

Financing Durable Assets

Adriano A. Rampini
Duke University

Hebrew University of Jerusalem
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Effect of Durability on Financing

■ Durability essential feature of capital

- Fixed assets comprise as much as 72% of aggregate capital stock
- Includes structures (both residential and non-residential), infrastructure, equipment, and consumer durable goods
- Focus on tangible durable assets but intangible assets similar

■ Variation in durability

- Depreciation rates vary from as low as 1% (new residential structures) to as high as 31% (computing equipment)

■ How does durability affect financing?

- Can durable assets serve as collateral and facilitate financing?
- *“Intuitively, as the assets become more durable, they provide the creditor with the security to wait longer before being repaid. ... And hence the debtor need not set aside as much of his initial borrowing to finance early debt repayments, leaving more to finance the initial investment” – Hart/Moore (1994)*

Effect of Durability on Financing (Cont'd)

- To the contrary: **durable assets are harder to finance**
 - Durability increases resale/collateral value allowing more borrowing
 - But durability also raises price of asset and hence financing need
 - Net effect: raises down payments

- **Implications**
 - **Durability choice**: more vs. less durable types of capital
 - Choice between **vintages**: new, durable vs. used, non-durable assets
 - **Legal enforcement** affects ...
 - **technology adoption**: dominated technologies may be adopted
 - **trade** in used capital goods
 - **Rent** vs. buy decision

- **Critical distinction between durability and pledgeability**
 - Durability impedes financing
 - Pledgeability facilitates financing
 - Hart/Moore (1994) is about pledgeability not durability

Model of Durable Asset Financing

- Discrete time, infinite horizon, deterministic productivity
- **Entrepreneurs** (or “firms”)
 - Measure $\rho \in (0, 1)$ enter each period; survive with probability $1 - \rho$; measure 1 alive
 - Preferences: $\sum_{t=0}^{\infty} \beta^t d_t$ with $\beta \in (0, 1)$, $d_t \geq 0$, and $\beta \equiv \hat{\beta}(1 - \rho)$
 - Initial net worth w_0 (cash-in-hand)
- In stationary equilibrium, unconstrained entrepreneurs are financiers
 - Equilibrium (gross) interest rate $R = \beta^{-1}$ (net: $r \equiv R - 1$)
- **Limited enforcement**
 - Entrepreneurs can default and abscond with all cash flows and fraction $1 - \theta$ of assets without exclusion where $\theta \in [0, 1)$
 - Rampini/Viswanathan (2010, 2013, 2018)
 - Optimal dynamic contract can be implemented with one-period ahead debt subject to **collateral constraints**

Modeling Durability: Two Approaches

- Capital with different **geometric depreciation rate**
 - Lower depreciation rate means more durable
- Capital with **finite useful life (one-horse shay depreciation)**
 - Capital provides same service flow for finite number of periods
 - New capital assets are more durable than used ones

Financing and Durability: Geometric Depreciation

■ Durability choice

- Two types of capital, more and less durable, $j \in J \equiv \{d, nd\}$
- **Geometric rate** δ_j per period, $\delta_d < \delta_{nd}$
- Cost q_j (in output goods) – linear capital production technology

■ Law of motion for capital

$$k_{j,t} = k_{j,t-1}(1 - \delta_j) + i_{j,t}$$

where $i_{j,t}$ is investment

- Perfect substitutes: $k_t \equiv \sum_{j \in J} k_{j,t}$
- Output $Af(k_t)$ next period where A is TFP and f is (strictly) increasing, concave, and $\lim_{k_t \rightarrow 0} f(k_t) = +\infty$

Financing and Durability: Firm's Problem

- **Firm's (recursive) problem:** Given w , q_d , q_{nd} , and R , solve

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b, w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w') \quad (1)$$

subject to budget constraints and collateral constraint

$$w + b \geq d + \sum_{j \in J} q_j k_j \quad (2)$$

$$Af(k) + \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb + w' \quad (3)$$

$$\theta \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb \quad (4)$$

and $k \equiv \sum_{j \in J} k_j$

- Endogenous state variable: net worth w

User Cost and Down Payment

■ Frictionless user cost

- Jorgenson (1963)'s frictionless one-period rental rate

$$u_j \equiv R^{-1}q_j(r + \delta_j)$$

- Paid at beginning of period

■ Down payment

- Minimal amount of internal funds needed to deploy unit of asset
- Present value of collateral value: $R^{-1}\theta q_j(1 - \delta_j)$
- **Down payment** per unit of capital

$$\varphi_j \equiv q_j - R^{-1}\theta q_j(1 - \delta_j)$$

- Relation to frictionless user cost

$$\varphi_j = \underbrace{u_j}_{\text{Jorgensonian user cost}} + \underbrace{R^{-1}(1 - \theta)q_j(1 - \delta_j)}_{\text{non-pledgeable part of resale value}} > u_j$$

