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Source: *Journal of Accounting Research*, Vol. 29, No. 2 (Autumn, 1991), pp. 302-321

Published by: Blackwell Publishing on behalf of The Institute of Professional Accounting,  
Graduate School of Business, University of Chicago

Stable URL: <http://www.jstor.org/stable/2491051>

Accessed: 05/09/2008 16:43

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# Trading Volume and Price Reactions to Public Announcements

OLIVER KIM\* AND ROBERT E. VERRECCHIA†

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## 1. Introduction

The purpose of this study is to investigate theoretically how the price and volume reactions to a public announcement are related to each other, to the announcement's characteristics, and to the traders' beliefs at the time of the announcement. Among many possible sources of (abnormal) trading volume at the time of a public announcement, our emphasis in this study is on differences in the quality of preannouncement information. The study uses a two-period rational expectations model. Traders achieve their optimal portfolios prior to the announcement by trading on what each knows in the preannouncement period. The public announcement changes traders' beliefs and induces them to engage in a new round of trade. It is assumed that traders are diversely informed and *differ in the precision* of their private prior information; they therefore respond differently to the announcement, and this leads to positive volume.

We obtain three results. First, the price change at the time of announcement is proportional to both the unexpected portion of the announcement and its relative importance across the posterior beliefs of traders. This relative importance is increasing in the precision of the announcement and decreasing in the precision of the preannouncement information.

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\* University of California, Los Angeles; †University of Pennsylvania. We gratefully acknowledge the comments of Bob Holthausen, Prem Jain, Rich Lambert, Bharat Sarath, Scott Stickel, and the workshop participants at Berkeley, Columbia, University of Michigan, University of Minnesota, M.I.T., Northwestern, University of Pittsburgh, University of Rochester, UCLA, Washington, and Yale. We also thank an anonymous referee for many helpful suggestions.

The second and main result is that trading volume is proportional to both the absolute price change and a measure of differential precision across traders. Price change, as Beaver [1968] points out, reflects the average change in traders' beliefs due to the announcement, whereas trading volume reflects traders' idiosyncratic reactions. In this study the different reactions of traders are caused by differing precisions of their private information. The newly announced information is relatively more important to traders with less precise private information and thus has a larger impact on their beliefs. Volume reflects the sum of differences in traders' reactions; the change in price measures only the average reaction. As a result, volume is proportional both to price change and to the degree of differential precision. If precision is unobservable, the first and the second results together suggest that trading volume may be a noisier indicator of the precision of the announcement, or the precision of the preannouncement information, than price change. Also, this result is consistent with the empirical findings that abnormal volume is positively correlated with absolute abnormal returns.

The third result is a generalization of Holthausen and Verrecchia [1988], who analyze price changes at public announcements in a two-period model. In their model investors do not possess private information and thus have homogeneous beliefs. They show that the price reaction to an announcement is, on average, increasing in its precision and decreasing in the amount of preannouncement information.<sup>1</sup> We show that the expected volume and the variance of price change are increasing functions of the precision of the announced information and decreasing functions of the amount of preannouncement public and private information. Therefore, the intuition and results of Holthausen and Verrecchia [1988] concerning price reaction extend to volume even when investors are informed diversely and with different precisions.

In related research, Pflleiderer [1984] and Holthausen and Verrecchia [1990] consider volume that arises due to differences in interpreting the announcement across traders.<sup>2</sup> Grundy and McNichols [1989] analyze volume arising from the correction of idiosyncratic errors induced by the revelation of information through prices.<sup>3</sup> Varian [1985] considers volume due to differences in prior beliefs.<sup>4</sup>

Our model should not be interpreted too broadly, although it provides

<sup>1</sup> Since traders have homogeneous beliefs, no trade occurs.

<sup>2</sup> See Indjejikian [1991] for an extension of this idea.

<sup>3</sup> Other rational expectations models that employ a two-period trading structure include Brown and Jennings [1987] and Krishnan [1987].

<sup>4</sup> We mentioned only those studies using Grossman-type rational expectations models. Studies which assume different market structures include Kyle [1985], Glosten and Milgrom [1985], Karpoff [1986], and Admati and Pflleiderer [1988]. Also, see Tauchen and Pitts [1983] and Karpoff [1987] for the relation between volume and price change not explicitly related to the arrival of new information and its properties, and Verrecchia [1981] for a discussion of what inferences can be drawn from volume.

insights into how public announcements affect price changes and volume through differing precisions in private prior information. For example, we abstract from trading based on liquidity considerations, portfolio rebalancing, tax effects, etc. We also assume that firms are cross-sectionally independent. In the empirical domain, it is necessary to control for these phenomena in assessing the effect of a public announcement on price changes and volume.

Section 2 describes the model and obtains market equilibrium. Section 3 contains the main results of the paper concerning the market reaction to public announcements. Section 4 summarizes our work with concluding remarks.

## 2. *The Model and Market Equilibrium*

The securities market model we suggest is one of pure exchange, a continuum of traders, and three time periods, referred to as periods 1, 2, and 3. Trading occurs in periods 1 and 2 and consumption in period 3. There are two assets in the economy, a risky asset and a riskless bond. One unit of riskless bond pays off one unit of consumption good in period 3. The return of the risky asset is a random variable, denoted by  $\tilde{u}$ , and is realized in period 3. It is assumed that  $\tilde{u}$  is normally distributed with mean  $\bar{u}$  and precision (inverse of variance)  $h$ .

Four events occur in period 1. First, trader  $i$ ,  $i \in [0, 1]$ , is endowed with  $E_i$  riskless bond and  $x_i$  risky asset.<sup>5</sup> The aggregate risky endowment, denoted by  $x \equiv \int_0^1 x_i di$ , is not known to individual traders and is normally distributed with mean 0 and precision  $t$ .<sup>6</sup> The randomness of the risky asset supply captures the fact that securities markets are generally subject to random demand and supply fluctuations arising from changing liquidity needs, weather, political situations, etc. In noisy rational expectations models this randomness serves as an additional source of uncertainty that prevents securities prices from revealing fully all private information; this, in turn, supports incentives to acquire costly private information.<sup>7</sup>

Second, all traders observe a public signal  $\tilde{y}_1 = \tilde{u} + \tilde{\eta}$ , where  $\tilde{\eta}$  is normally distributed with mean 0 and precision  $m$ . Third, trader  $i$  observes a private signal  $\tilde{z}_i = \tilde{u} + \tilde{\epsilon}_i$  where  $\tilde{\epsilon}_i$  is independently and normally distributed with mean 0 and precision  $s_i$ . It is assumed that the set  $\{s_i\}$  is uniformly bounded. Together with prior beliefs of  $\tilde{u}$ , the signals  $\tilde{y}_1$  and  $\tilde{z}_i$  represent the preannouncement public and private information, respectively, possessed by traders. The final event in period 1 is that the

<sup>5</sup> Assuming a  $[0, 1]$  continuum of traders is convenient because sums over traders are averages as well. The results of the paper are not affected by assuming a countably infinite number of traders, i.e.,  $i = 1, 2, \dots$ .

