Harvard University
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Topics in Finance
Spring 2013

Week 5: Collateral, Rental Markets, and Repo Markets

- Collateral key determinant of capital structure
  - Enforcement of repayment by borrower limited to tangible assets
  - Nature of assets required for production determine financing
  - Readings: Rampini/Viswanathan (2013), Collateral and capital structure

(Frictionless) Neoclassical Theory of Investment

- Environment
  - Time 0 and 1
  - Investor/owner
- Preferences
  - Investor is risk neutral and discounts time 1 payoffs at rate $R^{-1} < 1$
- Endowments
  - Investor net worth $w \gg 0$, i.e., deep pockets (Holmström/Tirole's (1997) called this A)
- Technology
  - Capital k invested at time 0
  - Payoff ("cash flow") at time 1
$$Af(k)$$
    where parameter A is "total factor productivity" (TFP)
  - Strict concavity $f'(k) > 0$ and $f''(k) < 0$; also: $\lim_{k\to 0} f'(k) = +\infty$; $\lim_{k\to\infty} f'(k) = 0$
  - Capital is durable and depreciates at rate δ; capital $(1-\delta)k$ remains at time 1

Neoclassical Investment and User Cost of Capital

- Investor's problem (objective: maximize "value" – present discounted value of dividends)
  - Choose dividends d_0 at time 0 and d_1 at time 1 and invest capital k to solve
  $$\max_{\{d_0,d_1,k\}} d_0 + R^{-1} d_1$$
  subject to budget constraints (but no limited liability constraints)
  $$\begin{aligned} w &\geq d_0 + k \\ Af(k) + k(1-\delta) &\geq d_1 \end{aligned}$$
  - First-order conditions (FOCs) (multipliers μ_0 and μ_1)
  $$\begin{aligned} 1 &= \mu_0 \\ R^{-1} &= \mu_1 \\ \mu_0 &= \mu_1[Af'(k) + (1-\delta)] \end{aligned}$$
- Optimal investment/capital k^* solves (combining FOCs)
  $$1 = R^{-1}[Af'(k) + (1-\delta)]$$
  or letting $R \equiv 1 + r$ and rewriting
  $$\underbrace{r+\delta}_{\text{user cost of capital}} = \underbrace{Af'(k)}_{\text{marginal product of capital}}$$
- **Jorgenson's (1963) user cost of capital**
  $$u \equiv \underbrace{r}_{\text{interest rate}} + \underbrace{\delta}_{\text{depreciation rate}}$$

Collateral Constraints as in Rampini/Viswanathan (2013)

- Environment with frictions (otherwise as before)
  - Environment: Two types of agents, owner/borrower and investor/lender
  - Borrower is risk neutral, impatient $\beta < R^{-1}$, and subject to limited liability
  - Borrower has limited funds $w > 0$; lender has deep pockets
  - **Collateral constraints:** Need to collateralize loan repayment with tangible assets
- **Financing problem with collateral constraints** (where $\theta < 1$)
  $$\max_{\{d_0,d_1,k,b\}} d_0 + \beta d_1$$
  subject to budget constraints and collateral constraint
  $$\begin{aligned} w + b &\geq d_0 + k \\ Af(k) + k(1-\delta) &\geq d_1 + Rb \\ \theta k(1-\delta) &\geq Rb \end{aligned}$$
  and limited liability $d_0,\ d_1 \geq 0$
- First-order conditions (FOCs) (multipliers μ_0, μ_1, and λ)
  $$\begin{aligned} 1 &\leq \mu_0, & \beta &= \mu_1 \\ \mu_0 &= \mu_1[Af'(k) + (1-\delta)] + \lambda\theta(1-\delta), & \mu_0 &= \mu_1 R + \lambda R \end{aligned}$$
- Optimal investment/capital k solves (combining FOCs)
  $$[1 - R^{-1}\theta(1-\delta)]\mu_0 = \beta[Af'(k) + (1-\theta)(1-\delta)]$$

