

# Constrained-Efficient Capital Reallocation

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# Motivation

- ▶ Financial frictions (specifically, **collateral constraints**) distort
  - ▶ level of aggregate investment
  - ▶ (re)allocation of capital across firms
- ▶ Analyze efficiency of capital allocation **subject to** financial frictions
- ▶ **Is resale price of capital (collateral) “too high” or “too low”?**

# This Paper

- ▶ Efficiency analysis in equilibrium model with
  - ▶ (macro) investment and capital reallocation
  - ▶ (heterogeneity) heterogeneous firms facing idiosyncratic shocks
  - ▶ (finance) collateral constraints
- ▶ Two types of pecuniary externalities through resale price of capital
  - ▶ Collateral externality
    - ▶ Higher collateral value facilitates new investment
  - ▶ Distributive externality
    - ▶ More constrained buy old capital from less constrained
    - ▶ Lower price of old capital facilitates purchases of old capital
- ▶ Insight: **Distributive externality dominates collateral externality**
  - ▶ New investment has positive externality: reduces price of old capital
  - ▶ Facilitates reallocation of old capital to more constrained firms
  - ▶ Both analytical and quantitative results

# Related Literature

## ▶ Capital Reallocation

- ▶ Eisfeldt/Rampini (2006, 2007); Lanteri (2018); Rampini (2019); Ma/Murfin/Pratt (2019); Gavazza/Lanteri (forthcoming)

## ▶ Pecuniary Externalities

- ▶ Lorenzoni (2008); Dávila/Hong/Krusell/Ríos-Rull (2012); Dávila/Korinek (2018); Bianchi/Mendoza (2018); Itskhoki/Moll (2019); Jeanne/Korinek (2019)

## ▶ Financial Frictions and Misallocation

- ▶ Kiyotaki/Moore (1997); Buera/Kaboski/Shin (2011); Midrigan/Xu (2014); Moll (2014)

# Outline

- (1) Stylized Model: Analytical Results
- (2) Quantitative Analysis

## **(1)** Stylized Model

# Capital Reallocation and Pecuniary Externalities

## Roadmap

- ▶ Environment
- ▶ First Best
- ▶ Competitive Equilibrium with Financial Frictions
- ▶ Constrained Efficiency
  - ▶ Distributive externality  $>$  collateral externality in comp. eqm.
  - ▶ Sustaining First Best
- ▶ Three Generalizations
- ▶ Essential Role of Heterogeneity and Reallocation

# Environment

- ▶ Time is discrete and horizon infinite
- ▶ Infinitely-lived representative household
  - ▶ Linear preferences

$$\sum_{t=0}^{\infty} \beta^t C_t$$

- ▶ Continuum of firms born at each date  $t$ ; live for two dates
  - ▶ Owned by representative household
  - ▶ Each firm draws initial net worth  $w$ 
    - ▶  $w \in W \equiv [w_{min}, w_{max}]$  with distribution  $\pi(w)$  (mass 1)
  - ▶ Invest at  $t$ , produce output at  $t + 1$
  - ▶ Maximize present value of dividends (net of financing costs)



# Capital Goods and Technology

- ▶ Capital goods
  - ▶ Last for two periods (so “new” and “old”)
  - ▶ New capital produced using output with linear technology at cost 1
  - ▶ (Standard) one period time to build
  - ▶ New and old capital perfect substitutes in production

- ▶ Firm production

$$y_t(w) = f \left( k_{t-1}^N(w) + k_{t-1}^O(w) \right)$$

with  $f_k > 0$  and  $f_{kk} < 0$

- ▶ Resource constraint (frictionless economy)

$$\int y_t(w) d\pi(w) = C_t + \int k_t^N(w) d\pi(w)$$

- ▶ Evolution of aggregate old capital

$$\int k_{t-1}^N(w) d\pi(w) = \int k_t^O(w) d\pi(w)$$

# First Best

Social planner maximizes household utility subject to resource constraints

- ▶ First-Best allocation satisfies

$$1 = \beta \left( f_k(k_t^{FB}) + q_{t+1}^{FB} \right)$$

$$q_t^{FB} = \beta f_k(k_t^{FB})$$

- ▶ Steady state

$$q^{FB} = \frac{1}{1 + \beta} \quad k^{FB} = f_k^{-1} \left( \frac{1}{\beta(1 + \beta)} \right)$$

