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# The Use of Mathematical Models in Financial Accounting

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## *1. Statement of Purpose*

Well over a year ago it was proposed that I write a survey paper for this conference which discussed the role of mathematical models in financial accounting. The difficulty with this assignment is that mathematical models have been used to address a wide variety of issues in financial accounting, and a paper which touched on all of them would probably be unable to examine any single one in depth. Therefore, I have chosen in financial accounting three popular mathematical modeling themes and have interpreted these themes in the context of a competitive equilibrium market model, to lend greater cohesiveness to the discussion.

I refer to the three topics chosen as: (1) assessing the consequences of public disclosure; (2) formalizing the notion that markets are efficient with respect to information; and (3) understanding the effect of increased public disclosure on private incentives to acquire information. Other areas in financial accounting which use mathematical models, but are not discussed here, include income measurement, valuation models (i.e., models which map accounting variables, or other information, into equi-

* University of Chicago. A preliminary version of this paper, entitled "An Integration of Accounting Theories of Financial Disclosure," was presented at Washington University (St. Louis), New York University, and the Universities of Chicago, Michigan, Pennsylvania, and Texas (Austin). I greatly appreciate the experience these seminars, and the people who participated in them, provided me in preparing this extensively revised draft. In addition, during the course of writing this paper at Chicago, detailed editorial suggestions were made by Robert Holthausen, Richard Leftwich, Jim Patell, Grace Pownall, Katherine Schipper, and Mark Wolfson.

librium market prices), and models of a competitive market used to derive testable implications.[1]

Among those papers in financial accounting that employ mathematical models as a methodology, perhaps the most popular theme during the past decade has been examining the consequences of public disclosure. In section 2, I survey this literature. Typically, papers written in this area posit a highly stylized model of an economy and attempt to determine when additional public disclosure does or does not have "social value." From a methodological perspective, I have two criticisms of this literature. First, there appear to be no general results in this literature other than the fact that public disclosure has the potential to redistribute wealth; furthermore, since the wealth redistributive effect was well established early in the literature's history, it is difficult to appreciate the contribution of subsequent articles. In particular, I hope to show that many papers are simply variations on the theme of wealth redistribution: they attempt to refine further the notion of a redistributive effect but toward no apparent greater objective.

My second criticism is that many of the models are perched so precariously on exogenous (and untestable) assumptions that their conclusions are highly suspect. For example, I develop a simple model of a competitive market and then, by varying assumptions about the economy, reach conflicting conclusions. (For purposes of this discussion, I regard the definition of "social value" used in each article as an assumption of the model.) This attempt at deriving multiple scenarios from a single universal example can be viewed as a type of sensitivity analysis; I claim the models are very sensitive to changing assumptions. (Another related criticism of this literature not discussed here is the extent to which the results obtained from mathematical models can be used to derive policy implications.)

In section 3, I review the literature which has attempted to formalize the notion that markets are efficient with respect to information. The objective of mathematical models here has been to conceptualize, or perhaps rationalize, the empirical evidence that capital markets are efficient. To my mind, this literature is still in transition and therefore it is difficult accurately to assess its impact. The interesting feature of this literature is that it answers economic questions about how prices and beliefs are formed endogenously in a competitive market. Furthermore, the mathematical models themselves have generally developed vertically: that is, each model subsumes previous ones. This is in contrast to the social welfare implications of public disclosure literature where the development has been horizontal: each model and its underlying assumptions tend to be incompatible with previous characterizations. In addition to a brief survey of the literature, section 3 reconciles different mathe-

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[1] I emphasize that the references at the end of this survey represent a selected bibliography and are not intended to be comprehensive. In particular, I do not refer to papers in accounting which are not particularly germane to the three themes I survey.

matical and empirical notions of an efficient market and suggests one which has been well received in the finance and economic literature.

In section 4, I examine the effect of additional public disclosure on the private incentives to acquire costly information. To do this, I propose a further refinement of the model of a competitive market developed in section 3. While I generally look favorably on this literature, the models that have been employed to study this question are limited in that they rely on exogenous, or partial equilibrium, assumptions to study a problem whose resolution clearly requires understanding endogenous phenomena. Because this criticism is obvious, rather than pursue it, I suggest my own model of costly private inquiry. The advantage of the model I propose is that it reconciles more endogenous phenomena than those characterizations previously offered. In addition, I derive some comparative statics concerning the relationship between exogenous, costless public disclosure, and costly private inquiry. The relationships are straightforward, and the comparative statics may be useful primarily for pointing out the types of questions we can answer given the limitations of a mathematical model.

In a concluding section of this paper, I offer my perspective on using mathematical models to examine accounting issues in general. It is difficult to articulate why some papers which employ this methodology are inherently more interesting and successful than others. In section 5, I suggest some reasons.

## 2. *Social Welfare and Public Disclosure*

### INTRODUCTION

While questions of social welfare are intimately related to general economic theory, and there have been important contributions in finance and economics to the issue of public disclosure, I limit the following survey as much as possible to those articles that have either been published in the accounting literature, or have been published outside this literature by individuals who view themselves primarily as accounting researchers and have used a mathematical model as an integral part of their discussions. This serves as more than simply a practical limit on those articles I select to survey, although this is clearly one benefit. My intention is to give this topic the perspective one has from reading the published *accounting* literature, since I view that as an important role of this conference.

I begin with an early paper by Feltham [1968], inspired by the work on the economic theory of teams by Marschak and Radner, who later expressed many of their ideas in their book on the topic (Marschak and Radner [1972]). While previous discussions of alternative accounting information systems were strictly qualitative, Feltham's purpose was to lay the foundation for a formalized approach for measuring the value of changes in an information system. In effect, the approach viewed a user of accounting information as an expected utility maximizer and an information system as a process which assigns probabilities to the occurrence of future events. This approach determines the values of alternative information systems by quantitatively measuring their effect on the user's expected utility.

The historical significance of Feltham's [1968] article cannot be overlooked. While it can be argued that all accounting research deals very broadly with the consequences of employing one method of reporting versus an alternative, it was Feltham's insight that led to the quantification of the values of alternative information systems. Subsequent articles by Feltham and Demski [1970], Butterworth [1972], and Marshall [1972] refined and expanded upon this theme. However, there is a clear suggestion in all of these discussions, perhaps attributable to the thrust of Feltham's [1967] doctoral dissertation, that the appropriate realm for this theory was evaluating changes in accounting information for *managerial* decisions, as opposed to financial disclosure alternatives.

Demski's [1973] paper marked an important shift in orientation from exclusively managerial to financial. Interpreting an information system more broadly as a financial accounting standard or disclosure alternative, Demski pointed out that rank-ordering all information systems, on the basis of one being at least as preferred by all expected utility maximizing users as an alternative regardless of users' preferences and choice problems, falls victim to the problem of ranking systems which are noncomparable. His argument is based on the fact that one system is universally at least as preferred as an alternative if it is at least as fine an information system as the alternative. However, Demski adds, fineness as a procedure for ranking yields incomplete relations; that is, two information systems can be noncomparable with respect to fineness, which implies that one is not necessarily universally at least as preferred as the other. This result, which derives from Blackwell [1953] and Blackwell and Girshick [1954], prompts Demski to remark that no set of standards exists that will always rank all alternatives in accordance with preferences and beliefs, regardless of these preferences and beliefs, in an expected utility characterization of the problem. Marshall [1972] makes many of the same points, but is less explicit in relating them to the choice among financial standards.

I interpret the choice among financial standards more narrowly as the choice among financial disclosure policies. Viewed with this interpretation, and the exalted perspective of perfect hindsight, there are two deficiencies with Demski's [1973] characterization of the problem. The first, and less significant, is that if the various financial disclosure policies are costless alternatives (as Demski assumes in his discussion), the most valuable information system (in the sense of fineness) can be achieved by disclosing more data. For example, faced with choosing either *LIFO* or *FIFO* to report inventories, one can always select a finer information system than either by simply disclosing both *LIFO* and *FIFO*. The point is that we can limit the choice among disclosure policies to a partition of progressively finer information systems with no loss of generality. In this characterization of the problem, the choice is determined by the information overload trade-off, as well as the (so far ignored) cost of implementing finer information systems.

The second, and more significant, problem with Demski's [1973] characterization is that each individual user rank-orders alternative information systems without considering competition among users of accounting data. The competitive economic process in which users employ financial accounting data has a profound influence on their choices and may cause information users not to prefer a finer information system to a coarser one if the data generated are to be released publicly in a competitive market. For example, Baiman [1975] makes the point that a coarser information system may be preferred by a user to a finer one if the system also changes the environment.

Contemporary with the Demski [1973] and Marshall [1972] articles, a watershed in accounting thought arose from a discussion in the economic literature by Hirshleifer [1971]. (See also Fama and Laffer [1971] and Marshall [1974].) Seizing upon an exchange between Arrow [1962] and Demsetz [1969] concerning whether society offers sufficient private incentives for inventive activity, Hirshleifer argued that circumstances arise in which plentiful private incentives to generate information exist despite the fact that the information yields no benefit to society (unlike true inventive activity). One example of this is an exchange economy in which investors generate information to benefit their individual trading activities without regard to increasing the aggregate wealth of society as a whole. In effect, information can have private value without having any "social value."

Demski [1974] introduced elements of the Hirshleifer discussion into the accounting literature by means of an example in which the simultaneous receipt of information failed to change traders' production and consumption schedules. While noting that in this instance the information is clearly socially useless, he acknowledged that what "social value" means in a more robust context is a deep issue. Demski also demonstrated that the information may have private value by showing that its *exclusive* use by a single trader leaves that trader better off. But to do this he employs a partial equilibrium argument in that the price taker's actions are assumed not to affect prices.

Demski [1974] also considered alternative disclosure policies, such as a firm disclosing information only to its owners. He observed that movement from one alternative to another may harm certain groups of traders while it benefits others. What crystallized is the recognition that different disclosure policies redistribute wealth in the economy. This is a significant step in the evolution of the choice among financial disclosure policies because Demski [1974] has turned from interpreting the choice as one of universal acceptance (as in Demski [1973]) to one of trade-offs among various categories of accounting data users. Furthermore, all changes away from the status quo in financial disclosure policies involve a potential redistribution of wealth among users.

Observing that there are trade-offs, Demski cited Arrow's [1963] well-known result that there exists no Pareto optimal method of selecting among social alternatives that is not dictatorial, is independent of irrelevant alternatives, and provides a complete, transitive, and reflexive ranking of the social alternatives. Policy implications of Arrow's result to financial accounting are discussed further in Beaver and Demski [1974]. In summary, Demski [1974, p. 232] remarks: "... evaluation of consequences ultimately must entail trading off one person's gains for another's. We cannot collapse the evaluation question into a generally usable, straightforward preferences relation without imposing how these trade-offs are to be accomplished. This imposition may take the form of a direct and obvious imposition or selection of some institutional method of choice, such as reliance on information markets, voting, or social control agencies."

In the light of contemporary thought in accounting, Demski's discussion may no longer seem revolutionary. However, conventional wisdom at that time appeared to be that more acounting data resulted in more "benefit" to society (ignoring the information overload phenomenon and the cost of producing that information). Therefore, the essential accounting question was how best to present this data. For example, during the same era as Demski [1974], May and Sundem [1973] argued that the more information available to market participants, the better the allocation of resources in the economy, without acknowledging that more information is also potentially harmful to certain groups of participants through the redistribution of their wealth.

Subsequent research could have simply accepted the fact that public disclosures resulted in the redistribution of wealth. Instead, it focused almost exclusively on scenarios in which additional data did, or did not, yield Pareto improvements. In retrospect, it appears that the realization among researchers that public disclosure can harm investors (through the redistribution of wealth) was both so puzzling and intriguing that it spawned a whole generation of dissertations, working papers, and publications on this single topic. One of my major criticisms is that the subsequent research uncovered no new truths. Instead, it simply promoted variations on the same theme that public disclosure redistributes wealth.

