Collateral and Intermediation in Equilibrium

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Research Agenda on Collateralized Finance

- Bulk of (external/debt) **financing collateralized in practice**
  - Households: mortgages, car loans, etc.
  - Firms: tangible assets primary determinant of leverage
  - Intermediaries: real estate finance (commercial and residential)
- Aim: tractable dynamic model of collateralized financing
- Macro finance (focus of today's talk)
  - Equilibrium consequences of collateral scarcity
  - Macro implications of collateralized intermediary finance
- Micro finance
  - Dynamic debt capacity and risk management; leasing
- Terminology
  - Micro/macro "corporate" finance – effects of financial frictions

Tractable Dynamic Model of Collateralized Financing

- Key friction: **limited enforcement**
  - Enforcement of repayment by borrower limited to tangible assets
  - Novel assumption: no exclusion
  - Implication: **collateral constraints**
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable!
- **Useful laboratory to study dynamics of financial constraints**

Collateral & Intermediation in Equilibrium – Macro Finance

- **Equilibrium household insurance**
  - Rampini/Viswanathan (2017a) Household risk management
  - Collateral scarcity lowers equilibrium interest rate limiting insurance
  - Insurance is state-contingent savings – crucial: intertemporal aspect
- **Financial intermediation**
  - Rampini/Viswanathan (2017b) Financial intermediary capital
  - Intermediaries with collateralization advantage require capital
  - Rich implications for economic dynamics

Dynamic Corporate Finance Implications – Micro Finance

(1) Capital structure

- Determinant: fraction tangible assets required for production

(2) Risk management

- Involves state contingent promises and needs collateral
- Opportunity cost: forgone investment
- Severely constrained firms do not hedge

(3) Leasing and rental markets

- Leasing has repossession advantage and permits greater borrowing
- Severely constrained firms lease

(4) Durability

- Durable assets have larger financing needs and are harder to finance

Dynamic Corporate Finance – Micro Finance: Papers

- **Corporate capital structure, risk management, and leasing**
  - Rampini/Viswanathan (*JF* 2010) Collateral, risk management and the distribution of debt capacity
  - Rampini/Viswanathan (*JFE* 2013) Collateral and capital structure
  - Rampini (2017) Financing durable assets
- **Empirical evidence**
  - Rampini/Sufi/Viswanathan (*JFE* 2014) Dynamic risk management
  - Rampini/Viswanathan/Vuillemey (2017) Risk management in financial institutions
  - Li/Whited/Wu (*RFS* 2016) Collateral, taxes, and leverage

(1) Dynamic Household Insurance – Synopsis

- Rampini/Viswanathan (2017a)

- Under stationary distribution, **household risk management is ...**
  - **incomplete** with probability 1; **absent** with positive probability
  - **globally increasing** in net worth and income
  - **precautionary** (increases when income gets riskier)

- **Insurance is state-contingent savings**
  - Insurance premia paid up front; intertemporal aspect to insurance

- **Collateral scarcity**
  - ... lowers equilibrium interest rate
  - ... reducing insurance & increasing inequality

Household Finance in an Endowment Economy: Model

- Discrete time, infinite horizon
- **Households**
  - Preferences: $E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$ where $\beta \in (0,1)$, $u(c)$ strictly increasing, strictly concave, continuously differentiable, $\lim_{c\to 0} u_c(c) = \infty$, $\lim_{c\to\infty} u_c(c) = 0$
  - Income $y(s)$ Markov chain on state space $s \in S$ with transition matrix $\Pi(s,s') > 0$ and $\forall s, s_+$, $s_+ > s$, $y(s_+) > y(s)$
  - Notation: $y' \equiv y(s')$, $\underline{s} = \min\{s : s \in S\}$, $\bar{s} = \max\{s : s \in S\}$, etc.
- **Lenders** (exogenous R for now)
  - Risk neutral, discount at $R^{-1} \in (\beta, 1)$, deep pockets, abundant collateral
- **Limited enforcement – default without exclusion**
  - Optimal dynamic contract can be implemented with **complete markets** in one-period ahead Arrow securities subject to short sale constraints (special case of **collateral constraints**)
  - Related: Rampini/Viswanathan (2010, 2013)

Household's Income Risk Management Problem

- Recursive formulation
- Given s and w, household solves

$$V(w,s) \equiv \max_{c,h',w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[V(w',s')|s] \quad (1)$$

subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + E[R^{-1}h'|s] \quad (2)$$

$$y' + h' \geq w' \quad (3)$$

and short sale constraints, $\forall s' \in S$

$$h' \geq 0 \quad (4)$$

- Arrow securities h' for each state s' (and associated net worth w')
- Endogenous state variable: net worth w (cum current income)

Characterization of Household Risk Management

- Well-behaved dynamic program
  - Return function concave; constraint set convex
  - Operator defined by (1) to (4) satisfies Blackwell's sufficient conditions
  - Solution: $\exists!$ value function v; strictly increasing; strictly concave
- **First order conditions**
  - Denote multipliers on (2) and (3) by μ and $\beta\Pi(s,s')\mu'$ and on (4) by $\beta\Pi(s,s')\lambda'$
  - Ignore non-negativity constraints on consumption (not binding)

$$\begin{aligned}
\mu &= u_c(c) \\
\mu' &= v_w(w',s') \\
\mu &= \beta R\mu' + \beta R\lambda'
\end{aligned}$$

- **Envelope condition:** $v_w(w,s) = \mu$
  - Value function continuously differentiable

Increasing Household Risk Management (Prop. 1)

- **Richer households hedge more states**
  - **(i)** Set of states that household hedges $S_h \equiv \{s' \in S : h(s') > 0\}$ is increasing in net worth w given current state s, $\forall s \in S$

- **Richer households better insured/spend more on hedging**
  - **(ii)** For $w_+ > w$ and denoting net worth next period associated with w_+ (w) by w'_+ (w'), we have
    - $w'_+ \geq w'$ and $c'_+ \geq c'$, $\forall s' \in S$, i.e., w'_+ and c'_+ statewise dominate and hence FOSD w' and c', respectively; moreover, $h'_+ \geq h'$, $\forall s' \in S$, and $E[h'_+|s] \geq E[h'|s]$
    - consumption across hedged states constant, i.e., $c' = c_h$, $\forall s' \in S_h$, and c_h is strictly increasing in w

