\frac{dw_i^*}{d\beta_r} < -\frac{(p-c_i)(p-c_{-i})}{(ad-bc)w_{-i}^*} a_i\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}} (\beta_\ell - 1)\left[1 + e^{-\frac{w_i^*}{w_{-i}^*}} (\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\right]\left(2 - \frac{w_i^*}{w_{-i}^*}\right) < 0$$

and

$$\frac{dw_{-i}^*}{d\beta_r} < -\frac{(p-c_i)(p-c_{-i})}{(ad-bc)w_i^*} a_{-i}\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}} (\beta_\ell - 1)\left[1 + e^{-\frac{w_i^*}{w_{-i}^*}} (\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\right]\left(2 - \frac{w_i^*}{w_{-i}^*}\right) < 0 \, .$$

Therefore the result holds for $\underline{w}_i > \overline{w}_{-i}$.

When $\underline{w}_{-i} \geq \overline{w}_i$, we can show that the unique equilibrium strategies $(w_i^*, w_{-i}^*)$ decrease with $\beta_r$ using the same method as in the last scenario.

When $\underline{w}_i \leq \overline{w}_{-i}$ and $\underline{w}_{-i} \leq \overline{w}_i$, the Pareto optimal equilibrium strategies $w_i^* = w_{-i}^* = \min(\overline{w}_i, \overline{w}_{-i})$ decrease with $\beta_r$ since both $\overline{w}_i$ and $\overline{w}_{-i}$ decrease with $\beta_r$.

Therefore, the unique (Pareto) equilibrium waiting times decrease with $\beta_r$.

We next show that the unique (Pareto) equilibrium strategies $(w_i^*, w_{-i}^*)$ are decreasing with $\beta_l$ when $\underline{w}_i > \overline{w}_{-i}$, $(\underline{w}_i \leq \overline{w}_{-i}) \cup \big(\underline{w}_{-i} \leq \overline{w}_i\big)$, $\underline{w}_{-i} \geq \overline{w}_i$, respectively.

We first show that when $\underline{w}_i > \overline{w}_{-i}$, the unique equilibrium strategies are decreasing with $\beta_l$. $(w_i^*, w_{-i}^*)$ can be determined from

$$\begin{cases} (p - c_i)\left[a_i\alpha + a_i e^{-1}\beta_r(\beta_\ell - 1) + a_{-i,i}\beta_r\left(1 + e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\right)\right] = \frac{c_i}{w_i^{*2}} \ , \\ (p - c_{-i})\left[a_{-i}\alpha + a_{-i}\beta_r + a_{-i}e^{-\frac{w_i^*}{w_{-i}^*}}\beta_r(\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\right] = \frac{c_{-i}}{w_{-i}^{*2}} \ . \end{cases}$$

According to implicit function theorem, we can derive the first-order derivative of $w_i^*$

and $w^*_{-i}$ with respect to $\beta_l$:

$$\begin{pmatrix} \frac{dw_i^*}{d\beta_l} \\ \frac{dw_{-i}^*}{d\beta_l} \end{pmatrix} = \begin{pmatrix} -(p-c_{-i})a_{-i}\beta_r e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^{*2}}{w_{-i}^{*3}} - \frac{2c_{-i}}{w_{-i}^{*3}} & (p-c_i)\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^*}{w_{-i}^{*2}} \\ -(p-c_{-i})a_{-i}\beta_r e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{w_i^*}{w_{-i}^{*2}} & (p-c_i)\beta_r a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\frac{1}{w_{-i}^*} - \frac{2c_i}{w_i^{*3}} \end{pmatrix}$$

$$\bullet \frac{1}{ad-bc} \begin{pmatrix} (p-c_i)\beta_r \left(a_i e^{-1} + a_{-i,i} e^{-\frac{w_i^*}{w_{-i}^*}}\right) \\ (p-c_{-i})a_{-i}\beta_r e^{-\frac{w_i^*}{w_{-i}^*}}\left(1 + \frac{w_i^*}{w_{-i}^*}\right) \end{pmatrix},$$

where $ad - bc > 0$. Since $w_i^* < w_{-i}^*$, we have

$$\frac{dw_i^*}{d\beta_l} < -\frac{(p-c_{-i})(p-c_i)\beta_r^2}{(ad-bc)w_{-i}^*} a_{-i}a_{-i,i} e^{-2\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\left(2 - \frac{w_i^*}{w_{-i}^*}\right) < 0$$

and

$$\frac{dw_{-i}^*}{d\beta_l} < -\frac{(p-c_{-i})(p-c_i)\beta_r^2}{(ad-bc)w_i^*} a_{-i}a_{-i,i} e^{-2\frac{w_i^*}{w_{-i}^*}}(\beta_\ell - 1)\left(1 + \frac{w_i^*}{w_{-i}^*}\right)\left(2 - \frac{w_i^*}{w_{-i}^*}\right) < 0 \ .$$

Therefore the result holds for $\underline{w}_i > \overline{w}_{-i}$.