Investment with Collateral Constraints

■ Investment Euler equation

- Combining first-order conditions

$$1 \geq \beta \frac{\mu'}{\mu} \frac{Af_k(k) + (1 - \theta)q_j(1 - \delta_j)}{\wp_j}$$

- With equality if $k_j > 0$ ($\nu_j = 0$)
- Firm's discount factor $\beta \frac{\mu'}{\mu}$

■ Determinants of investment

- First-order condition for investment with collateral constraints

$$u_j + \frac{\lambda'}{\mu'} \wp_j \geq \beta Af_k(k)$$

- Key determinants: (frictionless) user cost and down payment

Trade-off between User Cost and Down Payment

- Suppose **both two types of capital used in equilibrium**
- If $u_j \geq u_{j'}$ and $\wp_j \geq \wp_{j'}$, $j \neq j'$, type j capital weakly dominated
 - Therefore: $u_j > u_{j'}$ and $\wp_j < \wp_{j'}$, $j \neq j'$,
- Suppose $u_d > u_{nd}$
 - Then $R^{-1}q_d(r + \delta_d) > R^{-1}q_{nd}(r + \delta_{nd})$ implies $q_d > q_{nd}$
 - $\wp_d = u_d + R^{-1}(1-\theta)q_d(1-\delta_d) > u_{nd} + R^{-1}(1-\theta)q_{nd}(1-\delta_{nd}) = \wp_{nd}$
 - Contradiction

■ Durable capital has lower user cost but higher down payment

- $u_d < u_{nd}$ and $\wp_d > \wp_{nd}$
- More **durable capital must be more expensive** ($q_d > q_{nd}$)

$$q_d(1 - R^{-1}\theta(1 - \delta_d)) > q_{nd}(1 - R^{-1}\theta(1 - \delta_{nd}))$$

so effect of durability on price must exceed effect on collateral value

- “Harder to finance”

Constrained Firms Adopt Dominated Technologies

■ Unconstrained never use less durable “low quality” capital

- User cost of potentially constrained firm $u_j(w) \geq \beta \frac{\mu'}{\mu} Af_k(k)$:

$$u_j(w) \equiv u_j + \beta \frac{\lambda'}{\mu} (1 - \theta) q_j (1 - \delta_j)$$

- If $\lambda' = 0$, then $u_j(w) = u_j$.
- Unconstrained firms simply **compare user costs**

■ Severely constrained adopt dominated technology

- Alternatively, define user cost of potentially constrained firm as

$$u_j(w) \equiv \wp_j - \beta \frac{\mu'}{\mu} q_j (1 - \theta) (1 - \delta_j)$$

- Using the investment Euler equation,

$$1 \geq \beta \frac{\mu'}{\mu} \frac{Af_k(k) + (1 - \theta) q_j (1 - \delta_j)}{\wp_j}$$

as $w \rightarrow 0$, $\sum_j k_j \rightarrow 0$ and hence $\beta \frac{\mu'}{\mu} \rightarrow 0$, so $u_j(w) \rightarrow \wp_j$

- Severely constrained firms simply **compare down payments**

Dynamics of Firm Financing, Payout, and Investment

- Assume initial net worth w_0 low
- Young (severely constrained) firms
 - Pay no dividends
 - Compare down payments and invest only in less durable capital
- Unconstrained firms
 - Pay dividends
 - Compare user costs and invest only in more durable capital

Adopting Dominated Technologies and Legal Enforcement

- Two countries with strong and weak **legal enforcement**: $\theta_H > \theta_L$
- Technology choice: two types of capital with $\delta_d < \delta_{nd}$
 - Both used in equilibrium: $u_d < u_{nd}$; $\varphi_d(\theta_L) > \varphi_{nd}(\theta_L)$ ($q_d > q_{nd}$)

■ Less durable capital dominated with strong legal enforcement

- Note: $\varphi_j(\theta_L) > \varphi_j(\theta_H)$ and $\varphi_d(\theta_i) - \varphi_{nd}(\theta_i)$ decreasing in θ_i
- As $\theta_H \rightarrow 1$, $\varphi_d(\theta_H) - \varphi_{nd}(\theta_H) \rightarrow u_d - u_{nd} < 0$