- Remarks:
  - This does not say which states are hedged
  - All statements are conditional on state s

Income Processes with Positive Persistence

- **Stochastically monotone Markov chain**

- Consider Markov chains which exhibit a notion of positive persistence

  Definition 1 (Monotone Markov chain). *A Markov chain* $\Pi(s, s')$ *is* **stochastically monotone**, *if it displays first order stochastic dominance (FOSD) if* $\forall s, s_+, \hat{s}'$, $s_+ > s$, $\sum_{s' \leq \hat{s}'} \Pi(s_+, s') \leq \sum_{s' \leq \hat{s}'} \Pi(s, s')$

- Remarks:
  - Distribution of states next period conditional on current state s_+ FOSD distribution conditional on current state s for all $s_+ > s$
  - **IID is special case:** $\Pi(s, s') = \Pi(s')$, $\forall s \in S$, is stochastically monotone

Insurance with Stochastic Monotonicity (Prop. 2)

- Assume that $\Pi(s, s')$ is stochastically monotone
- **Key property**
  - **(i)** Marginal value of net worth $v_w(w, s)$ is decreasing in state s
- **Risk management is globally increasing**
  - **(ii)** Household hedges a lower interval of states, $S_h = \{\underline{s}', \ldots, s'_h\}$ given w and s; net worth next period w', hedging h', set of hedged states S_h, and hedged consumption c_h are all monotone increasing in w and s
- Intuition:
  - Higher current income means FOSD shift in income next period $\Rightarrow$ lower marginal value of current net worth
  - If property is satisfied, households hedge lower income realizations more

Insurance with Stochastic Monotonicity (Cont'd)

- **Proof of key property - Prop. 2**

- Define operator T as

$$Tv(w,s) \equiv \max_{c,h',w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w',s')|s]$$

subject to equations (2) through (4)

- Sketch: Show that if v has property that $\forall s, s_+$, $s_+ > s$, $v_w(w,s_+) \leq v_w(w,s)$, then Tv (and fixed point) inherit property

"Richer Households are Better Insured" (Prop. 2)

- **Decreasing variance of net worth and consumption with IID income**

- Assume income process independent: $\Pi(s, s') = \pi(s'), \forall s, s' \in S$

- **Richer households hedge more states/higher net worth**
  - **(ii)** Net worth in hedged states $w(s') = w_h, \forall s' \in S_h$, and w_h is increasing in w

- **Richer households lower variance of net worth and consumption**
  - **(iii)** Variance of net worth w' and consumption c' next period is decreasing in current net worth w

Incomplete Household Risk Management (Prop. 3)

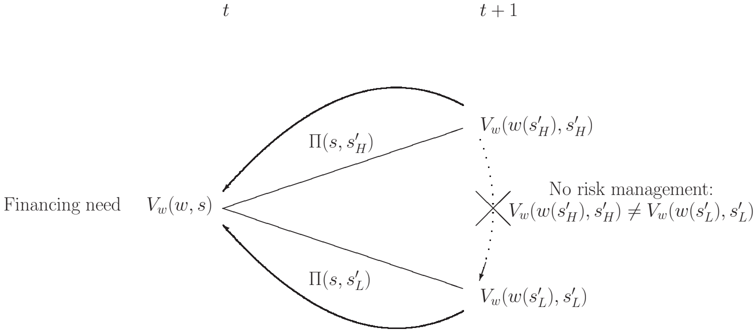
- Assume income process is stochastically monotone

- **"Poor households cannot afford insurance"**
  - **(i)** At net worth $w = \underline{y}$ in state $\underline{s}$, household does not hedge at all, that is, $\lambda' > 0$, $\forall s' \in \overline{S}$, and $S_h = \emptyset$

- **High income households are not completely hedged**
  - **(ii)** At net worth $w = \bar{y}$, household does not hedge highest state next period, that is, $\lambda(\bar{s}') > 0$ and $S_h \subsetneq S$, $\forall s \in S$

Households' Financing Risk Management Trade-off

- **Basic trade-off**
  - Poor households shift net worth to present, not across states next period



Household Risk Management in the Long Run

- **Household risk management under stationary distribution (Prop. 4)**

- Assume that $\Pi(s, s')$ is monotone

- **Existence and uniqueness**
  - **(i)** There exists a unique stationary distribution of net worth

- **Support of net worth distribution**
  - **(ii)** Support of stationary distribution is subset of $[\underline{w}, w_{bnd}]$ where $\underline{w} = \underline{y}$ and $w_{bnd} \geq \bar{y}$ with equality if $\Pi(s, s') = \pi(s'), \forall s, s' \in S$

- **Incomplete household risk management**
  - **(iii)** Under stationary distribution, household risk management is increasing, incomplete with probability 1, and completely absent with strictly positive probability

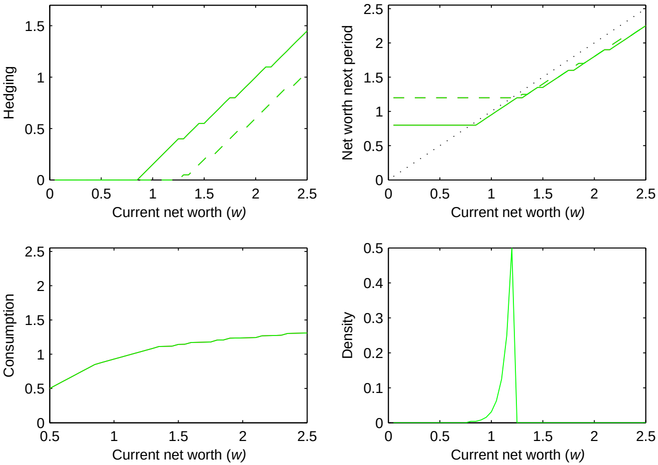
Increasing Household Risk Management

- **A theory of the insurance function?**

- Behavior of insurance similar to savings (see Friedman)
  - **Insurance is “state-contingent saving”**

- Friedman (1957), *A Theory of the Consumption Function*, page 39:
  - *“These regressions show* **savings to be negative at low measured income levels,** *and to be a successively larger fraction of income, the higher the measured income. If low measured income is identified with “poor” and high measured income with “rich,” it follows that the“poor” are getting poorer and the rich are getting richer. The identification of low measured income with “poor” and high measured income with “rich” is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof.”*

