The Origins and Effects of Macroeconomic Uncertainty

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Abstract

We construct and estimate a dynamic stochastic general equilibrium model that features demand- and supply-side uncertainty. Using term structure and macroeconomic data, we find sizable effects of uncertainty on risk premia and business cycle fluctuations. Both demand-side and supply-side uncertainty imply large contractions in real activity and an increase in term premia, but supply-side uncertainty has larger effects on inflation and investment. We introduce a novel analytical decomposition to illustrate how multiple distinct risk propagation channels account for these differences. Supply and demand uncertainty are strongly correlated in the beginning of our sample, but decouple in the aftermath of the Great Recession.

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1 Introduction

It is well-established that broad measures of macroeconomic and financial market uncertainty vary significantly over time.[1] There is also an emerging literature interested in studying how these changes in uncertainty affect business cycle fluctuations in micro-founded general equilibrium models. However, these papers typically only use macroeconomic data to pin down the effects of uncertainty, consider only one source of uncertainty, and estimate the process for uncertainty separately from the rest of the model.[2] In this paper, we use both macroeconomic and term structure data, distinguish between demand-side and supply-side uncertainty, and conduct a structural estimation of a micro-founded model in which the process for uncertainty and its effects are jointly estimated. Our results demonstrate that uncertainty matters. In particular, we uncover sizable effects of uncertainty shocks on business cycle and term premia dynamics. The specific effects of demand-side and supply-side uncertainty are examined through multiple endogenous risk propagation channels.

Asset prices contain valuable information about uncertainty, given that changes in macroeconomic uncertainty generate fluctuations in risk premia. We find that changes in nominal term premia contain key identifying information disciplining the effects of uncertainty and its propagation through various risk channels. At the same time, there is empirical and anecdotal evidence suggesting that changes in measures of uncertainty are related to heterogeneous sources (e.g., Bloom (2014)) and are also imperfectly correlated. Figure 1 plots various uncertainty measures whose pairwise correlations range between -0.30 to 0.85. We find it important to distinguish between different sources of uncertainty, and we explicitly model fluctuating demand and supply uncertainty. We identify demand uncertainty as originating from shocks to the time discount factor while supply uncertainty as emanating from shocks to TFP growth. In particular, we show that these two types of uncertainty act through distinct channels. Finally, jointly estimating the process for uncertainty and its effects on the economy has the important implication that uncertainty is not only identified via changes in stochastic volatility, but also through its first-order effects on the economy.

Our quantitative analysis is based on a dynamic stochastic general equilibrium (DSGE) model along the lines of Christiano, Eichenbaum, and Evans (2005), but with the following departures. First, we assume that the representative household has Epstein and Zin (1989) recursive preferences to capture sensitivity towards low-frequency consumption growth and discount rate risks. Second, we allow for stochastic volatility changes in TFP and preference shocks, both modeled as distinct Markov chains, estimated jointly within our DSGE

[1] See, for example, Baker, Bloom, and Davis (2016), Jurado, Ludvigson, and Ng (2015), and Berger, Dew-Becker, and Giglio (2017).

[2] Some examples include Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Basu and Bundick (2017).

model. Changes in stochastic volatility *and* the endogenous response of the economy to these changes both contribute to fluctuations in uncertainty. Third, we use an iterative solution method to endogenously capture sizable and time-varying risk premia. By modelling stochastic volatility as regime changes, we obtain a conditionally log-linear solution that facilitates an estimation using a modification of the standard Kalman filter. Lastly, we use data on nominal bond yields across different maturities in our estimation.

Our solution method captures the first- and second-order effects of uncertainty on agents' decision policies, as well as effects on conditional risk premia. We show that this feature of our solution method sharpens the identification of uncertainty dynamics. In addition, our solution method provides an approximate analytical risk decomposition that uncovers distinct risk propagation channels for which uncertainty affects macroeconomic fluctuations. We use the risk decomposition to illustrate how uncertainty shocks produce different effects depending on the origin (e.g., demand or supply). Our analysis therefore provides an economic interpretation for why there is not a consensus on the macroeconomic effects of uncertainty shocks. More broadly, our risk decomposition can be utilized in a wide range of dynamic stochastic models, and is therefore of independent interest.

Figure 2 illustrates the strong relation between real activity, measured as detrended GDP, the slope of the nominal yield curve, and macroeconomic volatility.[3] As the economy enters a recession, the slope of the yield curve and macroeconomic volatility both tend to rise. In our model, movements from low to high volatility regimes endogenously trigger a decline in real activity and a steepening of the yield curve, consistent with the data. We find that the effects of uncertainty are quantitatively significant. The two uncertainty shocks together explain over 14% of the variation in investment growth, around 10% for consumption growth, and 28% for the slope of the nominal yield curve. These shocks also produce significant countercyclical variation in the nominal term premium. The effects of uncertainty are even more sizable when focusing on fluctuations at business cycle frequencies. An economy that is exclusively affected by uncertainty shocks would generate business cycle fluctuations for consumption and investment as large as 24.5% and 31%, respectively, of an analogue economy with both uncertainty and traditional level shocks.

Both demand-side and supply-side uncertainty generate positive commovement between consumption and investment, which is often a challenge for standard macroeconomic models. Thus, uncertainty shocks emerge as an important source of business cycle fluctuations. However, the origin of uncertainty plays an important role, as the two types of uncertainty impact the economy in very distinct ways. Compared to demand-side uncertainty, supply-side uncertainty has larger effects on inflation and is relatively more important

[3] Detrended GDP is obtained by applying a bandpass filter. Similar results hold if GDP is detrended using an HP filter. The slope of the term structure is computed as the difference between the five-year yields and the one-year yield. Macroeconomic volatility is measured as a five-year moving average of the standard deviation of GDP growth.

for explaining fluctuations in investment. Furthermore, while in the first half of our sample, demand- and supply-side uncertainty tend to move together, they decouple in the second half of the sample.

Nominal term premia in our model is driven by time-varying demand and supply uncertainty. As such, using the term structure of interest rates as observables in our estimation is important for disciplining the effects of uncertainty. While both supply and demand uncertainty are important for the unconditional nominal term premia, we find that the conditional dynamics of nominal term premia are mostly attributed to variation in demand-side uncertainty through the inflation risk premia component. Therefore, the observed term structure dynamics help to sharpen the identification of the two different sources of uncertainty. Without using term structure data in our estimation, the timing of the uncertainty shocks is quite different, the volatility regimes are less persistent, and the effects of the uncertainty shocks on the macroeconomy are smaller.

To understand how uncertainty shocks affect the real economy and account for these differences, we use an approximate model solution method that allows us to identify and quantify five distinct *risk propagation channels* for uncertainty shocks that are labeled as precautionary savings, investment risk premium, inflation risk premium, nominal pricing bias, and investment adjustment channel. The *precautionary savings* term reflects the prudence of the representative household towards uncertainty about future income. This prudence term arises through the households' consumption-savings Euler equation. The *investment risk premium* term emerges through the investment Euler equation, which depends on the covariance between the pricing kernel and the return on investment. The *inflation risk premium* term shows up through the Fisherian equation, and the nominal term premium imposes strong restrictions on this channel. The *nominal pricing bias* arises in the Phillips curve due to the presence of nominal rigidities that makes firms more prudent when setting nominal goods prices. Finally, the *investment adjustment channel* arises because of rigidities in the household's ability to immediately adjust investment to the desired level.

Our decomposition of the risk propagation channels shows that different forces contribute to generating empirically realistic macroeconomic and asset pricing dynamics. The precautionary savings, investment risk premium, and the nominal pricing bias terms are the most quantitatively important risk propagation channels for business cycles. The parameters governing price stickiness, capital adjustment costs, and elasticity of labor supply are critical for determining the effects from these risk propagation channels. Price stickiness and labor supply elasticity are important for determining the sign and magnitude of the precautionary savings channel, while capital adjustment costs are important for determining the sign and magnitude of the investment risk premium channel. The degree of price stickiness and labor supply elasticity determines the sensitivity of labor demand shifts to uncertainty changes. The degree of capital adjustment costs affects the covariance of the return on investment and the stochastic discount factor, which determines the effect of the investment risk premium channel. The degree of price stickiness is important for determining the effects of the nominal pricing bias.

The *investment risk premium* channel plays a key role in amplifying the response of investment to changes in supply-driven uncertainty. The investment risk premium channel has opposite effects on investment for supply- and demand-side uncertainty. The underlying reason is that physical capital is a poor hedge against negative TFP shocks, but a good hedge for adverse preference shocks. In particular, demand and supply-side shocks produce different signs in the covariance between the pricing kernel and the return on investment. In response to a negative TFP shock, marginal utility increases but the value of physical capital decreases. Therefore, investment in physical capital commands a positive risk premium with respect to TFP shocks. In contrast, preference shocks produce the opposite pattern. A negative preference shock increases marginal utility and the value of capital. Therefore, investment commands a negative risk premium with respect to preference shocks. Consequently, when supply-side uncertainty increases, households have an incentive to lower investment so as to reduce exposure to TFP shocks. Instead, when demand-side uncertainty increases, households have an incentive to increase investment to hedge against preference shocks. Overall, this channel plays a key role in explaining why the cumulative decline in investment to an increase supply-side (demand-side) uncertainty is amplified (dampened).

We then use our decomposition to understand the small response of inflation to demand-driven uncertainty shocks, but a large response to supply-driven uncertainty. These inflation responses can be accounted for by differences in how the precautionary savings and nominal pricing bias channels operate under the two uncertainty shocks. Both demand- and supply-side uncertainty shocks trigger a strong precautionary savings channel effect, which generates downward pressure on inflation. However, for demand-driven uncertainty, another quantitatively important propagation channel is the nominal pricing bias, which is natural given that level preference shocks are one of the main drivers of inflation dynamics. For demand-side uncertainty shocks, the precautionary savings and nominal pricing bias channels have opposite effects on inflation that cancel each other out, and consequently, the cumulative effect on inflation is close to zero. In contrast, for supply-side uncertainty, the nominal pricing bias is not quantitatively important, since TFP growth shocks are not important for explaining inflation dynamics. Therefore, the cumulative effect of an increase in supply-side uncertainty is driven by the precautionary savings propagation channel, leading to a large decline in inflation.

Our paper relates to Basu and Bundick (2017) in that we also consider the role of the precautionary savings channel, in conjunction with sticky prices, for the propagation of demand-side uncertainty shocks.

In our estimation, we find that this channel is quantitatively important. Thus, we complement the findings of Basu and Bundick (2017), but differ along the following dimensions. First, we develop a novel analytical decomposition that unveils four additional risk propagation channels. In our estimation, we find that two of these four channels, the investment risk premium and nominal pricing bias, are as quantitatively important as the precautionary savings channel. Second, we conduct a structural estimation of our model using macroeconomic and bond yield data instead of calibration. In our structural estimation the process for uncertainty is not exogenously given, but jointly estimated with the rest of the model. We find that uncertainty plays a key role for both macro and term structure dynamics. Finally, we allow for both demand- and supply-side uncertainty changes, while Basu and Bundick (2017) only consider demand-side uncertainty shocks. While both types of uncertainty shocks are important for explaining business cycles, we find that the macroeconomic responses to these shocks to be quite different. For example, supply-side uncertainty changes generate more severe recessions, with significantly larger effects on inflation and investment. Our analytical decomposition allows us to carefully disentangle the economic margins that account for these different responses.

Our paper connects to the broader literature studying the impact of uncertainty shocks in macroeconomic models (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Bachmann and Bayer (2014), Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), Justiniano and Primiceri (2008), Bianchi, Ilut, and Schneider (2014), Schaal (2017), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), and Saijo (2017), etc.). We differ from these papers in that we (i) allow for multiple sources of uncertainty, (ii) conduct a structural estimation, (iii) use asset pricing data, in the form of nominal bond yields in the estimation and a prior on the investment risk premium, to discipline the effects of uncertainty, and (iv) do not deviate from the assumption of rational expectations. Christiano, Motto, and Rostagno (2014) build a general equilibrium model with financial frictions that features time-varying cross-sectional idiosyncratic uncertainty. They refer to stochastic disturbances to cross-sectional volatility as risk shocks, which they find to be important for explaining business cycle fluctuations.[4] In their estimation, this measure of risk is an unobserved latent variable. In contrast, our paper considers a smaller scale New Keynesian model without financial frictions, and instead focusing on aggregate uncertainty. In our setting, uncertainty is identified by both changes in first and second moments in the data.

The pricing of consumption and volatility risks builds on the endowment economy models of Bansal and

[4] Bachmann and Bayer (2014) highlight that nonconvex adjustment costs are important for jointly reconciling the procyclical dispersion in investment but countercyclical dispersion in productivity.

Yaron (2004), Piazzesi and Schneider (2007), and Bansal and Shaliastovich (2013). However, we differ by considering a general equilibrium framework with production, where the dynamics of stochastic consumption volatility risks are linked to the time-varying second moments of structural macroeconomic shocks and to the endogenous response of the macroeconomy to changes in the volatility of these shocks. Furthermore, our production-based setting allows us to consider the endogenous feedback between risk premia and business cycle fluctuations via uncertainty shocks. The role of preference shocks for generating a positive real term premia relates to the endowment economy model of Albuquerque, Eichenbaum, Luo, and Rebelo (2016). We build on this work, and show that time discount factor shocks also provide a novel endogenous source of inflation risk premia in a New Keynesian framework.

More broadly, our paper relates to an emerging literature studying asset prices in New Keynesian models (e.g., Bekaert, Cho, and Moreno (2010), Bikbov and Chernov (2010), Hsu, Li, and Palomino (2014), Rudebusch and Swanson (2012), Dew-Becker (2014), Bretscher, Hsu, and Tamoni (2017), Weber (2015), Kung (2015), and Campbell, Pflueger, and Viceira (2014)). With respect to these papers, we conduct a structural estimation of a micro-founded model assuming continuity between how assets are priced by the representative agent in the model and by the econometrician.

This paper is organized as follows. Section 2 presents the benchmark model used for the structural estimation. Section 3 illustrates the five risk propagation channels and our solution method using a simplified model. Section 4 contains the main results. Section 5 concludes.

2 Model

We use a dynamic stochastic general equilibrium (DSGE) model along the lines of Christiano, Eichenbaum, and Evans (2005), but with a number of important differences. One of the departures is that representative household has Epstein and Zin (1989) preferences, which is crucial for the asset pricing implications of the model. We allow for a rich set of shocks to show that even when additional disturbances are introduced, uncertainty plays a key role in explaining the bulk of business cycle and term structure fluctuations. Overall the estimated model has seven exogenous shocks to preferences, TFP growth, monetary policy, markups, relative price of investment, government spending, and liquidity. We also allow for two stochastic volatility processes to distinguish between supply-side (TFP) and demand-side (preferences) uncertainty. The volatility processes are modeled as two independent Markov-chains, ξ_t^S and ξ_t^D, with transition matrices H^S and H^D, where the letters, S and D, are used to label the supply- and demand-side shocks, respectively. We then obtain a combined chain, $\xi_t \equiv \left\{\xi_t^D, \xi_t^S\right\}$, with the corresponding transition matrix, $H \equiv H^D \otimes H^S$. A detailed description of the model is presented below.

Household Assume that the representative household has recursive utility over streams of consumption, C_t, and labor, L_t:

$$V_t \quad = \quad \left((1-\beta_t)\, u(C_t, L_t, B_{t+1})^{1-1/\psi} + \beta_t \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right)^{\frac{1}{1-1/\psi}},$$

where γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution.

We introduce habit formation in consumption and preference for liquidity, by specifying the utility kernel in the following form:

$$u(C_t, L_t, B_{t+1}) = (C_t - h\overline{C}_{t-1})e^{-\tau_0 \frac{L_t^{1+\tau}}{1+\tau}} e^{\zeta_{B,t} \frac{B_{t+1}}{R_t P_t Z_t^*}}, \tag{1}$$

where the variable, $\zeta_{B,t}$, shock captures time-variation in the liquidity premium on short-term government bonds. The average liquidity premium is determined by the steady-state value of this variable, ζ_B. The term Z_t^* is the stochastic trend of the economy, B_{t+1} is the amount of nominal one-period bonds held by household at time t, P_t is the nominal price of consumption good.

In the limit, when $\psi \to 1$, the preferences specified above become

$$V_t = u(C_t, L_t, B_{t+1})^{(1-\beta_t)} \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\beta_t}{1-\gamma}} \tag{2}$$

We focus on this unit elasticity case in what follows below.

The discount factor, β_t, is defined as $\beta_t \equiv \left(1 + \hat{\beta} e^{\tilde{b}_t}\right)^{-1}$, where $\tilde{b}_t$ is a preference shock

$$\tilde{b}_{t+1} = \rho_\beta \tilde{b}_t + \sigma_{\beta,\xi_{t+1}^D} \varepsilon_{\beta,t+1}, \;\; \varepsilon_{\beta,t+1} \sim N(0,1) \tag{3}$$

and ξ_t^D is a Markov-switching process with transition matrix, H^D, which determines the volatility regime for the preference shock. The liquidity shock $\widetilde{\zeta}_{B,t} \equiv \log(\zeta_{B,t}/\zeta_B)$ follows an AR(1) process:

$$\widetilde{\zeta}_{B,t+1} = \rho_{\zeta_B} \widetilde{\zeta}_{B,t} + \sigma_{\zeta_B} \varepsilon_{\zeta_B,t+1}, \;\; \varepsilon_{\zeta_B,t+1} \sim N(0,1). \tag{4}$$

The household supplies labor service, L_t, to a competitive labor market at the real wage rate, W_t. They also own the capital stock, $\overline{K}_{t-1}$, predetermined at time $t-1$, and rent out capital services, $K_t = U_t \overline{K}_{t-1}$, to a competitive capital market at the real rental rate, r_t^k, where U_t is capital utilization. Capital is accumulated according to:

$$\overline{K}_t \quad = \quad \overline{K}_{t-1}(1-\delta(U_t)) + \left[1 - S(I_t/I_{t-1})\right] I_t, \tag{5}$$

$$S(I_t/I_{t-1}) \quad = \quad \frac{\varphi_I}{2}\left(I_t/I_{t-1} - e^{\mu^*}\Upsilon\right)^2, \tag{6}$$

$$\delta(U_t) \quad = \quad \delta_0 + \delta_1(U_t - U_{ss}) + \frac{\delta_2}{2}(U_t - U_{ss})^2, \tag{7}$$

where $\delta(U_t)$ is the capital depreciation rate that varies depending on the utilization rate of capital, U_t, I_t is investment, the function $S(I_t/I_{t-1})$ captures capital adjustment costs, μ^* is a steady state growth rate of the economy.[5]

The time t budget constraint of the household is

$$P_tC_t + P_t(e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1}I_t + B_{t+1}/R_t = P_tD_t + P_tW_tL_t + B_t + P_t\overline{K}_{t-1}r_t^kU_t - P_tT_t, \tag{8}$$

where P_t is the nominal price of the consumption good, B_{t+1} is the amount of nominal one-period bonds held by household at time t that mature at $t+1$, R_t is the gross nominal interest rate set at time t by the monetary authority, D_t is the real dividend income received from the intermediate firms, and T_t denotes lump-sum taxes. The parameter, Υ, controls the average rate of decline in the price of the investment good relative to the consumption good, while $\zeta_{\Upsilon,t}$ is a shock to this relative price:

$$\zeta_{\Upsilon,t+1} = \rho_\Upsilon\zeta_{\Upsilon,t} + \sigma_{\zeta_\Upsilon}\varepsilon_{\zeta_\Upsilon,t+1},\ \varepsilon_{\zeta_\Upsilon,t+1} \sim N(0,1). \tag{9}$$

The household's problem and corresponding first-order conditions are contained in Appendix A.

