Collateral and Capital Structure[★]

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Abstract

We develop a dynamic model of investment, capital structure, leasing, and risk management based on firms' need to collateralize promises to pay with tangible assets. Both financing and risk management involve promises to pay subject to collateral constraints. Leasing is strongly collateralized costly financing and permits greater leverage. More constrained firms hedge less and lease more, both cross-sectionally and dynamically. Mature firms suffering adverse cash flow shocks may cut risk management and sell and lease back assets. Persistence of productivity reduces the benefits to hedging low cash flows and can lead firms not to hedge at all.

JEL classification: D24; D82; E22; G31; G32; G35

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1.   Introduction

We argue that collateral determines the capital structure and develop a dynamic agency based model of firm financing based on the need to collateralize promises to pay with tangible assets. We maintain that the enforcement of payments is a critical determinant of both firm financing and whether asset ownership resides with the user or the financier, i.e., whether firms purchase or lease assets. We study a dynamic neoclassical model of the firm in which financing is subject to collateral constraints derived from limited enforcement and firms choose between purchasing and renting assets. Our theory of optimal investment, capital structure, leasing, and risk management enables the first dynamic analysis of the financing vs. risk management trade-off and of firm financing when firms can rent capital.

In the frictionless neoclassical model asset ownership is indeterminate and firms are assumed to rent all capital. The recent dynamic agency models of firm financing ignore the possibility that firms rent capital. Of course, a frictionless rental market for capital would obviate financial constraints. We explicitly consider firms' dynamic lease vs. buy decision, modeling leasing as highly collateralized albeit costly financing. When capital is leased, the financier retains ownership which facilitates repossession and strengthens the collateralization of the financier's claim. Leasing is costly since the lessor incurs monitoring costs to avoid agency problems due to the separation of ownership and control.

We provide a definition of the user cost of capital in our model of investment with financial constraints that is similar in spirit to Jorgenson's (1963) definition in the frictionless neoclassical model. Our user cost of capital is the sum of Jorgenson's user cost and a term which captures the additional cost due to the scarcity of internal funds. In our model, firms require both tangible and intangible capital, but the enforcement constraints imply that only tangible capital can be pledged as collateral and borrowed against, resulting in a premium on internal funds; tangibility restricts external financing and hence leverage.

There is a fundamental connection between the optimal financing and risk management policy that has not been previously recognized. Both financing and risk management involve promises to pay by the firm, leading to a trade off when such promises are limited by collateral constraints. Indeed, firms with sufficiently low net worth do not engage in risk management at all because the need to finance investment overrides the hedging concerns. This result is in contrast to the extant theory, such as Froot, Scharfstein, and Stein (1993), and is consistent with the evidence that more constrained firms hedge less provided by Rampini, Sufi, and Viswanathan (2012), and the literature cited

therein, and with the strong positive relation between hedging and firm size in the data.

With constant investment opportunities, risk management depends only on firms' net worth and incomplete hedging is optimal, i.e., firms do not hedge to the point where the marginal value of net worth is equated across all states. In fact, firms abstain from risk management with positive probability under the stationary distribution. Thus, even mature firms that suffer a sequence of adverse cash flow shocks may see their net worth decline to the point where they find it optimal to discontinue risk management. We moreover characterize the comparative statics of firms' investment, financing, risk management, and dividend policy with respect to other key parameters of the model. Firms subject to higher risk can choose to hedge more and reduce investment due to the financing needs for risk management. Firms with more collateralizable or tangible assets can lever more and increase investment, while at the same time raising corporate risk management to counterbalance the increase in the volatility of net worth that higher leverage would otherwise imply. Firms with more curvature in their production function operate at smaller scale and may hence hedge less, not more as one might expect. Thus, our model has interesting empirical implications for firm financing and risk management both in the cross section and the time series.

With stochastic investment opportunities, risk management depends not only on firms' net worth but also on their productivity. If productivity is persistent, the overall level of risk management is reduced, because cash flows and investment opportunities are positively correlated due to the positive correlation between current productivity and future expected productivity. There is less benefit to hedging low cash flow states. Moreover, risk management is lower when current productivity is high, as higher expected productivity implies higher investment and raises the opportunity cost of risk management. With sufficient but empirically plausible levels of persistence, the firm abstains from risk management altogether, providing an additional reason why risk management is so limited in practice. Furthermore, when the persistence of productivity is strong, firms hedge investment opportunities, i.e., states with high productivity, as the financing needs for increased investment rise more than cash flows.

Leasing tangible assets requires less net worth per unit of capital and hence allows firms to borrow more. Financially constrained firms, i.e., firms with low net worth, lease capital because they value the higher debt capacity; indeed, severely constrained firms lease all their tangible capital. Over time, as firms accumulate net worth, they grow in size and start to buy capital. Thus, the model predicts that small firms and young firms lease capital. We show that the ability to lease capital enables firms to grow faster. Dynamically, mature firms that are hit by a sequence of low cash flows may sell assets and lease them back, i.e., sale-leaseback transactions may occur under the stationary

distribution. Moreover, leasing has interesting implications for risk management: leasing enables high implicit leverage; this may lead firms to engage in risk management to reduce the volatility of net worth that such high leverage would otherwise imply.

In the data, we show that tangible assets are a key determinant of firm leverage. Leverage varies by a factor 3 from the lowest to the highest tangibility quartile for Compustat firms. Moreover, tangible assets are an important explanation for the "low leverage puzzle" in the sense that firms with low leverage are largely firms with few tangible assets. We also take firms' ability to deploy tangible assets by renting or leasing such assets into account. We show that accounting for leased assets in the measurement of leverage and tangibility reduces the fraction of low leverage firms drastically and that firms with low lease adjusted leverage are firms with low lease adjusted tangible assets. Finally, we show that accounting for leased capital has a striking effect on the relation between leverage and size in the cross section of Compustat firms. This relation is essentially flat when leased capital is taken into account. In contrast, when leased capital is ignored, as is done in the literature, leverage increases in size, i.e., small firms seem less levered than large firms. Thus, basic stylized facts about the capital structure need to be revisited. Importantly, the lease adjustments to the capital structure we propose based on our theory are common in practice, and accounting rule changes are currently being considered by the US and international accounting boards that would result in the implementation of lease adjustments similar to ours throughout financial accounting. Our model and empirical evidence together suggest a collateral view of the capital structure.

Our paper is part of a recent and growing literature which considers dynamic incentive problems as the main determinant of the capital structure. The incentive problem in our model is limited enforcement of claims. Most closely related to our work are Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), and Rampini and Viswanathan (2010). Albuquerque and Hopenhayn (2004) study dynamic firm financing with limited enforcement. In their setting, the value of default is exogenous, albeit with fairly general properties, whereas in our model the value of default is endogenous as firms are not excluded from markets following default. Moreover, they do not consider the standard neoclassical investment problem.[1] Lorenzoni and Walentin (2007) consider limits on enforcement similar to ours in a model with constant returns to scale and adjustment costs on aggregate investment which implies that all firms are equally constrained at any given time. However, they assume that all enforcement constraints always bind, which

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[1] The aggregate implications of firm financing with limited enforcement are studied by Cooley, Marimon, and Quadrini (2004) and Jermann and Quadrini (2007). Schmid (2008) considers the quantitative implications for the dynamics of firm financing.

is not the case in our model, and focus on the relation between investment and Tobin's q rather than the capital structure. Rampini and Viswanathan (2010) consider a two period model in a similar setting with heterogeneity in firm productivities and focus on the distributional implications of limited risk management. While they consider the comparative statics with respect to exogenously given initial net worth, net worth is endogenously determined in our fully dynamic model, and our model moreover allows the analysis of the dynamics of risk management, risk management by mature firms in the long run, and the effect of persistence on the extent of risk management. Rampini, Sufi, and Viswanathan (2012) extend the model in this paper to analyze the hedging of a stochastic input price, e.g., airlines' risk management of fuel prices.

The rationale for risk management in our model is related to the one in Froot, Scharfstein, and Stein (1993) who show that firms subject to financial constraints are effectively risk averse and hence engage in risk management. Holmström and Tirole (2000) recognize that financial constraints may limit ex ante risk management. Mello and Parsons (2000) also argue that financial constraints can restrict hedging. None of these papers provide a dynamic analysis of the financing vs. risk management trade-off.[2] A literature, e.g., Leland (1998), argues that "[h]edging permits greater leverage" by allowing firms to avoid bankruptcy costs when they are financed with non-contingent debt. To the extent that low net worth firms have high leverage and a high probability of bankruptcy, this implies a negative relation between net worth and risk management, which is the opposite of our prediction and the relation in the data.

Capital structure and investment dynamics determined by incentive problems due to private information about cash flows or moral hazard are studied by Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), Biais, Mariotti, Rochet, and Villeneuve (2010), and DeMarzo, Fishman, He, and Wang (2012). Capital structure dynamics subject to similar incentive problems but abstracting from investment decisions are analyzed by DeMarzo and Fishman (2007b), DeMarzo and Sannikov (2006), and Biais, Mariotti, Plantin, and Rochet (2007).[3] In these models, collateral

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[2] Bolton, Chen, and Wang (2011) study risk management in a neoclassical model of corporate cash management; in contrast to our theory, they find that the hedge ratio decreases in firms' cash-to-capital ratio, and low cash firms hedge as much as possible while high cash firms do not hedge at all. The cost of hedging they consider is an inconvenience yield of posting cash to meet an exogenous margin requirement rather than the financing risk management trade-off we emphasize.

[3] Relatedly, Gromb (1995) analyzes a multi-period version of Bolton and Scharfstein (1990)'s two period dynamic firm financing problem with privately observed cash flows. Gertler (1992) considers the aggregate implications of a multi-period firm financing problem with privately

plays no role.

The role of secured debt is also considered in a literature which takes the form of debt and equity claims as given. Stulz and Johnson (1985) argue that secured debt can facilitate follow-on investment and thus ameliorate a Myers (1977) type underinvestment problem in the presence of debt overhang.[4] Morellec (2001) shows that secured debt prevents equityholders from liquidating assets to appropriate value from debtholders. Using an incomplete contracting approach, Bolton and Oehmke (2012) analyze the optimal priority structure between derivatives used for risk management and debt.

Several papers study the capital structure implications of agency conflicts due to managers' private benefits. Zwiebel (1996) argues that managers voluntarily choose debt to credibly constrain their own future empire-building in a model with incomplete contracts. Morellec, Nikolov, and Schürhoff (2012) study agency conflicts in a Leland (1998) type model in which managers divert a part of cash flows as private benefits leading them to lever less.

Moreover, none of these models consider intangible capital or the option to lease capital. An exception is Eisfeldt and Rampini (2009) who argue that leasing amounts to a particularly strong form of collateralization due to the relative ease with which leased capital can be repossessed, albeit in a static model. We are the first, to the best of our knowledge, to consider the role of leased capital in a dynamic model of firm financing and provide a dynamic theory of sale-and-leaseback transactions.

In Section 2 we report some stylized empirical facts about collateralized financing, tangibility, and leverage, taking leased capital into account. Section 3 describes the model, defines the user cost of tangible, intangible, and leased capital, and characterizes the payout policy. Section 4 analyzes optimal risk management and provides comparative statics with respect to key parameters. Section 5 characterizes the optimal leasing and capital structure policy and Section 6 concludes. All proofs are in Appendix A.

2. Stylized facts

This section provides some aggregate and cross-sectional evidence that highlights the first order importance of tangible assets as a determinant of the capital structure in the data. We first take an aggregate perspective and then document the relation between tangible assets and leverage across firms. We take leased capital into account

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observed cash flows. Atkeson and Cole (2008) consider a two period firm financing problem with costly monitoring of cash flows.

[4] Hackbarth and Mauer (2012) provide a Leland (1994) type model where priority rules mitigate this underinvestment problem.

explicitly and show that it has quantitatively and qualitatively large effects on basic stylized facts about the capital structure, such as the relation between leverage and size. Tangibility also turns out to be one of the few robust factors explaining firm leverage in the extensive empirical literature on capital structure, but we do not attempt to summarize this literature here.

2.1. Collateralized financing: the aggregate perspective

From the aggregate point of view, the importance of tangible assets is striking. Consider the balance sheet data from the Flow of Funds Accounts of the U.S. for (nonfinancial) corporate businesses, (nonfinancial) noncorporate businesses, and households reported in Table 1 for the years 1999 to 2008 (detailed definitions of variables are in the caption of the table). For businesses, tangible assets include real estate, equipment and software, and inventories, and for households mainly real estate and consumer durables.

Panel A documents that from an aggregate perspective, the liabilities of corporate and noncorporate businesses and households are less than their tangible assets and indeed typically considerably less, and in this sense all liabilities are collateralized. For corporate businesses, debt in terms of credit market instruments is 48.5% of tangible assets. Even total liabilities, which include also miscellaneous liabilities and trade payables, are only 83.0% of tangible assets. For noncorporate businesses and households, liabilities vary between 37.8% and 54.9% of tangible assets and are remarkably similar for the two sectors.

Note that we do not consider whether liabilities are explicitly collateralized or only implicitly in the sense that firms have tangible assets exceeding their liabilities. Our reasoning is that even if liabilities are not explicitly collateralized, they are implicitly collateralized since restrictions on further investment, asset sales, and additional borrowing through covenants and the ability not to refinance debt allow lenders to effectively limit borrowing to the value of collateral in the form of tangible assets. That said, households' liabilities are largely explicitly collateralized. Households' mortgages, which make up the bulk of households' liabilities, account for 41.2% of the value of real estate, while consumer credit amounts to 56.1% of the value of households' consumer durables.

Finally, aggregating across all balance sheets and ignoring the rest of the world implies that tangible assets make up 79.2% of the net worth of U.S. households, with real estate making up 60.2%, equipment and software 8.3%, and consumer durables 7.6% (see Panel B). While this provides a coarse picture of collateral, it highlights the quantitative importance of tangible assets as well as the relation between tangible assets and liabilities in the aggregate.

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2.2. Tangibility and leverage

To document the relation between tangibility and leverage, we analyze data for a cross section of Compustat firms. We sort firms into quartiles by tangibility measured as the value of property, plant, and equipment divided by the market value of assets. The results are reported in Panel A of Table 2, which also provides a detailed description of the construction of the variables. We measure leverage as long term debt to the market value of assets.

The first observation that we want to stress is that across tangibility quartiles, (median) leverage varies from 7.4% for low tangibility firms (i.e., firms in the lowest quartile) to 22.6% for high tangibility firms (i.e., firms in the highest quartile), i.e., by a factor 3. Tangibility also varies substantially across quartiles; the cut-off value for the first quartile is 6.3% and for the fourth quartile is 32.2%.

To assess the role of tangibility as an explanation for the observation that some firms have very low leverage (the so-called "low leverage puzzle"), we compute the fraction of firms in each tangibility quartile which have low leverage, specifically leverage less than 10%. The fraction of firms with low leverage decreases from 58.3% in the low tangibility quartile to 14.9% in the high tangibility quartile. Thus, low leverage firms are largely firms with relatively few tangible assets.

2.3. Leased capital and leverage

Thus far, we have ignored leased capital which is the conventional approach in the literature. To account for leased (or rented) capital, we simply capitalize the rental expense (Compustat item #47). This allows us to impute capital deployed via operating leases, which are the bulk of leasing in practice.[5] To capitalize the rental expense, recall that Jorgenson's (1963) user cost of capital is $u \equiv r + \delta$, i.e., the user cost is the sum of the interest cost and the depreciation rate. Thus, the frictionless rental expense for an amount of capital k is

$$\text{Rent} = (r + \delta)k.$$

Given data on rental payments, we can hence infer the amount of capital rented by capitalizing the rental expense using the factor $1/(r + \delta)$. For simplicity, we capitalize the rental expense by a factor 10. We adjust firms' assets, tangible assets, and liabilities by adding 10 times rental expense to obtain measures of lease adjusted assets, lease

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[5] Note that capital leases are already accounted for as they are capitalized on the balance sheet for accounting purposes. For a description of the specifics of leasing in terms of the law, accounting, and taxation see Eisfeldt and Rampini (2009) and the references cited therein.

adjusted tangible assets, and lease adjusted leverage.[6]

We proceed as before and sort firms into quartiles by lease adjusted tangibility. The results are reported in Panel B of Table 2. Lease adjusted debt leverage is somewhat lower as we divide by lease adjusted assets here. There is a strong relation between lease adjusted tangibility and lease adjusted leverage (as before), with the median lease adjusted debt leverage varying again by a factor of about 3. Rental leverage also increases with lease adjusted tangibility by about a factor 2 for the median and more than 3 for the mean. Similarly, lease adjusted leverage, which we define as the sum of debt leverage and rental leverage, also increases with tangibility by a factor 3.

Taking rental leverage into account reduces the fraction of firms with low leverage drastically, in particular for firms outside the low tangibility quartile. Lease adjusted tangibility is an even more important explanation for the "low leverage puzzle." Indeed, less than 4% of firms with high tangibility have low lease adjusted leverage.

It is also worth noting that the median rental leverage is on the order of half of debt leverage or more, and is hence quantitatively important. Overall, we conclude that tangibility, when adjusted for leased capital, emerges as a key determinant of leverage and the fraction of firms with low leverage.

2.4. Leverage and size revisited

Considering leased capital changes basic cross-sectional properties of the capital structure. Here we document the relationship between firm size and leverage (see Table 3 and Fig. 1). We sort Compustat firms into deciles by size. We measure size by lease adjusted assets here, although using unadjusted assets makes our results even more stark. Debt leverage is increasing in size, in particular for small firms, when leased capital is ignored. Rental leverage, by contrast, decreases in size, in particular for small firms.[7] Indeed, rental leverage is substantially larger than debt leverage for small firms. Lease adjusted leverage, i.e., the sum of debt and rental leverage, is roughly constant across Compustat size deciles. In our view, this evidence provides a strong case that leased capital cannot be ignored if one wants to understand the capital structure.

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[6] In accounting this approach to capitalization is known as constructive capitalization and is frequently used in practice, with "8 x rent" being the most commonly used. E.g., *Moody's* rating methodology uses multiples of 5x, 6x, 8x, and 10x current rent expense, depending on the industry. We discuss the calibration of the capitalization factor we use in footnote 12.

[7] Eisfeldt and Rampini (2009) show that this is even more dramatically the case in Census data, which includes firms that are not in Compustat and hence much smaller, and argue that for such firms renting capital may be the most important source of external finance.

3. Model

This section provides a dynamic agency based model to understand the first order importance of tangible assets and rented assets for firm financing and the capital structure documented above. Dynamic financing is subject to collateral constraints due to limited enforcement. We consider both tangible and intangible capital as well as firms' ability to lease capital. Moreover, we define the user cost of tangible, intangible, and leased capital. Finally, we characterize the dividend policy and show how tangibility and collateralizability of assets affect the capital structure in the special case without leasing.

3.1. Environment

A risk neutral firm subject to limited liability discounts the future at rate $\beta \in (0,1)$ and requires financing for investment. The firm's problem has an infinite horizon and we write the problem recursively. The firm starts the period with net worth w and has access to a standard neoclassical production function with decreasing returns to scale.

There are two types of capital, tangible capital and intangible capital. Tangible capital can be either purchased (k_p) or leased (k_l), while intangible capital (k_i) can only be purchased. The total amount of capital is $k \equiv k_i + k_p + k_l$ and we refer to total capital k often simply as capital. For simplicity, we assume that tangible and intangible capital are required in fixed proportions and denote the fraction of tangible capital required by φ.[8] Both tangible and intangible capital can be purchased at a price normalized to 1 and depreciate at the same rate δ. There are no adjustment costs. An amount of invested capital k yields stochastic cash flow $A(s')f(k)$ next period, where $A(s')$ is the realized total factor productivity of the technology in state s', which we assume follows a Markov process described by the transition function $\Pi(s,s')$ on $s' \in S$.