Cushing [1977] debated Demski's [1974] conclusion by pointing out that in the absence of the assumption of heterogeneous users, the problem of determining a mechanism of collective choice vanishes. (Arrow's discussion admits to the existence of heterogeneous users by requiring that the method of selection not be dictatorial: that is, there exists no single individual such that the group's decision is always consistent with his.) Assuming a single representative individual, however, also eliminates the problem Demski posed since financial disclosure alternatives no longer result in a redistribution of wealth among users of accounting data. Cushing also reviewed the economic literature which questions the appropriateness of Arrow's requirements of transitivity and independence of irrelevant alternatives (which Demski had labeled "innocuous"). Finally, Cushing defined an optimal accounting procedure as one which results in a Pareto move for all users vis-à-vis any alternative accounting principle, holding the principles used in all other areas of controversy constant. Limitations to this type of partial approach to standard setting were explored by Bromwich [1980].

Two points in Cushing's discussion are worth noting. First, with regard to the evolution of contemporary thought, Cushing did not challenge the fact and its impact on a single investor can be negative. This suggests that researchers became quickly resigned to this idea. Second, while Pareto optimality is an unassailable procedure for rank-ordering alternatives, it can result in noncomparability. This is a point emphasized in Beaver and Demski [1974], for example.

Hakansson [1977] considered more involved exchange economy scenarios. (See also Hakansson [1981] which appears to be a continuation and refinement of his 1977 paper.) He introduced groups of investors who differ in their endowments of superior information, skills in inferring it from the activities of the well-informed, and ability to purchase it. Hakansson defines one disclosure policy as superior to an alternative if its employment implies the existence of a redistribution of wealth which can leave everyone at least as well off regardless of which future state occurs. Whether this redistribution is to take place, and how it will be implemented, is left unresolved. It is difficult to arrive at any general conclusions from Hakansson's work because of the exogeneity of many of his assumptions; for example, one group of traders is exogenously endowed with superior information while another group can infer this information from the superior-informed group in some unspecified way. In the absence of empirical evidence, we have no knowledge about whether this scenario is representative of an actual securities market. This problem, however, is generic to much work done in this area and is, perhaps, the explanation for why so few insights (beyond those originally discussed in Hirshleifer [1971] and Demski [1974]) have trickled down from this steady stream of literature.

An interesting observation made by Hakansson was that using a type of Pareto criterion, as he and others proposed, to rank-order disclosure policies inevitably may preclude full disclosure. Hakansson [1977, pp. 413–14] remarks:

> The fact that wealth redistribution is accompanied by changes in output places those in charge of formulating disclosure requirements in a sensitive position. If the overriding goal of security regulation is to give everyone equal access to information, then requiring full disclosure of events, as they happen, appears necessary and may come close to accomplishing that task .... However, suppose that full disclosure leads to economic inefficiencies in the sense that a redistribution of wealth exists in the absence of full disclosure, which would make everyone better off .... In reality, of course, the redistribution in question is difficult to carry out, so it becomes hard to argue against overdisclosure in view of the fact that the

majority of investors are likely to benefit from it, at least in the short run. In addition, it is not entirely clear to what extent, if any, Congress intended security regulations to be concerned with redistribution per se as opposed to welfare in the larger sense.

I interpret these remarks as questioning whether it is reasonable that a disclosure policy achieve full disclosure even if the accounting data have no "social value." I add to this the observation that any type of Pareto criterion is likely to be sufficiently strong so as to preclude any movement in disclosure policies away from the status quo and leaves unresolved how the "status" should be set in new situations.

While an awareness of the existence of a redistributive effect may be vital to our understading of the problem, it was given a disproportionate amount of attention. Understanding circumstances in which information does, or does not, have "social value" seems to be of limited use if, at a practical level, those circumstances cannot be identified. A further criticism of this work is that there is not even agreement about what constitutes "social value."

There are some obvious candidates for a definition of "social value," but no clear consensus in the literature. The first candidate is that the information results in an increase in the aggregate quantity of goods and services supplied to society independent of its effect on particular individuals (e.g., the redistribution of wealth). This requirement seems to be implied by Hirshleifer [1971] and Hakansson [1977], and I refer to it as a circumstance in which information has aggregate benefit. The second candidate is that the information results in a Pareto improvement in the expected utilities of market participants. This requirement seems to be implied by Beaver and Demski [1974] and Cushing [1977], and I refer to it as a circumstance in which information has individual benefit. Carefully note that the first definition neither implies, nor is implied by, the second. The former requires that society is made better off in terms of increased total goods and services, while the latter requires that each individual within the society be made better off in terms of increased expected utility. It is a simple exercise to show that the former can be achieved in the absence of achieving the latter, and vice versa. (Of course, a third candidate is that a financial disclosure policy achieves both an aggregate and an individual benefit.)

Even if we accept the Pareto improvement criterion, there is the methodological problem of which beliefs to use in measuring an individual's change in expected utility: the ex ante beliefs which prevail before the information is revealed or the ex post beliefs which result after an individual knows the information. The latter seems to make more sense with regard to determining whether the information is beneficial (see, e.g., the discussion in Ng [1975]), but the former makes more sense if we explicitly consider the incentives to acquire information. The same controversy surfaces in the aggregate benefit requirement: is it sufficient that total goods and services are expected to increase, or must they indeed increase regardless of the outcome? (Hakansson [1977] adopts the latter.)

Perhaps for these reasons Beaver and Demski [1974] concur that the choice among financial disclosure policies requires a collective choice mechanism which gives explicit attention to the trade-offs across accounting users, despite the fact that the disclosed information may yield no "social value" using any of the definitions suggested above.

## AN ILLUSTRATION OF DIFFERENT PUBLIC DISCLOSURE SCENARIOS

What I attempt here is to illustrate aspects of the previous discussion by devising an example which, on the one hand, is practically transparent in its simplicity, while on the other hand, is sufficiently rich that I can distinguish a variety of scenarios by varying certain critical assumptions. My intent is to illustrate those scenarios which are suggested in the papers surveyed here by showing how they can be derived from a single example. Devising such an example is not without its shortcomings. For one thing, it does not capture the subtleties of the papers surveyed. Its advantage is twofold. First, many mathematical models oftentimes obfuscate simple points, and the advantage of my approach is that it aids in understanding general results in this literature. Second, it permits a sensitivity analysis in which I show how conclusions are changed (some might say dictated) by varying exogenously specified assumptions.

I do not claim that any of the scenarios I construct accurately reflect the consequences of public disclosure. However, we should not allow some obvious problems with these illustrations to obscure the more important weakness, which is the fact that by generating a variety of different scenarios, I do not know which conclusion to advocate in the absence of agreement about which model is the appropriate one. More fundamentally, I question whether assessing the social welfare of public disclosure is an appropriate use of a mathematical model given the many-faceted nature of this problem.

I begin my illustration with a simple model of a competitive equilibrium in which there are no public disclosure issues. As I proceed, I introduce variations on this example. Imagine that there are two farmers who live on a very isolated island off the Atlantic Coast and grow potatoes to sustain their survival. As it happens, a mountain range separates their respective properties. Esther, the first farmer, owns 3,000 acres of land on the east side of the mountain range, and Wesley, the second farmer, owns the same amount of acreage on the west side of the range. Esther and Wesley both have a utility for the consumption of $x$ bushels of potatoes given by the function:

$$U(x) = -e^{-x}.$$

This utility function is a negative exponential with a constant absolute risk-tolerance of one. It is a meteorological phenomenon of this island that during each summer growing season it has rained exclusively on either the east side of the mountain range or the west side, and the underlying meteorological process which produces this phenomenon is such that it does either with probability one-half. Where it rains, acreage produces one bushel of potatoes per acre, while on the other side no potatoes grow during that season.

Esther and Wesley recognize that setting up a contingent claims market can produce a Pareto improvement in their lives, since it results in risk sharing among risk-averse consumers. Therefore, after they have planted their respective acreage in the spring, but before it rains, each trades claims with the other for potatoes contingent upon rain on the east side or potatoes contingent upon rain on the west side. Let $P_E$ and $P_W$ denote the price of a contingent claims to a bushel of potatoes grown on the east and west sides of the mountain range, respectively. For convenience, I imagine that the farmers designate the price of a potato contingent upon rain on the east side of the mountain range to be a numeraire commodity, that is, $P_E \equiv 1$.

Upon entering the market, Esther's wealth is $3{,}000 \cdot P_E = 3{,}000$, and Wesley's wealth is $3{,}000 \cdot P_W$, since each farmer has 3,000 bushels of potatoes available for trade contingent upon rain on his respective property. Let $D_{EE}$ and $D_{EW}$ represent Esther's demand for $D$ bushels of potatoes contingent upon rain on the east and west sides of the mountain range, respectively; similarly, let $D_{WE}$ and $D_{WW}$ represent Wesley's demand for $D$ bushels of potatoes contingent upon rain on the east and west sides of the mountain range, respectively.

A competitive equilibrium in the market for contingent potatoes-claims is characterized by a circumstance in which there exists a set of prices $P_E$ and $P_W$, and a set of demands $D_{EE}$, $D_{EW}$, $D_{WE}$, and $D_{WW}$ whereby:
(1) Esther selects $D_{EE}$ and $D_{EW}$ to maximize her expected utility subject to her budget constraint, that is, she selects the $D_{EE}$ and $D_{EW}$ which:

$$\max_{\{D_{EE}, D_{EW}\}} \; -\tfrac{1}{2}e^{-D_{EE}} - \tfrac{1}{2}e^{-D_{EW}}$$

$$\text{subject to } 1 \cdot D_{EE} + P_W \cdot D_{EW} \leqslant 3{,}000;$$

(2) Wesley selects $D_{WE}$ and $D_{WW}$ to maximize his expected utility subject to his budget constraint, that is, he selects the $D_{WE}$ and $D_{WW}$ which:

$$\max_{\{D_{WE}, D_{WW}\}} \; -\tfrac{1}{2}e^{-D_{WE}} - \tfrac{1}{2}e^{-D_{WW}}$$

$$\text{subject to } 1 \cdot D_{WE} + P_W \cdot D_{WW} \leqslant 3{,}000 \cdot P_W;$$

(3) Markets clear, that is:

$$D_{EE} + D_{WE} = 3{,}000,$$

and:

$$D_{EW} + D_{WW} = 3{,}000.$$

It is a simple exercise to show that a competitive equilibrium is given by

$P_E \equiv 1$, $P_W = 1$, and $D_{EE} = D_{WE} = D_{EW} = D_{WW} = 1{,}500$. That is, because both farmers are identical except for their asymmetric endowments, the competitive equilibrium drives them toward complete symmetry characterized by each assuming a contingent potatoes-claim to one-half of his own potato yield and one-half the other farmer's potato yield. Consequently, in equilibrium each farmer's expected utility is:

$$-\tfrac{1}{2}e^{-1{,}500} - \tfrac{1}{2}e^{-1{,}500} = -e^{-1{,}500}.$$

As my first variation on this problem, I consider the effect of publicly available information on the competitive market. This variation is in the spirit of, for example, Baiman [1975], Ng [1975], and Verrecchia [1978]. That is, it makes the point that more information is not necessarily better than less, if, along with the increased information, the set of feasible actions is also changed. I refer to this variation as the "information paradox."

Suppose that the weather bureau installs (at no cost to Esther and Wesley) a meteorologist named Maurice on the island to record the prevailing direction of the Atlantic winds on the island each winter. It is a well-known meteorological phenomenon that each winter the winds blow either north or south with equal probability, and that when the prevailing winds blow north (south) in the winter, the conditional probability of rain on the east (west) side of the mountain range during the following spring–summer growing season is two-thirds, and when the winds blow south (north), the conditional probability of rain on the east (west) side during the growing season is one-third. After recording the direction of the prevailing winds each season, Maurice makes the information publicly available at no cost. Here I assume, and it is critical to this scenario, that the information is made available in the winter before the potato market meets in the spring. Note carefully that Maurice's information results in finer information about future rainfall than the null system (which results in traders associating equal probability to both events). Therefore, if provided for a farmer's exclusive use before he trades, this information would be of positive value to him.