Collateral Constraints: Tangible Assets and Capital Structure

- **"Minimal downpayment"** (per unit of capital)

$$\wp \equiv 1 - \underbrace{R^{-1}\theta(1-\delta)}_{\text{PV of } \theta\times \text{ resale value of capital}}$$

- **Capital structure**
  - Collateral constraints bind: Using FOCs and noting $\beta R < 1$

$$\beta R + \lambda R = \mu_0 \geq 1 \qquad \Rightarrow \qquad \lambda > 0, \qquad \text{i.e.,} \qquad Rb = \theta k(1-\delta)$$

  - Debt per unit of capital

$$R^{-1}\theta(1-\delta)$$

  - Internal funds per unit of capital

$$\wp = 1 - R^{-1}\theta(1-\delta)$$

- **Collateralizability** θ
  - Structures more collateralizable than equipment (composition varies by industry)
  - Financial development may raise θ and hence leverage
- **Tangibility** (includes mainly structures (incl. land) and equipment)
  - Suppose tangible assets are collateralizable (but not intangible assets)
  - Fraction tangible assets (φ) needed for production key: $\wp(\varphi) = 1 - R^{-1}\varphi\theta(1-\delta)$

Investment and Dividend Policy

- **Investment policy**
  - Investment FOC

$$1 \leq \beta\frac{Af'(k) + (1-\theta)(1-\delta)}{\wp}$$

    with equality if $d_0 > 0$
  - Dividend paying firm: Capital $\bar{k}$ solves equation above
    - Comparing FOCs can show $\bar{k} < k^*$ (**underinvestment**)
  - Non-dividend paying firm: $k = \frac{1}{\wp}w$ (invest all net worth and lever as much as possible)
- **Dividend policy** (threshold policy)
  - Pay out dividends today ($d_0 > 0$) if $w \geq \bar{w}$
    - Can we show threshold is optimal? Suppose pay dividends at w but not at $w^+ > w$
    - At w, invest $\bar{k}$; if not paying dividends at w^+, must invest *more;* can FOC hold?
- **Value of internal funds** μ_0 (remember the envelope condition?)
  - Premium on internal funds (unless firm pays dividends) since $\mu_0 \geq 1$
- **User cost** $u(w)$
  - User cost such that $u(w) = R\beta\frac{1}{\mu_0}Af'(k)$ where

$$u(w) \equiv r + \delta + \underbrace{R\frac{\lambda}{\mu_0}(1-\theta)(1-\delta)}_{\text{internal funds require premium}} > u$$

Limited Enforcement Implies Collateral Constraints

- Question: Why does borrower need to collateralize loans?
  - Enforcement is limited and it has to be incentive compatible for borrower to repay
- **Friction: Limited enforcement**
  - Borrower can abscond with all cash flows and fraction $1-\theta$ of (depreciated) capital
- **Limited enforcement implies collateral constraints**
  - **Enforcement constraint**
    - Ensure that borrower prefers to repay instead of absconding

$$\underbrace{Af(k)+k(1-\delta)-Rb}_{\text{payoff when repaying}} = d_1 \geq \underbrace{Af(k)+(1-\theta)k(1-\delta)}_{\text{payoff when defaulting}}$$

  - **Collateral constraint**
    - Canceling terms and rearranging enforcement constraint we obtain

$$\theta k(1-\delta) \geq Rb$$

Dynamic Financing

- **Dynamics**
  - Suppose financing problem repeats itself at $t = 0, 1, 2, \ldots$ (**infinite horizon**)
- **Dynamic programming** (Bellman (1953))
  - Suppose function $v(w)$ summarizes value to borrower from having net worth w
  - Problem: Function $v(w)$ is unknown!
    - Richard Bellman's key insight: $v(w)$ must solve a particular (functional) equation!
  - Financing problem with collateral constraints (**Bellman equation**)

$$v(w) \equiv \max_{\{d,k,b,w'\}} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d + k \\ Af(k) + k(1-\delta) &\geq w' + Rb \\ \theta k(1-\delta) &\geq Rb \end{aligned}$$

and limited liability $d \geq 0$

  - Net worth next period $w' = Af(k) + k(1-\delta) - Rb$
  - Note: Problem looks almost exactly as before!
- Dynamic programming is a remarkably powerful tool to solve dynamic problems