Tangible capital which the firm owns can be used as collateral for state-contingent one period debt up to a fraction $\theta \in (0,1)$ of its resale value. These collateral constraints are motivated by limited enforcement. We assume that enforcement is limited

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[8] We are implicitly using the Leontief aggregator of tangible and intangible capital $\min\{(k_p + k_l)/\varphi, k_i/(1 - \varphi)\}$ which yields $k_i = (1 - \varphi)k$, $k_p + k_l = \varphi k$, and $k = k_i + k_p + k_l$ as above, simplifying the firm's investment problem to the choice of capital k and leased capital k_l only. If tangible and intangible capital were not used in fixed proportions and had a constant elasticity of substitution $\gamma > -\infty$, i.e., the aggregator of tangible and intangible capital was $[\sigma(k_p + k_l)^\gamma + (1 - \sigma)k_i^\gamma]^{1/\gamma}$ with $\gamma \leq 1$, then the composition of capital would vary with firms' financial condition, with more financially constrained firms using a lower fraction of intangible capital.

in that firms can abscond with all cash flows, all intangible capital, and $1 - \theta$ of purchased tangible capital k_p. Further we assume that firms cannot abscond with leased capital k_l, i.e., leased capital enjoys a repossession advantage. It is easier for a lessor, who retains ownership of the asset, to repossess it, than for a secured lender, who only has a security interest, to recover the collateral backing the loan. Leasing enjoys such a repossession advantage under U.S. law and, we believe, in most legal systems. Importantly, we assume that firms who abscond cannot be excluded from any market: the market for intangible capital, tangible capital, loans, and rented capital. As we show in Appendix B, these dynamic enforcement constraints imply the above collateral constraints, which are similar to the ones in Kiyotaki and Moore (1997), albeit state contingent, and are described in more detail below.[9] We emphasize that any long-term contract that satisfies the enforcement constraints can be implemented with such one-period ahead state contingent debt subject to the above collateral constraints and hence long-term contracts are not ruled out. The motivation for our assumption about the lack of exclusion is two-fold. First, it allows us to provide a tractable model of dynamic collateralized firm financing. The equivalence of the two problems (with limited enforcement and collateral constraints, respectively) enables us to work directly with the problem with collateral constraints and use net worth as the state variable. In contrast, the outside options considered in the literature result in continuation utility being the appropriate state variable, which typically makes the dual problem easier to work with (see, e.g., Albuquerque and Hopenhayn, 2004). Second, a model based on this assumption has implications which are empirically plausible, in particular by putting the focus squarely on tangibility.

We assume that intangible capital can neither be collateralized nor leased. The idea is that intangible capital cannot be repossessed due to its lack of tangibility and can be deployed in production only by the owner, since the agency problems involved in separating ownership from control are too severe.[10]

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[9] These collateral constraints are derived from an explictly dynamic model of limited enforcement similar to the one considered by Kehoe and Levine (1993). The main difference to their limits on enforcement is that we assume that firms who abscond cannot be excluded from future borrowing whereas they assume that borrowers are in fact excluded from intertemporal trade after default. Similar constraints have been considered by Lustig (2007) in an endowment economy, by Lorenzoni and Walentin (2007) in a production economy with constant returns to scale, and by Rampini and Viswanathan (2010) in a production economy with a finite horizon. Krueger and Uhlig (2006) find that similar limits on enforcement in an endowment economy without collateral imply short-sale constraints, which would be true in our model in the special case where $\theta = 0$.

[10] The assumption that intangible capital cannot be collateralized or leased *at all* simplifies

Our model considers the role of leased capital in a dynamic model of firm financing subject to limited enforcement. The assumption that firms cannot abscond with leased capital captures the fact that leased capital can be repossessed more easily. This repossession advantage means that by leasing the firm can effectively borrow against the full resale value of tangible assets, whereas secured lending allows the firm to borrow only against a fraction θ of the resale value. The benefit of leasing is its higher debt capacity. However, leasing also has a cost: leased capital involves monitoring costs m per unit of capital incurred by the lessor at the end of the period, which are reflected in the user cost of leased capital u_l. Leasing separates ownership and control and the lessor must pay the cost m to ensure that the lessee uses and maintains the asset appropriately.[11] A competitive lessor with a cost of capital $R \equiv 1 + r$ charges a user cost of

$$u_l \equiv r + \delta + m$$

per unit of capital. Equivalently, we could assume that leased capital depreciates faster due to the agency problem at rate $\delta_l > \delta$ and set $m = \delta_l - \delta$. Due to the constraints on enforcement, the user cost of leased capital is charged at the beginning of the period and hence the firm pays $R^{-1}u_l$ per unit of leased capital up front. Specifically, since the lessor recovers the leased capital at the end of the period, no additional payments at the end of the period can be enforced and the leasing fee must hence be charged up front. Recall that in the frictionless neoclassical model, the rental cost of capital is Jorgenson (1963)'s user cost $u \equiv r + \delta$. Thus the only difference to the rental cost in our model is the positive monitoring cost m (or, equivalently, the costs due to faster depreciation $\delta_l - \delta$). Note that as in Jorgenson's definition, we define the user cost of capital in terms of value at the end of the period.[12]

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the analysis, but is not required for the main results. Assuming that intangible capital is less collateralizable and more costly to lease would suffice.

[11] In practice, there may be a link between the lessor's monitoring and the repossession advantage of leasing. In order to monitor the use and maintenance of the asset, the lessor needs to keep track of the asset which makes it harder for the lessee to abscond with it.

[12] To impute the amount of capital rented from rental payments, we should hence capitalize rental payments by $1/(R^{-1}(r + \delta + m))$. In documenting the stylized facts, we assumed that this factor takes a value of 10. This calibrated value is based on an approximate (unlevered) real cost of capital for tangible assets of 4%, an approximate depreciation rate for leased tangible assets of 5% (using a depreciation rate of 3% for structures and 15% for equipment, which are based on BEA data, and a weight of 80% (20%) on structures (equipment), since leased tangible assets are predominantly structures and structures are 66% of non-residential fixed assets overall), and an assumed monitoring cost m of 1%. The implicit debt associated with rented capital is $R^{-1}(1 - \delta)$ times the amount of capital rented, so in adjusting liabilities,

We assume that the firm has access to lenders who have deep pockets in all dates and states and discount the future at rate $R^{-1} \in (\beta, 1)$. These lenders are thus willing to lend in a state-contingent way at an expected return R. The assumption that firms are less patient than lenders, which is quite common, [13] implies that firms' financing policy matters even in the long run, i.e., even for mature firms, and that the financing policy is uniquely determined. Moreover, firms are never completely unconstrained and firms which currently pay dividends that are hit by a sequence of low cash flow shocks may eventually stop dividend payments, cut risk management, and switch back to leasing capital, implications that are empirically plausible. [14]

Firms in our model thus have access to two sources of external financing, state-contingent secured debt and leasing. In Section 4 we provide an alternative interpretation, which is equivalent, in which firms have access to non-contingent secured debt, leasing, and risk management using one period ahead Arrow securities subject to short sale constraints.

3.2. Firm's problem

The firm's problem can be written recursively as the problem of maximizing the discounted expected value of future dividends by choosing the current dividend d, capital k, leased capital k_l, net worth $w(s')$ in state s' next period, and state-contingent debt $b(s')$ given current net worth w and state s:

$$V(w,s) \equiv \max_{\{d,k,k_l,w(s'),b(s')\} \in \mathbb{R}^{3+S}_+ \times \mathbb{R}^S} d + \beta \sum_{s' \in S} \Pi(s,s') V(w(s'),s') \tag{1}$$

subject to the budget constraints for the current and next period

$$w + \sum_{s' \in S} \Pi(s,s')b(s') \geq d + (k - k_l) + R^{-1}u_l k_l \tag{2}$$

$$A(s')f(k) + (k - k_l)(1 - \delta) \geq w(s') + Rb(s'), \forall s' \in S, \tag{3}$$

we should adjust by $R^{-1}(1-\delta)$ times 10 to be precise. In documenting the stylized facts, we ignored the correction $R^{-1}(1-\delta)$, implicitly assuming that it is approximately equal to 1.

[13] E.g., this assumption is made by DeMarzo and Sannikov (2006), Lorenzoni and Walentin (2007), Biais, Mariotti, Plantin, and Rochet (2007), Biais, Mariotti, Rochet, and Villeneuve (2010), and DeMarzo, Fishman, He, and Wang (2012); DeMarzo and Fishman (2007a, 2007b) consider $\beta \leq R^{-1}$; in contrast, firms and lenders are assumed to be equally patient in Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), and Rampini and Viswanathan (2010).

[14] While we do not explicitly consider taxes here, our assumption about discount rates can also be interpreted as a reduced form way of taking into account the tax-deductibility of interest, which effectively lowers the cost of debt finance.

the collateral constraints

$$\theta(\varphi k - k_l)(1 - \delta) \geq Rb(s'), \qquad \forall s' \in S, \tag{4}$$

and the constraint that only tangible capital can be leased

$$\varphi k \geq k_l. \tag{5}$$

The program in (1)-(5) requires that dividends d and net worth $w(s')$ are non-negative which is due to limited liability. Furthermore, capital k and leased capital k_l have to be non-negative as well. We write the budget constraints as inequality constraints, despite the fact that they bind at an optimal contract, since this makes the constraint set convex as shown below. There are only two state variables in this recursive formulation, net worth w and the state of productivity s. This is due to our assumption that there are no adjustment costs of any kind and greatly simplifies the analysis. Net worth in state s' next period $w(s') = A(s')f(k) + (k - k_l)(1 - \delta) - Rb(s')$, i.e., equals cash flow plus the depreciated resale value of owned capital minus the amount to be repaid on state s' contingent debt. Borrowing against state s' next period by issuing state s' contingent debt $b(s')$ reduces net worth $w(s')$ in that state. In other words, borrowing less than the maximum amount which satisfies the collateral constraint (4) against state s' amounts to conserving net worth for that state and allows the firm to hedge the available net worth in that state.

We make the following assumptions about the stochastic process describing productivity and the production function:

Assumption 1. *For all $\hat{s}, s \in S$ such that $\hat{s} > s$, (i) $A(\hat{s}) > A(s)$ and (ii) $A(s) > 0$.*

Assumption 2. *f is strictly increasing, strictly concave, $f(0) = 0$, $\lim_{k \to 0} f_k(k) = +\infty$, and $\lim_{k \to +\infty} f_k(k) = 0$.*

We first show that the firm financing problem is a well-behaved dynamic programming problem and that there exists a unique value function V which solves the problem. To simplify notation, we introduce the shorthand for the choice variables x, where $x \equiv [d, k, k_l, w(s'), b(s')]'$, and the shorthand for the constraint set $\Gamma(w, s)$ given the state variables w and s, defined as the set of $x \in \mathbb{R}_+^{3+S} \times \mathbb{R}^S$ such that (2) to (5) are satisfied. Define operator T as

$$(Tg)(w, s) = \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s, s')g(w(s'), s').$$

We prove the following result about the firm financing problem in (1) to (5):

Proposition 1. **(i)** $\Gamma(w, s)$ *is convex, given* (w, s), *and convex in* w *and monotone in the sense that* $w \leq \hat{w}$ *implies* $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$. **(ii)** *The operator* T *satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point* V. **(iii)** V *is continuous, strictly increasing, and concave in* w. **(iv)** *Without leasing,* $V(w, s)$ *is strictly concave in* w *for* $w \in int\{w : d(w, s) = 0\}$. **(v)** *Assuming that for all* $\hat{s}, s \in S$ *such that* $\hat{s} > s$, $\Pi(\hat{s}, s')$ *strictly first order stochastically dominates* $\Pi(s, s')$, V *is strictly increasing in* s.

The proofs of part (i) to (iii) of the proposition are relatively standard. Part (iii) however only states that the value function is concave, not strictly concave. The value function is linear in net worth when dividends are paid. The value function may also be linear in net worth on some intervals where no dividends are paid, due to the linearity of the substitution between leased and owned capital. All our proofs below hence rely on weak concavity only. Nevertheless we can show that without leasing, the value function is strictly concave where no dividends are paid (see part (iv) of the proposition). [15]

Denote the multipliers on the constraints (2) to (5) by μ, $\Pi(s, s')\beta\mu(s')$, $\Pi(s, s')\beta\lambda(s')$, and $\bar{\nu}_l$. Let ν_d and $\underline{\nu}_l$ be the multipliers on the constraint $d \geq 0$ and $k_l \geq 0$. The first order conditions of the firm financing problem in Eqs. (1) to (5) are

$$\mu = 1 + \nu_d \tag{6}$$

$$\mu = \sum_{s' \in S} \Pi(s, s')\beta \{\mu(s') [A(s')f_k(k) + (1 - \delta)] + \lambda(s')\theta\varphi(1 - \delta)\} + \bar{\nu}_l\varphi \tag{7}$$

$$(1 - R^{-1}u_l)\mu = \sum_{s' \in S} \Pi(s, s')\beta \{\mu(s')(1 - \delta) + \lambda(s')\theta(1 - \delta)\} + \bar{\nu}_l - \underline{\nu}_l \tag{8}$$

$$\mu(s') = V_w(w(s'), s'), \qquad \forall s' \in S, \tag{9}$$

$$\mu = \beta\mu(s')R + \beta\lambda(s')R, \qquad \forall s' \in S, \tag{10}$$

where we use the fact that the constraints $k \geq 0$ and $w(s') \geq 0$, $\forall s' \in S$, are slack as Lemma 6 in Appendix A shows. [16] The envelope condition is $V_w(w, s) = \mu$; the marginal value of (current) net worth is μ. Similarly, the marginal value of net worth in state s' next period is $\mu(s')$.

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[15] Section 5 discusses how the linearity of the substitution between leased and owned capital may result in intervals on which the value function is linear and shows that when m is sufficiently high, the value function is strictly concave in the non-dividend paying region even with leasing.

[16] Since the marginal product of capital is unbounded as capital goes to zero by Assumption 2, capital is strictly positive. Because the firm's ability to issue promises against capital is limited, this in turn implies that the firm's net worth is positive in all states next period.

Using Eqs. (7) and (10), we obtain the investment Euler equation,

$$1 \geq \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{A(s') f_k(k) + (1 - \theta\varphi)(1 - \delta)}{1 - R^{-1}\theta\varphi(1 - \delta)}, \tag{11}$$

which holds with equality if the firm does not lease all its tangible assets (i.e., $\bar{\nu}_l = 0$). Notice that $\beta\mu(s')/\mu$ is the firm's stochastic discount factor; collateral constraints render the firm as if risk averse and hence provide a rationale for risk management.

3.3. User cost of capital

This section defines the user cost of (purchased) tangible and intangible capital in the presence of collateral constraints extending Jorgenson's (1963) definition. The definitions clarify the main economic intuition and allow a simple characterization of the leasing decision as we show in Section 5.

Denote the premium on internal funds for a firm with net worth w in state s by $\rho(w, s)$ and define it implicitly using the firm's stochastic discount factor as $1/(1+r+\rho(w, s)) \equiv \sum_{s' \in S} \Pi(s, s')\beta\mu(s')/\mu$; internal funds command a premium as long as at least one of the collateral constraints is binding. Define the user cost of tangible capital which is purchased $u_p(w, s)$ for a firm with net worth w in state s as

$$u_p(w, s) \equiv r + \delta + \frac{\rho(w, s)}{R + \rho(w, s)}(1 - \theta)(1 - \delta) \tag{12}$$

where $\rho(w, s)/(R + \rho(w, s)) = \sum_{s' \in S} \Pi(s, s')R\beta\lambda(s')/\mu$. The user cost of purchased tangible capital is the sum of the Jorgensonian user cost of capital and a second term, which captures the additional cost of internal funds for the fraction $(1 - \theta)(1 - \delta)$ of capital that has to be financed internally. Analogously, the user cost of intangible capital is $u_i(w, s) \equiv r + \delta + \rho(w, s)/(R + \rho(w, s))(1 - \delta)$.

Using our definitions, we can rewrite the first order condition for capital, Eq. (7), as

$$\varphi \min\{u_p(w, s), u_l\} + (1 - \varphi)u_i(w, s) = \sum_{s' \in S} \Pi(s, s')R\beta\frac{\mu(s')}{\mu}A(s')f_k(k).$$

Optimal investment equates the weighted average of the user cost of tangible and intangible capital with the expected marginal product of capital, where the applicable user cost of tangible capital is the minimum of the user cost of purchased and leased capital. [17]

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[17] We can rewrite Eq. (12) in a *weighted average (user) cost of capital* form as $u_p(w, s) = R/(R+\rho(w, s))((r+\rho(w, s))[1-R^{-1}\theta(1-\delta)]+r[R^{-1}\theta(1-\delta)]+\delta)$, where the fraction of capital that can be financed externally, $R^{-1}\theta(1 - \delta)$, is charged a cost of capital r, while the fraction

3.4. Dividend payout policy

We start by characterizing the firm's payout policy. The firm's dividend policy is very intuitive: there is a state-contingent cutoff level of net worth $\bar{w}(s)$, $\forall s \in S$, above which the firm pays dividends. [18] Moreover, whenever the firm has net worth w exceeding the cutoff $\bar{w}(s)$, paying dividends in the amount $w - \bar{w}(s)$ is optimal. All firms with net worth w exceeding the cutoff $\bar{w}(s)$ in a given state s, choose the same level of capital. Finally, the investment policy is unique in terms of the choice of capital k. The following proposition summarizes the characterization of firms' payout policy:

Proposition 2 (Dividend policy) *There is a state-contingent cutoff level of net worth, above which the marginal value of net worth is one and the firm pays dividends:* **(i)** *$\forall s \in S$, $\exists \bar{w}(s)$ such that, $\forall w \geq \bar{w}(s)$, $\mu(w, s) = 1$.* **(ii)** *For $\forall w \geq \bar{w}(s)$,*

$$[d_o(w,s), k_o(w,s), k_{l,o}(w,s), w_o(s'), b_o(s')] = [w - \bar{w}(s), \bar{k}_o(s), \bar{k}_{l,o}(s), \bar{w}_o(s'), \bar{b}_o(s')]$$

where $\bar{x}_o \equiv [0, \bar{k}_o(s), \bar{k}_{l,o}(s), \bar{w}_o(s'), \bar{b}_o(s')]$ attains $V(\bar{w}(s), s)$. Indeed, $k_o(w, s)$ is unique for all w and s. **(iii)** *Without leasing, the optimal policy x_o is unique.*

3.5. Effect of tangibility and collateralizability without leasing

We distinguish between the fraction of tangible assets required for production, φ, and the fraction of tangible assets θ that the borrower cannot abscond with and that is hence collateralizable. This distinction is important to understand differences in the capital structure across industries, as the fraction of tangible assets required for production varies considerably at the industry level whereas the fraction of tangible assets that is collateralizable primarily depends on the type of capital, such as structures versus equipment (which we do not distinguish here). Thus, industry variation in φ needs to be taken into account in empirical work. That said, in the special case without leasing, higher tangibility φ and higher collateralizability θ are equivalent in our model. This result is immediate as without leasing, φ and θ affect only (4) and only the product of the two matters. Thus, firms that operate in industries that require more intangible capital are more constrained, all else equal. Furthermore, the fact that firms can only borrow against a fraction $\varphi\theta$ of total capital is quantitatively relevant as the model predicts much lower, and empirically plausible, leverage ratios.

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that has to be financed internally, $1 - R^{-1}\theta(1 - \delta)$, is charged a cost of capital $r + \rho(w, s)$.

[18] Consistent with this prediction, DeAngelo, DeAngelo, and Stulz (2006) find a strong positive relation between the probability that firms pay dividends and their retained earnings.

4. Risk management and the capital structure

Our model allows an explicit analysis of dynamic risk management since firms have access to complete markets subject to the collateral constraints. We first show how to interpret the state-contingent debt in our model in terms of risk management and provide a general result about the optimal absence of risk management for firms with sufficiently low net worth. Next, we prove the optimality of incomplete hedging with constant investment opportunities, i.e., when productivity shocks are independently and identically distributed; indeed, firms abstain from risk management with positive probability under the stationary distribution, implying that even mature firms that experience a sequence of low cash flows eliminate risk management. Moreover, with stochastic investment opportunities persistent shocks further reduce risk management and may result in a complete absence of risk management for empirically plausible levels of persistence. Strong persistence of productivity may result in firms hedging higher productivity states, because financing needs for increased investment rise more than cash flows. Finally, we analyze the comparative statics of firms' investment, financing, risk management, and dividend policy with respect to key parameters of the model.

