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On the optimal use of loose monitoring in agencies

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Abstract We study the governance implications of firms being privately informed of their potential productivity before contracting with an agent to supply unobservable effort. We show that it can be optimal for high potential firms to have "loose monitoring" in the sense that the monitoring system is less perfect than what is implied by a standard agency model a la Holmstrom (The Bell J Econ 10:74–91, 1979). Loose monitoring is used to achieve separation among different types of firms such that firms with low potential do not have incentives to imitate contracts offered by high potential firms. Our findings imply that although loose monitoring may be a symptom of firms squandering scarce resources provided by investors, it can also arise as an optimal contracting arrangement.

Keywords Pay · Performance measurement · Information system · Monitoring

JEL Classification D82 · D86 · J33 · J41

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1 Introduction

We study why some firms may choose to commit to "loose monitoring" in the sense that the monitoring systems they implement are less informative than what is implied by a standard agency model a la Holmstrom (1979) while other firms that employ similar agents do not. Our basic idea is that identical agents may be differentially useful to different firms but, to some extent, such differential usefulness will be private information to firms' insiders. While many observably quite different firms can benefit from using the inputs of the same type of talent, it may not be obvious to prospective employees where they can best use their talents. Thus, it may be optimal for firms to find credible ways to communicate their differences in this respect to potential employees. This paper shows that such credible communication to prospective employees can take place via the terms of employment offered, specifically, and of particular interest from an accounting perspective, via the level of monitoring provided by their internal and external accounting systems.

A few anecdotal examples may be helpful for fixing ideas before proceeding. Consider a company such as Google that is currently enjoying significant successes. Despite being unique in many respects, there are still many large competitors for the talent that Google relies on for its successes. These competitors, like Google, face constant shifts in their potential fortunes. The issue of specific interest to us in this example, however, is that, at least anecdotally, Google is generally viewed as providing a work environment with less oversight and more individual freedom than its competitors.

As also seems to be the case, firms with a reputation for engaging in "loose monitoring" of employees often are ones that are viewed as having relatively higher potential for future successes. The seemingly "lax" oversight of employees appears to at least reinforce this perception by conveying a message of room for excesses. Firms with higher potential should be better able to afford not to scrutinize the activities and accomplishments of their employees. However, being lax in this way may also be perceived as not pursuing the interest of shareholders. The fact that stronger firms are better *able* to manage their workforce and related expenditures less vigorously does not imply that they should do so, however, as this would appear to be in direct conflict with the interest of shareholders.

A key result of our analysis is that offering terms of employment with features such as "loose monitoring" can actually be an effective as well as efficient way for high potential firms to communicate their productive advantage to employees. The alternative, which is to appear indistinguishable from low potential firms by utilizing the best monitoring available, may actually cost shareholders more. The intuition is the following. In traditional agency settings, the key binding constraint is the agent's incentive compatibility (IC) constraint, which implies inefficient risk sharing (relative to the first-best situation). There, the principal always chooses the best information system as better information reduces compensation-related risk for the agent and hence mitigates the adverse risk-sharing implications of the IC-constraint. Precisely because the best information system improves contracting efficiency, it also invites imitation.

Imitation arises because when the low potential firms' type is known, they need to expose their employees to more incentive related risk to motivate the same level of effort, which in turn results in higher expected wage payment than high potential firms. Thus low potential firms benefit if they cannot be distinguished from high potential ones. We show that in the optimal separating contract arrangement, the contracts that high potential firms offer generally deviate from the standard single-firm-type contracts. Specifically, the optimal separating contracts are characterized by overpayment to agents, or less emphasis on efficient risk-sharing in the sense that contracts may be based on a less precise monitoring system than those used by the low potential firms, or both.

While the deviations from the standard second-best contracts discourage profitable imitation by low potential firms, they nonetheless make high potential firms appear careless with their monitoring and their shareholders' money. What is important to note here is that, in the case of multiple unobservable firm types, such accusations could be unwarranted. The alternative to the separating contract is a contract which allows imitation by low potential firms. High potential firms find it optimal to separate because separation dominates the alternative of being viewed as low potential firms. Accordingly, our analysis suggests that firms that appear to squander shareholder value may be doing the opposite; the standard second best is unavailable due to the presence of firms with lesser potential.

Noteworthy from an accounting perspective, the optimality of a loose monitoring system implies an endogenous upper bound on the value of information system quality for high potential firms when there are multiple types of firms. To gain further insights into this issue, we consider how best to design lax monitoring systems. In particular we examine the role of biases and analyze what kind of news it is better to represent less accurately: good or bad. As we show, if the optimal employment arrangement involves both loose monitoring and overpayment, then the monitoring system should be designed to be "lenient" in that it is more likely to (mis-)report bad news as good news rather than the other way around. In other words, our theory suggests that firms with looser oversight should have more optimistic reporting systems in place than firms with more intense monitoring. As such, our paper also provides an empirically testable implication about the determinant of biases in accounting systems.

There are certainly other ways firms can communicate information credibly. Indeed, firms such as Google clearly differ in many observable ways from their competitors. However, many of these key observable differences among the prospective employers relate to their business models and cannot easily be changed for the purpose of attracting a particular employee. Furthermore, as suggested earlier, most companies operate in highly dynamic environments where the nature of the competition and the company's fortunes can change without notice so that differences in track-records provide little guidance on future fortunes. Therefore, many observable differences do not enable prospective employees to differentiate firms' potential productivity.[1] Such companies may therefore need to continuously

[1] For example, while in some labor markets Honda is competing for the same talent as Caterpillar, to the prospective employees it may be less than obvious which company offers the best overall opportunities. Yet Honda is Honda and Caterpillar is Caterpillar, and they are obviously different.

communicate to current and prospective employees their usefulness to their specific organization. Terms of employment (specific to individual employees) seem to be a natural choice for doing this. Our paper makes new and specific testable predictions about observable monitoring differences between firms.

Our paper is related to the informed principal framework pioneered by Myerson (1983) and Maskin and Tirole (1990, 1992). They study a general mechanism design setting where the principal has private information and the agent can infer this information from the mechanisms offered. Several papers subsequently apply the general framework to study contracting problems in an otherwise standard moral hazard setting (Beaudry 1994; Inderst 2001; Chade and Silvers 2002). These works focus on wage payments as the only separation tool and are closely related to the efficiency wage literature (e.g., Shapiro and Stiglitz 1984),[2] which argues that firms pay employees more than their competitive wages to provide incentives for employees to work. A limitation of the efficiency wage argument is that it assumes that firms cannot offer more sophisticated performance based compensation contracts. The extant informed principal literature shows that efficiency wage still arises even with firms utilizing performance based contracts. In contrast, our focus and innovation are on the use of information systems to achieve separation among firms.

Our paper also relates to the growing literature in accounting aimed at understanding when (more) noise in accounting information systems may be optimal. Prior studies have suggested that a noisy system may be optimal when it can be used as a substitute for commitment device in a multi-period setting (Arya et al. 1998; Demski and Frimor 1999; Christensen et al. 2002), or when it can reduce incentives for earnings management (Dye and Sridar 2002; Chen et al. 2007). We extend this line of inquiry by focusing on private information on the part of the principal and by allowing firms to design information systems with regard to the presence of their competitive peers in the labor market. Our paper provides an alternative explanation for the observed variations in the use of information systems and suggests that using loose monitoring as represented by a seemingly noisier-than-necessary information system, can be optimal for firms seeking to separate themselves from their labor market competitors in order to more efficiently recruit and motivate employees.[3]

The paper proceeds as follows. Section 2 lays out the basic model, derives its solution, and discusses its implications. Section 3 extends the analysis to study measurement biases and provides related empirical predictions. Section 4 provides concluding remarks.

2 Model set-up

Our model captures the core aspects of the general phenomenon that we have in mind as follows. A principal/firm contracts with an agent to supply a productive

[2] Also see Weiss (1990), which offers an excellent overview of the efficiency wage literature.

[3] Ayra and Mittendorf (2005) show that optimal contracts may vary in order to screen agents' talent. However, as discussed in Sect. 2.3.2, switching private information from the principal to agent in our set-up does not give rise to a noisy information system.

action, $a \in \{a_h, a_l\}$, which is only observable to the agent. Action a stochastically influences the realizations of the economic income, $x \in \{\overline{x}, \underline{x}\}$, with $\overline{x} > \underline{x} > 0$. Specifically, $\Pr(\overline{x}|a_l) = 0$ and $\Pr(\overline{x}|a_h) = p > 0$. Thus, picking a_l which we will refer to as working less hard or shirking leads to low economic income ($\underline{x}$) for sure, while working hard (a_h) generates good economic income ($\overline{x}$) with some strictly positive probability $p \in \{p_l, p_h\}$, where the sub-script refers to the exogenously given potential of the firm. The probability measures the marginal productivity of the agent's action conditional on the type of the firm the agent works for. It can be either low or high. Since the agent is more productive if he works for a high potential firm, $p_h > p_l$. We assume that p is privately observed by the firm (principal). The agent has access to the common prior $\Pr(p = p_h) = t$ and can also update his beliefs about the principal's type based on the contract offered by the principal.

For the principal, the only contractible variable available is a performance metric (call it accounting earnings) denoted by $e \in \{\overline{e}, \underline{e}\}$, with $\overline{e} > \underline{e}$, that is informative about the underlying economic income x. The principal designs and determines the accuracy of the performance evaluation system that generates the earnings number ex-ante. The accuracy of the system is denoted by $q \in (1/2, 1]$, with $\Pr(e = \overline{e}|x = \overline{x}) = \Pr(e = \underline{e}|x = \underline{x}) = q$. In essence, q is the probability with which the information system correctly captures the economic income and the choice of q is directly observable to the agent. To highlight the endogenous cost of a precise information system, we assume that there is no exogenous cost of implementing a perfect information system. Thus, a type $p_i \in \{p_h, p_l\}$ firm's contract specifies q_i as well as wage compensation to the agent $(\overline{w}_i, \underline{w}_i)$ as a function of realized accounting earnings with $\overline{w}_i$ ($\underline{w}_i$) paid to the agent when $\overline{e}$ ($\underline{e}$) is observed. For a schematic overview, see Fig. 1.