Net Worth Accumulation and Firm Growth

- First-order conditions (FOCs) for dynamic problem (multipliers μ, μ', and λ)

$$1 \leq \mu, \qquad \beta v'(w') = \mu'$$
$$\mu = \mu'[Af'(k) + (1-\delta)] + \lambda\theta(1-\delta), \qquad \mu = \mu' R + \lambda R$$

  - Note: FOCs look almost exactly as before!
  - Also: Envelope condition $v'(w) = \mu$
- **Dividend policy and net worth accumulation**
  - Dividend policy is threshold policy
    - For $w \geq \bar{w}$, pay dividends $d = w - \bar{w}$
    - For $w < \bar{w}$, pay no dividends and reinvest everything ("retain all earnings")
- **Investment policy and firm growth**
  - For $w \geq \bar{w}$, keep capital constant at $\bar{k}$ (no growth)
  - For $w < \bar{w}$, invest everything $k = \frac{1}{\wp} w$ resulting in net worth $w' > w$ next period
- **Firm age**
  - Young firms ($w < \bar{w}$) do not pay dividends, reinvest everything, and grow
  - Mature firms ($w \geq \bar{w}$) pay dividends and do not grow

Conclusions

- **Tangible assets as collateral**
  - If debt needs to be collateralized, type of assets required determines capital structure
- **Dynamics of financing**
  - Accumulate net worth over time
  - Young firms grow and retain all earnings
  - Mature firms pay dividends and grow less

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Week 5: Collateral, Rental Markets, and Repo Markets (Cont'd)

- Rental markets
  - Leasing has repossession advantage and permits greater borrowing
  - Severely constrained firms (and households) lease
  - Readings: Rampini/Viswanathan (2013), Collateral and capital structure
- Repurchase (Repo) agreements
  - Collateralized loans in which lender (temporarily) owns collateral
  - Key aspect of financial crisis?
  - Readings: Gorton/Metrick (2012), Securitized banking and the run on repo

Leasing as in Rampini/Viswanathan (2013)

- Environment with collateral constraints (as in last class) but firms can lease
  - Environment: Two types of agents, owner/borrower, investor/lender, and lessor
  - Borrower is risk neutral, impatient $\beta < R^{-1}$, and subject to limited liability
  - Borrower has limited funds $w > 0$; lender and lessor have deep pockets
- **Borrowing subject to collateral constraints**
  - Need to collateralize promises to pay with tangible assets (due to limited enforcement)
  - Promised repayment $\leq \theta \times$ resale value of tangible assets
- **Leasing:** Borrower can rent capital
  - **Repossession advantage:** Borrower cannot abscond with leased capital
    - In practice, repossession of rented capital easier than foreclosure on secured loan
    - Leasing allows borrower to borrow full resale value, not just fraction θ
  - **Monitoring cost** m (per unit of capital): Lessor needs to monitor to prevent abuse
    - Why? – Leasing separates ownership and control
  - **User cost of leased capital** (assuming lessors, like lenders, discount at R^{-1})

$$u_l \equiv r + \delta + m$$

needs to be paid in advance, i.e., at time 0

Lease or Buy?

- **Firm's problem with leasing** (k_o owned capital; k_l leased capital)

$$\max_{\{d_0,d_1,k_o,k_l,b\}} d_0 + \beta d_1$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d_0 + k_o + R^{-1}u_l k_l \\ Af(k_o+k_l) + k_o(1-\delta) &\geq d_1 + Rb \\ \theta k_o(1-\delta) &\geq Rb \end{aligned}$$

and non-negativity constraints k_o, $k_l \geq 0$, as well as limited liability d_0, $d_1 \geq 0$