Final Goods A representative firm produces the final (consumption) good in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods, $X_{i,t}$, as input in the following CES production technology

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{1}{1+\lambda_{p,t}}}\,di\right)^{1+\lambda_{p,t}}, \tag{10}$$

where $\lambda_{p,t}$ determines elasticity of substitution between intermediate goods and evolves as:

$$\log\lambda_{p,t} - \log\overline{\lambda_p} = \rho_\chi(\log\lambda_{p,t-1} - \log\overline{\lambda_p}) + \sigma_\chi\varepsilon_{\chi,t},\ \varepsilon_{\chi,t} \sim N(0,1). \tag{11}$$

The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity, $\nu_t = \frac{1+\lambda_{p,t}}{\lambda_{p,t}}$:

$$X_{i,t} = Y_t\left(P_{i,t}/P_t\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}},$$

where P_t is the nominal price of the final good and $P_{i,t}$ is the nominal price of the intermediate good i.

Intermediate Goods The intermediate goods sector is characterized by a continuum of monopolistic competitive firms. Each intermediate goods firm hires labor, $L_{i,t}$, and rents capital, $K_{i,t}$, in competitive markets and produces output, $X_{i,t}$, using a constant returns to scale technology:

$$X_{i,t} = K_{i,t}^\alpha(e^{n_t}L_{it})^{1-\alpha}, \tag{12}$$

[5] In the steady-state, the utilization rate of capital is equal to 1, $U_{ss} = 1$.

where n_t is a stochastic productivity trend with the following law of motion:

$$\Delta n_t = \mu + x_t,$$
$$x_t = \rho_x x_{t-1} + \sigma_{x,\xi_t^S}\varepsilon_{x,t},\ \varepsilon_{x,t} \sim N(0,1),$$

where μ is the unconditional mean of productivity growth, ρ_x is the persistence parameter of the autoregressive process x_t, and the Markov-switching process, ξ_t^S, controls the volatility of shocks to TFP growth. As explained above, this Markov-switching process is controlled by the transition matrix H^S, where we use the letter S to emphasize the supply-side nature of this shock.

The intermediate firms face a cost of adjusting the nominal price a lá Rotemberg (1982), measured in terms of the final good as:

$$G(P_{i,t}, P_{i,t-1}, Y_t) = \frac{\phi_R}{2}\left(\frac{P_{i,t}}{\Pi_{ss}^{\kappa_\pi}\Pi_{t-1}^{1-\kappa_\pi}P_{i,t-1}} - 1\right)^2 Y_t,$$

where $\Pi_{ss} \geqslant 1$ is the steady-state inflation rate, ϕ_R is the magnitude of the price adjustment costs, and the parameter κ_π controls price indexation to past inflation relative to steady-state inflation. The source of funds constraint is:

$$P_t D_{i,t} = P_{i,t}X_{i,t} - P_t W_t L_{i,t} - P_t r_t^k K_{i,t} - P_t G(P_{i,t}, P_{i,t-1}, Y_t),$$

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder value, $V_t^{(i)} = V^{(i)}(\cdot)$, taking the pricing kernel, M_t, competitive real wage, W_t, competitive real rental rate of capital, r_t^k, and vector of aggregate state variables, $\Psi_t \equiv (P_t, e^{n_t}, Y_t)$, as given.

The intermediate firm's problem and corresponding first-order conditions are contained in Appendix A.

Central Bank The central bank follows a modified Taylor rule that depends on output and inflation deviations:

$$\ln\left(\frac{R_t}{R_t^*}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R_t^*}\right) + (1-\rho_r)\left(\rho_\pi \ln\left(\frac{\Pi_t}{\Pi_{ss}e^{\overline{\pi}^*}}\right) + \rho_y \ln\left(\frac{\widehat{Y}_t}{\widehat{Y}_{ss}}\right)\right) + \sigma_R\varepsilon_{R,t},$$

where R_t is the gross nominal short rate, $\widehat{Y}_t \equiv Y_t/Z_t^*$ is detrended output, and $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate. Variables with an ss subscript denote deterministic steady-state values. We allow the inflation target to differ from the deterministic steady-state inflation to take into account that average inflation does not necessarily coincide with the deterministic steady-state when risk is taken into account in the solution method. The correction is controlled by the parameter, $\overline{\pi}^*$.

Symmetric Equilibrium In equilibrium, all intermediate firms make identical decisions $P_{i,t} = P_t$, $X_{i,t} = X_t = Y_t$, $K_{i,t} = K_t$, $L_{i,t} = L_t$, $D_{i,t} = D_t$, and nominal bonds are in zero net supply $B_t = 0$. The aggregate resource constraint is:

$$Y_t = C_t + (e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1} I_t + \frac{\phi_R}{2}\left(\Pi_t / \left(\Pi_{ss}^{\kappa_\pi}\Pi_{t-1}^{1-\kappa_\pi}\right) - 1\right)^2 Y_t + G_t,$$

where G_t are government spending, which follows exogenously specified AR(1) process in logs:

$$\log G_{t+1} - \log G_{ss} = \rho_g(\log G_t - \log G_{ss}) + \sigma_g \varepsilon_{g,t+1}.$$

Government spending is financed by lump-sum taxes on households: $G_t = T_t$.

3 Risk Propagation Channels

To describe the key model mechanisms more lucidly using our approximate analytical solutions, we consider a simplified version of the estimated benchmark model from Section 2. Here, we abstract from the following features: Changes in the price of investment, variable capital utilization, price indexation, habit formation, liquidity premium on short-term bonds and markup shocks. These features are useful to match macroeconomic dynamics in the estimation of the model, but they do not provide additional intuition for disentangling the five risk propagation channels that we want to focus on. The quantitative analysis using the full estimated model is presented in Section 4.

We find that in a New-Keynesian model, uncertainty shocks can be contractionary – even when the precautionary savings channel places upward pressure on investment – due to the presence of four other risk propagation channels, unveiled in our analytical decomposition characterized below. Thus, the overall effect of uncertainty is determined by how uncertainty propagates through the different channels. Analyzing uncertainty changes through the lens of these risk propagation channels helps us to understand (i) the heterogeneous effects of different uncertainty shocks on the macroeconomy, (ii) the role of risk premia for imposing restrictions on the propagation channels, and (iii) how various model frictions pin down the effect of the propagation channels. This approach can be applied to other models and it is therefore of independent interest.

3.1 Equilibrium conditions from the simplified model

We present equilibrium conditions from the simplified model that involve expectations of endogenous variables.

The optimization problem of the household results in the following intertemporal first-order condition:

$$1 = E_t \left[M_{t+1} P_t / P_{t+1} \right] R_t, \tag{13}$$

where

$$M_{t+1} = \frac{1-\beta_{t+1}}{1-\beta_t} \beta_t \left(\frac{V_{t+1}^{1-\gamma}}{E_t \left[V_{t+1}^{1-\gamma} \right]} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \tag{14}$$

is the Stochastic Discount Factor (SDF).

The first-order condition with respect to the investment decision is:

$$q_t \left[1 - \frac{\varphi_I}{2} \left(\frac{I_t}{I_{t-1}} - e^{\mu} \right)^2 - \varphi_I \left(\frac{I_t}{I_{t-1}} - e^{\mu} \right) \frac{I_t}{I_{t-1}} \right] + E_t \left[M_{t+1} q_{t+1} \varphi_I \left(\frac{I_{t+1}}{I_t} - e^{\mu} \right) \frac{I_{t+1}^2}{I_t^2} \right] = 1, \tag{15}$$

$$1 = E_t \left[M_{t+1} R_{t+1}^i \right], \tag{16}$$

where q_t is a shadow value of capital and the return on investment, R_{t+1}^i, is defined as:

$$R_{t+1}^i \equiv \frac{r_{t+1}^k + q_{t+1} \left(1 - \delta_0 \right)}{q_t}. \tag{17}$$

The price setting decision of the intermediate firm yields:

$$\begin{aligned} & (1-\nu) \left(\frac{P_{i,t}}{P_t} \right)^{-\nu} \frac{Y_t}{P_t} + \nu W_t \frac{L_{i,t}}{1-\alpha} \left(\frac{P_{i,t}}{P_t} \right)^{-1} \frac{1}{P_t} \\ & - \phi_R \left(\frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss} P_{i,t-1}} + E_t \left[M_{t+1} \phi_R \left(\frac{P_{i,t+1}}{\Pi_{ss} P_{i,t}} - 1 \right) \frac{Y_{t+1} P_{i,t+1}}{\Pi_{ss} P_{i,t}^2} \right] = 0. \end{aligned} \tag{18}$$

3.2 Log-linearization with risk-adjustment

Our goal is to study the effects of uncertainty on both asset prices and the macroeconomy. If standard log-linearization techniques were applied, all of the effects of uncertainty would be lost. Instead, we implement a risk-adjusted log-linearization of the model (e.g., Jermann (1998), Lettau (2003), Backus, Routledge, and Zin (2010), Uhlig (2010), Kaltenbrunner and Lochstoer (2010), Dew-Becker (2012), Borovička and Hansen (2014), and Malkhozov (2014)). This approximation approach exploits the fact that once the model is log-linearized, the log-variables follow a Normal distribution. This implies that the variables in levels follow a log-normal distribution. Thus, all the expectational equations in the standard log-linear approximation can be risk-adjusted to reflect that the variables are lognormal. In our model, we introduce stochastic volatility, which makes the variables conditionally log-normal. In the appendix, we explain in detail how to apply the method in this case. Then we solve a resulting system of linear expectational difference equations augmented with an iterative procedure designed to capture a risk-adjustment component. This procedure allows us to solve rational expectation models in which uncertainty is controlled by a Markov-switching process by using solution methods that have been developed for log-linear approximations. Worth emphasizing, our procedure allows risk to affect both asset prices *and* the policy functions controlling the macroeconomic variables – the latter of which is crucial to study the effects of uncertainty on the macroeconomy. Appendix B contains more details about our log-linearization and risk-adjustment approach.

We apply the risk-adjusted log-linearization to the first-order conditions and market clearing conditions presented above. Define the risk-free rate, $R_{f,t}$, as the return on a theoretical risk-free asset, which pays one unit of consumption good in every state of the world next period. The risk-free rate satisfies the following asset pricing equation:

$$1 = E_t\Big[M_{t+1}R_{f,t}\Big]. \tag{19}$$

As described above, the log-linearization approach that we are using approximates all expectational equations assuming that the variables are conditionally log-normal. Log-linearizing Eq. (19), we get

$$-\widetilde{r}_{f,t} = E_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big], \tag{20}$$

where variables with a tilde denote log-deviations from the deterministic steady state.[6] A log-linear approximation of the expression for the stochastic discount factor (Eq. (14)) using our risk adjustment approach yields:[7]

$$\widetilde{m}_{t+1} = \begin{bmatrix} \overline{\beta}\widetilde{b}_{t+1} - \widetilde{b}_t + (1-\gamma)(\widetilde{v}_{t+1} - E_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}]) - (\widetilde{c}_{t+1} - \widetilde{c}_t) \\ -\gamma\widetilde{x}_{t+1} - \frac{1}{2}(1-\gamma)^2 Var_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}] \end{bmatrix}. \tag{21}$$

Substituting this log-linear expression for stochastic discount factor in Eq. (20), we obtain:

$$\tilde{c}_t = E_t\Big[\tilde{c}_{t+1}\Big] - \widetilde{r}_{f,t} + (1-\overline{\beta}\rho_\beta)\tilde{b}_t + \rho_x\widetilde{x}_t \underbrace{-\frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}(1-\gamma)^2 Var_t\Big[\tilde{v}_{t+1} + \widetilde{x}_{t+1}\Big]}_{\text{Precautionary savings motive}}, \tag{22}$$

which is an Euler equation with respect to the risk-free rate. The risk adjustment component, $-\frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}(1-\gamma)^2 Var_t\Big[\tilde{v}_{t+1} + \widetilde{x}_{t+1}\Big]$, captures the precautionary savings motive. This term reflects the prudence of the household towards uncertainty about future income. Formally, the precautionary savings term relates to the convexity of marginal utility (e.g., Kimball (1990)).

Log-linearizing and risk-adjusting the intertemporal first-order condition of the household (Eq. (13)) and combining it with the expression for the log risk-free rate (Eq. (20)), we get:

$$\widetilde{r}_t = \widetilde{r}_{f,t} + E_t\Big[\widetilde{\pi}_{t+1}\Big] + \underbrace{Cov_t\Big[\widetilde{m}_{t+1};\widetilde{\pi}_{t+1}\Big] - \frac{1}{2}Var_t\Big[\widetilde{\pi}_{t+1}\Big]}_{\text{Inflation Risk Premium}}, \tag{23}$$

[6] For capital, $\widetilde{k}_t = \log K_t - \log K_{ss}$

[7] $E_t\Big[e^{(\tilde{v}_{t+1}+\tilde{x}_{t+1})(1-\gamma)}\Big]$ is approximated as $\exp\Big((1-\gamma)E_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}] + \frac{(1-\gamma)^2}{2}Var_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}]\Big)$

where $\widetilde{r}_t$ is the nominal short-term interest rate. The risk adjustment term, $Cov_t\Big[\widetilde{m}_{t+1};\widetilde{\pi}_{t+1}\Big]-\frac{1}{2}Var_t\Big[\widetilde{\pi}_{t+1}\Big]$, corresponds to an inflation risk premium, and it reflects the fact that the payoff of a nominal short-term bond in real terms is uncertain. The rate of return on this bond in consumption units depends on the realization of inflation next period. Therefore, the covariance of inflation with the real pricing kernel determines the inflation risk premium on the short-term nominal bond. If inflation tends to be high when the marginal utility of wealth is high, then nominal short-term bonds are risky and investors demand a risk premium for holding them.

We log-linearize and risk-adjust the equation characterizing the investment decision of the household, Eq. (16), and use Eq. (20) to obtain:

$$E_t[\widetilde{r}_{i,t+1}-\widetilde{r}_{f,t}] = \underbrace{-Cov_t[\widetilde{m}_{t+1};\widetilde{r}_{i,t+1}]-\frac{1}{2}Var_t[\widetilde{r}_{i,t+1}]}_{\text{Investment Risk Premium}}. \tag{24}$$

The risk adjustment component in brackets embodies an investment risk premium. If the return on investment is low when the marginal utility of wealth is high, then the return on investment in physical capital is risky and will command a risk premium. Therefore, in equilibrium, households will choose a level of investment such that the expected investment return will be higher than the risk-free rate by an amount sufficient to compensate them for the risk that they are exposed to.

The expression for $\widetilde{q}_t$ is obtained by log-linearizing Eq. (15):

$$\widetilde{q}_t-\varphi_I e^{2\mu}\Delta i_t+\varphi_I e^{2\mu}\overline{\beta}\Big(E_t\Big[\Delta i_{t+1}\Big]+\underbrace{Cov_t\Big[\widetilde{m}_{t+1}+\widetilde{q}_{t+1};\Delta i_{t+1}\Big]+\frac{5}{2}Var_t\Big[\Delta i_{t+1}\Big]}_{\text{Investment adjustment}}\Big)=0, \tag{25}$$

where $\Delta i_{t+1}=\widetilde{i}_{t+1}-\widetilde{i}_t+x_{t+1}$ is log investment growth. The risk adjustment term in this equation captures the fact that when making an investment decision at time t, households consider its impact on the capital adjustment costs at time $t+1$, which depends on investment growth Δi_{t+1}. Therefore, the household takes into account uncertainty about future investment growth and how it co-varies with the shadow value of capital and the pricing kernel.

We apply the same risk-adjustment technique to log-linearize the equation characterizing the price setting decision of the intermediate firms (Eq. (18)) to obtain the risk-adjusted Phillips Curve:

$$\widetilde{\pi}_t=\overline{\beta}E_t\left[\widetilde{\pi}_{t+1}\right]+\kappa_R(\widetilde{w}_t+\widetilde{l}_t-\widetilde{y}_t)+\underbrace{\frac{1}{2}\overline{\beta}^*\Big(2Cov_t\Big[\widetilde{m}_{t+1}+\widetilde{y}_{t+1}+\widetilde{x}_{t+1};\widetilde{\pi}_{t+1}\Big]+3Var_t\Big[\widetilde{\pi}_{t+1}\Big]\Big)}_{\text{Nominal Pricing Bias}}, \tag{26}$$

where the risk-adjustment component represents the nominal pricing bias and $\kappa_R=\frac{\nu-1}{\phi_R}$. The variance term captures a precautionary price setting motive due to the presence of the price adjustment costs. The covariance term between inflation and the pricing kernel relates to the inflation risk premium introduced above. In addition, the nominal pricing bias also depends on covariance terms between both output and TFP with inflation.

The rest of the equations, which are needed to close the system, do not have terms which depend on expectations of the endogenous variables. As a result, a simple log-linearization suffices and no additional risk adjustment terms are needed.

To summarize, based on the risk-adjusted log-linearization of the model above, we identify five risk propagation channels through which uncertainty affects the economy: A *precautionary savings motive channel* represented by the risk adjustment terms in the Eq. (22); an *inflation risk premium channel* represented by the risk-adjustment terms in the equation for short-term nominal interest rate (Eq. (23)); an *investment risk premium channel* captured by the risk adjustment terms in the intertemporal investment decision (Eq. (24)); a *nominal pricing bias channel* represented by the risk-adjustment terms in the Phillips Curve (Eq. (26)); a *investment adjustment channel* captured by the risk adjustment terms in Eq. (25).

