Financing Insurance

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Financing Insurance – Synopsis

- **Intertemporal aspect to insurance**
  - Premia paid up front; insurance is state-contingent savings
  - Smoothing over time and insurance across states linked
  - Obstacle as important as moral hazard and adverse selection?

(1) **Insurance demand:** in stationary equilibrium, insurance is ...
  - incomplete with probability 1; absent with positive probability
  - globally increasing in net worth and income
  - precautionary (increases when income gets riskier)

(2) **Insurance supply:** theory of insurance firms
  - Durable goods as collateral
  - Insurers’ balance sheets
    - Assets: collateralized loans
    - Liabilities: diversified portfolio of insurance claims
    - Consequence of intertemporal aspect to insurance
  - Collateral scarcity lowers equilibrium interest rate
    - ... reducing insurance and increasing inequality

Research agenda
Stylized Facts: Insurance & Net Worth – Extensive Margin

- Insurance by U.S. households
  - Cross section – Brown/Finkelstein (2007)
    - Health/long-term care coverage increase in income, wealth, age
    - Limited and heterogeneous insurance of health and mortality risk
  - **Within-household variation** – Fang/Kung (2021)
    - “[I]ndividuals who experience negative income shocks are more likely to lapse all [life insurance] coverage.”

- Consumption data – Blundell/Pistaferri/Preston (2008)
  - “full insurance of transitory shocks except among poor households”

- Insurance by farmers in developing economies
  - Consumption data – Townsend (1994)
    - “[some] evidence that the landless are less well insured”
  - Rainfall insurance – Giné et al. (2008), Cole et al. (2013)
    - Participation increases in wealth; decreases in borrowing constraints
    - Most common reason for lack: **insufficient funds to buy insurance**
  - Crop insurance – Casaburi/Willis (2018)
    - Take-up for pay-at-harvest (not up-front) insurance much higher
Stylized Facts: Insurance & Net Worth – Intensive Margin

- Recent – puzzling? – evidence on intensive margin of insurance
  - Armantier/Foncel/Treich (2020)
    - Car insurance coverage increases in measures of wealth and liquidity
  - Gropper/Kuhnen (2021)
    - Wealthier households have better term life, homeowners coverage
    - Wealthier households pay more per unit of coverage
  - Cohen/Einav (2007) – model of car insurance deductible choice
    - Low car insurance deductible positively related to wealth
    - “wealthier people have higher levels of absolute risk aversion”
    - Plausible? – Recall Arrow (1965)

- Counter to canonical theory (insurance is trade across states)
  - Mossin (1968) – static model
    - Coverage decreases and deductible increases with wealth
    - “only liquidity constrained households … purchase a generous … coverage”
    - “Wealthier people would mostly rely on … ability to time diversify”
(New) Stylized Fact: Insurance over the Business Cycle

- Lack of car and health insurance countercyclical
  - $\text{Corr}(\text{GDP pc}, \% \text{ uninsured motorists}): -0.21$
  - $\text{Corr}(\text{GDP pc}, \% \text{ without health insurance}): -0.58/-0.56/-0.12$

Panel A: Fraction of Uninsured Motorists

Panel B: Fraction of Population without Health Insurance

Adriano A. Rampini and S. Viswanathan
Financing Insurance
## Stylized Facts on Insurance Companies Balance Sheets

### Balance sheet of insurance companies

<table>
<thead>
<tr>
<th></th>
<th>Life</th>
<th>P&amp;C</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash &amp; invested assets</td>
<td>4,582</td>
<td>2,148</td>
<td>6,990</td>
</tr>
<tr>
<td>(% of industry overall)</td>
<td>66%</td>
<td>31%</td>
<td>100%</td>
</tr>
<tr>
<td>Bonds, mtgs., &amp; cash</td>
<td>3,916</td>
<td>1,239</td>
<td>5,347</td>
</tr>
<tr>
<td>(% of invested assets)</td>
<td>85%</td>
<td>58%</td>
<td>76%</td>
</tr>
<tr>
<td>Corp. bonds, MBS, &amp; ABS</td>
<td>2,764</td>
<td>670</td>
<td>3,537</td>
</tr>
<tr>
<td>(% of bonds)</td>
<td>86%</td>
<td>61%</td>
<td>79%</td>
</tr>
</tbody>
</table>