- ▶ Allocation of new vs. old capital at firm level is indeterminate

# Financial Frictions

- ▶ Collateral constraint

$$\theta q_{t+1} k_t^N \geq \beta^{-1} b_t$$

with  $\theta \in [0, 1)$

- ▶ Cost of equity issuance  $\phi(-d)$ 
  - ▶ increasing and convex for  $d < 0$
  - ▶ zero otherwise

# Competitive Equilibrium with Financial Frictions

- ▶ New firm's problem at time  $t$  in competitive equilibrium

$$\max_{\{d_{0t}, d_{1,t+1}, b_t, k_t^N, k_t^O\} \in \mathbb{R}^3 \times \mathbb{R}_+^2} d_{0t} - \phi(-d_{0t}) + \beta d_{1,t+1}$$

subject to budget constraints of new firm at  $t$  and old firm at  $t + 1$

$$w_{0t} + b_t = d_{0t} + k_t^N + q_t k_t^O$$

$$f(k_t^N + k_t^O) + q_{t+1} k_t^N = d_{1,t+1} + \beta^{-1} b_t$$

and collateral constraint

$$\theta q_{t+1} k_t^N \geq \beta^{-1} b_t$$

# Firm Optimality

- ▶ Firms maximize present value of dividends net of cost  $\phi$  subject to
  - ▶ budget constraints
  - ▶ collateral constraint ( $\beta\lambda_t$ )
  - ▶ non-negativity constraints on  $k_t^N, k_t^O$  ( $\underline{v}_t^N, \underline{v}_t^O$ )
- ▶ First-order conditions w.r.t.  $k_t^N, k_t^O, b_t$

$$1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta\theta\lambda_t q_{t+1} + \underline{v}_t^N$$

$$q_t(1 + \phi_{d,t}) = \beta f_k(k_t) + \underline{v}_t^O$$

$$1 + \phi_{d,t} = 1 + \lambda_t$$

- ▶ Marginal value of net worth  $1 + \phi_{d,t}$

# Stationary Competitive Equilibrium

- ▶ Definition: Stationary Competitive Equilibrium
  - ▶ Policy functions  $d_0(w)$ ,  $d_1(w)$ ,  $k^N(w)$ ,  $k^O(w)$ , and  $b(w)$
  - ▶ Price of old capital  $q$

such that

- ▶ Individual optimality
- ▶ Goods market clearing (including costs of equity issuance)

$$\int y(w)d\pi(w) = C + \int k^N(w)d\pi(w) + \int \phi(d_0(w))d\pi(w)$$

- ▶ Capital goods market clearing

$$\int k^O(w)d\pi(w) = \int k^N(w)d\pi(w)$$

# Characterization

## Proposition 1

*Stationary competitive equilibrium is characterized as follows*

- (i) New capital has higher down payment than old capital, but (weakly) lower user cost from perspective of unconstrained firm*
- (ii) Price of old capital exceeds price in frictionless economy:  $q \geq q^{FB}$*
- (iii) If  $q > q^{FB}$ , thresholds  $\underline{w}_N < \bar{w}_O < \bar{w}$  such that*
  - ▶ firms with  $w \leq \underline{w}_N$  invest only in old capital*
  - ▶ firms with  $w \in (\underline{w}_N, \bar{w}_O)$  invest  $\underline{k}$ ; invest in both new & old capital*
  - ▶ firms with  $w \geq \bar{w}_O$  invest only in new capital*
  - ▶ firms with  $w > \bar{w}$  pay dividends and invest  $\bar{k} > k^{FB} > \underline{k}$*

