# Dynamic Collateralized Finance

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CFAR, Washington University in St. Louis
August 13, 2015

# Aim: Tractable Dynamic Model of Collateralized Financing

- Key friction: **limited enforcement**
  - Enforcement of repayment by borrower limited to tangible assets
  - Implication: **collateral constraints**
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable!
- Key substantive implications
  (1) **Capital structure**
    - Determinant: fraction tangible assets required for production
  (2) **Risk management**
    - Involves state contingent promises and needs collateral
    - Opportunity cost: forgone investment
    - Severely constrained firms do not hedge
  (3) **Leasing and rental markets**
    - Leasing has repossession advantage and permits greater borrowing
    - Severely constrained firms lease
- **Useful laboratory to study dynamics of financial constraints**

# Papers on Dynamic Collateralized Finance

- **Corporate capital structure, risk management, and leasing**
  - Rampini, A.A., and S. Viswanathan, 2010, Collateral, risk management and the distribution of debt capacity, *Journal of Finance* 65, 2293-2322.
  - Rampini, A.A., and S. Viswanathan, 2013, Collateral and capital structure, *Journal of Financial Economics* 109, 466-492.
  - Rampini A.A., A. Sufi, and S. Viswanathan, 2014, Dynamic risk management, *Journal of Financial Economics* 111, 271-296.

- **Financial intermediation**
  - Rampini, A.A., and S. Viswanathan, 2015a, Financial intermediary capital, working paper.

- **Household insurance and other applications**
  - Rampini, A.A., and S. Viswanathan, 2015b, Household risk management, working paper.
  - Rampini, A.A., 2015, Financing durable assets, working paper.

# (1) Capital Structure

- **Collateral key determinant of capital structure**
  - Enforcement of repayment by borrower limited to tangible assets
  - Nature of assets required for production determines financing

- Key papers: Rampini/Viswanathan (2010, 2013)

# (Frictionless) Neoclassical Theory of Investment

- Environment
  - Discrete time, infinite horizon, deterministic (for now)
  - Investor/owner
- Preferences
  - Investor is risk neutral and discounts at rate $R^{-1} < 1$
- Endowments
  - Investor net worth $w \gg 0$, i.e., deep pockets
- Technology
  - Capital $k$ invested in current period
  - Payoff ("cash flow") next period $Af(k)$
    - Parameter $A > 0$ is "total factor productivity" (TFP)
  - Strict concavity $f_k(k) > 0$ and $f_{kk}(k) < 0$; also: $\lim_{k \to 0} f_k(k) = +\infty$; $\lim_{k \to \infty} f_k(k) = 0$
  - Capital is durable and depreciates at rate $\delta \in (0, 1]$
    - Depreciated capital $k(1 - \delta)$ remains next period

# Neoclassical Investment: Investor's Problem

- **Investor's objective**
  - Maximize "value" – present discounted value of dividends
- **Investor's problem - recursive formulation**
  - Choose current dividend $d$ and invest capital $k$ to solve

$$\max_{\{d,w',k\}} d + R^{-1}v(w')$$

subject to budget constraints (but no limited liability constraints)

$$\begin{aligned} w &\geq d + k \\ Af(k) + k(1-\delta) &\geq w' \end{aligned}$$

# Neoclassical Investment and User Cost of Capital

- First-order conditions ($FOC$s) (multipliers $\mu$ and $R^{-1}\mu'$)

$$\begin{aligned} 1 &= \mu \\ R^{-1} &= R^{-1}\mu' \\ \mu &= R^{-1}\mu'[Af_k(k) + (1-\delta)] \end{aligned}$$

- **Investment Euler Equation**
  - Optimal investment/capital $k^*$ solves (combining $FOC$s)

$$1 = R^{-1}[Af_k(k) + (1-\delta)]$$

  or letting $R \equiv 1 + r$ and rewriting

$$\underbrace{r + \delta}_{\text{user cost of capital}} = \underbrace{Af_k(k)}_{\text{marginal product of capital}}$$