Data: NAIC – Total U.S. Insurance Industry Cash and Invested Assets 2019; billion

### Bulk of assets are collateralized claims

- 76% of invested assets are bonds, mortgages, and cash
- Collateralized explicitly or implicitly (MBS/mtgs./ABS; corp. bonds)
(1) Dynamic Model of Insurance

- Discrete time, infinite horizon

- **Households**
  - Preferences: \( E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \) where \( \beta \in (0, 1) \), \( u(c) \) strictly increasing, strictly concave, continuously differentiable, \( \lim_{c \to 0} u_c(c) = \infty \), \( \lim_{c \to \infty} u_c(c) = 0 \)
  - Income \( y(s) \) Markov chain on state space \( s \in S \) with transition matrix \( \Pi(s, s') > 0 \) and \( \forall s, s_+, s_+ > s, y(s_+) > y(s) \)
  - Income net of non-discretionary spending for health, accidents, ...
  - Notation: \( y' \equiv y(s') \), \( \underline{s} = \min\{s : s \in S\} \), \( \bar{s} = \max\{s : s \in S\} \), etc.

- **Key friction**: limited enforcement without exclusion
  - Enforcement of repayment by borrower limited to tangible assets
  - Implication: collateral constraints
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable! (See also R/V (2010, 2013))

- **No collateral** (for now) so short-sale constraints

- **Partial equilibrium** (exogenous \( R \) for now): risk neutral lenders
  - Discount at \( R^{-1} \in (\beta, 1) \), deep pockets, abundant collateral
Household’s Dynamic Insurance Problem

- Recursive formulation

- Given $s$ and $w$, household solves

$$v(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^S} u(c) + \beta E[v(w', s') | s]$$  \hspace{1cm} (1)

subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + E[R^{-1}h'|s]$$  \hspace{1cm} (2)

$$y' + h' \geq w'$$  \hspace{1cm} (3)

and short sale constraints, $\forall s' \in S$

$$h' \geq 0$$  \hspace{1cm} (4)

- Arrow securities $h'$ for each state $s'$ (and associated net worth $w'$)

- Endogenous state variable: net worth $w$ (cum current income)
Characterization of Dynamic Insurance Problem

- Well-behaved dynamic program
  - Return function concave; constraint set convex
  - Operator defined by (1) to (4) satisfies Blackwell’s sufficient conditions
  - Solution: $\exists$! value function $v$; strictly increasing; strictly concave

- First-order conditions
  - Denote multipliers on (2) and (3) by $\mu$ and $\beta\Pi(s, s')\mu'$ and on (4) by $\beta\Pi(s, s')\lambda'$
  - Ignore non-negativity constraints on consumption (not binding)

$$
\mu = u_c(c) \\
\mu' = v_w(w', s') \\
\mu = \beta R \mu' + \beta R \lambda'
$$

- Envelope condition: $v_w(w, s) = \mu$ (marginal value of net worth)

- Value function continuously differentiable
Stochastically monotone Markov chain

Consider Markov chains which exhibit a notion of positive persistence

**Definition 1 (Monotone Markov chain).** A Markov chain \( \Pi(s, s') \) is stochastically monotone, if it displays first order stochastic dominance (FOSD) if \( \forall s, s_+, \hat{s}', s_+ > s \),
\[
\sum_{s' \leq \hat{s}'} \Pi(s_+, s') \leq \sum_{s' \leq \hat{s}'} \Pi(s, s')
\]

Remarks:

- Distribution of states next period conditional on current state \( s_+ \)
- FOSD distribution conditional on current state \( s \) for all \( s_+ > s \)
- IID is special case, that is, stochastically monotone
Insurance is Increasing (Prop. 2)

- Assume that $\Pi(s, s')$ is stochastically monotone.

- Key property
  - (i) Marginal value of net worth $v_w(w, s)$ is decreasing in state $s$.

- Insurance is globally increasing
  - (ii) Household insure lower interval of states $S_h = \{s', \ldots, s'_h\}$ given $w$ and $s$; net worth next period $w'$, insurance $h'$, set of insured states $S_h$, insured consumption $c_h$ monotone increasing in $w$ and $s$.

- Intuition
  - Higher current income means FOSD shift in income next period $\Rightarrow$ lower marginal value of current net worth.
  - If property satisfied, households insure lower income realizations more.