The principal is risk neutral and the agent is risk averse with an increasing concave utility function $U(w)$. Without loss of generality, we assume that the agent incurs an incremental disutility of m when working hard and has a reservation utility normalized to zero independent of the type of firm he works for. Finally, assume that $\overline{x}$ is sufficiently higher than $\underline{x}$ that the principal, regardless of his type, always wants to elicit high effort (a_h) from the agent. The last assumption allows us to concentrate on the use of the information system as a separation tool. If we were to allow multiple levels of effort, it is possible that the high potential firms might separate by reducing the level of effort required of the agent but keeping the information system intact. Reduced effort would lower the expected true output. When the impact of effort on output is high (which is the case when $\overline{x}$ is very large

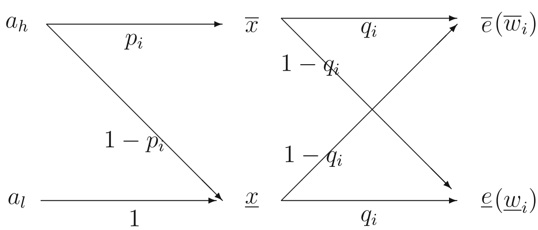

Fig. 1 p_i firm with contract of $(\overline{w}_i, \underline{w}_i, q_i)$

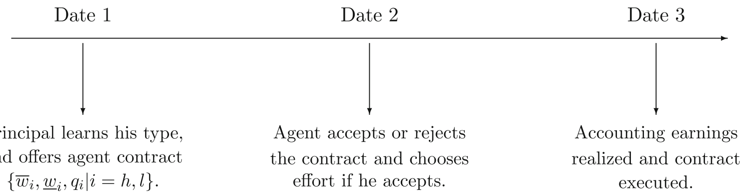

Fig. 2 Time line

relative to $\underline{x}$), then separation through effort reduction will be very costly for high potential firms. One neat feature of separation through information systems is that modifications to the information system change only the agent's wage payment and do not affect the actual output.

The principal's objective is to minimize the expected compensation to the agent while encouraging the agent to work hard. Figure 2 below provides the time line of the model. At Date 1 the principal becomes informed of his own type p_i after which he proposes a contract $(\overline{w}_i, \underline{w}_i, q_i)$ to the agent. At Date 2, the agent revises his belief about the principal's type based on the offered contract and decides whether to accept or reject the contract conditional on his revised belief. If the contract is accepted, the agent then decides whether to work or to shirk. At Date 3, earnings are realized and the contract in place is executed accordingly.

For notational ease, we define $\Pr(j|i)$, where $i,j \in \{h, l\}$, as the probability of observing a high earnings number $(\overline{e})$, conditional on the agent working hard (a_h) in a firm of type p_i which uses the accounting information system q_j and offers $\left(\overline{w}_j, \underline{w}_j\right)$ as the compensation payments, i.e.

$$\Pr(j|i) \equiv \Pr(\overline{e}|a_h, q_j, p_i) = p_i q_j + (1 - p_i)\left(1 - q_j\right).$$

Similarly we define $\Pr(i|i)$ as:

$$\Pr(i|i) \equiv \Pr(\overline{e}|a_h, q_i, p_i) = p_i q_i + (1 - p_i)(1 - q_i).$$

We focus on separating contracts and solve the following program:[4]

$$\min_{q_i, \overline{w}_i, \underline{w}_i, i \in \{h,l\}} \Pr(i|i)\overline{w}_i + [1 - \Pr(i|i)]\underline{w}_i$$

$$\begin{aligned}
s.t. \quad \Pr(h|l)\overline{w}_h + [1 - \Pr(h|l)]\underline{w}_h &\geq \Pr(l|l)\overline{w}_l + [1 - \Pr(l|l)]\underline{w}_l, && (PIC(L)) \\
\Pr(l|h)\overline{w}_l + [1 - \Pr(l|h)]\underline{w}_l &\geq \Pr(h|h)\overline{w}_h + [1 - \Pr(h|h)]\underline{w}_h, && (PIC(H)) \\
\Pr(i|i)U(\overline{w}_i) + [1 - \Pr(i|i)]U(\underline{w}_i) - m &\geq (1 - q_i)U(\overline{w}_i) + q_i U(\underline{w}_i), && (AIC(i)) \\
\Pr(i|i)U(\overline{w}_i) + [1 - \Pr(i|i)]U(\underline{w}_i) - m &\geq 0. && (AIR(i))
\end{aligned}$$

Let $\left(\overline{w}_i^*, \underline{w}_i^*, q_i^*\right)$, $i \in \{h, l\}$ be the solution to the above program. This solution constitutes the best (i.e. least costly) fully revealing equilibrium from the principal's perspective. The right-hand side (RHS) of $PIC(L)$ is the low type principal's expected compensation cost under contract $(\overline{w}_l, \underline{w}_l, q_l)$ and the left-hand side (LHS)

[4] We acknowledge the potential for the existence of other equilibria than this fully separating one. We provide further detailed discussions on this issue after Proposition 1 in the next section.

is his expected compensation cost if he pretends to be the high type and offers contract $(\overline{w}_h, \underline{w}_h, q_h)$ instead. The $PIC(L)$ constraint thus guarantees that the low type has no incentive to offer the high type's contract. Similarly, the $PIC(H)$ constraint guarantees that the high type has no incentive to adopt the low type's contract. Upon observing $\left(\overline{w}_i^*, \underline{w}_i^*, q_i^*\right)$, the agent then infers perfectly the principal's type. Conditional on the agent's perfect inference of the principal's type, the solution satisfies the agent's participation constraint ($AIR(i)$) to ensure that the agent receives at least his reservation utility as well as the relevant incentive compatibility constraint ($AIC(i)$) that ensures the agent prefers working hard to shirking.[5] The next section characterizes the solution to this program.

2.1 Basic analysis

We start by establishing that the solution to the above program differs from the standard solution when the principal and the agent are identically informed about the production function prior to contracting. Specifically, Lemma 1 shows that the standard second best contracts for each of the two types considered here do not satisfy the $PIC(i)$ constraints.

Lemma 1 (a) *If p_i is common knowledge, then the optimal contract is the second best contract given by $q_i^{SB} = 1$, $\overline{w}_i^{SB} = U^{-1}\left(\frac{m}{p_i}\right)$, and $\underline{w}_i^{SB} = U^{-1}(0)$, with $i \in \{h, l\}$;* (b) $\left\{(\overline{w}_i^{SB}, \underline{w}_i^{SB}, q_i^{SB}) | i = h, l\right\}$ *does not satisfy the principal's $PIC(L)$ constraint.*

Under the informativeness principle of Holmstrom (1979), it never helps to add noise to the information system in a standard moral hazard setting. This implies $q_i^{SB} = 1$ in the second best contracts with accounting earnings revealing the true economic outcome with 100% accuracy. The second best contracts also have both the agent's IR and IC constraints binding. Thus $\overline{w}_i^{SB}$ and $\underline{w}_i^{SB}$ can be solved for simply by substituting $q_i = 1$ into the agent's relevant IC and IR constraints.

For Lemma 1(b), notice that under the second best contracts, the high type firm offers the same payment as the low type firm when earnings are low ($\underline{w}_h^{SB} = U^{-1}(0) = \underline{w}_l^{SB}$) but pays the agent less $\left(\overline{w}_h^{SB} = U^{-1}\left(\frac{m}{p_h}\right) < \overline{w}_l^{SB} = U^{-1}\left(\frac{m}{p_l}\right)\right)$ when earnings are high. This suggests that, when p_i is privately observed by the principal, the low type principal has incentives to imitate the high type's contract offer. Indeed, the second best contracts do not satisfy the $PIC(L)$ constraint and therefore cannot be used to achieve separation when p_i is privately known by the firm.

Proposition 1 below presents a reduced program that greatly simplifies the task at hand and also sheds light on the primary tension in the model.

Proposition 1 *When the firm privately observes its type, the best separating contracts are the solution to the following program:*

[5] This formulation assumes that the principal has the bargaining power, which implies that in a standard single type model, the principal would never (need to) provide the agent with more than his reservation utility. Our results are not sensitive to this assumption. See Sect. 2.3.2 for detailed discussion.

$$\min_{q_h, \overline{w}_h, \underline{w}_h} \quad \Pr(h|h)\overline{w}_h + [1 - \Pr(h|h)]\underline{w}_h$$

$$\begin{aligned}
s.t. \quad & \Pr(h|l)\overline{w}_h + [1 - \Pr(h|l)]\underline{w}_h = p_l\overline{w}_l + (1 - p_l)\underline{w}_l && (PIC(L)) \\
& \Pr(h|h)U(\overline{w}_h) + [1 - \Pr(h|h)]U(\underline{w}_h) - m \geq 0 && (AIR(H)) \\
& U(\overline{w}_h) - U(\underline{w}_h) = \frac{m}{p_h(2q_h - 1)} && (AIC(H)) \\
& U(\overline{w}_l) = \frac{m}{p_l}, \quad U(\underline{w}_l) = 0, \quad q_l = 1.
\end{aligned}$$

Proof (All proofs are included in the appendix.) □

The program in Proposition 1 is designed to identify the contract that attains the best (i.e. least costly) fully revealing (separating) equilibrium from the high type principal's perspective. However, for this type of model, the existence of a particular equilibrium always depends on the specification of the off-equilibrium belief. To provide some sense of the validity and generality of the results in Proposition 1, we appeal to the Intuitive Criterion by Cho and Kreps (1987), which requires any off-equilibrium belief to be "reasonable." Specifically in our setting, if the low potential firm would not choose a given contract regardless of what the agent believes, a "reasonable" off-equilibrium belief by the agent is that any principal who does choose that contract cannot be (i.e., has zero probability of being) the low type. Proposition A1 of the appendix shows that under the Intuitive Criterion, all pooling equilibria are eliminated, and that the only separating equilibrium that survives is indeed the best separating one identified by the above program.[6,7]

Proposition 1 establishes that between $PIC(H)$ and $PIC(L)$, only $PIC(L)$ is binding. That is, only the low type firm has incentives to imitate the high type. Proposition 1 further reveals that the optimal separating contracts entail that the low type chooses the second best contract with the most accurate information system (i.e., $q_l = 1$). This result is reminiscent of the classic result in the mechanism design literature that there is no production distortion for the type that has incentives to imitate the others (see, e.g., Laffont and Tirole 1986), except that no distortion here means the contract is identical to the second best contract. Lastly, Proposition 1 shows that in both principals' contracts, the agent's IC constraint is binding. The intuition is the same as in Lemma 1. A slack *AIC* constraint means a larger than necessary spread in the agent's compensation to induce effort. The larger spread imposes unnecessary risk on the agent, which is always suboptimal for the principal when the agent is risk-averse.