- **Jorgenson's (1963) user cost of capital** (paid at end of period)

$$u \equiv \underbrace{r}_{\text{interest rate}} + \underbrace{\delta}_{\text{depreciation rate}}$$

# Collateral Constraints as in Rampini/Viswanathan

- Environment with frictions (otherwise as before)

- Two types of agents
  - Owner/borrower
  - Investor/lender

- Owner/borrower ("firm," "entrepreneur")
  - Preferences: risk neutral, impatient $\beta < R^{-1}$, subject to limited liability
  - Endowment: borrower has limited funds $w > 0$

- Investor/lender has deep pockets (as before)

- **Collateral constraints**
  - Need to collateralize loan repayment with tangible assets

# Collateral and Limited Enforcement

- Question: why does borrower need to collateralize loans?
  - Enforcement is limited and it has to be incentive compatible for borrower to repay
- **Friction: limited enforcement without exclusion**
  - Borrower can abscond with all cash flows and fraction $1 - \theta$ of (depreciated) capital

# Limited Enforcement Implies Collateral Constraints

- **Enforcement constraint**
  - Ensure that borrower prefers to repay instead of absconding; heuristically,

$$\underbrace{v(w')}_{\text{value when repaying}} \geq \underbrace{v(Af(k) + (1-\theta)k(1-\delta))}_{\text{value when defaulting}}$$

and since $v(\cdot)$ is strictly increasing

$$w' \geq Af(k) + (1-\theta)k(1-\delta)$$

and using budget constraint to substitute for $w'$ given borrowing $b$

$$\underbrace{Af(k) + k(1-\delta) - Rb}_{\text{payoff when repaying}} = w' \geq \underbrace{Af(k) + (1-\theta)k(1-\delta)}_{\text{payoff when defaulting}}$$

- **Collateral constraint**
  - Canceling terms and rearranging enforcement constraint we obtain

$$\theta k(1-\delta) \geq Rb$$

# Limited Enforcement – Collateral Constraints: Equivalence

- Proof (sketch) – see Rampini/Viswanathan (2013), Appendix B

- **Limited enforcement problem**
  - Start with limited enforcement problem in sequence formulation
  - **[Step 1]** Present value of remaining sequence of promises can never exceed current collateral value
    - Otherwise default and reissue same promises $\Rightarrow$ borrower better off
  - **[Step 2]** Any sequence of promises satisfying this condition can be implement with one-period ahead state-contingent claims subject to collateral constraints
  - Results in collateral constraint problem in sequence formulation

- **Collateral constraint problem – recursive formulation**
  - Define state variable (net worth $w$) appropriately

# Dynamic Financing Problem with Collateral Constraints

- **Firm's problem**

$$v(w) \equiv \max_{\{d,k,b,w'\}} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d + k \\ Af(k) + k(1-\delta) &\geq w' + Rb \\ \theta k(1-\delta) &\geq Rb \end{aligned}$$

and limited liability $d \geq 0$

- Net worth next period $w' = Af(k) + k(1-\delta) - Rb$

# First Order Conditions and Investment Euler Equation

- First-order conditions (multipliers $\mu$, $\beta\mu'$, and $\beta\lambda'$)

$$1 \leq \mu, \qquad v_w(w') = \mu'$$

$$\mu = \beta\mu'[Af_k(k) + (1-\delta)] + \beta\lambda'\theta(1-\delta), \qquad \mu = \beta\mu' R + \beta\lambda' R$$

  - Also: envelope condition $v_w(w) = \mu$

- **Investment Euler Equation**

$$1 = \beta\frac{\mu'}{\mu}\frac{Af_k(k) + (1-\theta)(1-\delta)}{1 - R^{-1}\theta(1-\delta)}$$

# Tangible Assets as Collateral and Capital Structure

- **"Minimal down payment"** (per unit of capital)

$$\wp \equiv 1 - \underbrace{R^{-1}\theta(1-\delta)}_{\text{PV of } \theta \times \text{ resale value of capital}}$$