<sup>6</sup> Assuming a nonzero mean of  $\tilde{x}$  does not affect the results.

<sup>7</sup> See Grossman and Stiglitz [1980] and Diamond and Verrecchia [1981] for detailed discussions of the role of noise.

market opens and traders buy and sell securities at the competitive market prices.

In period 2 there is a public announcement of a signal  $\tilde{y}_2 = \tilde{u} + \tilde{v}$ , where  $\tilde{v}$  is normally distributed with mean 0 and precision  $n$ . It is assumed that all random variables are mutually independent.<sup>8</sup> We study the market reaction to the announcement of  $\tilde{y}_2$  in period 2. The market opens again in period 2 and there is another round of trading. In period 3 the return of the risky asset is realized and consumption occurs.

Traders are risk averse and their preferences can be represented by negative exponential utility functions with risk tolerance  $r_i$ , i.e.,  $U_i(\tilde{W}_i) = -\exp(-(\tilde{W}_i/r_i))$ . Trader  $i$ 's final wealth  $\tilde{W}_i$  can be written as  $\tilde{W}_i = E_i + \tilde{P}_1 x_i + (\tilde{P}_2 - \tilde{P}_1)\tilde{D}_{1i} + (\tilde{u} - \tilde{P}_2)\tilde{D}_{2i}$ , where  $\tilde{P}_1$  and  $\tilde{P}_2$  are the prices of the risky asset in periods 1 and 2, and  $\tilde{D}_{1i}$  and  $\tilde{D}_{2i}$  are trader  $i$ 's holding of the risky asset at the end of periods 1 and 2, respectively. It is assumed that the set  $\{r_i\}$  is uniformly bounded.<sup>9</sup>

Traders are heterogeneous in terms of risk tolerances ( $r_i$ ) and they differ in terms of their private information in period 1 ( $\tilde{z}_i$ ) and its precision ( $s_i$ ). Thus, we model the simple observation that some traders are better informed than others and hold different expectations. This difference in information quality plays a central role in the trading volume reaction to public announcements analyzed later in the paper.

After observing available signals, traders also condition on the market price of the risky asset when choosing their demand. Each trader realizes that the prices for risky securities in the two trading periods,  $\tilde{P}_1$  and  $\tilde{P}_2$ , (potentially) reflect the information held by other traders. In a rational expectations equilibrium, traders make self-fulfilling conjectures about the relation between prices and traders information.

Let a linear conjecture of  $\tilde{P}_1$  and  $\tilde{P}_2$  be written as:

$$\begin{aligned} \tilde{P}_1 &= \alpha_1 \tilde{u} + \theta_1 \tilde{y}_1 + \beta_1 \int_0^1 \tilde{z}_i di - \gamma_1 \tilde{x} \\ &= \alpha_1 \tilde{u} + \theta_1 \tilde{y}_1 + \beta_1 \int_0^1 (\tilde{u} + \tilde{\epsilon}_i) di - \gamma_1 \tilde{x} \\ &= \alpha_1 \tilde{u} + \theta_1 \tilde{y}_1 + \beta_1 \tilde{u} - \gamma_1 \tilde{x} \end{aligned} \tag{1}$$

and, similarly:

$$\tilde{P}_2 = \alpha_2 \tilde{u} + \theta_{21} \tilde{y}_1 + \theta_{22} \tilde{y}_2 + \beta_2 \tilde{u} - \gamma_2 \tilde{x}, \tag{2}$$

<sup>8</sup> Assuming correlation between the error in the preannouncement public information,  $\tilde{y}_1$ , and that of the second-period announcement,  $\tilde{y}_2$ , does not qualitatively change the results.

<sup>9</sup> The uniform boundedness of  $\{s_i\}$  and  $\{r_i\}$  is assumed to have a well-defined integral  $\int r_i s_i di$ .

where (1) follows from the law of large numbers and the independence of the  $\tilde{\epsilon}_i$ s.  $\tilde{P}_1$  and  $\tilde{P}_2$  are linear functions of the average of the signals available at the time of trading and of the supply noise.  $\tilde{P}_1$  is also an available signal in period 2; expression (2) implicitly contains  $\tilde{P}_1$  because it contains all the variables of which  $\tilde{P}_1$  is a linear function, and because no restrictions are imposed on the coefficients. The constant terms of the two equations are written without loss of generality as multiples of  $\tilde{u}$ .

Given the conjectured behavior of prices outlined in (1) and (2), trader  $i$ 's problem is to choose the amount of the risky asset to hold at the end of periods 1 and 2. As in most dynamic programming problems, first the period 2 problem is analyzed and folded back into the period 1 problem. In period 2 trader  $i$ 's information consists of the first-period public signal  $\tilde{y}_1$  and his private signal  $\tilde{z}_i$ , the second-period public signal  $\tilde{y}_2$ , and the two price signals  $\tilde{P}_1$  and  $\tilde{P}_2$ . These signals can be written in normalized forms as  $\tilde{y}_1 = \tilde{u} + \tilde{\eta}$ ,  $\tilde{y}_2 = \tilde{u} + \tilde{\nu}$ ,  $\tilde{z}_i = \tilde{u} + \tilde{\epsilon}_i$ , and:

$$\tilde{q}_1 \equiv \frac{1}{\beta_1} (\tilde{P}_1 - \alpha_1 \tilde{u} - \theta_1 \tilde{y}_1) = \tilde{u} - B_1 \tilde{x}, \quad (3)$$

$$\tilde{q}_2 \equiv \frac{1}{\beta_2} (\tilde{P}_2 - \alpha_2 \tilde{u} - \theta_{21} \tilde{y}_1 - \theta_2 \tilde{y}_2) = \tilde{u} - B_2 \tilde{x}, \quad (4)$$

and the signals have precision (of error terms)  $m$ ,  $n$ ,  $s_i$ ,  $t/B_1^2$ , and  $t/B_2^2$ , respectively, where  $B_1 \equiv \gamma_1/\beta_1$  and  $B_2 \equiv \gamma_2/\beta_2$ . The information set  $\{\tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{q}_1, \tilde{q}_2\}$  is equivalent to  $\{\tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{P}_1, \tilde{P}_2\}$  because one can be generated from the other.

There are two possible types of equilibria in this market. In one, traders expect that the two prices fully reveal all private information and these expectations are fulfilled. In the other, equilibrium prices are not fully revealing.