It is difficult, if not impossible, to offer an explanation for the existence of Maurice. The literature I survey here typically eschews this whole issue by claiming to be concerned with the *consequences* of public disclosure and not its endogenous evolution. However, the *spirit* of the literature appears to suggest the following rationale (which I relate without comment). Each farmer knows that if information of the type Maurice offers were provided for his exclusive use before he trades, this information would be of value to him. Thus, each farmer may privately attempt to contract with Maurice for his services. But Maurice, for his part, realizes he can do better to sell his information to each farmer individually, since each farmer would pay to obtain identical information even though all other farmers (could and would) obtain it at the same cost.[2] Viewing this activity from afar, some ostensibly benevolent and unquestionably omnipotent observer (our *deus ex machina*) personally subsidizes Maurice to make his information publicly available at no cost to farmers. The extent to which this rationale has explanatory power in the real world is, at best, unclear.

Since the information is publicly available at no cost, both farmers will obtain it and face the following problem. With probability one-half, the information will disclose that the prevailing winds blow north. In this circumstance Esther will choose $D_{EE}$ and $D_{EW}$ such that:

$$\max_{\{D_{EE}, D_{EW}\}} \quad -\tfrac{2}{3}e^{-D_{EE}} - \tfrac{1}{3}e^{-D_{EW}}$$

$$\text{subject to } 1 \cdot D_{EE} + P_W \cdot D_{EW} \leq 3{,}000,$$

while Wesley will choose $D_{WE}$ and $D_{WW}$ such that:

$$\max_{\{D_{WE}, D_{WW}\}} \quad -\tfrac{2}{3}e^{-D_{WE}} - \tfrac{1}{3}e^{-D_{WW}}$$

$$\text{subject to } 1 \cdot D_{WE} + P_W \cdot D_{WW} \leq 3{,}000P_W.$$

A competitive equilibrium results in this case when $P_E \equiv 1$ and $P_W = ½$, and $D_{EE} = D_{EW} = 2{,}000$ while $D_{WE} = D_{WW} = 1{,}000$. The information that the prevailing winds blow north implies that Esther's wealth remains at 3,000, while her expected utility (in equilibrium) attains:

$$-\tfrac{2}{3}e^{-2{,}000} - \tfrac{1}{3}e^{-2{,}000} = -e^{-2{,}000},$$

which is greater than $-e^{-1{,}500}$. Poor Wesley, on the other hand, achieves a lower expected utility than previously because his wealth is lowered by the content of the information to $3{,}000 \cdot P_W = 1{,}500$. Furthermore, his expected utility in equilibrium is:

$$-\tfrac{2}{3}e^{-1{,}000} - \tfrac{1}{3}e^{-1{,}000} = -e^{-1{,}000},$$

which is less than $-e^{-1{,}500}$. This illustrates the redistribution of wealth through public disclosure. Esther's wealth has become relatively greater than Wesley's, and this is reflected in the level of expected utility each can achieve in a competitive market.

However, it is equally likely that the information will disclose that the prevailing winter winds blow south. In this circumstance, Esther's and Wesley's roles are reversed. Esther's expected utility (in equilibrium) is $-e^{-1{,}000}$ while Wesley's is $-e^{-2{,}000}$. Now consider the expected utilities of Esther or Wesley (they are identical) before Maurice records the wind direction. Let $EU$ denote the expected utility and $IP$ denote probability. Then, Esther's expected utility before Maurice records the wind direction

[2] A point which is oftentimes misunderstood is that even in the event that all farmers could do better by collectively agreeing not to acquire information, such a "solution" is unenforceable. The self-enforcing equilibrium is Nash and can be characterized typically as a situation in which each farmer acquires identical information, in effect to protect himself from other farmers behaving similarly.

is computed as:

$$\begin{aligned} EU[\text{Esther}] = {} & EU[\text{Esther} \mid \text{Prevailing northerly winds}] \\ & \cdot IP[\text{Prevailing northerly winds}] \\ & + EU[\text{Esther} \mid \text{Prevailing southerly winds}] \\ & \cdot IP[\text{Prevailing southerly winds}] \\ = {} & (-e^{-2{,}000}) \cdot (\tfrac{1}{2}) + (-e^{-1{,}000}) \cdot (\tfrac{1}{2}). \end{aligned}$$

But what is this value? Since the utility function $U(x) = -e^{-x}$ is a concave function, as are all utility functions that exhibit risk-aversion, it follows that for all $0 < \alpha < 1$ and all $x$ and $y$:

$$-\alpha e^{-x} - (1 - \alpha)e^{-y} < -e^{-\alpha x - (1-\alpha)y}.$$

Therefore, letting $\alpha = \frac{1}{2}$, $x = 2{,}000$, and $y = 1{,}000$, this relationship implies:

$$-\tfrac{1}{2}e^{-2{,}000} - \tfrac{1}{2}e^{-1{,}000} < -e^{-1{,}500}.$$

This, in turn, implies that publicly available information reduces the expected utility of both Esther and Wesley from what it would be in the absence of the information.

The moral of this story is that while we may take the availability of costless public information for granted, its effect on Esther and Wesley is no "small potatoes." When investors trade in a market, introducing additional information may be no more than introducing a lottery or an additional layer of uncertainty. Under the assumption that investors are risk-averse, this lottery makes them each worse off (i.e., it results in a strictly inferior Pareto move). This illustrates the problem with the Demski [1973] and Marshall [1972] characterization which relies exclusively on fineness as a criterion for rank-ordering information systems. As Baiman [1975, p. 9] remarks:

> Why does the Fineness Corollary not hold in a multiperson world as it does in a single-person world? The reason is because of the implicit assumption underlying the proof of Blackwell's Theorem and its Fineness Corollary that the environment faced by [an individual] does not depend upon his choice of information system. In a single-person world the informal argument for the nonnegative private value of a finer internal information system is that one can always ignore it. As a lower bound [an individual], can always choose the same action with the finer system as with the coarser one and be no worse off. But this presumes that [an individual's] choice of internal information system does not affect the environment or opportunity set (completely represented by state occurrence) he faces. Indeed the state occurrence is defined to be independent of any action choices of the individuals.

In the context of my example, Baiman's point is that by precluding the possibility of Esther and Wesley trading before the information is made publicly available, the one for whom the information is not favorable is deprived of the opportunity to assume a contingent potato-claims position which is feasible in the absence of the information. The publicly available information changes the opportunity set in a competitive market.

An obvious criticism of the "information paradox" scenario is that it precludes the possibility of Esther and Wesley trading before the information is available and thereby insuring themselves against its content. One rationale for this scenario was that it examined the consequences of information when markets were in disequilibrium. Another rationale for this was simply that an appropriate way to evaluate the "social value" of information was to consider two otherwise identical "would be" situations: one without publicly available information, and one with it and no possibility of insuring against its consequences before it arrived. The issue is moot as to whether this is an acceptable way to assess the effect of publicly available information; the point is that it appears repeatedly in the literature (see, e.g., Ng [1975], Ng [1977], and Hakansson et al. [1979]).

With regard to the sensitivity of this assumption, consider what happens if the farmers are allowed to trade before information about the direction of the prevailing winter winds is made publicly available. I refer to this scenario as the "null effect"; it is inspired by the Hirshleifer [1971] and Demski [1974] articles. Assume that at harvest time, Esther and Wesley initiate a market for next year's harvest with four contingent potato-claims: one for each wind direction and rain occurrence. For example, one potato-claim would be the one conditional on Maurice disclosing a prevailing northerly wind in winter and the event of rain on the east side of the mountain range during the growing season. But here we are back at the Demski [1974] scenario. By insuring themselves against public disclosure by setting up a richer contingent claims market, the farmers also insure that the information has no effect on consumption and investment plans. Thus, it can be argued (as Demski [1974] did) that the information has no "social value" since it causes no change. That is, Esther and Wesley preserve the expected utility positions they attained in the absence of public disclosure.

Two objections to the null effect scenario are that it avoids the problem of defining "social value" in a more robust setting, and it requires that the market be complete over both information disclosures and states. It has been argued that a truly complete contingent-claims market (as discussed, e.g., in Debreu [1959]) is not an assumption which corresponds to an actual market setting. Thus, an alternative is to consider the consequences of public disclosure after farmers trade to an equilibrium in a market which only allows contingent-claims to potatoes grown on the east and west sides of the mountain. My interpretation of Hakansson et al. [1979] is that this is what they have done. They define information to have "social value" if farmers trade after receipt of publicly available information, assuming that their endowment position happens to be their equilibrium position without information. In other words, if farmers are permitted to trade to an equilibrium position before information is made publicly available, and then trade after its arrival, Hakansson et al. [1979] can be interpreted as suggesting that the information has "social value." As I argued above, the difficulty with the Pareto criterion is that it is almost never satisfied and therefore makes it difficult to move away from the status quo. One difficulty with accepting the Hakansson et al. definition of "social value" is that it is generally satisfied when farmers are in equilibrium before the information is disclosed. That is, except for a set of circumstances which appear pathological, additional publicly available information will generally result in further trading. Thus, this definition of "social value" can potentially be used to justify an endless variety of disclosure requirements.

This reservation notwithstanding, I am not arguing that the Hakansson et al. definition is unacceptable, although neither do I find it compelling.[3] My point is that the literature surveyed here can lead to widely conflicting policy prescriptions depending upon how one defines "social value." Even Hakansson is not consistent in Hakansson [1977] and Hakansson et al. [1979].

But there are other ways to argue that public disclosure has "social value" by simply changing exogenously specified assumptions. Perhaps the most obvious one is to imagine that farmers can choose where to plant potatoes after they obtain public information. I refer to this variation as "production." No one in the literature has devoted much attention to it because it is so obvious. Suppose that when it becomes known that there is a prevailing northerly wind, Wesley plants 1,500 acres on his own land and then travels east to help Esther plant an additional 1,500 acres on the east side of the mountain range. This arrangement assumes a constant returns-to-scale for potato-acres-planted as a function of each farmer's seasonal labor. It increases the expected number of bushels of potatoes available to society from:

$$\tfrac{1}{3}\cdot 3{,}000 + \tfrac{2}{3}\cdot 3{,}000 = 3{,}000$$

to:

$$\tfrac{1}{3}\cdot 1{,}500 + \tfrac{2}{3}\cdot 4{,}500 = 3{,}500.$$

Even here, however, how we interpret this variation depends upon how we define "social value." The expected number of bushels of potatoes is increased, but if it happens to rain on the west side of the mountain (and it can with probability one-third), then there are fewer potatoes to harvest. Thus, the information does not necessarily result in an improve-

[3] At a practical level, what Hakansson et al. are suggesting is that the volume of trading subsequent to a disclosure is a proxy for the disclosure's "social value" (see, e.g., the discussion in Lev and Ohlson [1982]). Volume could potentially be a useful proxy if (ignoring various frictions in a market) there happens to be an unambiguous relationship between the extent to which newly disclosed information altered traders' expectations and the level of volume which resulted from these altered expectations. The relationship is, however, ambiguous (see, e.g., Verrecchia [1981]) and therefore I question the usefulness of volume as a proxy even in a relatively unencumbered, frictionless model of a market.

ment in the aggregate supply of potatoes *post facto*. Using Hakansson's [1977] definition of a superior disclosure policy, which requires an improvement regardless of which future state occurs, the disclosure of this information would not necessarily be superior. (To achieve superiority, I would have to assume an increasing returns-to-scale for potato-acres-planted as a function of information.)