- **Capital structure**
    - In deterministic case, collateral constraints always bind
    - Debt per unit of capital

$$R^{-1}\theta(1-\delta)$$

    - Internal funds per unit of capital

$$\wp = 1 - R^{-1}\theta(1-\delta)$$

# Investment Policy

- **Investment Euler Equation** for dividend paying firm

$$1 = \beta \frac{Af_k(k) + (1-\theta)(1-\delta)}{\wp}$$

- Dividend paying firm: capital $\bar{k}$ solves equation above
  - Comparing $FOC$s can show $\bar{k} < k^*$ (**underinvestment**)
- Non-dividend paying firm: $k = \frac{1}{\wp} w$ (invest all net worth and lever as much as possible)

# Dividend Policy

- Threshold policy
- Pay out dividends today $(d' > 0)$ if $w \geq \bar{w}$
- Can we show threshold is optimal?
  - Suppose pay dividends at $w$ but not at $w^+ > w$
  - At $w$, invest $\bar{k}$
  - If not paying dividends at $w^+$, must invest *more;* can $IEE$ hold?

# Value of Internal Funds

- **Value of internal funds** $\mu$ (remember the envelope condition?)
  - Premium on internal funds (unless firm pays dividends) since $\mu \geq 1$

- **User cost** $u(w)$
  - User cost such that $u(w) = R\beta\frac{\mu'}{\mu}Af_k(k)$ where

$$u(w) \equiv r + \delta + \underbrace{R\beta\frac{\lambda}{\mu}(1-\theta)(1-\delta)}_{\text{internal funds require premium}} > u$$

# Net Worth Accumulation and Firm Growth

- **Dividend policy and net worth accumulation**
  - Dividend policy is threshold policy
  - For $w \geq \bar{w}$, pay dividends $d = w - \bar{w}$
  - For $w < \bar{w}$, pay no dividends and reinvest everything ("retain all earnings")
- **Investment policy and firm growth**
  - For $w \geq \bar{w}$, keep capital constant at $\bar{k}$ (no growth)
  - For $w < \bar{w}$, invest everything $k = \frac{1}{\wp}w$ resulting in net worth $w' > w$ next period
- **Firm age**
  - Young firms ($w < \bar{w}$) do not pay dividends, reinvest everything, grow
  - Mature firms ($w \geq \bar{w}$) pay dividends and do not grow

# Dynamic Debt Capacity Management: Stochastic Case

- Environment as before but here with uncertainty
  - Uncertainty: Markov chain state $s' \in \mathcal{S}$ next period – transition probability $\Pi(s, s')$
  - Two types of agents, owner/borrower and investor/lender

- **Preferences**
  - Borrower is risk neutral, impatient $\beta$, and subject to limited liability
  - Lender is risk neutral and discounts at $R^{-1} \in (\beta, 1)$

- **Endowments**
  - Borrower has limited funds $w > 0$
  - Lender has deep pockets

# Dynamic Debt Capacity Management (Cont'd)

- **Technology**
  - Capital $k$ invested in current period yields stochastic payoff ("cash flow") in state $s'$ next period

$$A(s')f(k)$$

    where $A' \equiv A(s')$ is realized "total factor productivity" (TFP)
  - Strict concavity $f_k(k) > 0$; $f_{kk}(k) < 0$; also: $\lim_{k\to 0} f_k(k) = +\infty$; $\lim_{k\to\infty} f_k(k) = 0$
  - Capital is durable and depreciates at rate $\delta$
    - Depreciated capital $k(1-\delta)$ remains next period

- **Collateral constraints**
  - Need to collateralize all promises to pay with tangible assets
  - Can pledge up to fraction $\theta < 1$ of value of depreciated capital