■ More dominated technology use with weak legal enforcement

- Firm indifferent between two types of capital

$$\frac{Af_k(k) + (1 - \theta_i)q_d(1 - \delta_d)}{\varphi_d(\theta_i)} = \frac{Af_k(k) + (1 - \theta_i)q_{nd}(1 - \delta_{nd})}{\varphi_{nd}(\theta_i)},$$

- Lower θ_i implies indifferent at higher total investment $\underline{k}(\theta_i)$ and hence higher net worth

Durability and Composition of Investment

- Suppose **two types of capital are imperfect substitutes**
 - Aggregator for capital with constant elasticity of substitution (CES)

$$k \equiv \left(\sum_{j \in J} \sigma_j k_j^\gamma \right)^{1/\gamma}$$

- Type- j capital k_j , $j \in J$; factor shares $\sigma_j > 0$, $\forall j \in J$; $\sum_{j \in J} \sigma_j = 1$
- Substitution coeff. γ ; $-\infty < \gamma < 1$; elasticity of subst. $1/(1 - \gamma)$
- First order conditions yield ▶ Firm's problem

$$\frac{u_{nd}(w)}{u_d(w)} = \left(\frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d}$$

- **Composition of investment** determined by
 - ... **(frictionless) user costs** $u_j(w) = u_j$ **for unconstrained firms**
 - ... **down payments** $u_j(w) = \wp_j$ **for severely constrained firms**
- Constrained firms substitute away from durable assets ▶ Composition

Composition of Investment and Legal Enforcement

- **Legal enforcement affects investment composition**
 - Ratio of down payments $\varphi_{nd}(\theta_i)/\varphi_d(\theta_i)$ and hence k_d/k_{nd} increasing in θ_i
 - No effect on unconstrained firms

Hart/Moore (1994): Pledgeability not Durability

- *"We say that the assets become longer lived, or more durable, if [the liquidation value] $L(t)$ rises for all $0 \leq t \leq T$."*
- **Durability \approx liquidation value** (price and use value fixed)
- Interpretation of their liquidation value L in our model

$$L_j k_j \equiv \theta q_j k_j (1 - \delta_j)$$

- Effect of L_j is effect of pledgeability θ in our model not durability

Durability and Financing: Two-Period Assets

■ Technology

- New assets last for two periods (one-horse shay depreciation)
- **New, durable assets** k_d last two periods; price $q_d = 1$ (exogenous)
- **Used, non-durable assets** k_{nd} one period of useful life left; price q_{nd}
- Perfect substitutes in production: $k \equiv k_d + k_{nd}$; output $Af(k)$ where A is TFP; f is (strictly) increasing, concave, $\lim_{k \rightarrow 0} f_k(k) = +\infty$

One-Horse Shay Depreciation



- “The Deacons Masterpiece: or the Wonderful ‘One-Hoss-Shay’ ”
 - Oliver Wendell Holmes (1858)

Durability and Financing: Two-Period Assets

■ Technology

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- **New, durable assets** k_d last two periods; price $q_d = 1$ (exogenous)
- **Used, non-durable assets** k_{nd} one period of useful life left; price q_{nd}
- Perfect substitutes in production: $k \equiv k_d + k_{nd}$; output $Af(k)$ where A is TFP; f is (strictly) increasing, concave, $\lim_{k \rightarrow 0} f_k(k) = +\infty$

- Price of used capital q_{nd} determined in (stationary) **equilibrium**

■ In equilibrium,

- **New assets have lower frictionless user cost:** $u_d \leq u_{nd}$
- **New assets have larger down payments:** $\wp_d > \wp_{nd}(= q_{nd})$

■ Properties

- (Un)constrained firms buy used (new) assets
- Weak legal enforcement economies are net importers of used assets

Entrepreneur's Problem with Two-Period Assets

- Given net worth w , entrepreneur solves

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b, w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w') \quad (5)$$

subject to budget constraints for current and next period

$$w + b \geq d + k_d + q_{nd}k_{nd} \quad (6)$$

$$Af(k) + q_{nd}k_d \geq Rb + w' \quad (7)$$

and the collateral constraint

$$\theta q_{nd}k_d \geq Rb \quad (8)$$

and $k \equiv k_d + k_{nd}$

- Endogenous state variable: net worth w

Equilibrium

- **Market clearing condition in used capital market**

$$\underbrace{(1 - \rho) \sum p(w)k_d(w)}_{\text{supply of used assets}} = \underbrace{\sum p(w)k_{nd}(w)}_{\text{demand for used assets}}$$