Sketch of proof, Example, Figure, Without stochastic monotonicity, Comparison to Bewley model.
Assume income process \textbf{independent}: \( \Pi(s, s') = \pi(s'), \forall s, s' \in S \)

Richer households insure more states/higher net worth
- Net worth in insured states \( w(s') = w_h, \forall s' \in S_h \), i.e., constant
- \( w_h \) is increasing in \( w \)

Richer households lower variance of net worth/consumption
- Variance of net worth \( w' \) and consumption \( c' \) next period is decreasing in current net worth \( w \)
Insurance is Incomplete (Prop. 3)

- Assume income process is stochastically monotone

- “Poor households cannot afford insurance”
  - (i) At net worth \( w = y \) in state \( s \), household does not insure at all, that is, \( \lambda' > 0 \), \( \forall s' \in \overline{S} \), and \( S_h = \emptyset \)

- High income households are not completely insured
  - (ii) At net worth \( w = \bar{y} \), household does not insure highest state next period, that is, \( \lambda(\bar{s}') > 0 \) and \( S_h \subsetneq S \), \( \forall s \in S \)
Insurance under Stationary Distribution (Prop. 4)

- Assume that $\Pi(s, s')$ is stochastically monotone

- Existence and uniqueness
  - (i) There exists a unique stationary distribution of net worth

- Support of net worth distribution
  - (ii) Support of stationary distribution is subset of $[\underline{w}, \overline{w}_{bnd}]$ where $\underline{w} = \underline{y}$ and $\overline{w}_{bnd} \geq \overline{y}$ with equality if $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$

- Insurance increasing, incomplete, absent with positive prob.
  - (iii) Under stationary distribution, insurance is increasing, incomplete with probability 1, and completely absent with strictly positive prob.

- Empirical prediction: within-household variation in insurance
  - Limited evidence to date
  - Prediction on intensive margin (coverage and deductibles)
A Theory of the Insurance Function?

- Behavior of insurance similar to savings
  - **Insurance is “state-contingent saving”**
  - Buffer stock savings with state-contingent buffer stocks

- Friedman (1957), *A Theory of the Consumption Function* (page 39)
  - *“These regressions show savings to be negative at low measured income levels, and to be a successively larger fraction of income, the higher the measured income. If low measured income is identified with ‘poor’ and high measured income with ‘rich,’ it follows that the ‘poor’ are getting poorer and the ‘rich’ are getting richer. The identification of low measured income with ‘poor’ and high measured income with ‘rich’ is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof.”*
Precautionary Nature of Insurance (Prop. 5)

- **Definition:** Behavior is precautionary if it increases when risk increases (MPS on $y'$).

- Assume income process independent: $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$.

- **Insurance is precautionary**
  - $y(s')$ given $\pi(s')$ mean-preserving spread (MPS) of $y(s')$ given $\pi(s')$.
  - Then $\tilde{E}[\tilde{h}'] \geq E[h']$.

- Remarkably: **Risk aversion sufficient**
  - No assumptions on prudence ($u_{ccc}(c)$) required.
  - Contrast: Classic precautionary savings result in models with incomplete markets (Bewley (1977), Aiyagari (1994), Leland (1968)).


Precautionary savings in Bewley model

Example
Insurance with Durable Goods as Collateral

- Environment: two goods – consumption and durables
- Preferences: \( u(c) + g(k) \) where \( k \) is durable good (e.g., housing)
- Endowment: income process as before

Durable goods

- ... depreciate at rate \( \delta \)
- ... as collateral for state-contingent debt \( b' \)

\[
\theta k(1 - \delta) \geq Rb'
\]

where \( \theta \in [0,1) \) is collateralizability
- ... imply additional financing needs

Equivalent insurance formulation

- Fully lever durables: set \( \hat{b}' = R^{-1} \theta k(1 - \delta) \) and pay down

\[
\varphi \equiv 1 - R^{-1} \theta (1 - \delta)
\]

- Insure with Arrow securities \( h' \) subject to short sale constraints

\[
h' \equiv \theta k(1 - \delta) - Rb'
\]
Household’s Problem with Durable Goods