To explain the fully-revealing equilibrium, suppose that traders conjecture that  $B_1 \neq B_2$ . Then, from (3) and (4),  $\tilde{u} = (B_2 \tilde{q}_1 - B_1 \tilde{q}_2)/(B_2 - B_1)$ . Since  $\tilde{q}_1$  and  $\tilde{q}_2$  are known in period 2,  $\tilde{u}$  is also known. Once the return of the risky asset is perfectly revealed, the equilibrium price,  $\tilde{P}_2$ , must equal the return,  $\tilde{u}$ .<sup>10</sup> At  $\tilde{P}_2 = \tilde{u}$ , traders have no incentive to trade (or not to trade). In period 1 traders know that the risky return will be revealed in period 2, and thus the equilibrium in period 1 is the same as that in the one-period model of Hellwig [1980] and others. As a result, the market price reacts to the announcement in period 2 and volume is indeterminate in the sense that any level of trading volume (including

<sup>10</sup> Otherwise, traders will either buy if  $\tilde{u} > \tilde{P}_2$ , or sell if  $\tilde{u} < \tilde{P}_2$ , an infinite amount because there is no risk.

zero) supports the equilibrium.<sup>11</sup> The fact that prices fully reveal *all* private information in this equilibrium (and, as a result, traders' beliefs become homogeneous) lacks institutional appeal given what we observe about how markets work. For this reason the rest of the analysis in this paper is based on the second equilibrium in which prices only partially reveal traders' private information.<sup>12</sup>

Suppose that traders conjecture that  $B_1 = B_2$ . This implies  $\tilde{q}_1 = \tilde{q}_2$ , and the two price signals are perfect substitutes. At the end of this section it will be verified that there is a unique equilibrium in which the condition,  $B_1 = B_2$ , is satisfied. Let  $B \equiv B_1 = B_2$  and  $\tilde{q} \equiv \tilde{q}_1 = \tilde{q}_2$ . The error terms of the signals  $\tilde{y}_1, \tilde{y}_2, \tilde{z}_i$ , and  $\tilde{q}$  are mutually independent and therefore it is straightforward to calculate:

$$\begin{aligned} K_{2i} &\equiv \text{Var}^{-1}(\tilde{u} \mid \tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{P}_1, \tilde{P}_2) \\ &= \text{Var}^{-1}(\tilde{u} \mid \tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{q}) \\ &= h + m + n + s_i + \frac{t}{B^2}, \\ \tilde{\mu}_{2i} &\equiv E(\tilde{u} \mid \tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{P}_1, \tilde{P}_2) \\ &= E(\tilde{u} \mid \tilde{y}_1, \tilde{y}_2, \tilde{z}_i, \tilde{q}) \\ &= \frac{h\tilde{u} + m\tilde{y}_1 + n\tilde{y}_2 + s_i\tilde{z}_i + (t/B^2)\tilde{q}}{h + m + n + s_i + (t/B^2)}. \end{aligned} \tag{5}$$

By convenient properties of the normal distribution, the precision of trader  $i$ 's total information at the end of period 2, denoted by  $K_{2i}$ , is simply the sum of the precisions of his prior and observed signals. His posterior expectation of  $\tilde{u}$  at the end of period 2, denoted by  $\tilde{\mu}_{2i}$ , is the

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<sup>11</sup> Prior work that is similar in part to ours is Grundy and McNichols [1989]. Both use two-period noisy rational expectations models in order to capture the price and volume reactions to the second-period public announcement and both obtain a fully revealing and a partially revealing equilibrium. The major difference in the two models is in the preannouncement information structure. In Grundy and McNichols [1989], traders' preannouncement information consists of a common prior and private signals with a common error as well as idiosyncratic errors. The idiosyncratic errors have the same precision. As a result, there is no volume in the partially-revealing equilibrium. In the fully-revealing equilibrium traders observe the market price and correct their idiosyncratic errors which results in positive volume.

<sup>12</sup> It is difficult to suggest which equilibrium is more interesting on purely theoretical grounds. One possible approach is to consider a sequence of finite economies of which the present economy is the limit and to see which equilibrium is the limit of the equilibria of the sequence of economies. Another is to consider a sequence of economies in which the correlation between the supplies in the two trading periods converges to one as in the present model. Both approaches are difficult to implement, however, because they do not appear to yield linear equilibria.

average of his prior expectation and observed signals weighted by the precision.

Let  $\tilde{D}_{2i}$  be trader  $i$ 's desired holding (gross demand) of the risky asset in period 2. It is well known that the normality of distributions in conjunction with the exponential utility function allows for a simple expression for  $\tilde{D}_{2i}$ .<sup>13</sup>

$$\begin{aligned} \tilde{D}_{2i} &= r_i K_{2i} (\tilde{\mu}_{2i} - \tilde{P}_2) \\ &= r_i (h\tilde{u} + m\tilde{y}_1 + n\tilde{y}_2 + s_i\tilde{z}_i + (t/B^2)\tilde{q} - K_{2i}\tilde{P}_2). \end{aligned} \tag{6}$$

Trader  $i$ 's demand decision is based on his market opportunity, which is the difference between his assessment of the risky return,  $\tilde{\mu}_{2i}$ , and the market price,  $\tilde{P}_2$ . The degree of aggressiveness with which he exploits his market opportunity is determined by his risk tolerance,  $r_i$ , and the precision of his information,  $K_{2i}$ .

Equating the aggregate supply to the aggregate gross demand of the risky asset:

$$\begin{aligned} \tilde{x} &= \tilde{D}_2 \\ &\equiv \int \tilde{D}_{2i} di \\ &= \int r_i [h\tilde{u} + m\tilde{y}_1 + n\tilde{y}_2 + s_i(\tilde{u} + \tilde{\epsilon}_i) + (t/B^2)\tilde{q} - K_{2i}\tilde{P}_2] di \\ &= r[h\tilde{u} + m\tilde{y}_1 + n\tilde{y}_2 + s\tilde{u} + (t/B^2)\tilde{q} - K_2\tilde{P}_2], \end{aligned}$$

where  $r \equiv \int r_i di$ ,  $s \equiv (1/r) \int r_i s_i di$ , and  $K_2 \equiv (1/r) \int r_i K_{2i} di$ .  $s$  and  $K_2 = h + m + n + s + (t/B^2)$  are, respectively, the averages of  $s_i$  and  $K_{2i}$  weighted by  $r_i$ . The term  $\int r_i s_i \tilde{\epsilon}_i di$  vanishes by the law of large numbers. Rewriting the above using the definition of  $\tilde{q}$ :

$$\tilde{P}_2 = \frac{1}{K_2} \left[ h\tilde{u} + m\tilde{y}_1 + n\tilde{y}_2 + \left( s + \frac{t}{B^2} \right) \tilde{u} - \left( \frac{1}{r} + \frac{t}{B} \right) \tilde{x} \right]. \tag{7}$$

The equilibrium condition that the linear price conjecture is self-fulfilling dictates that (2) and (7) are identical. Therefore:

$$\alpha_2 = \frac{h}{K_2}, \theta_{21} = \frac{m}{K_2}, \theta_2 = \frac{n}{K_2}, \beta_2 = \frac{s + (t/B^2)}{K_2}, \gamma_2 = \frac{r^{-1} + (t/B)}{K_2}.$$

From  $B \equiv \gamma_2/\beta_2 = [r^{-1} + (t/B)]/[s + (t/B^2)]$ ,  $B = 1/rs$ . The total precision of trader  $i$  in period 2,  $K_{2i}$ , and the average total precision in period 2,

<sup>13</sup> Maximizing utility with respect to  $\tilde{D}_{2i}$  conditional on  $\tilde{y}_1$ ,  $\tilde{y}_2$ ,  $\tilde{z}_i$ , and  $\tilde{q}$  generates the result.