Optimal Household Risk Management



- Parameters: $y' \in \{0.8, 1.2\}$, $p = 0.5$, CRRA with $\gamma = 2$

Precautionary Nature of Household Insurance (Prop. 5)

- **Definition:** Behavior is **precautionary** if it increases when risk increases (MPS on y')
- Assume income process independent: $\Pi(s, s') = \pi(s'), \forall s, s' \in S$
- **Risk management is precautionary**
  - $\tilde{\pi}(s')$ is a mean-preserving spread (MPS) of $\pi(s')$
  - Then $\tilde{E}[\tilde{h}'] \geq E[h']$
- Remarkably: **Risk aversion sufficient**
  - No assumptions on prudence ($u_{ccc}(c)$) required
  - Contrast: Classic precautionary savings result in models with incomplete markets (Bewley (1977), Aiyagari (1994), Leland (1968))

(Novel) Global Monotonicity – Why?

- **Household finance**
  - Positive persistence of income further lowers marginal value of net worth when current income realization is high
  - Yields **increasing household risk management** theorem
- **Corporate finance**
  - Positive persistence in cash flows due to productivity shocks
    - Positive persistence productivity shocks implies conditional expected productivity higher
    - **Investment opportunities** raise marginal value of net worth when current productivity is high
  - Effects go in opposite direction; no global monotonicity result

Household Finance with Durable Goods

- Preferences: $u(c) + g(k)$ where k is durable good (e.g., housing)

- **Durable goods**
  - ... **as collateral** for state-contingent debt b'

$$\theta k(1-\delta) \geq Rb'$$

  - ... imply additional financing needs

- Equivalent **risk management formulation**
  - Fully lever durables: set $\hat{b}' = R^{-1}\theta k(1-\delta)$ and pay down

$$\wp \equiv 1 - R^{-1}\theta(1-\delta)$$

  - Hedging with Arrow securities h' subject to short sale constraints

$$h' \equiv \theta k(1-\delta) - Rb'$$

Household's Problem with Durable Goods

- Given s and w, household solves

$$v(w,s) \equiv \max_{c,k,h',w' \in \mathbb{R}^2_+ \times \mathbb{R}^{2S}} u(c) + \beta g(k) + \beta E[v(w',s')|s] \qquad (5)$$

subject to the budget constraints for the current and next period, $\forall s' \in S$

$$\begin{aligned} w &\geq c + \wp k + E[R^{-1}h'|s] \qquad (6) \\ y' + (1-\theta)k(1-\delta) + h' &\geq w' \qquad (7) \end{aligned}$$

and the short sale constraints, $\forall s' \in S$

$$h' \geq 0 \qquad (4)$$

- Remarks
  - Net worth w is cum income and durable goods net of borrowing
  - Investment Euler equation for durable goods

$$1 = \beta \frac{g_k(k)}{\mu} \frac{1}{\wp} + E\left[\beta \frac{\mu'}{\mu} \frac{(1-\theta)(1-\delta)}{\wp} \middle| s \right]$$

Risk Management with Durable Goods: Properties

- **Properties generalize** (Prop. 6)

- **Monotonicity**
  - Net worth next period w' strictly increases in w, given s
    - Hedging h' does not necessarily increase in w

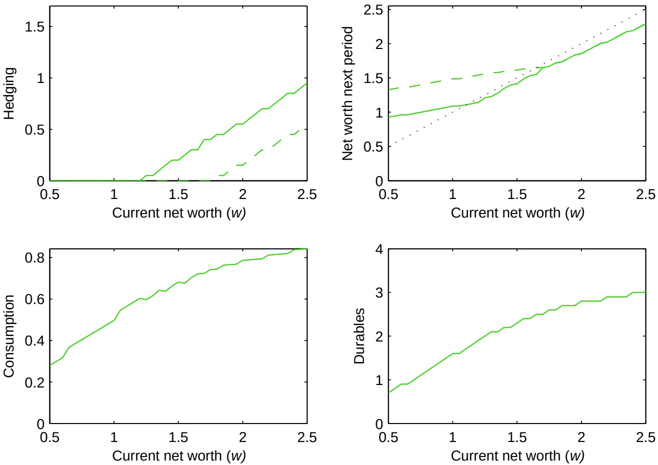
- **Incomplete hedging** (with $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$)
  - Household never hedges the highest state next period

- **Increasing household risk management with stochastic monotonicity**
  - Household hedges lower interval of states
    - Marginal value of net worth $v_w(w, s)$ is decreasing in s

- **Absence of risk management for poor households**
  - For sufficiently low net worth, household is constrained against all states next period
    - Net worth next period

$$w' \geq y' + (1 - \theta)k(1 - \delta) > y'$$

Risk Management with Durable Goods: Example



- Financing needs for durables reduce hedging and increase net worth accumulation
- Parameters: $y' \in \{0.8, 1.2\}$, $p = 0.5$, CRRA with $\gamma = 2$ and $g = 2$, $\theta = 0.8$

Equilibrium and Effect of Collateral on Insurance

- Determine R in equilibrium to clear market for collateralized claims
- In equilibrium, $\beta R \leq 1$, with equality iff $\theta \geq \bar{\theta}$
- When $\theta < \bar{\theta}$, collateral is scarce and $\beta R < 1$ as previously assumed

Stationary Equilibrium

- Definition (Stationary Equilibrium) *A* **stationary equilibrium** *is*
  - *allocation* $x(z) \equiv \{c(z), k(z), h'(z), w'(z)\}$ *for each household given* $z \equiv \{w, s\}$
  - *interest rate* R
  - *stationary distribution* $F(z)$

  such that
  - $x(z)$ *solves each household's problem in (5)-(7) and (4), given* z
  - *market for state-contingent promises clears*

$$\int_z E[b'(z)|s]dF(z) = 0 \tag{12}$$

or equivalently, the supply of collateralized claims equals the demand for state-contingent claims $h'(z)$

$$\int_z \theta k(z)(1-\delta)dF(z) = \int_z E[h'(z)|s]dF(z). \tag{13}$$

- Interpretation: representative insurance company – assets = mortgages; liabilities = insurance claims

Aggregation and Resource Constraints

- Define $W \equiv \int_z w(z)dF(z)$, $C \equiv \int_z c(z)dF(z)$, $K \equiv \int_z k(z)dF(z)$, $H' \equiv \int_z h'(z)dF(z)$.