[6] If firms can choose an equilibrium before they learn their types, they strictly prefer pooling. This can be easily established due to the agent's risk aversion and the need for firms to (costly) signal. However, this commitment to pooling requires that firms resist the temptation to deviate after their types are privately known.

[7] In a two-sided adverse selection setting, Cella (2005) finds that pooling always dominates separation. Our setting differs from his in two significant ways. First, our setting is a case of common value while his deals with private value. Second, we impose ex post IC and IR constraints while his pooling equilibrium is obtained under interim IC and IR constraints. Ex post IC and IR imply interim IC and IR but the reverse is not true.

Together with Lemma 1, Proposition 1 implies that in order to create the separation, the high type principal has to offer a different contract than the second best to satisfy *PIC*(*L*). Specifically, he can do so by choosing an imprecise accounting system ($q_h < 1$). To capture this more succinctly, we re-parameterize the simplified program in Proposition 1 as follows:

$$\min_{q_h, U_0} \quad \Pr(h|h)U^{-1}\left(\overline{U}_h\right) + [1 - \Pr(h|h)]U^{-1}\left(\underline{U}_h\right)$$

$$s.t. \quad \Pr(h|l)U^{-1}\left(\overline{U}_h\right) + [1 - \Pr(h|l)]U^{-1}\left(\underline{U}_h\right)$$

$$= p_l U^{-1}\left(\overline{U}_l\right) + (1 - p_l)U^{-1}\left(\underline{U}_l\right) \qquad (PIC(L)')$$

$$\overline{U}_h \equiv U(\overline{w}_h) = U_0 + \frac{mq_h}{p_h(2q_h - 1)}$$

$$\underline{U}_h \equiv U(\underline{w}_h) = U_0 + \frac{m(q_h - 1)}{p_h(2q_h - 1)}$$

$$U_0 \geq 0$$

$$\overline{U}_l = \frac{m}{p_l}, \quad \underline{U}_l = 0, \quad q_l = 1.$$

Here U_0 denotes the agent's expected utility under the optimal contract in the high type firm, i.e., $U_0 = \Pr(h|h)U(\overline{w}_h) + [1 - \Pr(h|h)]U(\underline{w}_h) - m$. This equation, together with the fact that *AIC*(*H*) is binding, enables us to uniquely pin down $\overline{U}_h$ and $\underline{U}_h$ in terms of q_h and U_0. Consequently, we can express the $AIR(H)$ constraint as $U_0 \geq 0$ and replace the $AIC(H)$ constraint with the implied expressions for $\overline{U}_h$ and $\underline{U}_h$. The principal's choice variables are now q_h and U_0, and the re-parameterized problem has the same solution as the original program.

Lemma 2 below provides some preliminary insights on how $PIC(L)'$ could be satisfied with U_0 and q_h and on the relation between these two tools.

Lemma 2 (a) *Holding* q_h constant, the *LHS of* $PIC(L)'$ *strictly increases in* U_0. (b) *Holding* U_0 *constant, the LHS of* $PIC(L)'$ *strictly decreases in* q_h. (c) *Holding* $PIC(L)'$ *at equality implicitly defines* U_0 as a function of q_h, *with* $\frac{dU_0}{dq_h} > 0$.

Recall that the *LHS* of $PIC(L)'$ represents the expected wage cost for the low type firms when they imitate the high types' contract offer. Lemma 2(a) shows that a higher U_0 increases this cost and therefore helps satisfy the $PIC(L)'$ constraint by increasing its *LHS* without changing the *RHS*. Lemma 2(b) implies that the low type's cost to imitate the high type's contract is higher when q_h is lower (i.e., when the high type adopts a less than perfect information system). The intuition is that introducing noise into the information system weakens the link between effort and performance measurement. As a result, a risk averse agent demands higher expected wage payment, thus making it costly for the low type principal to misrepresent his type.

Lastly, Lemma 2(c) suggests that the two tools are substitutes for each other. A small increase in U_0 frees up the principal's $PIC(L)'$ constraint (as implied by Lemma 2(a)) and enables q_h to increase as an optimal response (as implied by Proposition 1 and Lemma 2(b)). This substitution can potentially create a net benefit to the principal when the gains from the latter outweigh the losses caused by the former.

To summarize, the key new ingredient of our model derives from the $PIC(L)'$ constraint. In the traditional agency setting, the optimal q is always set at the upper bound of 1 (assuming zero out-of-pocket implementation cost) as the principal always benefits from achieving better risk sharing with the agent. However, Lemma 2 indicates that, for a high type firm, a higher q brings an endogenous cost in our setting: it induces the low type to imitate by making $PIC(L)'$ harder to satisfy. As a result the optimal q_h may not be 1 anymore. Similarly, in the traditional agency setting, paying the agent higher than his reservation utility has no benefits but only costs to the principal. If the principal has the bargaining power, the optimal contract would keep the agent's IR constraint binding. In contrast, providing the agent with excess utility has a benefit here by making $PIC(L)'$ easier to satisfy. Thus, when the principal is privately informed of his type, the optimal contract for high type incorporates a trade-off between the relative costs and benefits of these two means of separation. In the next section we explore this trade-off further by identifying sufficient conditions for when the high type firm optimally chooses $q_h < 1$ (in Proposition 2) or sets $U_0 > 0$ (in Proposition 3).

2.2 Sufficient conditions for separation

We first investigate the determinants for the quality of the monitoring system adopted by the high type firm. The principal here is always in a position to choose $q_h = 1$ with no direct cost. A sufficient condition for him to choose $q_h < 1$ would be that if the expected compensation embedded among all possible contracts that satisfy the constraints of the simplified program in Proposition 1 is increasing in q_h at $q_h = 1$, i.e.,

$$\frac{dE(w_h)}{dq_h}\Big|_{q_h=1} > 0.$$

Proposition 2 identifies a sufficient condition under which this is the case.

Proposition 2 *A sufficient condition for the optimal $q_h < 1$ is*

$$\frac{U\left(\overline{w}^*_{h,q_h=1}\right) - U\left(\underline{w}^*_{h,q_h=1}\right)}{\overline{w}^*_{h,q_h=1} - \underline{w}^*_{h,q_h=1}} < \frac{U'\left(\overline{w}^*_{h,q_h=1}\right) + U'\left(\underline{w}^*_{h,q_h=1}\right)}{2} \tag{1}$$

where $\overline{w}^*_{h,q_h=1}$ and $\underline{w}^*_{h,q_h=1}$ are the optimal wage compensation to the agent by the high type firm when q_h is restricted to 1. A sufficient condition for (1) is $U'''(w) > 0$.

To understand the intuition behind Proposition 2, we first need to recognize the potential benefit of $q_h < 1$ in our setting. One might think that $q_h < 1$ can never be optimal because it introduces noise and increases the risk premium demanded by the risk-averse agent. This intuition turns out to be incomplete. In a standard agency model where the principal has no private information, the principal's problem is to trade off between the risk imposed on the agent and the need to motivate the agent to work. Then it is true that distorting the information system has no value. (This is also established in Lemma 1.) However, in our setting, on top of a traditional moral hazard problem the principal also desires to separate among different types. In this case, as Lemma 2 shows, a noisy system with $q_h < 1$ can potentially help separation.

The next step in understanding the intuition behind Proposition 2 is to recognize that a noisy system hurts the high type relatively less on the margin than it does the low type. To illustrate this, let us first pick any contract that satisfies the agent's *AIC* constraint. Note that slightly decreasing q_i while holding the agent's expected utility constant increases the principal's expected wage payment. In formal terms,

$$\begin{aligned}\frac{\partial E(w_i)}{\partial q_i} &= \frac{\partial[\Pr(i|i)\overline{w}_i + (1-\Pr(i|i))\underline{w}_i]}{\partial q_i} \\ &= \left[(2p_i - 1)(\overline{w}_i - \underline{w}_i) - \Pr(i|i)\frac{m}{U'(\overline{w}_i)(2q_i-1)^2}\right. \\ &\quad \left. +(1-\Pr(i|i))\frac{m}{U'(\underline{w}_i)(2q_i-1)^2}\right] < 0.\end{aligned}$$

The usual single crossing property here calls for $\frac{\partial^2 E(w_i)}{\partial q_i \partial p_i} > 0$. That is, although a smaller q_h increases the expected wage payment for both types, it increases less for the high type principal. Note that $\frac{\partial^2 E(w_i)}{\partial q_i \partial p_i} > 0$ if and only if

$$\frac{U(\overline{w}) - U(\underline{w})}{\overline{w} - \underline{w}} < \frac{2U'(\overline{w})U'(\underline{w})}{U'(\overline{w}) + U'(\underline{w})}. \tag{2}$$

It is easy to verify that (1) is a more general condition than (2) because (2) is derived by holding the agent's expected utility constant, which is more restricted than Proposition 2.