We can easily show that when $(\underline{w}_i \leq \overline{w}_{-i}) \cup (\underline{w}_{-i} \leq \overline{w}_i)$ or $\underline{w}_{-i} \geq \overline{w}_i$, the unique (Pareto) equilibrium strategies decrease with $\beta_l$. ■

# F Proof of Proposition 6

*Proof.* Firm $i$'s profit function is:

$$P_i(w_i, w_{-i}) = (p - c_i)\left[a_0 + a_i S_i(w_i, r) - a_{-i,i}S_{-i}(w_{-i}, r)\right] - \frac{c_i}{w_i} \ ,$$

where

$$S_i(w_i, r) = -\alpha w_i + \beta_r \left[\frac{w_i + w_{-i}}{2} - w_i - e^{-\frac{w_i + w_{-i}}{2w_i}}(\beta_l - 1)w_i\right] \ .$$

If

$$\frac{\partial P_i}{\partial w_i} = -(p-c_i)\left[a_i\alpha + \frac{a_i + a_{-i,i}}{2}\beta_r + a_i\beta_r e^{-\frac{r}{w_i}}(\beta_l - 1)\left(1 + \frac{w_{-i}}{2w_i}\right) + a_{-i,i}\beta_r e^{-\frac{r}{w_{-i}}}(\beta_l - 1)\frac{1}{2}\right] + \frac{c_i}{w_i^2} = 0 \ ,$$

then

$$\begin{aligned}
\frac{\partial^2 P_i}{\partial w_i^2} &= (p-c_i)\beta_r(\beta_l-1)\frac{1}{4w_{-i}}\left(-a_i e^{-\frac{r}{w_i}}\frac{w_{-i}^3}{w_i^3}+a_{-i,i}e^{-\frac{r}{w_{-i}}}\right)-\frac{2c_i}{w_i^3}\\
&= -(p-c_i)\frac{2}{w_i}\left\{a_i\alpha+\frac{a_i+a_{-i,i}}{2}\beta_r+\beta_r(\beta_l-1)\left[a_i e^{-\frac{r}{w_i}}\left(\frac{w_{-i}^2}{8w_i^2}+\frac{w_{-i}}{2w_i}+1\right)+a_{-i,i}e^{-\frac{r}{w_{-i}}}\left(\frac{1}{2}-\frac{w_i}{8w_{-i}}\right)\right]\right\}\\
&< -(p-c_i)\frac{2}{w_i}\left\{a_i\alpha+\frac{a_i+a_{-i,i}}{2}\beta_r+\beta_r(\beta_l-1)a_{-i,i}\left[e^{-\frac{r}{w_i}}\left(\frac{w_{-i}^2}{8w_i^2}+\frac{w_{-i}}{2w_i}+1\right)+e^{-\frac{r}{w_{-i}}}\left(\frac{1}{2}-\frac{w_i}{8w_{-i}}\right)\right]\right\}\\
&< 0\ .
\end{aligned}$$

The second equality is supported by $\partial P_i/\partial w_i = 0$, while the last inequality is because

$$e^{-\frac{1+x}{2}}\left(1+\frac{x}{2}+\frac{x^2}{8}\right)+e^{-\frac{1+x}{2x}}\left(\frac{1}{2}-\frac{1}{8x}\right)>0$$

for any $x = w_{-i}/w_i \geq 0$.