- First-order conditions (FOCs) (multipliers μ_0, μ_1, and λ; let $k \equiv k_o + k_l$): As before,

$$1 \leq \mu_0, \qquad \beta = \mu_1, \qquad \mu_0 = \mu_1 R + \lambda R$$

and almost as before (except inequality as borrower might not own any assets)

$$\mu_0 \geq \mu_1[Af'(k) + (1-\delta)] + \lambda\theta(1-\delta) \qquad \Leftrightarrow \qquad u(w) \geq R\beta\mu_0^{-1}Af'(k)$$

and finally new

$$R^{-1}u_l\mu_0 \geq \mu_1 Af'(k) \qquad \Leftrightarrow \qquad u_l \geq R\beta\mu_0^{-1}Af'(k)$$

- **Leasing policy**
  - Lease if $u_l < u(w)$ and buy otherwise ("choose capital with lower user cost")
  - Recall: $u_l = r + \delta + \underbrace{m}_{\text{monitoring cost}}$ and $u(w) = r + \delta + \underbrace{R\lambda/\mu_0(1-\theta)(1-\delta)}_{\text{premium on internal funds required}}$

Leasing as Costly Way to Borrow More

- **Incremental cash flows of buying vs. leasing**

Time		0		1
Buying (with secured loan)		$1 - R^{-1}\theta(1-\delta)$		$(1-\theta)(1-\delta)$
Leasing		$R^{-1}u_l$		
Difference buying - leasing	=	$\underbrace{\wp - R^{-1}u_l}_{\text{extra funds required up front}}$	=	$\underbrace{(1-\theta)(1-\delta)}_{\text{extra amount recovered}}$

- **Implicit interest rate on additional amount borrowed when leasing**

$$R_l \equiv \frac{(1-\theta)(1-\delta)}{\wp - R^{-1}u_l} = R\frac{1}{1 - \frac{m}{(1-\theta)(1-\delta)}} > R$$

  - Leasing is costly financing since $R_l > R$

- **Implicit "down payment" when leasing**

$$R^{-1}u_l = 1 - \underbrace{R^{-1}\theta(1-\delta)}_{\text{financed at } R} - \underbrace{R_l^{-1}(1-\theta)(1-\delta)}_{\text{financed at } R_l}$$

- **Who leases?**
  - Severely constrained firms do!
  - As $w \to 0$, $k \to 0$ and $f'(k) \to +\infty$; hence, using FOCs, $\mu_0 \to +\infty$ and

$$R\lambda/\mu_0 = 1 - \beta/\mu_0 R \to 1 \qquad \Rightarrow u(w) \to r + \delta + (1-\theta)(1-\delta)$$

  - Assuming $(1-\theta)(1-\delta) > m$, borrowers with sufficiently low w lease all their capital!

Repo Markets as discussed in Gorton/Metrick (2012)

- **Collateralized financing**
  - Securitization
    - Pooling of assets, sold to separate legal entities (special purpose vehicles (SPVs))
    - SPVs are financed with (mostly) debt of different seniority (tranching)
  - **Repo ("repurchase agreements")**
    - Agreement to sell and repurchase security; form of (*super*-)collateralized financing
- **Cost of financing during crisis**
  - Spreads (on asset backed securities) and repo rates (interest rate on repos) "blow out"
- **Facts**
  - Spreads on various asset classes correlated with
    - ... LIBOR-OIS spread but *not* ABX (Asset-Backed subprime RMBS Index)
  - **Haircuts** (essentially "down-payment requirements") on repos are correlated with
    - ... volatility but *not* LIBOR-OIS spread or ABX
- **Questions**
  - What type of collateralized financing are repos arguably (and remarkably) similar too?
  - Repos are collateralized – why would there be a run?
  - Is there a "run on repo" during the financial crisis and, if so, in what sense?

Conclusions

- **Rental markets**
  - Renting capital facilitates repossession
  - Lessor is financier but retains ownership
  - Leasing permits greater leverage which is beneficial for severely constrained firms
- **Repo markets**
  - Collateralized loans in which lender owns collateral
  - Haircuts vary with volatility - but why?