To see why a noisy system hurts the high type less on the margin, note that *conditional on the agent exerting high effort and holding wage payments constant*, a noisy system decreases the expected wage payment of the high type but increases that of the low type. Specifically, fix any two wage payments of $\overline{w}$ and $\underline{w}$ with $\overline{w} > \underline{w}$. Under the perfect information system, the agent is paid $\overline{w}$ when the true output is high and $\underline{w}$ when the output is low. As Fig. 1 shows, a noisy system ($q < 1$) increases the expected wage payment for both types when the true output is low, but decreases the payment when the true output is high. Since the low type firm is more likely to end up with true low output, its expected wage payment increases more than the high type under the noisy system.

The above illustration holds $\overline{w}$ and $\underline{w}$ constant and does not consider the effect of introducing noise on the optimal wage payments to motivate effort. While a noisy system costs the high type less on expected wage payment when the agent works, it makes it harder to motivate effort to begin with. Condition (1) ensures that this incentive cost of noisy systems does not overwhelm their benefit in helping the high type separate. While seemingly technical, Condition (1) actually speaks to the agent's risk preference. As shown in the appendix, it is satisfied for all utility functions that exhibit non-increasing absolute risk aversion, such as logarithm, power, and negative exponential utility functions.

This is not surprising given that risk aversion is the fundamental reason for the high type to be able to use noisy systems to separate. In particular, the high type's advantage in separation with $q_h < 1$ is higher when the agent is more risk averse; in the extreme case when the agent is risk-neutral, the high type cannot use $q < 1$ to separate at all. In Proposition 2, we start at the highest $q_h = 1$ and therefore the highest wealth given to the agent (i.e., rely solely on overpayment to separate by Lemma 2). While decreasing the agent's wealth benefits the principal, it makes $PIC(L)'$ harder to satisfy and therefore invites imitation by the low type. With decreasing risk aversion, however, reducing the agent's wealth increases his risk aversion, which provides the high type the comparative advantage in reducing q to keep $PIC(L)'$ binding.

To complete the characterization of the optimal contract, we note that the agent's IR constraint may not be binding. Our next proposition identifies sufficient conditions for that to be the case.

Proposition 3 *Let $\overline{w}^*_{h,U_0=0}$ and $\underline{w}^*_{h,U_0=0}$ be the wage payments to the agent and $q^*_{h,U_0=0}$ be the optimal q_h in the high type firm holding $U_0 = 0$. Then a sufficient condition for the optimal $U_0 > 0$ is*

$$\frac{U\left(\overline{w}^*_{h,U_0=0}\right) - U\left(\underline{w}^*_{h,U_0=0}\right)}{\overline{w}^*_{h,U_0=0} - \underline{w}^*_{h,U_0=0}} > \frac{U'\left(\overline{w}^*_{h,U_0=0}\right) + U'\left(\underline{w}^*_{h,U_0=0}\right)}{2}. \qquad (3)$$

A sufficient condition for (3) *is the agent is $U'''(w) < 0$.*

Similar to Proposition 2, to prove Proposition 3, we start with a feasible contract where $U_0 = 0$ and then slightly increase U_0 while maintaining all the constraints. Under Condition (3), the principal's expected wage payment strictly decreases with U_0 at $U_0 = 0$, i.e., $\frac{dE(w_h)}{dU_0}|_{U_0=0} < 0$. Then, it has to be case that the optimal U_0 is strictly higher than the agent's reservation utility of 0.

A sufficient condition for Condition (3) is $U'''(w) < 0$, which implies that the agent exhibits increasing risk aversion. To see the intuition, note that at $U_0 = 0$, the agent exhibits the least risk aversion, which means that the high type's advantage in signaling through q is the weakest. When $U'''(w) < 0$, increasing his wealth would make him more risk averse, which makes separation through q relatively more attractive.

It may seem counter-intuitive to set $U_0 > 0$ as the single crossing property does not appear to be satisfied for U_0. One can easily verify that $\frac{\partial[\Pr(h|i)\overline{w}_h + (1-\Pr(h|i))\underline{w}_h]}{\partial U_0} > 0$ and $\frac{\partial^2[\Pr(h|i)\overline{w}_h + (1-\Pr(h|i))\underline{w}_h]}{\partial U_0 \partial p_i} > 0$. This means everything else equal, a higher U_0 increases the expected wage payment for both types of principals, and the increase is more for the high type than for the low type principal. This suggests that, compared with q_h,U_0 would be a costly, and hence an inferior separation device for the high type principal. However, this intuition fails to recognize the subtle interaction between U_0 and q_h, which comes from the fact that increasing U_0 can help relax the $PIC(L)'$ constraint and thus makes room for potential improvement on q_h. However, the improvement in q_h will invite imitation by the low type. In order to keep the low type at bay, it has to be that the high type maintains his comparative advantage in signaling through q. As discussed earlier, this advantage is high when the agent is more risk averse. Therefore, when the agent exhibits increasing risk aversion, increasing U_0 help maintain the high type's advantage.

We want to reiterate here that Conditions (1) and (3) are sufficient conditions. Moreover, they are not complement sets to each other although they do have the appearance of being so. The values both sides of these conditions take vary, depending on where these expressions are evaluated. Condition (1) is evaluated at the point where $q_h = 1$ and $U_0 > 0$, while Condition (3) is evaluated at the point where $U_0 = 0$ and $q_h < 1$. In general, whether Condition (1) or (3) is met depends on the local curvature of the utility function, which relates to the agent's risk preference as well as his wealth level.

2.3 Additional analyses and discussions

2.3.1 Numerical examples

Two numerical examples below help provide more intuitive illustrations of our results. The first one shows that loose monitoring (i.e. the optimal $q_h < 1$) can arise in the best separating contract. Assume the following parameter values:

$$U(w) = Ln(w); \quad p_h = 0.4; \quad p_l = 0.2; \quad m = 0.3.$$

when $q_h = 1$, from $PIC(L)'$ binding, we can get $\underline{U}_h = 0.32684$ and $U_0 > 0$. Plug these values in the objective function, we have the expected wage payment at 2.0061. Now, reduce q_h from 1 to 0.99. Again, a binding $PIC(L)'$ implies that $\underline{U}_h = 0.31592$, under which $U_0 > 0$ still holds. The new expected wage payment, however, is 2.0054, strictly less than the expected wage payment when $q_h = 1$. Hence, the optimal contract must have $q_h^* < 1$.

The next numerical example shows how increasing U_0 can benefit the principal. Assume the following parameter values:

$$U(w) = 0.5 - 0.5(2 - w^2)^2 \text{ for } w \in \left[\sqrt{2/3}, \sqrt{2}\right]; \quad p_h = 0.4; \quad p_l = 0.2; \\ m = 0.06.$$

when $U_0 = 0$, from AIR, AIC, and $PIC(L)'$ binding, we can solve for the optimal contract as $\left\{\overline{w}_h^* = 1.22876, \underline{w}_h^* = 0.88983, q_h^* = 0.6236\right\}$, under which the high type's expected wage payment is 1.0507. Now increase U_0 a little bit to 0.005 and solve for the optimal contract again. We have $\left\{\overline{w}_h^* = 1.21582, \underline{w}_h^* = 0.90195, q_h^* = 0.6321\right\}$. Under the new contract, the information system becomes more accurate, the spread (hence risk imposed on the agent) between $\overline{w}$ and $\underline{w}$ is lower, and the principal's expected wage payment is also lower at 1.0506. Hence, the optimal contract must have $U_0^* > 0$.

2.3.2 Sensitivity to alternative assumptions

We now discuss the robustness of our results to alternative assumptions.[8] The first is when the agent, not the principal, privately observes the productivity parameter p. In this case, the principal needs to offer a menu of contracts to separate different types of agents. As is standard in the adverse selection literature, the high type agent has incentives to imitate the low type, ceteris paribus, and the optimal screening contract leaves positive rent to the high type agent. However, the optimal information system is the most accurate one regardless of the agent's type. That is, loose monitoring does not arise as part of the optimal screening contract when agents privately observe p. The intuition is that adding noise to the low type agent's contract would lead to an even higher award when the performance signal is good, which is a more likely event for the high type agent and thus generates more incentives for the high type to imitate.[9]

Second, our main set-up follows the standard assumption by giving the principal the bargaining power in designing the contract. What we have in mind is a situation where a relatively large number of otherwise identical job candidates seek employment with a limited supply of firms with differing levels of productivity. For example, sales associates at SAP and Oracle probably come from a potentially large pool of candidates with similar backgrounds. But there are only a few firms like SAP and Oracle. Therefore, it is not unreasonable to assume that an industry leader such as SAP has a much larger share of the bargaining power against its employees and at the same time needs to signal its productivity relative to its competitors.

Nonetheless, it is interesting to see whether our results are sensitive to the assumption about bargaining power. We therefore consider a setting where the agent offers a menu of contracts to the principal such that the expected wage payment from the principal cannot exceed a reservation level, i.e., let the IR constraint be for the firm such that the firm does not exceed its (exogenous) budget for compensation. As a benchmark, if the firm's productivity is publicly known, the optimal (second-best) information system is a perfect one. However, when productivity is privately observed by the firm, under the second-best contract, much like in our existing setting, the low type principal has incentives to imitate the high type.

The intuition here is that if correctly identified, the low (high) type firm needs to offer a contract with a greater (smaller) spread in compensation, leading to a higher (lower) expected wage payment. Thus, to restore incentive compatibility in a separating equilibrium, the principal's IC constraint needs to be satisfied. It can be shown that adding noise to the information system in the high type firm can benefit the agent, for intuitions similar to what we established earlier in the main model. Hence, our main implication that the high type introduces noise to signal his type is not sensitive to shifts in bargaining power.

[8] Detailed proof for this section is available from the authors upon request.