3.3 Solution Method

The key step for implementing our solution method is realizing that in a model in which stochastic volatility is modeled as a Markov-switching process, uncertainty at time t only depends on the regime in place at time t, denoted by ξ_t. Thus, the system of equations presented above can be written by using matrix notation as in a standard log-linearization:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Gamma_\sigma Q_{\xi_t} \varepsilon_t + \Gamma_\eta \eta_t + \Gamma_{c,\xi_t}, \tag{27}$$

where the DSGE state vector S_t contains all variables of the model known at time t, Q_{ξ_t} is a regime-dependent diagonal matrix with all of the standard deviations of the shocks on the main diagonal, ε_t is a vector with all structural shocks, η_t is a vector containing the expectation errors, and the Markov-switching constant Γ_{ξ_t} captures the effects of uncertainty:

$$\Gamma_{c,\xi_t} = \begin{pmatrix} a_1 Cov_t[c_1' S_{t+1}; d_1' S_{t+1}] \\ a_2 Cov_t\left[c_2' S_{t+1}; d_2' S_{t+1}\right] \\ \vdots \end{pmatrix}, \tag{28}$$

where we have used the fact that uncertainty at time t only depends on the regime in place at time t, denoted by ξ_t. Elements of Γ_{c,ξ_t} represent risk adjustment terms, c_i and d_i are vectors of coefficients, and a_i are constants implied by our risk adjustment technique.

However, we cannot compute the volatility terms in Γ_{c,ξ_t} without knowing the solution for S_t. This is because to compute the one-step-ahead variance and covariance terms, we need to know how the economy reacts to the exogenous shocks, ε_t, and to the regime changes themselves. Therefore, we employ the following iterative procedure. First, given some $\Gamma_{c,\xi_t} = \widetilde{\Gamma}_{c,\xi_t}$, the solution to Eq. (27) can be characterized as a Markov Switching Vector Autoregression (Hamilton (1989), Sims and Zha (2006)):

$$S_t = T\left(\theta^p\right) S_{t-1} + R\left(\theta^p\right) Q\left(\xi_t, \theta^v\right) \varepsilon_t + C\left(\xi_t, \theta^v, \theta^p, H\right), \tag{29}$$

where θ^p is the vector structural parameters, θ^v is the vector containing the stochastic volatilities, H is the probability transition matrix, and $Q_{xi_t} \equiv Q\left(\xi_t, \theta^v\right)$. Taking (29) as given, we can now compute the implied level of uncertainty (i.e., the implied $\widetilde{\Gamma}_{c,\xi_t}$). In particular,

$$\begin{aligned} Cov_t\left[c_1' S_{t+1}; d_1' S_{t+1}\right] &= E_t\left\{Cov_t\left[c_1' S_{t+1}; d_1' S_{t+1} | \xi_{t+1}\right]\right\} + Cov_t\left\{E_t\left[c_1' S_{t+1} | \xi_{t+1}\right]; E_t\left[d_1' S_{t+1} | \xi_{t+1}\right]\right\} \\ &= c_1' E_t\left[R Q_{\xi_{t+1}} (R Q_{\xi_{t+1}})'\right] d_1 + c_1' Var_t\left[C_{\xi_{t+1}}\right] d_1, \end{aligned} \tag{30}$$

where we used the law of total covariance: $Cov(X,Y) = E(Cov(X,Y|Z)) + Cov(E(X|Z), E(Y|Z))$. Note that the changes in the Markov-switching constant, induced by the risk adjustment, are themselves a source of uncertainty. Given the new value for $\widetilde{\Gamma}_{c,\xi_t}$, we repeat the iteration: First, compute a new solution to (27), and then update $\widetilde{\Gamma}_{c,\xi_t}$. This iterative procedure continues until the desired level of accuracy is reached. It is worth emphasizing that only C_{ξ_t} depends on Γ_{c,ξ_t}, while the matrices, T and R, do not depend on it, so we only need to iterate on C_{ξ_t}. Furthermore, standard conditions for the existence and uniqueness of a stationary solution apply, given that regime changes enter the model additively. Thus, we know that a finite level of uncertainty exists, as long as a solution exists and the shocks are stationary.

In the solution (Eq. 29), the matrices, T and R, are equivalent to a standard log-linear solution. Therefore, conditional on the volatility regime, the dynamics of the model are the same as in a standard log-linear solution. Volatility matters in two ways. First, like in log-linearized models, volatility affects the size of the innovations, captured by Q_{ξ_t}. Second, volatility affects the level of uncertainty in endogenous variables. Changes in uncertainty, in turn, impact the risk adjustment term, C_{ξ_t}, which is not present in a standard log-linear approximation. This term reflects the endogenous response of the economy to uncertainty and it is a source of uncertainty itself. Overall, the risk adjustment term adjusts the levels of the variables, determines model dynamics in response to a volatility regime change, and produces additional uncertainty.

Importantly, the Markov-switching constant, $C_{\xi_t} = C\left(\xi_t, \theta^v, \theta^p, H\right)$ depends on the structural parameters, because for a given volatility of the exogenous disturbances, different structural parameters determine the various levels of uncertainty. In a standard log-linearization, this term would always be zero. As shown below, this approach allows us to capture salient asset pricing features despite having approximated a model with a conditionally linear solution. Furthermore, given that agents are aware of the possibility of regime changes, uncertainty also depends on the transition matrix, H. Finally, given that regime changes enter the system of equations additively, the conditions for the existence and uniqueness of a solution are not affected by the presence of regime changes. The model can then be solved by using solution algorithms developed for fixed coefficient general equilibrium models (Blanchard and Kahn (1980) and Sims (2002)). The model can also be solved by using the solution algorithms explicitly developed for MS-DSGE models (e.g., Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), Cho (2016), and Foerster, Rubio-Ramírez, Waggoner, and Zha (2016)), but these methods are more computationally expensive. Appendix C shows that our risk-adjusted log-linearization provides an accurate approximation of the model solution.

3.4 Nominal Bond Yields

This section characterizes how bond yields are determined. Let $P_t^{(n)}$ be the n-period nominal bond price at time t. This bond price satisfies the following asset pricing Euler equation:

$$P_t^{(n)} = E_t\Big[M_{t+1}P_{t+1}^{(n-1)}/\Pi_{t+1}\Big]. \tag{31}$$

Applying the same log-linearization and risk-adjustment technique described above, we get

$$\widetilde{p}_t^{(n)} = E_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big] + .5Var_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big]. \tag{32}$$

Using this equation, we solve for nominal bond prices iteratively, starting from $n = 2$. Note that the gross short-term nominal interest rate is an inverse of the price of a one-period nominal bond, $R_t = 1/P_t^{(1)}$, and therefore, $\widetilde{p}_t^{(1)} = -\widetilde{r}_t$. Given Eq. (29), the solution to Eq. (32) is given by:

$$\widetilde{p}_t^{(n)} = T_p S_{t-1} + R_p Q_{\xi_t}\varepsilon_t + C_{p,\xi_t}. \tag{33}$$

Having solved for $\widetilde{p}_t^{(n-1)}$ and knowing the solution of the model (29), we can compute $Var_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big]$ in a way similar to Eq. (30) to get the solution for $\widetilde{p}_t^{(n)}$. Given a price of the n-period nominal bond $P_t^{(n)} = P_{ss}^{(n)}e^{\widetilde{p}_t^{(n)}}$, the yield on this bond is given by:

$$y_t^{(n)} = -\frac{1}{n}\log P_t^{(n)},$$

where $P_{ss}^{(n)}$ is the price of the n-period nominal bond in the deterministic steady state. Importantly, the pricing of bonds is internally consistent, in the sense that the econometrician and the agent in the model price bonds in the same way.

4 Empirical Analysis

We estimate the model by using Bayesian methods using the sample period 1984:Q2-2015:Q4. The model solution retains the key non-linearity represented by regime changes, but it is linear conditional on a regime sequence. Thus, Bayesian inference can be conducted using Kim's modification of the basic Kalman filter to compute the likelihood (i.e., Kim and Nelson (1999)). In addition to the priors on the single model parameters, we also have priors on the unconditional means of inflation, the real interest rate, the slope of the nominal yield curve, and the investment risk premium. Unlike in a linear model, the unconditional means of these variables are not pinned down by a single parameter. Thus, these priors induce a joint prior on the parameters of the model, in a way similar to Del Negro and Schorfheide (2008). The priors for the model parameters are combined with the likelihood to obtain the posterior distribution.

Eleven observables are used: GDP per-capita growth, inflation, FFR, consumption growth, investment growth, price of investment growth, one-year yield, two-year yield, three-year yield, four-year yield, and five-year yield (all variables are annualized). Given that there are more observables than shocks (i.e., eleven variables compared to seven shocks), we allow for observation errors on all variables, except for the FFR. We also repeated our estimation excluding the zero-lower-bound period, with no significant changes in the results. Finally, alternative versions of the model are estimated, such as, a specification in which both volatility processes are perfectly correlated and another specification where all shocks exhibit stochastic volatility that are perfectly correlated, but these versions did not lead to a better fit of the data. As it will become clear below, the data ostensibly favors a separation between supply- and demand- side uncertainty shocks.

4.1 Parameter Estimates and Model Fit

Table 1 reports the posterior mean for the structural parameters together with the 90% error bands and the priors. A few comments are in order. First, we fix the elasticity of intertemporal substitution to 1. Second, the parameters controlling the magnitude of the price adjustment cost, ϕ_R, and the average markup, ν, cannot be separately identified. Thus, when solving the model, we define and estimate the parameter, $\kappa_R = \frac{\nu-1}{\phi_R}$, while we fix the parameter, ν.[8] The resulting estimated value for κ_R implies an elevated level of price stickiness, in line with the existing New Keynesian literature. Third, in accordance with previous results in the literature, we find a more than one-to-one response of the FFR to inflation, despite the long time spent at the zero lower bound. The fact that the response is well above 1 guarantees that the Taylor principle is satisfied.

Table 1 reports estimates for the volatilities of the shocks and the persistence of the two regimes. Figure 3 reports the probability of the High volatility regimes (Regime 2 for each chain) for the preference shock (top panel) and the TFP shock (bottom panel). The high volatility regime for the preference shock is less persistent than the low volatility regime, while the opposite is true for the high TFP volatility regime.

Figure 4 compares the variables as implied by our model with the observed variables. The figure shows

[8]The average markup (ν) affects the steady state of the model. For the purpose of computing the steady state we fix this parameter to 6, a value that implies an average net markup of 20% and that is considered in the ballpark (see Gali (1999)).

that the model does a very good job in matching the behavior of both the macro variables and the term structure. We observe some visible deviations between model-implied and observed variables only for the growth rate of the price of investment. Thus, observation errors do not play a key role in matching the observed path for yields and macro variables. The last panel of the figure also shows that the model tracks the behavior of the slope of the yield curve quite well, defined as the difference between the one-year and five-year yields. As we will see below, variations of the term premium over the business cycle play a key role in generating such a close fit.

4.2 The Effects of Uncertainty

Given that the model allows for two TFP volatility regimes and two preferences volatility regimes, there are a total of four regimes labeled as follows: (i) Low Preference - Low TFP volatility; (ii) Low Preference - High TFP volatility; (iii) High Preference - Low TFP volatility; and (iv) High Preference - High TFP volatility. We are interested in characterizing the level of uncertainty across the four regimes. Uncertainty is computed taking into account the possibility of regime changes, following the methods developed in Bianchi (2016). For each variable, z_t, we measure uncertainty by computing the conditional standard deviation, $sd_t(z_{t+s}) = \sqrt{\mathbb{V}_t(z_{t+s})} = \sqrt{\mathbb{E}_t[z_{t+s} - \mathbb{E}_t(z_{t+s})]^2}$, where $\mathbb{E}_t(\cdot) \equiv \mathbb{E}(\cdot|\mathbb{I}_t)$ and $\mathbb{I}_t$ denotes the information available at time t. We assume that $\mathbb{I}_t$ includes knowledge of the regime in place at time t, the data up to time t, and the model parameters for each regime, while future regime realizations are unknown. These assumptions are consistent with the information set available to agents in our model, and so our measure of uncertainty reflects uncertainty supposedly faced by the agent in the model across the four regimes.

Overall macroeconomic uncertainty is influenced through two general effects. The first one is direct: As the size of the Gaussian shocks hitting the economy increases, uncertainty goes up. The second one is more subtle: The endogenous response of the macroeconomy to uncertainty – through the five risk propagation channels – is in itself a source of uncertainty. Thus, the magnitude of the response to uncertainty and the frequency of regime changes matter for the overall level of uncertainty. The relative contribution of these two sources of uncertainty are described in detail below.

Uncertainty and business cycles. Figure 5 reports the levels of uncertainty across the different regimes. The time horizon s appears on the x-axis. Solid and dashed lines are used to denote low and high preference shock volatility regimes, respectively. Conditional on these line styles, we use lines with dots and without dots to denote low and high TFP shock volatility, respectively. When both demand-side and supply-side volatilities are high (dashed-line with dots), uncertainty is high for all variables at all horizons. When only one of the shocks is in the high volatility regime, the effects differ across the variables. For inflation, the FFR, and the slope of the yield curve, the main driver of uncertainty is the volatility of the preference shock. Instead, uncertainty about the growth rate of the real variables is higher when TFP is in the high volatility regime. It is also interesting to notice that uncertainty for consumption and GDP is slightly hump-shaped when the high TFP volatility regime prevails. In other words, when TFP volatility is high, uncertainty is not monotonically increasing with respect to the time horizon, as agents are more uncertain about the short-run than the long-run. This is because of two competing forces. On the one hand, events that are further into the future are naturally harder to predict, as the possibility of shocks and regime changes increase. On the other hand, in the long run, the probability of still being in the high volatility regime declines.

Figure 6 presents a simulation to understand the impact of these changes in uncertainty on business cycle fluctuations and the term structure. We take the most likely regime sequence, as presented in Figure 3, and simulate the economy based on the parameters at the posterior mode, setting all Gaussian shocks to zero. The top left panel reports the cyclical behavior of GDP and the slope of the yield curve implied by the model. An increase in uncertainty produces a drop in real activity and an increase in the slope of the yield curve, which consequently generates negative comovement between the slope of the yield curve and real activity, as in the data (e.g., Ang, Piazzesi, and Wei (2006)). The four panels in the second and third row of the figure compare the movements in the slope, GDP, consumption, and investment, induced by the increase in uncertainty, with the business cycle fluctuations of the actual series. The estimated sequence of the volatility regimes produces business cycle fluctuations and changes in the slope of the yield curve in a way that closely tracks the observed fluctuations in the data.

The fluctuations in uncertainty also lead to significant breaks in the term premium. Term premium is defined as the difference between the yield on a 5-year bond and the expected average short-term yield (1 quarter) over the same five years (following Rudebusch, Sack, and Swanson (2006)). The expected value is computed taking into account the possibility of regime changes using the methods developed in Bianchi (2016). The top-right panel of Figure 6 shows that both supply-side and demand-side uncertainty lead to an increase in the term premium. Specifically, Table 5 shows that the nominal (real) term premia associated with the different regimes are: (1) Low Preference - Low TFP volatility: 0.58% (0.33%); (2) Low Preference - High TFP volatility: 0.84% (0.60%); (3) High Preference - Low TFP volatility: 1.03% (0.51%); and (4) High Preference - High TFP volatility: 1.29% (0.78%). In Subsection 4.4, the mechanisms that lead to these sizable premia are explored in detail. For now, we are highlighting that term premia are large and vary considerably in response to changes in uncertainty.

Variance decomposition. Our estimated model allows for a rich set of shocks to avoid forcing the estimation to artificially attribute a large role to the uncertainty shocks. The results presented above suggest that uncertainty shocks can in fact lead to sizable fluctuations for both the macroeconomy and bond risk premia. In order to formally quantify the importance of uncertainty shocks with respect to the other disturbances, we proceed in two steps. First, we compute a variance decomposition by comparing the unconditional variance, as implied by the model when only one shock is active, to the overall variance. Second, we explore how much variation in endogenous variables at business cycle frequencies can be generated by uncertainty shocks. We do this by computing the volatility of business cycle fluctuations in an economy where only uncertainty shocks are present and comparing it to the volatility of business cycle fluctuations in an economy where both uncertainty and level shocks are active.[9]

The decomposition of the unconditional variance for the observables is reported in the left panel of Table 2. The results confirm that uncertainty shocks play an important role in explaining fluctuations in the slope of the yield curve (28% of the unconditional variance), but they also account for a large fraction of the variability of consumption and investment growth (14.26% and 9.67%, respectively). The right panel of Table 2 highlights that uncertainty shocks appear even more important if we focus on their ability to generate sizable business cycle fluctuations. Uncertainty shocks explain a substantial part of the variation in consumption, investment, and output over the business cycle. In particular 24.52% of the variation in consumption and around 31% of the variation in investment at business cycle frequencies can be explained by uncertainty shocks. Finally, uncertainty shocks also explain 38.44% of business cycle variation in the slope of the yield curve, confirming the evidence presented in Figure 6.

Finally, the variance decomposition in the left panel of Table 2 shows that the combination of TFP shocks, preference shocks, and their corresponding volatility shocks accounts for a very large fraction of the volatility of the macroeconomy and bond yields. Specifically, these shocks combined account for more than 80% of the variance of GDP growth, for more than 90% of the variance of consumption growth, for more than 85% of the variance of investment growth, and for almost 60% of the variance of inflation and the slope of the yield curve. The only other shock that plays a significant role is the markup shock. However, this shock only appears to account for high-frequency movements in the volatility of inflation, as typical in estimated New-Keynesian models. Thus, the combination of first and second moments shocks to TFP and preferences account for the bulk of the volatility of the observed variables, despite the fact that we allow for a series of other shocks, like the liquidity shock, that generally play a significant role in the estimation of New-Keynesian DSGE models without the risk-adjustment. This suggests that extending standard estimation technique to include the first-order effects of uncertainty shocks can significantly change the importance of the other shocks, possibly allowing for more parsimonious models to explain the observed fluctuations.