# Firm's Dynamic Debt Capacity Management Problem

- **State-contingent borrowing** $b' \equiv b(s')$
  - Collateral constraint for state-contingent borrowing $b'$

$$\theta k(1-\delta) \geq Rb'$$

- **Firm's debt capacity use problem**

$$\max_{\{d,w',k,b'\}} d + \beta \sum_{s' \in \mathcal{S}} \Pi(s,s') v(w',s')$$

subject to budget constraints and **collateral constraints**, $\forall s' \in \mathcal{S}$,

$$\begin{aligned}
\underbrace{w + \sum_{s' \in \mathcal{S}} \Pi(s,s') b'}_{\text{total borrowing}} &\geq d + k \\
A' f(k) + k(1-\delta) &\geq Rb' + w' \\
\theta k(1-\delta) &\geq Rb'
\end{aligned}$$

and limited liability $d \geq 0$

# Dynamic Debt Capacity Choice – Optimality Conditions

- First-order conditions (multipliers $\mu$, $\Pi(s,s')\beta\mu(s')$, and $\Pi(s,s')\beta\lambda(s')$)

$$1 \leq \mu, \qquad v_w(w',s') = \mu'$$

$$\wp\mu = \sum_{s' \in \mathcal{S}} \Pi(s,s')\beta\mu'[A'f_k(k) + (1-\theta)(1-\delta)], \qquad \mu = \beta\mu' R + \beta\lambda' R$$

- **Investment Euler equation**

$$1 = \sum_{s' \in \mathcal{S}} \Pi(s,s')\beta\frac{\mu'}{\mu}\frac{A'f_k(k) + (1-\theta)(1-\delta)}{\wp}$$

- Firms do not exhaust debt capacity against all states
  - Debt capacity use/leverage: $\theta(1-\delta) \geq R\sum_{s' \in \mathcal{S}} \Pi(s,s')b'/k$
  - Recall: equality in deterministic case

# Stationary Distribution of Net Worth

- **Induced transition function $P$**
  - Optimal policy together with Markov process induce transition function $P$ on $(W, \mathcal{W})$
    - Induced state space of net worth $W = [\varepsilon_w, w_{bnd}] \subset \mathbb{R}$
  - Operator on bounded, cont. functions $T : B(W, \mathcal{W}) \to B(W, \mathcal{W})$
  - Operator on probability measures $T^* : \mathcal{P}(W, \mathcal{W}) \to \mathcal{P}(W, \mathcal{W})$
  - Show that $P$ satisfies properties such that $\exists !$ stationary distribution

- **Stationary distribution allows computation of moments**
  - Computation of steady-state moments
  - Characterization of cross-sectional and time-series properties
  - Simulation and analysis using simulated data

# Structural/Quantitative Work: Li/Whited/Wu (2015)

- Li, S., T.M. Whited, and Y. Wu, 2015, Collateral, taxes, and leverage, working paper.

- **Structural estimation of Rampini/Viswanathan (2013)**
  - Simulated Method of Moments (SMM)
  - Data: non-financial Compustat firms; 1965-2012
  - Assumptions:
    - $f(k) = k^{\alpha}$; $\beta$ calibrated; 12 steady-state moments matched
    - $z \equiv \log(A)$ with $z' = \rho_z z + \varepsilon'$; discrete-state approximation to AR(1)
  - Estimated parameter values

| Parameter | $\delta$ | $\alpha$ | $\rho_z$ | $\sigma_z$ | $R^{-1} - \beta$ | $\hat{\theta}$ |
|---|---|---|---|---|---|---|
| Estimate | 0.081 | 0.782 | 0.631 | 0.418 | 0.032 | 0.365 |
| | (0.005) | (0.034) | (0.027) | (0.019) | (0.016) | (0.007) |

  - Firms conserve some debt capacity, albeit limited amount
    - Simulated debt (incl. interest) is 0.304; roughly 90% of debt capacity
  - Remarkable: adding taxes to model leaves capital structure largely unchanged

# Collateralizability vs. Tangibility

- **Collateralizability** $\theta$
  - Structures more collateralizable than equipment (composition varies by industry)
  - Financial development may raise $\theta$ and hence leverage
- **Tangibility** $\varphi$
  - Includes mainly structures (incl. land) and equipment
  - Suppose tangible assets are collateralizable (but not intangible assets)
  - Fraction tangible assets ($\varphi$) needed for production key

$$\wp(\varphi) = 1 - R^{-1}\varphi\theta(1-\delta)$$

- Interpretation of $\hat{\theta}$ in Li/Whited/Wu (2015)
  - $\hat{\theta}$ should be interpreted as $\varphi\theta$
  - Substantial variation in estimated $\hat{\theta}$ across 24 industries
  - Correlation of estimated $\hat{\theta}$ with industry asset tangibility $\varphi$: 0.53
    - Slope in cross-industry regression: 0.99