[9] Correspondingly, because the low type does not want to imitate the high type to begin with, adding noise in the high type's contract only increases the principal's cost to induce effort from the high type without added benefits.

Third, our current setting only allows a binary signal. Given q and U_0 , the binary wage payments are uniquely pinned down by the agent's binding IC constraint. However, when there are more than two levels of signal realizations, the agent's IC can no longer uniquely determine all wage payments. Thus, in this case, the principal may have an additional tool to use for separation. That is, for fixed q and U_0 and a binding agent's IC constraint, the principal now has the ability to fine-tune wage payments across different signal realizations to achieve more efficient separation.[10] However, the key message of our paper that a noisy information system can serve as a separation device remains unaltered as the low type will not choose any $q < 1$ to begin with.

Finally, throughout the paper, we have assumed that there are no exogenous out-of-pocket costs to implement an information system for stewardship purpose. This assumption allows us to focus on the main insight of our analysis, i.e., there is an endogenous cost of having an accurate information system for stewardship purposes. Information systems are of course not costless. When it is sufficiently costly to set up a perfect stewardship system, loose monitoring may not be an effective separating tool to the extent that the low type firms find it easier to imitate by adopting a noisier system to reduce their monitoring cost. However, we note that firms often install information systems (potentially with exogenous set-up cost) for other purposes such as decision making and yet these systems can provide stewardship uses on the margin. It is conceivable that, in order to separate, firms simply do not utilize the best available technology that's already acquired but instead noise up the performance measure in order to credibly convey private information to agents. In this sense, "loose monitoring" means that high potential firms are less likely to utilize the best available existing technology for monitoring than low potential firms.

2.3.3 Alternative interpretation

In our analysis we interpret p as the exogenous difference in firms' productivity that is unobservable to potential employees. This difference can be more broadly interpreted in other ways, one of which may be that high p firms have better baseline information systems in place (for example, better information infrastructures that provide feedback to employees to improve their performance) but the quality of such systems is unobservable to potential employees. In this case, q can be interpreted as some observable changes to the baseline information system that is specific for stewardship purposes. For example, in a school setting where schools compete for talented students, how well exams can test students' learning and provide students with useful feedback to improve their human capital (i.e., the quality of the school's baseline information system) may sometimes be difficult for students to observe. However, the grade assignment scheme (whether to use an A/B/C/D system or simply a pass/fail system) is usually publicly observable and committed to beforehand.

[10] We thank an anonymous referee for this observation.

While we have interpreted q as the degree of monitoring intensity or the quality of performance measurement system, one could equivalently interpret the combination of $(q, \overline{w}, \underline{w})$ as the principal offering a randomized contract to the agent. Since randomized contracts do not correspond well to what casual empiricism suggests as used in practice, the information system or the degree of monitoring intensity can be viewed as realistic ways of implementing randomized contracts. Whatever the interpretation, however, our main point stands. That is, when firms need to convey their private information to potential employees in the presence of imitating competitive peers, the terms of employment contracts in high potential firms may appear to be inefficient, especially if one insists on comparing them with the standard second-best contracts.

3 Biases in information systems

The structure analyzed in the previous section assumes that any noise introduced is symmetric, i.e., the probability of misrepresenting a good economic income ($\overline{x}$) as a low performance measure ($\underline{e}$) is the same as that of misreporting a bad economic income ($\underline{x}$) as a high performance measure ($\overline{e}$). In this section, we relax this assumption and analyze the situation where the principal can choose both $\Pr(e = \overline{e}|x = \overline{x}) = q^1$ and $\Pr(e = \underline{e}|x = \underline{x}) = q^2$ individually and is not constrained to set $q^1 = q^2$ (with $q^1, q^2 \in [\frac{1}{2}, 1]$ and cannot both be equal to $\frac{1}{2}$). Figure 3 illustrates the modified information system.

With this slight change in the model setup, the principal's problem is to choose a contract $\left(q_i^1, q_i^2, \overline{w}_i, \underline{w}_i\right)$, where $i \in \{h, l\}$, to minimize his expected payment:

$$\min_{q_i^1, q_i^2, \overline{w}_i, \underline{w}_i, i \in \{h,l\}} \quad \Pr(i|i)\overline{w}_i + [1 - \Pr(i|i)]\underline{w}_i$$

subject to the $AIC(i)$, $AIR(i)$, $PIC(L)$, and $PIC(H)$ constraints as shown in Sect. 2 , with the probabilities modified as follows:

$$\text{where,}\ \Pr(j|i) = p_i q_j^1 + (1 - p_i)(1 - q_j^2),$$
$$\Pr(i|i) = p_i q_i^1 + (1 - p_i)(1 - q_i^2).$$

Similar to the symmetric case, we can show (details available upon request) that the optimal contracts have the same general properties as those characterized in the prior section. Furthermore, the conditions in Propositions 2 and 3 for the symmetric case clearly still apply here: the sufficient conditions under which a perfect information system is dominated and where the IR constraint (i.e., $U_0 > 0$) is slack are still sufficient conditions here.

We call an information system an unbiased system if $q^1 = q^2$, a harsh system if $q^1 < q^2$ (because it is more likely to represent a good economic outcome as bad performance than it is to represent a bad economic outcome as good performance), and a lenient system if $q^1 > q^2$ (the opposite of the harsh system). A priori, it is not clear which of these three possibilities is optimal as a device to make $PIC(L)$ satisfied, because all three systems introduce noise into the information system that will increase the low type's cost to imitate. The following proposition

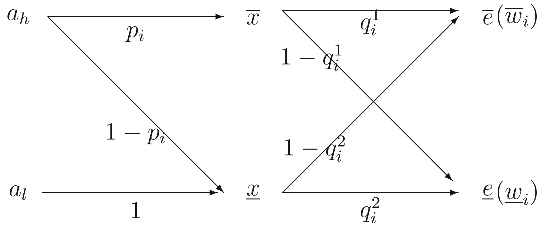

Fig. 3 p_i firm with contract of $(\overline{w}_i, \underline{w}_i, q_i^1, q_i^2)$

shows that if the optimal contract involves loose monitoring and if the IR constraint is not binding, then the loose monitoring has to be lenient, i.e., $q_h^1 > q_h^2$.

Proposition 4 *If the optimal contract involves both loose monitoring and* $U_0 > 0$ *then it must be either* $\frac{1}{2} \leq q_h^2 < 1 = q_h^1$ *or* $\frac{1}{2} = q_h^2 < q_h^1 < 1$.

The intuition can be gleaned from Fig. 3. Observe that decreasing q_h^2 increases the probability of classifying a bad economic income $\underline{x}$ as good performance $\overline{e}$ and hence increases the expected payment to the agent. A low type principal (p_l) is more likely to reach this node (i.e., having a bad economic income) and hence more likely to end up paying a high wage to the agent. A smaller q_h^2 therefore affects the high type principal adversely but to a lesser extent than it does the low type principal. In contrast, decreasing q_h^1 increases the probability of classifying a good economic income $\overline{x}$ as bad performance measure $\underline{e}$. This would hurt the high type principal more than the low type because the former is more likely to have a good economic income and thus has to compensate the agent for the increased compensation risk.

The intuition can also be shown by the single crossing property. Start with a feasible contract where $q_h^1 = q_h^2 = 1$ and $U_0 > 0$. Slightly decrease q_h^2 while keeping the agent's expected utility constant. The marginal change in the expected wage payment for a type i principal is $\frac{\partial E(w_i)}{\partial q_h^2}$ and can be easily shown to be negative. Further $\frac{\partial^2 E(w_i)}{\partial q_h^2 \partial p_i} > 0$. That is, while a lower q_h^2 hurts both principals, it hurts the low type more than it hurts the high type. In other words, the single crossing property for q_h^2 is satisfied. However, the same property doesn't hold for q_h^1: if we slightly decrease q_h^1 while keeping the agent's expected utility constant at 0, $\frac{\partial E(w_i)}{\partial q_h^1} < 0$ and $\frac{\partial^2 E(w_i)}{\partial q_h^1 \partial p_i} < 0$. Thus, in fact, it is the low type principal that has an advantage when decreasing q_h^1.

To the extent that external financial accounting earnings are an important input in the compensation contracts to firm management, the above result may be of particular relevance for the contemporary literature on accounting conservatism. Our results suggest that firms with brighter prospects may not just pay higher compensation and be more lax in terms of measuring performance; their performance measure will also exhibit more optimistic (or less conservative) bias. Other implications include using market-to-book as a measure of individual firms growth prospects. While firms with high growth potential should enjoy relatively high market valuation, our result suggests that they may also exhibit relatively more liberal accounting and thus report higher book values as well.

4 Conclusion

We study a setting where a firm (principal) is privately informed of the firm's potential and contracts with an agent to supply unobservable effort. We show that it can be optimal for the firm to have "loose monitoring" in the sense that the monitoring system is less perfect than what is implied by a standard agency model a la Holmstrom (1979). Further, it may be optimal also to provide the agent with higher expected utility than his reservation level. These contractual features are used to achieve separation among different types of firms such that firms with low potential do not imitate contracts offered by high potential firms. Our findings imply that although loose monitoring and seemingly excessive compensation to employees may be symptoms of firms squandering scarce resources provided by investors, they can also arise as an optimal contracting arrangement to maximize shareholders' wealth.

Of further interest when we can identify strict preferences for biases, we show that firms that optimally provide loose monitoring may prefer a lenient information system in the sense that representing bad news as good is preferred to representing good as bad. Particularly, this is the case when the conditions favor slack in the IR constraint. In other words our theory suggests that firms with higher pay and less focus on tight oversight should have more optimistic or less conservative monitoring systems than their more stringent counterparts. Our paper provides direct testable implications about the determinants of reporting biases and their relation to compensation arrangements and market values.

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Appendix

Proof of Proposition 1

Proof: We start with a relaxed program where we ignore $PIC(H)$. Later, we'll show that the optimal solution under the relaxed program satisfies $PIC(H)$. The proof proceeds in several steps.