What drives the large effects of uncertainty? From a methodological point of view, uncertainty matters in our setting because we are estimating the sources and effects of uncertainty jointly, instead of using a two-step procedure. Thus, uncertainty is not exclusively identified by movements in second moments, but also through its first-order effects on risk premia and business cycle fluctuations. More practically, there are a series of parameters that play an important role. To make this point, Table 3 reports the variance decomposition at business cycle frequencies for different levels of risk aversion (increasing in γ) and nominal rigidities (decreasing in κ). Low levels of risk aversion (low γ) imply a large reduction in the importance for uncertainty shocks. Similarly, more flexible prices also reduce the importance of uncertainty shocks. Of

[9] From a technical point of view the contribution of uncertainty shocks is given by the amount of volatility generated by the Markov-switching constant.

course, all the parameters matter to pin down the importance of uncertainty shocks, but these two channels appear to be particularly relevant. This also highlights an important difference with respect to previous work such as Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2015). Using Epstein-Zin preferences allow us to separate risk aversion from the intertemporal elasticity of substitution, while they use log utility which implies a risk aversion of one. Note that when we fix the coefficient controlling risk aversion to 1 (i.e., the log utility case given that the intertemporal elasticity of substitution is also set to 1), we get very small effects of uncertainty.

4.3 Inspecting the Mechanism

To better understand the mechanisms at work, we decompose the effects of the uncertainty shocks into the five risk propagation channels that were discussed in the context of the simplified model from Section 3.2: Precautionary savings, investment risk premium, inflation risk premium, nominal pricing bias, and investment adjustment. The results here show that the origins of uncertainty are important to understand both the qualitative and quantitative effects.

Figure 7 presents the median and 90% error bands for the impulse responses to a demand-side (dashed line) and a supply-side (solid line) uncertainty shock, while Figure 8 presents the median and 90% error bands for the *difference* between these impulse responses. Impulse responses are computed as the change in the expected path of the endogenous variables following an initial impulse, in line with the way impulse responses are computed for shocks to levels. Specifically, these impulse responses assume a shift from low to high uncertainty in the first period, but from that point on they are computed integrating out future regime changes. Thus, the impulse responses are conceptually different from the simulations reported in Figure 6 where the posterior mode regime sequence was imposed.

Despite these technical differences that take into account uncertainty about the future regime path, uncertainty shocks still emerge as a driving force of business cycle fluctuations. Both demand- and supply-side uncertainty shocks generate positive comovement between consumption, investment, output, as there is an economic contraction following heightened macroeconomic uncertainty. Also, higher uncertainty increases the nominal and real term premia, consistent with the observed dynamics in the data. However, a supply-side uncertainty shock leads to a much larger decline in inflation. Furthermore, the recession generated by a supply-side uncertainty shock is visibly larger, as confirmed by the first row of Figure 7. The effects on term premia are also quantitatively different with the supply-side uncertainty shock generating a smaller increase in the nominal term premium and a larger increase in the real term premium. Figure 9 decomposes the effects of demand- (Panel a) and supply-side (Panel b) uncertainty through our risk propagation channels. The effects of the individual propagation channels on each variable differ depending on the origin of uncertainty.

The dashed line shows the contribution of the precautionary savings channel of uncertainty. With higher supply or demand uncertainty, the precautionary savings channel increases the desire for saving. This effect is reflected in the variance of marginal utility growth, given by Eq. (22). Note that the precautionary savings channel generates positive comovement between consumption, investment, and output. The reason is that the estimated model has a sufficiently high degree of price stickiness for higher uncertainty to generate a large enough downward shift in labor demand that translates to a fall in investment, labor hours, and output. This is the mechanism that Basu and Bundick (2017) uses to produce positive comovement between macroeconomic aggregates. However, while this channel plays a key role in driving consumption down following an uncertainty shock, other channels play an equally important role to understand the effects of uncertainty on the other macroeconomic variables.

A line with circle markers on the Figure 9 shows the contribution of the investment risk premium channel. We find that when the economy experiences a supply-side uncertainty shock, the investment risk premium channel is equally (and at certain horizons more) important than the precautionary savings channel in determining a decline in investment. On the other hand, when the economy experiences a demand-side uncertainty shock, the risk propagation channel works in the opposite direction and mitigates the decline in investment. Thus, demand and supply uncertainty propagate differently through the investment risk premium channel. The direction of the investment risk premium channel depends mainly on the covariance between the return on investment and the pricing kernel (see Eq. (24)). This covariance is determined by the response to the level shocks, and the impulse responses are depicted in the Figure 10. The difference between the supply- and demand-side uncertainty is determined by how the shadow value of capital responds to adverse demand and supply shocks. For the household, capital works as a hedge against adverse preference shocks, because the return on investment is positive in a state of the world with high marginal utility of wealth (high SDF). The opposite is true for a negative TFP shock, as the return on investment is negative in the high SDF state. So, when supply side uncertainty increases, the effect of the investment risk premium channel is driven by investment becoming riskier and households, *keeping all else equal*, optimally choosing to cut investment. In contrast, when demand uncertainty increases, investment becomes less risky and the effect of this channel is determined by the household choosing a relatively higher level of investment than it would choose if the investment risk premium remained constant. Importantly, the net effect of demand-side uncertainty on investment is still negative because of the combined effect of the precautionary savings and nominal pricing bias channels.

Another important channel that affects the response of investment to higher supply-side uncertainty is the investment adjustment channel (dotted line with diamond markers on Figure 9). The investment adjustment channel depends on the volatility of future investment growth, and how it comoves with the real stochastic discount factor and marginal q (see Eq. (25)).

The contribution of the nominal pricing bias channel of uncertainty is illustrated in Figure 9 by a dotted line. When the economy experiences a demand-side uncertainty shock, the nominal pricing bias channel contributes to the decline in consumption and investment. In response to an increase in demand-side uncertainty, the nominal pricing bias determines effects similar to a markup shock, given that it enters the New-Keynesian Phillips curve in an isomorphic way (see Eq. (26)). Inflation goes up while consumption and investment go down. This contributes to a deepening of the recession, while on the other hand, mitigating the effects on inflation.

In general, the magnitude and direction of the nominal pricing bias channel depends on (i) the variance of inflation and (ii) the covariance between inflation and the real stochastic discount factor, output, and TFP growth (see Eq. (26)). The variance term in Eq. (26) relates to a precautionary price setting effect highlighted in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) that creates a desire for firms to increase prices more when uncertainty is higher in the presence of nominal rigidities.

Our decomposition also helps in understanding why the response of inflation is so muted with respect to both demand and supply uncertainty shocks. In both cases, the precautionary savings channel generates deflationary pressure. However, following a demand-side uncertainty shock the pricing bias channel essentially nullifies the effects on inflation, while in the case of the supply-side uncertainty shock this channel plays a very little role. To understand why, it is useful to revisit the impulse responses presented in Figure 10. Inflation experiences a persistent increase in response to a demand shock, while a supply shock has very little quantitative impact on inflation dynamics. As a result, the nominal pricing bias channel is not quantitatively important for supply side uncertainty: Firms do not adjust their price setting decision, as supply side uncertainty has limited impact on uncertainty about future inflation. In contrast, the preference shock is an important driver of inflation dynamics, and demand-side uncertainty directly translates into uncertainty about future inflation. Hence, the nominal pricing bias is an important determinant of the economy's response to demand-side uncertainty.

Finally, we find that the inflation risk premium channel (a solid line with cross markers on the Figure 9) has small quantitative effects. Both the nominal pricing bias and the inflation risk premium channels depend on the covariance between the real pricing kernel and inflation, and are therefore tightly linked to nominal term premia. Hence, in the estimation we discipline these channels using asset pricing data, namely, nominal bond yields across different maturities. As we show below, in the estimated model, both supply-side and demand-side uncertainty contribute positively to term premia, albeit through two very different mechanisms.

In addition, the equity premium is closely tied to the investment risk premium in the model. Consequently, in our structural estimation we discipline the investment risk premium channel by requiring investment risk premium to be positive on average, and we verify that it increases with an increase in investment risk.

4.4 Yield Curve

In this section, we provide more details about the fit of the model and the mechanisms at play by inspecting the ability of the model to match movements in the term structure. We already conducted a first check on the fit of the model in Figure 4 showing that the yields across various maturities, as implied by our model, track very closely observed yields despite allowing for observation errors on all variables, except for the FFR. In what follows, we analyze how the model is able to match the dynamics of yields and the slope so closely.

Table 4 reports the nominal and real yield curve as implied by our estimated model. Our model generates both an upward-sloping real and nominal yield curve with sizable average term spreads. Both preference and TFP shocks are important for generating the unconditional real term premium, while the preference shock is important for generating the unconditional inflation risk premium. It is worth emphasizing that in our estimated model, the endogenous risk premium is significantly more important than the liquidity premium in generating nominal term premia and the only determinant of *real* term premia. The liquidity premium is the premium arising from a linear term that captures the preference of the household for short-term bonds, and it is controlled by the parameter, ζ_B. Thus, liquidity shocks seem to play only a small role for explaining business cycle fluctuations; and moreover, the liquidity premium seems less important in determining term premia compared to the risk-based channels.

The right side of Table 4 shows that the overall nominal term premium is 0.96%, generating an unconditional slope of the term structure very much in line with the data (1.05%). The risk premium accounts for the bulk of the term premium: 0.90% Vs. 0.06%. The real term premium is 0.56% and it is all due to the risk premium arising from the preference and TFP shocks. To understand the relative importance of demand-side versus supply-side uncertainty in generating the premia, we consider a counterfactual simulation in which the standard deviations of all shocks are set to zero, except for preference (TFP) shocks. When only preference (TFP) shocks are allowed, the nominal term premium is 0.77% (0.29%), while with only preference (TFP) shocks, the real term premium is 0.26% (0.34%). These results show that demand-side uncertainty is relatively more important in determining the nominal term premium, while the two sources of uncertainty contribute to the real term premia of a similar magnitude. Next, we study how the two sources of uncertainty lead to sizable risk premia.

Persistent shocks to time discount rates coupled with recursive preferences contributes significantly to both the real term premia and inflation risk premia. To understand the mechanism behind this finding, Figure 11 presents the impulse responses to such a shock for key variables. A negative preference shock (less patience) induces household to consume more and save less, which decreases the wealth-to-consumption ratio. A drop in the wealth-to-consumption ratio implies a decline in the return on a claim to aggregate consumption. When agents prefer an early resolution of uncertainty ($\psi > 1/\gamma$), a decrease in the return on the consumption claim increases marginal utility. When the shock is persistent, this leads to a sharp increase in marginal utility. A persistent negative time preference shock also increases the real rate persistently, which erodes the payoffs of long real bonds more than short ones. Given that a negative time preference shock is associated with high marginal utility, long real bonds provide less insurance against bad states of the world relative to short real bonds. In equilibrium, this contributes to an upward-sloping real yield curve and a positive real term premia, in a way similar to Albuquerque, Eichenbaum, Luo, and Rebelo (2016).

The time preference shock endogenously generates a negative relation between marginal utility and inflation, which translates into positive inflation risk premia increasing with maturity. A persistent negative time preference shock increases aggregate demand, which raises inflation persistently. The negative time preference shock is also associated with high marginal utility as discussed above. Persistently higher inflation erodes the value of long nominal bonds more than short nominal bonds during high marginal utility states. Consequently, the nominal yield curve is upward-sloping.

The persistent TFP growth shocks in conjunction with habits contributes positively to real term premia. A negative TFP growth shock decreases consumption today relative to habit (proportional to lagged consumption), decreasing surplus consumption (i.e., the difference between consumption and habits), and raising marginal utility. However, next period, the habit catches up and increases expected surplus consumption growth. This induces a borrowing motive to smooth surplus consumption, which therefore increases the real rate akin to Wachter (2006). A persistent increase in the real rate erodes the value of long-term real bonds more than short-term ones. Therefore, long-term real bonds provide less insurance against high marginal utility states induced by negative TFP shocks, which contributes to the upward-sloping real yield curve and positive real term premia. However, the TFP shocks do not generate significant inflation risk premia as TFP shocks have a very small effect on inflation (see Figure 10). Therefore, the impact of TFP shocks on the nominal term premia are primarily through the real term premia component.

Table 5 illustrates how the dynamics of real and nominal term premia are driven by the preference and TFP uncertainty shocks. As preference uncertainty shocks contribute significantly to the unconditional real term premia and inflation risk premia, changes in demand-side uncertainty generates sizable variation in the conditional real term premia and the conditional inflation risk premia. In contrast, TFP uncertainty shocks mainly contribute to real term premia and not towards inflation risk premia since the level TFP shocks mainly contribute to the unconditional real term premia. Quantitatively, changes in demand-side uncertainty produce large fluctuations in term premia through the effects on inflation risk premia.

4.5 Informational Content of the Term Structure

Given the importance of demand- and supply-side uncertainty for term premia movements, the use of bond yield data in our estimation is crucial for identifying the overall effects of uncertainty and distinguishing between the two types of uncertainty. Figures 12 and 13 plot the impulse response functions for demand and supply uncertainty shocks from our benchmark estimation using term structure data (solid line) and an estimation without using term structure data (dashed line). Interestingly, the estimated effects of demand-side uncertainty are significantly amplified using term structure data than without, while the effects are muted for supply-side uncertainty. As demand-side uncertainty is more important for nominal term premia compared to supply-side uncertainty, including term structure data in the estimation therefore increases the relative importance of demand-side uncertainty.

Figure 14 illustrates that the inclusion of term structure data in the estimation affects the timing, duration, and importance of uncertainty shocks. In particular, comparing this figure with Figure 6, it is evident that using term structure data provides valuable information for the role of uncertainty in explaining business cycle fluctuations. When the term structure is not included, periods of high uncertainty have a shorter duration and produce smaller effects. Furthermore, in the 1991 recession there is no visible effect from the increase in demand-side uncertainty, consistent with the impulse responses from Figure 12. Overall, when the term structure is not included, liquidity shocks become more important for explaining business cycle fluctuations as they account for around 77% and 29% of investment and consumption volatility, respectively, compared to 2.31% and 0.94% in the benchmark estimation. On the other hand, the estimation excluding the term structure also implies a counterfactual yield curve, as the unconditional nominal slope is only 0.27%, with most of the nominal spread coming from the real curve (see Table 6). This result is due to the fact that demand shocks are a key source of inflation risk premia, but when term structure data is excluded, the role of demand shocks is significantly reduced as illustrated in Figure 12. Thus, the term structure encodes important information about uncertainty and macroeconomic fluctuations while disciplining the relative importance of liquidity shocks. The joint estimation exploits the strong relation between the slope of the yield curve, business cycle fluctuations, and uncertainty.

5 Conclusion

This paper quantitatively explores the effects of different macroeconomic uncertainty shocks on business cycle and asset pricing fluctuations. We build and estimate a DSGE model that features realistic bond risk premia. We estimate the model using macroeconomic data, the term structure of interest rates, and imposing restrictions on the average investment risk premium. Our model allows for stochastic changes in the volatility of demand-side (preferences) and supply-side (TFP) shocks, while at the same time controlling for other disturbances often included in the estimation of New-Keynesian DSGE models. Uncertainty shocks are triggered by changes in stochastic volatility, but the endogenous response of the macroeconomy to these changes is in itself an important determinant of overall uncertainty.

We study the effects of uncertainty through the lens of a novel decomposition that identifies five endogenous risk propagation mechanisms: Precautionary savings, investment risk premium, inflation risk premium, nominal pricing bias, and investment adjustment channels. The effects arising from the investment and inflation risk premia channels are disciplined by the investment risk and nominal term premia, respectively.

We find sizable effects of changes in uncertainty. Both demand-side and supply-side generate a positive comovement in consumption, investment, and output. The responses of inflation and term premia differ depending on the source of uncertainty. Supply-side uncertainty leads to larger contractions in both investment and consumption. These differences are explained in light of the way uncertainty propagates through the real economy. In response to an increase in supply-side uncertainty, an increase in the risk of investing in physical capital contributes to a larger recession. Instead, when demand-side uncertainty is high, investment in capital becomes more attractive, reducing the fall in investment. In response to an increase in demand-side uncertainty, the negative effects on inflation from the precautionary savings channel are nullified by a nominal bias in pricing. The joint estimation of macro and yield curve variables put additional discipline on the relative importance of these channels, as the model is also asked to account for the negative comovement between term premia and the macroeconomy. Overall, our results highlight the importance of accounting for the origins of macroeconomic uncertainty and for using asset prices to discipline the various risk propagation channels for uncertainty.

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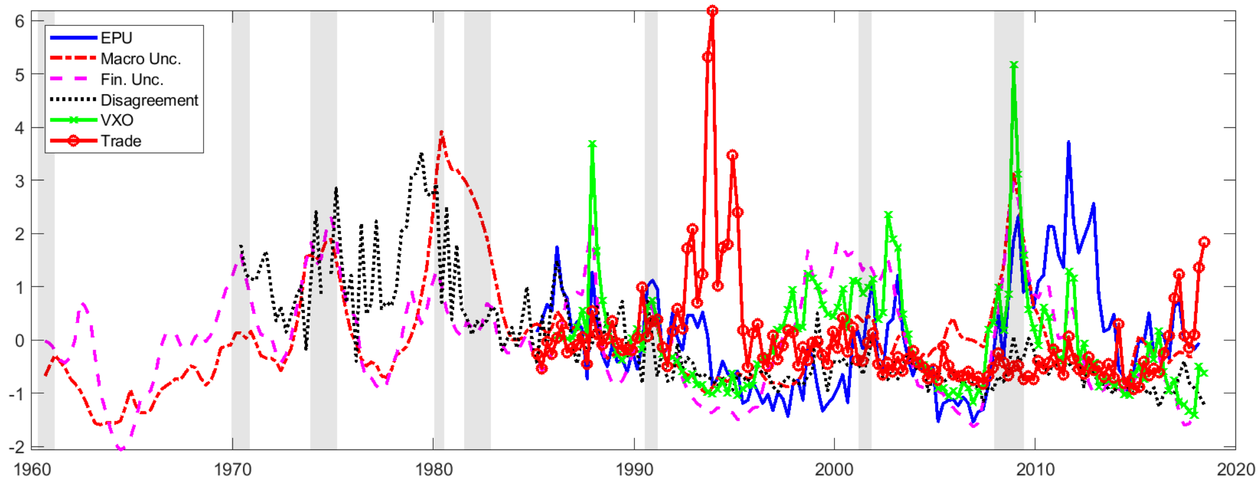


Figure 1: This figure plots various uncertainty measures. All measures are demeaned and normalized to have standard deviation equal to 1. 'EPU' - Economic Policy Uncertainty Index (Baker, Bloom, and Davis (2016)), 'Macro Unc.' - Macroeconomic uncertainty index for 12 month horizon (Jurado, Ludvigson, and Ng (2015)). 'Fin Unc.' - Financial uncertainty index for 12 month horizon (Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2018)). 'Disagreement' - Forecast disagreement about real GDP growth. 75^{th} percentile minus 25^{th} percentile of the forecast for growth rate at 4 quarters horizon. 'VXO' - CBOE S&P 100 Volatility Index. 'Trade' - Trade policy uncertainty (a component of Economic Policy Uncertainty Index). The pairwise correlations range from -0.30 to 0.85.