# Choice between New and Old Capital

- ▶ New and old capital differ in terms of

- ▶ down payments  $\wp_N$  and  $\wp_O$

$$\wp_N \equiv 1 - \beta\theta q > \wp_O \equiv q$$

- ▶ user cost (for unconstrained firm)

$$u_N \equiv 1 - \beta q \leq u_O \equiv q$$

- ▶ Investment Euler equations for new and old capital

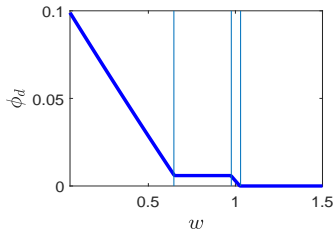
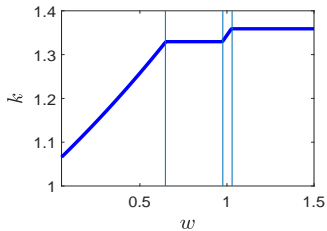
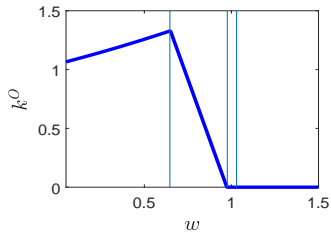
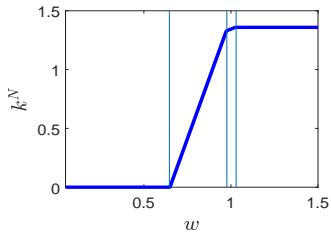
$$u_N(w) \equiv u_N + \phi_d \wp_N \geq \beta f_k(k)$$

$$u_O(w) \equiv u_O + \phi_d \wp_O \geq \beta f_k(k)$$

- ▶ Sufficiently (un)constrained firms invest in old (new) capital



# Policy Functions



# Constrained-Efficient Allocation

- ▶ **Planner** chooses allocations and price to maximize household utility subject to
  - ▶ technological constraintsand
  - ▶ individual budget and financial constraints
  - ▶ market clearing condition for old capital ( $\eta_t$ )
- ▶ First-order conditions w.r.t.  $k_t^N, k_t^O, b_t$

$$1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta\theta\lambda_t q_{t+1} + \underline{v}_t^N + \beta\eta_{t+1}$$

$$q_t(1 + \phi_{d,t}) = \beta f_k(k_t) + \underline{v}_t^O - \eta_t$$

$$1 + \phi_{d,t} = 1 + \lambda_t$$

## Constrained-Efficient Price

- ▶ First-order condition w.r.t. price  $q_t$

$$\int k_t^O(w) (1 + \phi_{d,t}(w)) d\pi(w) = \int k_{t-1}^N(w) (1 + \theta\lambda_{t-1}(w)) d\pi(w)$$

or

$$\begin{aligned} \int k_t^O(w) (1 + \phi_{d,t}(w)) d\pi(w) - \int k_{t-1}^N(w) d\pi(w) \\ = \theta \int k_{t-1}^N(w) \lambda_{t-1}(w) d\pi(w) \end{aligned}$$

- ▶ Using market clearing for capital goods ( $\int k_t^O d\pi = \int k_{t-1}^N d\pi$ )

$$\int k_t^O(w) \phi_{d,t}(w) d\pi(w) = \theta \int k_{t-1}^N(w) \lambda_{t-1}(w) d\pi(w)$$

- ▶ Two types of pecuniary externalities

- ▶ Distributive externality:  $k_t^O \phi_{d,t}$

- ▶ Collateral externality:  $\theta k_{t-1}^N \lambda_{t-1}$

# Externalities in Competitive Equilibrium

## Proposition 2

*In stationary competitive equilibrium*

- ▶ *Distributive externality is larger than collateral externality*

$$\int k^O(w)\phi_d(w)d\pi(w) > \theta \int k^N(w)\lambda(w)d\pi(w)$$

- ▶ Competitive-equilibrium price of old capital is **higher** than constrained-efficient one
- ▶ Intuition: (recall  $\lambda(w) = \phi_d(w)$ )
  - ▶ cov. between mrg. value of net worth and **old** capital investment exceeds
  - ▶ cov. between mrg. value of net worth and **new** capital investment

# Constrained-Efficient Allocation: Sustaining First Best

Stationary constrained-efficient allocation achieves First-Best welfare

$$q^* = \frac{w_{min}}{k^{FB}} < q^{FB} \leq q$$

- ▶ Price of old capital is low enough that
  - ▶ even firms with net worth  $w_{min}$  achieve scale  $k^{FB}$
  - ▶ without issuing equity:  $\phi_d = \lambda = 0$  for all  $w$