4.1. Optimal absence of risk management

Our model with state-contingent debt $b(s')$ is equivalent to a model in which firms borrow as much as they can against each unit of tangible capital which they purchase, i.e., borrow $R^{-1}\theta\varphi(1-\delta)$ per unit of capital, and keep additional net worth in a state contingent way by purchasing Arrow securities with a payoff of $h(s')$ for state s'. This formulation allows us to characterize the corporate hedging policy. Specifically, we define risk management in terms of purchases of Arrow securities for state s' as

$$h(s') \equiv \theta(\varphi k - k_l)(1-\delta) - Rb(s'). \tag{13}$$

We say a firm does not engage in risk management when the firm does not buy Arrow securities for any state next period. Under this interpretation, firms' debt is not state-contingent and hence risk-free, as are lease contracts, since we assume that the price of capital is constant for all states. We denote the amount firms pay down per unit of capital by $\wp(\varphi) \equiv 1 - R^{-1}\theta\varphi(1-\delta)$ and the amount firms pay down per unit of tangible capital by $\wp \equiv \wp(1) \equiv 1 - R^{-1}\theta(1-\delta)$. Using this notation, we can write the budget constraints for the current and next period (2) and (3) for this implementation as

$$w \geq d + \sum_{s'\in S} \Pi(s,s')R^{-1}h(s') + \wp(\varphi)k - (\wp - R^{-1}u_l)k_l \tag{14}$$

$$A(s')f(k) + [(1-\varphi)k + (1-\theta)(\varphi k - k_l)](1-\delta) + h(s') \geq w(s'), \tag{15}$$

and the collateral constraints (4) reduce to short sale constraints

$$h(s') \geq 0, \qquad \forall s' \in S, \qquad (16)$$

implying that holdings of Arrow securities have to be non-negative. The budget constraint (14) makes the trade-off between financing and risk management particularly apparent; the firm can spend its net worth w on purchases of Arrow securities, i.e., hedging, or to buy fully levered capital; leasing frees up net worth as long as $\wp - R^{-1}u_l > 0$ which we assume (see Assumption 3 in Section 5) as leasing is otherwise dominated. Eq. (15) states that net worth $w(s')$ in state s' next period is the sum of cash flows, the value of intangible capital and owned tangible capital not pledged to the lenders, and the payoffs of the Arrow securities, if any. Our model with state-contingent borrowing is hence a model of financing and risk management.

In the implementation considered in this section, firms pledge as much as they can against their tangible assets to lenders leaving no collateral to pledge to derivatives counterparties. The holdings of Arrow securities are hence subject to short sale constraints and the cost of risk management is the net worth required in the current period to purchase them. That said, since firms in our model have access to complete markets, subject to collateral constraints, firms can replicate any type of derivatives contract, including forward contracts or futures that do not involve payments up front, implying that our results hold for forward contracts and futures, too. Such contracts involve promises to pay in some states next period which count against the collateral constraints in those states. There is an opportunity cost for such promises, because promises against these states could alternatively be used to finance current investment. Thus, there are no constraints on the type of hedging instruments firms can use in our model, and the only constraint on risk management is that promised payments need to be collateralized, which is identical to the constraint imposed on financing.

The next proposition states that for severely constrained firms all collateral constraints bind, which means that such firms do not purchase any Arrow securities at all, and, in this sense, do not engage in risk management. The numerical examples in Sections 4.2 and 4.4 show that the extent to which firms hedge low states is in fact increasing in net worth. [19]

Severely constrained firms optimally abstain from risk management altogether:

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[19] In our model, we do not take a stand on whether the productivity shocks are firm specific or aggregate. Since all states are observable, as the only friction considered is limited enforcement, our analysis applies either way. Hedging can hence be interpreted either as using loan commitments, e.g., to hedge idiosyncratic shocks to firms' net worth or as using traded assets to hedge aggregate shocks which affect firms' cash flows.

Proposition 3 (Optimal absence of risk management) *Firms with sufficiently low net worth do not engage in risk management, i.e., $\forall s \in S$, $\exists \underline{w}_h(s) > 0$, such that $\forall w \leq \underline{w}_h(s)$, all collateral constraints bind, $\lambda(s') > 0$, $\forall s' \in S$.*

Collateral constraints imply that there is an opportunity cost to issuing promises to pay in high net worth states next period to hedge low net worth states, as such promises can also be used to finance current investment. The proposition shows that when net worth is sufficiently low, the opportunity cost of risk management due to the financing needs must exceed the benefit. Hence, firms optimally do not hedge at all. Notice that the result obtains for a general Markov process for productivity. The result is consistent with the evidence that firms with low net worth hedge less, and is in contrast to the conclusions from static models in the extant literature, such as Froot, Scharfstein, and Stein (1993). The key difference is that our model explicitly considers dynamic financing needs for investment as well as the limits on firms' ability to promise to pay.

4.2. Risk management with constant investment opportunities

With independent productivity shocks, risk management only depends on the firm's net worth, because the expected productivity of capital is independent of the current state s, i.e., investment opportunities are constant. More generally, both cash flows and investment opportunities vary, and the correlation between the two affects the desirability of hedging, as we show in Section 4.4 below.

With constant investment opportunities, the marginal value of net worth is higher in states with low cash flows and complete hedging is never optimal:

Proposition 4 (Optimality of incomplete hedging) *Suppose that $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$. **(i)** The marginal value of net worth next period $\mu(s') = V_w(w(s'))$ is (weakly) decreasing in the state s', and the multipliers on the collateral constraints are (weakly) increasing in the state s', i.e., $\forall s', s'_+ \in S$ such that $s'_+ > s'$, $\mu(s'_+) \leq \mu(s')$ and $\lambda(s'_+) \geq \lambda(s')$. **(ii)** Incomplete hedging is optimal, i.e., $\exists s' \in S$, such that $\lambda(s') > 0$. Indeed, $\exists s', \hat{s}' \in S$, such that $w(s') \neq w(\hat{s}')$. Moreover, the firm never hedges the highest state, i.e., is always borrowing constrained against the highest state, $\lambda(\bar{s}') > 0$ where $\bar{s}' = \max\{s' : s' \in S\}$. The firm hedges a lower interval of states, $[\underline{s}', \ldots, s'_h]$, where $\underline{s}' = \min\{s' : s' \in S\}$, if at all.*

Complete hedging would imply that all collateral constraints are slack and consequently the marginal value of net worth is equalized across all states next period. But hedging involves conserving net worth in a state-contingent way at a return R. Given the firm's relative impatience, it can never be optimal to save in this state-contingent way for all

states next period. Thus, incomplete hedging is optimal. Further, since the marginal value of net worth is higher in states with low cash flow realizations, it is optimal to hedge the net worth in these states, if it is optimal to hedge at all. Firms' optimal hedging policy implicitly ensures a minimum level of net worth in all states next period.

We emphasize that our explicit dynamic model of collateral constraints due to limited enforcement is essential for this result. If the firm's ability to pledge were not limited, then the firm would always want to pledge more against high net worth states next period to equate net worth across all states. However, in our model the ability to credibly pledge to pay is limited and there is an opportunity cost to pledging to pay in high net worth states next period, since such pledges are also required for financing current investment. This opportunity cost implies that the firm never chooses to fully hedge net worth shocks.

To illustrate the interaction between financing needs and risk management, we compute a numerical example. We assume that productivity is independent and, for simplicity, that productivity takes on two values only, $A(s_1) < A(s_2)$, and that there is no leasing. The results and details of the parameterization are reported in Fig. 2.

Investment as a function of net worth is shown in Panel A, which illustrates Proposition 2. Above a threshold $\bar{w}$, firms pay dividends and investment is constant. Below the threshold, investment is increasing in net worth and dividends are zero.

The dependence of risk management on net worth is illustrated in Panel B. With independent shocks, the firm never hedges the high state, i.e., $h(s_2') = 0$, where $h(s')$ is defined as in Eq. (13) (see Proposition 4). Panel B thus displays only the payoff of the Arrow securities that the firm purchases to hedge the low state, $h(s_1')$. Importantly, note that the hedging policy is increasing in firm net worth, i.e., better capitalized firms hedge more. This illustrates the main conclusion from our model for risk management. Above the threshold $\bar{w}$, hedging is constant (as Proposition 2 shows). Below the threshold, hedging is increasing, and for sufficiently low values of net worth w, the firm does not hedge at all, as Proposition 3 shows more generally. Indeed, hedging is zero for a sizable range of values of net worth (up to a value of around 0.1 in the example).

The values of net worth next period are displayed in Panel C and illustrate the endogenous dynamics of net worth. Since optimal hedging is incomplete, net worth next period is higher in state s_2' than in state s_1'. The figure moreover plots the 45 degree line (dotted), where $w = w'$, to facilitate the characterization of the dynamics of net worth and the stationary distribution. Net worth next period is higher than current net worth, i.e., increases, when $w(s')$ is above the 45 degree line and decreases otherwise. In the low (high) state next period net worth decreases (increases) for all levels of net worth above (below) the intersection of $w(s_1')$ ($w(s_2')$) and the 45 degree line, which

we denote $\underline{w}(s_1)$ ($\bar{w}(s_2)$). These transition dynamics of net worth together with the policy functions describe the dynamics of investment, financing, and risk management fully; if the low (high) state is realized, investment and risk management both decrease (increase) as long as current net worth is in the interval $[\underline{w}(s_1), \bar{w}(s_2)]$.

Since levels of net worth below $\underline{w}(s_1)$ and above $\bar{w}(s_2)$ are transient, the ergodic set must be bounded below by $\underline{w}(s_1)$ and above by $\bar{w}(s_2)$, and the support of the stationary distribution is a subset of the interval $[\underline{w}(s_1), \bar{w}(s_2)]$. Indeed, firms with net worth above $\bar{w}(s_2)$ will pay out the extra net worth and start the next period within the ergodic set. Moreover, evaluating the first order condition (10) for $b(s')$ at $\underline{w}(s_1)$ and $s' = s_1$, we have $V_w(\underline{w}(s_1)) = R\beta V_w(\underline{w}(s_1)) + R\beta\lambda(\underline{w}(s_1))$ and thus $\lambda(\underline{w}(s_1)) > 0$. This means that the firm abstains from risk management altogether at $\underline{w}(s_1)$. By continuity, the absence of risk management is optimal for sufficiently low values of net worth in the stationary distribution, a result that is more generally true (see Proposition 5 below).

The multipliers on the collateral constraints $\lambda(s')$ are shown in Panel D. The first order conditions (10) for $b(s')$ imply that $\mu(s_1') + \lambda(s_1') = \mu(s_2') + \lambda(s_2')$; firms do not simply equate the marginal value of net worth across states, but the sum of the marginal value of net worth and the multiplier on the collateral constraint. The figure shows that $\lambda(s_2') > 0$ for all w, whereas the multiplier on the collateral constraint for the low state $\lambda(s_1')$ is zero for levels of w at which the firm hedges and positive for lower levels at which the firm abstains from risk management. Collateral constraints result in a trade off between financing and risk management.

4.3. Risk management under the stationary distribution

We now show that firms do not hedge at the lower bound of the stationary distribution as we observed in the example above. Indeed, firms abstain from risk management with positive probability under the (unique) stationary distribution.

Proposition 5 (Absence of risk management under the stationary distribution) *Suppose $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$, and $m = +\infty$ (no leasing).* **(i)** *For the lowest state $\underline{s}'$, the wealth level $\underline{w}$ for which $w(\underline{s}') = \underline{w}$ is unique and the firm abstains from risk management at $\underline{w}$.* **(ii)** *There exists a unique stationary distribution of net worth and firms abstain from risk management with positive probability under the stationary distribution.*

Proposition 5 implies that even if a firm is currently relatively well capitalized and paying dividends, a sufficiently long sequence of low cash flows will leave the firm so

constrained that it chooses to stop hedging.[20] Thus, it is not only young firms with very low net worth that abstain from risk management, but also mature firms that suffer adverse cash flow shocks. Consistently, Rampini, Sufi, and Viswanathan (2012) show that airlines that hit financial distress dramatically cut their fuel price risk management.

4.4. Risk management with stochastic investment opportunities

With stochastic investment opportunities, risk management depends not only on net worth but also on the firm's productivity, since the conditional expectation of future productivity varies with current productivity when productivity is persistent. Positive autocorrelation reduces the benefit to hedging and can eliminate the need to hedge completely.

We first show that incomplete hedging is optimal even when investment opportunities are stochastic:

Proposition 6 (Optimality of incomplete hedging with persistence) *Suppose* $m = +\infty$ *(no leasing). Optimal risk management is incomplete with positive probability, i.e.,* $\exists s$ *such that for all* w, $\lambda(s') > 0$, *for some* s'.

Thus, firms engage in incomplete risk management for any Markov process of productivity, generalizing the results from Proposition 4. As in the case without persistence, firms engage in risk management to transfer funds into states in which the marginal value of net worth is highest (if at all). With persistence, however, the marginal value of net worth depends on both net worth and investment opportunities going forward. Positive autocorrelation, which is typical in practice, implies that investment opportunities are worse (better) when productivity is low (high), decreasing (increasing) the marginal value of net worth in states with low (high) realizations. This reduces the benefit to hedging and thus firms hedge less or not at all. Indeed, if this effect is strong enough, firms may hedge states with high productivity, despite the fact that cash flows are high then, too. Current investment opportunities also affect the benefits to investing and the opportunity cost of hedging, and thus the extent of risk management depends on current productivity as well.

To study the effect of persistence, we reconsider the example with a two state symmetric Markov process for productivity from the previous section. The results are reported in Fig. 3. We increase the transition probabilities $\Pi(s_1, s_1) = \Pi(s_2, s_2) \equiv \pi$, which are

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[20] Eq. (10) implies that there is an upper bound on the extent to which the marginal value of net worth can increase as $V_w(w(s'), s')/V_w(w, s) \leq (\beta R)^{-1}$; however, the marginal value of net worth could increase, e.g., when cash flows are sufficiently low.

0.5 when investment opportunities are constant, progressively to 0.55, 0.60, 0.75, and 0.90. Since the autocorrelation of a symmetric two state Markov process is $2\pi - 1$, this corresponds to progressively raising the autocorrelation from 0 to 0.1, 0.2, 0.5, and 0.8. Given the symmetry, the stationary distribution of the productivity process over the two states is 0.5 and 0.5 across all our examples, and the unconditional expected productivity is hence the same as well.

Panels A and B display the investment and hedging policies in the case without persistence from the previous section (see also Fig. 2). Since these policy functions are independent of the state of productivity s, there is only one function for each policy. In the panels with persistence, the solid lines denote the policies when current productivity is low (s_1) and the dashed lines the ones when current productivity is high (s_2).

The left panels show that as persistence increases, investment is higher when current productivity is high, because the conditional expected productivity is higher. The right panels show the effect of persistence on the hedging policy. Panel D shows that hedging (for the low state) decreases relative to the case without persistence, but more so when current productivity is high. Most notably, for given net worth, the firm hedges the low state less when current productivity is high than when it is low. The economic intuition has two aspects. First, persistent shocks reduce the marginal value of net worth when productivity is low (since the conditional expected productivity is low then too) while raising the marginal value of net worth when productivity is high. Thus, there is less reason to hedge. Second, high current productivity leads to higher investment when productivity is persistent (and higher opportunity cost of risk management), which raises net worth next period and further reduces the hedging need. This second effect goes the other way when current productivity is low, but the first effect dominates in our example even in this case.

When persistence is raised further, the benefit to hedging is still lower and the firm abstains from hedging completely when current productivity is high and only hedges (the low state) when current productivity is low (Panel F); indeed, the firm stops hedging completely at an autocorrelation of productivity of 0.5 (see Panel H). This suggests that, for empirically plausible parameterizations (see, e.g., the calibrated autocorrelation of 0.62 in Gomes, 2001), even firms with high net worth may engage in only limited risk management or none at all.

For the persistence levels considered thus far, firms hedge the low cash flow state, if at all. With severe persistence (see Panels I and J), the difference in investment is very large across the two productivity states, and firms have an incentive to hedge the high state due to its substantially greater investment opportunities when current productivity is low (see the dash-dotted line in Panel J). But notice that even in this

case hedging is increasing in net worth.

This example illustrates the dynamic trade-off between financing current investment and risk management. First, current expected productivity affects the benefits to investing and hence the opportunity cost of risk management. Second, expected productivity next period affects the benefits to hedging and which states the firm hedges; for plausible levels of persistence firms abstain from risk management altogether. However, whatever the persistence, firms do not hedge at all when they are severely constrained.

4.5. Effect of risk, tangibility, and collateralizability

We now study the comparative statics of the firm's investment, financing, risk management, and dividend policy with respect to key parameters of the model. Specifically we consider how the firm's optimal policy varies with the risk of the productivity process $A(s')$, the tangibility φ and collateralizability θ, and the curvature of the production function α when $f(k) = k^\alpha$. For simplicity, we consider the case without leasing, which implies that the effects of tangibility and collateralizability are identical. Moreover, we assume that investment opportunities are constant.

First, consider how the firm responds to an increase in risk of the productivity process $A(s')$ in the Rothschild and Stiglitz (1970) sense. An increase in risk decreases firm value. Intuitively, an increase in productivity risk results in an increase in risk in net worth, given the optimal policy, which reduces value since the value function is concave in net worth. In contrast, in a frictionless world firm value would be unaffected by such an increase in risk as expected cash flows are unchanged.

The increase in risk also affects the firm's dividend and investment policy. Relative to the deterministic case, a firm subject to risk pays dividends only at a higher level of net worth since there is a precautionary motive to retaining net worth. Such a firm also invests more in the dividend paying region essentially because of a precautionary motive for investment. That said, when the firm engages in risk management, the financing needs for risk management can reduce investment given net worth. The following proposition summarizes these results:

Proposition 7 (Effect of risk) *Suppose that $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$, and $m = +\infty$ (no leasing). (i) Valuation: Suppose $A_+(s')$ is an increase in risk from $A(s')$ in the Rothschild and Stiglitz (1970) sense and let V (V_+) be the value function associated with $A(s')$ $(A_+(s'))$; then $V(w) \geq V_+(w)$, $\forall w$, i.e., an increase in risk reduces the value of the firm. (ii) Investment and dividend policy: Suppose A'_0 is a constant and $A_\sigma(s')$ is an increase in risk from A'_0 and denote the associated optimal policy by x_0 (x_σ); then $\bar{k}_\sigma \geq \bar{k}_0$ and $\bar{w}_\sigma \geq \bar{w}_0$, i.e., the investment of a dividend paying firm and the cutoff*

net worth at which the firm starts to pay dividends are higher in the stochastic case. Moreover, suppose that $S = \{\underline{s}, \bar{s}\}$; if a dividend paying firm is hedging, then $\bar{k}_\sigma$ is decreasing in risk.

Fig. 4 illustrates the comparative statics with respect to risk. Panel A plots firms' investment policy for different levels of risk. The precautionary motive for risk management increases the optimal investment of dividend paying firms. At the same time, the financing needs for risk management reduce investment given net worth when firms do not pay dividends and also reduce investment for dividend paying firms that hedge. Since investment is constant in the dividend paying region, Panel A also shows that as risk increases, firms postpone paying dividends until a higher cutoff of net worth is reached. Panel B shows that as risk increases, firms start to hedge earlier and do more risk management given net worth. [21]

Second, consider how collateralizability θ and tangibility φ affect the firm's optimal policy. We emphasize that in the absence of leasing the effects of collateralizability and tangibility are the same, and that collateralizability and tangibility are primarily determined by the nature of the assets used in a particular industry; tangibility φ is determined by the extent to which tangible assets (structures and equipment) are used and collateralizability θ is determined by the extent to which structures, which are arguably more collateralizable, are used instead of equipment. We discuss the effects in terms of collateralizability. Firm value increases in collateralizability; intuitively, a firm with higher collateralizability could choose the same policy as a firm with lower collateralizability, but chooses not to do so. A larger fraction of the capital can be finance with debt. Moreover, and most directly, collateralizability and tangibility increase leverage. These results are stated formally in the following proposition:

Proposition 8 (Effect of tangibility and collateralizability) *Suppose $m = +\infty$ (no leasing).* **(i)** *Valuation: Suppose $\theta_+ > \theta$ and let V (V_+) be associated with θ (θ_+); then $V_+(w, s) > V(w, s)$, $\forall w$ and s, i.e., an increase in collateralizability (or tangibility) strictly increases the value of the firm.* **(ii)** *Investment policy: Suppose that $S = \{\underline{s}, \bar{s}\}$ and $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$; if a dividend paying firm is hedging, then $\bar{k}$ is increasing in collateralizability (or tangibility).*

Fig. 5 illustrates these comparative statics. Panel A shows that higher collateralizability θ increases investment given net worth, because it allows higher leverage and thus a

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[21] Variation in risk across firms thus induces a negative relation between value and risk management given net worth, all else equal, in contrast to the hypothesis of a positive relation typically explored in empirical work.

large fraction of capital can be financed with debt, which is relatively cheap. Moreover, firms with higher collateralizability start paying dividends at a lower level of net worth since less net worth is required to run a firm of a given size. Panel B shows that collateralizability increases risk management, because it allows higher leverage which makes the net worth of the firm more volatile, all else equal.