- Aggregate budget constraints (6) and (7) $\forall s' \in S$

$$W = C + \wp K + E[R^{-1}H'] \quad (14)$$

$$E[y'] + (1-\theta)K(1-\delta) + E[H'] = W \quad (15)$$

- Using market clearing condition $\theta K(1-\delta) = E[H']$, (14) and (15) imply that

$$W = C + K = E[y'] + K(1-\delta) \quad (16)$$

so aggregate wealth = consumption + capital stock or output + depreciated capital stock or

$$E[y'] = C + \delta K$$

Full Insurance in Economy with Abundant Collateral

- Suppose $\beta R = 1$; then $\mu = \mu' + \lambda' \geq \mu'$, $s' \in S$, so $\mu = \mu' = 1$ (full insurance)
- Consumption c^*, net worth w^*, and durable goods k^* constant and investment Euler equation

$$r^* + \delta = \frac{g_k(k^*)}{u_c(c^*)} \tag{17}$$

- Using $\wp^* = 1 - R^{*-1}\theta(1-\delta)$, write (6) and (7) as

$$w^* = c^* + \wp^* k^* + E[R^{*-1}h'^*], \tag{18}$$

$$y' + (1-\theta)k^*(1-\delta) + h'^* = w^*, \qquad \forall s' \in S. \tag{19}$$

- Feasible? – As long as $h'^* \geq 0$, $\forall s' \in S$; using (16) to substitute for w^* in (19)

$$h'^* = \theta k^*(1-\delta) - (y' - E[y']) \geq 0, \qquad \forall s' \in S$$

that is, **sufficient pledgeability**

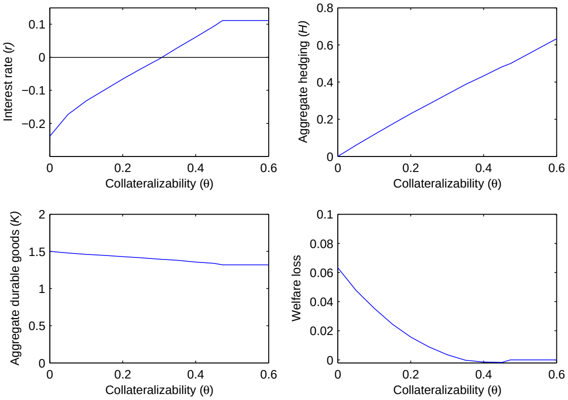
$$\theta \geq \bar{\theta} \equiv \frac{y(\bar{s}') - E[y']}{k^*(1-\delta)} \geq 0$$

Effect of Collateral Scarcity on Interest Rate and Insurance

- Collateral scarcity: $\theta < \bar{\theta}$
- Then $\beta R < 1$ as previously assumed (at $\beta R = 1$ excess demand for collateralized claims)
- Therefore, incomplete insurance and previous characterization applies
- $R < 1$ is possible, that is, negative interest rates $r = R - 1$

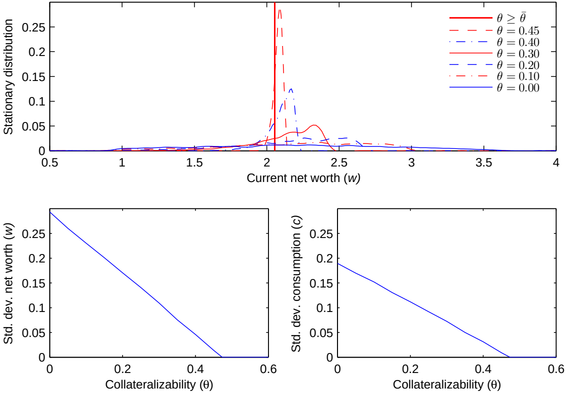
Effect of Collateral Scarcity on Interest Rate and Insurance

- Effect on interest rate, hedging, durable stock, and welfare loss



Effect of Collateral Scarcity on Inequality

- When collateral is scarce, wealth distribution fans out



Conclusion

- When collateral is scarce, **equilibrium insurance**
  - **... is globally incomplete, increasing, and precautionary**
- **Intuition**
  - Intertemporal aspect to insurance
  - Key: **Collateral scarcity ($\beta R < 1$) and collateral constraints**
- Explains basic patterns in insurance and risk management

(2) Financial Intermediation with Collateral Constraints

- Rampini/Viswanathan (2017b) Financial Intermediary Capital
- **Aim: Tractable dynamic theory of financial intermediation**
  - Building on dynamic model of collateralized finance
- **Motivation**
  - Financial crisis and its aftermath
  - Intermediary capitalization critical for macro fluctuations and growth

Collateral and Financial Intermediation – Synopsis

- **Economy with limited enforcement and limited participation**
  - Two sub periods
    - Morning: cash flows realized; more (θ_i) capital collateralizable
    - Afternoon: investment/financing; only fraction $\theta < \theta_i$ collateralizable
  - Limited participation with two types of lenders
    - Households present only in afternoons; intermediaries always
  - Optimal contract implemented with two sets of one-period Arrow securities (for morning and afternoon)
- **Financial intermediaries with collateralization advantage**
  - Intermediaries need to enforce morning claims
  - Intermediaries need to finance morning claims out of own net worth
  - Intermediated finance is short term
- **Role for intermediary capital**
  - Economy with two state variables: firm and intermediary net worth

Our Theory of Financial Intermediary Capital: Implications

- **Relatively slow accumulation of intermediary net worth**

- **Compelling dynamics in model with two state variables**
  - When corporate sector is very constrained,
    - ... **intermediaries "hold cash" at low interest rates**
  - When intermediaries are very constrained,
    - ... **firms' investment stays low even as firms pay dividends**
  - Tentative and halting nature of recoveries from crises

- Consistent with key **stylized facts about macro downturns with credit crunch** (Reinhart and Rogoff (2014) and related literature)
  - **Fact 1: Severity**
  - **Fact 2: Protractedness** ("**halting, tentative**... recoveries")
  - **Fact 3: Severity of credit crunch affects severity/protractedness**