# Constrained-Efficient Allocation: Implementation

- ▶ Competitive equilibrium with taxes  $\tau_t^N(w)$ ,  $\tau_t^O(w)$

$$\tau^N = -\beta\eta = -\beta(q^{FB} - q^*) < 0$$

$$\tau^O = \frac{\eta}{q^*} = \frac{q^{FB}}{q^*} - 1 > 0$$

- ▶ Tax rates independent of net worth  $w$
- ▶ Taxes rebated lump-sum so as to respect each budget constraint
- ▶ Under additional restriction  $\tau_t^O(w) = 0$ , we show  $\tau_t^N(w) < 0$

# Three Generalizations

Sign of inefficiency obtains in three generalizations of the model:

- ▶ Risk-averse entrepreneurs (Proposition 3)

Risk-averse entrepreneurs

- ▶  $u(c_{0t}) + \beta u(c_{1,t+1}), u_c > 0, u_{cc} < 0$

- ▶ Heterogeneity in productivity (Proposition 4)

Heterogeneity in productivity

- ▶  $y_t(w) = s f(k_{t-1}(w))$

- ▶  $\frac{\partial \phi_d(w,s)}{\partial s} \geq 0$

- ▶ Long-lived firms and capital (Proposition 5)

# Long-Lived Firms and Capital

- ▶ Stochastic firm life cycle
  - ▶ Probability of firm death  $\rho$
  - ▶ Net worth is endogenous state variable
- ▶ Long-lived capital
  - ▶ Fraction  $\delta^N$  of new capital becomes old
  - ▶ Fraction  $\delta^O$  of old capital is destroyed
  - ▶ Both new and old capital serve as collateral
- ▶ Stylized model is special case:  $\rho = \delta^N = \delta^O = 1$

## Proposition 5

*In stationary competitive equilibrium*

- ▶ *Distributive externality is larger than collateral externality*



# Essential Role of Heterogeneity and Reallocation

- ▶ Distributive externality hinges on reallocation in equilibrium
  - ▶ Stationary equilibrium with reallocation
- ▶ Representative entrepreneur in steady state – Kiyotaki/Moore (1997)
  - ▶ Assets in fixed supply (land)
  - ▶ Entrepreneur has constant amount of land in steady state
  - ▶ Misallocation, but no reallocation
  - ▶ Change in price of land has no effect on budget constraints
  - ▶ Only collateral externality
- ▶ Our result obtains with assets in fixed supply and OLG firms
  - ▶ Heterogeneity between young and old firms
  - ▶ Reallocation of land from old to young firms
  - ▶ Distributive externality dominates collateral externality

## (2) Quantitative Analysis

# Quantitative Model

We generalize assumptions as follows:

- ▶ Stochastic life cycle (prob. of death  $\rho$ )
- ▶ Long-lived capital ( $\delta^N, \delta^O$ )
- ▶ Persistent **idiosyncratic productivity shocks**  $s$
- ▶ New and old capital **imperfect substitutes** in production

$$y = s f \left( g(k^N, k^O) \right)$$

where  $g$  CES aggregator

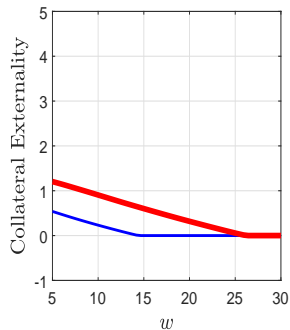
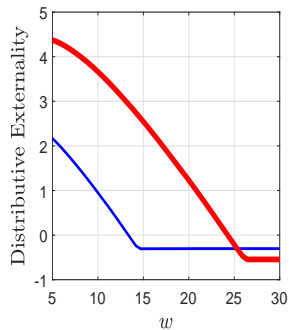
- ▶ **Scrapage value** of old capital  $\underline{q} \geq 0$