But it is not necessary to introduce production to justify public disclosure in an expected utility context. Consider a fourth variation on my original example. Suppose that Esther's prior beliefs about the probability of rain on the east and west sides of the mountain range are 0.9 and 0.1, respectively, while Wesley's are 0.1 and 0.9, respectively. In this scenario, a competitive equilibrium in the absence of public disclosure results in no trading. Esther and Wesley are each sufficiently convinced that it will rain only on their own respective sides of the mountain range that they see no advantage to trading. Of course, Esther and Wesley do not have "rational expectations" about the possibility of rain because I continue to assume that the underlying meteorological process which generates rain is such that it rains half the time on the east side and half the time on the west. I refer to this variation as "irrational expectations." At a very simple level, this scenario captures the essence of Ng [1977] (although he endows it with a sexier title, the "Pareto-optimality of authentic information"). This variation is also inspired by researchers (e.g., Downs and Dyckman [1973]) who argue that despite the fact that markets are efficient, market efficiency is an aggregate phenomenon, and each trader can still benefit from increased disclosure by improving his ability to form correct expectations.

Now, imagine that a disclosure (in the form of an explanation) by Maurice about how the underlying meteorological process operates convinces both Esther and Wesley that it does indeed rain on either side of the mountain range with equal probability. This causes each farmer to revise his beliefs to reflect the correct probabilities; it also induces them to trade between themselves and thereby insure their survival. Consequently, even in the absence of a productive benefit, the information has made society better off. The intuition behind this variation is that the beneficial role of public disclosure may be that it prods investors to have "rational expectations" about whatever uncertainty underlies the market. This is an idea alluded to by Downs and Dyckman [1973, p. 316]: ". . . even if we accept the efficient-markets hypothesis as a reasonable approximation, there are a host of individual effects caused by the heterogeneity of individual expectations that deserve consideration from the accounting viewpoint. Accounting information may have a significantly greater societal impact than current writers on efficient markets would have us believe." I interpret this remark (in the context of their entire discussion) as suggesting that the only thing the efficient markets literature demonstrates is that markets are rational in aggregate. The role of financial disclosure in benefiting society may be in motivating "rational expectations" on an individual basis among investors.

CONCLUSION

In simplest terms, the fact that the implication of each of the papers in this survey relies critically upon either exogenously specified assumptions or a definition of "social value" demonstrates the essential failure of this literature. The relationship between public disclosure and social welfare can be discussed *ad absurdum* by introducing more complicated scenarios. With equal facility and no claim to offer resolution, we can debate the number of angels who can dance on the head of a pin. In the absence of empirical evidence there seems to be no way to determine which set of exogenously specified assumptions is the most reasonable, and even if we could there appears to be no consensus in the literature about what constitutes "social value."

I want to be careful in my criticism. Each of the papers I mentioned is interesting in its own regard and some, such as Baiman [1975], never get entangled in social welfare issues. Furthermore, understanding how public disclosure would evolve endogenously in a mathematical model of a competitive market remains an interesting puzzle. However, I do not believe this literature will ultimately solve this puzzle. I have two specific reservations. First, little is known about how a variety of more primitive economic phenomena (e.g., the production, dissemination, and acquisition of information) evolve endogenously in the simplest scenarios. Therefore, it seems unreasonable to expect to solve the puzzle when these other endogenous phenomena are ignored in its solution. This fact has been obscured by those papers that compensate for their lack of endogeneity by introducing progressively more complicated exogenously specified economic environments. Second, whatever solution is offered of necessity will rely on a series of simplifying assumptions, the effect of which will be difficult to gauge. The previous discussion suggests that conclusions are very sensitive to assumptions over which there is no clear agreement.

This section, and especially the previous paragraph, invite comparisons with Ohlson and Buckman [1980]. They remark (Ohlson and Buckman [1980, pp. 538–39]):

> ... basic insights with respect to the economic role of information-production and dissemination have to be developed within the context of general equilibrium analysis.... Depending upon the nature of the mechanism in question, the preferences and beliefs of individuals, the distribution of endowments across individuals, the number and types of commodities, the opportunities created by the productive technologies, etc., information can, presumably, have substantially different welfare implications. This kind of perspective raises a rather difficult yet basic question: what should be the "essential" ingredients, or model-characteristics, of any analysis if its basic purpose pertains to the welfare analysis of information? As is usually the case, it makes no sense to consider a model of great generality and "realism"; the sheer complexity will virtually guarantee the absence of any discernible implications. The alternative approach is one of making "strategically simplifying assumptions." Yet, any such model must capture attributes of the economic environment which cannot be dispensed with if meaningful conclusions about the role of information are to be drawn.

Ohlson and Buckman use this argument to justify more research in this area (as suggested in their title, "Toward a Theory of Financial Accounting"). However, I would argue that this passage points to the fact that questions of social welfare and public disclosure are too broad in scope, complicated in detail, and multifaceted in nature to be successfully broached by a methodology as simple as that of a mathematical model. Wolfson [1980], in discussing Ohlson and Buckman [1980], mentions even more endogenous phenomena which should be considered in solving the puzzle of social welfare and public disclosure; it is unclear to me whether Wolfson intends for his remarks to be interpreted as a criticism of this literature. My point is that they should be. In my opinion it is unlikely that future research in this area will provide useful insights.

## *3. The Informational Efficiency of Capital Markets*

### INTRODUCTION

In this section, I survey several theoretical papers that attempt to formalize the concept of market efficiency in an informational sense. At the conclusion of this survey, I suggest a rational expectations model of a market as an interpretation of an informationally efficient market. Specifically, a rational expectations model requires that traders' conjectures about the behavior of an endogenously determined price are fulfilled in equilibrium. This topic is a departure from the previous one. However, because the entire paper is restricted to models of a competitive market, the methodology I use in this section is a refinement of that used in the previous section. In the next section, I integrate the characterization of an efficient market proposed here into the problem of costly private acquisition of information.

The idea that equilibrium prices in competitive security markets largely reflect the information possessed by various traders is both widely accepted and often analyzed by researchers in accounting, economics, and finance. Empirically, evidence shows that predictions conditioned on market prices are not dominated by predictions using many other sources of information (see, e.g., Fama [1970; 1976] and Gonedes and Dopuch [1974]).

The empirical literature has inspired several theoretical papers. Lintner [1969] demonstrates that equilibrium prices can be characterized as a type of geometric averaging of traders' heterogeneous beliefs, implying that markets are efficient by virtue of how prices are formed. Extending Lintner's idea, Rubinstein [1975] suggests that a necessary and sufficient condition for an individual to perceive that all of his information is fully reflected in prices is that he have "consensus beliefs." That is, consensus beliefs are those beliefs which, if held by all individuals in an otherwise similar economy, would generate the same equilibrium prices as in the actual heterogeneous economy. (See also, Verrecchia [1979*b*].) However, the notion of information efficiency embodied in Rubinstein's discussion has been more broadly interpreted (see, e.g., Beaver [1981*a*; 1981*b*]) as follows. A market is efficient with respect to an information set $A$, say, if the prices it generates are identical to those generated in an otherwise identical economy in which the set $A$ accurately describes the information available to each and every market participant. The common, or homogeneous, belief induced by knowledge of $A$ is the consensus belief.

One of the major problems with Rubinstein's [1975] discussion is that his definition of a consensus belief is so thoroughly couched in a stylized economy that what he means to suggest more generally is subject to a variety of interpretations. For example, Beaver [1981*a*, pp. 160–61] remarks: "The crux of a theory of market efficiency which does not rely upon the existence of a set of "experts" is that the level of knowledge reflected in prices is greater than merely the "*average*" level of knowledge among investors in the market . . . the quality of the knowledge reflected in prices is considerably higher than the *average* quality of knowledge across the individuals who comprise the market" [emphasis added]. Later, in discussing the nature of a consensus assessment, Beaver [1981*a*, p. 161] remarks: "The consensus can predict better than the *average* of the individuals who comprise it . . . " [emphasis added].

The problem here is that a consensus belief, as suggested by Rubinstein, is a particular type of average: the one implied by the market clearing price. It is unclear how the "average" to which Beaver alludes, and which he suggests is inferior to the consensus, is determined. I interpret a consensus belief as a belief which in some sense is superior to the individual heterogeneous beliefs from which it is composed. The difficulty with accepting *my* interpretation at face value is that there is no intrinsic reason that this should be the case. If two bettors have an opinion about which horse will win a race, and one bettor's opinion is vastly superior to the other's (because of the time and expense invested in formulating that opinion), there may be no "averaging" of their opinions that will be superior to that of the more sagacious bettor.[4]

Verrecchia [1979*a*] suggests one explanation. As the number of traders who participate in the market for a security becomes large, the variations in the price of a security caused by the variations in traders' beliefs make the market price vary as if all traders knew the unobservable distribution of returns for the security. The fact that efficiency is predicated on some large number of traders has been alluded to by various authors. For example, Fama [1970] suggests that "the market may be efficient if 'sufficient numbers' of investors have ready access to available information." Beaver [1981*a*, p. 160] has an analogous interpretation:

> Consider each individual containing a "small" amount of knowledge and a considerable amount of idiosyncratic behavior. This can be modeled as each individual receiving a garbled signal from an information system that provides an ungarbled signal disguised by

[4] Perhaps the most reasonable interpretation of the use of the word "average" in Beaver's discussion is that it is the average over time. That it, over time or in repeated trials, no single individual can outperform the consensus. But this interpretation leaves unclear how market efficiency is defined in a single trial, or how many trials are required before the consensus dominates. My interpretation of a consensus belief requires that a consensus belief dominates even in the event of a single trial. Thus, while my interpretation is well defined, it is, perhaps, too strong a requirement.

a "noise" component. The garbling is so large that any inspection of that individual's behavior provides little indication that such an individual is contributing to the efficiency of the market with respect to the ungarbled information system. Moreover, assume that this is true for every individual who comprises the market. However, the idiosyncratic behavior, by definition, is essentially uncorrelated among individuals. As a result, security price, which can be viewed as a "consensus" across investors, is effectively able to diversify away the large idiosyncratic component, such that only the knowledge (i.e., the ungarbled signal) persists in terms of explaining the security price.

Viewed in a statistical context, Beaver's language suggests that either price itself, or some element implicit in price, is an unbiased estimator of whatever unobservable causal random variable underlies the economy. This would lead to the following definition of an efficient market: a market is efficient if price is a more efficient estimator (i.e., has a smaller variance) of the unobservable random variable than the estimator used by any single individual (assuming that all these estimators are unbiased). Verrecchia [1980] explores the robustness of this definition: specifically, he suggests that price is a more efficient estimator by virtue of traders' information acquisition activities.

But a logical dilemma results from the refinement offered by either Beaver [1981*a*] or Verrecchia [1979*a*; 1980] of the original Rubinstein notion. If price is such a good estimator, why don't competitive traders condition their beliefs at least in part on the behavior of price? For example, when an investor observes the change in the price of a security while his own beliefs remain fixed, he learns something about the expectations of other traders. Therefore, in an efficient market, each trader's belief should somehow be endogenously determined through both his private information and the behavior of price, which, in turn, is endogenously determined through traders' demands for a security.

To examine this point in greater detail, I briefly survey that body of literature which gives explicit attention to the idea that traders condition their beliefs and actions on a conjecture about the behavior of price; furthermore, the behavior they conjecture about price is self-fulfilling in equilibrium. This notion is referred to as "rational expectations," an unfortunate term because it in no way captures the subtlety of the issue and prejudices the case (since, after all, who would want to defend a "nonrational expectations" model of a market?). Simply stated, the rational expectations scenario requires that traders understand their economic environment sufficiently that their conjectures about the behavior of prices are fulfilled in equilibrium. A useful by-product of this understanding is that traders learn from prices, since prices are an aggregator of information.