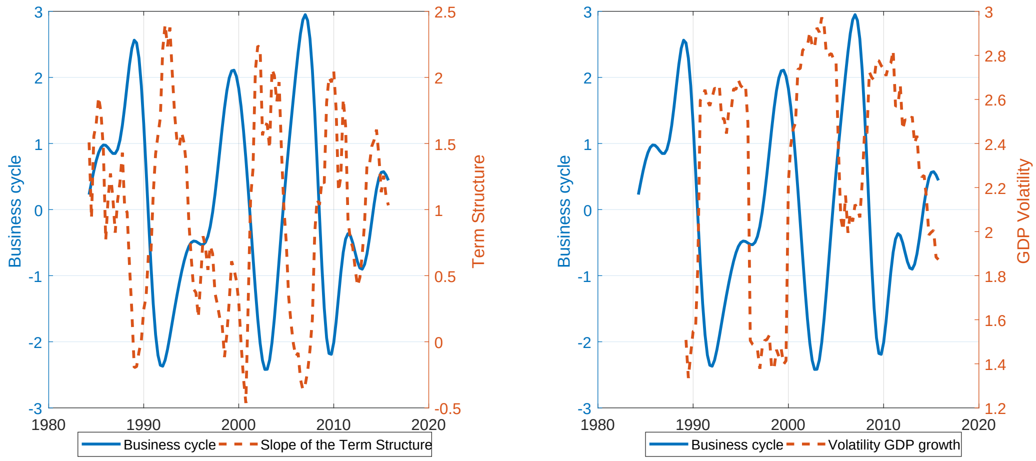


Figure 2: Slope and volatility over the business cycle. Panel A plots the comovement between the slope of the yield curve (dashed line) and the cyclical component of GDP (solid line) and Panel B plots the comovement between the volatility of GDP growth (dashed line) and the cyclical component of GDP (solid line) from the data.

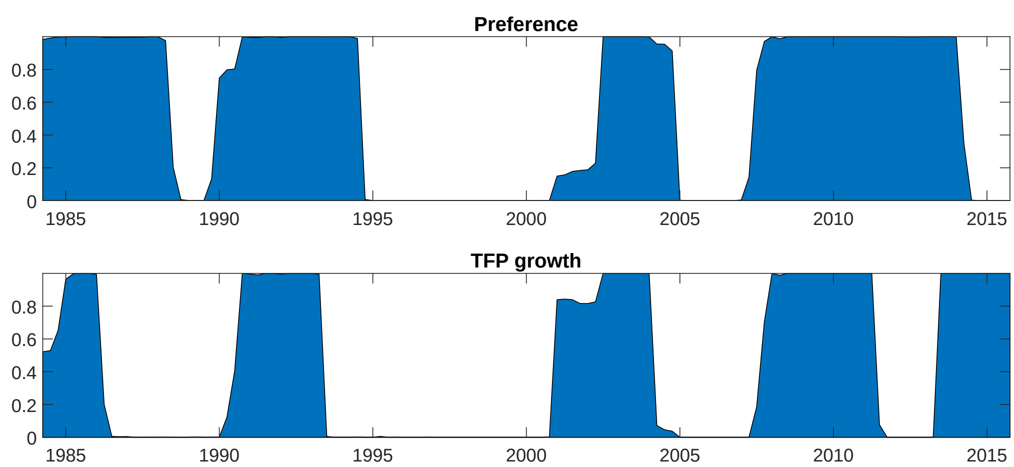


Figure 3: Regime probabilities. The figure plots the probability of the high uncertainty regime for the preference shock (top panel) and the TFP growth shock (bottom panel).

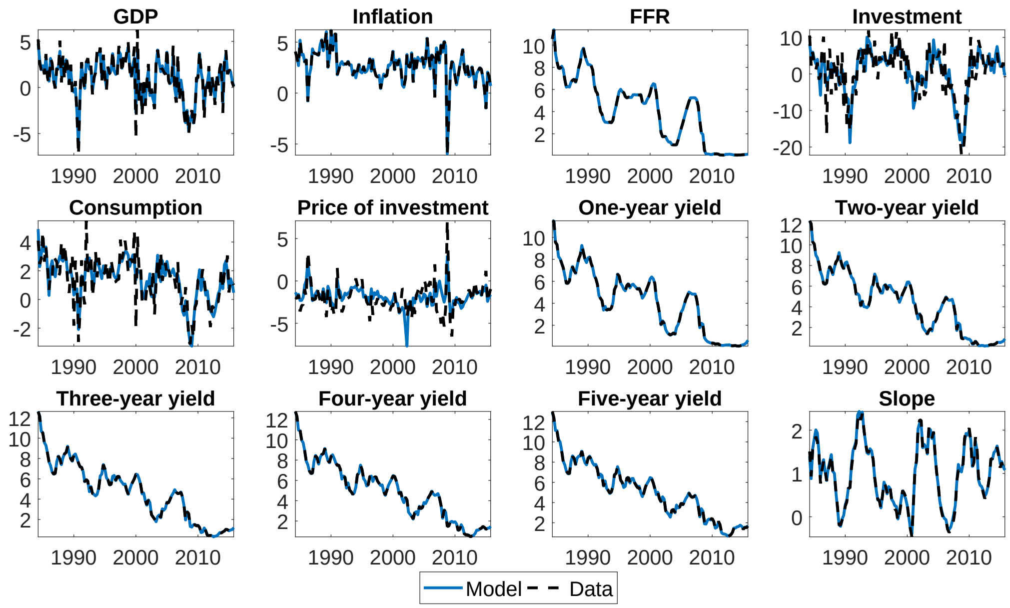


Figure 4: Actual and fitted series. The figure compares the fluctuations of the macroeconomy and the term structure of interest rates implied by our model (blue solid line) with the fluctuations observed in the data (black dashed line).

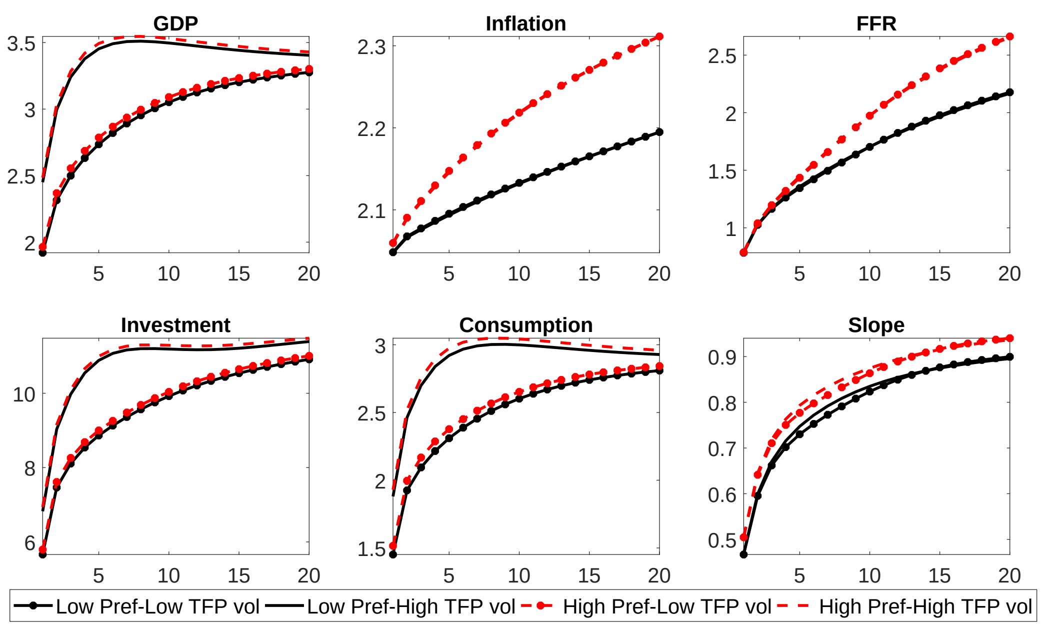


Figure 5: Uncertainty. The figure reports the level of uncertainty at different horizons. Uncertainty is computed taking into account the possibility of regime changes. All variables are annualized.

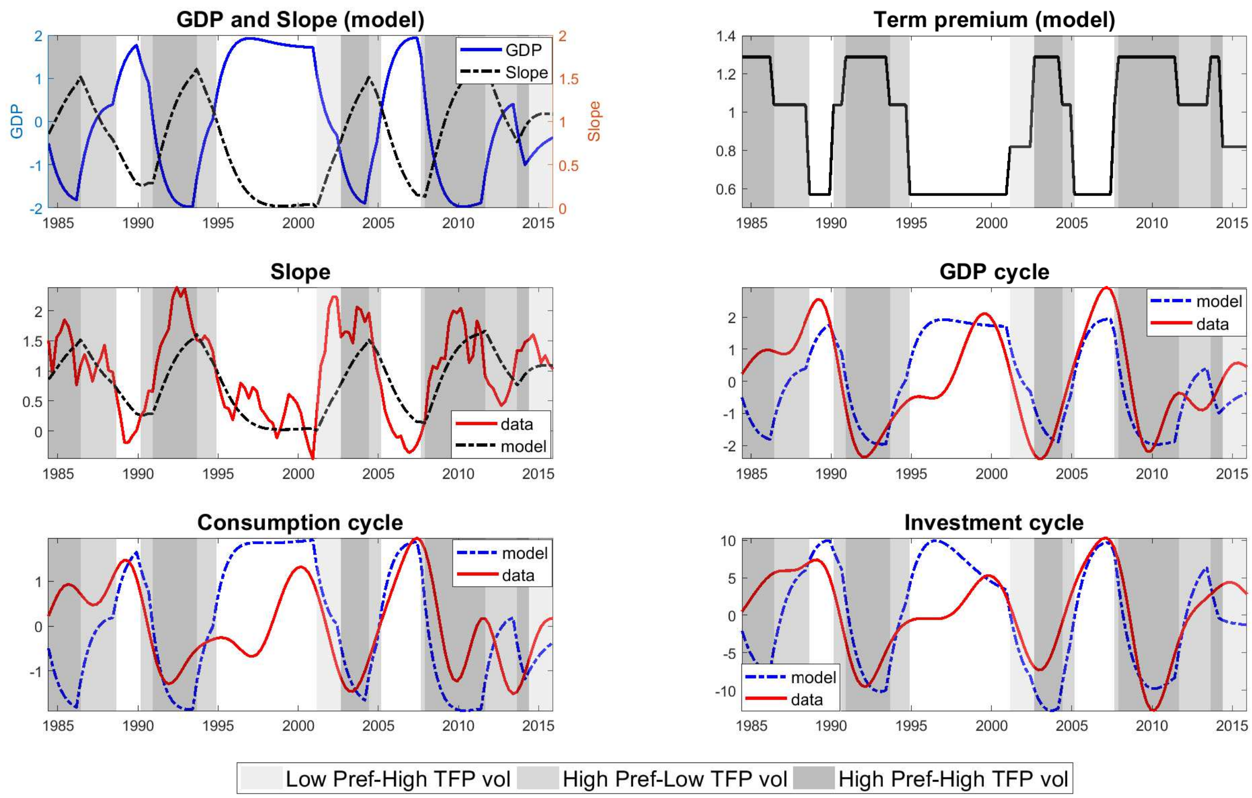


Figure 6: Uncertainty-driven fluctuations. The figure plots selected variables from the simulation of the model with estimated volatility regime sequence (all Gaussian shocks are set to zero in this simulation). Top left panel: simulated path of GDP, expressed in log-deviations from steady state, and slope of the yield curve, expressed as a difference between 5-year yield and 1-year yield. Top right panel: simulated dynamic of nominal term premium in the model, expressed as a difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters. Middle left panel: simulated slope of the yield curve and slope of the yield curve observed in the data. The subsequent panels plot the model-implied path of GDP, consumption, and investment in response to changes in uncertainty and the cyclical components of the corresponding series in the data (obtained using bandpass filter). Units on the y-axis for macro variables are percentage points (model and data). Units on the y-axis for Term premium and Slope are annualized percent (data and model).

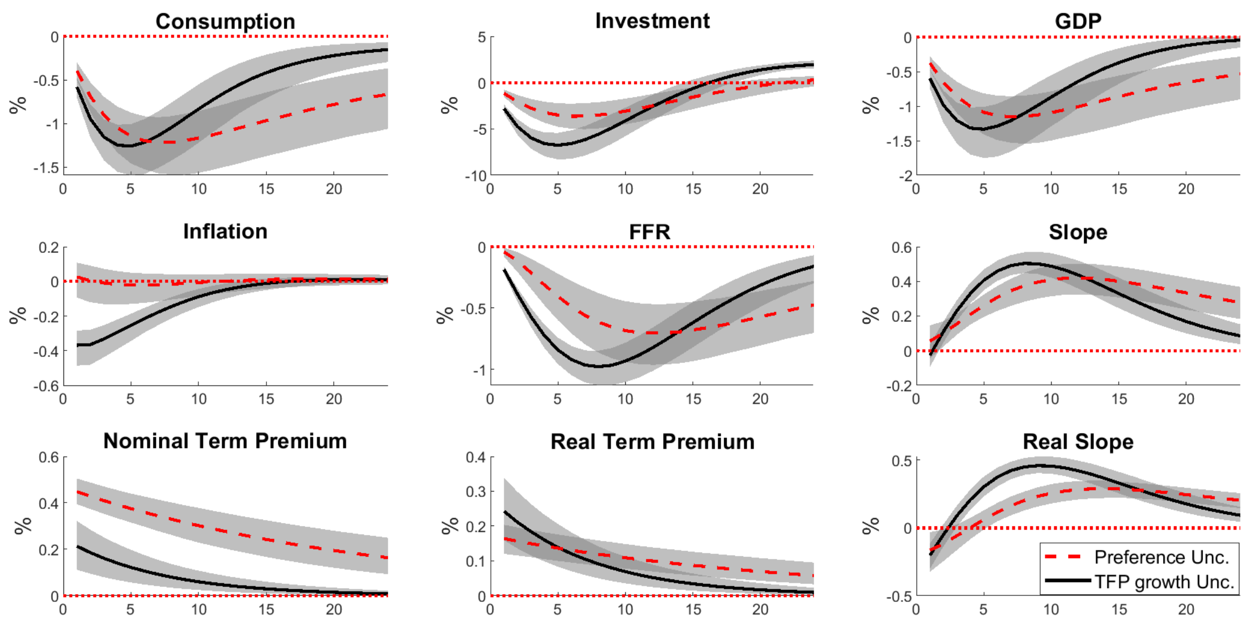


Figure 7: Responses to uncertainty shocks. This figure plots impulse responses to a change from low uncertainty regime to high uncertainty regime for preference and TFP growth shocks. The gray areas represent 90% credible sets. The impulse responses are computed as the change in the expected path of the corresponding variables when the volatility regime changes. The figure plots impulse responses of consumption, investment, GDP, inflation, Fed Funds Rate (1-quarter nominal interest rate), the slope of the yield curve expressed as the difference between 5-year and 1-year nominal yields, nominal term premium defined as the difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters, the real term premium defined as the difference between 5-year real yield and an expected average yield on 1-quarter real bond over the next 20 quarters, the real slope expressed as the difference between 5-year and 1-year real yields. The units of the y-axis are percentage deviations from a steady state (values for inflation, interest rates and term premia are annualized). Units on the x axis are quarters.

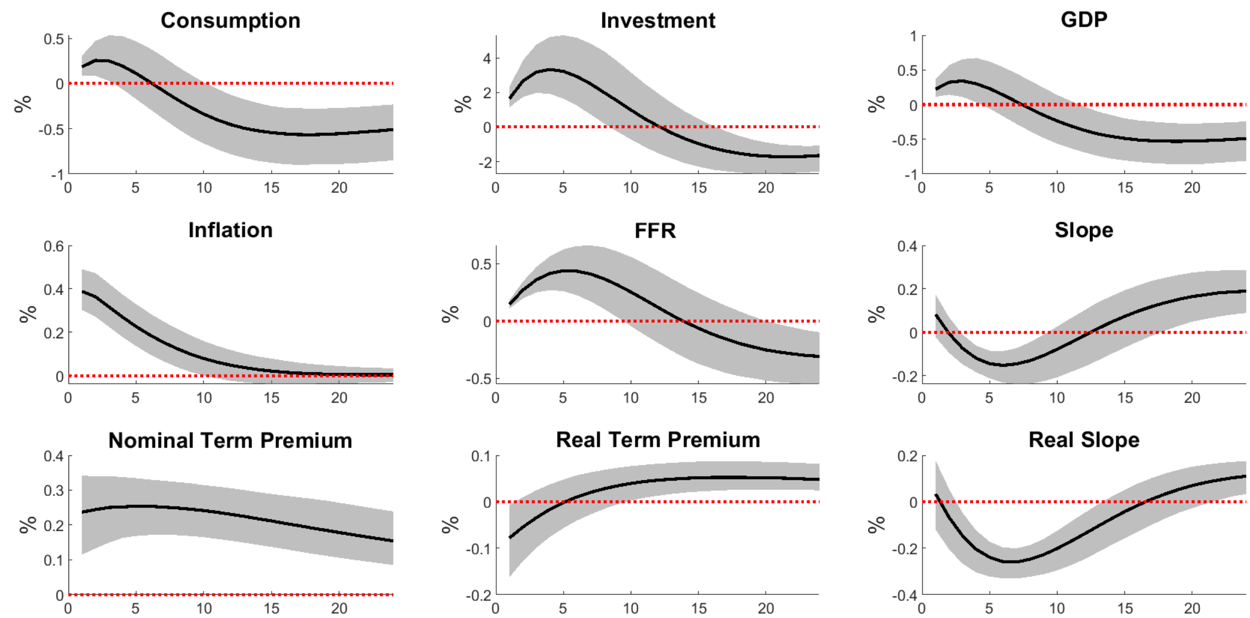
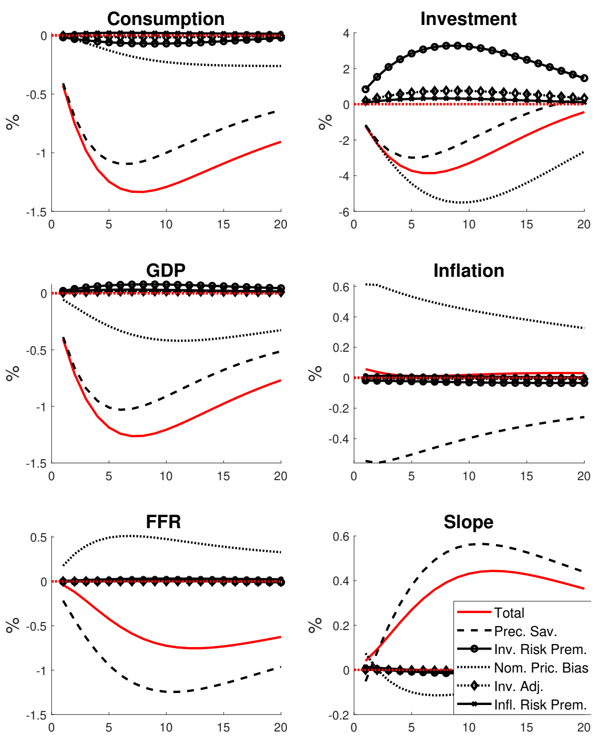
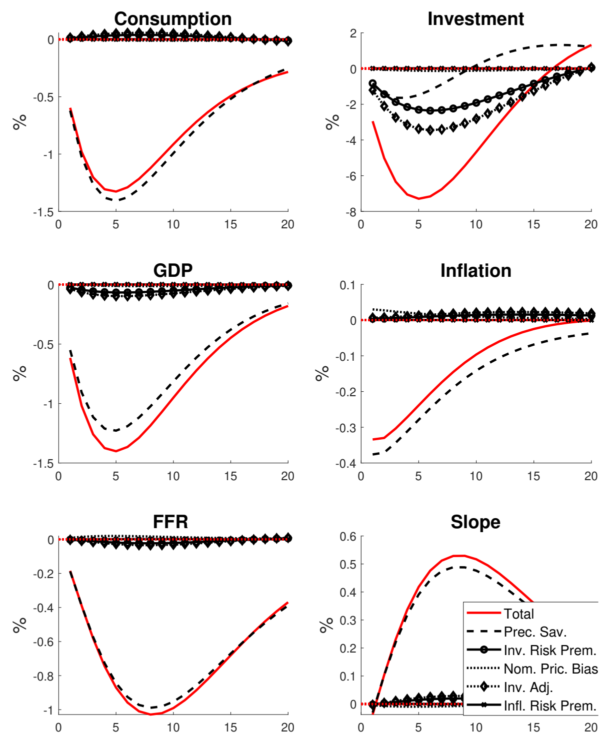