Finally, consider the effect of the curvature of the production function α when $f(k) = k^\alpha$. One might expect that higher curvature of the production function, i.e., lower α, would induce more curvature in the value function and result in increased risk management. However, when the curvature is higher, the marginal product of capital decreases faster, which reduces the optimal size of the firm. This in turn reduces the financing needs for investment and the cash flow risk and can result in reduced hedging. In fact, Fig. 6 illustrates that both investment and risk management decrease as the curvature increases given our parameterization. The comparative statics of our model imply interesting empirical predictions for firm financing and risk management in both the cross section and the time series.

5. Leasing and the capital structure

This section analyzes the dynamic leasing decision. We first prove that leasing is optimal for firms with sufficiently low net worth. We then analyze the dynamic choice between leasing and secured financing in the deterministic case, which facilitates explicit characterization because the collateral constraint binds throughout, to highlight the economic intuition; leasing allows firms to grow faster. In the stochastic case, leasing and risk management are jointly determined and firms that lease may engage in risk management because leasing enables higher leverage but reduces net worth in low cash flow states. Mature firms may engage in sale-and-leaseback transactions when they experience adverse cash flow shocks. Moreover, the model's implications for the capital structure are consistent with the empirical facts documented in Section 2.

5.1. Optimality of leasing

Using the definitions of the user cost of tangible capital and (10), the first order condition with respect to leased capital, (8), simplifies to

$$u_p(w, s) = u_l + \frac{R}{\mu}(\bar{\nu}_l - \underline{\nu}_l). \tag{17}$$

The decision between purchasing capital and leasing reduces to a straight comparison of the user costs. If the user cost of leasing exceeds the user cost of purchased capital,

which depends on the firm's net worth w and state s, $\underline{\nu}_l > 0$ and the firm purchases all capital. If the reverse is true, $\bar{\nu}_l > 0$ and all capital is leased. When $u_p(w, s) = u_l$, the firm is indifferent between leasing and purchasing capital at the margin.

Using Eqs. (8) and (10) we moreover obtain an Euler equation for leasing capital

$$1 = \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{(1 - \theta)(1 - \delta)}{\wp - R^{-1} u_l} + \frac{1}{\mu} \frac{\bar{\nu}_l - \underline{\nu}_l}{\wp - R^{-1} u_l}. \tag{18}$$

Leasing instead of purchasing a unit of tangible capital allows the firm to borrow the additional amount $\wp - R^{-1} u_l$, which is the difference between the minimal down payment required to purchase a unit of capital and the leasing fee, in return for giving up the incremental payoff of $(1 - \theta)(1 - \delta)$ next period, which is the additional amount that the firm can pledge to lessors. We define the implied interest rate paid on the additional amount the firm can borrow by leasing as

$$R_l \equiv \frac{(1 - \theta)(1 - \delta)}{\wp - R^{-1} u_l} = R \frac{1}{1 - \frac{m}{(1 - \theta)(1 - \delta)}} > R. \,^{22}$$

Effectively, leasing is a costly way to borrow more. Indeed, using the premium on internal funds $\rho(w, s)$ defined in Section 3 the leasing Euler equation (18) simplifies to

$$1 \gtrless \frac{1}{1 + r + \rho(w, s)} R_l,$$

and the firm leases some or all of its tangible capital if $R_l \leq 1 + r + \rho(w, s)$, i.e., if the cost of leasing is less than or equal to the cost of internal finance, and does not lease otherwise. In a sense there is a pecking order among the two modes of external financing in our model: firms use secured financing and use lease financing only if their internal funds are sufficiently scarce.

The fact that leasing allows additional borrowing is evident when one writes the up front user cost of leased capital using this notation as

$$R^{-1} u_l = 1 - R^{-1} \theta (1 - \delta) - R_l^{-1} (1 - \theta)(1 - \delta).$$

By leasing, the firm implicitly borrows $\theta(1 - \delta)$ at an interest rate R and the additional amount $(1 - \theta)(1 - \delta)$ at the implied interest rate R_l. The cost of leasing to implicitly borrow more is constant, whereas the return on investment is decreasing as one can see from the investment Euler equation (11). The basic tradeoff is that when the firm's net worth is low, investment is low and hence the marginal return on investment is high whereas the cost of leasing is unchanged, and hence the firm leases all its tangible

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[22] Assumption 3 below implies that the difference $\wp - R^{-1} u_l$ is positive and that $R_l > \beta^{-1}$.

capital. In contrast, when the firm's net worth is high, investment is high and the marginal return on investment low, and the firm may purchase some or all of its tangible capital as the cost of borrowing more by leasing is too high.

The following assumption ensures that the monitoring costs are such that leasing is neither dominated nor dominating, which rules out the uninteresting special cases in which firms never lease or always lease tangible assets:

Assumption 3. *Leasing is neither dominated nor dominating, i.e.,*

$$(1 - \theta)(1 - \delta) > m > (1 - R\beta)(1 - \theta)(1 - \delta).$$

We maintain this assumption throughout. The condition $(1 - \theta)(1 - \delta) > m$ ensures that leasing is not dominated. There are two ways to derive this condition. First, the condition is equivalent to assuming that $\wp > R^{-1}u_l$, which means that the minimal down payment required to purchase a unit of tangible capital is larger than the up front payment required to lease the same capital, i.e., the additional outlay for purchasing capital instead of leasing it is strictly positive. Otherwise there would be no benefit to leasing at all. Second, the user cost of purchased capital for a severely constrained firm is $\lim_{w\to 0} u_p(w, s) = r + \delta + (1 - \theta)(1 - \delta)$, which, given the assumption, strictly exceeds the user cost of leased capital $u_l = r + \delta + m$.

The condition $m > (1 - R\beta)(1 - \theta)(1 - \delta)$ ensures that dividend paying firms do not lease. This is because for a dividend paying firm, the user cost of purchased capital is bounded above by $r + \delta + (1 - R\beta)(1 - \theta)(1 - \delta)$, which, given the assumption, is strictly less than the user cost of leased capital.[23] Thus, leasing is not dominating secured financing either. In other words, the condition implies that $R_l > \beta^{-1}$ making borrowing more by leasing too costly for dividend paying firms.

We now prove that severely constrained firms lease all their tangible assets, while dividend paying firms, and firms with sufficiently high net worth, do not lease:

Proposition 9 (Optimality of leasing) *Suppose m satisfies Assumption 3.* **(i)** *Firms with sufficiently low net worth lease all tangible capital, i.e., $\forall s \in S$, $\exists \underline{w}_l(s) > 0$, such that $\forall w \leq \underline{w}_l(s)$, $k_l = \varphi k$.* **(ii)** *Firms with sufficiently high net worth do not lease any tangible capital, i.e., $\forall s \in S$, $\exists \bar{w}_l(s) < \bar{w}(s)$, such that $\forall w \geq \bar{w}_l(s)$, $k_l = 0$.*

The proposition holds for any Markov process for productivity, and hence cash flows, and does not require any further assumptions. The intuition is that when net worth is

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[23] For a dividend paying firm, $\mu = 1$, and, using (10), we have $R\beta\lambda(s') = 1 - R\beta\mu(s') \leq 1 - R\beta$, $\forall s'$, and thus $u_p(w, s) = r + \delta + \sum_{s'\in S} \Pi(s, s')R\beta\lambda(s')(1-\theta)(1-\delta) \leq r + \delta + (1-R\beta)(1-\theta)(1-\delta)$.

sufficiently low, the firm's investment must be very low and hence its marginal product very high. But then the firm's financing need must be so severe, that it must find the higher debt capacity of leasing worthwhile. Firms with sufficiently high net worth, in contrast, do not value the higher debt capacity enough and hence choose to buy all their capital instead of leasing it. [24]

5.2. Dynamic deterministic choice between leasing and financing

We consider the capital structure dynamics in the deterministic case next. To start, consider the deterministic dynamics of firm financing without leasing. As long as net worth is below a cutoff $\bar{w}$, firms pay no dividends and accumulate net worth over time which allows them to increase the amount of capital they deploy. Once net worth reaches $\bar{w}$, dividends are positive and firms no longer grow.

When leasing is an option, firms have to choose a leasing policy in addition to the investment, financing, and payout policy. In this case, the financing dynamics are as follows: when firms have low net worth, they lease all the tangible capital and purchase only the intangible capital. Over time, firms accumulate net worth and increase their total capital. When they reach a certain net worth threshold, they start to substitute owned capital for leased capital, continuing to accumulate net worth. Once firms own all their tangible and intangible capital, they further accumulate net worth and increase the capital stock until they start to pay dividends. At that point, capital stays constant. The following proposition summarizes the deterministic dynamics:

Proposition 10 (Deterministic capital structure dynamics) (i) *Suppose $m = +\infty$ (no leasing). For $w \leq \bar{w}$, no dividends are paid and capital is strictly increasing in w and over time. For $w > \bar{w}$, dividends are strictly positive and capital is constant at a level $\bar{k}$.* **(ii)** *Suppose m satisfies Assumption 3. For $w \leq \bar{w}$, no dividends are paid and capital is increasing in w and over time. For $w > \bar{w}$, dividends are strictly positive and capital is constant at a level $\bar{k}$. There exist $\underline{w}_l < \bar{w}_l < \bar{w}$, such that for $w \leq \underline{w}_l$ all tangible capital is leased and for $w \in (\underline{w}_l, \bar{w}_l)$ capital is constant at $\underline{k}$ and some but not all tangible capital is leased with the fraction of capital leased linearly decreasing in w between $\underline{w}_l$ and $\bar{w}_l$.*

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[24] If one does not impose the second inequality in Assumption 3, then even dividend paying firms may lease capital. In practice, there arguably are different types of tangible assets which differ in their monitoring costs and so Assumption 3 may be satisfied for some types of assets but not others. This means that there are some types of assets that firms never lease, others that they only lease when severely constrained, and yet others that they lease even when their net worth is quite high or they pay dividends.

Leasing allows constrained firms to grow faster. To see this note that the minimum amount of internal funds required to purchase one unit of capital is $\wp(\varphi) = 1 - R^{-1}\theta\varphi(1-\delta)$, since the firm can borrow against fraction θ of the resale value of tangible capital, which is fraction φ of capital. The minimum amount of internal funds required when tangible capital is leased is $1 - \varphi + R^{-1}u_l\varphi = \wp(\varphi) - R_l^{-1}(1-\theta)\varphi(1-\delta)$, since the firm has to finance all intangible capital with internal funds $(1-\varphi)$ and has to pay the leasing fee on tangible capital up front $(R^{-1}u_l\varphi)$. Per unit of internal funds, the firm can hence buy capital in the amount of one over these minimum amounts of internal funds. Under Assumption 3, leasing allows higher leverage, i.e., $1/(\wp(\varphi) - R_l^{-1}(1-\theta)\varphi(1-\delta)) > 1/\wp(\varphi)$. Thus, leasing allows firms to deploy more capital and hence to grow faster.

Corollary 1 (Leasing and firm growth) *Leasing enables firms to grow faster.*

The same economic intuition carries over to the stochastic case analyzed below. However, the high leverage that leasing entails can change firms' hedging policy considerably.

Finally, we briefly discuss the possibility of linear segments of the value function on $[\underline{w}_l, \bar{w}_l]$. Considering the limit as $m \to (1 - \beta R)(1 - \theta)(1 - \delta)$, which implies that $\beta R_l \to 1$, we have $\underline{k} \to \bar{k}$ and $\bar{w}_l \to \bar{w}$; when leasing is not much more expensive than the firm's discount rate, the firm stops leasing at a level of net worth $\bar{w}_l$ which is close to the level at which the firm starts to pay dividends, $\bar{w}$. But then a firm with current net worth $\bar{w}_l$ has net worth next period $w'(\bar{w}_l) \to \bar{w}_{cum} = \bar{w} + \bar{d}$, and analogously $w'(w) > \bar{w}$ for $w \leq \bar{w}_l$ and close to $\bar{w}_l$. Thus the firm pays dividends next period, implying that $\mu' = 1$ and, using the Euler equation for leasing $\mu = \beta R_l > 1$. Therefore, there is an interval $[\hat{w}, \bar{w}_l] \subseteq [\underline{w}_l, \bar{w}_l]$ on which the value function is linear with slope $\beta R_l > 1$.

We can moreover show that large enough m is a sufficient condition to ensure strict concavity of the value function $\forall w \leq \bar{w}(s)$ in the general stochastic case, by considering the limit as $m \to (1-\theta)(1-\delta)$, implying that $R_l \to +\infty$. The leasing Euler equation (18) at an interior solution implies that $\mu(s')/\mu \to 0$, and hence $\lambda(s') > 0$, $\forall s' \in S$. The investment Euler equation (11) in turn implies that $k \to 0$ when leasing is interior and $w(s') = A(s')f(k) + (1 - \theta\varphi)k(1-\delta) \to 0 < \bar{w}(s')$, i.e., the firm does not pay dividends in any state next period, and hence the firm's net worth is in the strictly concave part of the value function next period. But then the value function is strictly concave this period as well and is strictly concave for all $w \leq \bar{w}(s)$, $\forall s \in S$, for m sufficiently high.

5.3. Leasing, leverage, and risk management

To study the leasing, financing, and risk management policy jointly, we consider the stochastic example without persistence analyzed in Section 4.2 and introduce leasing.

The parameters are mostly as before, with details in the caption of Fig. 7 which displays the results. Panel A displays the investment and leasing policy, which is very similar to the one in the deterministic case; more constrained firms lease more, if not all, of their tangible capital; further, as net worth increases, firms substitute owning and borrowing for leasing and eventually stop leasing altogether.

Particularly noteworthy are the implications for risk management in Panel B. For high values of net worth the figure shows the by now familiar pattern for risk management, with risk management increasing in net worth until the dividend paying region is reached and constant from there on. However, for lower values of net worth at which the firm leases a substantial amount of capital, risk management first increases and then drops quite dramatically and in fact drops back to zero. To understand this result, recall that leasing allows higher leverage and hence firms which are very constrained choose to lease to be able to lever up more. But the high implicit leverage reduces firms' net worth in the low state, and, if this effect is sufficiently strong, firms undo some of it by purchasing some Arrow securities for the low cash flow state next period. Effectively, firms use leasing to borrow more from the high state next period while at least partially undoing the effects of higher leverage for the low state next period via risk management. Panel D shows that the multiplier on the collateral constraint for the low state next period $(\lambda(s_1))$ is non-monotone. It is positive for low values of net worth, consistent with Proposition 3, zero for an interval in which firms hedge when leasing a substantial amount of capital, again positive when firms purchase enough of their capital, and finally goes back to zero for sufficiently well capitalized firms.

The transition function of net worth is displayed in Panel C and is reminiscent of Panel C of Fig. 2. Note that the solid line which denotes $w(s_1')$ crosses the 45 degree line below the point where firms stop leasing. Recall that this intersection is the lower bound of the support of the stationary distribution. This implies that even mature firms that are hit by a sequence of low productivity shocks eventually will return to leasing capital, i.e., firms engage in sale-leaseback transactions under the stationary distribution. Sale-leaseback transactions by financially constrained firms are relatively common in practice. In our model, such transactions enable firms that are hit by adverse shocks to free up net worth and cut investment by less than they would have to otherwise.

Panel E illustrates the implication that rental leverage is decreasing in net worth while debt leverage is increasing and total leverage is approximately constant, consistent with the empirical leverage size relation documented in Section 2. Thus, our model matches these basic cross sectional facts on leasing and the capital structure.

6. Conclusion

We argue that collateral determines the capital structure. We provide a dynamic agency based model of the capital structure of a firm with a standard neoclassical production function subject to collateral constraints due to limited enforcement. In the model firms require both tangible and intangible capital, and the fraction of tangible assets required is a key determinant of leverage and the dynamics of firm financing.

There is a fundamental connection between firms' financing and risk management policy, since both involve promises to pay by firms, and financing needs can override hedging concerns. In fact, poorly capitalized firms optimally do not engage in risk management and firms abstain from risk management with positive probability under the stationary distribution. Our dynamic model which allows explicit analysis of the financing needs for investment and the limits on firms' ability to promise to pay is critical for this result. Moreover, we prove the optimality of incomplete hedging. It is not optimal for the firm to hedge to the point that the marginal value of internal funds is equal across all states. Our dynamic analysis of risk management shows that for plausible levels of the autocorrelation of productivity, firms may not hedge at all and that even dividend-paying firms that are hit by a sequence of adverse shocks eventually become so constrained that they cut risk management. An increase in risk can raise the amount of risk management and reduce investment due to the financing needs for risk management. An increase in collateralizability or tangibility allows firms to lever more and increase investment, and firms raise corporate risk management to counterbalance the increase in the volatility of net worth that higher leverage would otherwise imply. Thus, the comparative statics of our model imply interesting empirical predictions for firm financing and risk management both in the cross section and the time series.

Firms' ability to lease capital is explicitly taken into account in contrast to previous dynamic models of firm financing and investment with financial constraints. The extent to which firms lease is determined by firms' financial condition, and more constrained firms lease more. Indeed, severely constrained firms find it optimal to lease all their tangible capital. Sale-leaseback transactions free up net worth and can be an optimal response to adverse cash flow shocks. Moreover, leasing enables firms to grow faster. Leased capital is an important mode of financing, in particular for constrained firms, and should be taken into account not only in corporate finance, but also in studies of the effect of financing on development and growth. Indeed, changes to financial accounting standards are currently being considered by accounting boards that would result in adjustments to firms' capital structure similar to the ones suggested here.

We also provide stylized empirical facts that highlight the importance of tangibility as

a determinant of the capital structure in the data. Firm leverage changes substantially with the fraction of assets which is tangible. Moreover, the lack of tangible assets largely explains why some firms have low leverage, and hence addresses the "low leverage puzzle." Leased capital is quantitatively important and further reduces the fraction of firms with low leverage.

We conclude that the tangibility of assets and firms' ability to lease capital are critical determinants of the capital structure. The simple form of the optimal contract in our dynamic agency based capital structure model should facilitate the calibration and empirical implementation, which has remained a challenge for other such agency based models. Moreover, due to its simplicity, our model may also prove to be a useful framework to address other theoretical questions in dynamic corporate finance and financial macroeconomics.

Appendix A: Proofs

Proof of Proposition 1. The proposition is proved in Lemma 1-5 below.

Lemma 1. $\Gamma(w, s)$ *is convex, given* (w, s)*, and convex in* w *and monotone in the sense that* $w \le \hat{w}$ *implies* $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$.

Proof of Lemma 1. Suppose $x, \hat{x} \in \Gamma(w, s)$. For $\phi \in (0, 1)$, let $x_\phi \equiv \phi x + (1 - \phi)\hat{x}$. Then $x_\phi \in \Gamma(w, s)$ since Eqs. (2), (4), and (5), as well as the right hand side of Eq. (3) are linear and, since f is concave,

$$A(s')f(k_\phi) + (k_\phi - k_{l,\phi})(1 - \delta) \ge \phi[A(s')f(k) + (k - k_l)(1 - \delta)]$$
$$+ (1 - \phi)[A(s')f(\hat{k}) + (\hat{k} - \hat{k}_l)(1 - \delta)].$$

Let $x \in \Gamma(w, s)$ and $\hat{x} \in \Gamma(\hat{w}, s)$. For $\phi \in (0, 1)$, let $x_\phi \equiv \phi x + (1 - \phi)\hat{x}$. Since Eqs. (3), (4), and (5) do not involve w and $\hat{w}$, respectively, and $\Gamma(w, s)$ is convex given w, x_ϕ satisfies these equations. Moreover, since x and $\hat{x}$ satisfy Eq. (2) at w and $\hat{w}$, respectively, and Eq. (2) is linear in x and w, x_ϕ satisfies the equation at w_ϕ. Thus, $x_\phi \in \Gamma(\phi w + (1 - \phi)\hat{w}, s)$. In this sense, $\Gamma(w, s)$ is convex in w.