$K_2$ , can now be written as:

$$K_{2i} = h + m + n + s_i + r^2s^2t,$$

$$K_2 = h + m + n + s + r^2s^2t,$$

where  $r^2s^2t$  is the precision of the price signal which is common across traders.

Trader  $i$ 's problem in period 1 is to choose his demand given signals  $\tilde{y}_1$ ,  $\tilde{z}_i$ , and  $\tilde{P}_1$ . He also knows (6) and (7). That is, he knows the exact future relations among his demand, the price, and the available signals in period 2. Formally, his problem is to:

$$\begin{aligned} \max_{\tilde{D}_{1i}} &= E[U_i(W_i) \mid \tilde{y}_1, \tilde{z}_i, \tilde{P}_1] \\ &= E[U_i(W_i) \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}] \\ &= E\left[-\exp\left\{-\frac{1}{r_i}[E_i + \tilde{P}_1\tilde{x}_i + (\tilde{P}_2 - \tilde{P}_1)\tilde{D}_{1i} + (\tilde{u} - \tilde{P}_2)\tilde{D}_{2i}]\right\} \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}\right] \\ &= E\left[-\exp\left\{-\frac{1}{r_i}[E_i + \tilde{P}_1\tilde{x}_i + (\tilde{P}_2 - \tilde{P}_1)\tilde{D}_{1i} \right. \right. \\ &\quad \left. \left. - K_{2i}(\tilde{u} - \tilde{P}_2)(\tilde{\mu}_{2i} - \tilde{P}_2)\right\} \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}\right] \end{aligned}$$

subject to (7).

The solution to this problem is calculated in Appendix A and can be written as:

$$\begin{aligned} \tilde{D}_{1i} = \frac{r_i}{n} &[K_2h\tilde{u} + K_2m\tilde{y}_1 + ns_i\tilde{z}_i + \{K_2(s + r^2s^2t) - ns\}\tilde{q} \\ &- \{n(s_i - s) + K_1K_2\}\tilde{P}_1], \quad (8) \end{aligned}$$

where:

$$\begin{aligned} K_{1i} &\equiv \text{Var}^{-1}(\tilde{u} \mid \tilde{y}_1, \tilde{z}_i, \tilde{P}_1) \\ &= \text{Var}^{-1}(\tilde{u} \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}) \\ &= h + m + s_i + r^2s^2t, \end{aligned}$$

and:

$$\begin{aligned} K_1 &\equiv \frac{1}{r} \int r_i K_{1i} di \\ &= h + m + s + r^2s^2t. \end{aligned}$$

Applying the market clearing condition:

$$\begin{aligned}
 \tilde{x} &= \tilde{D}_1 \\
 &\equiv \int \tilde{D}_{1i} \, di \\
 &= \int \frac{r_i}{n} \left[ K_2 h \bar{u} + K_2 m \tilde{y}_1 + n s_i (\tilde{u} + \tilde{\epsilon}_i) + \{K_2(s + r^2 s^2 t) - ns\} \right. \\
 &\quad \left. \cdot \left( u - \frac{\tilde{x}}{rs} \right) - \{n(s_i - s) + K_1 K_2\} \tilde{P}_1 \right] di \\
 &= \frac{r}{n} \left[ K_2 h \bar{u} + K_2 m \tilde{y}_1 + K_2(s + r^2 s^2 t) \tilde{u} \right. \\
 &\quad \left. - \left\{ K_2 \left( \frac{1}{r} + rst \right) - \frac{n}{r} \right\} \tilde{x} - K_1 K_2 \tilde{P}_1 \right].
 \end{aligned}$$

The above can be rewritten as:

$$\tilde{P}_1 = \frac{1}{K_1} \left[ h \bar{u} + m \tilde{y}_1 + (s + r^2 s^2 t) \tilde{u} - \left( \frac{1}{r} + rst \right) \tilde{x} \right]. \tag{9}$$

In equilibrium (1) and (9) are identical and thus:

$$\alpha_1 = \frac{h}{K_1}, \theta_1 = \frac{m}{K_1}, \beta_1 = \frac{s + r^2 s^2 t}{K_1}, \gamma_1 = \frac{r^{-1} + rst}{K_1}.$$

For (6), (7), (8), and (9) to be established as an equilibrium, it has to be verified that the assumption  $B = B_1 = B_2$  is true.  $B_1 = \gamma_1/\beta_1 = (r^{-1} + rst)/(s + r^2 s^2 t) = 1/rs$ , and it was shown that  $B_2 = 1/rs$ . Therefore, (6), (7), (8), and (9) together with  $B = 1/rs$  characterize a unique rational expectations equilibrium in which prices only partially reveal traders' private information.

### 3. Price and Volume Reactions to Public Announcements

This section contains the analysis of trading volume and price change at the time of public announcement. From (7) and (9) the price reaction to the announcement of  $\tilde{y}_2$  is:

$$\begin{aligned}
 \tilde{P}_2 - \tilde{P}_1 &= \frac{n\tilde{v}}{K_2} + \left( \frac{1}{K_2} - \frac{1}{K_1} \right) [h(\bar{u} - \tilde{u}) + m\tilde{\eta} - (rst + r^{-1})\tilde{x}] \\
 &= \frac{n}{K_1 K_2} [h(\tilde{u} - \bar{u}) - m\tilde{\eta} + (rst + r^{-1})\tilde{x} + K_1 \tilde{v}] \\
 &= \frac{n}{K_1 K_2} [(K_1 - m - s - r^2 s^2 t) \tilde{u} + K_1 \tilde{v} - h\bar{u} - m\tilde{\eta} \\
 &\quad + (rst + r^{-1})\tilde{x}]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{n}{K_1 K_2} \left[ K_1(\tilde{u} + \tilde{v}) - h\tilde{u} - m(\tilde{u} + \tilde{\eta}) - s\tilde{u} \right. \\
 &\quad \left. - r^2 s^2 t \left( \tilde{u} - \frac{\tilde{x}}{rs} \right) + \frac{\tilde{x}}{r} \right] \\
 &= \frac{n}{K_2} \left[ \tilde{y}_2 - \frac{h\tilde{u} + m\tilde{y}_1 + s\tilde{u} + r^2 s^2 t\tilde{q}}{K_1} + \frac{\tilde{x}}{rK_1} \right]. \tag{10}
 \end{aligned}$$

Equation (10) is now restated as a proposition.