Literature: Models of Financial Intermediaries

- **Intermediary capital**
  - Holmström/Tirole (1997) – need capital at stake to commit to monitor
  - Diamond/Rajan (2000), Diamond (2007) – ability to enforce claims due to better monitoring
- **Other theories of financial intermediation - no role for capital**
  - Liquidity provision theories – Diamond/Dybvig (1983)
  - Diversified delegated monitoring theories – Diamond (1984), Ramakrishnan/Thakor (1984), Williamson (1986)
  - Coalition based theories – Townsend (1978), Boyd/Prescott (1986)

Literature: Dynamic Models with Net Worth Effects

- **Firm net worth**
  - Bernanke/Gertler (1989), Kiyotaki/Moore (1997a)
- **Intermediary net worth**
  - Gertler/Kiyotaki (2010), Brunnermeier/Sannikov (2014)
- **Firm and intermediary net worth**
  - This paper

Model: Environment

- Discrete time
- Infinite horizon
- 3 types of agents
  - Households
  - Financial intermediaries
  - Firms

Model: Households

- Risk neutral, discount at $R^{-1} > \beta$ where firms' discount rate is β
- Large endowment of funds (and collateral) in all dates and states

Model: Financing Subject to Collateral Constraints

- **Collateral constraints**
  - Complete markets in one period ahead Arrow securities
    - subject to collateral constraints
  - Firms can issue state-contingent promises
    - ... up to fraction θ of resale value of capital to households
    - ... up to fraction θ_i of resale value of capital to intermediaries
  - Related: Kiyotaki/Moore (1997a); but two types of lenders and allow risk management

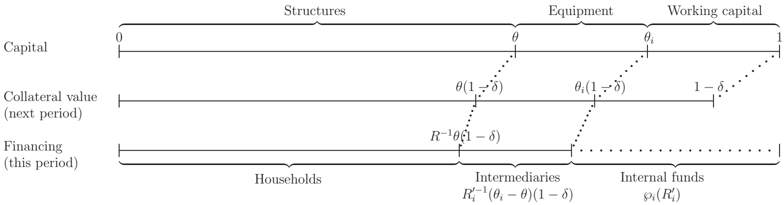
- **Limited enforcement**
  - We derive such collateral constraints from limited enforcement without exclusion - different from Kehoe/Levine (1993)
  - Related: Rampini/Viswanathan (2010, 2013)

Model: Financial Intermediaries

- Risk neutral, discount at $\beta_i \in (\beta, R^{-1})$
- **Collateralization advantage**
  - Ability to seize up to fraction $\theta_i > \theta$ of (resale value of) collateral
- **Refinancing collateralized loans**
  - Idea: Intermediaries can borrow against their (collateralized) loans
    - ... but only to extent households can collateralize assets backing loans
  - Households can collateralize up to θ of collateral backing loans (**"structures"**)
  - Intermediaries need to finance $\theta_i - \theta$ out of own net worth (**"equipment"**)

Model: Collateral and Financing

- Capital, collateral value, and financing



Model: Representative Firm (or "Corporate Sector")

- Risk neutral, limited liability, discount at $\beta < 1$
- Capital k
  - Depreciation rate δ; no adjustment costs
- Standard neoclassical production function
  - Cash flows $A'f(k)$ where $A' \equiv A(s')$ is (stochastic) Markov productivity with transition probability $\Pi(s, s')$
  - Strictly decreasing returns to scale ($f(\cdot)$ strictly concave)
- Two sources of outside finance
  - Households
  - Financial intermediaries

Firm's Problem

- Firm solves following dynamic program

$$v(w,z)=\max_{\{d,k,b',b'_i,w'\}\in\mathbb{R}^2_+\times\mathbb{R}^S\times\mathbb{R}^{2S}_+} d+\beta E\left[v(w',z')\right] \quad (20)$$

subject to budget constraints

$$w \geq d+k-E\left[b'+b'_i|z\right] \quad (21)$$

$$A'f(k)+k(1-\delta) \geq w'+Rb'+R'_ib'_i \quad (22)$$

and **two types of collateral constraints**

$$(\theta_i-\theta)k(1-\delta) \geq R'_ib'_i \quad (23)$$

$$\theta k(1-\delta) \geq Rb' \quad (24)$$

- State-contingent interest rates R'_i determined in equilibrium

Firm's Problem: Comments

- Two sets of state-contingent collateral constraints restricting
  - ... borrowing from households b'
  - ... borrowing from financial intermediaries b_i'
- **State variables:** net worth w and state of economy $z = \{s, w, w_i\}$
  - **Net worth of representative firm w and intermediary w_i**

Characterization of Firm's Problem

- Multipliers
  - ... on (21) through (24): μ, $\Pi(z,z')\beta\mu'$, $\Pi(z,z')\beta\lambda_i'$, and $\Pi(z,z')\beta\lambda'$
  - ... on $d' \geq 0$ and $b_i' \geq 0$: ν_d and $\Pi(z,z')R_i'\beta\nu_i'$
  - (Redundant: $k \geq 0$ and $w' \geq 0$)

- **First order conditions**

$$\mu = 1 + \nu_d \quad (25)$$

$$\mu = E\left[\beta\mu'\left(\left[A'f_k(k) + (1-\delta)\right] + \left[\lambda'\theta + \lambda_i'(\theta_i - \theta)\right](1-\delta)\right)|z\right] \quad (26)$$

$$\mu = R\beta\mu' + R\beta\lambda' \quad (27)$$

$$\mu = R_i'\beta\mu' + R_i'\beta\lambda_i' - R_i'\beta\nu_i' \quad (28)$$

$$\mu' = v_w(w', z') \quad (29)$$

- Envelope condition

$$v_w(w, Z) = \mu$$

Intermediary's Problem

- Representative intermediary solves

$$v_i(w_i, z) = \max_{\{d_i, l', l'_i, w'_i\} \in \mathbb{R}_+^{1+3S}} d_i + \beta_i E\left[v_i(w'_i, z')|z\right] \tag{30}$$

subject to budget constraints

$$w_i \geq d_i + E[l' + l'_i|z] \tag{31}$$

$$Rl' + R'_i l'_i \geq w'_i \tag{32}$$

- State-contingent loans to households l' and to firms l'_i

Characterization of Intermediary's Problem

- Multipliers
  - ... on (31) through (32): μ_i and $\Pi(z,z')\beta_i\mu_i'$,
  - ... on $d_i' \geq 0$, $l' \geq 0$, and $l_i' \geq 0$: η_d, $\Pi(z,z')R\beta_i\eta'$, and $\Pi(z,z')R_i'\beta_i\eta_i'$
  - (Redundant: $w_i' \geq 0$)