For example, Grossman [1976; 1978] analyzed an economy in which traders have diverse pieces of information about the return of risky assets, and he claimed that the rational expectations equilibrium price reveals to all traders all of the information of the traders taken together. A major implication of this result is that when traders take prices as given, they have no economic incentive to acquire information since price costlessly reveals to all traders whatever is known by a single trader. Analyses suggesting the full revelation of aggregate information (see also, Kihlstrom and Mirman [1975]) are subject to the objection that they produce results which are too strong on empirical grounds. For example, at a minimum, one would have to provide empirical evidence that markets were "strong-form" efficient before claiming that prices were fully revealing (I discuss this below).

Therefore, an alternative is to study markets in which an equilibrium results in some aggregation of individuals' information without revealing all of it. This circumstance is informally referred to as a "noisy rational expectations economy." (A paper by Green [1973] is one of the earliest examples of this idea.) The important feature of models of partial revelation is that when an individual trader uses observable endogenous variables, such as prices, as pieces of information in addition to his privately collected information, he benefits from the information collected by others without believing that his own is superfluous. This allows private incentives to collect information (I explore this issue in greater detail in section 4).

In the following section, I introduce results from some recent work by Hellwig [1980] and Diamond and Verrecchia [1981] concerning the partial aggregation of diverse information. The purpose of this exercise is to better understand what is meant by an efficient market in a formal mathematical model of a competitive equilibrium.

## AN ILLUSTRATION OF A PARTIALLY REVEALING RATIONAL EXPECTATIONS ECONOMY

I return to the island of Esther and Wesley in an attempt to illustrate some of the concepts discussed above. I change the previous scenario considerably. Irrigation systems have been built so that regardless of on which side of the mountain range it rains, both Esther and Wesley have an adequate supply of water to grow potatoes. In fact, there is such a bountiful harvest that they set aside surplus potatoes for future consumption. The surplus consists of two distinct types of potatoes. One type is suitable only for storing at harvest time in underground cellars. This preserves the potatoes and can be thought of as a process in which each potato stored at harvest time yields one potato for consumption in the spring. The second type is suitable only to grow additional spuds in a winter greenhouse. However, the greenhouse technology has not been entirely perfected, and the potato yield per surplus potato employed is best represented by a random variable $\tilde{u}$. (I adopt the convention that a tilde over a symbol denotes that it is a random variable, while realizations of the random variable are represented by the same symbol without the tilde.)

Each farmer has identical prior beliefs about $\tilde{u}$. Specifically, each believes that $\tilde{u}$ is distributed normally with mean ten and precision one (where precision is defined to be the inverse of variance).[5] In order to analyze the ability of markets to aggregate diverse information, I assume that each farmer is costlessly endowed with information about what the realization of $\tilde{u}$ will be at the end of the winter growing season. Specifically, each farmer observes the temperature only on his side of the mountain range, and he knows that observing the temperature tells him something about the success of growing potatoes during the winter. That is, observing temperature is equivalent to observing the realization of $\tilde{u}$ perturbed by some noise, or random error. Let $\tilde{y}_E$ and $\tilde{y}_W$ denote the temperature information observed by Esther and Wesley, respectively. Given the description above:

$$\tilde{y}_E = \tilde{u} + \tilde{\epsilon}_E,$$

and:

$$\tilde{y}_W = \tilde{u} + \tilde{\epsilon}_W,$$

where $\tilde{\epsilon}_E$ and $\tilde{\epsilon}_W$ are independent random variables each distributed normally with mean zero and precision one. Each farmer uses his own observation (i.e., $\tilde{y}_E = y_E$ for Esther and $\tilde{y}_W = y_W$ for Wesley) in conjunction with his prior beliefs to form posterior judgments about $\tilde{u}$, but he does not observe the other farmer's information.

In addition to the information observed privately by each farmer, another stochastic factor influences their environment. This factor introduces noise into the relationship between farmers' demands and equilibrium prices, and results in prices revealing only part of the aggregate information. The noise is represented by uncertainty about the aggregate supply of potatoes dedicated to the greenhouse process. I imagine that the number of potatoes Esther and Wesley grow each season that are suitable for the greenhouse process are random variables denoted by $\tilde{x}_E$ and $\tilde{x}_W$, respectively. The $\tilde{x}_E$ and $\tilde{x}_W$ are independent, identically distributed normal random variables, each with a mean of zero and a variance $V$, and are both independent of $\tilde{u}$, $\tilde{\epsilon}_E$, and $\tilde{\epsilon}_W$. (A mean of zero is assumed for convenience with no loss of generality.) Let $\tilde{X} = \tilde{x}_E + \tilde{x}_W$ denote the total supply of greenhouse input potatoes. Each farmer knows how many potatoes he personally contributes to the greenhouse process, but not the amount contributed by the other. Because $\tilde{x}_E$ and $\tilde{x}_W$ are independent,

[5] The fact that the potato yield employed, $\tilde{u}$, can assume negative values sometimes causes readers to pause. An important implicit assumption is that Esther and Wesley assume unlimited liability for the crop, and a negative realization of $\tilde{u}$ can be thought of as "blighted potatoes," the removal of which is both required and costly (as is, for example, the case with garbage or hazardous waste). Similarly, the price of a claim to the yield $\tilde{u}$ can be negative if it is reasonably thought that blighted potatoes are all that will result from the greenhouse process. Here, it must be assumed that in this event, Esther and Wesley could not simply destroy the crop instead of awaiting its consequences. In any event, the normal distribution is assumed to ensure a facile analysis and should not cause undue concern.

each farmer's contribution provides some imperfect information about the total contribution $\tilde{X}$.

Suppose now that Esther and Wesley set up a market to exchange shares in claims to potatoes stored underground and claims to the winter greenhouse potato yield. The price of a claim to a potato stored underground is maintained at one (i.e., it represents the numeraire commodity); the price of a claim to the yield $\tilde{u}$, which is the yield per potato used in the greenhouse process, is denoted by $P$. The expressions $B_E$ and $D_E$ denote Esther's demand for a claim to an underground stored potato and a claim to the greenhouse yield, respectively; similarly, $B_W$ and $D_W$ denote Wesley's demand for each, respectively. Finally, I continue to assume that Esther's and Wesley's preferences for potatoes are characterized by the negative exponential utility function introduced earlier.

Esther and Wesley now enter the market to exchange claims. A rational expectations competitive equilibrium can be loosely characterized as a circumstance in which markets clear, traders' beliefs about the distribution of all observable random variables are fulfilled, and prices depend on information through supply and demand. In the simple market I describe, I can solve for rational expectations equilibrium price random variable $\tilde{P}$ through a series of steps. First, imagine that Esther and Wesley conjecture that the behavior of price is described by:

$$\tilde{P} = \alpha 10 + \frac{\beta}{2}\{\tilde{y}_E + \tilde{y}_W\} - \frac{\gamma}{2}\tilde{X}, \tag{1}$$

where $\alpha + \beta = 1$.[6] This implies that $\tilde{P}$ has a normal distribution with mean ten and variance $\beta^2 + \frac{\beta^2}{2} + \frac{\gamma^2 V}{2}$. I solve for $\alpha$, $\beta$, and $\gamma$.

Consider Esther's perspective. If she conjectures that price behaves as in equation (1), then she believes that the vector of random variables peculiar to her situation, $(\tilde{u}, \tilde{X}, \tilde{x}_E, \tilde{y}_E, \tilde{P})$, has a five-variate normal distribution with mean (10, 0, 0, 10, 10) and covariance matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & \beta \\ 0 & 2V & V & 0 & -\gamma V \\ 0 & V & V & 0 & -(\gamma/2)V \\ 1 & 0 & 0 & 2 & (3/2)\beta \\ \beta & -\gamma V & -(\gamma/2)V & (3/2)\beta & \beta^2 + (\beta^2/2) + \gamma^2(V/2) \end{pmatrix}. \tag{2}$$

[6] Obviously, I assume that Esther and Wesley make this conjecture because it turns out to be the correct one (i.e., it is self-fulfilling). Even though they may not be born with this intuition, over time presumably, Esther and Wesley would figure this out. However, this raises a subtle technical point about whether other conjectures exist which are also self-fulfilling. The equilibrium I derive is unique for the class of all prices having this (conjectured) linear form. Whether equilibria exist for some other conjectured form is not, as yet, known, but appears unlikely.

Wesley has an identical representation of the vector of random variables peculiar to his situation, $(\tilde{u}, \tilde{X}, \tilde{x}_W, \tilde{y}_W, \tilde{P})$.

This implies, however, that upon observing $\tilde{x}_E = x_E$, $\tilde{y}_E = y_E$, and $\tilde{P} = P$ (the only random elements Esther actually observes) and using properties of conditional distributions, Esther determines that $\tilde{u}$ has a normal distribution with mean:

$$E[\tilde{u} \mid \tilde{x}_E = x_E, \tilde{y}_E = y_E, \tilde{P} = P] = 10 + \frac{\beta\gamma x_E + \gamma^2 V(y_E - 10) + 2\beta(P - 10)}{2\gamma^2 V + 3\beta^2} \tag{3}$$

and variance:

$$\text{Var}[\tilde{u} \mid \tilde{x}_E = x_E, \tilde{y}_E = y_E, \tilde{P} = P] = \frac{\gamma^2 V + \beta^2}{2\gamma^2 V + 3\beta^2}. \tag{4}$$

Furthermore, preferences for potatoes represented by an exponential utility function imply that Esther's demand for the claims to the risky process are given by:

$$D_E = \frac{E[\tilde{u} \mid \tilde{x}_E = x_E, \tilde{y}_E = y_E, \tilde{P} = P] - P}{\text{Var}[\tilde{u} \mid \tilde{x}_E = x_E, \tilde{y}_E = y_E, \tilde{P} = P]}. \tag{5}$$

Similarly, Wesley's demand $D_W$ is determined by an expression which is identical to equation (5) except that Wesley observes $\tilde{x}_W = x_W$ and $\tilde{y}_W = y_W$ and the same price $\tilde{P} = P$.

The market-clearing requirement for claims to the risky asset is given by:

$$\begin{aligned} \tilde{X} &= \tilde{x}_E + \tilde{x}_W = D_E + D_W \\ &= \frac{E[\tilde{u} \mid x_E, y_E, P] - P}{\text{Var}[\tilde{u} \mid x_E, y_E, P]} + \frac{E[\tilde{u} \mid x_W, y_W, P] - P}{\text{Var}[\tilde{u} \mid x_W, y_W, P]}. \end{aligned} \tag{6}$$

Let $h^{-1} = \text{Var}[\tilde{u} \mid x_E, y_E, P] = \text{Var}[\tilde{u} \mid x_W, y_W, P] = \dfrac{\gamma^2 V + \beta^2}{2\gamma^2 V + 3\beta^2}$. Rearranging equation (6) yields:

$$\tilde{P} = \frac{1}{2}\{E[\tilde{u} \mid x_E, y_E, P] + E[\tilde{u} \mid x_W, y_W, P]\} - \frac{1}{2h}\tilde{X},$$

or:

$$\begin{aligned} \tilde{P} = &\left\{\frac{\gamma^2 V + 3\beta^2 - 2\beta}{2\gamma^2 V + 3\beta^2 - 2\beta}\right\}10 + \left\{\frac{\gamma^2 V}{2\gamma^2 V + 3\beta^2 - 2\beta}\right\}\left\{\frac{\tilde{y}_E + \tilde{y}_W}{2}\right\} \\ &- \left\{\frac{\gamma^2 V + \beta^2 - \gamma\beta}{2\gamma^2 V + 3\beta^2 - 2\beta}\right\}\left\{\frac{\tilde{X}}{2}\right\}. \end{aligned} \tag{7}$$

However, from equation (1), $\tilde{P} = \alpha 10 + \beta\left\{\dfrac{\tilde{y}_E + \tilde{y}_W}{2}\right\} - \gamma\left\{\dfrac{\tilde{X}}{2}\right\}$. Thus, the original conjecture about the behavior of price is self-fulfilling whenever equation (1) and equation (7) are identical; but this occurs only if:

$$\alpha = \frac{V+1}{2V+3}, \qquad \beta = \frac{V+2}{2V+3}, \qquad \gamma = \frac{V+2}{2V+3}.$$

Furthermore:

$$h = 2 + \frac{1}{2V+1}. \tag{8}$$

It is easy to allow the algebraic manipulation to obscure the economic intuition. The distinguishing features of a rational expectations equilibrium are that beliefs are formed endogenously by conditioning expectations on price and that traders' conjectures about the behavior of price (which is also endogenously determined through supply and demand) are fulfilled. In short, traders are never "fooled" by the behavior of price, and they use it as an auxiliary source of information. This is fundamentally different from assuming that either beliefs or prices are exogenously specified in equilibrium. (Private data, however, are exogenously specified in this model.)