**PROPOSITION 1.** The price reaction to a public announcement is proportional to the importance of the announced information relative to the average posterior beliefs of traders and the surprise contained in the announced information plus noise. That is:

$$\tilde{P}_2 - \tilde{P}_1 = \frac{n}{K_2} (\text{Surprise} + \text{Noise}),$$

where:

$$\text{Surprise} \equiv \tilde{y}_2 - \frac{h\tilde{u} + m\tilde{y}_1 + s\tilde{u} + r^2 s^2 t\tilde{q}}{K_1}, \quad \text{Noise} \equiv \frac{\tilde{x}}{rK_1}.$$

The surprise in the announced information as defined in Proposition 1 is the difference between the announced signal,  $\tilde{y}_2$ , and an average of traders' expectations of the risky return  $\tilde{u}$  and, at the same time, of the announcement  $\tilde{y}_2$ .<sup>14</sup>

The relation in Proposition 1 captures the spirit of event studies, which are conducted typically to examine the information content of particular announcements. In the case of earnings announcements the change in price and the surprise in this model correspond to abnormal returns and unexpected earnings, respectively. The multiple in the relation,  $n/K_2 = n/(h + m + n + s + r^2 s^2 t)$ , is an increasing function of the precision of the announced information,  $n$ , which can be interpreted as the information content of the announced information. A greater  $n$  implies a more sensitive price reaction to the announcement. When  $n$  is zero, there is no price change. On the other hand,  $n/K_2$  is a decreasing function of the precision of *other* information available prior to the announcement. A greater amount of preannouncement information implies that the price reacts less sensitively to the surprise in the announcement.

Using (9) equation (10) can be rewritten as:

$$\tilde{P}_2 - \tilde{P}_1 = \frac{n}{K_2} (\tilde{y}_2 - \tilde{P}_1). \tag{11}$$

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<sup>14</sup> The term,  $(h\tilde{u} + m\tilde{y}_1 + s\tilde{u} + r^2 s^2 t\tilde{q})/K_1$ , is the weighted average of  $\tilde{\mu}_{1i} = (h\tilde{u} + m\tilde{y}_1 + s_i\tilde{z}_i + r^2 s^2 t\tilde{q})/K_{1i}$ , which is trader  $i$ 's expectation of  $\tilde{u}$  or  $\tilde{y}_2$  conditional on available information in period 1. The weight is  $r_i K_{1i}$  which measures the degree of traders' aggressiveness in exploiting their market opportunities in period 1.

This relation is without noise because  $\tilde{P}_1$  itself contains the noise defined in Proposition 1. Both the left-hand and the right-hand side are in principle observable.

For an analysis of trading volume, first rewrite trader  $i$ 's demand of risky asset in the two trading periods expressed in (8) and (6) using (9) and (7) as:

$$\tilde{D}_{1i} = r_i s_i \tilde{\epsilon}_i + \frac{r_i(s_i - s)}{K_1} [h(\tilde{u} - \bar{u}) - m\tilde{\eta} + rst\tilde{x}] + \frac{r_i K_{1i}}{rK_1} \tilde{x},$$

and:

$$\tilde{D}_{2i} = r_i s_i \tilde{\epsilon}_i + \frac{r_i(s_i - s)}{K_2} [h(\tilde{u} - \bar{u}) - m\tilde{\eta} - n\tilde{\nu} + rst\tilde{x}] + \frac{r_i K_{2i}}{rK_2} \tilde{x}.$$

Therefore:

$$\begin{aligned} \tilde{D}_{2i} - \tilde{D}_{1i} &= r_i(s_i - s) \left( \frac{1}{K_2} - \frac{1}{K_1} \right) [h(\tilde{u} - \bar{u}) - m\tilde{\eta} + rst\tilde{x}] \\ &\quad - \frac{r_i(s_i - s)n\tilde{\nu}}{K_2} + \frac{r_i}{rK_1K_2} (K_1K_{2i} - K_{1i}K_2)\tilde{x} \\ &= -r_i(s_i - s) \frac{n}{K_1K_2} [h(\tilde{u} - \bar{u}) - m\tilde{\eta} + (r^{-1} + rst)\tilde{x} + K_1\tilde{\nu}] \\ &= -r_i(s_i - s)(\tilde{P}_2 - \tilde{P}_1). \end{aligned} \tag{12}$$

The volume reaction to the announcement of  $\tilde{y}_2$  can now be calculated using (12) and the definition of trading volume,  $\text{Volume} = \frac{1}{2} \int |\tilde{D}_{2i} - \tilde{D}_{1i}| di$ , as in the following proposition.

**PROPOSITION 2.** The volume reaction to a public announcement is proportional both to the absolute price change at the time of the announcement and to a measure of differential precision across traders. That is:

$$\begin{aligned} \text{Volume} &= \left( \frac{1}{2} \int r_i |s_i - s| di \right) |\tilde{P}_2 - \tilde{P}_1| \\ &= \left( \frac{1}{2} \int r_i |s_i - s| di \right) \frac{n}{K_2} |\text{Surprise} + \text{Noise}|. \end{aligned}$$

The multiple  $\int r_i |s_i - s| di$  in the above relation is the weighted average of the absolute deviations of the precision of traders' private information,  $s_i s$ , from the average precision,  $s$ , weighted by the  $r_i s$ .

Intuitively, when the new public information,  $\tilde{y}_2$ , is released in period 2, all traders revise their beliefs, and this revision is reflected in the change in market price. Relatively better informed traders revise their beliefs less because the new information is relatively less important to

them than to those who are more poorly informed. The presence of differential precision thus causes *differential belief revision* among traders which, in turn, creates trading volume. When there is no difference in precision, i.e., when  $s_i = s$  for all  $i$ , then traders' belief revisions and the price movement are parallel and there is no volume. Note that this is true even when  $r_i$ s differ among traders. Thus differences in risk aversion alone do not result in a positive trading volume in the present model (although such differences can affect volume in the presence of differential precision).<sup>15</sup>

Proposition 2 can be related to an event study context. In tests of the information content of particular events, such as earnings announcements, Proposition 2 suggests that volume may be a noisier indicator than price change of the information content of the announcement,  $n$ , and of the amount of preannouncement information,  $h$ ,  $m$ , and  $s$ , which correspond to the prior, public, and private information held at the time of announcement.<sup>16</sup> If the measure of differential precision, which functions as noise if it is not observable, is uncorrelated with the information variable of interest, the results of a study using volume will not be biased. However, if the measure of differential precision is systematically related to the information variable of interest, then the use of volume may distort results. For example, if more risk-tolerant traders tend to prefer stocks of smaller firms, the multiple in the relation in Proposition 2 will be greater for smaller firms. Consequently, a volume study which tests the difference in the amount of preannouncement information between large and small firms will produce results that exaggerate the difference.<sup>17</sup>