# Conclusions for Capital Structure

- **Tangible assets as collateral**
  - If debt needs to be collateralized, type of assets required determines capital structure
- **Dynamics of financing**
  - Accumulate net worth over time
  - Young firms grow and retain all earnings
  - Mature firms pay dividends and grow less
  - Firms conserve debt capacity to some extent

# (2) Corporate Risk Management

- Financial constraints give rationale for corporate risk management
  - If firms' net worth matters, then firms are as if risk averse
  - Collateral constraints link financing and risk management
  - More constrained firms hedge less and often not at all
- Key papers: Rampini/Viswanathan (2010, 2013)
  - Rampini/Sufi/Viswanathan (2014) consider input price risk management (see below)

# Collateral and Corporate Risk Management

- Why should firms hedge?
  - Firms are risk neutral, why hedge?
  - Financial constraints make firms risk averse
    - Firms' value function concave in net worth

- **Financing vs. risk management trade-off**
  - Limited enforcement: need to collateralize promises to financier and counterparties
  - Collateral constraints link financing and risk management
  - More constrained firms hedge less as financing needs dominate hedging concerns

- Relatedly for **households:** financing vs. insurance trade-off
  - "The poor can't afford insurance"
  - Rampini/Viswanathan (2015b) (see (5) below)

# Corporate Risk Management Problem

- **Equivalent risk management formulation**
  - Collateral constraint for state-contingent borrowing $b'$
$$\theta k(1-\delta) \geq Rb'$$
  - Equivalently, borrow as much as possible and hedge $h' \equiv \theta k(1-\delta) - Rb' \geq 0$
- **Firm's risk management problem**
$$\max_{\{d,w',k,h'\}} d + \beta \sum_{s' \in \mathcal{S}} \Pi(s,s') v(w',s')$$
subject to budget constraints and **short sale constraints**, $\forall s' \in \mathcal{S}$,
$$\begin{aligned} w &\geq d + \wp k + \underbrace{R^{-1} \sum_{s' \in \mathcal{S}} \Pi(s,s') h'}_{\text{cost of hedging portfolio}} \\ A' f(k) + (1-\theta)k(1-\delta) + h' &\geq w' \\ h' &\geq 0 \end{aligned}$$
and limited liability $d \geq 0$

# Financing vs. Risk Management Trade-off

- **Investment Euler equation**

$$
\begin{aligned}
1 &= \sum_{s' \in \mathcal{S}} \Pi(s,s')\beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp} \\
&\geq \Pi(s,s')\beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp}
\end{aligned}
$$

  - As $w \to 0$, capital $k \to 0$ and marginal product $f_k(k) \to \infty$
  - Therefore, marginal value of net worth in state $s'$ (relative to current period) $\mu'/\mu \to 0$
  - Using first order condition for hedging

$$\lambda'/\mu = (\beta R)^{-1} - \mu'/\mu > 0$$

    so **severely constrained firms do not hedge at all**

- **Financing vs. risk management trade-off**
  - Hedging uses up net worth which is better used to purchase additional capital/downsize less

- IID case: if firms hedge, they hedge states with low net worth due to low cash flows

# Why Was This Not Previously Recognized?

- **Reasons for incomplete hedging – as in Tirole (2006)**
  - 5 reasons provided (beyond "transactions costs")
    - (i) market power; (ii) serial correlation of profits; (iii) aggregate risk; (iv) asymmetric information; (v) incentives
  - Fact that hedging uses up net worth is not listed
    - That said, Holmström/Tirole (2000) come close
- **No financing risk management trade-off in previous models**
  - Models consider risk management using frictionless markets
    - Without imposing same frictions on financing and hedging, no trade-off
  - Models have no financing in first period where firms hedge
    - Without investment which requires financing, no trade-off
- **Intuitive, but counterfactual, prediction: more constrained firms hedge more**
  - Froot/Scharfstein/Stein (1993)
- In practice, more constrained (and smaller) firms hedge less!