### RECONCILIATING NOTIONS OF MARKET EFFICIENCY

I attempt here to reconcile the various interpretations of market information efficiency discussed earlier. First, consider the expression for the rational expectations competitive equilibrium price in equation (7):

$$\tilde{P} = \frac{10(V+1)}{V+3} + \frac{V+2}{V+3}\left(\frac{\tilde{y}_E + \tilde{y}_W}{2}\right) - \frac{V+2}{V+3}\tilde{X}.$$

Suppose that some trader always knew the realization $\tilde{X} = X$, that is, he had "insider information." Then, through the algebraic manipulation (noting that all other parameters on the left-hand side of equation (9) are common knowledge):

$$\frac{\tilde{P} - \dfrac{10(V+1)}{V+3} + \dfrac{V+2}{V+3}\tilde{X}}{\dfrac{V+2}{V+3}} = \frac{\tilde{y}_E + \tilde{y}_W}{2} \tag{9}$$

he would be able to infer $\frac{\tilde{y}_E + \tilde{y}_W}{2}$; this inference yields information which is superior (in a statistical sense) to the information available to either Esther or Wesley. This is because $\frac{\tilde{y}_E + \tilde{y}_W}{2}$ is an unbiased statistic of $\tilde{u}$ and has a variance of three-halves. Therefore, Fischer Black has suggested to me the following interpretation of "consensus beliefs" in a rational expectations context: a market is efficient if the consensus belief implicit in price $\left(\text{i.e., } \frac{\tilde{y}_E + \tilde{y}_W}{2}\right)$ is no less efficient an estimator than the estimator available to any single trader in the market in the absence of the information available in price.

Consider, for example, the information available to either Esther or Wesley in the absence of the information available in price. In each case, it would be just the signal $\tilde{y}_E$ or $\tilde{y}_W$, respectively. As each signal has a variance of two, it represents an estimator which is inferior to the consensus belief implicit in price, $\frac{\tilde{y}_E + \tilde{y}_W}{2}$, which has a variance of three-halves. In other words, if the consensus belief were noiselessly revealed through price, it would represent an estimator that was at least as good as that estimator available to each trader privately.

Of course, in a rational expectations economy, traders condition their beliefs on price. Recalling equation (8), the precision each trader achieves in equilibrium is equal to:

$$h = 2 + \frac{1}{V + 1}.$$

That is, the precision in equilibrium is the sum of the precision of his prior beliefs, one, the precision of his private source of information, one, and the precision of the additional information he gleans from price, $\frac{1}{V+1}$. By conditioning his beliefs on price, each trader profits from the information acquisition of both traders. The expression $\frac{1}{V+1}$ is interpreted as a measure of the informedness of price.

The polar cases are particularly interesting. Observe that $\lim_{V\to\infty} h \to 2$ and $\lim_{V\to 0} h \to 3$. The level of precision $h = 2$ is that level achieved without using prices as an auxiliary source of information. It represents the level attained by using only prior beliefs about $\tilde{u}$ and the private source of information $\tilde{y}$. In effect, as the level of noise becomes infinite, traders learn nothing from price: prices can be described as "nonrevealing." Alternatively, the level of precision $h = 3$ is achieved when each trader knows all the information available in the market. It represents the level attained by using prior beliefs about $\tilde{u}$ in conjunction with the union of all the private sources of information $(\tilde{y}_E, \tilde{y}_W)$. Thus, as the level of noise approaches zero, traders learn everything from price: prices can be described as "fully revealing."

In addition to reconciling theoretical notions of market efficiency, there is the problem of reconciling mathematical models of market efficiency with notions in the empirical literature. The interface between these two bodies of literatures is poor, due perhaps to the semantic problem of using the word "efficiency" to describe a variety of economic phenomena.

It is often suggested that people confuse the interpretation of an efficient market. For example, Kanodia [1980, pp. 923–24] remarks:

> Extant beliefs regarding accounting information are crystallized at two polar extremes. Policy-making bodies like the SEC and the FASB and most practitioners believe that investors are naive in processing information, and are principally concerned with the earnings per share figure as reported by accountants. Investor decision rules are invariant to the rules for computing income, and therefore if the "right" or "proper" measure of income were not adopted, investors would be misled. In recent years stock price researchers have rapidly accumulated evidence against this naive investor hypothesis and in favor of the efficient markets hypothesis .... Armed with these and other empirical findings, some accounting academics have, at the theoretical plane, swung to the polar extreme opposite to the naive investor hypothesis. Implicitly, it has been assumed that at any moment of time the distribution of future security prices is independent of the accounting system and is somehow implied by the economy. The role of accounting information is to reveal this exogenous true distribution to investors. Thus Beaver [1972] argues that the only potential value of accounting information to the individual investor is in the assessment of the systematic risk of securities. Gonedes [1976] theorizes that newly produced information provides signals on the "exact distribution of returns."

> If expectations are formed rationally, asset prices will "fully reflect" all available information and traders' decisions will be optimal in the relation to the behavior of asset prices, *for any* accounting disclosure system. Rather than determining whether or not markets are efficient, the accounting system determines which of several efficient markets equilibria will occur.

To restate Kanodia's point, if by "market efficiency" what is meant precisely is a rational expectations economy (and certainly this is Kanodia's interpretation), then asset prices and beliefs are determined endogenously by the accounting information system itself. Therefore, the accounting data that are produced determine the distribution of prices that are observed. Furthermore, it makes little sense to imagine that the role of accounting information is to make the market "efficient" since, in a rational expectations economy, the market is "efficient" by definition regardless of the accounting disclosure system. However, changing the accounting disclosure system can change endogenously formed expectations, and to that extent can change the nature of the equilibria that result.

A market in which prices are fully revealing is analogous to the "strong-form" efficiency described by Fama [1970]. Beyond that obvious relationship, however, I encounter difficulty relating Fama's [1970] description of "semi-strong" and "weak-form" efficiency to a particular characterization of a rational expectations economy. Regardless of the level of informedness of price, rational expectations competitive equilibria operate "efficiently": traders make conjectures about the behavior of price and their conjectures are fulfilled through their own competitive price-taking behavior. Independent of the level of noise which pervades the economy, traders are never fooled by the behavior of price because their conjectures are fulfilled in equilibrium. Thus, the market is efficient.

No noise implies that the market is efficient in the strong-form. But the presence of noise does not render the market inefficient. It simply reduces the informedness of price (i.e., the extent to which each trader knows the information available to all traders). Consequently, the only unresolved empirical issue is the extent to which prices partially reveal the total of the aggregate information. The point is, if we accept a literal interpretation of a rational expectations economy, there are no efficiency issues to resolve.

The problem here is that in the real world of a security market, where empirical tests are performed, there are frictions, such as transaction costs and information-processing costs. Thus, a literal interpretation of an efficient market is inappropriate. I offer my own interpretation of tests of "semi-strong form" efficiency and/or "consensus beliefs" in a rational expectations economy. Suppose there was a piece of publicly available information in the market. In the (frictionless) mathematical model of a rational expectations economy I present above, all traders would condition their beliefs on this piece of publicly available information, and, since price aggregates the information of traders, price would "accurately reflect" this piece of data. However, price also aggregates other information as well (e.g., traders' private information); therefore, expectations conditioned on price alone would dominate expectations conditioned on the piece of publicly available data alone. When there are frictions present, however, it is not necessarily the case that each trader's demand function will incorporate a particular piece, or pieces, of publicly available information. This, in turn, implies that price will not necessarily "accurately reflect" this piece of data. Thus, expectations conditioned on price alone could potentially be dominated by expectations conditioned on a particular piece of publicly available data. In effect, a test of "semi-strong form" efficiency is a test of the level of friction in a market, since in the absence of friction there is nothing to test (i.e., price would always dominate whatever is known publicly).

## CONCLUSION

To digress briefly on a methodological point, the model of a rational expectations economy Kanodia [1980] introduces differs considerably in detail from the one I present. Kanodia's descends from work by Lucas [1972], whereas mine evolves from work by, for example, Green [1973] and Grossman [1976]. The Lucas-type formulation can be thought of as macroeconomic. For example, Kanodia's model considers production but does not capture the interactive effect among investors: there exists only a single representative individual in the economy. (In his model the choice among costless information systems is dictated solely by fineness, since a single representative individual will always "unanimously" prefer a finer information system.) While the model I present does not introduce production, it explicitly considers the interactive effect created by heterogeneous traders.

An obvious question at this stage is that once we have a mathematical model in which prices are determined endogenously and the beliefs are formed endogenously by conditioning on prices, what other endogenous phenomena would we like to add? In a recent *Wall Street Journal* (March 8, 1982) editorial, S. Burton remarks: "... 'efficient markets' research has had a profound effect on the way that accounting setters have pushed for increased disclosure." In the next section, I examine the relationship between costless, exogenous public disclosure and costly, endogenous private inquiry in an "efficient market" (i.e., a noisy rational expectations model of a competitive market). The objective here is to make costly private information acquisition endogenous, along with prices and belief formation.

## 4. *The Relationship Between Public Disclosure and Private Information Acquisition*

### INTRODUCTION

In the discussions in sections 2 and 3, investors and/or society were always assumed to be exogenously endowed with information. This ignores the obvious relationship between public disclosure and costly private inquiry. This element of the financial disclosure controversy has been expounded upon at length in a series of articles: Gonedes and Dopuch [1974], Gonedes [1975], and Gonedes, Dopuch, and Penman [1976]. While many subtle economic points were raised in those articles, the major conceptual issue seemed to revolve around a single theme. Specifically, disclosure laws may induce a suboptimal allocation of resources for at least two reasons. (1) They may lead to the production of information that would not otherwise be produced in the absence of disclosure requirements; and (2) they may not lead to the production of information by the most efficient producer.

This theme is further refined in Gonedes [1980]. My own liberal interpretation of Gonedes [1980] is that, among other things, he raised three primitive, interrelated questions. (1) How does public disclosure affect private information acquisition? (2) How does public disclosure affect the overall level of informedness of investors (i.e., their knowledge conditional on both public and private sources of information)? And (3) what do the first two questions add to our understanding of the choice among financial disclosure alternatives? The salient feature of Gonedes' [1980] work is that it pointed out that the third question cannot be addressed without answering the first two (i.e., by explicitly recognizing the complementarity effect between public disclosure and privately produced information).