Reversing the above argument, the use of volume and returns together could potentially generate insights about the multiple, which depends on traders' risk attitudes and the degree of differential precision among them. If there are reasons to believe that these variables are different across firms, industries, or types of announcements, then one could use volume data to test such conjectures. This line of thinking also offers an alternative way to understand observed differences in volume relative to returns. For example, Jain [1988] reports that the announcements of certain macroeconomic variables such as money supply and consumer price index induce significant abnormal returns but no abnormal volume. On the other hand, many studies document that there are both significant

<sup>15</sup> The fact that differences in risk aversion alone do not result in volume is an artifact of the exponential utility function. Differences in risk aversion in conjunction with diverse information generally lead to volume; see, for example, Verrecchia [1981].

<sup>16</sup> Atiase [1985], Bamber [1986; 1987], Freeman [1987], and Grant [1980], among others, compare the extent of market reactions between large and small firms to test the difference in the amount of preannouncement information.

<sup>17</sup> Our results extend (trivially) to a multiasset model in which asset returns and aggregate supplies are mutually independent. Extending the model to a general correlation structure is much more complicated; see Admati [1985]. Therefore, our insights are limited to empirical studies in which cross-sectional differences are investigated and these differences do not depend on cross-sectional correlations.

price and volume reactions to earnings announcements.<sup>18</sup> Jain [1988] interprets the difference in volume relative to the absolute price change between the two types of announcements as caused by differences in the degree of differential interpretation among traders. However, even without differential interpretations, we show that volume is influenced by the level of differential precision. Consequently, one must be careful to consider the roles of *both* differential interpretations *and* differential precision in making inferences about volume.

Finally, the second equation of Proposition 2 implies that the volume reaction to a public announcement is proportional both to the relative importance of the announced information and to the absolute value of the surprise (plus noise) as defined in Proposition 1. This relation is intuitive and consistent with Bamber's [1987] result that volume is positively associated with the absolute value of unexpected earnings. If size is positively associated with the amount of preannouncement information (which is in turn negatively related to the relative importance of the announcement), volume will be negatively associated with size. Such a relation is reported by Bamber [1987].

The average magnitude of market reaction is often compared among different firms or different types of announcements without considering whether the announced news is good or bad. Comparable theoretical measures are variance of price change and expected volume. The following lemma calculates the variance of price change from (10).

LEMMA 1. The variance of price change at the time of public announcement is:

$$\begin{aligned}\Delta &\equiv \text{Var}(\tilde{P}_2 - \tilde{P}_1) \\ &= \frac{n}{K_2^2} + \frac{n^2}{K_1^2 K_2^2} (K_1 + s + r^{-2}t^{-1}) \\ &= \frac{n}{K_2^2} (1 + nL_1),\end{aligned}$$

where:

$$L_1 \equiv \text{Var}(\tilde{u} - \tilde{P}_1) = (K_1 + s + r^{-2}t^{-1})/K_1^2.$$

The expected volume is calculated in the following lemma using Proposition 2, Lemma 1, and the fact that the expectation of the absolute value of a normally distributed random variable with zero mean is  $\sqrt{2/\pi}$  times its standard deviation.

LEMMA 2. The expected volume at the time of public announcement is:

$$\begin{aligned}\bar{V} &\equiv E[\text{Volume}] \\ &= \sqrt{\frac{\Delta}{2\pi}} \int r_i |s_i - s| di.\end{aligned}$$

<sup>18</sup> These include Beaver [1968], Morse [1981], Pincus [1983], and Bamber [1986; 1987].

The following proposition is an immediate result of Lemmas 1 and 2 and shows how the magnitude of market reaction is associated with the precisions of the announced and preannouncement information.

**PROPOSITION 3.** The magnitudes of both volume and price change at the time of public announcement are on average associated positively with the precision of the announced information and negatively with the precision of the preannouncement prior, public, and private information. That is:

1.  $\frac{\partial \Delta}{\partial n} > 0, \frac{\partial \bar{V}}{\partial n} > 0;$
2.  $\frac{\partial \Delta}{\partial h} < 0, \frac{\partial \bar{V}}{\partial h} < 0;$
3.  $\frac{\partial \Delta}{\partial m} < 0, \frac{\partial \bar{V}}{\partial m} < 0;$
4.  $\frac{\partial \Delta}{\partial s} < 0, \frac{\partial \bar{V}}{\partial s} \Big|_{\int r_i |s_i - s| di = \text{constant}} < 0.$

The proof is provided in Appendix A. The results of Proposition 3 are intuitive. On the one hand, as the quality of an announcement increases, traders react to the announcement with greater conviction. On the other hand, as the quality of preannouncement information increases, the relative importance of the announcement to traders decreases, so they respond less strongly to the announcement. Holthausen and Verrecchia [1988] formalize this intuition for price changes in a two-period rational expectations model and show that this intuition is valid for homogeneous expectations. Proposition 3 shows that the intuition concerning price changes remains valid and also applies to volume even when traders are diversely informed and have different precisions. Furthermore, the results are also consistent with the intuition and empirical results of Atiase [1985] and others.

#### 4. Conclusion

We have examined Beaver's [1968] intuition that the change in price reflects the average change in traders' beliefs, while volume reflects the sum of the differences in traders' reactions to an announcement, using a highly stylized model with strong assumptions. The relatively clean and specific results obtained in this study should thus be interpreted with care, although the general intuition in most of the results is clear and does not seem to depend critically on the simplifying assumptions made. They are also largely consistent with existing empirical findings.