# Calibration Strategy

- ▶ Technology and shocks
  - ▶ Evidence on investment and reallocation dynamics of US firms (Khan/Thomas, 2013; Lanteri, 2018)
- ▶ Financial frictions
  - ▶ Estimates of financing costs from corporate-finance literature (Hennessy/Whited, 2007; Catherine/Chaney/Huang/Sraer/Thesmar, 2020; Li/Whited/Wu, 2016)
- ▶ Capital reallocation
  - ▶ Joint distribution of firm age and capital age (Ma/Murfin/Pratt, 2020)

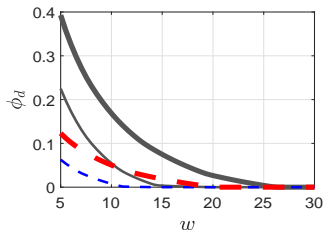
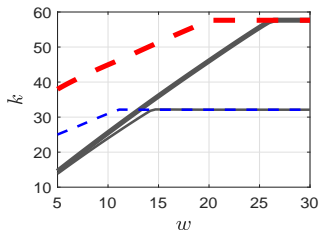
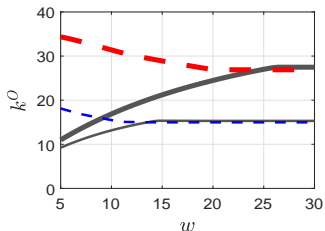
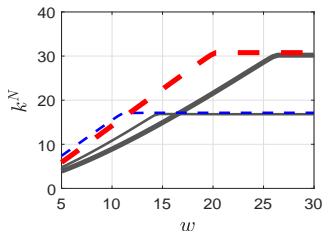
# Cross Section of Externalities

$$|\text{Distributive externality}| \approx 2.3 \times |\text{Collateral externality}|$$



Thick red: High productivity. Thin blue: low productivity.

# Constrained-Efficient Reallocation



Solid: Competitive Equilibrium. Dashed: Constrained Efficient.

# Aggregate Outcomes

Variable	Comp. Eqm.	Constr. Eff.	Constr. Eff. ( $\tau^O = 0$ )
Output	0.899	0.973	0.921
Investment	0.857	0.962	0.893
Consumption	0.933	0.983	0.943
Price $q$	1.010	0.184	0.987
Average tax $\tau^N$	0	-8.6%	-0.6%
Average tax $\tau^O$	0	103.8%	n.a.

- ▶ Allocations and price expressed as fractions of First-Best value

# Additional Analyses and Robustness

- ▶ Transition dynamics
  - ▶ Compute optimal “simple” policy starting from competitive eqm.
  - ▶ Optimal time-invariant  $\tau^N$  for all firms  $\approx -0.3\%$  Figure
- ▶ Benchmarking gains from capital reallocation
  - ▶ Consider restriction:  $\frac{k^O(s^a)}{k^N(s^a)} = \omega$
  - ▶ Going from restricted to unrestricted competitive equilibrium
  - ▶ Approximately 0.4% consumption gain
- ▶ Sensitivity analysis
  - ▶ Collateralizability  $\theta$ , elasticity of substitution  $\epsilon$ , scrap value  $\underline{q}$



# Conclusion

- ▶ Gains from reallocation of old capital
  - ▶ High-MPK firms buy old capital
- ▶ **Price of old capital in competitive equilibrium is too high**
  - ▶ Distributive externality dominates collateral externality
- ▶ New investment today makes old capital less scarce in future
  - ▶ Positive externality on constrained firms in future
  - ▶ Novel rationale for subsidies on new investment

Extra

## Lagrangian for Constrained Efficiency

$$\begin{aligned}\mathcal{L} \equiv & \sum_{t=0}^{\infty} \beta^t \left\{ \int (d_{0t} - \phi(-d_{0t}) + d_{1t}) d\pi \right. \\ & - \int \mu_{0t} (d_{0t} - w + k_t^N + q_t k_t^O - b_t) d\pi \\ & - \int \mu_{1t} (d_{1t} - f(k_{t-1}^N + k_{t-1}^O) - q_t k_{t-1}^N + \beta^{-1} b_{t-1}) d\pi \\ & \quad \left. + \int \lambda_t (\beta \theta q_{t+1} k_t^N - b_t) d\pi \right. \\ & \left. + \int \underline{v}_t^N k_t^N d\pi + \int \underline{v}_t^O k_t^O d\pi - \eta_t \left( \int k_t^O d\pi - \int k_{t-1}^N d\pi \right) \right\}\end{aligned}$$