Figure 8: Heterogenous effects of uncertainty. This figure plots the difference between the impulse responses to demand and supply uncertainty. The gray areas represent 90% credible sets. The impulse responses are computed as the change in the expected path of the corresponding variables when the volatility regime changes. The figure plots impulse responses of consumption, investment, GDP, inflation, Fed Funds Rate (1-quarter nominal interest rate), the slope of the yield curve expressed as the difference between 5-year and 1-year nominal yields, nominal term premium defined as the difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters, the real term premium defined as the difference between 5-year real yield and an expected average yield on 1-quarter real bond over the next 20 quarters, the real slope expressed as the difference between 5-year and 1-year real yields. The units of the y-axis are percentage deviations from a steady state (values for inflation, interest rates and term premia are annualized). Units on the x axis are quarters.

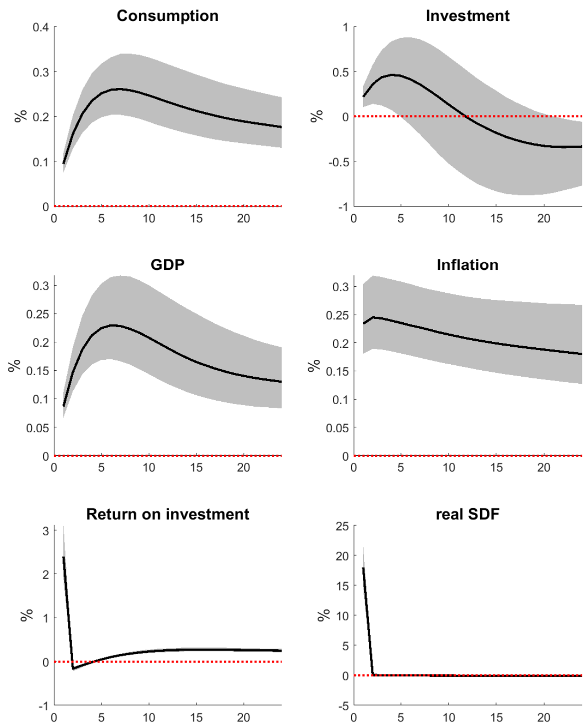


(a) Increase in volatility of a preference shock

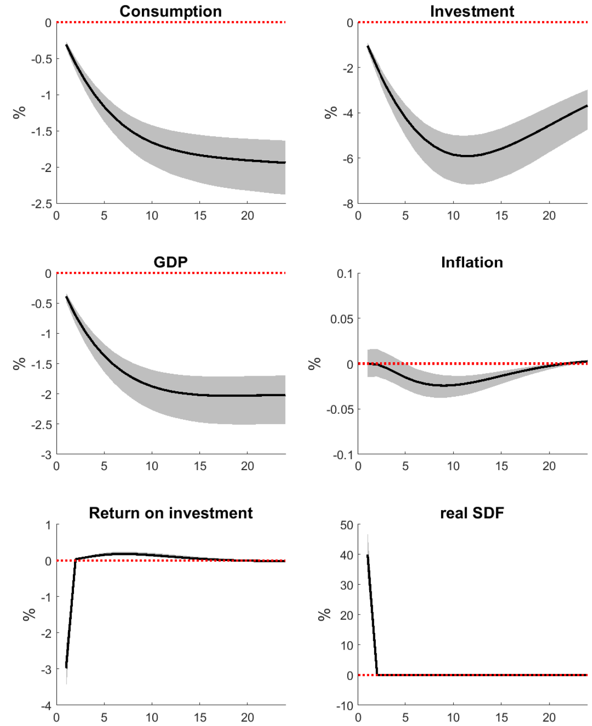


(b) Increase in vol. of a TFP growth shock

Figure 9: Inspecting the mechanism. The impule responses represent a change in the expected path of corresponding variables when volatility regime changes. The units of the y-axis are percentage deviations from a steady state (values for inflation and FFR are annualized). Units on the x axis are quarters. The red solid line depicts an IRF to volatility regime change in a benchmark model. The black dashed line shows the contribution of a precautionary savings motive. The black line with circles shows the contribution of the channel operating through change in the risk premium on investment return. The black line with crosses shows the contribution of inflation risk premium channel. The black dotted line shows the contribution of the nominal pricing bias channel. The line with diamond markers shows the contribution of the investment adjustment channel



(a) IRF to a preference shock



(b) IRF to a negative TFP growth shock

Figure 10: Impulse responses to level preference and TFP shocks. The units of the y-axis are percentage deviations from a steady state (values for inflation and return on investment are annualized). Units on the x axis are quarters.

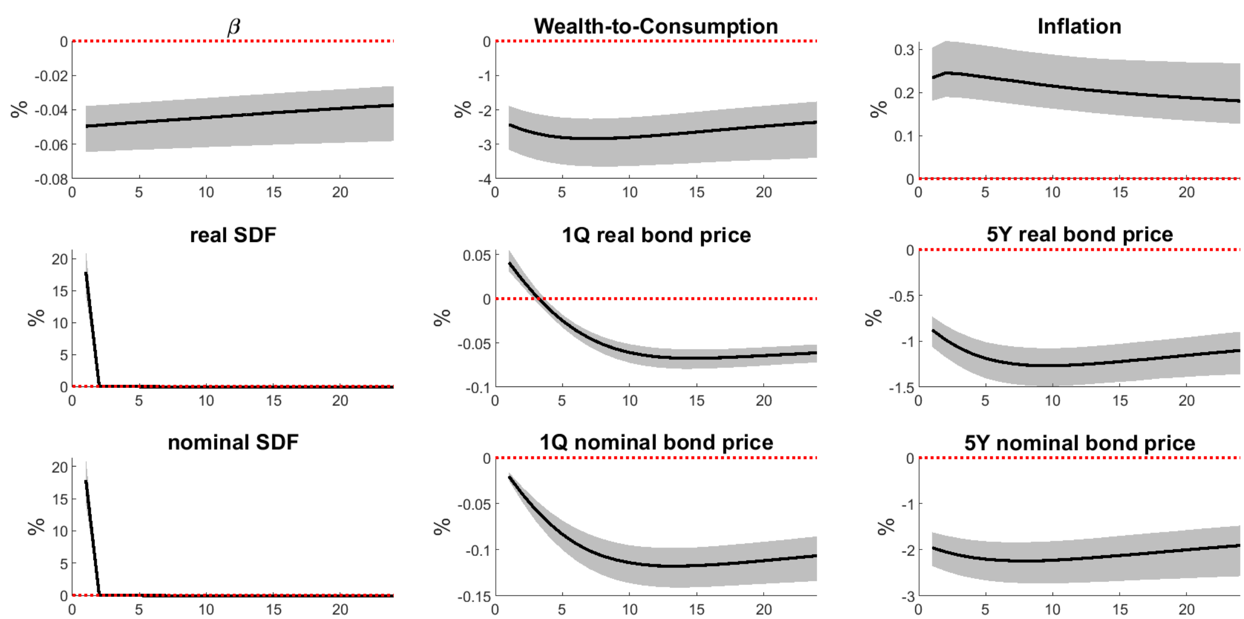


Figure 11: IRF to a preference shock and term premium. The units of the y-axis are percentage deviations from a steady state (values for inflation are annualized). Units on the x axis are quarters. The top left panel plots β_t - loading on continuation utility in Epstein - Zin value function.

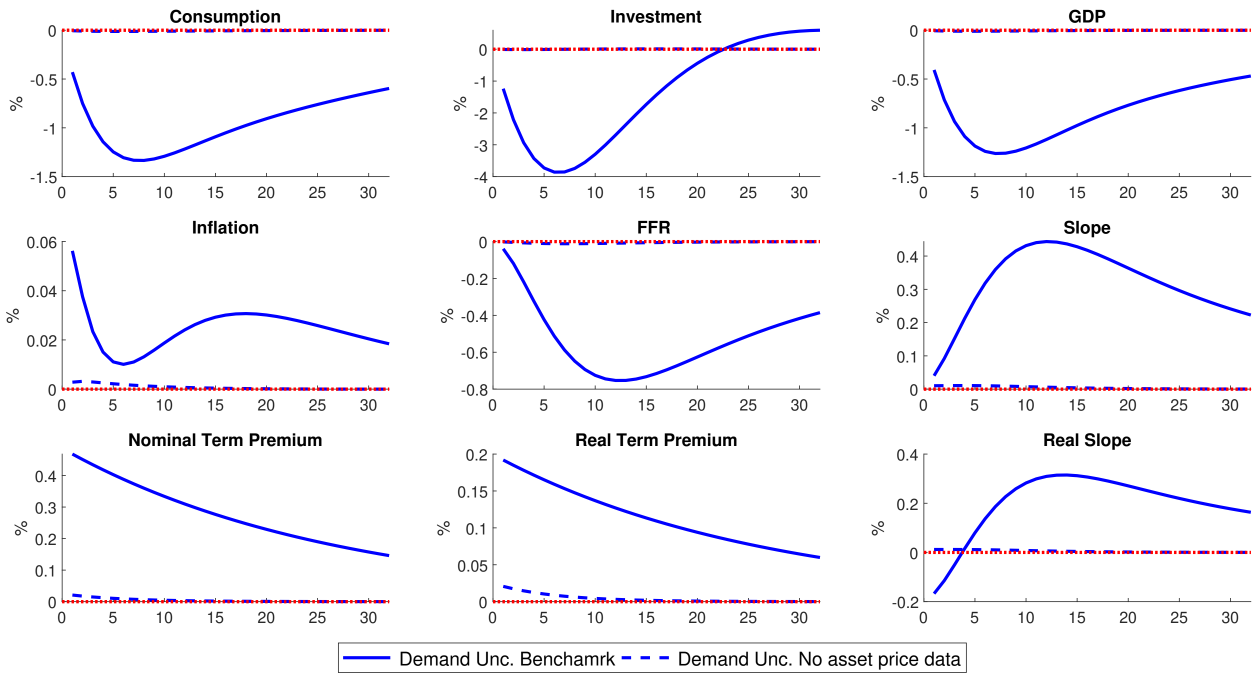


Figure 12: Effects of demand-side uncertainty when removing the term structure. This figure plots the impulse responses to a demand-side uncertainty shock based on the benchmark estimation (solid line) and in an alternative estimation without the term structure (dashed line).

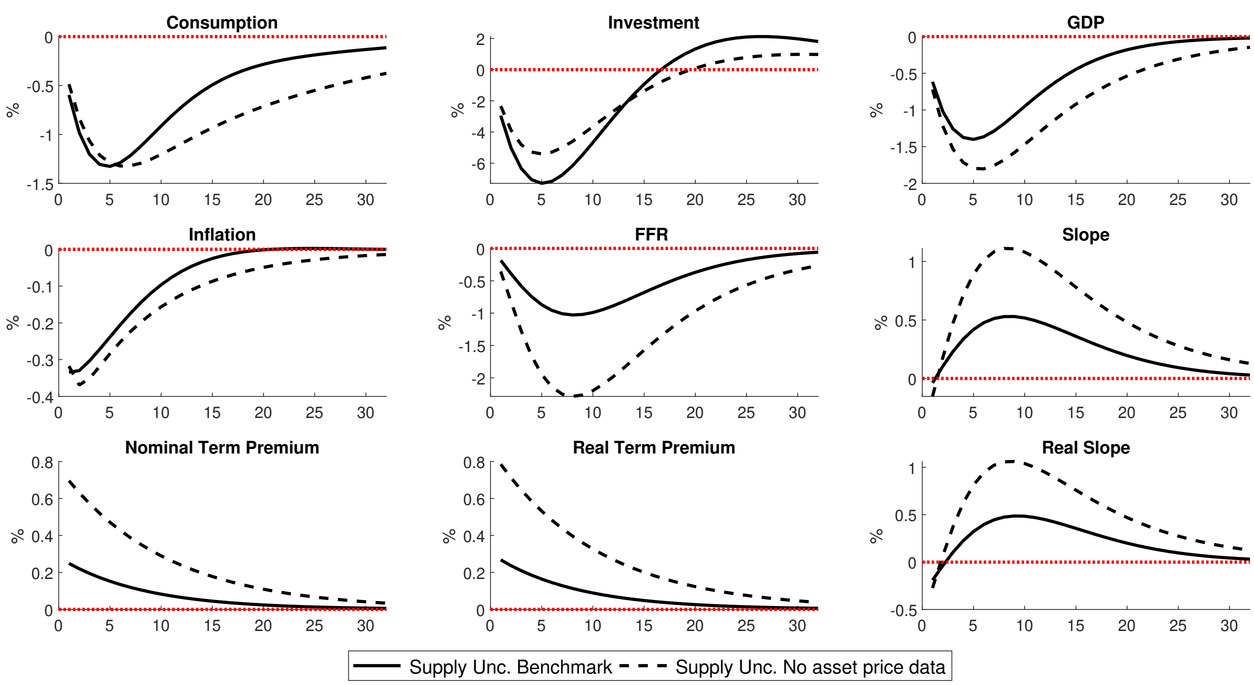


Figure 13: Effects of supply-side uncertainty when removing the term structure. This figure plots the impulse responses to a supply-side uncertainty shock based on the benchmark estimation (solid line) and in an alternative estimation without the term structure (dashed line).

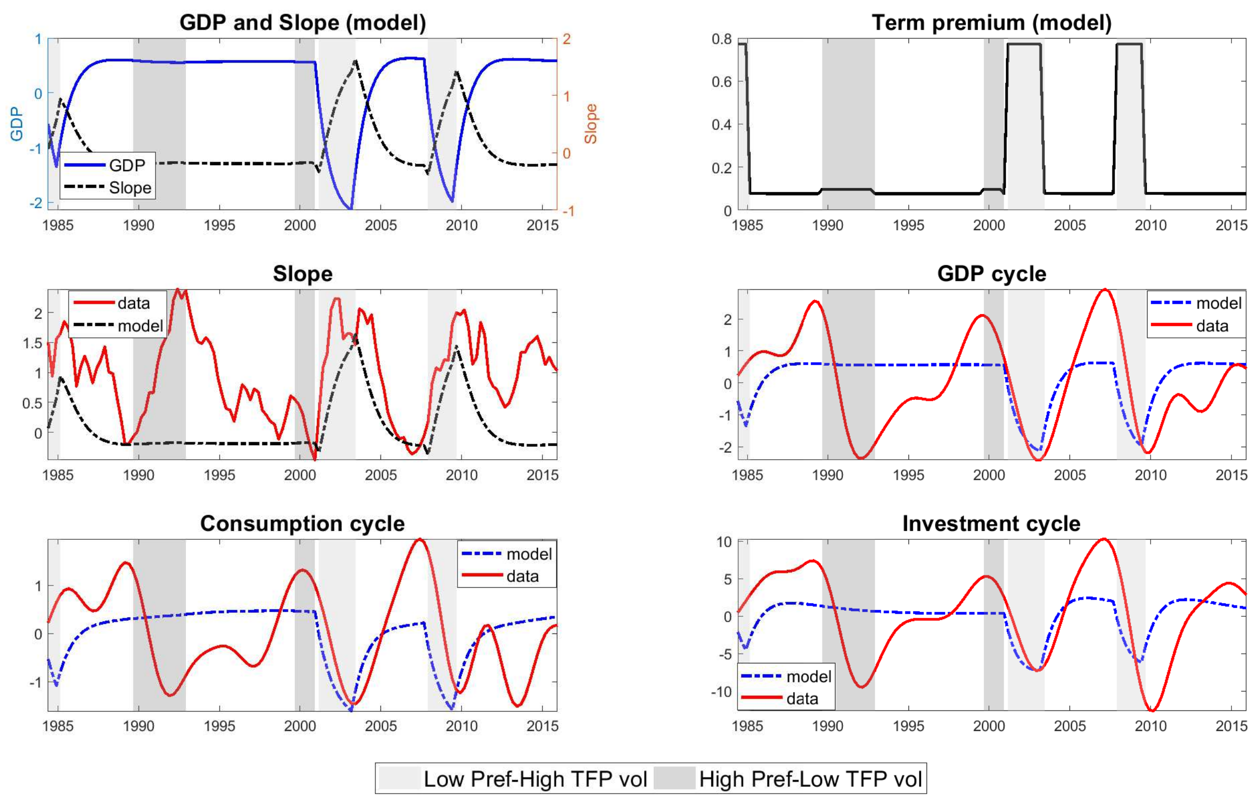


Figure 14: Uncertainty-driven fluctuations in an estimated model without the term structure. The figure plots selected variables from the simulation of the model estimated without asset pricing data. The simulation only considers the effects of uncertainty based on the estimated regime sequence (all Gaussian shocks are set to zero in this simulation). Top left panel: simulated path of GDP, expressed in log-deviations from steady state, and slope of the yield curve, expressed as a difference between 5-year yield and 1-year yield. Top right panel: simulated dynamic of nominal term premium in the model, expressed as a difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters. Middle left panel: simulated slope of the yield curve and slope of the yield curve observed in the data. The subsequent panels plot the model-implied path of GDP, consumption, and investment in response to changes in uncertainty and the cyclical components of the corresponding series in the data (obtained using bandpass filter). Units on the y-axis for macro variables are percentage points (model and data). Units on the y-axis for Term premium and Slope are annualized percent (data and model).