- Given \( s \) and \( w \), household solves

\[
v(w, s) \equiv \max_{c, k, h', w' \in \mathbb{R}_+^2 \times \mathbb{R}^2_s} u(c) + g(k) + \beta E[v(w', s') | s]
\]

subject to budget constraints for current and next period, \( \forall s' \in S \)

\[
w \geq c + \varphi k + E[R^{-1} h' | s]
\]

\[
y' + (1 - \theta)k(1 - \delta) + h' \geq w'
\]

and short sale constraints, \( \forall s' \in S \)

\[
h' \geq 0
\]

- Remarks
  - Net worth \( w \) is cum income and durable goods net of borrowing
  - Investment Euler equation for durable goods

\[
1 = \frac{g(k)}{\mu} \frac{1}{\varphi} + E \left[ \frac{\beta \mu'}{\mu} \frac{(1 - \theta)(1 - \delta)}{\varphi} \middle| s \right]
\]
Insurance with Durable Goods: Properties (Prop. 6)

- **Properties generalize**

- **Monotonicity**
  - Net worth next period $w'$ strictly increases in $w$, given $s$
  - Insurance $h'$ does not necessarily increase in $w$

- **Incomplete insurance** (with $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$)
  - Household never insures the highest state next period

- **Increasing insurance with stochastic monotonicity**
  - Household insures lower interval of states
    - Marginal value of net worth $v_w(w, s)$ is decreasing in $s$

- **Poor households do not participate in insurance markets**
  - For sufficiently low net worth, household is constrained against all states next period: Net worth next period
    \[ w' \geq y' + (1 - \theta)k(1 - \delta) > y' \]
Determine $R$ in equilibrium to clear market for collateralized claims

**Theory of insurance firms**
- Asset-and-liability structure reflects intertemporal nature of insurance

**Collateral scarcity**
- In equilibrium, $\beta R \leq 1$, with equality iff $\theta \geq \bar{\theta}$
- When $\theta < \bar{\theta}$, collateral is scarce and $\beta R < 1$ as previously assumed

**IID shocks (for simplicity)**
Stationary Equilibrium

Definition (Stationary Equilibrium) A stationary equilibrium is

- allocation \( x(w) \equiv \{c(w), k(w), h'(w), w'(w)\} \) for each household
- interest rate \( R \)
- stationary distribution \( F(w) \)

such that

- \( x(w) \) solves each household’s problem in (5)-(7) and (4), given \( w \)
- market for state-contingent promises clears

\[
\int_w E[b'(w)]dF(w) = 0 \quad (12)
\]

or equivalently, supply of collateralized claims equals demand for state-contingent claims \( h'(w) \)

\[
\int_w \theta k(w)(1 - \delta)dF(w) = \int_w E[h'(w)]dF(w). \quad (13)
\]
Theory of Insurance Firms

- Interpretation of market clearing condition
  - Representative insurance company

- **Insurers’ balance sheet**

\[ \int_{w} \theta k(w)(1 - \delta) dF(w) = \int_{w} E[h'(w)] dF(w) \]

- Assets: mortgages
- Liabilities: diversified insurance claims

- Assets: collateralized loans ("mortgages")
- Liabilities: diversified portfolio of insurance claims

- Insurers as diversified financial intermediaries
  - Diversification key as in Diamond (1984) and Boyd/Prescott (1986)

- **Why do insurers/insurance sector have positive assets?**
  - Consequence of intertemporal nature of insurance
  - Insurance premia are paid up-front
Aggregation and Resource Constraints

- Define $W \equiv \int w \, w dF(w)$, $C \equiv \int w \, c(w) dF(w)$, ...

- Aggregate budget constraints (6) and (7) $\forall s' \in S$

  $$W = C + \varphi K + E[R^{-1}H'] \quad (14)$$

  $$E[y'] + (1 - \theta)K(1 - \delta) + E[H'] = W \quad (15)$$

- Using market clearing $\theta K(1 - \delta) = E[H']$, (14) and (15) imply

  $$W = C + K = E[y'] + K(1 - \delta) \quad (16)$$

  so aggregate wealth $=$ consumption $+$ capital stock or

  $$E[y'] = C + \delta K$$

  so aggregate income $=$ consumption $+$ investment
Full Insurance in Economy with Abundant Collateral