According to the theorem in Crouzeix (1980), if function $P_i$ is twice differentiable, and if $\partial P_i/\partial w_i = 0$ implies $\partial^2 P_i/\partial w_i^2 < 0$, then $P_i$ is a quasi-concave function of $w_i$. ■

# G Proof of Proposition 7

*Proof.* Since $P_i$ is quasi-concave in $w_i$ and $(\partial P_i/\partial w_i)|_{w_i=0} = \infty > 0$ and

$$\left.\frac{\partial P_i}{\partial w_i}\right|_{w_i=\infty} = -(p-c_i)\left[a_i\alpha+\frac{a_i+a_{-i,i}}{2}\beta_r+a_i\beta_r(\beta_l-1)e^{-\frac{1}{2}}\right]<0\ ,$$

given firm $-i$'s waiting time standard $w_{-i}$, firm $i$'s unique best response $w_i^*$ can be determined by the following equation:

$$\frac{\partial P_i}{\partial w_i} = -(p-c_i)\left[a_i\alpha+\frac{a_i+a_{-i,i}}{2}\beta_r+a_i\beta_r e^{-\frac{r}{w_i}}(\beta_l-1)\left(1+\frac{w_{-i}}{2w_i}\right)+a_{-i,i}\beta_r e^{-\frac{r}{w_{-i}}}(\beta_l-1)\frac{1}{2}\right]+\frac{c_i}{w_i^2}=0\ .$$

We can infer that if $(\partial P_i/\partial w_i)|_{w_i=w_{-i}} > 0$, then $w_i^*$ is on the right of $w_{-i}$ and $w_i^* > w_{-i}$. The condition is

$$\left.\frac{\partial P_i}{\partial w_i}\right|_{w_i=w_{-i}} = -(p-c_i)\left[a_i\alpha+\frac{a_i+a_{-i,i}}{2}\beta_r+\frac{3a_i+a_{-i,i}}{2}e^{-1}(\beta_l-1)\beta_r\right]+\frac{c_i}{w_{-i}^2}>0\ ,$$

which is equivalent to

$$w_{-i} < \widehat{w}_{-i} = \left(\rho_i\widehat{\delta}_i\right)^{-\frac{1}{2}}\ .$$

Similarly, if $w_{-i} = \widehat{w}_{-i}$, then $(\partial P_i/\partial w_i)|_{w_i=w_{-i}} = 0$, which means that $w_i^* = w_{-i}$; while if $w_{-i} > \widehat{w}_{-i}$, then $(\partial P_i/\partial w_i)|_{w_i=w_{-i}} < 0$, which implies that $w_i^*$ is on the left of $w_{-i}$ and $w_i^* < w_{-i}$.

We next analyze the structure of the best response curve $w_i^*(w_{-i})$. The first order derivative of $w_i^*$ with respect to $w_{-i}$ is:

$$\frac{dw_i^*}{dw_{-i}} = \frac{\beta_r(\beta_l-1)\frac{w_i^{*2}}{4w_{-i}^2}\left(a_i e^{-\frac{r}{w_i^*}}\frac{w_{-i}^3}{w_i^{*3}} - a_{-i,i}e^{-\frac{r}{w_{-i}}}\right)}{2a_i\alpha + \beta_r(a_i + a_{-i,i}) + \beta_r(\beta_l-1)\left[a_i e^{-\frac{r}{w_i^*}}\left(2 + \frac{w_{-i}}{w_i^*} + \frac{w_{-i}^2}{4w_i^{*2}}\right) + a_{-i,i}e^{-\frac{r}{w_{-i}}}\frac{3}{4}\right]} = \frac{A}{B},$$

where $A$ and $B$ denote the numerator and the denominator respectively. Clearly, $B > 0$.

If $dw_i^*/dw_{-i} = 0$, which is equivalent to $A = 0$ and $a_i e^{-r/w_i^*} w_{-i}^3/w_i^{*3} = a_{-i,i}e^{-r/w_{-i}}$, then

$$\begin{aligned}\frac{d^2w_i^*}{dw_{-i}^2} &= -\frac{A}{B^2}\frac{dB}{dw_{-i}} - \frac{2A}{Bw_i^*w_{-i}} + \frac{\beta_r(\beta_l-1)\frac{w_i^*}{4w_{-i}^3}\left[a_i e^{-\frac{r}{w_i^*}}\frac{w_{-i}^3}{w_i^{*3}}\left(3 - \frac{w_{-i}}{2w_i^*}\right) - a_{-i,i}e^{-\frac{r}{w_{-i}}}\frac{w_i^*}{2w_{-i}}\right]}{B} \\ &= \frac{\beta_r(\beta_l-1)\frac{w_i^*}{4w_{-i}^3}a_{-i,i}e^{-\frac{r}{w_{-i}}}\left(3 - \frac{w_{-i}}{2w_i^*} - \frac{w_i^*}{2w_{-i}}\right)}{B}.\end{aligned}$$