Gonedes initiated his discussion by suggesting that an important implicit assumption commonly made in both academic and nonacademic work is that a financial disclosure is valuable (Gonedes used the word "effective" or "informative") if it alters an investor's assessed distribution of a future event. However, Gonedes questioned the extent to which one can employ a ceteris paribus argument when determining the effect of additional information. He remarked (Gonedes [1980, pp. 442–43]):

> Overlooked by this perspective are the characteristics of information production decisions made by market agents on personal account—via individual actions or private forms of collusion (e.g., information-production activities carried out by firms for subscribers or by trade associations for members). If there are any substitution or complementarity relationships among the signals produced on private account and the signals covered by disclosure rules, then the "effectiveness" of the rules cannot, in general, be assessed independently of private information-production activities. Thus, empirical evidence dealing with the effects of new disclosure rules will, in general, actually reflect both (1) the direct (or "own") effects of the rules on produced information and (2) the effects of any changes in private information-production activities induced by the rules' enactment. In short, there will be a confounding of the rules' "own" effects and induced effects, even though only the former are declared or implied to be of interest.

A resolution of the question Gonedes raised is difficult. Gonedes offered conjectures about the relationship between public disclosure and private information acquisition in a general equilibrium setting on the basis of a partial equilibrium argument, but this approach has obvious limitations.

In the remainder of this section I suggest a model of endogenous, costly information acquisition when public disclosure is exogenous and costless. My objective is to propose a model which is in the spirit of Gonedes' work. In particular, I am curious about the relationship between publicly available information and costly private inquiry in a model in which markets are efficient with respect to information (i.e., I assume a noisy rational expectations model of a competitive market). For example, I derive some general (as opposed to partial) equilibrium comparative statics concerning the relationship between publicly disclosed and privately produced information.

I want to emphasize the limited scope of the following analysis. The model I propose is limited to observing how costless changes in exogenously specified levels of financial disclosure affect endogenously determined levels of costly enquiry. Furthermore, the comparative statics offered later in this section follow fairly straightforwardly from the assumptions of my model, and are introduced more as a pedagogical device than as provocative new results. The more interesting, and clearly more controversial, question of what level of financial disclosure prevails in the first place, or why it exists at all, cannot be addressed in a model as simple as the one I propose. For all intents and purposes, the level of financial disclosure in my model is a parameter which I assume can be set at any value, including zero. Obviously, I am trying to disassociate the analysis done here from the work surveyed in section 2. I think this disassociation is legitimate because here I am not concerned with the "social value" of information (however it is defined). However, I acknowledge that this topic suffers from many of the same conceptual problems that plague the social welfare implications of public disclosure literature.

FIG. 1.—The sequence of events.

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Traders receive endowments | Public disclosure of information | Private information purchased | Trading occurs | Portfolios consumed |

A DESCRIPTION OF A MARKET

In this section, I propose a model to capture certain elements of Gonedes' [1980] discussion. The model used here is a further development of that introduced in previous sections and is a variation on the one suggested by Verrecchia [1982]. It has two assets, a riskless bond and a risky asset, both of which pay off in a single consumption good in the economy (potatoes, for example). It should be clear that the storage process and winter greenhouse process discussed in section 3 were serving the role of a riskless bond and risky asset, respectively. Therefore, in this regard the model described here is identical to the previous one. I clone Esther and Wesley, however, to yield $T$ traders with an identical preference for potatoes given by the exponential utility function (and absolute risk tolerance of one).[7] Five discrete events, or time periods, occur during the operation of the market. First, traders are endowed with assets. Second, there is a public disclosure concerning the return on the risky asset: remember, this is exogenously specified. Third, each trader decides how much costly private information to acquire. Fourth, he observes the costly private information he acquired in the third period and exchanges his endowment with other traders, but does not consume. Finally, trading ceases, and each trader consumes the return realized from his portfolio (see fig. 1).

The numeraire in the market is the price of a bond which all traders know returns one in the final period (i.e., the bond is tantamount to riskless storage). The liquidating dividend on the risky asset is unknown until the final period and is represented by a random variable denoted by $\tilde{u}$, whose realization is $u$. Each trader's prior belief about $\tilde{u}$ is that it has a normal distribution with mean ten and precision one.

The public disclosure in the second period is the announcement of an observation of a random variable $\tilde{y}_0$. This random variable communicates the true return $\tilde{u}$ perturbed by some noise $\tilde{\epsilon}_0$:

$$\tilde{y}_0 = \tilde{u} + \tilde{\epsilon}_0.$$

The random variable $\tilde{\epsilon}_0$ has a normal distribution with mean zero and precision $s_0 \geq 0$. The parameter $s_0$ represents the amount, or quality, of the public information. For convenience I assume the public disclosure is costless to traders. This assumption was arbitrarily chosen and is not intended to portray a realistic scenario.

[7] This is a simplification of the more general case (considered in Verrecchia [1982]) in which traders are heterogeneous.

In the third period each trader can acquire costly private information which he receives in the fourth period. The information trader $t$ receives is also the observation of a random variable $\tilde{y}_t$, which communicates the true return $\tilde{u}$ perturbed by some noise $\tilde{\epsilon}_t$:

$$\tilde{y}_t = \tilde{u} + \tilde{\epsilon}_t.$$

Each $\tilde{\epsilon}_t$ is a random variable which has a normal distribution with mean zero and precision $s_t \geq 0$ and is independent of the perturbations of other traders, as well as the perturbation in the public disclosure, $\tilde{\epsilon}_0$. The parameter $s_t$ represents the amount, or quality, of the private information. The cost of acquiring information with precision $s_t$ is $c(s_t)$, where $c(\cdot)$ is a continuous function with continuous positive first derivative $c'$ and nonnegative second derivative $c''$ for all $s_t \geq 0$. (This implies that $c(s_t)$ is a strictly increasing convex function.)

With regard to costly inquiry, trader $t$ faces the following problem in the third period. His prior beliefs about $\tilde{u}$ are that it has a normal distribution with mean ten and precision one. In the second period he observes $\tilde{y}_0$ and thereby updates his beliefs such that his posterior belief about $\tilde{u}$ is that it has a normal distribution with mean:

$$\frac{10 + s_0 \cdot y_0}{1 + s_0},$$

and precision:

$$1 + s_0.$$

In the fourth period each trader knows that he will costlessly learn something about $\tilde{u}$ by observing price, which aggregates and partially reveals the total amount of information observed by all traders. Let $\Delta$ denote the level of informedness of price, that is, the additional precision each trader gains by conditioning his beliefs on price. Therefore, in the third period each trader decides how much precision $s_t$ to demand at a cost $c(s_t)$ such that the total level of informedness he achieves when he exchanges assets is:

$$1 + s_0 + s_t + \Delta$$

(i.e., the precision of his posterior beliefs); he bases his decision on maximizing his expected utility conditional on observing the public disclosure $\tilde{y}_0$, his private (costly) information $\tilde{y}_t$, and price.

In a world of rational expectations, however, beliefs in the third period (when a trader makes his private information acquisition decision) about the level of informedness in price must be fulfilled in the fourth period (when a market-clearing price obtains). Therefore, for a rational expectations equilibrium to result, traders' conjectures about how much information will be (partially) revealed by price are fulfilled by their own costly information acquisition activities in the third period.

In summary, a rational expectations equilibrium in which the amount of costly private information each trader acquires is endogenously determined requires that markets clear, traders maximize their expected utilities conditional on what they observe and how much costly information they acquire, and traders' beliefs about the behavior of price (and, in particular, its level of informedness) are fulfilled.

Before I discuss such an equilibrium, I need to introduce "noise" into the market, since in its absence prices are fully revealing and the private incentives to acquire costly information are eliminated. (There are alternative methods of introducing noise into the model, but this is one of the most convenient.) Uncertainty about the aggregate supply of risky assets creates noise in the relationships between traders' demands and prices, and results in prices revealing only part of the aggregate information.

Trader $t$ is endowed with $\bar{B}_t$ riskless bonds and $x_t$ risky assets, where $x_t$ is the realization of a random variable $\tilde{x}_t$. The $\tilde{x}_t$ are mutually independent normal random variables with mean zero and variance $TV > 0$, and are independent of the $\tilde{y}_0$ and $\tilde{y}_t$ random variables. Per-capita aggregate supply of risky assets is represented by the random variable $\tilde{X} \equiv \frac{1}{T} \sum_{t=1}^{T} \tilde{x}_t$, which has a normal distribution with mean zero and variance $V > 0$, independent of the number of traders in the market. The parameter $V$ is the level of noise in the market.

In this description of a market, certain assumptions are critical for insuring a tractable analysis. The use of the (negative) exponential utility function implies a linear demand for the risky asset which does not depend on wealth. The assumption that random variables have a multivariate normal distribution considerably simplifies the derivation of conditional distributions. Together, these assumptions permit me to derive a closed-form expression for a rational expectations competitive equilibrium price. The assumption that information acquisition costs are continuous and convex allows a straightforward proof for the existence of an equilibrium (in the theorem below) and also facilitates the comparative statics. While none of these assumptions is necessary for the existence of an equilibrium, they appear to be a requirement for the closed-form characterizations of an equilibrium I derive.

On the basis of these assumptions, I compute a unique rational expectations competitive equilibrium in which the amount of costly private information each trader acquires is endogenously determined. I have done this using methodology developed in Verrecchia [1982]. The derivation is complicated, however, and I obtain results here by adapting Verrecchia's [1982] model to suit my present needs.

This adaptation allows me to specify the amount of costly information each trader acquires in equilibrium.

PROPOSITION 0. Each trader acquires the amount of costly information given by a level of precision $s$, where $s$ is determined by:

$$s = \text{maximum}\left\{0, \hat{s} \,|\, \hat{s} + 1 + s_0 + \Delta = \frac{1}{2c'(\hat{s})}\right\}. \tag{10}$$

*Proof.* This follows from Lemma 2 in Verrecchia [1982], allowing for the fact that in that paper $h_0$, which is the precision of prior beliefs, assumes the role of both the precision of prior beliefs and the precision of publicly available information, that is, $h_0 = 1 + s_0$. Also note that the level of the informedness of price works out to be $\Delta = \frac{s^2}{V}$ by appropriate substitution. Q. E. D.

Notice that since traders have identical preferences, they each purchase an identical level $s$ of precision. However, each observes diverse information since the perturbations in traders' observations are independent. In short, the amount of information each trader acquires is the same, but the actual observations are different.

In addition, for this model the level of informedness of price is given by:

$$\Delta = \frac{s^2}{V}. \tag{11}$$

Notice that $\Delta$ is endogenously determined as well. It is governed by the information acquisition activities of all traders through $s$. Therefore, $s$ is endogenously determined by $\Delta$ through equation (10), whereas $\Delta$ is endogenously determined by $s$ through equation (11). An equilibrium is an $s$, $\Delta$ pair such that when $s$ satisfies equation (10) for a given $\Delta$, and $\Delta$ satisfies equation (11) for a given $s$, $s$ and $\Delta$ are the same in both cases.

To illustrate the nature of this equilibrium, I consider two cases in which traders' overall level of informedness remains the same. Let the cost of acquiring private information be $c(s) = s/6$ and the level of noise be $V = 1$. In the first case imagine that there is no publicly available information, that is, $s_0 = 0$. To satisfy equations (10) and (11), each trader acquires an amount $s = 1$ of costly information, and the level of informedness of price is $\Delta = \frac{s^2}{V} = 1$ (notice that here $\hat{s} + 1 + s_0 + \Delta = 3$, which is also the value of $\frac{1}{2}c'(\hat{s})$; furthermore, $\Delta = s^2/V = 1$). In brief, in the absence of publicly available information each trader acquires one unit of precision at a cost $c(1) = \frac{1}{6}$, and the level of informedness of price is one unit. The overall level of a trader's informedness is $s + 1 + s_0 + \Delta = 3$.

In the second case imagine that the level of publicly available information is such that it precludes costly private inquiry. Specifically, let $s_0 = 2$, which implies that equations (10) and (11) are satisfied when traders acquire no information privately, that is, $s = 0$, and the level of informedness of price is also zero, that is, $\Delta = 0$, since here $\hat{s} + 1 + s_0 + \Delta = 3$ which is also the value of $\frac{1}{2}c'(\hat{s})$; furthermore, $\Delta = s^2/V = 0$. In brief, an amount of publicly available information equal to two units of precision induces traders to refrain from costly private inquiry. The overall level of a trader's informedness is also three in this case.