## Lagrangian for Constrained Efficiency: New Subsidies Only

$$\begin{aligned}\mathcal{L} \equiv & \sum_{t=0}^{\infty} \beta^t \left\{ \int (d_{0t} - \phi(-d_{0t}) + d_{1t}) d\pi \right. \\ & - \int \mu_{0t} (d_{0t} - w + k_t^N + q_t k_t^O - b_t) d\pi \\ & - \int \mu_{1t} (d_{1t} - f(k_{t-1}^N + k_{t-1}^O) - q_t k_{t-1}^N + \beta^{-1} b_{t-1}) d\pi \\ & + \int \lambda_t (\beta \theta q_{t+1} k_{t+1}^N - b_t) d\pi \\ & + \int \psi_t (q_t (1 + \phi_{d,t}) - \beta f_k(k_t)) d\pi \\ & \left. + \int \underline{v}_t^N k_t^N d\pi + \int \underline{v}_t^O k_t^O d\pi - \eta_t \left( \int k_t^O d\pi - \int k_{t-1}^N d\pi \right) \right\}\end{aligned}$$

# Numerical Example

Table: PARAMETER VALUES

		Parameter	Value
Preferences	Discount rate	$\beta$	0.96
Net worth	Uniform $\pi(w)$	$w_{min}$	0.05
		$w_{max}$	1.5
Technology	Curvature of $f$	$\alpha$	0.6
Financial constraints	Collateralizability	$\theta$	0.5
	Cost of equity	$\phi_0$	0.1
		$\phi_1$	2

► Functional forms:  $f(k) = k^\alpha$ ;  $\phi(-d) = \phi_0(-d)^{\phi_1}$  for  $d < 0$

# Covariance Interpretation

Interpretation: covariance between net expenditure and marginal value of net worth

- ▶ We show that

$$\text{Cov}(k^O, \phi_d) > \text{Cov}(k^N, \phi_d)$$

- ▶ Also,  $\text{Cov}(k^N, \phi_d) < 0$
- ▶ Typically (and in our example),  $\text{Cov}(k^O, \phi_d) > 0$

# Implementation

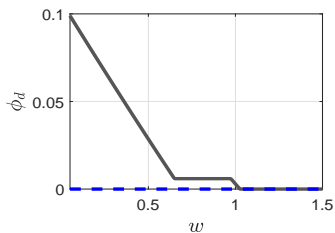
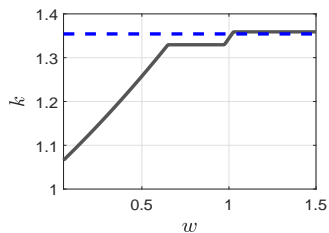
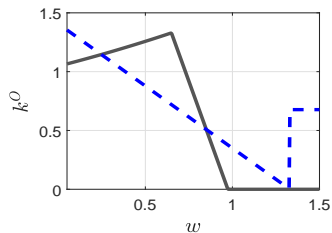
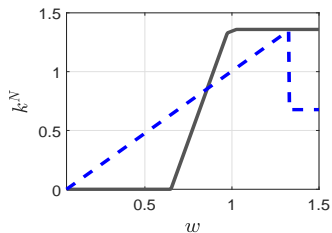
- ▶ Competitive equilibrium with
  - ▶ proportional taxes on new and old capital  $\tau_t^N(w)$ ,  $\tau_t^O(w)$
  - ▶ rebated lump-sum to each firm, so as to not to redistribute resources
- ▶ Budget constraint

$$w + b_t = d_{0t} + k_t^N(1 + \tau_t^N) + q_t k_t^O(1 + \tau_t^O) - T_t$$

- ▶ Lump-sum transfer

$$T_t = \tau_t^N k_t^N + \tau_t^O q_t k_t^O$$

# Constrained-Efficient Reallocation



Solid: Competitive Equilibrium. Dashed: Constrained Efficient.