Numerical tests show that $3 - w_{-i}/2w_i^* - w_i^*/2w_{-i} > 0$ if $a_i e^{-r/w_i^*} w_{-i}^3/w_i^{*3} = a_{-i,i}e^{-r/w_{-i}}$ and $w_i^* > w_{-i}$; while $3 - w_{-i}/2w_i^* - w_i^*/2w_{-i} < 0$ if $a_i e^{-r/w_i^*} w_{-i}^3/w_i^{*3} = a_{-i,i}e^{-r/w_{-i}}$ and $w_i^* < w_{-i}$. Therefore, when $w_i^* > w_{-i}$, if $dw_i^*/dw_{-i} = 0$, we have $d^2w_i^*/dw_{-i}^2 > 0$; when $w_i^* < w_{-i}$, if $dw_i^*/dw_{-i} = 0$, we have $d^2w_i^*/dw_{-i}^2 < 0$. According to the theorem in Crouzeix (1980), given that function $w_i^*(w_{-i})$ is twice differentiable, $w_i^*$ is quasi-convex in $w_{-i}$ when $w_i^* > w_{-i}$ and is quasi-concave in $w_{-i}$ when $w_i^* < w_{-i}$. ■

# H Proof of Theorem 2

*Proof.* Since firm $i$'s profit $P_i$ is quasi-concave in waiting time $w_i$, there exists a pure strategy Nash Equilibrium (Fudenberg and Tirole (1991)), which means that there are intersections of the two best-response curves $w_i^*(w_{-i})$ and $w_{-i}^*(w_i)$.

To prove scenario 1, we need only show that when $\widehat{w}_i > \widehat{w}_{-i}$, the intersections of the two response curves are below the forty-five degree line in the $(w_{-i}, w_i)$ space, which is equivalent to no intersection above the forty-five degree line.

The best response curve $w_i^*(w_{-i})$ above the forty-five degree line can be denoted as

| $c_i$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $a_i$ | 0.6 | 0.8 | 1 | 1.2 | 2.1 |
| $\beta_r$ | 1 | 4 | 7 | 10 | 13 |
| $\beta_l$ | 2 | 4 | 6 | 8 | 10 |

Table 1: Model parameters.

$(w_{-i}, w_i^*)$ with $w_{-i} < \widehat{w}_{-i}$; while the best response curve $w_{-i}^*(w_i)$ above the forty-five degree line can be denoted as $(w_{-i}^*, w_i)$ with $w_i > \widehat{w}_i$.

According to Proposition 7, $w_{-i}^*(w_i)$ is quasi-concave in $w_i$ when $w_i > \widehat{w}_i$, so we have $w_{-i}^* \geq \min\left(w_{-i}^*(\widehat{w}_i), w_{-i}^*(\infty)\right)$, where $w_{-i}^*(\widehat{w}_i) = \widehat{w}_i > \widehat{w}_{-i}$ and

$$w_{-i}^*(\infty) = \left(\frac{c_{-i}}{(p - c_{-i})\left[a_{-i}\alpha + \frac{a_{-i}+a_{i,-i}}{2}\beta_r + \frac{a_{i,-i}}{2}e^{-\frac{1}{2}}(\beta_l - 1)\beta_r\right]}\right)^{\frac{1}{2}} > \widehat{w}_i > \widehat{w}_{-i} \ .$$

Therefore, when $w_i > \widehat{w}_i$, we have $w_{-i}^* > \widehat{w}_{-i}$. As a result, the curve $(w_{-i}^*, w_i)$ does not intersect with the curve $(w_{-i}, w_i^*)$ where $w_{-i} < \widehat{w}_{-i}$. So there is no intersection of the two response curves above the forty-five degree line when $\widehat{w}_i > \widehat{w}_{-i}$, which implies that in the Nash Equilibrium, $w_i^* < w_{-i}^*$.

Scenarios 2 and 3 can be proved in the same way. ■

# I Uniqueness of the Nash equilibrium when $r = (w_i + w_{-i})/2$

In this appendix we numerically study whether the pure strategy Nash equilibrium is unique when $r = (w_i + w_{-i})/2$. We conduct an extensive computational study with varied model parameter as indicated in Table 1.

Specifically, we fix firm $-i$'s parameters and change firm $i$'s parameters according to Table 1. While firm $-i$'s capacity cost, $c_{-i}$, is assumed to be 3, we vary firm $-i$'s capacity cost, $c_i$, to be $1, 2, 3, 4$ and 5. We set $\alpha = 2$. Firm $-i$'s parameter $a_{-i}$ is set to be 1, while firm $i$'s parameter $a_i$ is assumed to be $0.6, 0.8, 1, 1.2$ or 2.1. In addition, we vary the reference effect parameter, $\beta_r$, to be $1, 4, 7, 10$ and 13. The loss aversion parameter, $\beta_l$ is assumed to be $2, 4, 6, 8$ and 10. Finally, both $a_{-i,i}$ and $a_{i,-i}$ take the value of 0.2. The above choices of parameters cover all three scenarios in Theorem 2 and all three scenarios in Proposition 2.