- **First order conditions**

$$\begin{aligned} \mu_i &= 1+\eta_d, && (33) \\ \mu_i &= R\beta_i\mu_i' + R\beta_i\eta', && (34) \\ \mu_i &= R_i'\beta_i\mu_i' + R_i'\beta_i\eta_i', && (35) \\ \mu_i' &= v_{i,w}(w_i',z'), && (36) \end{aligned}$$

- Envelope condition

$$v_{i,w}(w_i,z) = \mu_i$$

Model with Limited Enforcement and Limited Participation

- **Timing**
  - Afternoon: repayments, investment, consumption
  - Morning: cash flows, repayments
- **Limited participation**
  - Afternoon: Firms, intermediaries, and households present
  - Morning: Firms and intermediaries present, not households
- **Limited enforcement**
  - Afternoon
    - Firms can abscond with cash flows and $1 - \theta$ of capital (not structures)
    - Intermediaries can abscond with funds paid in morning
  - Morning
    - Firms can abscond with cash flows and $1 - \theta_i$ of capital (not structures and equipment)

Equivalence: Limited Enforcement & Collateral Constraints

- Loans against $\theta_i - \theta$ ("equipment") only enforceable in morning
  - Intermediaries must extend such loans
  - Loans must be repaid each morning (no rollover) – **new model of short term intermediated finance**
- Loans up to θ ("structures") enforceable in afternoon
  - Households extend such loans w.l.o.g.
  - Rollover possible
- Two equivalent implementations with collateral constraints
  - Direct implementation
    - Households lend to firms directly
  - Indirect implementation
    - Households lend to intermediaries
    - Intermediaries lend to firms and borrow from households against collateralized corporate loans

Limited Enforcement and Limited Participation – Timeline

- Limited participation by households affords intermediaries advantage

	Current period: Afternoon	Next period: Morning	Next period: Afternoon
	Consume d Invest k	State s' Cash flow $A'f(k)$	
Payments			
… from/to intermediaries	Borrow $E[b_i'] + E[b_a']$	Repay $R_i'b_i'$	Repay Rb_a'
… from/to households	Borrow $E[b']$		Repay Rb'
Net worth if firm defaults		$A'f(k) + (1-\theta_i)k(1-\delta)$	$A'f(k) + (1-\theta)k(1-\delta)$

Endogenous Minimum Down Payment Requirement

- **Minimum down payment requirement $\wp$ (or margin)**
  - Borrowing from households only

$$\wp = 1 - R^{-1}\theta(1-\delta)$$

  - Borrowing from households and financial intermediaries

$$\wp_i(R_i') = \wp - E[(R_i')^{-1}](\theta_i - \theta)(1-\delta)$$

- **Firm's investment Euler equation**

$$1 \geq E\left[\beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta_i)(1-\delta)}{\wp_i(R_i')}\right] \quad (37)$$

User Cost of Capital with Intermediated Finance

- **Extension of Jorgenson's (1963) user cost of capital definition**

$$u \equiv r + \delta$$

  - User cost would be rental cost in frictionless economy

- Premium on internal funds ρ: $1/(R+\rho) \equiv E[\beta\mu'/\mu]$

- Premium on intermediated finance ρ_i: $1/(R+\rho_i) \equiv E[(R_i')^{-1}]$

- **Firm's user cost of capital** u is

$$u \equiv r + \delta + \frac{\rho}{R+\rho}(1-\theta_i)(1-\delta) + \frac{\rho_i}{R+\rho_i}(\theta_i - \theta)(1-\delta),$$

where $1 + r \equiv R$

Premia on Internal and Intermediated Finance

- **Internal and intermediated funds are scarce**
- Proposition 1 (Premia on internal and intermediated finance)
  - *Premium on internal finance ρ (weakly) exceeds premium on intermediated finance ρ_i*

$$\rho \geq \rho_i \geq 0,$$

  - *Premia equal, $\rho = \rho_i$, iff $E[\lambda_i'] = 0$.*
  - *Premium on internal finance strictly positive, $\rho > 0$, iff $E[\lambda'] > 0$.*

Equilibrium

- Definition 1 (Equilibrium) *An* **equilibrium** *is*
  - *allocation* $x \equiv [d, k, b', b_i', w']$ *(for firm) and* $x_i \equiv [d_i, l', l_i', w_i']$ *(for intermediary)*
  - *interest rate process* R_i' *for intermediated finance*

such that

  - *(i)* x *solves firm's problem in (20)-(24) and* x_i *solves intermediary's problem (30)-(32)*
  - *(ii) market for intermediated finance clears in all dates and states*

$$l_i' = b_i'. \tag{38}$$

Essentiality of Financial Intermediation

- Definition 2 (Essentiality of intermediation) *Intermediation is* **essential** *if an allocation can be supported with a financial intermediary but not without.*
  - Analogous: Hahn's (1973) definition of essentiality of money

- **Intermediaries are essential**

- Proposition 3 (Positive intermediary net worth) *Financial intermediaries always have positive net worth in a deterministic or eventually deterministic economy.*

- Proposition 4 (Essentiality of intermediaries) *In any deterministic economy, financial intermediaries are always essential.*
  - Intuition: Without intermediaries, shadow spreads would be "high."

Deterministic Steady State

- **Steady state spread and intermediary capitalization**

- Definition 3 (Steady state) *A deterministic steady state equilibrium is an equilibrium with constant allocations, that is,* $x^* \equiv [d^*, k^*, b'^*, b_i'^*, w'^*]$ *and* $x_i^* \equiv [d_i^*, l'^*, l_i'^*, w_i'^*]$.