- The IC constraint for the agent in the low type firm ($AIC(L)$) is binding at the optimal solution. Suppose not. Then, by continuity, there exists a $\epsilon > 0$ such that we can decrease $\overline{w}_l$ by ϵ and increase $\underline{w}_l$ by $\frac{\Pr(l|l)}{1-\Pr(l|l)}\epsilon$ and still maintain $AIC(L)$ slack. This operation maintains the low type principal's expected payoff. At the same time, it relaxes the agent's participation constraint ($AIR(L)$), which means the low type principal can then be made strictly better off by reducing $\overline{w}_l$

and $\underline{w}_l$ with the same amount, while satisfying all the constraints. A contradiction.

- The agent's *IR* constraint in the low type firm (*AIR*(*L*)) is binding at the optimal solution. Suppose not. Decreasing $U(\overline{w}_l)$ and $U(\underline{w}_l)$ by the same amount makes the low type principal strictly better off while satisfying all the constraints. A contradiction.
- $q_l = 1$ at the optimal solution. Suppose not. Increasing q_l reduces the low type principal's expected payment to the agent, i.e., *PIC*(*L*) becomes slack. A contradiction. This also implies that $q_h < 1$. Otherwise, $q_h = 1$ would make the high type's contract the same as that in the second best situation, which as shown in Lemma 1 does not satisfy the principal's IC constraints.
- Combining the above three points, we obtain $\overline{w}_l = U^{-1}(\frac{m}{p_l})$ and $\underline{w}_l = U^{-1}(0)$. That is, at the optimal solution, the low type firm's contract is the same as the second best contract in Lemma 1.
- The IC constraint for the agent in the high type firm (*AIC*(*H*)) is binding at the optimal solution. Suppose not. Then we can decrease $\overline{w}_h$ by ϵ and increase $\underline{w}_h$ by $\frac{\Pr(h|h)}{1-\Pr(h|h)}\epsilon$ such that *AIC*(*H*) is still slack. By construction, this maintains the high type principal's expected wage payment, makes *AIR*(*H*) slack (as the same expected payment has a smaller spread for the agent) and *PIC*(*L*) slack (as $[1-\Pr(h|l)]\frac{\Pr(h|h)}{1-\Pr(h|h)}\epsilon - \Pr(h|l)\epsilon > 0$). Thus, the high type principal can be made strictly better off by reducing $\overline{w}_h$ and $\underline{w}_h$ with the same amount, while satisfying all the constraints. A contradiction. At equality, *AIC*(*H*) implies $U(\overline{w}_h) - U(\underline{w}_h) = \frac{m}{p_h(2q_h-1)}$.
- We have two more constraints: the low type principal's *IC* constraint (*PIC*(*L*)) and the agent's *IR* constraint in the high type firm (*AIR*(*H*)). We now show that *PIC*(*L*) is binding at the optimal solution. Suppose not. There are two cases. First, when *AIR*(*H*) is not binding at the optimal solution. A slight decrease of $U(\overline{w}_h)$ and $U(\underline{w}_h)$ by the same amount leads to less expected compensation to the agent, while satisfying all the constraints. A contradiction. The second case is when *AIR*(*H*) is binding at the optimal solution. Because $q_h < 1$ (as shown in bullet point #2 above), one can increase q_h, which leads to a higher payoff for the high type principal while maintaining all the constraints. A contradiction.
- Now, we show that *PIC*(*H*) is indeed satisfied by the solution from the relaxed program. Note that any pooling contract where the high type firm adopts the second best contract for the low type (i.e., $q_h = q_l^{SB} = 1$, $U(\overline{w}_h) = U(\overline{w}_l^{SB}) = \frac{m}{p_l}$, and $U(\underline{w}_h) = U(\underline{w}_l^{SB})$) satisfies all the constraints in the relaxed program. Thus the optimal contract $(\overline{w}_h^*, \underline{w}_h^*, q_h^*)$ under the relaxed program must be

$$\begin{aligned}
&\Pr(q_h = q_h^*|h)(\overline{w}_h = \overline{w}_h^*) + [1-\Pr(q_h = q_h^*|h)](\underline{w}_h = \underline{w}_h^*) \\
&\quad \le \Pr(q_h = q_l^{SB}|h)(\overline{w}_h = \overline{w}_l^{SB}) + [1-\Pr(q_h = q_l^{SB}|h)](\underline{w}_h = \underline{w}_l^{SB}) \\
&\quad = \Pr(q_l^*|h)\overline{w}_l^* + [1-\Pr(q_l^*|h)]\underline{w}_l^*.
\end{aligned}$$

Obviously, the first and last line in the above expression are exactly the $PIC(H)$ constraint. □

Proposition A1 The only equilibrium that survives "Intuitive Criterion" is the best separating equilibrium identified by Proposition 1.

Proof of the Proposition A1 Note in our setting, if a low potential firm would not choose a contract regardless of what the agent thinks, "Intuitive Criterion" requires that all "reasonable" off-equilibrium beliefs specify that such a contract is offered by the low potential firm with probability zero. Now, consider any other separating equilibria. It's easy to see that in any separating equilibrium, the low potential firm offers the second best contract. Clearly, "Intuitive Criterion" requires that any contract that satisfies $AIC(H)$ and $AIR(H)$ and strictly satisfies $PIC(L)$ should be offered with zero probability by the low potential firm.[11] Since the best separating equilibrium gives the highest payoff to the high potential principal among all contracts that satisfies $AIC(H)$, $AIR(H)$, and $PIC(L)$, it is the only separating equilibrium that survives the "Intuitive Criterion." Next, consider pooling equilibria where both types of the firms offer the same contract to the agent. Without loss of generality, let's assume that the pooling contract has a perfect information system and the agent's IC and IR constraints are binding. Denote the wage payments of the pooling contract as $(1, \overline{w}, \underline{w})$, with $\overline{w}$ ($\underline{w}$) being the payment when accounting signal is high (low). Clearly, the incentive compatibility constraint binding implies $\overline{w} = U^{-1}(\frac{m}{\widetilde{p}}) > \underline{w} = U^{-1}(0)$, where $\widetilde{p} \equiv tp_h + (1-t)p_l$ is the ex ante unconditional probability of obtaining $\bar{x}$ given high effort. Denote the solution to the following program as $(q'_h, \overline{w}'_h, \underline{w}'_h)$.

$$\begin{aligned}
&\min_{q_h, \overline{w}_h, \underline{w}_h} \quad \Pr(h|h)\overline{w}_h + [1 - \Pr(h|h)]\underline{w}_h \\
&s.t. \quad \Pr(h|l)\overline{w}_h + [1 - \Pr(h|l)]\underline{w}_h \geq p_l\overline{w} + (1-p_l)\underline{w} \qquad (PIC(L)) \\
&\qquad \Pr(h|h)U(\overline{w}_h) + [1 - \Pr(h|h)]U(\underline{w}_h) - m \geq 0 \qquad (AIR(H)) \\
&\qquad U(\overline{w}_h) - U(\underline{w}_h) \geq \frac{m}{p_h(2q_h - 1)} \qquad (AIC(H))
\end{aligned}$$

- $AIC(H)$ is binding at the optimal solution. Suppose not. Then we can decrease $\overline{w}_h$ by ϵ and increase $\underline{w}_h$ by $\frac{\Pr(h|h)}{1-\Pr(h|h)}\epsilon$ such that $AIC(H)$ is still slack. By construction, this maintains the principal's expected wage payment, makes $AIR(H)$ slack (as the same expected payment has a smaller spread for the agent) and $PIC(L)$ slack (as $[1 - \Pr(h|l)]\frac{\Pr(h|h)}{1-\Pr(h|h)}\epsilon - \Pr(h|l)\epsilon > 0$). Thus, the high type principal can be

[11] To see this, note that PIC(L) strictly satisfied means the low potential firm doesn't want to choose this contract even if the agent believes such a contract is offered by the high potential firm with probability one.

made strictly better off by reducing $\overline{w}_h$ and $\underline{w}_h$ with the same amount, while satisfying all the constraints. A contradiction. At equality, $AIC(H)$ implies $U(\overline{w}_h) - U(\underline{w}_h) = \frac{m}{p_h(2q_h-1)}$.

- At $(q'_h, \overline{w}'_h, \underline{w}'_h)$, $PIC(H)$ is satisfied, i.e.

$$p_h\overline{w} + (1-p_h)\underline{w} > \Pr(h|h)\overline{w}'_h + [1-\Pr(h|h)]\underline{w}'_h. \qquad (PIC(H))$$

Note that any pooling contract where the high potential firm adopts $(1, \overline{w}, \underline{w})$ satisfies all the constraints in the above program. Thus the optimal contract $(q'_h, \overline{w}'_h, \underline{w}'_h)$ under the program must be

$$\begin{aligned}
&\Pr(h|h)\overline{w}'_h + [1-\Pr(h|h)]\underline{w}'_h \\
&\quad < \Pr(\overline{e})|_{q_h=1}\overline{w} + [1-\Pr(\underline{e})|_{q_h=1}]\underline{w} \\
&\quad = \Pr(l|h)\overline{w} + [1-\Pr(l|h)]\underline{w}.
\end{aligned}$$

Note the inequality is strict because, if the high potential firm offers $(1, \overline{w}, \underline{w})$, $AIC(H)$ is slack and hence not the optimal solution by the first bullet point. Obviously, the first and last line in the above expression are exactly the $PIC(H)$ constraint.