# New-Capital Subsidies Only

Introduce additional constraint: old capital cannot be taxed

$$q_t(1 + \phi_{d,t}) \geq f_k(k_t)$$

with multiplier  $\psi_t$

Lagrangian

Back

# Constrained Efficiency: New-Capital Subsidies Only

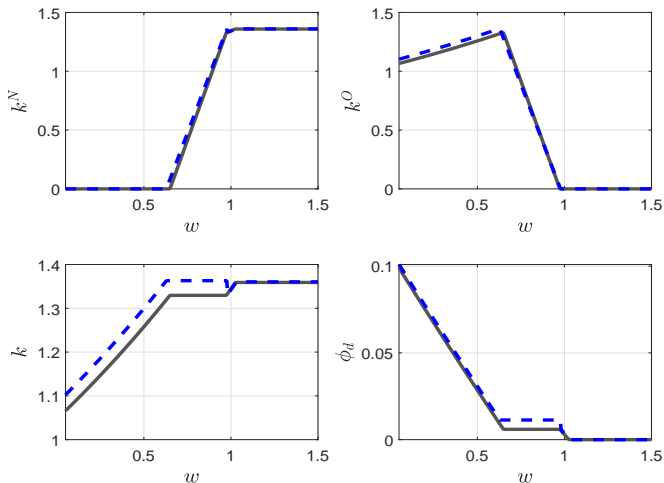
- ▶ FOC wrt  $k_{t+1}^N$

$$1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta \theta \lambda_t q_{t+1} + \beta \eta_{t+1} + \psi_t (q_t \phi_{dd,t} - \beta f_{kk}(k_t))$$

- ▶ FOC wrt  $q_t$

$$\int k_t^O (1 + \phi_{d,t}) d\pi = \int k_{t-1}^N (1 + \theta \lambda_{t-1}) d\pi + \int \psi_t (1 + \phi_{d,t} + q_t \phi_{dd,t} k_t^O) d\pi$$

# Constrained-Eff. Reallocation: New-Capital Subsidies Only



Solid: Competitive Equilibrium. Dashed: Constrained-Efficient.

# Risk-Averse Entrepreneurs

- ▶ Entrepreneurs maximize  $u(c_{0t}) + \beta u(c_{1,t+1})$ ,  $u_c > 0$ ,  $u_{cc} < 0$

## Proposition 6

*In stationary competitive equilibrium*

- ▶ *Distributive externality is larger than collateral externality*

$$\int k^O(w) u_c(c_0(w)) d\pi(w) > \int k^N(w) [u_c(c_1(w)) + \theta\lambda(w)] d\pi(w)$$

# Heterogeneity in Productivity

- ▶ Joint distribution of net worth  $w$  and productivity  $s$ ,  $\pi(w, s)$
- ▶ Production  $y_t = s f(k_{t-1}^N + k_{t-1}^O)$
- ▶ We show  $\frac{\partial \phi_d(w, s)}{\partial s} \geq 0$

## Proposition 7

*In stationary competitive equilibrium*

- ▶ *Distributive externality is larger than collateral externality*

$$\int k^O(w, s) \phi_d(w, s) d\pi(w, s) > \theta \int k^N(w, s) \lambda(w, s) d\pi(w, s)$$

# Quantitative Model: Net Worth and Collateral Constraint

► Net worth evolution

$$w_t(s^a) = s_a f(k_{t-1}(s^{a-1})) + (1 - \delta^N(1 - q_t))k_{t-1}^N(s^{a-1}) \\ + q_t(1 - \delta^O)k_{t-1}^O(s^{a-1}) - \beta^{-1}b_{t-1}(s^{a-1})$$

► Collateral constraint

$$\theta \left[ (1 - \delta^N(1 - q_{t+1}))k_t^N(s^a) + q_{t+1}(1 - \delta^O)k_t^O(s^a) \right] \geq \beta^{-1}b_t(s^a)$$