- Proposition 5 (Steady state) *In steady state*
  - *Intermediaries essential; positive net worth; pay positive dividends*
  - *Spread on intermediated finance* $R_i'^* - R = \beta_i^{-1} - R > 0$
  - *(Ex dividend) intermediary net worth (relative to firm's net worth)*

$$\frac{w_i^*}{w^*} = \frac{\beta_i(\theta_i - \theta)(1-\delta)}{\wp_i(\beta_i^{-1})}$$

  (ratio of intermediary's financing to firm's down payment requirement)

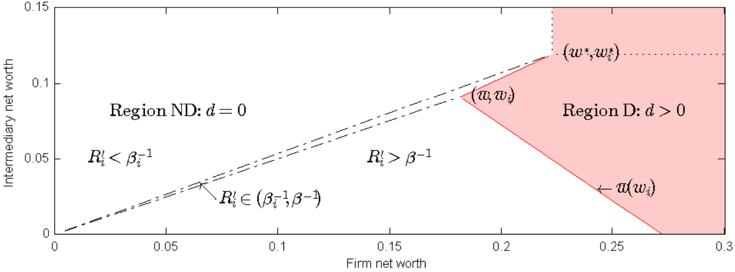
Deterministic Equilibrium Dynamics

- Two main phases: **no dividend phase and dividend phase**
- Proposition 6 (Deterministic dynamics) *Given w and w_i, there exists a unique deterministic dynamic equilibrium which converges to the steady state characterized by a no dividend region (ND) and a dividend region (D) (which is absorbing) as follows: [Region ND] $w_i \leq w_i^*$ (w.l.o.g.) and $w < \bar{w}(w_i)$, and (i) $d = 0$ ($\mu > 1$), (ii) the cost of intermediated finance is*

$$R_i' = \max \left\{ R, \min \left\{ \frac{(\theta_i - \theta)(1-\delta)\left(\frac{w}{w_i} + 1\right)}{\wp}, \frac{A' f_k\left(\frac{w+w_i}{\wp}\right) + (1-\theta)(1-\delta)}{\wp} \right\} \right\},$$

(iii) investment $k = (w + w_i)/\wp$ if $R_i' > R$ and $k = w/\wp_i(R)$ if $R_i' = R$, and (iv) $w'/w_i' > w/w_i$, that is, firm net worth increases faster than intermediary net worth. [Region D] $w \geq \bar{w}(w_i)$ and (i) $d > 0$ ($\mu = 1$). For $w_i \in (0, \bar{w}_i)$, (ii) $R_i' = \beta^{-1}$, (iii) $k = \bar{k}$ which solves $1 = \beta[A' f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp\bar{k} - w_i$. For $w_i \in [\bar{w}_i, w_i^*)$, (ii) $R_i' = (\theta_i - \theta)(1-\delta)k/w_i$, (iii) k solves $1 = \beta[A' f_k(k) + (1-\theta)(1-\delta)]/(\wp - w_i/k)$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp_i(R_i')k$. For $w_i \geq w_i^*$, $\bar{w}(w_i) = w^*$ and the steady state of Proposition 5 is reached with $d = w - w^*$ and $d_i = w_i - w_i^*$.

Joint Dynamics of Intermediary and Corporate Net Worth



Slow Intermediary Net Worth Accumulation (Region ND)

- Law of motion (as long as no dividends)

$$w_i' = R_i' w_i$$

  - Intermediaries lend out all funds at interest rate R_i' ($\geq R$)
  - When firm's collateral constraint binds,

$$R_i' = \frac{(\theta_i - \theta)(1-\delta)\left(\frac{w}{w_i} + 1\right)}{\wp}$$

  - When collateral constraint slack,

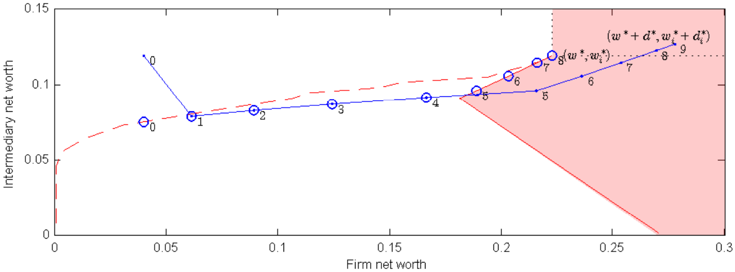
$$R_i' = \frac{A' f_k\left(\frac{w+w_i}{\wp}\right) + (1-\theta)(1-\delta)}{\wp}$$

- **Relatively slow accumulation of intermediary net worth**
  - Intermediaries earn R_i' which is at most marginal return on capital (**collateral constraint**)
  - Firms earn average return (**decreasing returns to scale**)

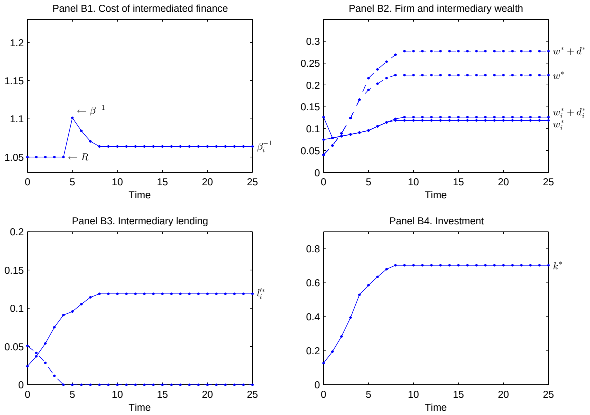
Dynamics of Downturn without Credit Crunch

- **Downturn without credit crunch**
  - Unanticipated drop in firm (but not intermediary) net worth from steady state (say due to surprise drop in productivity A')

- Dynamics of net worth, spread, and investment



Dynamics of Downturn without Credit Crunch (Cont'd)



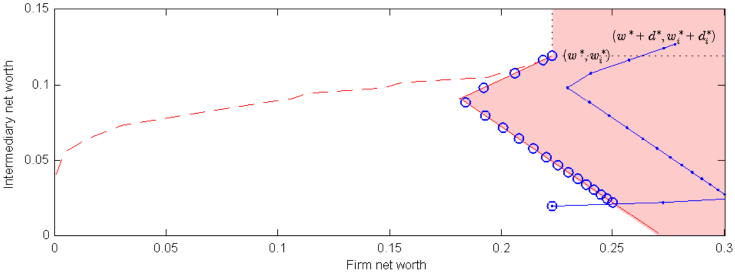
- Low firm net worth $\Rightarrow$ drop in real investment $k = w/\wp_i(R)$
- Lack of collateral/low loan demand $\Rightarrow$ spread on intermediated finance may fall
  - **Intermediaries save at low interest rate** $R_i' = R$ (lend to households) to meet future loan demand