where $p(w)$ is stationary net worth distribution

User Cost and Down Payment

■ User cost for unconstrained firm

- New, durable assets

$$u_d \equiv 1 - R^{-1}q_{nd} = R^{-1}(r + (1 - q_{nd}))$$

- Used, non-durable assets

$$u_{nd} \equiv q_{nd} = R^{-1}q_{nd}(r + 1)$$

■ Down payment

- New, durable assets

$$\varphi_d \equiv 1 - R^{-1}\theta q_{nd}$$

- Used, non-durable assets

$$\varphi_{nd} \equiv q_{nd}$$

- Relation to frictionless user cost

$$\varphi_d = u_d + R^{-1}(1 - \theta)q_{nd} > u_d$$

Investment with Collateral Constraints

■ Investment Euler equation

- New, durable assets

$$1 \geq \beta \frac{\mu'}{\mu} \frac{Af_k(k) + (1 - \theta)q_{nd}}{\varphi_d} + \frac{\nu_d}{\mu\varphi_d}$$

- Used, non-durable assets

$$1 \geq \beta \frac{\mu'}{\mu} \frac{Af_k(k)}{\varphi_{nd}} + \frac{\nu_{nd}}{\mu\varphi_{nd}}$$

■ Determinants of investment (unchanged)

- First-order condition for investment with collateral constraints

$$u_j + \frac{\lambda'}{\mu'} \varphi_j \geq \beta Af_k(k)$$

- Key determinants: (frictionless) user cost and down payment

Equilibrium in Used Asset Market

- **In equilibrium** $u_d \leq u_{nd}$
 - Otherwise $\wp_d > u_d > u_{nd} = \wp_{nd}$, so no new investment
- **In equilibrium** $\wp_d > \wp_{nd}$
 - Otherwise $u_{nd} = \wp_{nd} \geq \wp_d > u_d$, so used assets dominated
 - Intuition: if $\wp_d \leq \wp_{nd}$, new assets require less funds up front and give extra payoff next period (“arbitrage”)
- **New assets have lower user cost but higher down payment**
 - $u_d \leq u_{nd}$ and $\wp_d > \wp_{nd}$
 - Clearly new assets more expensive: $1 > q_{nd}$
 - Recall q_{nd} endogenous

Durability and Financing Need

- Financing need of new, durable and used, non-durable assets

Time	t	$t + 1$	$t + 2$
Used, non-durable assets			
Value		q_{nd}	0
Collateral value			0
Borrowing		0	
Internal funds required (\wp_{nd})		q_{nd}	
New, durable assets			
Value	1	q_{nd}	0
Collateral value		θq_{nd}	
Borrowing	$R^{-1}\theta q_{nd}$		
Internal funds required (\wp_d)	$1 - R^{-1}\theta q_{nd}$		

- **New, durable assets require more internal funds $\wp_d > \wp_{nd}$**
 - ... despite ability to borrow $R^{-1}\theta q_{nd} > 0$
 - ... because they cost more $1 > q_{nd} \Rightarrow$ larger financing need

► Example

Equilibrium Price of Used Capital

- In frictionless economy **frictionless user cost** u^* would be

$$1 = u^* + R^{-1}u^* \quad \Rightarrow \quad q_{nd}^* = u^* = \frac{R}{1+R}$$

■ Marginal investor in used capital

- Indifferent between investing in new and used capital: $\nu_d = \nu_{nd} = 0$
- Investment Euler equations imply

$$\underbrace{1 - R^{-1}\theta q_{nd} - q_{nd}}_{\text{Incr. cost of new capital}} = \underbrace{\beta \frac{\mu'}{\mu} (1 - \theta) q_{nd}}_{\text{Valuation of resale value}}$$

Down payment φ_d

- **If constrained firm prices used assets, then** $q_{nd} > q_{nd}^* = \frac{R}{1+R}$

- If $\beta \frac{\mu'}{\mu} = R^{-1}$, $q_{nd} = q_{nd}^*$ and $u_d = u_{nd} = u^*$
- If $\beta \frac{\mu'}{\mu} < R^{-1}$, $q_{nd} > q_{nd}^*$ and $u_d < u_{nd}$

Dividend Paying Firm

- Once firm starts to pay dividends, it always pays dividends and is unconstrained
 - Marginal value of net worth $v_w(w) = \mu = 1$
 - Firm is unconstrained going forward $v_w(w') = \mu' = 1$ and $\lambda' = 0$
- If $q_{nd} > \frac{R}{1+R}$, **unconstrained firm purchases only new assets**

$$\begin{aligned}u_d &= \beta A f_k(k) + \nu_d \\u_{nd} &= \beta A f_k(k) + \nu_{nd}\end{aligned}$$