# Input Price Risk Management

- **Profit functions are convex in prices** – basic microeconomics
  - In practice, many firms hedge input prices (e.g., airlines)
  - Say additional input $x'$ needed for production with stochastic price $p'$
  - Induced within-period profit function (with $\hat{\alpha} > 0$, $\phi > 0$, $\hat{\alpha} + \phi < 1$)

$$\pi(k) \equiv \max_{x'} \hat{A}' k^{\hat{\alpha}} x'^{\phi} - p'x' \equiv A'k^{\alpha}$$

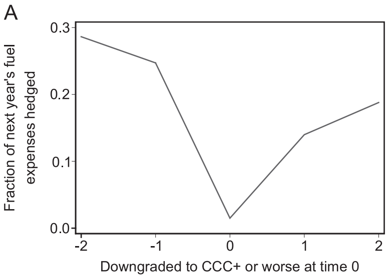
where $\alpha \equiv \frac{\hat{\alpha}}{1-\phi}$ and $A' \equiv \hat{A}'^{\frac{1}{1-\phi}}(1-\phi)\phi^{\frac{\phi}{1-\phi}} p'^{-\frac{\phi}{1-\phi}}$; **convex in** $p'$

- But: **firms as if risk averse in net worth**
  - Hedging does not change spot price $p'$; convexity irrelevant
  - Hedging shifts net worth across states; value function **concave in** $w'$

- **Ad-hoc approach to modeling risk management fails**
  - Ad-hoc model: hedging means buying input at expected price $E[p'|s]$
  - Fails given convexity of profit function!

# Empirical: Rampini/Sufi/Viswanathan (2014)

- **Fuel price risk management** by airlines
- Why useful empirical laboratory? – Panel data on hedging intensity
  - Fraction of next year's expected fuel expenses hedged
  - Most other studies
    - Dummies for derivatives use – extensive margin only
    - Single cross section – no within-firm variation
- Evidence in cross section and time series consistent with theory
  - **More constrained airlines hedge less – across and within airlines**
  - Hedging around distress – within-airline variation



# Ad-hoc Ex-ante Collateral Constraints

- **Ex-ante collateral constraints and limited enforcement**
  - Literature at times imposes ex-ante collateral constraints

$$\hat{\theta}k \geq \sum_{s' \in S} \Pi(s, s')b'$$

    instead of our state-by-state ex-post constraints, $\forall s' \in S$,

$$\theta k(1-\delta) \geq Rb'$$

  - Ex-ante limited enforcement: abscond ex ante with dividend and $1-\hat{\theta}$ of capital and borrow from other lender

$$v(w) \geq v(d_0 + (1-\hat{\theta})k)$$

    implies ex-ante collateral constraints using budget constraint

- **Equivalence in deterministic case or with non-contingent debt**
  - Setting $\hat{\theta} \equiv R^{-1}\theta(1-\delta)$ equivalent under these conditions
- But: **no constraints on risk management**
  - Only one collateral constraint so $\mu = \beta\mu' R + \beta\lambda R$; all $\mu'$ equalized
  - Counterfactual implications – complete hedging!

# Conclusions for Corporate Risk Management

- Rationale for **corporate risk management**
  - Financial constraints make firms as if risk averse

- Trade-off between financing and risk management
  - Promises to financiers and hedging counterparties need to be collateralized
  - Severely constrained firms hedge less or not at all
    - ... both in theory and in practice
  - Such firms may be more susceptible to downturns

# (3) Leasing and Rental Markets

- Leasing has repossession advantage and permits greater borrowing
- Severely constrained firms (and households) lease
- Key papers: Rampini/Viswanathan (2013, 2015b); Eisfeldt/Rampini (2009)