Deterministic Dynamics: Initial Dividend

- **Intermediaries may pay initial dividend when downturn hits!**
- Lemma 2 (Initial intermediary dividend) *The representative intermediary pays at most an initial dividend and no further dividends until the steady state is reached. If $w_i > w_i^*$, the initial dividend is strictly positive.*
- Intuition: **Low firm net worth limits loan demand**
  - Intermediaries save only part of net worth to meet future loan demand

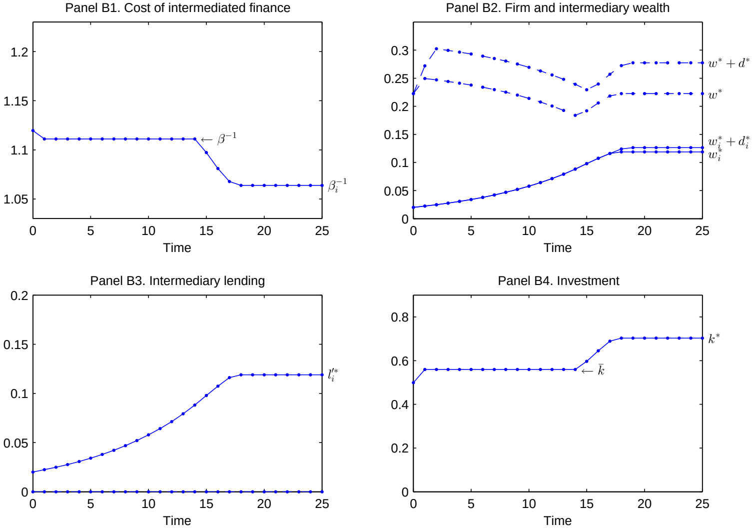
Dynamics of Credit Crunch

- **Credit crunch**
  - Unanticipated drop in intermediary net worth w_i from steady state
- Joint dynamics of firm and intermediary net worth



Dynamics of Credit Crunch (Cont'd)

- Dynamics of net worth, spread, and investment



Dynamics of a Credit Crunch (Cont'd)

- **Fact 2: Protractedness** – slow/delayed recovery

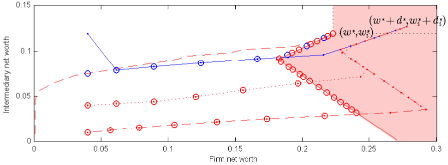
- **"Delayed or stalled recovery"** (until intermediaries accumulate sufficient capital)
  - Reinhart/Rogoff (2014): "halting, tentative nature of the post-crisis recoveries (even in cases where there is a sharp – but not sustained – growth rebound)"
  - Partial recovery until $R'_i = \beta^{-1}$ when firms reinitiate dividends
  - Corporate investment remains depressed at $\bar{k}$ as firms pay dividends and stop growing, **waiting for intermediary capital to catch up**

$$R'_i = \left(\beta \frac{\mu'}{\mu}\right)^{-1} = \beta^{-1} = \frac{A' f_k(\bar{k}) + (1-\theta)(1-\delta)}{\wp}$$

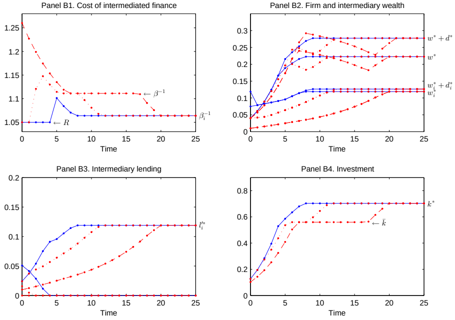
- Corporate deleveraging (and eventual releveraging when intermediaries catch up)

Effect of Severity of Credit Crunch

- Joint dynamics of firm and intermediary net worth

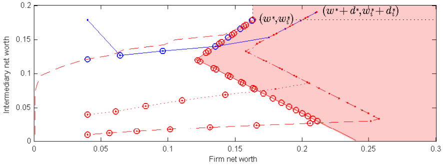


- Fact 3: Impact of severity of financial crises; halting recovery – stalls

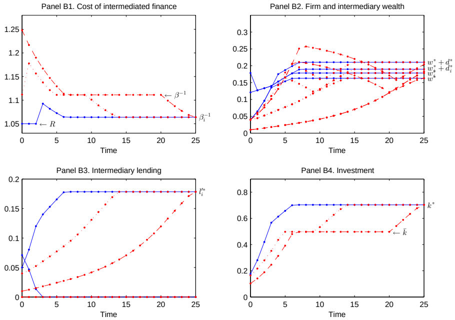


Downturn with Credit Crunch – Bank-Dependent Economy

- Bank dependence: higher θ_i



- More severe, more protracted, longer stalls; Europe/Japan?



Conclusions

- **Theory of intermediaries with collateralization advantage**
  - Better ability to enforce claims
    - ... implies **role for financial intermediary capital**
  - Tractable dynamic model with (two types of) collateralized finance
- **Dynamics of intermediary capital**
  - Economic activity and spreads **determined by firm and intermediary net worth jointly**
  - **Slow accumulation** of intermediary net worth
  - Downturns associated with credit crunch are
    - (1) **severe** and (2) **protracted** (and **"tentative"/"halting"**)
    - and (3) **severity of credit crunch affects severity & protractedness**,

  particularly so in bank-dependent economies

Models of Dynamic Collateralized Financing – Conclusion

- **Useful laboratory to study dynamic micro & macro finance**
  - Tractability allows explicit theoretical analysis of dynamics
  - Insights for macro-finance/general equilibrium
    - Collateral scarcity affects interest rate and equilibrium insurance
    - Intermediation requires net worth, key additional state variable

    and (previously) micro/corporate finance
    - Capital structure/debt capacity
    - Risk management/insurance
    - Leasing
    - Durability
  - Dynamic models facilitate quantitative work/structural estimation
- **Empirically/quantitatively plausible class of models**