If $w \le \hat{w}$, then $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$. $\square$

Lemma 2. *The operator* T *satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point* V.

Proof of Lemma 2. Suppose $\hat{g}(w, s) \ge g(w, s)$, for all $(w, s) \in \mathbf{R}_+ \times S$. Then, for any $x \in \Gamma(w, s)$,

$$(T\hat{g})(w, s) \ge d + \beta \sum_{s' \in S} \Pi(s, s')\hat{g}(w(s'), s') \ge d + \beta \sum_{s' \in S} \Pi(s, s')g(w(s'), s').$$

Hence,

$$(T\hat{g})(w, s) \ge \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s, s')g(w(s'), s') = (Tg)(w, s)$$

for all $(w, s) \in \mathbb{R}_+ \times S$. Thus, T satisfies monotonicity.

Operator T satisfies discounting since

$$T(g + a)(w, s) \le \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s, s')(g + a)(w(s'), s') = (Tg)(w, s) + \beta a.$$

Therefore, operator T is a contraction and, by the contraction mapping theorem, has a unique fixed point V. $\square$

Lemma 3. V *is strictly increasing and concave in* w.

Proof of Lemma 3. Let $x_o \in \Gamma(w, s)$ and $\hat{x}_o \in \Gamma(\hat{w}, s)$ attain $(Tg)(w, s)$ and $(Tg)(\hat{w}, s)$, respectively. Suppose g is increasing in w and suppose $w \le \hat{w}$. Then,

$$(Tg)(\hat{w}, s) = \hat{d}_o + \beta \sum_{s' \in S} \Pi(s, s')g(\hat{w}_o(s'), s') \ge d + \beta \sum_{s' \in S} \Pi(s, s')g(w(s'), s').$$

Hence,

$$(Tg)(\hat{w}, s) \geq \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s, s') g(w(s'), s') = (Tg)(w, s),$$

i.e., Tg is increasing in w. Moreover, suppose $w < \hat{w}$, then

$$(Tg)(\hat{w}, s) \geq (\hat{w} - w) + d_o + \beta \sum_{s' \in S} \Pi(s, s') g(w_o(s'), s') > (Tg)(w, s),$$

implying that Tg is strictly increasing. Hence, T maps increasing functions into strictly increasing functions, which implies that V is strictly increasing.

Suppose g is concave. Then, for $\phi \in (0, 1)$, let $x_{o,\phi} \equiv \phi x_o + (1 - \phi)\hat{x}_o$ and $w_\phi \equiv \phi w + (1 - \phi)\hat{w}$, we have

$$(Tg)(w_\phi, s) \geq d_{o,\phi} + \beta \sum_{s' \in S} \Pi(s, s') g(w_{o,\phi}(s'), s')$$

$$\geq \phi \left[d_o + \beta \sum_{s' \in S} \Pi(s, s') g(w_o(s'), s') \right] + (1 - \phi) \left[\hat{d}_o + \beta \sum_{s' \in S} \Pi(s, s') g(\hat{w}_o(s'), s') \right]$$

$$= \phi(Tg)(w, s) + (1 - \phi)(Tg)(\hat{w}, s).$$

Thus, Tg is concave, and T maps concave functions into concave functions, which implies that V is concave. Since V is increasing and concave in w, it must be continuous in w. $\square$

Lemma 4. *Without leasing, $V(w, s)$ is strictly concave in w for $w \in int\{w : d(w, s) = 0\}$.*

Proof of Lemma 4. Without leasing, k_l is set to zero throughout and all the prior results continue to hold. Suppose $w, \hat{w} \in int\{w : d(w, s) = 0\}$, $\hat{w} > w$. There must exists some state s_*^t, where $s^t = \{s_0, s_1, \ldots, s_t\}$, which has strictly positive probability and in which the capital stock choice at $\hat{w}$ is different from the choice at w, i.e., $\hat{k}(s_*^t) \neq k(s_*^t)$. Suppose instead that $\hat{k}(s^t) = k(s^t)$, $\forall s^t \in S^t, t = 0, 1, \ldots$. Then there must exist some state s_{**}^t with strictly positive probability in which $\hat{d}_o(s_{**}^t) > d_o(s_{**}^t)$ and for which borrowing is not constrained along the path of s_{**}^t. Reducing $\hat{d}_o(s_{**}^t)$ by η and paying out the present value at time 0 instead changes the objective by $(R^{-t} - \beta^t)(\hat{d}_o(s^t) - d_o(s^t)) > 0$, contradicting the optimality of $d(\hat{w}, s) = 0$.

Assume, without loss of generality, that $\hat{k}_o(s'_*) \neq k_o(s'_*)$, for some $s'_* \in S$. Rewrite the Bellman equation as

$$V(w, s) = \max_{\substack{x \in \Gamma(w,s), \\ x(s') \in \Gamma(w(s'), s'), \forall s' \in S}} d + \beta \sum_{s' \in S} \Pi(s, s') \left\{ d(s') + \beta \sum_{s'' \in S} \Pi(s', s'') V(w(s''), s'') \right\}$$

and note the convexity of the constraint set. Using the fact that $\hat{k}_o(s'_*) \neq k_o(s'_*)$, that V is concave and strictly increasing, and that $f(k)$ is strictly concave, we have, for $\phi \in (0, 1)$, and denoting $x_{o,\phi} = \phi x_o + (1 - \phi)\hat{x}_o$ and analogously for other variables,

$$V(w_\phi, s) > d_{o,\phi} + \beta \sum_{s' \in S} \Pi(s, s') \left\{ d_{o,\phi}(s') + \beta \sum_{s'' \in S} \Pi(s', s'') V(w_{o,\phi}(s''), s'') \right\}$$
$$\geq \phi V(w, s) + (1 - \phi) V(\hat{w}, s).$$

The first (strict) inequality is due to the fact that for s'' following s'_* Eq. (3) is slack and hence a net worth $w(s'') > w_{o,\phi}(s'')$ is feasible. The second inequality is due to concavity of V. $\square$

Lemma 5. *Assuming that for all $\hat{s}, s \in S$ such that $\hat{s} > s$, $\Pi(\hat{s}, s')$ strictly first order stochastically dominates $\Pi(s, s')$, V is strictly increasing in s.*

Proof of Lemma 5. Let $S = \{s_1, \ldots, s_n\}$, with $s_{i-1} < s_i$, $\forall i = 2, \ldots, n$ and $N = \{1, \ldots, n\}$. Define the step function on the unit interval $b : [0, 1] \to \mathbb{R}$ as $b(\omega) = \sum_{i=1}^n b(s'_i) \mathbf{1}_{B_i}(\omega)$, $\forall \omega \in [0, 1]$, where $\mathbf{1}$ is the indicator function, $B_1 = [0, \Pi(s, s'_1)]$, and

$$B_i = \left(\sum_{j=1}^{i-1} \Pi(s, s'_j), \sum_{j=1}^i \Pi(s, s'_j) \right], \qquad i = 2, \ldots, n.$$

For $\hat{s}$, define $\hat{B}_i$, $\forall i \in N$, analogously. Let $B_{ij} = B_i \cap \hat{B}_j$, $\forall i, j \in N$, of which at most $2n - 1$ are non-empty. Then, we can define the step function $\hat{b} : [0, 1] \to \mathbb{R}$ as

$$\hat{b}(\omega) = \sum_{j=1}^n \sum_{i=1}^n b(s'_i) \mathbf{1}_{B_{ij}}(\omega), \qquad \forall \omega \in [0, 1].$$

We can then define the stochastic debt policy for $\hat{B}_j$, $\forall j \in N$, with positive Lebesgue measure $(\lambda(\hat{B}_j) > 0)$, as $\hat{b}(s'_i | s'_j) = b(s'_i)$ with conditional probability $\pi(s'_i | s'_j) = \lambda(B_{ij}) / \lambda(\hat{B}_j)$. Moreover, this implies a stochastic net worth

$$\hat{w}(s'_i | s'_j) = A(s'_j) f(k) + (k - k_l)(1 - \delta) - R \hat{b}(s'_i | s'_j)$$
$$\geq A(s'_i) f(k) + (k - k_l)(1 - \delta) - R \hat{b}(s'_i) = w(s'_i), \qquad a.e.,$$

with strict inequality when $i < j$, since under the assumption in the statement of the lemma, $\lambda(B_{ij}) = 0$ whenever $i > j$. Moreover, $\hat{w}(s'_i | s'_j) > w(s'_i)$ with positive probability given that assumption.

Now suppose $\hat{s} > s$ and $g(w, \hat{s}) \geq g(w, s)$, $\forall w \in \mathbb{R}_+$. Let x_o attain the $(Tg)(w, s)$. Then

$$(Tg)(w, \hat{s}) \geq d_o + \beta \sum_{\hat{s}' \in S} \Pi(\hat{s}, \hat{s}') \sum_{s' \in S} \pi(s' | \hat{s}') g(\hat{w}_o(s' | \hat{s}'), \hat{s}')$$
$$> d_o + \beta \sum_{s' \in S} \Pi(s, s') g(w_o(s'), s') = (Tg)(w, s).$$

Thus, T maps increasing functions into strictly increasing functions, implying that V is strictly increasing in s. $\square$

Lemma 6. *Under Assumption 2, capital and net worth in all states are strictly positive, $k > 0$ and $w(s') > 0$, $\forall s' \in S$.*

Proof of Lemma 6. We first show that if $k > 0$, then $w(s') > 0$, $\forall s' \in S$. Note that (3) holds with equality. Using (4) we conclude

$$w(s') = A(s')f(k)+(k-k_l)(1-\delta)-Rb(s') \geq A(s')f(k)+((k-k_l)-\theta(\varphi k-k_l))(1-\delta) > 0.$$

To show that $k > 0$, note that (7) and (10) imply that

$$\mu(1 - R^{-1}\theta\varphi(1-\delta)) \geq \sum_{s'\in S} \Pi(s,s')\beta\mu(s')\left[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)\right]. \tag{19}$$

Suppose that $\mu = 1$. Then $k > 0$ since $\mu(s') = V_w(w(s'),s') \geq 1$ and hence the right hand side goes to $+\infty$ as $k \rightarrow 0$, a contradiction. Suppose instead that $\mu > 1$ and hence $d = 0$. For k sufficiently small, $\exists \hat{s}' \in S$, such that $\mu(\hat{s}') = (R\beta)^{-1}\mu$. But then

$$0 \geq \sum_{s'\in S\backslash \hat{s}} \Pi(s,s')\beta\mu(s')\left[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)\right]$$
$$+ \left\{\Pi(s,\hat{s}')R^{-1}[A(\hat{s}')f_k(k) + (1-\theta\varphi)(1-\delta)] - (1 - R^{-1}\theta\varphi(1-\delta))\right\}\mu.$$

where the first term is positive and the second term goes to $+\infty$ as $k \rightarrow 0$, a contradiction. □

Proof of Proposition 2. Part (i): By the envelope condition, $\mu(w,s) = V_w(w,s)$. By Lemma 3, V is concave in w and hence $\mu(w,s)$ is decreasing in w. The first order condition (6) implies that $\mu(w,s) \geq 1$. If $d(\hat{w},s) > 0$, then $\mu(\hat{w},s) = 1$ and $\mu(w,s) = 1$ for all $w \geq \hat{w}$. Let $\bar{w}(s) = \inf\{w : d(w,s) > 0\}$.

Part (ii): Suppose $w > \hat{w} \geq \bar{w}(s)$ and let $\hat{x}_o$ attain $V(\hat{w},s)$. Since $V_w(w,s) = 1$ for $w \geq \bar{w}(s)$, $V(w,s) = V(\hat{w},s) + \int_{\hat{w}}^w dv$. The choice $x_o = [w - \hat{w} + \hat{d}_o, \hat{k}_o, \hat{k}_{l,o}, \hat{w}_o(s'), \hat{b}_o(s')]$ attains $V(w,s)$ and thus is weakly optimal.

The optimal choice $\hat{x}_o$ is unique in terms of the capital stock $\hat{k}_o$. To see this, suppose instead that $\hat{x}_o$ and $\tilde{x}_o$ both attain $V(\hat{w},s)$, but $\hat{k}_o \neq \tilde{k}_o$. Recalling that $\Gamma(\hat{w},s)$ is convex and noting that

$$A(s')f(k_{o,\phi}) + (k_{o,\phi} - k_{l,o,\phi})(1-\delta) > \phi[A(s')f(\hat{k}_o) + (\hat{k}_o - \hat{k}_{l,o})(1-\delta)]$$
$$+(1-\phi)[A(s')f(\tilde{k}_o) + (\tilde{k}_o - \tilde{k}_{l,o})(1-\delta)],$$

where $x_{o,\phi}$ is defined as usual, we conclude that at $x_{o,\phi}$, (3) is slack, and hence there exists a feasible choice that attains a strictly higher value, a contradiction. Indeed, $x_o(w,s)$ is unique in terms of $k_o(w,s)$, for all w and s.

Now take $w > \hat{w}$ and let x_o attain $V(w,s)$. By part (i) of Proposition 1, $x_{o,\phi} \in \Gamma(w_\phi,s)$. Moreover, if $k_o \neq \hat{k}_o$, then there exists a feasible choice such that $V(w_\phi) > \phi V(w,s) + (1-\phi)V(\hat{w},s)$ contradicting the linearity of V. Thus, $k_o(w,s) = \bar{k}_o(s)$, $\forall w \geq \bar{w}(s)$.

Part (iii): We now show that without leasing the optimal policy is unique also in terms of state-contingent net worth, state-contingent borrowing, and the dividend. Define $\hat{S}^0 = \{s' : \hat{w}_o(s') < \bar{w}(s')\}$ and $\hat{S}^+ = S \setminus \hat{S}^0$. Then $\forall s' \in \hat{S}^0$, $\hat{w}_o(s')$ is unique. To see this suppose instead that there is a $\tilde{x}_o$ with $\tilde{w}_o(s') \neq \hat{w}_o(s')$ for some $s' \in \hat{S}^0$ that also attains $V(\hat{w},s)$. Then a convex combination $x_{o,\phi}$ is feasible and attains a

37

strictly higher value due to strict concavity of $V(w, s')$ for $w < \bar{w}(s')$ (part (iv) of Proposition 1). For the alternative optimal policy $\tilde{x}_o$ define $\tilde{S}^0$ and $\tilde{S}^+$ analogously to $\hat{S}^0$ and $\hat{S}^+$. By above, $\tilde{S}^0 \supseteq \hat{S}^0$. For any $s' \in \hat{S}^+$, $\tilde{w}_o(s') \geq \bar{w}(s')$. For suppose instead that $\tilde{w}_o(s') < \bar{w}(s')$, then by strict concavity of V for $w < \bar{w}(s')$ a convex combination would again constitute a feasible improvement. Thus, $\tilde{S}^+ \supseteq \hat{S}^+$ and as a consequence $S^+ \equiv \tilde{S}^+ = \hat{S}^+$ and $S^0 \equiv \tilde{S}^0 = \hat{S}^0$. For $s' \in S^0$, $b_o(s')$ is uniquely determined by (3). For $s' \in S^+$, Eq. (4) holds with equality and determines $b_o(s')$ uniquely, and $w_o(s')$ is then uniquely determined by (3). Hence, the optimal policy is unique. Moreover, the policy determined by part (ii) (with $k_{l,o}(w, s)$ set to 0) is the unique optimal policy for $w > \bar{w}(s)$. $\square$

Proof of Proposition 3. Using the first order conditions for investment (7) and substituting for $\lambda(s')$ using (10) we have

$$
\begin{aligned}
1 &\geq \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{[A(s') f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)} \\
&\geq \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{A(s') f_k(k)}{\wp(\varphi)}.
\end{aligned} \tag{20}
$$

Using the budget constraint (2) and the collateral constraints (4), we have

$$
w \geq (1 - \varphi)k + (\varphi k - k_l)\wp + R^{-1} u_l k_l,
$$

and thus as $w \to 0$, investment $k \to 0$. But then the marginal product of capital $f_k(k) \to +\infty$, which implies by (20) that $\mu(s')/\mu \to 0$, and using (10), that $\lambda(s')/\mu = (R\beta)^{-1} - \mu(s')/\mu \to (R\beta)^{-1} > 0$, $\forall s' \in S$. Therefore, by continuity, $\forall s \in S$, $\exists \underline{w}_h(s) > 0$, such that $\forall w \leq \underline{w}_h(s)$, $\lambda(s') > 0$, $\forall s' \in S$. $\square$

Proof of Proposition 4. Part (i): If $w(s') \leq w(s'_+)$, then $\mu(s') \geq \mu(s'_+)$ by concavity. Moreover, $\mu(s') + \lambda(s') = \mu(s'_+) + \lambda(s'_+)$, so $\lambda(s') \leq \lambda(s'_+)$. Suppose instead that $w(s') > w(s'_+)$. Then $\lambda(s') = 0$ since otherwise net worth in state s' could not be larger than in state s'_+. But then $\mu(s') = \mu(s'_+) + \lambda(s'_+)$, implying $\mu(s'_+) \leq \mu(s')$. If $\mu(s'_+) = \mu(s')$, then $\lambda(s'_+) = \lambda(s')$ and the assertion is true. If instead $\mu(s'_+) < \mu(s')$, then by concavity $w(s'_+) \geq w(s')$, a contradiction.

Part (ii): Suppose that $\lambda(s') = 0$, $\forall s' \in S$. Then (9), (10), and the envelope condition imply that $V_w(w) = \mu = \beta\mu(s')R = R\beta V_w(w(s')) < V_w(w(s'))$ and, by concavity, $w > w(s')$, $\forall s' \in S$. If $d = 0$, then saving the entire net worth w at R would imply net worth $Rw > w(s')$ in all states next period and hence attain a higher value of the objective, contradicting optimality. Suppose $d > 0$ and hence $w > \bar{w}$ as defined in Proposition 2. That proposition also implies that $V(w)$ can be attained by the same optimal policy as at $\bar{w}$ except that $d = w - \bar{w}$. Since $V_w(w(s')) > 1$, we conclude that $w(s') < \bar{w}$. But then paying out $d = w - \bar{w}$ as before and saving $\bar{w}$ at R raises net worth in all states next period and hence improves the value of the objective, a contradiction.

Hence, $\exists s' \in S$ such that $\lambda(s') > 0$, and, since $\lambda(s')$ is increasing in s' by part (i), $\lambda(\bar{s}') > 0$ where $\bar{s}' = \max\{s' : s' \in S\}$. If $\lambda(s') > 0$, $\forall s'$, then $w(s') = A(s')f(k) + k(1 - \theta\varphi)(1 - \delta) - k_l(1 - \theta)(1 - \delta)$ and hence $w(s') \neq w(\hat{s}')$, $s \neq \hat{s}'$. If $\lambda(\hat{s}') = 0$ for some $\hat{s}'$, then $\mu(\hat{s}') = \mu(\bar{s}') + \lambda(\bar{s}') > \mu(\bar{s}')$ and $w(\hat{s}') < w(\bar{s}')$.

Suppose $\lambda(s') = 0$ for some $s' \in S$. For any $s'_- < s'$, $\mu(s'_-) \geq \mu(s')$ by part (i), and $\mu(s'_-) \leq \mu(s'_-) + \lambda(s'_-) = \mu(s')$, implying $\mu(s'_-) = \mu(s')$. Thus, the firm hedges all states below $s'_h = \max\{s' : \lambda(s') = 0\}$. Note that the set may be empty, i.e., the firm may not hedge at all. $\square$

Lemma 7 (Net worth transition dynamics) *Suppose $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$, and $m = +\infty$ (no leasing).* **(i)** *$\forall s', s'_+ \in S$, such that $s'_+ > s'$, $w(s'_+) \geq w(s')$, with strict inequality iff $s'_+ > s'_h$ where s'_h is defined in Proposition 4.* **(ii)** *$w(s')$ is increasing in w, $\forall s' \in S$; for w sufficiently small, $w(s') > w$, $\forall s' \in S$; and for w sufficiently large, $w(s') < w$, $\forall s' \in S$.* **(iii)** *$\forall s' \in S$, $\exists w$ dependent on s' such that $w(s') = w$.*

Proof of Lemma 7. Part (i): By part (iv) of Proposition 1 $V(w)$ is strictly concave unless $w > \bar{w}$. By Proposition 4, the firm hedges a lower set of states $[\underline{s}', \ldots, s'_h]$ if at all. If $s'_+ \leq s'_h$, then $\mu(s') = \mu(s'_+) > 1$ and hence $w(s') = w(s'_+) < \bar{w}$. If $s'_+ > s'_h$, then either $\lambda(s') = 0$ and $\mu(s') > \mu(s'_+)$, implying $w(s'_+) > w(s')$, or $\lambda(s') > 0$, which together with (3) and (4) at equality implies $w(s'_+) > w(s')$.