# Financing Subject to Collateral Constraints

- Environment with collateral constraints but firms can lease
  - Three types of agents, owner/borrower, investor/lender, and lessor
  - Borrower is risk neutral, impatient $\beta < R^{-1}$, and subject to limited liability
  - Borrower has limited funds $w > 0$
  - Lender and lessor have deep pockets, discount at $R^{-1}$
  - For simplicity, deterministic case here

- **Borrowing subject to collateral constraints**
  - Need to collateralize promises to pay with tangible assets (due to limited enforcement)
  - Promised repayment $\leq \theta \times$ resale value of tangible assets

# Leasing as in Eisfeldt/Rampini and Rampini/Viswanathan

- **Leasing:** borrower can rent capital
- **Repossession advantage**
  - Borrower cannot abscond with leased capital
  - In practice, repossession of rented capital easier than foreclosure on secured loan
  - Leasing allows borrower to borrow full resale value, not just fraction $\theta$
- **Monitoring cost** $m$ (per unit of capital)
  - Lessor needs to monitor to prevent abuse
  - Why? – Leasing separates ownership and control
- **User cost of leased capital**

$$u_l \equiv r + \delta + m$$

needs to be paid in advance (i.e., at beginning of period)

# Firm's Problem with Leasing and Secured Lending

- **Firm's problem with leasing** ($k_o$ owned capital; $k_l$ leased capital)

$$\max_{\{d,w',k_o,k_l,b\}} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d + k_o + R^{-1}u_l k_l \\ Af(k_o + k_l) + k_o(1-\delta) &\geq Rb + w' \\ \theta k_o(1-\delta) &\geq Rb \end{aligned}$$

non-negativity constraints $k_o$, $k_l \geq 0$, and limited liability $d \geq 0$

# Leasing and Secured Lending – Optimality Conditions

- First-order conditions (multipliers $\mu$, $\beta\mu'$, and $\beta\lambda$; let $k \equiv k_o + k_l$)
- As before,

$$1 \leq \mu, \qquad v_w(w') = \mu', \qquad \mu = \beta\mu' R + \beta\lambda R$$

and almost as before (except inequality as borrower might not own any assets)

$$\mu \geq \beta\mu'[Af_k(k)+(1-\delta)]+\beta\lambda\theta(1-\delta) \qquad \Leftrightarrow \qquad u(w) \geq R\beta\frac{\mu'}{\mu}Af_k(k)$$

and finally new

$$R^{-1}u_l\mu \geq \beta\mu' Af_k(k) \qquad \Leftrightarrow \qquad u_l \geq R\beta\frac{\mu'}{\mu}Af_k(k)$$

# Lease or Buy?

- Lease if $u_l < u(w)$ and buy otherwise ("choose capital with lower user cost")
- Recall

$$u_l = r + \delta + \underbrace{m}_{\text{monitoring cost}}$$

and

$$u(w) = r + \delta + \underbrace{\beta R\lambda/\mu(1-\theta)(1-\delta)}_{\text{premium on internal funds required}}$$

# Leasing as Costly, Highly Collateralized Financing

- **Incremental cash flows of buying vs. leasing**

| | | | | |
|---|---|---|---|---|
| Time | | $0$ | | $1$ |
| Buying (secured loan) | | $1 - R^{-1}\theta(1-\delta)$ | | $(1-\theta)(1-\delta)$ |
| Leasing | | $R^{-1}u_l$ | | |
| Diff buying - leasing | $=$ | $\underbrace{\wp - R^{-1}u_l}_{\text{extra funds required up front}}$ | $=$ | $\underbrace{(1-\theta)(1-\delta)}_{\text{extra amount recovered}}$ |

- **Implicit interest rate on additional amount borrowed by leasing**

$$R_l \equiv \frac{(1-\theta)(1-\delta)}{\wp - R^{-1}u_l} = R\frac{1}{1-\frac{m}{(1-\theta)(1-\delta)}} > R$$

- Leasing is costly financing since $R_l > R$

# Financially Constrained Firms Lease

- **Implicit "down payment" when leasing**

$$R^{-1}u_l = 1 - \underbrace{R^{-1}\theta(1-\delta)}_{\text{financed at } R} - \underbrace{R_l^{-1}(1-\theta)(1-\delta)}_{\text{financed at } R_l}$$