- Unconstrained firms simply **compare user costs**
 - Unconstrained firms sell assets once they are old/used
-
- Capital stock of unconstrained firm \bar{k}_d solves

$$1 = \beta [A f_k(k_d) + q_{nd}]$$

Investment Euler Equation and Constrained Investment

■ Severely constrained firm purchases only used capital

- As $w \rightarrow 0$ so does $k_d + k_{nd}$ and hence $f_k \rightarrow +\infty$
- Using **Investment Euler equations**

$$1 = \beta \frac{\mu'}{\mu} \frac{A f_k(k) + (1 - \theta) q_{nd}}{\wp_d} + \frac{\nu_d}{\mu \wp_d}$$

$$1 = \beta \frac{\mu'}{\mu} \frac{A f_k(k)}{q_{nd}} + \frac{\nu_{nd}}{\mu q_{nd}}$$

so as $w \rightarrow 0$, $\beta \frac{\mu'}{\mu} \rightarrow 0$ and combining Euler equations $\Rightarrow \nu_d > 0$

$$\underbrace{\wp_d - \wp_{nd}}_{\text{Incr. cost of new capital}} = \underbrace{\beta \frac{\mu'}{\mu} (1 - \theta) q_{nd}}_{\text{Valuation of resale value}} + \frac{\nu_d}{\mu} - \frac{\nu_{nd}}{\mu}$$

- Severely constrained firms simply **compare down payments**

▶ User cost with financial constraints

▶ Imperfect substitutes

Trade

- **World market for used capital** (at price $q_{nd} > q_{nd}^*$)
- Strong vs. weak **legal enforcement** economies: $\theta_H > \theta_L$
- **Used capital may be dominated with strong legal enforcement**
 - In equilibrium, $u_d \leq u_{nd}$ and $\varphi_d(\theta_L) > \varphi_{nd}$
 - But: if $\theta_H \rightarrow 1$, $\varphi_d(\theta_H) \rightarrow u_d < \varphi_{nd}$ (all used capital shipped)
- **Weak legal enforcement economies net used capital importers**
 - Unconstrained firms investment identical: $1 = \beta[Af(\bar{k}_d) + q_{nd}]$
 - Constrained firms indifferent between new and used assets when
$$\frac{Af_k(\underline{k}(\theta_i)) + (1 - \theta)q_{nd}}{\varphi_d(\theta_i)} = \frac{Af_k(\underline{k}(\theta_i))}{q_{nd}}$$
 - Lower θ_i implies indifferent at higher capital of capital $\underline{k}(\theta_i)$ and higher level of net worth
 - More firms use used assets in weak legal enforcement economies

Renting Durable Assets

- **Renting/leasing**

- Benefit: higher leverage due to repossession advantage
- Cost: monitoring cost

- Two types of capital k_j (different δ_j) required (fixed proportions)

- **Severely constrained firms rent both types of assets**

- **Moderately constrained rent only more durable assets**

Literature

- Hart/Moore (1994)
 - See above

- Eisfeldt/Rampini (2007)
 - Used capital is cheaper but requires ex-post maintenance cost
 - Durability plays no role

- Empirical evidence on used capital
 - International trade: Sen (1962), Navaretti/Soloaga/Takacs (2000)
 - Legal enforcement: Benmelech/Bergman (2011)

Conclusion

- **Durable assets are harder to finance**
 - Durability raises financing need and required down payments
- **Financial constraints especially salient for durable assets**
 - Structures (residential and non-residential) and infrastructure
 - Equipment, especially aircraft and ships
 - Consumer durables, especially motor vehicles and household durables
 - Similarly, durable intangible capital ($\theta = 0$), e.g., organization capital
- Constrained firms (and **households**) buy used assets, adopt dominated technologies, and rent durable assets
- Weak **legal enforcement** economies invest in dominated technologies and import used assets
- **Distinguish durability from pledgeability**

Characterization of Entrepreneur's Problem

- Well-behaved dynamic program
 - Return function concave; constraint set convex
 - Operator defined by (1) to (4) satisfies Blackwell's sufficient conditions
 - Solution: $\exists!$ value function v ; strictly increasing; concave

■ First order conditions

- Denote multipliers on (2) and (3) by μ and $\beta\mu'$ and on (4) by $\beta\lambda'$

$$\begin{aligned}\mu &= 1 + \nu \\ \mu q_j &= \beta\mu'[Af_k(k) + q_j(1 - \delta_j)] + \beta\lambda'\theta q_j(1 - \delta_j) + \nu_j, \quad \forall j \in J \\ \mu &= \mu' + \lambda' \\ \mu' &= v_w(w')$$

- **Envelope condition:** $v_w(w) = \mu$
 - Value function continuously differentiable