The main result of this paper, that volume may be a noisier indicator of information variables than the change in price, does not necessarily imply that volume studies are redundant or inferior. First, volume studies

can to a large extent substitute for returns studies. More important, since volume contains the differences among traders which are averaged out in the returns data, the use of volume in conjunction with returns could identify systematic differences in investors' knowledge or other characteristics which result in different reactions to public announcements across firms or across types of announcements. This paper identifies differences in precision across traders as a potentially important factor influencing volume relative to price change. This intuition could shed light on other interesting issues in accounting and finance related to differences in the quality of investors' information.<sup>19</sup>

APPENDIX A

Calculation of  $\tilde{D}_{1i}$

Omitting terms unrelated to  $\tilde{D}_{1i}$ , the objective function is written as:

$$E_{\tilde{P}_2, \tilde{\mu}_{2i}, \tilde{u}} \left[ -\exp \left\{ \frac{1}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} - K_{2i} (\tilde{u} - \tilde{P}_2) (\tilde{\mu}_{2i} - \tilde{P}_2) \right\} \middle| \tilde{y}_1, \tilde{z}_i, \tilde{q} \right].$$

Using the law of iterated expectations, this becomes:

$$\begin{aligned} & E_{\tilde{P}_2, \tilde{\mu}_{2i}} \left[ E_{\tilde{u}} \left[ -\exp \left\{ \frac{1}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} - K_{2i} (\tilde{u} - \tilde{P}_2) (\tilde{\mu}_{2i} - \tilde{P}_2) \right\} \right. \right. \\ & \quad \cdot \left. \left. \middle| \tilde{y}_1, \tilde{z}_i, \tilde{q}, \tilde{P}_2, \tilde{\mu}_{2i} \right] \middle| \tilde{y}_1, \tilde{z}_i, \tilde{q} \right] \\ & = E_{\tilde{P}_2, \tilde{\mu}_{2i}} \left[ -\exp \left\{ \frac{1}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} - \frac{K_{2i}}{2} (\tilde{\mu}_{2i} - \tilde{P}_2)^2 \right\} \middle| \tilde{y}_1, \tilde{z}_i, \tilde{q} \right] \end{aligned}$$

because:

$$\begin{aligned} E[\tilde{u} \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}, \tilde{P}_2, \tilde{\mu}_{2i}] &= \tilde{\mu}_{2i}, \\ \text{Var}[\tilde{u} \mid \tilde{y}_1, \tilde{z}_i, \tilde{q}, \tilde{P}_2, \tilde{\mu}_{2i}] &= \frac{1}{K_{2i}}, \end{aligned}$$

and thus:

$$\begin{aligned} & E_{\tilde{u}} \left[ -\exp \left\{ -K_{2i} (\tilde{u} - \tilde{P}_2) (\tilde{\mu}_{2i} - \tilde{P}_2) \right\} \middle| \tilde{y}_1, \tilde{z}_i, \tilde{q}, \tilde{P}_2, \tilde{\mu}_{2i} \right] \\ & = -\exp \left\{ -K_{2i} (\tilde{\mu}_{2i} - \tilde{P}_2) (\tilde{\mu}_{2i} - \tilde{P}_2) + \frac{K_{2i}^2 (\tilde{\mu}_{2i} - \tilde{P}_2)^2}{2K_{2i}} \right\} \\ & = -\exp \left\{ -\frac{K_{2i}}{2} (\tilde{\mu}_{2i} - \tilde{P}_2)^2 \right\} \end{aligned}$$

using the moment-generating function of a normal random variable.

<sup>19</sup> Studies that utilize different properties of volume and returns for analyzing other issues include Morse [1980], Lakonishok and Vermaelen [1986], and Richardson, Sefcik, and Thompson [1986].



(7) and (5) are now written as:

$$\begin{aligned}
 \tilde{P}_2 &= \frac{1}{K_2} [h\bar{u} + m\tilde{y}_1 + n\tilde{y}_2 + (s + r^2s^2t)\bar{u} - (rst + r^{-1})\bar{x}] \\
 &= \frac{1}{K_2} [h\bar{u} + m\tilde{y}_1 + n\tilde{y}_2 + (s + r^2s^2t)\bar{q}], \\
 \tilde{\mu}_{2i} &= \frac{1}{K_{2i}} [h\bar{u} + m\tilde{y}_1 + n\tilde{y}_2 + s_i\tilde{z}_i + r^2s^2t\bar{q}] \\
 &= \frac{s_i}{K_{2i}} \tilde{z}_i + (\tilde{P}_2 - \frac{s}{K_2} \bar{q}) \frac{K_2}{K_{2i}} \\
 &= \frac{1}{K_{2i}} [K_2\tilde{P}_2 + s_i\tilde{z}_i - s\bar{q}].
 \end{aligned}$$

Therefore:

$$\tilde{\mu}_{2i} - \tilde{P}_2 = \frac{1}{K_{2i}} [-(s_i - s)\tilde{P}_2 + s_i\tilde{z}_i - s\bar{q}].$$

Using this relation, the objective function above can be written as:

$$E \left[ -\exp \left\{ \frac{1}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} - \frac{1}{2K_{2i}} [-(s_i - s)\tilde{P}_2 + s_i\tilde{z}_i - s\bar{q}]^2 \right\} \middle| \tilde{y}_1, \tilde{z}_i, \bar{q} \right].$$

The only random variable in this expression given  $\tilde{y}_1$ ,  $\tilde{z}_i$ ,  $\bar{q}$ , and thus  $\tilde{P}_1$  is  $\tilde{P}_2$  in a quadratic form.

First, calculate the conditional expectation and variance of  $\tilde{P}_2$ .

$$E[\tilde{P}_2 | \tilde{y}_1, \tilde{z}_i, \bar{q}] = \frac{1}{K_{1i}K_2} [K_{2i}(h\bar{u} + m\tilde{y}_1) + s_in\tilde{z}_i + (r^2s^2tK_{2i} + sK_{1i})\bar{q}],$$

$$\text{Var}[\tilde{P}_2 | \tilde{y}_1, \tilde{z}_i, \bar{q}] = \frac{nK_{2i}}{K_{1i}K_2^2}.$$

The objective function can now be rewritten as:

$$\begin{aligned}
 &E \left[ -\exp \left\{ \frac{1}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} - \frac{1}{2K_{2i}} [-(s_i - s)\tilde{P}_2 + s_i\tilde{z}_i - s\bar{q}]^2 \right\} \middle| \tilde{y}_1, \tilde{z}_i, \bar{q} \right] \\
 &\propto - \int \exp \left[ -\frac{1}{2} \left\{ -\frac{2}{r_i} (\tilde{P}_1 - \tilde{P}_2) \tilde{D}_{1i} + \frac{1}{K_{2i}} \{-(s_i - s)\tilde{P}_2 + s_i\tilde{z}_i - s\bar{q}\}^2 \right. \right. \\
 &\quad \left. \left. + \frac{K_{1i}K_2^2}{nK_{2i}} \left[ \tilde{P}_2 - \frac{1}{K_{1i}K_2} \{K_{2i}(h\bar{u} + m\tilde{y}_1) \right. \right. \right. \\
 &\quad \left. \left. \left. + s_in\tilde{z}_i + (r^2s^2tK_{2i} + sK_{1i})\bar{q}\} \right]^2 \right\} \right] d\tilde{P}_2
 \end{aligned}$$