- Suppose $\beta R = 1$; then

$$v_w(w) = v_w(w') + \lambda' \geq v_w(w'), \quad s' \in S$$

so $v_w(w) = v_w(w') = v_w(w^*)$; full insurance

- Let $R^* = \beta^{-1}$ and $r^* = R^* - 1$; consumption $c^*$, net worth $w^*$, and durable goods $k^*$ constant and investment Euler equation

$$R^*^{-1}(r^* + \delta) = \frac{g_k(k^*)}{u_c(c^*)}$$  \hspace{1cm} (17)

- Using $\phi^* = 1 - R^*^{-1}\theta(1 - \delta)$, write (6) and (7) as

$$w^* = c^* + \phi^*k^* + E[R^*^{-1}h'^*], \quad (18)$$

$$y' + (1 - \theta)k^*(1 - \delta) + h'^* = w^*, \quad \forall s' \in S. \quad (19)$$
Feasible? – As long as $h'^* \geq 0$, $\forall s' \in S$

Using (16) to substitute for $w^*$ in (19)

$$h'^* = \theta k^*(1 - \delta) - (y' - E[y']) \geq 0, \quad \forall s' \in S$$

that is, sufficient collateralizability

$$\theta \geq \bar{\theta} \equiv \frac{y(s') - E[y']}{k^*(1 - \delta)} \geq 0$$
Collateral scarcity: $\theta < \bar{\theta}$

Then $\beta R < 1$ as previously assumed

- At $\beta R = 1$ excess demand for collateralized claims

Incomplete insurance and previous characterization applies

$R < 1$ is possible, that is, negative interest rates $r = R - 1$
Effect of Collateral Scarcity on Interest Rate and Insurance

- Effect on interest rate, insurance, durable stock, and welfare loss

- **Collateral scarcity** lowers interest rate, reducing insurance
Effect of Collateral Scarcity on Inequality

When collateral is scarce, wealth distribution fans out

![Graph showing the effect of collateral scarcity on inequality]

- **Collateral scarcity increases inequality**
Insurance with Durable Goods as Collateral (Version 2.0)

- Economy with two goods: consumption and durables
  \[ u(c) + \kappa g(k) \]

- Durables are fully pledgeable: \( \theta = 1 \) (Version 2.0)

- Indirect utility function \( \hat{u} \) inherits properties of \( u \) and \( g \)
  \[ \hat{u}(\hat{c}) = \max_{c,k} u(c) + \kappa g(k) \]
  subject to
  \[ \hat{c} \geq c + \varphi k \]

- Key Insight 1: Prop. 1-5 apply without change – insurance is...
  - monotone increasing, incomplete, precautionary, absent w. pos. prob.

- Key Insight 2 (conjecture): With isoelastic utility \( u(\cdot) = g(\cdot) \)
  - \( \exists ! \) equilibrium; interest rate \( R \) is monotone in \( \kappa \)
  - For \( \kappa \) suff. high, \( R = \beta^{-1} \); as \( \kappa \to 0 \), \( R \to 1 - \delta \) but \( K >> 0 \)
  - As \( \kappa \) increases, FOSD shift in wealth and consumption distribution
Conclusion – Focus: Intertemporal Nature of Insurance

- Financing aspect of insurance crucial

- Variation in insurance demand with net worth
  - Empirical predictions at intensive margin within households
  - Consistent with evidence in contrast to received theory

- Asset-and-liability structure of insurance firms
  - Theory of insurers as diversified financial intermediaries

- Friction as important as moral hazard and adverse selection?
Research Agenda on Collateralized Finance

- Bulk of (external/debt) **financing collateralized in practice**
  - Households: mortgages, car loans, etc.
  - Firms: tangible assets primary determinant of leverage
  - Intermediaries: real estate finance (commercial and residential)

- Tractable dynamic model of collateralized financing

- Key friction: **limited enforcement**
  - Enforcement of repayment by borrower limited to tangible assets
  - Novel assumption: **no exclusion**
  - Implication: **collateral constraints**
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable!
Dynamic Collateralized Finance – Implications

(1) **Risk management** – Rampini/Viswanathan (2010, 2013)
- Involves state contingent promises and needs collateral
- Opportunity cost: forgone investment
- Severely constrained firms do not hedge

(2) **Capital structure and leasing/rental markets** – R/V (2013)
- Determinant: fraction tangible assets required for production
- Leasing has repossession advantage and permits greater borrowing
- Severely constrained firms lease

(3) **Durability** – Rampini (2019)
- Durable assets have larger financing needs and are harder to finance