In total, we vary 4 model parameters. There are altogether 625 possible combinations of model parameters. All the cases are such that the Nash equilibrium is unique.

# J Proof of Corollary 2

*Proof.* From Theorem 2 we know that in order to determine the order of the two firms' equilibrium strategies $w_i^*$ and $w_{-i}^*$, we need only compare $\widehat{w}_i$ and $\widehat{w}_{-i}$. If $\widehat{w}_i > \widehat{w}_{-i}$, then the equilibrium strategies $w_i^* < w_{-i}^*$. The condition corresponds to

$$\frac{\widehat{w}_i}{\widehat{w}_{-i}} = \left(\frac{\rho_{-i}}{\rho_i} \cdot \frac{2\alpha + (1+\theta_2)\beta_r + (3+\theta_2)e^{-1}(\beta_l - 1)\beta_r}{2\theta_3\alpha + (\theta_1+\theta_3)\beta_r + (\theta_1+3\theta_3)e^{-1}(\beta_l - 1)\beta_r}\right)^{-\frac{1}{2}} > 1 \ ,$$

which is equivalent to $\rho_{-i}/\rho_i < \widehat{R}$, where

$$\widehat{R} = \frac{2\theta_3\alpha + (\theta_1+\theta_3)\beta_r + (\theta_1+3\theta_3)e^{-1}(\beta_l - 1)\beta_r}{2\alpha + (1+\theta_2)\beta_r + (3+\theta_2)e^{-1}(\beta_l - 1)\beta_r} \ .$$

If $\widehat{w}_i = \widehat{w}_{-i}$, then the equilibrium strategies $w_i^* = w_{-i}^*$. The condition corresponds to $\rho_{-i}/\rho_i = \widehat{R}$. If $\widehat{w}_i < \widehat{w}_{-i}$, then the equilibrium strategies $w_i^* > w_{-i}^*$. The condition corresponds to $\rho_{-i}/\rho_i > \widehat{R}$. ■

# K Proof of Proposition 8

When we fix $\beta_l$, to analyze $\widehat{R}$, we need to compare $\theta_3$ and

$$E = \frac{\theta_1 + \theta_3 + (\theta_1 + 3\theta_3)e^{-1}(\beta_l - 1)}{1 + \theta_2 + (3+\theta_2)e^{-1}(\beta_l - 1)} \ .$$

Since $a_{-i,i}/a_{i,-i} < a_i/a_{-i}$, we have $\theta_3 > E$, hence $\widehat{R}$ is decreasing in $\beta_r$ and bounded from below by $E$.

When we fix $\beta_r$, to analyze $\widehat{R}$, we need to compare

$$F = \frac{2\theta_3\alpha + (\theta_1 + \theta_3)\beta_r}{2\alpha + (1 + \theta_2)\beta_r}$$

and $G = (\theta_1 + 3\theta_3)/(3 + \theta_2)$. If $\beta_r < \alpha$, we have $F > G$, so $\widehat{R}$ is decreasing in $\beta_l$ and bounded from below by $G$; if $\beta_r = \alpha$, we have $F = G$, so $\widehat{R} = G$; if $\beta_r > \alpha$, we have $F < G$, so $\widehat{R}$ is increasing in $\beta_l$ and bounded from above by $G$. ■

# L Proof of Proposition 9

When $\beta_r = 0$, we have $\widehat{R} = \theta_3$; when $\beta_r > 0$, $\widehat{R}$ stays below $\theta_3$ and is bounded from below by $(\theta_1 + \theta_3)/(1 + \theta_2)$, given that $a_{-i,i}/a_{i,-i} < a_i/a_{-i}$ $(\theta_1/\theta_2 < \theta_3)$. Therefore, as long as $\rho_{-i}/\rho_i$ is within the interval of $((\theta_1 + \theta_3)/(1 + \theta_2), \theta_3)$, there always exist values of $\beta_r$ and $\beta_l$ for the reversal effect to happen. On the other hand, if we observe the reversal effect, then $\rho_{-i}/\rho_i$ is always within the interval. ■