Characterization of Entrepreneur's Problem

■ First order conditions

- Denote multipliers on (2) and (3) by μ and $\beta\mu'$ and on (4) by $\beta\lambda'$

$$\begin{aligned}\mu &= 1 + \nu \\ \mu q_j &= \beta\mu'[Af_k(k) + q_j(1 - \delta_j)] + \beta\lambda'\theta q_j(1 - \delta_j) + \nu_j, \quad \forall j \in J \\ \mu &= \mu' + \lambda' \\ \mu' &= v_w(w')\end{aligned}$$

■ Envelope condition: $v_w(w) = \mu$

- Value function continuously differentiable

Effect of Durability on Down Payment and User Cost

■ Effect of durability on down payment

- When collateral constraint binds, $b = R^{-1}\theta qk(1 - \delta)$
- Minimal **down payment** per unit of capital

$$\wp = q - R^{-1}\theta q(1 - \delta)$$

- $\frac{\partial \wp}{\partial \delta} = R^{-1}\theta q > 0$; more durable capital has lower down payment?
- But keeping q fixed, lower δ reduces frictionless user cost of capital!

$$u \equiv R^{-1}q(r + \delta)$$

Effect of Durability on Down Payment and User Cost

■ More durable capital requires larger down payment

- Fixing frictionless user cost u^* instead, using $u^* = R^{-1}q^*(\delta)(r + \delta)$ price is

$$q^*(\delta) = \frac{Ru^*}{r + \delta}$$

- Down payment keeping user cost fixed

$$\varphi^*(\delta) = q^*(\delta) - R^{-1}\theta q^*(\delta)(1 - \delta)$$

- $\frac{\partial \varphi^*(\delta)}{\partial \delta} = -q^*(\delta) \frac{1-\theta}{r+\delta} < 0$

- Down payment – decomposition

$$\varphi(\delta) = \underbrace{u^*}_{\text{Jorgensonian user cost}} + \underbrace{R^{-1}q(\delta)(1 - \theta)(1 - \delta)}_{\text{non-pledgeable part of resale value}}$$

■ In general, arguably more durable capital is more expensive

- $q(\delta)$ with $\frac{\partial q(\delta)}{\partial \delta} < 0$
- $\varphi(\delta) = q(\delta) - R^{-1}\theta q(\delta)(1 - \delta)$
- $\frac{\partial \varphi(\delta)}{\partial \delta} = \frac{\partial q(\delta)}{\partial \delta} (1 - R^{-1}\theta(1 - \delta)) + qR^{-1}\theta \leq 0$ (ambiguous)

▶ User cost and sensitivity to financial constraints

Effect of Durability on Down Payment and User Cost

- **User cost depends on firm's discount factor**

$$\begin{aligned}u(w)(\delta) &= \wp(\delta) - \beta \frac{\mu'}{\mu} q(\delta)(1 - \theta)(1 - \delta) \\ &= u^* + \beta \frac{\lambda'}{\mu} \frac{Ru^*}{r + \delta} (1 - \theta)(1 - \delta)\end{aligned}$$

- **Durable capital user cost sensitive to financial constraints**

- $\frac{\partial u(w)(\delta)}{\partial \delta} = -q(\delta) \frac{\lambda'}{\mu} \frac{1 - \theta}{r + \delta} < 0$
- $u(w)(1) = u^*$ independent of w (also if $\theta = 1$ but we assume $\theta < 1$)

Continuum of Capital Types with Varying Durability

- **Continuum of technologies with durability $1 - \delta$**

- Production cost (and price) for type δ : $\phi(\delta)$ with $\phi_\delta(\delta) < 0 < \phi_{\delta\delta}(\delta)$

- **Unconstrained firms minimize frictionless user cost $u(\delta)$**

$$\delta^* \in \arg \max_{\delta \in [0,1]} R^{-1} \phi(\delta)(r + \delta)$$

- In frictionless economy (or if $\theta = 1$), only durability $1 - \delta^*$ produced

- **Constrained firms purchase less durable types of capital**

- Possibly continuum of less durable types of capital produced

Model with Vintage Capital and Geometric Depreciation

■ Vintage capital with geometric depreciation

- New capital: cost q_d ; depreciation δ_d ; **fraction η becomes used**
- Used capital: price q_{nd} ; depreciation $\delta_{nd} > \delta_d$
- Previous vintage model: $\delta_d = 0$, $\delta_{nc} = 1$, $\eta = 1$, $q_d = 1$, $q_{nd} = q$

■ Endogenous price of used capital q_{nd}

- If marginal investor is unconstrained, $u_d = u_{nd}$; frictionless price

$$q_{nd}^* = \frac{r + \delta_d + \eta(1 - \delta_d)}{r + \delta_{nd} + \eta(1 - \delta_d)} q_d < q_d$$