(4) **Financial intermediation** – Rampini/Viswanathan (2019)
- Intermediaries with collateralization advantage require capital
- Rich implications for economic dynamics
Insurance is Increasing (Prop. 1)

- **Richer households insure more states**
  - *(i)* Set of states that household insures \( S_h \equiv \{ s' \in S : h(s') > 0 \} \) is increasing in net worth \( w \) given current state \( s \), \( \forall s \in S \)

- **Richer households better insured/spend more on insurance**
  - *(ii)* For \( w_+ > w \) and denoting net worth next period associated with \( w_+ (w) \) by \( w'_+ (w') \), we have
    - \( w'_+ \geq w' \) and \( c'_+ \geq c' \), \( \forall s' \in S \), i.e., \( w'_+ \) and \( c'_+ \) statewise dominate and hence FOSD \( w' \) and \( c' \), respectively; moreover, \( h'_+ \geq h' \), \( \forall s' \in S \), and \( E[h'_+|s] \geq E[h'|s] \)
    - consumption across insured states constant, i.e., \( c' = c_h \), \( \forall s' \in S_h \), and \( c_h \) is strictly increasing in \( w \)

- **Remarks:**
  - This does not say which states are insured
  - All statements are conditional on state \( s \)
Proof of key property - Prop. 2

Define operator $T$ as

$$Tv(w, s) \equiv \max_{c,h',w' \in \mathbb{R}^+ \times \mathbb{R}^2} u(c) + \beta E[v(w', s')|s]$$

subject to equations (2) through (4)

Sketch: Show that if $v$ has property that $\forall s, s_+, s_+ > s, v_w(w, s_+) \leq v_w(w, s)$, then $Tv$ (and fixed point) inherit property
Basic Financing Insurance Trade-off

- Poor shift net worth to present not across states next period

\[ \Pi(s, s') \]

\[ v_w(w, s) \]

\[ v_w(w(s'), s') \]

\[ v_w(w(s'), s') \neq v_w(w(s'), s') \]

Current consumption

No insurance
**Optimal Insurance**

- Parameters: \( y' \in \{0.8, 1.2\}, \ p = 0.5, \) CRRA with \( \gamma = 2 \)
(Novel) Global Monotonicity – Why?

■ **Household finance**
  ■ Positive persistence of income further lowers marginal value of net worth when current income realization is high
  ■ Yields *globally increasing insurance* theorem

■ **Corporate finance**
  ■ Positive persistence in cash flows due to productivity shocks
    ■ Positive persistence productivity shocks implies conditional expected productivity higher
    ■ **Investment opportunities** raise marginal value of net worth when current productivity is high
  ■ Effects go in opposite direction; no global monotonicity result
Comparison to Bewley (1977) Economy

- **Precautionary savings** $h$ instead of insurance $h'$, $\forall s' \in S$

- Given $s$ and $w$, household solves

$$V(w, s) \equiv \max_{c, h, w' \in \mathbb{R}_+ \times \mathbb{R}^{S+1}} u(c) + \beta E[V(w', s')|s]$$  \hspace{1cm} (20)

subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + R^{-1} h$$  \hspace{1cm} (21)

$$y' + h \geq w'$$  \hspace{1cm} (22)

and short sale constraint

$$h \geq 0$$  \hspace{1cm} (23)

- Same key property: $v_w(w, s)$ decreasing in $w$ and $s$ (same proof strategy)

- With $FOSD$, $h$ (weakly) increasing in $w$ given $s$, but (weakly) decreasing in $s$ given $w$

- “Insurance” in Bewley model not increasing in $s$ (Prop. D.1)
Comparison to Bewley (1977) Economy (Cont’d)

- **Precautionary savings vs. insurance**

- **Parallels**
  - Since \( v_w(w, s) \) decreasing in \( w \) and \( s \), envelope condition implies **consumption** \( c \) **increasing in** \( w \) **and** \( s \)
  - Thus precautionary savings \( h \) and insurance expenditure \( E[R^{-1}h'|s] \) both increasing in \( w \) (given \( s \)) and **decreasing in** \( s \) (given \( w \))