- $AIC(H)$ binding implies $(q'_h, \overline{w}'_h, \underline{w}'_h)$ is different from $(1, \overline{w}, \underline{w})$ because at $(1, \overline{w}, \underline{w})$ the high potential firm's AIC constraint is slack, and $PIC(H)$ slack implies that the high potential firm incurs a smaller expected wage payment under $(q'_h, \overline{w}'_h, \underline{w}'_h)$ relative to $(1, \overline{w}, \underline{w})$. Finally, since $PIC(L)$ is satisfied by $(q'_h, \overline{w}'_h, \underline{w}'_h)$, the low type firm has no (strict) incentive to offer $(q'_h, \overline{w}'_h, \underline{w}'_h)$. Thus, by "Intuitive Criterion," the agent should believe any principal that offers $(q'_h, \overline{w}'_h, \underline{w}'_h)$ is a high potential firm with probability one, and the high potential is strictly better off by deviating and offering $(q'_h, \overline{w}'_h, \underline{w}'_h)$, implying that pooling equilibria do not survive "Intuitive Criterion." □

Proof of Lemma 2

Part 1 of Lemma 2 is straightforward. For Part 2, since

$$\overline{U}_h = U_0 + \frac{mq_h}{p_h(2q_h-1)} \quad \text{and} \quad \underline{U}_h = U_0 - \frac{m(1-q_h)}{p_h(2q_h-1)},$$

we have,

$$\begin{aligned}
\frac{\partial LHS \quad \text{of} \quad \text{PIC(L)}'}{\partial q_h} &= \frac{\partial \Pr(h|l)\overline{w}_h + [1-\Pr(h|l)]\underline{w}_h}{\partial q_h} \\
&= (2p_l - 1)(\overline{w}_h - \underline{w}_h) + \Pr(h|l)\frac{\partial \overline{w}_h}{\partial q_h} + [1-\Pr(h|l)]\frac{\partial \underline{w}_h}{\partial q_h} \\
&= (2p_l - 1)(\overline{w}_h - \underline{w}_h) + \frac{-m\Pr(h|l)}{p_h(2q_h-1)^2}\frac{1}{\overline{U}'_h} + \frac{m[1-\Pr(h|l)]}{p_h(2q_h-1)^2}\frac{1}{\underline{U}'_h}.
\end{aligned}$$

Concavity of the utility function implies that the above expression is

$$
\begin{aligned}
&< (2p_l - 1)(\overline{w}_h - \underline{w}_h) + \frac{1}{\overline{U}'_h}\frac{m[1 - 2\Pr(h|l)]}{p_h(2q_h - 1)^2} \\
&= (2p_l - 1)(\overline{w}_h - \underline{w}_h) + \frac{1}{\overline{U}'_h}\frac{m(1 - 2p_l)}{p_h(2q_h - 1)} \\
&\overset{*}{=} (2p_l - 1)(\overline{w}_h - \underline{w}_h) - \frac{\overline{U}_h - \underline{U}_h}{\overline{U}'_h}(2p_l - 1) \\
&= (2p_l - 1)(\overline{w}_h - \underline{w}_h)\underbrace{\left[1 - \frac{\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)}}{\overline{U}'_h}\right]}_{<0} < 0 \text{ if } p_l > 1/2.
\end{aligned}
$$

Note, * utilizes the fact that at the optimal solution $\overline{U}_h - \underline{U}_h = \frac{m}{p_h(2q_h - 1)}$. Also, we can express the inequality above as

$$
\begin{aligned}
&< (2p_l - 1)(\overline{w}_h - \underline{w}_h) + \frac{1}{\underline{U}'_h}\frac{m(1 - 2p_l)}{p_h(2q_h - 1)} \\
&\leq (2p_l - 1)(\overline{w}_h - \underline{w}_h)\underbrace{\left[1 - \frac{\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)}}{\underline{U}'_h}\right]}_{>0} < 0 \text{ if } p_l < 1/2.
\end{aligned}
$$

Thus, for all values of p_l, *LHS* of $PIC(L)'$ is decreasing in q_h. Hence choosing $q_h < 1$ could help achieve separation.

For Part 3, since $PIC(L)'$ constraint is binding, using implicit function theorem, we have

$$
\begin{aligned}
\frac{dq_h}{dU_0} &= -\frac{\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1 - \Pr(h|l)}{\underline{U}'_h}}{\frac{\partial LHS \text{ of } PIC(L)}{\partial q_h}} \\
&= -\frac{\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1 - \Pr(h|l)}{\underline{U}'_h}}{(2p_l - 1)(\overline{w}_h - \underline{w}_h) + \frac{-m\Pr(h|l)}{p_h(2q_h - 1)^2}\frac{1}{\overline{U}'_h} + \frac{m[1 - \Pr(h|l)]}{p_h(2q_h - 1)^2}\frac{1}{\underline{U}'_h}}.
\end{aligned}
$$

The denominator is negative, as shown earlier, and the numerator is positive. Therefore we have $\frac{dq_h}{dU_0} > 0$. □

Proof of Proposition 2

The strategy to prove Proposition 2 is to find conditions under which the high type firm's expected payment is increasing in q_h at $q_h = 1$, i.e.,

$$
\frac{dE(w_h)}{dq_h}\Big|_{q_h = 1} > 0
$$

while satisfying all other constraints. Obviously, if the above holds, the optimal $q_h^* < 1$.

Since

$$\overline{w} = U^{-1}\left(U_0 + \frac{mq_h}{p_h(2q_h - 1)}\right), \quad \underline{w}_h = U^{-1}\left(U_0 - \frac{m(1-q_h)}{p_h(2q_h - 1)}\right),$$

we have

$$\frac{d\overline{w}_h}{dq_h} = \frac{1}{\overline{U}'_h}\left[\frac{dU_0}{dq_h} - \frac{m}{p_h(2q_h-1)^2}\right]$$

and

$$\frac{d\underline{w}_h}{dq_h} = \frac{1}{\underline{U}'_h}\left[\frac{dU_0}{dq_h} + \frac{m}{p_h(2q_h-1)^2}\right].$$

Substituting the above into $\frac{dE(w_h)}{dq_h}$ yields

$$\begin{aligned}\frac{dE(w_h)}{dq_h} &= \frac{d[\Pr(h|h)\overline{w}_h + [1-\Pr(h|h)]\underline{w}_h]}{dq_h} \\ &= \left[(2p_h - 1)(\overline{w}_h - \underline{w}_h) + \frac{-m\Pr(h|h)}{p_h(2q_h-1)^2}\frac{1}{\overline{U}'_h} + \frac{m[1-\Pr(h|h)]}{p_h(2q_h-1)^2}\frac{1}{\underline{U}'_h}\right] \\ &\quad + \left(\frac{\Pr(h|h)}{\overline{U}'_h} + \frac{1-\Pr(h|h)}{\underline{U}'_h}\right)\frac{dU_0}{dq_h}.\end{aligned}$$

Recall $\frac{dU_0}{dq_h} = -\frac{\frac{\partial LHS \text{ of PIC(L)}}{\partial q_h}}{\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1-\Pr(h|l)}{\underline{U}'_h}}$. Notice that the term in the parenthesis on the last line above is the same expression as the denominator for $\frac{dU_0}{dq_h}$ except that all p_l's are replaced by p_h's. Further, the term in the bracket in the second line above is just the expression for the numerator of $\frac{dU_0}{dq_h}$ except that all p_l's are replaced by p_h's. Therefore, the conditions for $\frac{dE(w_h)}{dq_h}|_{q_h=1} > 0$ are essentially the same as the condition for $\frac{\partial\left(\frac{dU_0}{dq_h}\right)}{\partial p_l}|_{q_h=1} < 0$.

The sign for $\frac{\partial\left(\frac{dU_0}{dq_h}\right)}{\partial p_l}$, by quotient rule, is the same as that for

$$\left(\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1-\Pr(h|l)}{\underline{U}'_h}\right)\frac{\partial\left(-\frac{\partial LHS \quad \text{of} \quad \text{PIC(L)}}{\partial q_h}\right)}{\partial p_l}$$
$$- \left(-\frac{\partial LHS \quad \text{of} \quad \text{PIC(L)}}{\partial q_h}\right)\frac{\partial\left(\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1-\Pr(h|l)}{\underline{U}'_h}\right)}{\partial p_l},$$

which, after some algebra, can be simplified to

$$\frac{2}{\overline{U}'_h \underline{U}'_h} \frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)} - \left(\frac{1}{\overline{U}'_h} + \frac{1}{\underline{U}'_h}\right),$$

which is negative if and only if

$$\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)} < \frac{1}{2}\left(\overline{U}'_h + \underline{U}'_h\right),$$

which is the condition in Proposition 2 evaluated at $\overline{w}^*_{h,q_h=1}$ and $q_h = 1$.

Define

$$F(w_1, w_2) = (U'(w_1) + U'(w_2))(w_1 - w_2) - 2(U(w_1) - U(w_2)).$$

The condition in Proposition 2 is guaranteed if $F(w_1, w_2) > 0$ for $w_1 > w_2$. We next show that $U''' > 0$ is a sufficient condition for $F(w_1, w_2) > 0$ for $w_1 > w_2$.

Note $F(w_1, w_2) = 0$ when $w_1 = w_2$. Thus $F(w_1, w_2) > 0$ $\forall$ $w_1 > w_2$ if $\frac{\partial F(w_1,w_2)}{\partial w_1} > 0$ for $w_1 > w_2$. Define $F_1(w_1, w_2) = \frac{\partial F(w_1,w_2)}{\partial w_1}$. Then,

$$\begin{aligned} F_1(w_1, w_2) &= U''(w_1)(w_1 - w_2) + U'(w_1) + U'(w_2) - 2U'(w_1) \\ &= U''(w_1)(w_1 - w_2) - U'(w_1) + U'(w_2) \\ &= 0_{|w_1=w_2}. \end{aligned}$$

Furthermore, $F_1(w_1, w_2) > 0 \forall w_1 > w_2$ if $\frac{\partial F_1(w_1,w_2)}{\partial w_1} > 0$ for $w_1 > w_2$. Note

$$\begin{aligned} \frac{\partial F_1(w_1, w_2)}{\partial w_1} &= U'''(w_1)(w_1 - w_2) + U''(w_1) - U''(w_1) \\ &= U'''(w_1)(w_1 - w_2). \end{aligned}$$

Since $w_1 > w_2$, then if $U''' > 0$, $F_1(w_1, w_2)$ is an increasing function of w_1. Since $F_1 = 0$ at $w_1 = w_2$, we have $F_1 > 0$ for $w_1 > w_2$. This means that F is an increasing function of w_1 and is positive for $w_1 > w_2$.