		Posterior			Prior		
		Mean	5%	95%	Type	Param. 1	Param. 2
Model parameters:							
Subjective discount factor	β	0.9843	0.9818	0.9867	B	0.9800	0.0100
Persist. of preference shock	ρ_β	0.9893	0.9816	0.9965	B	0.5000	0.2000
Degree of habit formation	h	0.8735	0.8508	0.8990	B	0.5000	0.2000
Risk aversion	γ	18.4716	12.6640	24.2366	G	10.0000	5.0000
Elasticity of labor supply	τ	8.2173	5.8233	11.1089	G	5.0000	4.0000
Liquidity preference param.	$100\zeta_B$	0.1557	0.0804	0.2548	G	0.1500	0.0500
Persistence of liquidity shock	ρ_{ζ_B}	0.8580	0.8253	0.8872	B	0.5000	0.2000
Average economic growth	$100\mu^*$	0.1334	0.0268	0.2543	N	0.4000	0.1250
Persist. of TFP growth shock	ρ_x	0.6693	0.5825	0.7373	B	0.1500	0.1000
Capital share in production	α	0.0881	0.0605	0.1160	B	0.3500	0.1000
Average capital depreciation	δ_0	0.0163	0.0139	0.0188	B	0.0350	0.0050
Capital depreciation param.	δ_2	7.4286	3.6509	12.1433	G	10.0000	5.0000
Capital adj. cost parameter	φ_I	7.1240	5.8048	8.7090	G	5.0000	3.0000
Persist. price of invest. shock	ρ_Υ	0.9556	0.9334	0.9762	B	0.5000	0.2000
Slope of phillips curve	$100\kappa_R$	0.0809	0.0599	0.1090	G	5.0000	4.0000
Persistence of markup shock	ρ_χ	0.0406	0.0164	0.0718	B	0.2500	0.1000
Indexation to past inflation	κ_π	0.9439	0.8901	0.9821	B	0.5000	0.2000
Monetary policy inertia	ρ_r	0.8246	0.8004	0.8473	B	0.5000	0.2000
Taylor rule param., inflation	ρ_π	1.6850	1.5089	1.8967	N	2.0000	0.5000
Taylor rule param., output	ρ_y	0.1841	0.1258	0.2457	G	0.5000	0.2000
Inflation in steady state	π_{ss}	0.0091	0.0072	0.0110	N	0.0070	0.0013
Risk adj. of inflation target	$\overline{\pi}^*$	0.0214	0.0161	0.0268	N	0.0050	0.0050
Share of gov.spending	η_g	0.1355	0.0834	0.1999	B	0.1500	0.0500
Standard deviations of shocks:							
Preference, low unc.	$100\sigma_\beta(\xi^D = 1)$	2.4172	1.9365	2.9388	IG	0.0016	3.2652
Preference, high unc.	$100\sigma_\beta(\xi^D = 2)$	4.0592	3.4989	4.7359	IG	0.0016	3.2652
TFP growth, low unc.	$100\sigma_x(\xi^S = 1)$	0.4106	0.3096	0.5349	IG	0.0001	2.5891
TFP growth, high unc.	$100\sigma_x(\xi^S = 2)$	0.8695	0.6599	1.1849	IG	0.0001	2.5891
Monetary policy	$100\sigma_R$	0.1312	0.1162	0.1477	IG	0.0000	2.5891
Markup	$100\sigma_\chi$	0.5077	0.4581	0.5622	IG	0.0016	3.2652
Price of invest.	$100\sigma_{\zeta_\Upsilon}$	0.3456	0.2899	0.4053	IG	0.0000	2.5891
Gov. spending	$100\sigma_g$	2.8867	1.8351	4.3727	IG	0.0016	3.2652
Liquidity	$100\sigma_{\zeta_B}$	0.0899	0.0796	0.1010	IG	0.0000	2.5891
Regime persistence:							
Low demand uncertainty	$H^D_{1,1}$	0.9804	0.9668	0.9913	D	0.8889	0.0721
High demand uncertainty	$H^D_{2,2}$	0.9759	0.9605	0.9878	D	0.8889	0.0721
Low supply uncertainty	$H^S_{1,1}$	0.9270	0.9083	0.9432	D	0.8889	0.0721
High supply uncertainty	$H^S_{2,2}$	0.9413	0.9062	0.9691	D	0.8889	0.0721
Standard deviations of observation errors:							
GDP	σ_y	0.1534	0.0660	0.2323	IG	0.0000	2.0000
Inflation	σ_π	0.0475	0.0260	0.0806	IG	0.0000	2.0000
Investment	σ_i	0.7410	0.6286	0.8738	IG	0.0000	2.0000
Consumption	σ_c	0.2063	0.1709	0.2484	IG	0.0000	2.0000
Price of investment	σ_{π_I}	0.3476	0.2885	0.4115	IG	0.0000	2.0000
1-year yield	σ_{y_1}	0.0103	0.0061	0.0148	IG	0.0000	2.0000
2-year yield	σ_{y_2}	0.0090	0.0069	0.0111	IG	0.0000	2.0000
3-year yield	σ_{y_3}	0.0063	0.0048	0.0079	IG	0.0000	2.0000
4-year yield	σ_{y_4}	0.0090	0.0068	0.0110	IG	0.0000	2.0000
5-year yield	σ_{y_5}	0.0154	0.0127	0.0180	IG	0.0000	2.0000
Priors and posteriors on endogenous variables:							
Inflation	π	2.2564	1.6634	2.8103	N	2	0.5
Equity premium	$E(r^i - r_f)$	0.8761	0.7113	1.0423	N	1	0.1
Real interest rate	$r - \pi$	0.4962	-0.0984	1.0811	N	2	0.5
Slope	$y_5 - y_1$	0.8403	0.7644	0.9172	N	0.9	0.05

Table 1: Mean, 90% error bands and prior distributions of the DSGE model parameters. Column 6 reports type of the prior distribution: B - beta, G - gamma, N - normal, IG - inverse gamma, D - dirichlet. For all distribution types, except inverse gamma, columns 7 and 8 report mean (Param. 1) and standard deviation (Param. 2) of the corresponding distribution. For inverse gamma distribution columns 7 and 8 report shape and scale parameters.

	Unconditional variance decomposition					Uncertainty and business cycle	
	Preference	TFP growth	Monetary	Markup	Uncertainty	Median	Conf. Inter.
GDP	2.60	71.67	1.10	1.78	6.69	23.19	(16.49; 31.15)
Inflation	31.62	0.06	0.04	67.72	0.49	14.47	(11.60; 17.88)
FFR	79.13	1.65	4.46	7.22	6.42	38.26	(29.58; 48.10)
Investment	2.56	70.27	2.56	4.40	16.05	33.76	(24.15; 45.88)
Consumption	4.27	78.64	1.48	2.36	9.05	23.86	(17.35; 32.50)
Slope	16.41	9.43	16.29	27.70	24.75	38.73	(30.43; 47.61)

Table 2: The left panel presents the contribution of the different shocks to the unconditional variance of the macroeconomic variables and the slope of the yield curve. The right panel analyzes the importance of uncertainty shocks in generating business cycle fluctuations with respect to the traditional level shocks. Specifically, we use the posterior mode parameter values to simulate two economies 1,000 times. In the first economy, only uncertainty shocks occur. In the second economy, we have level shocks on top of the same uncertainty shocks. For each simulation and for each variable we extract business cycle fluctuations using a bandpass filter. Finally, for each simulation we compute the ratio between the volatilities of the business cycle fluctuations for the two economies.

	Benchmark	Counterfactuals			
	$\gamma = 21.35$ $100\kappa_R = 0.0725$	$\gamma = 1$ $100\kappa_R = 0.0725$	$\gamma = 10$ $100\kappa_R = 0.0725$	$\gamma = 10$ $100\kappa_R = 0.7250$	$\gamma = 21.35$ $100\kappa_R = 0.7250$
GDP	23.19	0.76	11.15	7.25	15.31
Inflation	14.47	0.44	7.23	27.68	50.56
FFR	38.26	1.14	19.64	31.58	56.89
Investment	33.76	0.40	16.80	5.59	12.62
Consumption	23.86	0.93	11.57	8.67	18.18
Slope	38.73	1.19	19.32	35.08	62.43

Table 3: Counterfactual variance decomposition for different values of risk aversion and nominal rigidities. The first column reports the benchmark decomposition, obtained using the posterior mode parameter values. The other columns consider counterfactual parameterizations by varying the degree of risk aversion (γ) and nominal rigidities (κ)

	Yields						Slope				
	1Q	1Y	2Y	3Y	4Y	5Y	Total	Risk	Liquidity	Only Pref.	Only TFP
Nominal	2.77	2.88	3.08	3.31	3.53	3.72	0.95	0.89	0.06	0.63	0.41
Real	0.57	0.58	0.71	0.89	1.06	1.21	0.63	0.63	-	0.22	0.43

Table 4: The left panel reports unconditional means of nominal and real yields in the estimated model for the following maturities: 1-quarter and 1,2,3,4,5 years. The right panel reports the slopes of the corresponding term structures, defined as the difference between yields on 5-year and 1-quarter bonds. The first column in the right panel reports the total value, while the next two columns decompose the difference between 5-year and 1-quarter yield into risk premium and liquidity premium. The last two columns report the slope of the term structure in a model with only preference shocks and only TFP growth shocks. Values are annualized percent. The 1-quarter real yield corresponds to the risk free rate $r_{f,t}$ in the model. Real bond prices are computed as $P_{r,t}^{(n)} = E_t[M_{t+1}P_{r,t+1}^{(n-1)}]$, where M_{t+1} is a real SDF.

Uncertainty regime				
Preference uncertainty	Low	Low	High	High
TFP growth uncertainty	Low	High	Low	High
Nominal Term Premium	0.57	0.82	1.04	1.29
Real Term Premium	0.40	0.67	0.59	0.86
Inflation Risk Premium	0.17	0.15	0.44	0.42

Table 5: This table reports nominal and real term premia conditional on the uncertainty regime. The term premium in the model is computed as the difference between 5-year yield and the expected average yield on 1-quarter bond over the next 20 quarters. The inflation risk premium refers to the difference between nominal and real term premia

	Term Premia				Average Slope
Preference Unc.	Low	Low	High	High	
TFP growth Unc.	Low	High	Low	High	
Nominal	0.08	0.77	0.10	0.79	0.27
Real	-0.07	0.72	-0.05	0.74	0.10

Table 6: This table reports results from the model estimated without using asset price data. The left panel reports nominal and real term premia conditional on the uncertainty regime. The term premium in the model is computed as the difference between 5-year yield and the expected average yield on the 1-quarter bond over the next 20 quarters. The right panel reports the unconditional slopes of the corresponding term structures, defined as the difference between yields on 5-year and 1-quarter bonds. Values are annualized percent.

Appendices

A First-order conditions from the estimated model

Household's problem. Household solves the following constrained optimization problem. It maximizes its value function

$$V(\overline{K}_{t-1}, I_{t-1}, B_t) = \max_{C_t, L_t, B_{t+1}, I_t, \overline{K}_t, U_t} u(C_t, L_t, B_{t+1})^{(1-\beta_t)} \left(E_t \left[V(\overline{K}_t, I_t, B_{t+1})^{1-\gamma}\right]\right)^{\frac{\beta_t}{1-\gamma}},$$

where

$$u(C_t, L_t, B_{t+1}) = (C_t - h\overline{C}_{t-1}) e^{-\tau_0 \frac{L_t^{1+\tau}}{1+\tau}} e^{\zeta_{B,t} \frac{B_{t+1}}{R_t P_t Z_t^*}},$$

subject to the following constraints:

$$\begin{aligned}
\overline{K}_t &= \overline{K}_{t-1}\left(1 - \delta(U_t)\right) + \left[1 - S\left(I_t/I_{t-1}\right)\right] I_t, \\
S\left(I_t/I_{t-1}\right) &= \frac{\varphi_I}{2}\left(I_t/I_{t-1} - e^{\mu^*}\Upsilon\right)^2, \\
\delta(U_t) &= \delta_0 + \delta_1(U_t - U_{ss}) + \frac{\delta_2}{2}(U_t - U_{ss})^2, \\
P_t C_t + P_t (e^{\zeta_{\Upsilon,t}} \Upsilon^t)^{-1} I_t + B_{t+1}/R_t &= P_t D_t + P_t W_t L_t + B_t + P_t \overline{K}_{t-1} r_t^k U_t - P_t T_t.
\end{aligned}$$

From the household's optimization problem, we can derive the following first-order intertemporal condition:

$$1 = E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}\right] R_t + \frac{1}{Z_t^*}\bar{\zeta}_B e^{\tilde{\zeta}_{B,t}}(C_t - h\overline{C}_{t-1}), \tag{34}$$

where

$$M_{t+1} = \frac{1-\beta_{t+1}}{1-\beta_t}\beta_t \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} \left(\frac{u(C_{t+1}, L_{t+1}, B_{t+2})}{u(C_t, L_t, B_{t+1})}\right)^{-1} \left(\frac{u_1'(C_{t+1}, L_{t+1}, B_{t+2})}{u_1'(C_t, L_t, B_{t+1})}\right) \tag{35}$$

is the stochastic discount factor.

The intratemporal condition is

$$W_t = \tau_0 L_t^\tau (C_t - h\overline{C}_{t-1}).$$

The first-order conditions of the household with respect to a capital utilization choice and investment decision result in the following two equations, respectively:

$$\frac{r_t^k}{\delta'(U_t)}\left[1-\frac{\varphi_I}{2}\left(\frac{I_t}{I_{t-1}}-e^{\mu^*}\Upsilon\right)^2-\varphi_I\left(\frac{I_t}{I_{t-1}}-e^{\mu^*}\Upsilon\right)\frac{I_t}{I_{t-1}}\right]+$$
$$+E_t\Big[M_{t+1}\frac{r_{t+1}^k}{\delta'(U_{t+1})}\varphi_I\left(\frac{I_{t+1}}{I_t}-e^{\mu^*}\Upsilon\right)\frac{I_{t+1}^2}{I_t^2}\Big]=(e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1},$$

and

$$\frac{r_t^k}{\delta'(U_t)}=E_t\Big[M_{t+1}\Big(r_{t+1}^kU_{t+1}+\frac{r_{t+1}^k}{\delta'(U_{t+1})}\Big(1-\delta(U_{t+1})\Big)\Big)\Big].$$

Intermediate firm's problem. Intermediate firm i maximizes the present value of current and future cash flows:

$$V^{(i)}\left(P_{i,t-1}\right)=\max_{P_{i,t},K_{i,t},L_{i,t}}\left\{D_{i,t}+E_t\left[M_{t+1}\;V^{(i)}\left(P_{i,t}\right)\right]\right\},$$

subject to the following constraints:

$$P_tD_{i,t}=P_{i,t}X_{i,t}-P_tW_tL_{i,t}-P_tr_t^kK_{i,t}-P_tG(P_{i,t},P_{i,t-1},Y_t),$$
$$X_{i,t}=Y_t\left(P_{i,t}/P_t\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}},$$
$$X_{i,t}=K_{i,t}^{\alpha}(e^{n_t}L_{it})^{1-\alpha},$$
$$G(P_{i,t},P_{i,t-1},Y_t)=\frac{\phi_R}{2}\left(\frac{P_{i,t}}{\Pi_{ss}^{\kappa_\pi}\Pi_{t-1}^{1-\kappa_\pi}P_{i,t-1}}-1\right)^2Y_t.$$

The first-order condition of the intermediate firm with respect to the price setting decision is given by:

$$\left(1-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}\frac{Y_t}{P_t}+W_t\frac{L_{i,t}}{1-\alpha}\left(\frac{1+\lambda_{p,t}}{\lambda_{p,t}}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-1}\frac{1}{P_t}$$
$$-\phi_R\left(\frac{P_{i,t}}{\Pi_{ss}^{\kappa_\pi}\Pi_{t-1}^{(1-\kappa_\pi)}P_{i,t-1}}-1\right)\frac{Y_t}{\Pi_{ss}^{\kappa_\pi}\Pi_{t-1}^{(1-\kappa_\pi)}P_{i,t-1}}+E_t\left[M_{t+1}\phi_R\left(\frac{P_{i,t+1}}{\Pi_{ss}^{\kappa_\pi}\Pi_t^{(1-\kappa_\pi)}P_{i,t}}-1\right)\frac{Y_{t+1}P_{i,t+1}}{\Pi_{ss}^{\kappa_\pi}\Pi_t^{(1-\kappa_\pi)}P_{i,t}^2}\right]=0.$$

Combining the first-order conditions of the intermediate firm with respect to the capital and labor choice, we get:

$$r_t^k=\frac{\alpha}{1-\alpha}W_t\frac{L_{i,t}}{K_{i,t}}.$$

B Details about the solution method

This section provides more details about our log-linearization approach. As explained in the main text, our approach is quite common in the asset pricing and macro-finance literatures (e.g., Jermann (1998), Lettau (2003), Backus, Routledge, and Zin (2010), Uhlig (2010), Dew-Becker (2012), Malkhozov (2014), and

Bianchi, Ilut, and Schneider (2014)). This appendix is meant to provide more details about the method in order to make the paper self-contained. In particular, we aim to make the following points:

1. The method can be characterized as a guess-and-verify approach. This is because once the model is log-linearized and solved, *with or without risk-adjustment*, the variables of the model follow a linear process in logs and are therefore log-normal in levels. Thus, the method exploits this property of the solution when log-linearizing the model and implements a risk-adjusted log-linearization. This affects only the equilibrium conditions in which an expectational term appears. Note that log-normality does not affect the rest of the log-linearized equations. When introducing stochastic volatility, the process becomes conditionally log-normal. We explain how this affects the method and the quality of the approximation below.