- **Key distinction**
  - Insurance \( h' \) **increasing in** \( s \) (although insurance expenditure \( E[R^{-1}h'|s] \) **decreasing in** \( s \))
    - Insurance is increasing in \( s \) \((w' \text{ increasing in } s)\)
  - Precautionary savings \( h \) decreasing in \( s \) implies \( w(s') \) decreasing in \( s \)
    - “Insurance” in Bewley model not increasing in \( s \)
Precautionary Savings in Bewley Model (Prop. D.2)

- **Key condition:** Convexity of marginal utility \( u_c(c) \)

- Assume income process independent: \( \Pi(s, s') = \pi(s'), \forall s, s' \in S \)

- **Marginal value of net worth**
  - If \( u_c(c) \) is convex, then \( v_w(w) \) is convex

- **Precautionary savings guaranteed only if** \( u_c(c) \) **convex**
  - Suppose \( \tilde{\pi}(s') \) MPS of \( \pi(s') \)
  - If \( u_c(c) \) convex, then \( \tilde{h} \geq h \)

- New proof of classic result
  - Leland (1968), Sandmo (1970), Sibley (1975), and Kimball (1990)

- Contrast: Precautionary insurance requires only risk aversion
Risk and Precautionary Behavior of Insurance and Saving

Parameters: $y(s') \in \{0.8, 1, 1.2\}$; $\pi(s') = \pi_\sigma, 1 - 2\pi_\sigma, \pi_\sigma$; with $\pi_\sigma = 0, 0.2, 0.5$. 
Insurance with Durable Goods: Example

■ Financing needs for durables reduce insurance and increase net worth accumulation

■ Parameters: $y' \in \{0.8, 1.2\}$, $p = 0.5$, CRRA with $\gamma = 2$, $g = 2$, $\theta = 0.8$
Durable Goods Price Risk Management

- Stochastic durable goods price $q(s)$
- Down payment $\varphi(s) \equiv q(s) - R^{-1} \theta E[q'|s](1 - \delta)$

- **Household’s problem:** Given $s$ and $w$, household maximizes (5) subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + \varphi(s)k + E[R^{-1}h'|s]$$

$$y' + (1 - \theta)q'k(1 - \delta) + h' \geq w'$$

and the short sale constraints, $\forall s' \in S$

$$h' \geq 0$$

- Effect of durable goods price
  - Investment Euler equation for durable goods

$$1 = \beta g_k(k) \frac{1}{\mu s} + E \left[ \beta \frac{\mu'}{\mu} \frac{(1 - \theta)q'(1 - \delta)}{\varphi(s)} \bigg| s \right]$$

- Price affects down payment $\varphi(s)$ and net worth $w'$ directly
Durable Goods Price Risk: Example

- Hedging durable goods prices \( (q' \in \{0.95, 1.05\}) \) (and income)

- Durable goods price affects cost of durables, net worth, and required down-payments
Increasing Durable Goods Price Risk Management

- **Full pledgeability of resale value** ($\theta = 1$) (Prop. 8)

- With logarithmic preferences, household income insurance is increasing and household does not hedge price risk
  - Why? - Price risk separable; does not affect marginal utility of net worth

- With isoelastic preferences and $\gamma < 1$, risk management is increasing

  - **Intuition**
    - Equivalent to economy with preference shocks
    - With $\gamma < 1$, $q(s)$ lowers marginal utility (similar: Campbell (1996))
Risk Management and Rent vs. Buy Decision

- **Renting:** Ease of repossession allows higher leverage
  - See Eisfeldt/Rampini (2009) and R/V (2013)
  - Pay rental fee in advance

\[
 u_l(s) \equiv rq(s) - (E[q'|s] - q(s)) + E[q'|s](\delta + m)
\]

where \(m\) is monitoring cost

- Financially constrained households rent

- **Renting affects incentives to hedge**
  - High (implicit) leverage induces hedging
  - Sign of hedging demand can flip (related: Sinai/Souleles (2005))
Household’s problem: Given $s$ and $w$, household maximizes (5) subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + \phi(s)k_o + R^{-1}u_l(s)k_l + E[R^{-1}h'|s]$$

$$y' + (1 - \theta)q'k_o(1 - \delta) + h' \geq w'$$

non-negativity constraints on owned and rented durables,

$$k_o, k_l \geq 0$$

and short sale constraints, $\forall s' \in S$

$$h' \geq 0$$

with $k = k_o + k_l$
High leverage of renting induces risk management \( (m = 0.02) \)
Renters hedge states with high house prices!