Part (ii): If $d > 0$, then $w(s')$ is constant by Proposition 2 and hence (weakly) increasing. If $d = 0$, then $w(s')$ is strictly increasing in w for $\{s'|\lambda(s') = 0\}$ using strict concavity of V and the fact that $V_w(w) = R\beta V_w(w(s'))$. For $\{s'|\lambda(s') > 0\}$, (3) and (4) hold with equality and hence $w(s')$ is increasing in w if k is. We now show that k is strictly increasing in w for $w \leq \bar{w}$. If $\lambda(s') > 0$, $\forall s \in S$, then $k = w/\wp(\varphi)$ and k is hence strictly increasing. Suppose $\lambda(s') = 0$, some $s \in S$. Then using the first order conditions for investment (7) and substituting for $\lambda(s')$ using (10) we have

$$
\begin{aligned}
1 &= \sum_{s' \in S} \Pi(s')\beta\frac{\mu(s')}{\mu}\frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)} \\
&= \sum_{\{s'|\lambda(s')>0\}} \Pi(s')\beta\frac{\mu(s')}{\mu}\frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)} \\
&\quad + \sum_{\{s'|\lambda(s')=0\}} \Pi(s')\beta\frac{\mu(s')}{\mu}\frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)}.
\end{aligned}
\tag{21}
$$

Take $w^+ > w$ and suppose that $k^+ \leq k$ with the usual abuse of notation. Then $f_k(k^+) \geq f_k(k)$. Moreover, for $\{s'|\lambda(s') = 0\}$, $\mu(s')/\mu = (R\beta)^{-1}$. Since $\mu^+ < \mu$, (21) implies that $\exists s'$ such that $\mu^+(s') < \mu(s')$. But $k^+ \leq k$ implies that for $\{s'|\lambda(s') > 0\}$, $w^+(s') \leq w(s')$ and hence $\mu^+(s') \geq \mu(s')$, a contradiction. Hence, k and $w(s')$ are strictly increasing in w for $w \leq \bar{w}$.

To show that $\exists w$ such that $w(s') > w$, $\forall s' \in S$, note that Proposition 3 implies that for w sufficiently small, $\lambda(s') > 0$, $\forall s' \in S$, and thus $w = k/\wp(\varphi)$ and $w(s') = A(s')f(k) + k(1 - \theta\varphi)(1 - \delta)$. But then

$$
\frac{dw(s')}{dw} = \frac{A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)}{\wp(\varphi)} = \frac{A(s')f_k(k)k + k(1 - \theta\varphi)(1 - \delta)}{\wp(\varphi)k} < \frac{w(s')}{w},
$$

where the inequality uses the strict concavity of $f(\cdot)$. Moreover, as $w \to 0$, $f_k(k) \to +\infty$ and thus $dw(s')/dw \to +\infty$ and $w(s')/w > 1$ for w sufficiently low.

To show that $\exists w$ such that $w(s') < w, \forall s' \in S$, it is sufficient to show that such a w exists for $w(\bar{s}')$ given part (i). By Proposition 2, $\forall w \geq \bar{w}$, the optimal policy x_o is independent of w (except for the current dividend), and thus $w_o(\bar{s}') = A(\bar{s}')f(\bar{k}_o) + \bar{k}_o(1-\delta) - R\bar{b}_o(\bar{s}') < +\infty$, and hence for $w > w_o(\bar{s}')$ the assertion holds.

Part (iii): By the theorem of the maximum $w(s')$ is continuous in w, and the intermediate value theorem and part (ii) hence imply the result. $\square$

Proof of Proposition 5. Part (i): Denoting the wealth level as defined in part (iii) of Lemma 7 for the lowest state $\underline{s}'$ by $\underline{w}$ and using $w(\underline{s}') = \underline{w}$, (9), (10), and the envelope condition, we have $V_w(\underline{w}) = R\beta V_w(\underline{w}) + R\beta\lambda(\underline{s}')$ and thus $\lambda(\underline{s}') > 0$, and by Proposition 4 the firm abstains from risk management altogether at $\underline{w}$. The level of net worth $\underline{w}$ is unique, since either $d > 0$ at $\underline{w}$, and hence $w(\underline{s}')$ is constant, or $d = 0$ and then using $\lambda(s') > 0, \forall s \in S$, and evaluating $dw(\underline{s}')/dw$ as in part (ii) at $\underline{w}$

$$\left.\frac{dw(\underline{s}')}{dw}\right|_{w=\underline{w}} = \left.\frac{A(\underline{s}')f_k(k)k + k(1-\theta\varphi)(1-\delta)}{\wp(\varphi)k}\right|_{w=\underline{w}} < \frac{w(\underline{s}')}{\underline{w}} = 1.$$

Thus the locus of $w(\underline{s}')$ crosses the 45 degree line from above, i.e., at most once. Moreover, $w(\underline{s}')/w < 1, \forall w > \underline{w}$.

Part (ii): We adapt Theorem 12.12 from Stokey, Lucas, and Prescott (1989). Let $\varepsilon_w > 0$ and $w_{\text{bnd}} = A(\bar{s}')f(k_{\text{bnd}}) + k_{\text{bnd}}(1-\delta)$ where k_{bnd} such that $A(\bar{s}')f_k(k_{\text{bnd}}) = r+\delta$. Define the induced state space $W = [\varepsilon_w, w_{\text{bnd}}] \subset \mathbb{R}$ with its Borel subsets $\mathcal{W}$. Take P to be the induced transition function on $(W, \mathcal{W})$, with the associated operator on bounded continuous functions $T : B(W, \mathcal{W}) \to B(W, \mathcal{W})$ and the associated operator on probability measures $T^* : \mathcal{P}(W, \mathcal{W}) \to \mathcal{P}(W, \mathcal{W})$.

We need to show that P is monotone (i.e., for any bounded, increasing function g, the function Tg defined by $(Tg)(w) = \int g(w')P(w, dw'), \forall w$, is also increasing), has the Feller property (i.e., for any bounded, continuous function g, the function Tg is also continuous), and $\exists w^o \in W, \varepsilon > 0$, and $N \geq 1$, such that $P^N(\varepsilon_w, [w^o, w_{\text{bnd}}]) \geq \varepsilon$ and $P^N(w_{\text{bnd}}, [\varepsilon_w, w^o]) \geq \varepsilon$.

Take any bounded, increasing function g. Then $(Tg)(w) = \sum_{s' \in s} \Pi(s')g(w(s')(w))$ is increasing since $w(s')(w)$ is increasing by part (ii) of Lemma 7. For any bounded, continuous function g, $(Tg)(w)$ is moreover continuous as $w(s')(w)$ is continuous by the theorem of the maximum.

From Lemma 7 and part (i) we know that levels of net worth below $\underline{w}$ and above $w_o(\bar{s}')$ are transient. We now provide an explicit characterization of the stationary solution when $\bar{w} \leq \underline{w}$ and then show that otherwise w^o can be set to $w^o = \bar{w}$, where $\bar{w}$ is the level of net worth above which the firm pays dividends (see Proposition 2).

If $\bar{w} \leq \underline{w}$, then the stationary distribution is a subset of the dividend paying set and the solution is quasi-deterministic, in the sense that capital $\bar{k}_o$ is constant under the stationary distribution. In this case, $\bar{k}_o$ solves

$$1 = \beta\frac{\sum_{s' \in S} \Pi(s')A(s')f_k(\bar{k}_o) + (1-\theta\varphi)(1-\delta)}{\wp(\varphi)}.$$

Then $\bar{w} = \wp(\varphi)\bar{k}_o$ and

$$\underline{w} = w(\underline{s}') = A(\underline{s}')f(\bar{k}_o) + \bar{k}_o(1 - \theta\varphi)(1 - \delta)$$

$$\geq \bar{w} = \wp(\varphi)\bar{k}_o = \beta \left(\sum_{s' \in S} \Pi(s')A(s')f_k(\bar{k}_o)\bar{k}_o + \bar{k}_o(1 - \theta\varphi)(1 - \delta) \right).$$

The condition for $\bar{w} \leq \underline{w}$ is thus

$$(1 - \beta)(1 - \theta\varphi)(1 - \delta) \geq \beta \sum_{s' \in S} \Pi(s')A(s')f_k(\bar{k}_o) - A(\underline{s}')\frac{f(\bar{k}_o)}{\bar{k}_o}. \tag{22}$$

Concavity implies that $f(\bar{k}_o)/\bar{k}_o > f_k(\bar{k}_o)$ and thus a sufficient condition is that $A(\underline{s}') \geq \beta \sum_{s' \in S} \Pi(s')A(s')$. If $f(k) = k^\alpha$, a sufficient condition is $A(\underline{s}') \geq \alpha\beta \sum_{s' \in S} \Pi(s')A(s')$. Note that in the quasi-deterministic case the firm abstains from risk management with probability 1 under the stationary distribution.

If (22) is violated, then $\underline{w} < \bar{w}$. Moreover, the firm cannot be paying dividends in the lowest state next period if it currently is paying dividends, for if it paid dividends, it would choose $\bar{k}_o$ as determined above and

$$w(\underline{s}') = A(\underline{s}')f(\bar{k}_o) + \bar{k}_o(1 - \theta\varphi)(1 - \delta)$$

$$< \beta \left(\sum_{s' \in S} \Pi(s')A(s')f_k(\bar{k}_o)\bar{k}_o + \bar{k}_o(1 - \theta\varphi)(1 - \delta) \right) = \wp(\varphi)\bar{k}_o = \bar{w},$$

a contradiction. Thus, if $w^o = \bar{w}$, then $P(w_{\mathrm{bnd}}, [\varepsilon_w, \bar{w}]) \geq \Pi(\underline{s}')$ and $N_1 = 1$ and $\exists \varepsilon_1 > 0$ such that $\Pi(\underline{s}') > \varepsilon_1 > 0$.

Now given $w = \varepsilon_w$, and since the net worth could be paid out, the objective has to exceed the value of net worth,

$$0 < \varepsilon_w = w \leq E\left[\sum_{t=0}^{\infty} \beta^t d_t \right] = E\left[\sum_{t=0}^{N} \beta^t d_t \right] + E\left[\sum_{t=N+1}^{\infty} \beta^t d_t \right].$$

Note that $d_t \leq d_o(\bar{s}') = w_o(\bar{s}') - \bar{w}$ and the last expectation above is thus bounded by $\beta^{N+1}d_o(\bar{s}')/(1 - \beta)$. For any $\varepsilon_d > 0$ such that $\varepsilon_w > \varepsilon_d$, $\exists N_2 < \infty$ such that the last expectation is less than ε_d. But then $P^{N_2}(\varepsilon_w, [\bar{w}, w_{\mathrm{bnd}}]) \geq (\varepsilon_w - \varepsilon_d)/d_o(\bar{s}') > 0$. Let $\varepsilon_2 \equiv (\varepsilon_w - \varepsilon_d)/d_o(\bar{s}')$, $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, and $N = \max\{N_1, N_2\}$. Finally, when $\underline{w} < \bar{w}$, $dw(\underline{s}')/dw < 1$ at $\underline{w}$ and $w(\underline{s}') < w$ for all $w \geq \underline{w}$, and thus a sufficiently long sequence of the lowest productivity realization results in a net worth in a neighborhood of $\underline{w}$ and hence the firm abstains from risk management with positive probability. $\square$

Proof of Proposition 6. We first show that, for each s, $k \leq k_{fb}(s)$ where $k_{fb}(s)$ solves $r + \delta = \sum_{s' \in S} \Pi(s, s')A(s')f_k(k_{fb}(s))$, i.e., $k_{fb}(s)$ is the capital level in the frictionless case when cash flows are discounted at r. Using the first order conditions (7) and (10), the investment Euler equation implies

$$1 = \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)} \tag{23}$$

$$\leq R^{-1} \frac{\sum_{s' \in S} \Pi(s, s') A(s') f_k(k) + (1 - \theta\varphi)(1 - \delta)}{\wp(\varphi)}.$$

Since the term on the right of the inequality equals 1 at $k_{fb}(s)$ and decreases in k the result is immediate.

Second, we show that, given s, k is weakly increasing in w. We focus on the non-dividend paying region (as k is constant otherwise). Let $w^+ > w$. By the concavity of the value function (Proposition 1, Part (iii)), $\mu^+ \leq \mu$. Suppose $k^+ < k$. If $\lambda(s') = 0$, $\forall s' \in S$, at w and w^+, then $k = k_{fb}(s)$ at both levels of wealth, a contradiction. Thus, suppose $\exists s' \in S$, such that $\lambda(s') > 0$ at w and assume that $\{s' : \lambda(s') > 0\}$ is the same at w and w^+. Using (23) we have

$$1 = \sum_{\{s':\lambda(s')>0\}} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)}$$

$$+ \sum_{\{s':\lambda(s')=0\}} \Pi(s, s') R^{-1} \frac{[A(s')f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{\wp(\varphi)}.$$

The second summation must be higher at w^+ and hence $\exists s' \in \{s' : \lambda(s') > 0\}$, such that $\mu^+(s') < \mu(s')$. But for such an s', $w^+(s') < w(s')$ and hence $\mu^+(s') \geq \mu(s')$, a contradiction. Therefore, $k^+ \geq k$, i.e., investment is weakly increasing in w.

Finally, we show that $\exists s$, such that $\forall w$, there is an s' for which $\lambda(s') > 0$, i.e., incomplete hedging in state s is optimal for all wealth levels. To see this note that there must be a current state (w, s) and state next period s' such that $d = 0$ at (w, s) and $d(s') > 0$, implying that $\lambda(s') > 0$ at (w, s). (Otherwise, the firm would either never pay a dividend, which is not possible, or would always pay a dividend in which case $\lambda(s') > 0$, $\forall s'$, in any state (w, s).) Since the firm pays a dividend in state s' next period, $w(s') > \bar{w}(s')$. Take $w^+ > w$. Since k is weakly increasing in w, $w^+(s') \geq w(s') > \bar{w}(s')$, i.e., the firm still pays dividends in s' and is hence constrained against s' at (w^+, s), too. Picking $w^+ \geq \bar{w}(s)$ implies that $\lambda(s') > 0$ and thus $\bar{k}(s) < k_{fb}(s)$ arguing as in the first step of the proof. But then, given s, $k \leq \bar{k}(s) < k_{fb}(s)$, which implies that $\exists s'$ such that $\lambda(s') > 0$, $\forall w$ (using the argument in the first step of the proof again). Note however that the state s' against which the firm is constrained in state s may not be the same for wealth levels below the w we started with. Since state s has positive probability, hedging is incomplete with positive probability. (We can moreover conclude that k is strictly increasing in w for $w \leq \bar{w}(s)$ in state s. To see this use the fact that the value function is strictly concave (Proposition 1, Part (iv)) and the fact that $\{s' : \lambda(s') > 0\} \neq \emptyset$ and argue as in the second step of the proof.) $\square$

Proof of Proposition 7. Part (i): Let S be the support of s' for $A(s')$. Without loss of generality, consider $A_+(s')$ such that $A_+(s') = A(s')$, $\forall s' \in S \setminus \hat{s}'$. For $\hat{s}$, let $A_+(\hat{s}') = A(\hat{s}') + \tilde{a}$ where $E[\tilde{a}|\hat{s}] = 0$. Thus $A_+(s')$ is obtained from $A(s')$ by a mean preserving spread. Note that given net worth w, the set of feasible choices coincide for the problem with $A(s')$ and $A_+(s')$, and let x (x_+) attain $V(w)$ $(V_+(w))$ respectively.

By concavity of the value function,

$$
\begin{aligned}
V_+(w) = d_+ + \beta E[V_+(w'_+)] &\le d_+ + \beta E[V_+(E[w'_+|s'])] \\
&\le d_+ + \beta\{E[d'_+(E[w'_+|s'])] + \beta E[V_+(E[w''_+(E[w'_+|s'])])]\}
\end{aligned}
\tag{24}
$$

and so forth, where $d'_+(E[w'_+|s'])$ denotes the optimal dividend given net worth $E[w'_+|s']$ and analogously for other variables. Now consider $\hat{x}$ such that $\hat{d} = d_+$, $\hat{k} = k_+$, and $\hat{b}(s') = E[b'_+|s']$, implying that $\forall s'$, $\hat{w}(s') = A(s')f(\hat{k}) + \hat{k}(1-\delta) - R\hat{b}(s') = E[w'_+|s']$. Therefore,

$$
V(w) \ge \hat{d} + \beta E[V(\hat{w}')] \ge \hat{d} + \beta\{E[\hat{d}'(\hat{w}')] + \beta E[V(\hat{w}''(\hat{w}'))]\},
\tag{25}
$$

where $\hat{d}'(\hat{w}')$ is chosen analogously to match the optimal policy for $A_+(s')$ given net worth $\hat{w}(s')$, and so forth. The right hand side of (24) and (25) coincide in the limit. Therefore, $V(w) \ge V_+(w)$.

Part (ii): For a dividend paying firm, the investment Euler Eq. (23) in the deterministic case determines $\bar{k}_0$ as $1 = \beta(A'_0 f_k(k) + (1-\theta)(1-\delta))/\wp$, and in the stochastic case implies that $\bar{k}_\sigma$ solves

$$
1 = \sum_{s' \in S} \pi(s')\beta\mu_\sigma(s')\frac{A_\sigma(s')f_k(k) + (1-\theta)(1-\delta)}{\wp}.
\tag{26}
$$

Suppose $\mu_\sigma(s') = 1$, $\forall s' \in S$. Then $\bar{k}_0$ solves (26) and hence $\bar{k}_0 = \bar{k}_\sigma$. Suppose instead $\exists s' \in S$ such that $\mu_\sigma(s') > 1$, then $\bar{k}_\sigma > \bar{k}_0$.

In the deterministic case, investment is $k = w/\wp$ for $w \le \bar{w}_0$ and $\bar{k}_0 = \bar{w}_0$ otherwise. Since $\bar{k}_0 \le \bar{k}_\sigma$, we have $\bar{w}_0/\wp = \bar{k}_0 \le \bar{k}_\sigma \le \bar{w}_\sigma/\wp$, implying that $\bar{w}_0 \le \bar{w}_\sigma$ (with strict inequality iff $\bar{k}_\sigma > \bar{k}_0$). Using the fact that k is continuous and strictly increasing in w (see the proof of Proposition 6) we can provide a more explicit comparison: for $w \le \underline{w}_{h\sigma}$, $k_\sigma = k_0$, and for $w \in [\underline{w}_{h\sigma}, \bar{w}]$, $k_\sigma \le w/\wp = k_0$; indeed, if $\bar{w}_\sigma > \bar{w}_0$, $\exists \hat{w} \in [\bar{w}_0, \bar{w}_\sigma]$ such that $k_\sigma < k_0$, $\forall w < \hat{w}$, and $k_\sigma \ge k_0$, $\forall w \le \hat{w}$.