$$\begin{aligned} \propto & - \int \exp \left[ -\frac{1}{2} \left\{ \left( \frac{(s_i - s)^2}{K_{2i}} + \frac{K_{1i}K_2^2}{nK_{2i}} \right) \tilde{P}_2^2 - 2\tilde{P}_2 \left( \frac{(s_i - s)(s_i\tilde{z}_i - s\tilde{q})}{K_{2i}} \right) \right. \right. \\ & \left. \left. + \frac{K_2}{nK_{2i}} \{ K_{2i}(h\bar{u} + m\tilde{y}_1) + s_i n\tilde{z}_i + (r^2s^2tK_{2i} + sK_{1i})\tilde{q} \} - \frac{\tilde{D}_{1i}}{r_i} \right\} \right. \\ & \left. + \tilde{P}_1 \left( \frac{\tilde{D}_{1i}}{r_i} \right) \right] d\tilde{P}_2, \end{aligned}$$

omitting terms unrelated to  $\tilde{D}_{1i}$  or  $\tilde{P}_2$ . This is simplified to:

$$\begin{aligned} - \int \exp \left[ -\frac{1}{2} \left\{ \left( s_i - s + \frac{K_1K_2}{n} \right) \tilde{P}_2^2 - 2\tilde{P}_2 \left( \frac{K_2}{n} (h\bar{u} + m\tilde{y}_1) + s_i z_i \right) \right. \right. \\ \left. \left. + \left\{ \frac{K_2}{n} (s + r^2s^2t) - s \right\} \tilde{q} - \frac{\tilde{D}_{1i}}{r_i} \right\} + \tilde{P}_1 \left( \frac{\tilde{D}_{1i}}{r_i} \right) \right] d\tilde{P}_2 \end{aligned}$$

because:

$$\begin{aligned} \frac{1}{nK_{2i}} \{ n(s_i - s)^2 + K_{1i}K_2^2 \} &= \frac{1}{nK_{2i}} \{ n(K_{2i} - K_2)^2 + K_{1i}K_2^2 \} \\ &= \frac{1}{nK_{2i}} (nK_{2i}^2 - 2nK_{2i}K_2 + K_2K_2^2) \\ &= K_{2i} - K_2 + \frac{K_2^2 - nK_2}{n} \\ &= s_i - s + \frac{K_1K_2}{n}, \end{aligned}$$

and:

$$\begin{aligned} & \frac{(s_i - s)(s_i\tilde{z}_i - s\tilde{q})}{K_{2i}} + \frac{K_2}{nK_{2i}} \{ K_{2i}(h\bar{u} + m\tilde{y}_1) + s_i n\tilde{z}_i + (r^2s^2tK_{2i} + sK_{1i})\tilde{q} \} \\ &= \frac{K_2}{n} (h\bar{u} + m\tilde{y}_1) + \frac{s_i\tilde{z}_i}{K_{2i}} (K_{2i} - K_2 + K_2) + \frac{\tilde{q}}{nK_{2i}} \\ & \quad \cdot \{ K_2r^2s^2tK_{2i} + s(-nK_{2i} + nK_2 + K_2K_{1i}) \} \\ &= \frac{K_2}{n} (h\bar{u} + m\tilde{y}_1) + \frac{s_i\tilde{z}_i}{K_{2i}} (K_{2i} - K_2 + K_2) + \frac{\tilde{q}}{nK_{2i}} \\ & \quad \cdot \{ K_2r^2s^2tK_{2i} + s(-nK_{2i} + K_2K_{2i}) \} \\ &= \frac{K_2}{n} (h\bar{u} + m\tilde{y}_1) + s_i\tilde{z}_i + \left\{ \frac{K_2}{n} (s + r^2s^2t) - s \right\} \tilde{q}. \end{aligned}$$

The above integral is written as:

$$\begin{aligned}
 & -\exp\left[\tilde{P}_1 \left(\frac{\tilde{D}_{1i}}{r_i}\right)\right. \\
 & \quad \left. + \frac{\{K_2/n(h\bar{u} + m\tilde{y}_1) + s_i\tilde{z}_i + \{(K_2/n)(s + r^2s^2t) - s\} \tilde{q} - (\tilde{D}_{1i}/r_i)\}^2}{2(s_i - s + (K_1K_2/n))}\right] \\
 & \int \exp\left[-\frac{1}{2}\left(s_i - s + \frac{K_1K_2}{n}\right)\left[\tilde{P}_2 - \left(\frac{K_2}{n}(h\bar{u} + m\tilde{y}_1) + s_i\tilde{z}_i\right.\right.\right. \\
 & \quad \left.\left.\left. + \left\{\frac{K_2}{n}(s + r^2s^2t) - s\right\} \tilde{q} - \frac{\tilde{D}_{1i}}{r_i}\right) / \left(s_i - s + \frac{K_1K_2}{n}\right)\right]^2\right] d\tilde{P}_2.
 \end{aligned}$$

The integral in this expression is a multiple of a cumulative normal density, which is one, with mean  $(\cdot)/(\cdot)$  and variance  $1/(s_i - s + (K_1K_2/n))$ . Since the multiple is only a function of the variance which does not include  $\tilde{D}_{1i}$ , the whole integral can be ignored for the analysis of the choice of  $\tilde{D}_{1i}$ . Maximizing the objective function is now equivalent to minimizing the exponent in the above expression. Differentiating the exponent with respect to  $\tilde{D}_{1i}$  and setting it equal to zero yields:

$$\begin{aligned}
 \frac{\tilde{D}_{1i}}{r_i} = \frac{K_2}{n} (h\bar{u} + m\tilde{y}_1) \\
 + s_i\tilde{z}_i + \left\{\frac{K_2}{n}(s + r^2s^2t) - s\right\} \tilde{q} - \left(s_i - s + \frac{K_1K_2}{n}\right) \tilde{P}_1
 \end{aligned}$$

which can be easily rewritten as (8).

*Proof of Proposition 3*

From Lemma 1:

$$\begin{aligned}
 \frac{\partial \Delta}{\partial n} &= \frac{1}{K_2^3} [K_2 - 2n + 2n(K_2 - n)L_1] \\
 &= \frac{1}{K_2^3} [K_2 + 2n(K_1L_1 - 1)] > 0
 \end{aligned}$$

because:

$$K_1L_1 - 1 = (s + r^{-2}t^{-1})/K_1 > 0.$$

Also:

$$\begin{aligned}
 \frac{\partial \Delta}{\partial h} &= \frac{\partial \Delta}{\partial m} \\
 &= -\frac{2n}{K_2^3} + \frac{n^2}{K_1^3K_2^3} [K_1K_2 - 2(K_1 + s + r^{-2}t^{-1})(K_1 + K_2)] < 0,
 \end{aligned}$$

and:

$$\frac{\partial \Delta}{\partial s} = \frac{-2n}{K_1^3 K_2^3} [(1 + 2r^2 st)K_1^3 - n(1 + r^2 st)K_1 K_2 + n(1 + 2r^2 st)(K_1 + K_2)(K_1 + s + r^{-2}t^{-1})] < 0.$$

The partial derivatives of  $\bar{V}$  have the same signs as those of  $\Delta$  by Lemma 2.

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