2. To understand why the solution without risk adjustment already implies lognormality, it is important to notice that all shocks are specified in logs. Thus, when taking a lognormal approximation, the solution of the model implies a linear process in logs with Gaussian innovations. Note that when the variance of a shock increases, the mean of the shock is unchanged. The mean of the exponential of the shock would change, but this is not what is used in the log-linear approximation. Thus, without the risk-adjusted log-linearization, the increase in the variance of the shocks would translate into an increase in the variance of the variables expressed in logs, but it would not have first-order effects. The mean of the level of the variables would change, but this is not how we measure the effects of uncertainty. For example, the mean of log-consumption would not be affected, so we would conclude that there are no effects of uncertainty on consumption. Importantly, the mean of consumption in levels would change independently from using or not the risk-adjusted log-linearization.

3. The solution with risk-adjustment allows us to take into account the effects of uncertainty on the economy. As explained above, the method exploits the fact that even without risk adjustment, the log-linearized solution implies that the variables have a lognormal distribution (i.e., they are linear in log-deviations from the deterministic steady state). While the risk-adjusted log-linearization allows us to take into account the effects of uncertainty, the effects of uncertainty are not automatically large in this setting. Instead, the effects of uncertainty depend on the model and the estimated parameters. In the paper, we show that nominal rigidities and Epstein-Zin preferences are important. Below we consider a very simple example to make the same point in an even simpler setting.

B.1 A simple model

To illustrate the points above and the approximation method used in the paper, consider the simple Fisherian model:

$$R_t = E_t \left[I_t / \Pi_{t+1} \right],$$

where R_t is the gross real interest rate (the notation here is different with respect to the paper), I_t is the gross nominal interest rate, and $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate. Assume a Taylor rule for the nominal interest rate:

$$I_t/I = (\Pi_t/\Pi)^{\psi_\pi},$$

and a normal process for the log of the real interest rate:

$$\log(R_t) = r_t \sim N\left(0, \sigma_r^2\right).$$

Thus, in this simple model, the real interest rate is follows an exogenous process. Furthermore, r_t is approximately equal to the net real interest rate: $\log(R_t) = \log(1 + r_t) \cong r_t$. The assumption that the exogenous shock specified in logs follows a Normal distribution is standard in the applied macro literature, where all shocks are specified as log deviations from a steady state. In this case, the steady state for the log of the real interest rate is zero. The mean of the gross real interest R_t depends on σ_r^2, however, note that σ_r^2 does not affect the mean of r_t.

In the zero (net) inflation deterministic steady state we have:

$$\Pi = 1, \ R = 1, \ I = 1.$$

The standard log-approximation would give us:

$$\begin{aligned} r_t &= i_t - E_t\left[\pi_{t+1}\right] \\ i_t &= \psi_\pi \pi_t, \end{aligned}$$

where all variables are now expressed in logs. Given that all variables are zero in the steady state, the lower case letters also denote log deviations from steady state. The solution to the model is given by:

$$\begin{aligned} \pi_t &= \psi_\pi^{-1} r_t + \psi_\pi^{-1} E_t\left[\pi_{t+1}\right] \\ &= \psi_\pi^{-1} r_t + \psi_\pi^{-2} E_t\left[r_{t+1}\right] + ... \\ &= \psi_\pi^{-1} r_t, \end{aligned}$$

where we have used the fact that the one-step-ahead expected value of the real interest rate is zero. Note that in this case, changes in the variance of the exogenous shock (σ_r^2) do not affect the solution. However, given that π_t is a linear transformation of the normally distributed shock, r_t, it also has a normal distribution. Thus, Π_t is lognormal and its mean depends on the variance of r_t. Note that this is true even if we have used the standard log-linearization without risk-adjustment. But, again, this is not how we asses the effects of uncertainty. We work with logs and we look at the behavior of π_t, not Π_t. With standard log-linearization there are no effects of σ_r^2 on the mean of π_t. Thus, we conclude that in the standard log-linear approximation approach, we cannot capture the effects of uncertainty on inflation, despite that the mean of gross inflation varies with σ_r^2.

Now, consider the risk-adjusted log-linearization used in the paper. As explained above, π_t is a linear transformation of a normal variable (r_t), so it also has a normal distribution. Thus Π_t has a log-normal distribution. We can then use a guess-and-verify approach and use a risk-adjusted log-linearization that takes into account that the solution satisfies log-normality. We then have:

$$\begin{aligned} r_t &= i_t - E_t\left[\pi_{t+1}\right] - .5V_t\left[\pi_{t+1}\right] \\ i_t &= \psi_\pi \pi_t \end{aligned}$$

Note that $V_t\left[\pi_{t+1}\right] = \sigma_\pi^2$ is a *constant* that depends on the volatility of the real interest *and* the policy parameter ψ_π. We can then start with a guess on its value, solve the model, and then replace σ_π^2 with the value implied by the solution. The solution now becomes:

$$\pi_t = \psi_\pi^{-1} r_t + \psi_\pi^{-1} E_t\left[\pi_{t+1}\right] + .5\psi_\pi^{-1}\sigma_\pi^2$$

Solving forward, we have:

$$\begin{aligned} \pi_t &= \psi_\pi^{-1} r_t + .5\frac{\psi_\pi^{-1}}{1-\psi_\pi^{-1}}\sigma_\pi^2 \\ &= \psi_\pi^{-1} r_t + .5\frac{\psi_\pi^{-1}}{1-\psi_\pi^{-1}} V_t\left[\psi_\pi^{-1} r_{t+1}\right] \\ &= \psi_\pi^{-1} r_t + .5\frac{\psi_\pi^{-1}}{1-\psi_\pi^{-1}}\left[\psi_\pi^{-2}\sigma_r^2\right], \end{aligned}$$

where we have used the fact $\sigma_\pi^2 = V_t\left[\pi_{t+1}\right] = V_t\left[\psi_\pi^{-1} r_{t+1}\right] = \psi_\pi^{-2}\sigma_r^2$. Now, if we vary σ_r^2, the mean of net inflation, π_t, also varies. But, interestingly, this is not because we are varying the level of the real interest rate: The shock to r_t only presents a change in variance, while its mean is still zero. In other words, the mean of the log of the gross real interest rate, r_t, is not changing.

Instead, the effect on the level of inflation is endogenous and depends on how strongly the nominal interest rate reacts to inflation. To see this, note that as we increase the response to inflation in the Taylor rule, the variance of the real interest rate becomes less and less relevant for average inflation. Consistent with the fact that shocks to r_t are in levels, while shocks to σ_r^2 is a second moment shock, the importance of the latter decays faster. As an example, suppose that we double the size of the response to inflation from 2 to 4:

$$\begin{aligned}
\pi_t &= \psi_\pi^{-1} r_t + .5\frac{1}{\psi_\pi - 1}\left[\psi_\pi^{-2}\sigma_r^2\right] \\
&= .5 r_t + .125\sigma_r^2 \\
\pi_t &= .25\psi_\pi^{-1} r_t + .125\frac{1}{2\psi_\pi - 1}\left[\psi_\pi^{-2}\sigma_r^2\right] \\
&= .25\psi_\pi^{-1} r_t + 0.015625\left[\sigma_r^2\right]
\end{aligned}$$

the response to r_t is cut in half, while the response is divided by 8.

B.2 Adding regime changes

In the presence of Markov-switching volatility regimes, the model solution is log-normal conditional on the regime. In this subsection, we discuss in more details how our approximation method compares to the one by Bansal and Zhou (2002).

To study the difference between the two approaches, consider a univariate Markov-switching process:

$$z_{t+1} = c_{\xi_{t+1}} + a z_t + \sigma_{\xi_{t+1}} \varepsilon_{t+1} \tag{36}$$

where ξ_{t+1} denotes the volatility regime at time $t+1$. The solution of the model, presented in the main text, has this form. When we log-linearize the system of model equations, we are facing log-linearization equations of the following form:

$$E_t[e^{z_{t+1}}]. \tag{37}$$

We first summarize the approach in Bansal and Zhou (2002), where they utilize conditional log-normality of the process in equation (36). In particular:

$$E_t[e^{z_{t+1}}|\xi_{t+1}] = e^{E_t[z_{t+1}|\xi_{t+1}] + 0.5 Var_t[z_{t+1}|\xi_{t+1}]}.$$

Therefore, using the law of iterated expectations:

$$\begin{aligned}
E_t[e^{z_{t+1}}] = E_t[E_t[e^{z_{t+1}}|\xi_{t+1}]] = E_t[e^{E_t[z_{t+1}|\xi_{t+1}] + 0.5 Var_t[z_{t+1}|\xi_{t+1}]}] = \\
= E_t[e^{c_{\xi_{t+1}} + a z_t + 0.5\sigma^2_{\xi_{t+1}}}].
\end{aligned}$$

To proceed forward, Bansal and Zhou (2002) use an approximation: $e^{c_{\xi_{t+1}} + a z_t + 0.5\sigma^2_{\xi_{t+1}}} \approx 1 + c_{\xi_{t+1}} + a z_t + 0.5\sigma^2_{\xi_{t+1}}$. This procedure implies a linearization under the expectation sign. Due to this approximation, the expectation becomes linear in the MS constant, $c_{\xi_{t+1}}$, and the final expression does not depend on the volatility of $c_{\xi_{t+1}}$. Indeed,

$$E_t[e^{z_{t+1}}] = E_t[E_t[e^{z_{t+1}}|\xi_{t+1}]] \approx E_t[1 + c_{\xi_{t+1}} + az_t + 0.5\sigma^2_{\xi_{t+1}}] = \tag{38}$$

$$= 1 + E_t[c_{\xi_{t+1}}] + az_t + 0.5E_t[\sigma^2_{\xi_{t+1}}] \tag{39}$$

Next, we compare this procedure with our log-linearization and risk-adjustment approach. We approximate $E_t[e^{z_{t+1}}]$ as if z_{t+1} is log-normally distributed (note that process in Eq. (36) implies only conditional log-normality, so our procedure is an approximation)

$$E_t[e^{z_{t+1}}] \approx e^{E_t[z_{t+1}]+0.5Var_t[z_{t+1}]} \approx 1 + E_t[z_{t+1}] + 0.5Var_t[z_{t+1}]$$

Then, using law of total covariance, we compute the risk adjustment term, $Var_t[z_{t+1}]$:

$$Var_t[z_{t+1}] = E_t[Var_t[z_{t+1}|\xi_{t+1}]] + Var_t[E_t[z_{t+1}|\xi_{t+1}]]$$
$$= E_t[\sigma^2_{\xi_{t+1}}] + Var_t[c_{\xi_{t+1}} + az_t] = E_t[\sigma^2_{\xi_{t+1}}] + Var_t[c_{\xi_{t+1}}]$$

As a result:

$$E_t[e^{z_{t+1}}] \approx 1 + E_t[z_{t+1}] + 0.5(E_t[\sigma^2_{\xi_{t+1}}] + Var_t[c_{\xi_{t+1}}]) =$$
$$1 + E_t[c_{\xi_{t+1}}] + az_t + 0.5E_t[\sigma^2_{\xi_{t+1}}] + 0.5Var_t[c_{\xi_{t+1}}].$$

The difference with the approach described in Bansal and Zhou (2002) (see Eq. (38)) is the presence of the term, $0.5Var_t[c_{\xi_{t+1}}]$. So, our log-linearization and risk adjustment procedure takes into account the uncertainty that comes from the Markov-switching constant. If we were to disregard this term, the two solutions would be identical. The presence of this term affects the level of the risk adjustment terms, but it has very small effect on the model dynamics. To demonstrate this point, we solve the model ignoring the uncertainty that comes from the Markov-switching constant. Table 7 reports the moments obtained from such solution and compares them to our benchmark solution. Figure 15 plots a simulation of the model and compares it to a simulation of the model that was solved using our benchmark solution method. It is easy to see that the two methods return very similar results, especially when it comes to the model dynamics at business cycle frequencies, the focus of our paper. More generally, both methods have their pros and cons. In one case, the effects of what we call endogenous uncertainty, captured by the MS constant are lost. In the other case, conditional log-normality only holds approximately. We decided to retain the effects of endogenous uncertainty, but it is important to verify that the approaches do not lead to very different conclusions.

	Benchmark	No unc. MS const
Std(Δy)	3.24	3.25
Std(Δi)	10.78	10.89
Std(Δc)	2.67	2.68
E(π)	2.27	2.79
Std(π)	2.48	2.48
E(r)	2.77	3.64
Std(r)	3.56	3.57
Slope	0.95	1.06

Table 7: This table compares moments from the model, solved using our benchmark approximation method (column 2), and an approximation method, that ignores uncertainty about MS constant (column 3). The table reports volatilities of output (Δy), investment (Δi) and consumption growth (Δc); moments of inflation π, fed fund rate r and nominal slope of the yield curve. All variables are annualized.

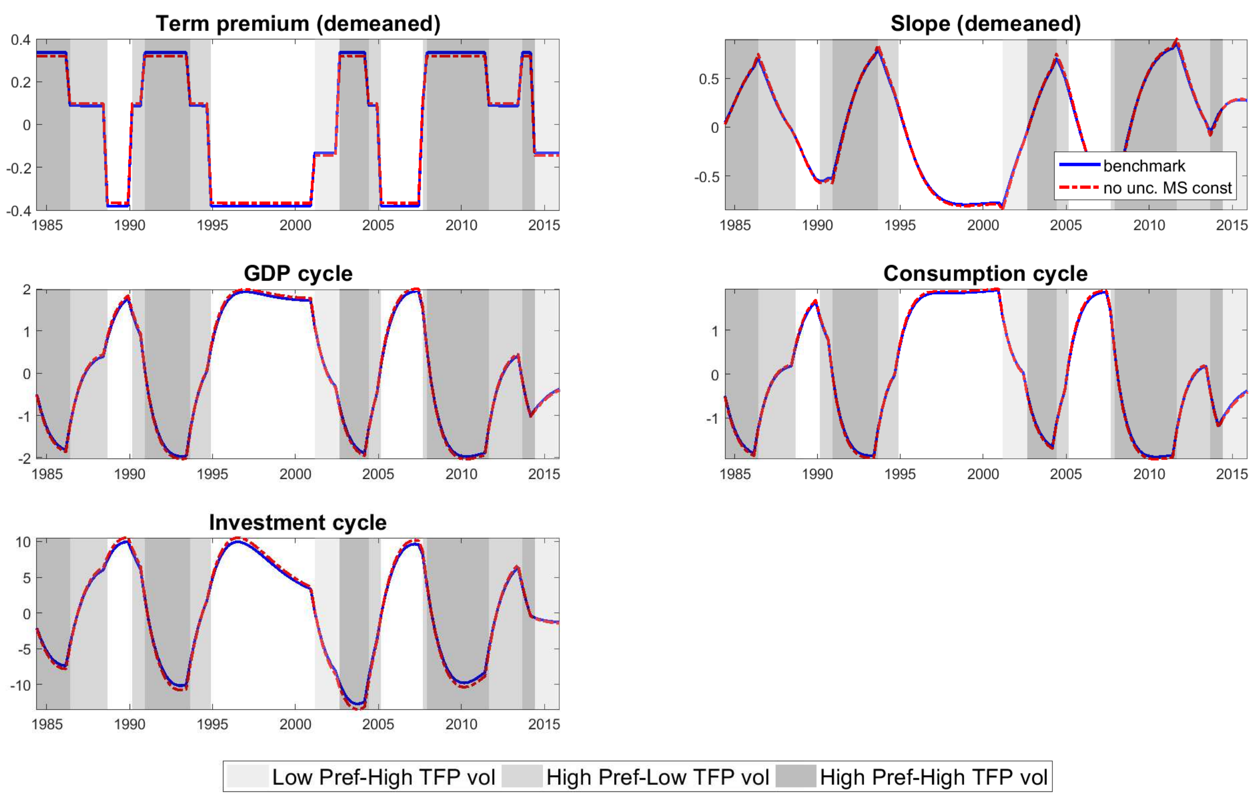


Figure 15: This figure plots simulation of the model. Blue solid line corresponds to the benchmark log-linearization approach, red dotted line corresponds to the approximate solution, that ignores uncertainty about the MS constant.

C Accuracy test

To assess the accuracy of the log-linear solution with risk adjustment employed in this paper, we conduct a Den Haan and Marcet (1994) test for the estimated model. We simulate 5000 economies for 3500 periods and drop the first 500 observations using the posterior mode for the parameter values. We use the conditionally linear policy functions for consumption, the value function, and the nominal interest rate to compute the time path of the corresponding variables. We then use the original non-linear Euler equation (34) to compute the realized Euler equation errors:

$$err_{t+1} = M_{t+1}\frac{P_t}{P_{t+1}}R_t + \bar{\zeta}_B e^{\tilde{\zeta}_{B,t}}(\hat{C}_t - \frac{1}{\Delta Z_t^*}h\hat{C}_{t-1}) - 1, \tag{40}$$

where the stochastic discount factor M_{t+1} is given by Eq. (35) and $\hat{C}_t = C_t/Z_t^*$. Under the null hypothesis that the approximation is exact, the Euler equation (Eq. (34)) implies $E_t(err_{t+1}) = 0$.

We then compute the Den Haan-Marcet statistic:

$$DM = \left[T\left(\sum_{s=1}^{T}(err_s)/T\right)^2\right] / \left[\sum_{s=1}^{T}(err_s^2)/T\right].$$

Under the null hypothesis, this statistic has a chi-squared distribution. We obtain $5,000$ statistics, one for each simulated economy and we check how many of them are above the 95% and below the 5% chi-squared critical values. Table 8 shows that the percentages of realized test statistics below 5% and above 95% critical values of a χ^2 distribution are very close to the theoretical ones. This result shows that our log-linearization approach with risk adjustment terms provides a good approximation of the model solution.

	Below 5%	Above 95%
Approximate solution	5.40%	5.92%

Table 8: This table reports the proportion of realized Den Haan, Marcet (1994) test statistics below 5% and above 95% critical values of χ^2 distribution. We simulate 5000 economies for 3500 periods and discard the first 500 observations.