- **Who leases?**
  - Severely constrained firms do!
  - As $w \to 0$, $k \to 0$ and $f_k(k) \to +\infty$; using $FOC$s, $\mu'/\mu \to 0$ and

$$\beta R\lambda/\mu = 1 - \beta R\mu'/\mu \to 1 \qquad \Rightarrow u(w) \to r + \delta + (1-\theta)(1-\delta)$$

  - Assuming $(1-\theta)(1-\delta) > m$, borrowers with sufficiently low $w$ lease all their capital!

# Conclusions for Leasing and Rental Markets

- Renting capital facilitates repossession
- Lessor is financier but retains ownership
- **Leasing permits greater leverage – beneficial for severely constrained firms**
- Despite quantitative importance, rental markets largely ignored in theoretical and empirical economics (finance, macro, development)

# (4) Financial Intermediation

- Rampini/Viswanathan (2015a)
- **Economy with limited enforcement and limited participation**
  - Two sub periods
    - Morning: cash flows realized; more ($\theta_i$) capital collateralizable
    - Afternoon: investment/financing; only fraction $\theta < \theta_i$ collateralizable
  - Limited participation with two types of lenders
    - Households present only in afternoons; intermediaries always
  - Optimal contract implemented with two sets of one-period Arrow securities (for morning and afternoon)
- **Financial intermediaries as collateralization specialists**
  - Intermediaries need to enforce morning claims
  - Intermediaries need to finance morning claims out of own net worth
  - Intermediated finance is short term
- **Role for intermediary capital**
  - Economy with two state variables: firm and intermediary net worth

# (5) Dynamic Household Insurance

- Rampini/Viswanathan (2015b)
- **Risk-averse household with stochastic income $y'$**

$$\max_{\{c,w',h'\}} u(c) + \beta \sum_{s' \in \mathcal{S}} \Pi(s,s')v(w',s')$$

subject to budget constraints and **short sale constraints**, $\forall s' \in \mathcal{S}$,

$$\begin{aligned} w &\geq c + R^{-1} \sum_{s' \in \mathcal{S}} \Pi(s,s')h' \\ y' + h' &\geq w' \\ h' &\geq 0 \end{aligned}$$

- Under stationary distribution, **household risk management is ...**
  - **incomplete** with probability 1; **absent** with positive probability
  - **globally increasing** in net worth and income
  - **precautionary** (increases when income gets riskier)
- **Insurance is state-contingent savings**
  - Insurance premia paid up front; intertemporal aspect to insurance

# (6) Durability and Financing – Rampini (2015)

- **Durability facilitates financing** – Hart/Moore (1994)
  - Define higher durability as lower depreciation rate $\delta$

$$\frac{\partial \wp}{\partial \delta} = \frac{\partial}{\partial \delta}\left\{1 - R^{-1}\theta(1-\delta)\right\} = R^{-1}\theta > 0$$

  - Durable assets easier to finance due to higher collateral value

- To the contrary: **durability impedes financing**
  - Keep frictionless user cost $u = r + \delta$ constant not price; so $q = \frac{u}{r+\delta}$

$$\frac{\partial \wp}{\partial \delta} = \frac{\partial}{\partial \delta}\left\{\frac{u}{r+\delta}(1 - R^{-1}\theta(1-\delta))\right\} = -q\frac{1-\theta}{r+\delta} < 0$$

  - Durable assets cost more and require larger down-payments

- Implications for technology adoption, incidence of financial constraints, choice of capital vintage

# Models of Dynamic Collateralized Financing – Conclusion

- **Useful laboratory to study dynamic financing problems**
  - Tractability allows explicit theoretical analysis of dynamics
  - Insights yielded so far
    - Capital structure/debt capacity
    - Risk management/insurance
    - Leasing
    - Intermediation
    - Durability
  - Dynamic models facilitate quantitative work/structural estimation
- **Empirically/quantitatively plausible class of models**