Suppose that $S = \{\underline{s}, \bar{s}\}$ and that a dividend paying firm is hedging, then $1 = \beta R\mu(\underline{s}')$, so the investment Euler Eq. (26) can be written as

$$
1 = \pi(\underline{s}')R^{-1}\frac{A(\underline{s}')f_k(k) + (1-\theta)(1-\delta)}{\wp} + \pi(\bar{s}')\beta\frac{A(\bar{s}')f_k(k) + (1-\theta)(1-\delta)}{\wp}.
\tag{27}
$$

Totally differentiating given a mean preserving spread in the productivity process $dA(\underline{s}') < 0 < dA(\bar{s}')$ with $\sum_{s' \in S}\pi(s')A(s') = 0$, we have by continuity

$$
0 = [\pi(\underline{s}')dA(\underline{s}') + \pi(\bar{s}')\beta RdA(\bar{s}')]f_k(k) + [\pi(\underline{s}')A(\underline{s}') + \pi(\bar{s}')\beta RA(\bar{s}')]f_{kk}(k)dk.
$$

The first term in brackets is negative and since $f_{kk}(k) < 0$ we must have $dk < 0$. $\square$

Proof of Proposition 8. Part (i): Let $x \in \Gamma_\theta(w, s)$ be the set of $x = \{d, k, w(s'), b(s')\} \in \mathbb{R}_+^{2+S} \times \mathbb{R}^S$ such that for all $s' \in S$

$$w + \sum_{s' \in S} \Pi(s, s')b(s') \geq d + k \tag{28}$$

$$A(s')f(k) + k(1 - \delta) \geq w(s') + Rb(s') \tag{29}$$

and

$$\theta k(1 - \delta) \geq Rb(s'), \tag{30}$$

given θ. Let x (x_+) be optimal at θ (θ_+). Note that $\Gamma_\theta(w, s)$ is increasing in the sense that $\Gamma_\theta(w, s) \subseteq \Gamma_{\theta_+}(w, s)$ and hence $x \in \Gamma_{\theta_+}(w, s)$ implying that $V_+(w, s) \geq V(w, s)$. But at x (30) given θ_+ is slack, so x cannot be optimal if s is the state from Proposition 6 for which $\lambda(s') > 0$ for some $s' \in S$. Otherwise, let the firm with θ_+ choose the optimal policy of the firm with θ until state $\hat{s}$ from Proposition 6 is reached, for which $\{s' : \lambda(s') > 0\} \neq \emptyset$. In state $\hat{s}$, (30) is slack and hence choosing x cannot be optimal. By uniqueness of the optimal policy, we must have $V_+(w, s) > V(w, s)$.

Part (ii): If $S = \{\underline{s}, \bar{s}\}$ and $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$, the investment Euler equation of a dividend paying firm which hedges (27) totally differentiated w.r.t. θ implies that

$$0 = \sum_{s' \in S} \pi(s')\beta\mu(s')\frac{A(s')f_{kk}(k)}{\wp}\frac{dk}{d\theta}$$
$$+ \sum_{s' \in S} \pi(s')\beta\mu(s')\frac{R^{-1}(1 - \delta)}{\wp}\left(\frac{A(s')f_k(k) + (1 - \theta)(1 - \delta)}{\wp} - R\right).$$

Using the investment Euler Eq. (26) and the fact that $1 > \sum_{s' \in S} \pi(s')\beta\mu(s')R$, since $\exists s' \in S$, such that $\lambda(s') > 0$, the second term must be strictly positive and therefore $dk/d\theta > 0$. $\square$

Proof of Proposition 9. Part (i): Proceeding as in the proof of Proposition 3, we conclude that as $w \to 0$, investment $k \to 0$ and $\lambda(s')/\mu \to (R\beta)^{-1} > 0$, $\forall s' \in S$. Therefore, given Assumption 3, the user cost of owned tangible capital exceeds the user cost of leased capital in the limit as $u_p(w, s) \equiv r + \delta + \sum_{s' \in S} \Pi(s, s')R\beta\lambda(s')/\mu(1 - \theta)(1 - \delta)$ goes to $r + \delta + (1 - \theta)(1 - \delta) > r + \delta + m = u_l$. Consequently, using (17) we obtain $R/\mu(\bar{\nu}_l - \underline{\nu}_l) = u_p(w, s) - u_l > 0$, so $\bar{\nu}_l > 0$ (and $\underline{\nu}_l = 0$); all tangible capital is leased in the limit. By continuity, $\forall s \in S$, $\exists \underline{w}_l(s) > 0$, such that $\forall w \leq \underline{w}_l(s)$, $u_p(w, s) > u_l$ and all tangible capital is leased.

Part (ii): Suppose the firm pays dividends and hence $\mu = 1$. Taking the first term on the right hand side of the Euler equation for leasing capital (18) and using the fact that $\mu(s') \geq 1$, the definition of $\wp$ and u_l, and the second inequality from Assumption 3, we have

$$\sum_{s' \in S} \Pi(s, s')\beta\frac{\mu(s')}{\mu}\frac{(1 - \theta)(1 - \delta)}{\wp - R^{-1}u_l} \geq \beta\frac{(1 - \theta)(1 - \delta)}{R^{-1}((1 - \theta)(1 - \delta) - m)} > \beta R\frac{1}{1 - (1 - \beta R)} = 1,$$

which implies that $\underline{\nu}_l > 0$. Therefore, firms with net worth $w \geq \bar{w}(s)$ do not lease. Moreover, since $\underline{\nu}_l > 0$ at $\bar{w}(s)$, by continuity $\exists \varepsilon > 0$ such that for $w \in (\bar{w}(s) - \varepsilon, \bar{w}(s)]$ the multiplier $\underline{\nu}_l > 0$, and thus $\exists \bar{w}_l(s) < \bar{w}(s)$ such that $k_l = 0$ for $w \geq \bar{w}_l(s)$, $\forall s \in S$. $\square$

Proof of Proposition 10. Part (i): Denote with a prime variables which in the stochastic case were a function of the state tomorrow, i.e., w', b', μ', and λ'. We first characterize a steady state. From (9) and the envelope condition we have $\mu' = \mu$. Then (10) implies $\lambda' = ((R\beta)^{-1} - 1)\mu > 0$, i.e., the firm is constrained in the steady state, and (7) can be written as $1 - [R^{-1}\theta\varphi + \beta(1 - \theta\varphi)](1 - \delta) = \beta A' f_k(k)$, which implicitly defines the steady state value of capital $\bar{k}$. Denoting steady state variables with a bar, using (4) and (3) at equality, we have $\bar{b} = R^{-1}\theta\varphi\bar{k}(1 - \delta)$ and the cum-dividend net worth in the steady state $\bar{w}_{cum} = A'f(\bar{k}) + \bar{k}(1 - \theta\varphi)(1 - \delta)$. Dividends in the steady state are

$$
\begin{aligned}
\bar{d} &= A'f(\bar{k}) - \bar{k}(1 - [R^{-1}\theta\varphi + (1 - \theta\varphi)](1 - \delta)) \\
&> A'f(\bar{k}) - \beta^{-1}\bar{k}(1 - [R^{-1}\theta\varphi + \beta(1 - \theta\varphi)](1 - \delta)) \\
&= \int_0^{\bar{k}} \left\{ A'f_k(k) - \beta^{-1}(1 - [R^{-1}\theta\varphi + \beta(1 - \theta\varphi)](1 - \delta)) \right\} dk > 0
\end{aligned}
$$

and hence $\bar{\mu} = 1$. The lowest level of net worth for which $\bar{k}$ is feasible is $\bar{w} \equiv \bar{w}_{cum} - \bar{d}$, and $\bar{w}$ is the ex-dividend net worth in the steady state. Thus, for $w < \bar{w}$, $k < \bar{k}$. Using the first order conditions and the envelope condition we have

$$
\frac{V_w(w)}{V_w(w')} = \frac{\mu}{\mu'} = \beta \frac{A'f_k(k) + (1 - \theta\varphi)(1 - \delta)}{\wp(\varphi)}.
$$

Note that the right hand side equals 1 at $\bar{k}$ and is decreasing in k. Thus, if $k < (>) \bar{k}$, $V_w(w) > (<) V_w(w')$ and $w < (>) w'$. Since $k < \bar{k}$ for $w < \bar{w}$, $w < w'$ and w increases over time. If $w > \bar{w}$, then either $d > 0$ (and $V_w(w) = 1$) or $d = 0$ and $k > \bar{k}$. In the first case, concavity and the fact that $V_w(w') \geq 1$ imply $V_w(w') = 1$ and hence $k = \bar{k}$. In the second case, $w > w'$, but simply saving w at R would result in higher net worth and hence $k > \bar{k}$ cannot be optimal.

Part (ii): Consider the optimal policy without leasing from part (i). The user cost of tangible capital at $\bar{w}$ is $\bar{u}_p = r + \delta + (1 - R\beta)(1 - \theta)(1 - \delta) < u_l$ under Assumption 3. Thus, there is no leasing at $\bar{w}$ and the solution is as before as long as w is sufficiently high. Recall that as w decreases μ'/μ decreases and hence λ'/μ increases. Note also that under Assumption 2, as w goes to zero, k and μ'/μ go to zero and hence λ'/μ goes to $(R\beta)^{-1}$ and $u_p(w)$ goes to $r + \delta + (1 - \theta)(1 - \delta) > u_l$ given Assumption 3. When $\lambda'/\mu = (R\beta)^{-1}m/((1 - \theta)(1 - \delta))$, $u_l = u_p(w)$ and (7) simplifies to

$$
\wp(\varphi) = R^{-1}\left(1 - \frac{m}{(1 - \theta)(1 - \delta)}\right)[A'f_k(k) + (1 - \theta\varphi)(1 - \delta)],
$$

which defines $\underline{k}$. At $\bar{w}_l \equiv \wp(\varphi)\underline{k}$ all the tangible capital is owned and at $\underline{w}_l \equiv (1 - \varphi + R^{-1}u_l\varphi)\underline{k}$ all the tangible capital is leased. For $w \in [\underline{w}_l, \bar{w}_l]$, leased capital is

$$
k_l = \frac{\wp(\varphi)\underline{k} - w}{\wp - R^{-1}u_l}
$$

which is linear and decreasing in w. Moreover, w' is linearly decreasing in k_l and hence linearly increasing in w. For $w < \underline{w}_l$, $k = w/(1 - \varphi + R^{-1}u_l\varphi)$ and $w' = A'f(k) + k(1 - \varphi)(1 - \delta)$. $\square$

45

Appendix B: Enforcement versus collateral constraints

In this appendix we prove the equivalence of enforcement constraints and collateral constraints. For simplicity, we abstract from leasing, but it is straightforward to extend the proof by recognizing that the firm cannot abscond with leased capital. The firm's problem with limited enforcement at any time $\tau \geq 0$, denoted $P_\tau(w(s^\tau))$, is to choose the sequence of dividends, capital levels, and net payments to the lender $\{x(s^t)\}_{t \geq \tau}$, where $x(s^t) = \{d(s^t), k(s^t), p(s^t)\}$ and $s^t \equiv \{s_0, s_1, \ldots, s_t\}$, given current net worth $w(s^\tau)$ and history s^τ, to maximize the discounted expected value of future dividends

$$E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{(t-\tau)} d_t \right] \tag{31}$$

subject to the budget constraints

$$w(s^\tau) \geq d(s^\tau) + k(s^\tau) + p(s^\tau), \tag{32}$$
$$A(s^t)f(k(s^{t-1})) + k(s^{t-1})(1-\delta) \geq d(s^t) + k(s^t) + p(s^t), \qquad \forall t > \tau, \tag{33}$$

the lender's participation constraint,

$$E_\tau \left[\sum_{t=\tau}^{\infty} R^{-(t-\tau)} p_t \right] \geq 0 \tag{34}$$

and the enforcement constraints

$$E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{(t-\tau')} d_t \right] \geq E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{(t-\tau')} \hat{d}_t \right], \quad \forall \tau' \geq \tau, \text{ and } \forall \{\hat{d}(s^t)\}_{t=\tau'}^{\infty}, \tag{35}$$

where $\{\hat{d}(s^t)\}_{t=\tau'}^{\infty}$ together with $\{\hat{k}(s^t)\}_{t=\tau'}^{\infty}$ and $\{\hat{p}(s^t)\}_{t=\tau'}^{\infty}$ solve $P_{\tau'}(w(s^{\tau'}))$ given net worth $w(s^{\tau'}) = A(s^{\tau'})f(k(s^{\tau'-1})) + (1-\theta\varphi)k(s^{\tau'-1})(1-\delta)$, i.e., the same problem with a different level of net worth. We say a sequence of net payments is implementable if it satisfies the lender's participation constraint and the enforcement constraints.

Proposition 11 (Equivalence of enforcement and collateral constraints) (i) *Any sequence of net payments $\{p(s^t)\}_{t=\tau}^{\infty}$ is implementable in problem $P_\tau(w(s^\tau))$ iff*

$$\theta\varphi k(s^{\tau'-1})(1-\delta) \geq E_{\tau'} \left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')} p_t \right], \qquad \forall \tau' \geq \tau, \tag{36}$$

i.e., the present value of the remaining net payments never exceeds the current collateral value. **(ii)** *Moreover, the set of sequences of net payments that satisfy (36) is equivalent to the set of sequences of one period state contingent claims $\{b(s^t)\}_{t=\tau}^{\infty}$ which satisfy*

$$\theta\varphi k(s^{t-1})(1-\delta) \geq Rb(s^t), \qquad \forall t > \tau. \tag{37}$$

Proof of Proposition 11. Part (i): Suppose the sequence $\{p(s^t)\}_{t=\tau}^\infty$ is such that (36) is violated for some $s^{\tau'}$, $\tau' > \tau$, i.e.,

$$\theta \varphi k(s^{\tau'-1})(1-\delta) < E_{\tau'}\left[\sum_{t=\tau'}^\infty R^{-(t-\tau')}p_t\right].$$

W.l.o.g., assume $\tau' = \tau + 1$. Suppose the firm defaults in state $s^{\tau+1}$ at time $\tau + 1$ and issues a new sequence of net payments $\{\hat{p}(s^t)\}_{t=\tau+1}^\infty$ such that $E_{\tau+1}\left[\sum_{t=\tau+1}^\infty R^{-(t-(\tau+1))}\hat{p}_t\right] = 0$ with $\hat{p}(s^t) = p(s^t)$, $\forall t > \tau+1$, and $\hat{p}(s^{\tau+1}) = -E_{\tau+1}\left[\sum_{t=\tau+2}^\infty R^{-(t-(\tau+1))}\hat{p}_t\right]$ (which has zero net present value by construction), keeping the dividend and investment policies the same, except for the dividend in state $s^{\tau+1}$ at time $\tau + 1$. This dividend increases since the firm makes payment $\hat{p}(s^{\tau+1})$ instead of $p(s^{\tau+1})$ while buying back the tangible assets which have been seized, i.e., $\theta \varphi k(s^\tau)(1-\delta)$, and thus

$$\hat{d}(s^{\tau+1}) = d(s^{\tau+1}) + \left(p(s^{\tau+1}) - \hat{p}(s^{\tau+1}) - \theta\varphi k(s^\tau)(1-\delta)\right)$$

$$= d(s^{\tau+1}) + \left(E_{\tau+1}\left[\sum_{t=\tau+1}^\infty R^{-(t-(\tau+1))}p_t\right] - \theta\varphi k(s^\tau)(1-\delta)\right) > d(s^{\tau+1}).$$

Such a deviation would hence constitute an improvement, a contradiction. Conversely, if (36) is satisfied $\forall \tau' \geq \tau$, then defaulting cannot make the firm better off.

Part (ii): Take any sequence of net payments $\{p(s^t)\}_{t=\tau}^\infty$ and define

$$Rb(s^{\tau'}) \equiv E_{\tau'}\left[\sum_{t=\tau'}^\infty R^{-(t-\tau')}p_t\right] \leq \theta\varphi k(s^{\tau'-1})(1-\delta), \qquad \forall \tau' > \tau.$$

Then $Rb(s^{\tau'}) = p(s^{\tau'}) + R^{-1}E_{\tau'}\left[Rb(s^{\tau'+1})\right]$ and thus $p(s^{\tau'}) = Rb(s^{\tau'}) - E_{\tau'}\left[b(s^{\tau'+1})\right]$ and Eq. (33) can be rewritten as

$$A(s^t)f(k(s^{t-1})) + k(s^{t-1})(1-\delta) + E_t\left[b(s^{t+1})\right] \geq d(s^t) + k(s^t) + Rb(s^t), \qquad \forall t > \tau. \quad (38)$$

Similarly, setting $b(s^\tau) = 0$ yields $p(s^\tau) = -E_\tau\left[b(s^{\tau+1})\right]$. Thus, any sequence of net payments satisfying (36) can be implemented with a sequence of one period contingent claims satisfying (37).

Conversely, take any sequence $\{b(s^t)\}_{t=\tau}^\infty$ satisfying (37) and define $p(s^t) = Rb(s^t) - E_t\left[b(s^{t+1})\right]$, $\forall t \geq \tau$. Then, $\forall \tau' > \tau$,

$$E_{\tau'}\left[\sum_{t=\tau'}^\infty R^{-(t-\tau')}p_t\right] = E_{\tau'}\left[\sum_{t=\tau'}^\infty R^{-(t-\tau')}(Rb^t - b^{t+1})\right] = Rb(s^{\tau'}) \leq \theta\varphi k(s^{\tau'-1})(1-\delta),$$

i.e., the sequence of one period contingent claims satisfying (37) can be implemented with a sequence of net payments satisfying (36). $\square$

Thus, the sequence problem P_τ in Eqs. (31) to (35) is equivalent to maximizing (31) subject to $w(s^\tau) \geq d(s^\tau) + k(s^\tau) - E_\tau\left[b(s^{\tau+1})\right]$, (37), and (38), which can be written recursively as in Eqs. (1) to (4) by defining net worth after repayment of one period claims issued previously as $w(s^t) \equiv A(s^t)f(k(s^{t-1})) + k(s^{t-1})(1-\delta) - Rb(s^t)$, $\forall t > \tau$.

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Table 1
Tangible assets and liabilities.

This table reports balance sheet data on tangible assets and liabilities from the Flow of Funds Accounts of the United States for the 10 years from 1999 to 2008 [Federal Reserve Statistical Release Z.1], Tables B.100, B.102, B.103, and L.229. Panel A measures liabilities two ways. Debt is *Credit Market Instruments* which for (nonfinancial) businesses are primarily corporate bonds, other loans, and mortgages and for households are primarily home mortgages and consumer credit. Total liabilities are *Liabilities,* which, in addition to debt as defined before, include for (nonfinancial) businesses primarily miscellaneous liabilities and trade payables and for households primarily the trade payables (of nonprofit organizations) and security credit. For (nonfinancial) businesses, we subtract *Foreign Direct Investment in the U.S.* from Table L.229 from reported miscellaneous liabilities as Table F.229 suggests that these claims are largely equity. For households, real estate is mortgage debt divided by the value of real estate, and consumer durables is consumer credit divided by the value of consumer durables. Panel B reports the total tangible assets of households and noncorporate and corporate businesses relative to the total net worth of households. The main types of tangible assets, real estate, consumer durables, equipment and software, and inventories are also separately aggregated across the three sectors.

Panel A: Liabilities (% of tangible assets)

Sector	Debt	Total liabilities
(Nonfinancial) corporate businesses	48.5%	83.0%
(Nonfinancial) noncorporate businesses	37.8%	54.9%
Households and nonprofit organizations		
Total tangible assets	45.2%	47.1%
Real estate	41.2%	
Consumer durables	56.1%	

Panel B: Tangible assets (% of household net worth)

Assets by type	Tangible assets
Total tangible assets	79.2%
Real estate	60.2%
Equipment and software	8.3%
Consumer durables	7.6%
Inventories	3.1%

Table 2
Tangible assets and debt, rental, and lease adjusted leverage.

Panel A displays the relation between tangibility and (debt) leverage and Panel B displays the relation between tangibility and leverage adjusted for rented assets. Annual firm level Compustat data for 2007 are used excluding financial firms. Panel A: Tangibility: *Property, Plant, and Equipment–Total (Net)* (Item #8) divided by Assets; Assets: *Assets–Total* (Item #6) plus *Price–Close* (Item #24) times *Common Shares Outstanding* (Item #25) minus *Common Equity–Total* (Item #60) minus *Deferred Taxes* (Item #74); Leverage: *Long-Term Debt–Total* (Item #9) divided by Assets. Panel B: Lease Adjusted Tangibility: *Property, Plant, and Equipment–Total (Net)* plus 10 times *Rental Expense* (Item #47) divided by Lease Adjusted Assets; Lease Adjusted Assets: Assets (as above) plus 10 times *Rental Expense*; Debt Leverage: *Long-Term Debt–Total* divided by Lease Adjusted Assets; Rental Leverage: 10 times *Rental Expense* divided by Lease Adjusted Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage.