- If marginal investor is constrained, $q_{nd} > q_{nd}^*$

■ Down payments $\wp_d > \wp_{nd}$

- $\wp_d \equiv q_d - R^{-1}\theta q_d(1 - \delta_d) + R^{-1}\theta\eta(q_d - q_{nd})(1 - \delta_d)$
- $\wp_{nd} \equiv q_{nd} - R^{-1}\theta q_{nd}(1 - \delta_{nd})$

Composition Determined by User Cost or Down Payments

- **Firm's problem with two types of capital:** Given w , solve

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b, w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d + \sum_{j \in J} q_j k_j \\ Af\left(\left(\sum_{j \in J} \sigma_j k_j^\gamma\right)^{1/\gamma}\right) + \sum_{j \in J} q_j k_j (1 - \delta_j) &\geq Rb + w' \\ \theta \sum_{j \in J} q_j k_j (1 - \delta_j) &\geq Rb \end{aligned}$$

- First order condition for type j capital

$$u_j(w) = \beta \frac{\mu'}{\mu} Af_k(k) \frac{\partial k}{\partial k_j}$$

Distortion of Financially Constrained Investment

- Investment distorted away from durable assets if

$$\frac{u_{nd}}{u_d} > \frac{\wp_{nd}}{\wp_d} \Leftrightarrow \frac{\wp_d}{u_d} > \frac{\wp_{nd}}{u_{nd}}$$

- Down payment to user cost ratio higher for durable assets

$$\frac{\wp_j}{u_j} = \frac{R^{-1}q_j(r + \delta_j) + R^{-1}q_j(1 - \theta)(1 - \delta_j)}{R^{-1}q_j(r + \delta_j)} = 1 + (1 - \theta)\frac{1 - \delta_j}{r + \delta_j}$$

- Decreasing in δ_j : $\frac{\partial}{\partial \delta_j} \left(\frac{\wp_j}{u_j} \right) < 0$
- Why? – **Residual value is larger fraction of value**

$$\frac{\wp_j - u_j}{\wp_j} = \frac{1}{1 + \left((1 - \theta) \frac{1 - \delta_j}{r + \delta_j} \right)^{-1}}$$

Characterization of Entrepreneur's Problem

- Well-behaved dynamic program
 - Return function concave; constraint set convex
 - Operator defined by (5) to (8) satisfies Blackwell's sufficient conditions
 - Solution: $\exists!$ value function v ; strictly increasing; concave

■ First order conditions

- Denote multipliers on (6) and (7) by μ and $\beta\mu'$ and on (8) by $\beta\lambda'$

$$\mu = 1 + \nu$$

$$\mu = \beta\mu'[Af_k(k) + q_{nd}] + \beta\lambda'\theta q_{nd} + \nu_d$$

$$\mu q_{nd} = \beta\mu' Af_k(k) + \nu_{nd}$$

$$\mu = \mu' + \lambda'$$

$$\mu' = v_w(w')$$

- **Envelope condition (marginal value of net worth):** $v_w(w) = \mu$
 - Value function continuously differentiable

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Durability and Financing Need

- Financing need of new, durable and used, non-durable assets

Time	t	$t + 1$	$t + 2$
Used, non-durable assets			
Value		q_{nd}	0
Collateral value			0
Borrowing		0	
Internal funds required (φ_{nd})		q_{nd}	
New, durable assets			
Value	1	q_{nd}	0
Collateral value		θq_{nd}	
Borrowing		$R^{-1}\theta q_{nd}$	
Internal funds required (φ_d)	$1 - R^{-1}\theta q_{nd}$		

- **New, durable assets require more internal funds** $\varphi_d > \varphi_{nd}$
 - ... despite ability to borrow $R^{-1}\theta q_{nd} > 0$
 - ... because they cost more $1 > q_{nd} \Rightarrow$ larger financing need
- Example: If $R \approx 1$, $\theta = 0$, and $q_{nd} = 0.5$, then $\varphi_d = 1$

Durability and Depreciation Rate

- In first period, new assets depreciate at **depreciation rate**

$$\delta_d \equiv \frac{1 - q_{nd}}{1} \leq 1 - \frac{R}{1 + R} = \frac{1}{1 + R} < 50\%$$

- ... and in second period (when used) at rate

$$\delta_{nd} \equiv \frac{q_{nd} - 0}{q_{nd}} = 100\%$$

User Cost of New Capital for Financially Constrained Firm

- User cost of durable assets depends on firm's discount factor

$$\begin{aligned}u_d(w) &= \wp_d - \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd} \\ &= u_d + \beta \frac{\lambda'}{\mu} (1 - \theta) q_{nd}\end{aligned}$$