# Quantitative Model: Market Clearing

- ▶ Market clearing for old capital

$$\begin{aligned} \sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) k_t^O(s^a) \\ = \sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) \left[ \delta^N k_{t-1}^N(s^a) + (1 - \delta^O) k_{t-1}^O(s^a) \right] \end{aligned}$$

# Quantitative Model: Constrained-Efficient Allocation

- ▶ FOC wrt  $k_t^N(s^a)$

$$1 + \phi_{d,t} = \beta \mathbb{E}_t \left[ s_{a+1} f_k(k_t) g_{N,t} + (1 - \delta^N (1 - q_{t+1})) \right] (1 + (1 - \rho) \phi_{d,t+1}) + \beta \theta \lambda_t (1 - \delta^N (1 - q_{t+1})) + \beta \delta^N \eta_{t+1}$$

- ▶ FOC wrt  $k_t^O(s^a)$

$$q_t (1 + \phi_{d,t}) = \beta \mathbb{E}_t \left[ s_{a+1} f_k(k_t) g_{O,t} + (1 - \delta^O) q_{t+1} \right] (1 + (1 - \rho) \phi_{d,t+1}) + \beta \theta (1 - \delta^O) \lambda_t q_{t+1} - \eta_t + \beta (1 - \delta^O) \eta_{t+1}$$

- ▶ FOC wrt  $b_t(s^a)$

$$\phi_{d,t} = (1 - \rho) \mathbb{E}_t \phi_{d,t+1} + \lambda_t$$



# Quantitative Model: Constrained-Efficient Price

► FOC wrt  $q_t$

$$\sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) k_t^O (1 + \phi_{d,t}) \geq$$
$$\sum_{a=0}^{\infty} \gamma_a \sum_{s^{a+1}} p(s^{a+1}) \left( \delta^N k_{t-1}^N + (1 - \delta^O) k_{t-1}^O \right) (1 + (1 - \rho)\phi_{d,t} + \theta\lambda_{t-1})$$

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# Solving for Constrained Efficiency

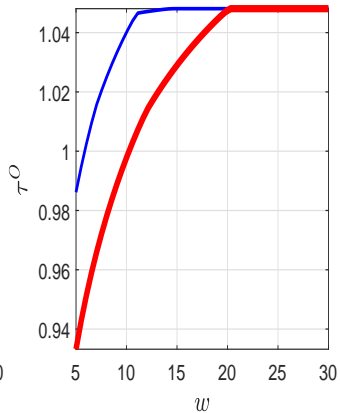
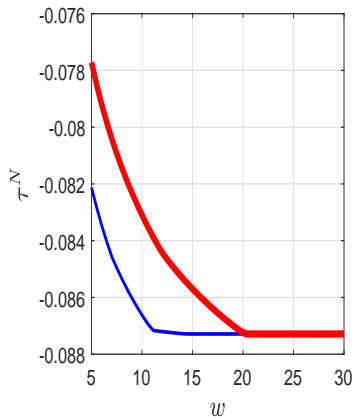
- ▶ Guess shadow value of old capital  $\eta$ 
  - ▶ Guess price  $q$
  - ▶ Compute policy functions solving investment FOCs on grid for  $(w, s)$
  - ▶ Compute stationary distribution  $\pi(w, s)$
  - ▶ Check market clearing and update  $q$
  
- ▶ Evaluate externalities (FOC wrt  $q$ ) and update  $\eta$

# Calibration

	Parameter	Value
Discount rate	$\beta$	0.96
Initial net worth	$w_0$	5
Death probability	$\rho$	0.1
Curvature of production function	$\alpha$	0.6
CES elasticity of substitution	$\epsilon$	5
CES new share	$\nu$	0.5
Depreciation new	$\delta^N$	0.2
Depreciation old	$\delta^O$	0.2
Scrap value	$\underline{q}$	0.1
Productivity persistence	$\chi_s$	0.7
Productivity st. dev. of innovations	$\sigma_s$	0.12
Collateralizability	$\theta$	0.5
Cost of raising equity	$\phi_0$	0.1
	$\phi_1$	5

- Functional forms:  $f(k) = k^\alpha$ ;  $\phi(-d) = \phi_0(-d)^{\phi_1}$  for  $d < 0$

# Implementation



# Transition Dynamics