In other words, a sufficient condition is $U''' > 0$, which is a necessary condition for the agent to be decreasingly risk averse. A sufficient condition for decreasing risk aversion is $U''' > -A(w)U'' > 0$ where $A(w) = -U''(w)/U'(w)$ is the absolute risk aversion parameter. Using a similar approach, we can show that negative exponential utility function, which exhibits constant risk aversion, also satisfies condition (1). □

Proof of Proposition 3

To prove Proposition 3, we first find the optimal solutions holding U_0 at an arbitrary level. We then show that the principal's expected payment can be reduced when we increase U_0 from $U_0 = 0$. That is, we will show that

$$\begin{aligned} \frac{dE(w_h)}{dU_0}\Big|_{U_0=0} &= \frac{d[\Pr(h|h)\overline{w}_h + [1-\Pr(h|h)]\underline{w}_h]}{dU_0}\Big|_{U_0=0} \\ &= \left[(2p_h - 1)(\overline{w}_h - \underline{w}_h)\frac{dq_h}{dU_0} + \Pr(h|h)\frac{d\overline{w}_h}{dU_0} + (1-\Pr(h|h))\frac{d\underline{w}_h}{dU_0}\right]\Big|_{U_0=0} < 0. \end{aligned}$$

Since

$$\overline{w} = U^{-1}\left(U_0 + \frac{mq_h}{p_h(2q_h-1)}\right), \quad \underline{w}_h = U^{-1}\left(U_0 - \frac{m(1-q_h)}{p_h(2q_h-1)}\right),$$

we have

$$\frac{d\overline{w}_h}{dU_0}|_{U_0=0} = \frac{1}{\overline{U}'_h}\left[1 - \frac{m\frac{dq_h}{dU_0}}{p_h(2q_h-1)^2}\right]$$

and

$$\frac{d\underline{w}_h}{dU_0}|_{U_0=0} = \frac{1}{\underline{U}'_h}\left[1 + \frac{m\frac{dq_h}{dU_0}}{p_h(2q_h-1)^2}\right].$$

Substituting these expressions into $\frac{dE(w_h)}{dU_0}$, after some combining of terms and simplifications, we arrive at

$$\frac{dE(w_h)}{dU_0} = \frac{\Pr(h|h)}{\overline{U}'_h} + \frac{1-\Pr(h|h)}{\underline{U}'_h} + \left[(2p_h-1)(\overline{w}_h - \underline{w}_h) + \frac{-m\Pr(h|h)}{p_h(2q_h-1)^2}\frac{1}{\overline{U}'_h} + \frac{m[1-\Pr(h|h)]}{p_h(2q_h-1)^2}\frac{1}{\underline{U}'_h}\right]\frac{dq_h}{dU_0}$$

Notice that the bracket term in the second line above is the same expression as the denominator for $\frac{dq_h}{dU_0}$ except that all p_l's are replaced by p_h's. Also, notice that $\frac{\Pr(h|h)}{\overline{U}'_h} + \frac{1-\Pr(h|h)}{\underline{U}'_h}$ is just the expression for the numerator of $\frac{dq_h}{dU_0}$ except that all p_l's are replaced by p_h's. Therefore, the conditions for $\frac{dE(w_h)}{dU_0} < 0$ is essentially the same as the condition for $\frac{\partial\left(\frac{dq_h}{dU_0}\right)}{\partial p_l}|_{U_0=0} < 0$.

The sign for $\frac{\partial\left(\frac{dq_h}{dU_0}\right)}{\partial p_l}$, by quotient rule, is the same as that for

$$\left(-\frac{\partial LHS \quad \text{of} \quad \text{PIC(L)}}{\partial q_h}\right)\frac{\partial\left(\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1-\Pr(h|l)}{\underline{U}'_h}\right)}{\partial p_l} - \left(\frac{\Pr(h|l)}{\overline{U}'_h} + \frac{1-\Pr(h|l)}{\underline{U}'_h}\right)\frac{\partial\left(-\frac{\partial LHS \quad \text{of} \quad \text{PIC(L)}}{\partial q_h}\right)}{\partial p_l},$$

which, after some algebra, can be simplified to

$$\frac{1}{\overline{U}'_h} + \frac{1}{\underline{U}'_h} - \frac{2}{\overline{U}'_h\underline{U}'_h}\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)},$$

which is negative if and only if

$$\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)} > \frac{1}{2}\left(\overline{U}'_h + \underline{U}'_h\right),$$

which is the condition in Proposition 3 evaluated at $U_0^* = 0$ and $q^*_{h,u_0=0} < 1$.

Based on the second half of the proof for Proposition 2, it is clear that $U'''(w) > 0$ is a sufficient condition for $\frac{\overline{U}_h - \underline{U}_h}{(\overline{w}_h - \underline{w}_h)} > \frac{1}{2}\left(\overline{U}'_h + \underline{U}'_h\right)$.

Proof of Proposition 4

Employing approaches similar to the proof for Proposition 1, we can simplify the program into the following:

$$\min_{\overline{w}_h, \underline{w}_h, q^1, q^2} \Pr(h|h)\overline{w} + [1 - \Pr(h|h)]\underline{w}$$

$$s.t: \ \Pr(h|l)\overline{w}_h + [1 - \Pr(h|l)]\underline{w}_h - E(w_l) \geq 0 \quad (\mu) \quad (PIC(L)'')$$

$$U(\overline{w}_h) - U(\underline{w}_h) - \frac{m}{p_h(q_h^1 + q_h^2 - 1)} \geq 0 \quad (\gamma) \qquad (AIC(H)'')$$

$$\Pr(h|h)U(\overline{w}) + [1 - \Pr(h|h)]U(\underline{w}) - m \geq 0 \quad (\lambda) \qquad (AIR(H)'')$$

$$\text{where, } E(w_l) = p_l\overline{w}_l + (1 - p_l)\underline{w}_l,\ U(\overline{w}_l) = \frac{m}{p_l}, \quad U(\underline{w}_l) = 0.$$

We use μ, γ, and λ to denote the Lagrangian multipliers for the $PIC(L)''$, $AIC(H)''$ and $AIR(H)''$. Note that we write the constraints in inequality terms as we know that these multipliers measure the shadow prices of the constraints and have to be nonnegative. The first order conditions are

$$\frac{\partial L}{\partial \overline{w}} = \Pr(h|h) - \mu \Pr(h|l) - \gamma U'(\overline{w}) - \lambda \Pr(h|h)U'(\overline{w}) = 0$$

$$\frac{\partial L}{\partial \underline{w}} = [1 - \Pr(h|h)] - \mu[1 - \Pr(h|l)] + \gamma U'(\underline{w}) - \lambda(1 - \Pr(h|h))U'(\underline{w}) = 0$$

$$\frac{\partial L}{\partial q^1} = p_h(\overline{w} - \underline{w}) - \mu p_l(\overline{w} - \underline{w}) - \gamma\frac{2m}{p_h(q^1 + q^2 - 1)^2} - \lambda p_h[U(\overline{w}) - U(\underline{w})]$$

$$\frac{\partial L}{\partial q^2} = (p_h - 1)(\overline{w} - \underline{w}) - \mu(p_l - 1)(\overline{w} - \underline{w}) - \gamma\frac{2m}{p_h(q^1 + q^2 - 1)^2} - \lambda(p_h - 1)[U(\overline{w}) - U(\underline{w})].$$

$AIR(H)''$ not binding implies $\lambda = 0$, under which

$$\frac{\partial L}{\partial q^1} - \frac{\partial L}{\partial q^2} = (1 - \mu)(\overline{w} - \underline{w}),$$

$$\frac{\partial L}{\partial \overline{w}} + \frac{\partial L}{\partial \underline{w}} = 1 - \mu + \gamma[U'(\underline{w}) - U'(\overline{w})] = 0.$$

Case 1: both q^1 and q^2 are interior. Then both $\frac{\partial L}{\partial q^1}$ and $\frac{\partial L}{\partial q^2}$are equal to zero. Hence $\frac{\partial L}{\partial q^1} - \frac{\partial L}{\partial q^2} = 0$ and $1 - \mu = 0$. Together with $\frac{\partial L}{\partial \overline{w}} + \frac{\partial L}{\partial \underline{w}} = 0$, this implies $\gamma = 0$. But substitute $\gamma = \lambda = 0$ and $\mu = 1$ back into the expressions for $\frac{\partial L}{\partial q^1}$ and $\frac{\partial L}{\partial q^2}$, we see that both would be positive, a contradiction. That is, one of the q's has to be at the corner. Case 2: Suppose $\frac{1}{2} \leq q^1 < 1$ interior and $q^2 = 1$. Then $\frac{\partial L}{\partial q^2} < 0 \leq \frac{\partial L}{\partial q^1}$ and thus $\frac{\partial L}{\partial q^1} - \frac{\partial L}{\partial q^2} > 0$, which implies $1 - \mu > 0$. However, with $1 - \mu > 0$, $\frac{\partial L}{\partial \overline{w}} + \frac{\partial L}{\partial \underline{w}} = 0$ implies that $\gamma < 0$. A contradiction. Case 3: $q^1 = \frac{1}{2}$ and q^2 interior. Then $\frac{\partial L}{\partial q^2} = 0 < \frac{\partial L}{\partial q^1}$, which implies $1 - \mu > 0$ and thus $\gamma < 0$. A contradiction. Thus the only cases that survive are $\frac{1}{2} \leq q^2 < 1 = q^1$ and $\frac{1}{2} = q^2 < q^1 < 1$. □

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