- User cost of new, durable assets for severely constrained firm

- As $w \rightarrow 0$, $\beta \frac{\mu'}{\mu} \rightarrow 0$ so

$$u_d(w) \rightarrow \wp_d$$

- Severely constrained firms simply compare down payments

Composition of Investment with Two Period Assets

- Suppose two period assets are **imperfect substitutes**
- First order conditions yield (as in case with geometric depreciation)

$$\frac{u_{nd}}{u_d(w)} = \left(\frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d}$$

- Composition of investment
 - $\varphi_{nd}/\varphi_d (u_{nd}/u_d)$ determines composition for (un)constrained firms
 - For used, non-durable assets: $\frac{\varphi_{nd}}{u_{nd}} = \frac{q_{nd}}{q_{nd}} = 1$
 - For new, durable assets: $\frac{\varphi_d}{u_d} = \frac{1-R^{-1}\theta q_{nd}}{1-R^{-1}q_{nd}} > 1$
 - Constrained firms substitute away from new capital
 - Intuition: $\frac{\varphi_d - u_d}{\varphi_d} = \frac{R^{-1}(1-\theta)q_{nd}}{1-R^{-1}\theta q_{nd}} > 0 = \frac{\varphi_{nd} - u_{nd}}{\varphi_{nd}}$
- Especially with weak legal enforcement

Renting Durable Assets

■ Capital goods with different durability

- Capital k_j , $j \in J$, with depreciation rate $\delta_d < \delta_{nd}$ and price q_j
- Leontief aggregator $k = \min\left\{\frac{k_d+k_d^l}{\sigma_d}, \frac{k_{nd}+k_{nd}^l}{\sigma_{nd}}\right\}$
 - Implies $k_j + k_j^l = \sigma_j k$ where k is aggregate capital
- Rental price $u_j^l \equiv R^{-1}q_j(r + \delta_j + m)$ where m is monitoring cost
- Firm cannot abscond with rented capital (repossession advantage)

■ Entrepreneur's problem with rentals: Given w , solve

$$v(w) \equiv \max_{\{d, \{k_j, k_j^l\}_{j \in J}, b, w'\} \in \mathbb{R}_+^5 \times \mathbb{R}^2} d + \beta v(w') \quad (9)$$

subject to budget, collateral, and technological constraints

$$w + b \geq d + \sum_{j \in J} q_j k_j + \sum_{j \in J} u_j^l k_j^l \quad (10)$$

$$Af(k) + \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb + w' \quad (11)$$

$$\theta \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb \quad (12)$$

$$k_j + k_j^l \geq \sigma_j k \quad (13)$$

Renting Durable Assets (Cont'd)

- First order conditions with multipliers as before and $\beta\mu'\eta_j$ on (13)

$$\begin{aligned}\sum_{j \in J} \sigma_j \eta_j &= Af_k(k) \\ \mu q_j &= \beta\mu'\eta_j + \beta\mu'q_j(1 - \delta_j) + \beta\lambda'\theta q_j(1 - \delta_j) + \nu_j \\ \mu u_j^l &= \beta\mu'\eta_j + \nu_j^l\end{aligned}$$

- Investment Euler equation (IEE)

$$1 = \beta \frac{\mu' \eta_j + (1 - \theta)q_j(1 - \delta_j)}{\mu \wp_j} + \frac{\nu_j}{\mu \wp_j}$$

- **More constrained firms rent less durable assets**

- Suppose rent and own some type j capital, so $\nu_j = \nu_j^l = 0$ and IEE

$$\frac{\lambda'}{\mu}(1 - \theta)(1 - \delta_j) = m$$

- If δ_j higher, so multiplier on collateral constraint $\frac{\lambda'}{\mu}$ higher

Literature: Durable Goods Theory

■ Optimal durability and planned obsolescence

- Conventional wisdom: monopolists produce less durable goods
- Fallacy: Swan (1970, 1972), Sieper/Swan (1973): cost minimization
- Resuscitated: Barro (1972): impatience; Rust (1986): scrappage
- Bulow (1982, 1986): escape Coase (1972) time-inconsistency

■ Role of rental markets

- Coase (1972), Bulow (1982), Stokey (1981): Coasian dynamics
- Hendel/Lizzeri (2002): leasing ameliorates adverse selection

■ Key driving force: market power

- Our theory: optimal competitive response to **financial constraints**

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- Our theory: optimal competitive response to **financial constraints**

■ Other features

- Adverse selection: Akerlof (1970)
- Illiquidity