My objective in providing equations (10) and (11) is to perform comparative statics. (Notice that to achieve these comparative statics I require only equations (10) and (11), as well as my earlier stated assumptions.) Consider the first of Gonedes' questions: how does the amount of traders' costly private information, $s$, change as the amount of public information, $s_0$, changes?

PROPOSITION 1. The amount of traders' costly private information acquisition is nonincreasing as the amount of public information increases.

*Proof.* Let "$*$" denote the derivative with respect to $s_0$, that is, $s^* = \frac{\partial s}{\partial s_0}$. From equation (10):

$$\frac{\partial}{\partial s_0}\left\{\hat{s} + 1 \ + s_0 + \Delta = \frac{1}{2c'(\hat{s})}\right\} = \hat{s}^* + 1 + \Delta^* = \frac{-c''\hat{s}^*}{2c'^2}.$$

This implies

$$\hat{s}^* = -\frac{1 + \Delta^*}{1 + \dfrac{c''}{2c'^2}} \qquad (12)$$

Because $c'' \geq 0$, $\hat{s}^*$ is nonnegative provided that $\Delta^* \geq -1$. Suppose that $\Delta^* < -1$. This implies from (12) that $\hat{s}^* > 0$, which in turn implies from (10) that $s^* \geq 0$. However, equation (11) implies that:

$$\Delta^* = \frac{2s \cdot s^*}{V}. \qquad (13)$$

Thus $s^* \geq 0$ implies that $\Delta^* \geq 0$: an obvious contradiction. Therefore, it must be that $\Delta^* \geq -1$, which implies from (12) that $\hat{s}^* \leq 0$, which in turn implies from (10) that $s^* \leq 0$. Q. E. D.

Stated differently, increasing publicly available information (generally) decreases the amount of costly private information traders acquire on their own. This result should come as no surprise: it is "economically appealing" (this is my somewhat lavish way of saying it makes sense). Of course, an important assumption here is that public information is a perfect substitute for private information. Furthermore, this result says nothing about whether traders want additional publicly available information, simply that additional disclosure motivates them to cut back on their costly private acquisition activities.

A more subtle comparative static concerns how changes in the amount of public information affect changes in the overall level of informedness of traders. The overall level of a trader's informedness is defined to be the sum of the precisions of his prior beliefs, the publicly available information, his costly private information acquisition, and the informedness of price: $1 + s_0 + s + \Delta$. This is a major issue in Gonedes [1980] because he recognizes the interactive effect between increases in publicly available information (which lessen the motivation to acquire private information) and overall level of informedness (which benefits from additional public disclosure). I offer the following comparative static.

PROPOSITION 2. The overall level of traders' informedness is nondecreasing as the amount of public information increases.

*Proof.* The overall level of a trader's informedness is $1 + s_0 + s + \Delta$. Its derivative with respect to $s_0$ is given by:

$$\frac{\partial(1 + s_0 + s + \Delta)}{\partial s_0} = 1 + s^* + \Delta^*.$$

Since $s = \max\left\{0,\ \hat{s}\,|\,\hat{s} + 1 + s_0 + \Delta = \frac{1}{2c'(s)}\right\}$, from the proof to Proposition 1, $s^*$ is either zero or $-\frac{(1+\Delta^*)}{1+\frac{c''}{2c'^2}}$. If $s^*$ is zero, equation (13) implies $\Delta^*$ is zero, which implies $1 + s^* + \Delta^* = 1$, that is, the overall level of informedness is increasing. If $s^* = -\frac{(1+\Delta^*)}{1+\frac{c''}{2c'^2}}$, then:

$$1 + s^* + \Delta^* = 1 - \left[\frac{1+\Delta^*}{1+\frac{c''}{2c'^2}}\right] + \Delta^* = \frac{(1+\Delta^*)\frac{c''}{2c'^2}}{1+\frac{c''}{2c'^2}},$$

which is nonnegative since $c'' \geq 0$ and, from the proof to Proposition 1, $\Delta^* \geq -1$. Q. E. D.

Stated differently, increased publicly available information (generally) increases traders' informedness despite the fact that it also (generally) decreases the amount of costly private information each trader acquires. This means that while more publicly available information lessens the amount of costly private inquiry, it does not do so to the extent that the overall level of informedness decreases. Traders substitute public information for private information, but only to a degree that their overall level of informedness increases. In interpreting this result, however, it should be kept in mind that since public information is costless by assumption, its increase does not jeopardize traders' private inquiry opportunities.

## CONCLUSION

The deficiencies in my model are easy to point out. Since publicly available information is exogenously specified and costless, and traders face a homogeneous information acquisition cost function, my model does not really address whether information is produced either optimally or by the most efficient producer. This was, perhaps, Gonedes' major concern. In a richer setting we can imagine that firms face one technology concerning the production of information about themselves, financial analysts face another, investors face a third, and so forth. Furthermore, by assuming a continuous cost function which is zero at zero (i.e., $c(0) = 0$), I have eschewed all of the problems of information acquisition when discontinuities and fixed costs (i.e., $c(0) > 0$) arise. Finally, I assume that traders make independent observations which are, in turn, independent of the publicly available information. This assumption disregards an important potential inefficiency associated with the fact that traders and/or society may be paying for information which is already known in part.

I could continue to point out deficiencies (but, with two such illustrious discussants to perform this task, what is my incentive to ridicule my own work?). A defense against this type of criticism is that if there is a specific phenomenon we want to study (such as discontinuous cost functions), we can probably adapt the model to handle that assumption. If an adaptation is not possible, this strongly suggests we are using the wrong model! The advantage of the model I propose in this section is that it can be used to tell us something about endogenous costly private inquiry, and what it does tell us might be difficult to understand in the absence of a model because prices and belief formation are also endogenous.

But all this raises an issue which is central to this paper. A mathematical model is most useful when it is employed to examine a particular relationship between two elements of a problem, such as deriving a comparative static: specifying a model to derive a comparative static is much like designing a controlled experiment. On the other hand, I question the appropriateness of mathematical models to study complicated and convoluted economic phenomena such as the social welfare implications of public disclosure. There are too many elements of the problem which we cannot model to be able to answer anything effectively. I feel more comfortable with a model that provides a simple comparative static than one which attempts to determine whether information is produced optimally or by the most efficient producer. (This last topic comes perilously close to assessing the "social value" of public disclosure.) This is not to say that optimal information production is not an important economic concern, but rather to say that it is a topic which is too broad in scope to lend itself to a model. Therefore, those readers who are disappointed by the fact that the model I propose does not seem to answer either enough questions, or at least a single dramatic question, may not fully appreciate the extent to which this methodology has been abused in an attempt to provide these answers.

## *5. A Perspective*

It is apparent at this stage that this paper has not been a survey of mathematical models per se. By judiciously choosing my three topics, I have been able to discuss each from the perspective of a single model, that is, a competitive market. The primary advantage of this approach is that it permits a cohesive discussion, as well as an opportunity to introduce greater sophistication gradually into a single paradigm as the analysis progressed. Some readers may be disappointed that I did not consider alternative constructs that are potentially useful in financial accounting, such as models of game theory, competitive bidding, and optimal contracting. On the other hand, I would argue that to discuss mathematical models in financial accounting alone may focus myopically on a methodology and ignore the more important question of whether the methodology is being used correctly. Stated differently, there is rarely a problem with a methodology (assuming that it has been correctly interpreted); the problem is with how it is applied.

There are three (not necessarily mutually exclusive) criteria that can be suggested to determine whether a mathematical model is being usefully employed in financial accounting. First, does the model address an interesting economic or financial accounting question?[8] I no longer believe that models which assess the consequences of publicly available information address an interesting question. Obviously, this is an observation made from hindsight, and even at that I admit that many of the papers on this topic were clever in a mathematical, puzzle-solving way. It is interesting to try to understand how prices are determined or beliefs are formed in an efficient market. Much work remains in this area, and there is the potential to achieve testable implications.

Second, is the answer the mathematical model provides something that we would not have been able to discern as straightforwardly solely on the basis of our economic intuition? All too often we observe models that use intuitively plausible assumptions to arrive at intuitively obvious conclusions. I refer to this as a model which tells a "dog bites man" story.[9] One

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[8] In the version of the paper presented at the conference, I originally used the word "answer" instead of "address" in this sentence. Ross Watts was quick to point out that the word "answer" was somewhat presumptuous and suggested the word "address" (see his discussion accompanying this paper). I agree, and I am grateful that he brought this to my attention and that I had the opportunity to change it in the final draft.

[9] In his accompanying discussion of my paper (of which I have benefit in preparing this final draft), Ross Watts comments that my section 4 does not survey the literature on the relationship between public disclosure and private information acquisition as much as it presents a model of this phenomenon. Furthermore, the results I derive using this model fall victim to the "dog bites man" criticism I offer here. Finally, Watts remarks, this literature can be criticized using the same criticism I apply to the social welfare implications of public disclosure literature, which I do not do. I agree with the thrust of Watt's comments. Rather than attempt a detailed survey per se, I thought it better to present a relatively clean model of the relationship between public disclosure and private information acquisition. Furthermore, my model does use plausible assumptions to arrive at fairly straightforward conclusions. My intention here was to describe how one might go about modeling this question and using comparative statics. That is, the primary function of this section was pedagogical, and I would not claim that the comparative statics were particularly provocative. Finally, one can criticize this literature in much the same way I criticize the social welfare implications of public disclosure literature. Perhaps, on a more positive note, I tried

of the advantages of models that examine endogenous phenomena (e.g., costly private inquiry in a noisy rational expectations economy) is that, with sufficient endogeneity, our intuition is overextended. Thus, the solution which is offered is typically one which we could not have reliably determined in the absence of a model, because of interactive effects.

Third, is the model elegant in the sense of being at a level of complexity appropriate to the problem it seeks to resolve? The purpose behind using a mathematical model in the first case should be to make transparent a phenomena which is relatively complicated; it should not obfuscate a phenomenon which is obvious. My point is that oftentimes we see models that introduce mathematical sophistication either for its own benefit or to intimidate the reader. One example of this is authors who couch their mathematical model in measure theory constructs, only to end with a result which does not require that level of generality. Understanding how much mathematical sophistication is appropriate, and how much is gratuitous, is important. For example, many of the social welfare implications of public disclosure scenarios can be illustrated through an example of an economy as simple as that of Esther and Wesley. To discuss efficient markets in any meaningful way, however, some mathematical sophistication is unavoidable.

It is disturbing to hear general criticism of mathematical models in financial accounting when the problem is with the application of the methodology, and not the methodology per se. In concept, a mathematical model is very simple. Used properly, it is little more than a device to make a good economic argument better through the imposition of rigor. As with any argument, it quickly becomes useless (some might say dysfunctional) when overburdened with convoluted constructs and ad hoc assumptions. But as a tool, its importance should not be overlooked. The three criteria I suggest represent a primer for success, and I look forward to future applications. For example, some of the most interesting research currently being attempted in financial accounting is that which uses mathematical models to derive testable implications and then performs these tests. It is, perhaps, also the most challenging since it requires compromising the model to derive tests and compromising the tests to be consistent with the model. Despite this difficulty, I think this has to be the direction of future research if we reasonably expect theory and evidence to have a material impact in the field of accounting. I hope that future accounting researchers, "brute force empiricists" as well as mathematical modelers, will heed this recommendation.

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to suggest that there are some very narrow issues that can be addressed without necessarily becoming embroiled in social welfare questions. I liken the relationship between public disclosure and private information acquisition literature to a narrow path up the face of a sheer and forbidding cliff. I am not entirely sure where the path is going to take us, but what is, perhaps, more disturbing, one slip along the path into the abyss of discussing social welfare implications will certainly prove fatal.

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