Panel A: Tangible assets and debt leverage

Tangibility quartile	Quartile cutoff (%)	Leverage (%) med.	Leverage (%) mean	Low leverage firms (%) (leverage $\leq$ 10%)
1	6.3	7.4	10.8	58.3
2	14.3	9.8	14.0	50.4
3	32.2	12.4	15.5	40.6
4	n.a.	22.6	24.2	14.9

Panel B: Tangible assets and debt, rental, and lease adjusted leverage

| Lease adjusted tangibility quartile	Quartile cutoff (%)	Debt med.	Debt mean	Rental med.	Rental mean	Lease adj. med.	Lease adj. mean	Debt	Rental	Lease adj.
1	13.2	6.5	10.4	3.7	4.2	11.4	14.6	61.7	97.7	46.0
2	24.1	9.8	12.9	6.9	8.1	18.4	21.0	50.1	68.2	16.1
3	40.1	13.1	14.8	8.0	10.5	24.2	25.3	41.7	60.6	12.0
4	n.a.	18.4	20.4	7.2	13.8	32.3	34.2	24.4	57.3	3.7

Table 3
Leverage and size revisited.

This table displays debt and rental leverage across size deciles (measured by lease adjusted book assets). Lease Adjusted Book Assets: Assets – Total plus 10 times Rental Expense; Debt Leverage: Long-Term Debt – Total divided by Lease Adjusted Book Assets; Rental Leverage: 10 times Rental Expense divided by Lease Adjusted Book Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage. For details of data and variables used see caption of Table 2.

| | Size deciles | | | | | | | | | |
Median leverage	1	2	3	4	5	6	7	8	9	10
Debt	6.0	7.3	7.4	14.1	19.5	22.6	20.6	20.2	21.6	17.8
Rental	21.8	14.6	10.8	11.1	11.2	9.1	9.7	9.1	7.8	7.3
Lease adjusted	30.6	24.2	21.0	28.8	36.4	37.7	33.4	36.6	31.7	26.3
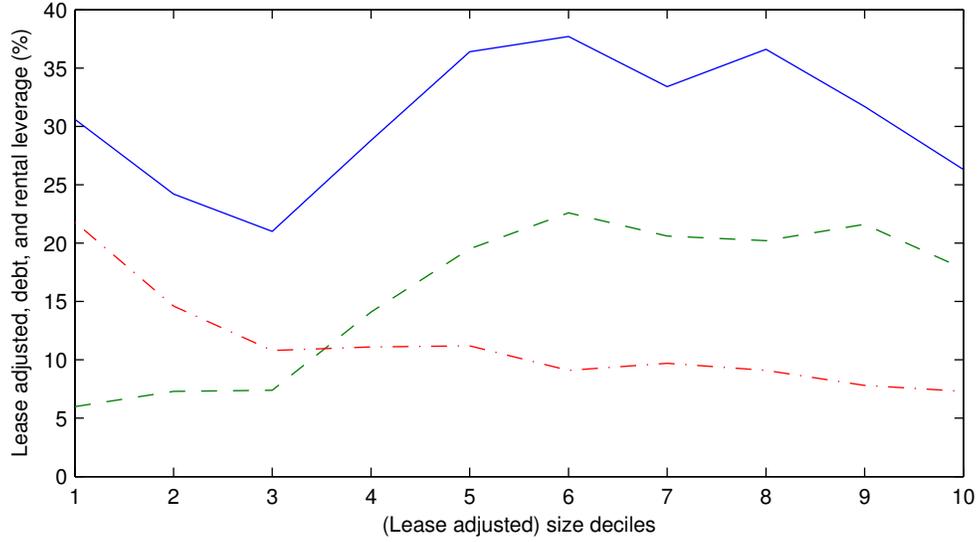
Fig. 1. Leverage versus size revisited. Lease adjusted leverage (solid), debt leverage (dashed), and rental leverage (dash dotted) across size decicles for Compustat firms. For details see caption of Table 3.
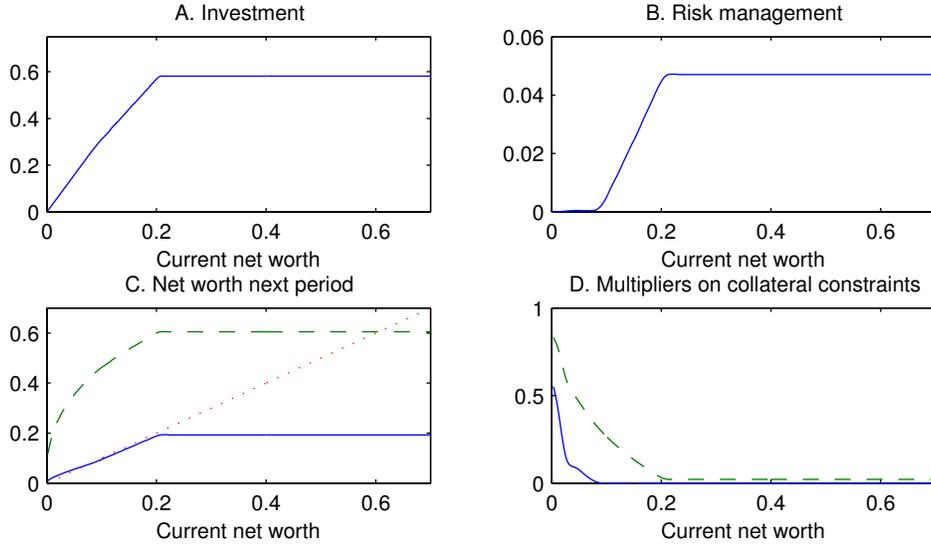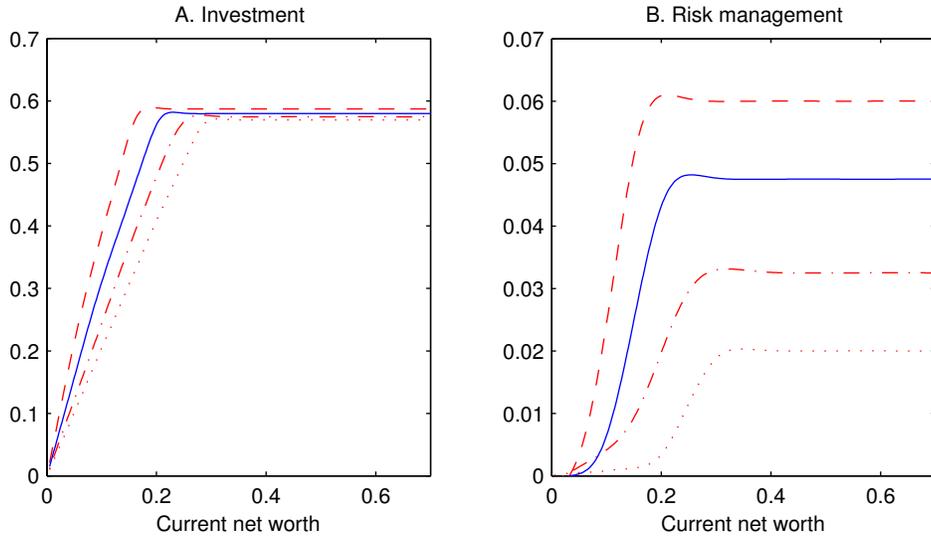

Fig. 2. Investment and risk management. Panel A shows investment k; Panel B shows risk management for the low state $h(s'_1)$; Panel C shows net worth in low state next period $w(s'_1)$ (solid) and in high state next period $w(s'_2)$ (dashed); Panel D shows scaled multipliers on the collateral constraint for the low state next period $\beta\lambda(s'_1)/\mu$ (solid) and for the high state next period $\beta\lambda(s'_2)/\mu$ (dashed); all as a function of current net worth w. The parameter values are: $\beta = 0.93$, $r = 0.05$, $\delta = 0.10$, $m = +\infty$, $\theta = 0.80$, $\varphi = 1$, $A(s_2) = 0.6$, $A(s_1) = 0.05$, and $\Pi(s, s') = 0.5$, $\forall s, s' \in S$, and $f(k) = k^\alpha$ with $\alpha = 0.333$.
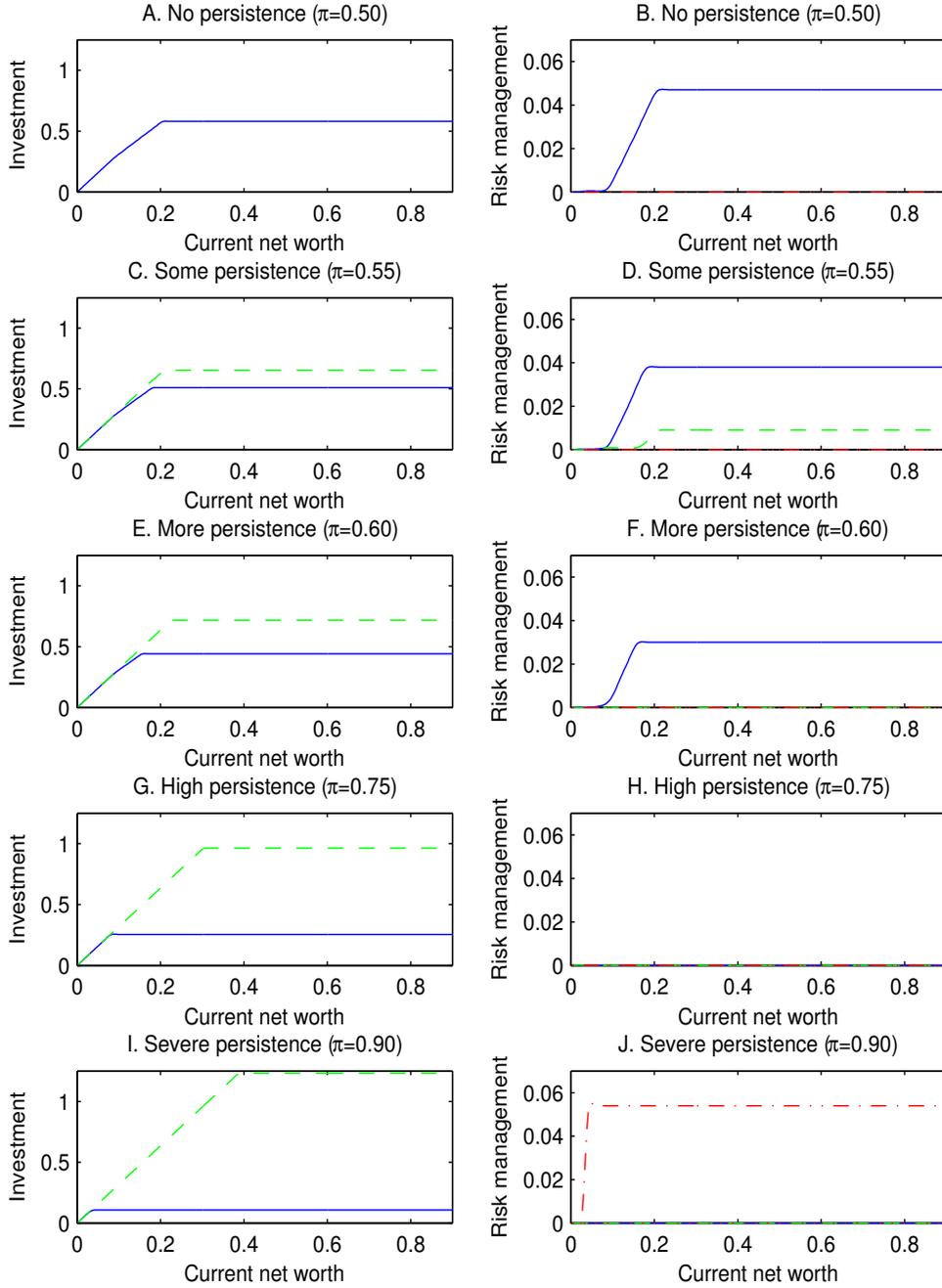
Fig. 3. Risk management with stochastic investment opportunities. Panels A through I: Investment (k) and risk management for the low state ($h_1(s')$) as a function of current net worth w for low current productivity (s_1) (solid) and high current productivity (s_2) (dashed). Panel J: Risk management for the high state ($h_2(s')$) as a function of current net worth w for low current productivity (s_1) (dash dotted). Persistence measured by $\Pi(s_1, s_1) = \Pi(s_2, s_2) \equiv \pi$ is 0.50, 0.55, 0.60, 0.75, and 0.90 in Panels A/B (no persistence), Panels C/D (some persistence), Panels E/F (more persistence), Panels G/H (high persistence), and Panels I/J (severe persistence). For other parameter values see the caption of Fig. 2.
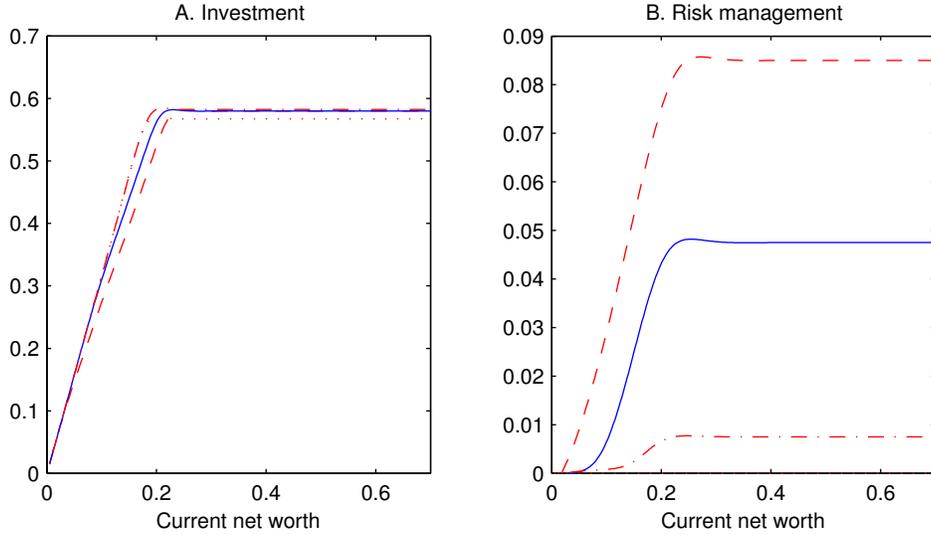
Fig. 4. Investment and risk management: Effect of risk. Panel A shows investment k and Panel B shows risk management for the low state $h(s_1')$ as a function of current net worth w as risk varies. The parameter values are as in Fig. 2 except that $(A(s_2), A(s_1))$ vary from $(0.625, 0.025)$ (dashed), $(0.600, 0.050)$ (solid), $(0.575, 0.075)$ (dash dotted), to $(0.550, 0.100)$ (dotted).

Fig. 5. Investment and risk management: Effect of tangibility and collateralizability. Panel A shows investment k and Panel B shows risk management for the low state $h(s_1')$ as a function of current net worth w as tangibility or collateralizability varies. The parameter values are as in Fig. 2 except that θ varies from 0.9 (dashed), 0.8 (solid), 0.7 (dash dotted), to 0.6 (dotted).
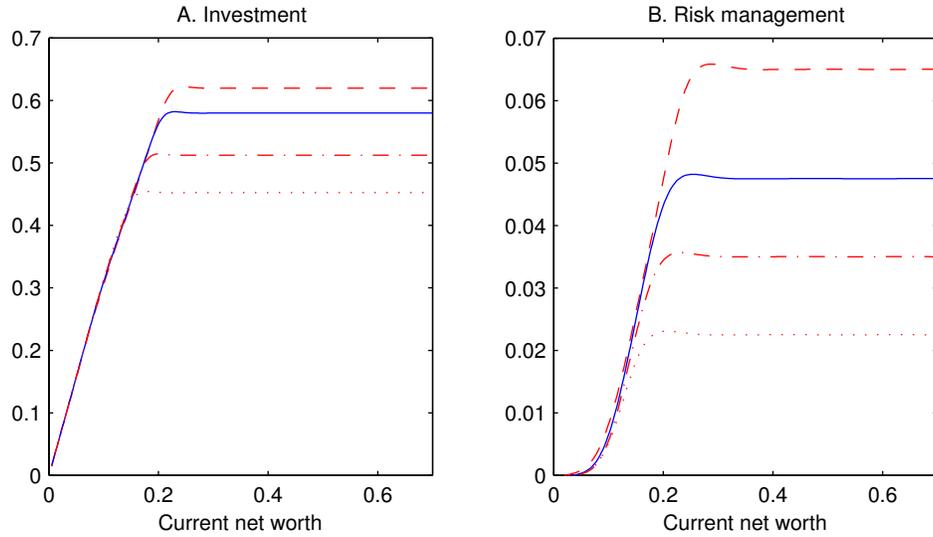
Fig. 6. Investment and risk management: Effect of curvature of production function. Panel A shows investment k and Panel B shows risk management for the low state $h(s_1')$ as a function of current net worth w as the curvature of the production function α varies. The parameter values are as in Fig. 2 except that α varies from 0.366 (dashed), 0.333 (solid), 0.300 (dash dotted), to 0.267 (dotted).
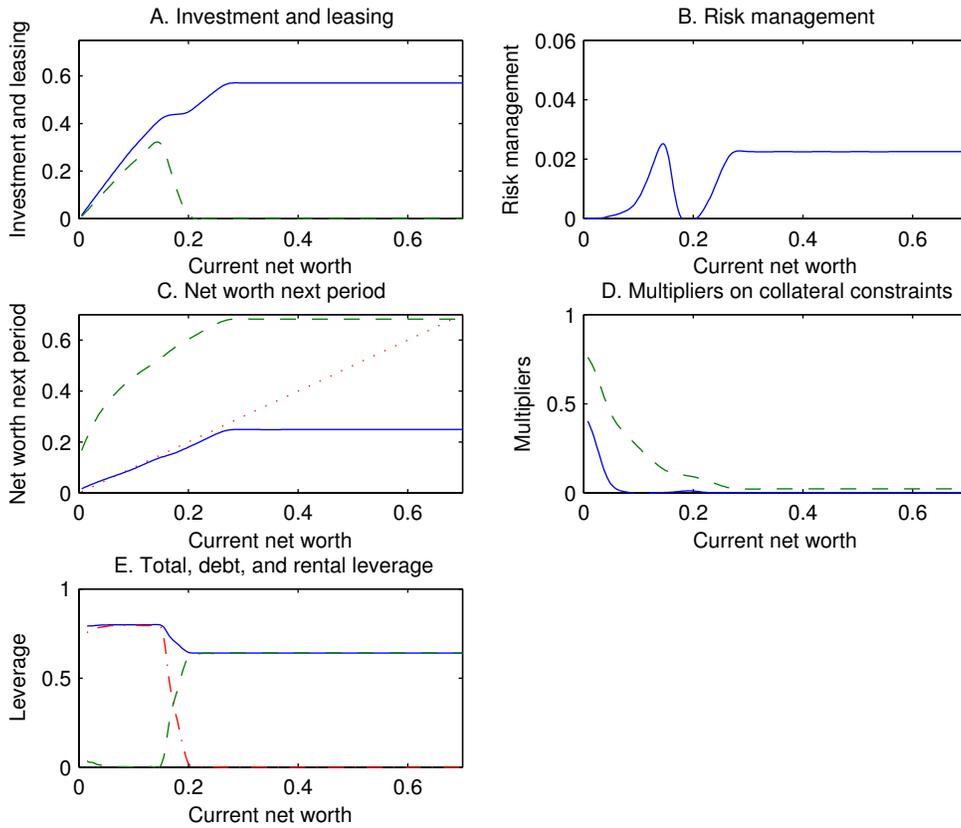
Fig. 7. Leasing, leverage, and risk management. Panel A shows investment (k) (solid) and leasing (k_l) (dashed); Panel B shows risk management for the low state ($h_1(s')$); Panel C shows net worth next period in the low state next period ($w(s_1')$) (solid) and in the high state next period ($w(s_2')$) (dashed); Panel D shows the multipliers on the collateral constraints for the low state ($\beta\lambda(s_1')$) (solid) and for the high state ($\beta\lambda(s_2')$) (dashed); and Panel E shows total leverage ($\theta(\varphi k - k_l) + k_l)/k$ (solid), debt leverage $\theta(\varphi k - k_l)/k$ (dashed), and rental leverage k_l/k (dash dotted); all as a function of current net worth w. For other parameter values see the caption of Fig. 2 except that $m = 0.01$ and $\varphi = 0.8$.