Appendix to "New or Used? Investment with Credit Constraints" by Andrea L. Eisfeldt and Adriano A. Rampini

This appendix provides the characterization of an economy where credit constraints do not affect pricing, i.e., where $p_u = 1 - \beta m_u$. The characterization is quite similar to the case where $p_u > 1 - \beta m_u$. In particular, agents with few internal funds are credit constrained, buy only used capital, and start smaller firms. There are two main differences, however. The first difference is that investment in new capital is not uniquely determined for all agents. The minimum amount an agent invests in used capital, i_u^{min}, however, is uniquely determined and has the same properties as before. The minimum investment in used capital is 100% of investment for agents with internal funds below some threshold $\bar{e}_u$, then decreases over an interval of intermediate values of internal funds between $\bar{e}_u$ and $\bar{e}_n$, and is zero above $\bar{e}_n$. The second difference is that an agent's total investment is strictly increasing below $\bar{e}_u$ only. The third region of Proposition 3 in the paper thus collapses, i.e., $\bar{e}_n = \bar{\bar{e}}$ using the notation of that proposition.[31]

To see that investment in new capital is not uniquely determined consider the following argument: Any agent who is willing to invest a positive amount in new capital would be indifferent between doing so and raising the investment in used capital by a small amount while reducing his investment in new capital by the same small amount and reducing borrowing (or increasing savings) by the difference. Specifically, increasing i_u by Δ and reducing i_n by the same amount frees up $(1 - p_u)\Delta$ units of consumption at date 0. Reducing borrowing (or increasing savings) b by that amount leaves consumption at date 0 unchanged, and pays off $R(1 - p_u)\Delta$ at date 1. Maintenance costs at date 1 increase by $m_u\Delta$, but the reduction in borrowing (or increase in savings) exactly covers that. Thus consumption at date 1 is not affected either.

We denote the minimum investment in used capital, which is determined uniquely given the agent's internal funds e, by i_u^{min} and the corresponding maximum investment in new capital by i_n^{max}. Similarly, we denote the implied maximum borrowing by b^{max}. The solution can then be characterized as follows:

Proposition 1 *Suppose $p_u + \beta m_u = 1$. There exist two cutoff levels of internal funds $\bar{e}_u < \bar{e}_n$ and a level of capital $\bar{k}$ such that the solution to the agent's problem satisfies:*

(i) *For $e \le \bar{e}_u$, $i_u > 0$, $i_n = 0$, and $b = \beta\theta p_u i_u(1-\delta)$. Moreover, $\frac{di_u}{de} > 0$.*

(ii) *For $\bar{e}_u < e < \bar{e}_n$, $i_u^{min} > 0$, $i_n^{max} > 0$, and $b^{max} = \beta\theta p_u(i_u + i_n)(1-\delta)$. Moreover, $i_u + i_n = \bar{k}$, $\frac{di_u^{min}}{de} < 0$, and $\frac{di_n^{max}}{de} > 0$.*

(iii) *For $e \ge \bar{e}_n$, $i_n^{max} > 0$, $i_u^{min} = 0$, and, for $e > \bar{e}_n$, $b^{max} < \beta\theta p_u(i_u + i_n)(1-\delta)$. Moreover, $i_u + i_n = \bar{k}$ and $\frac{di_n^{max}}{de} = 0$.*

Proof of Proposition 1. Recall from equation (6) in the paper that $\mu_1 m_u + \lambda_n - \lambda_u = \mu_0(1 - p_u) = \mu_1 m_u + \lambda_b m_u$, where the second equality uses the fact that $\mu_0 = \mu_1\beta^{-1} + \lambda_b\beta^{-1}$ and $p_u + \beta m_u = 1$. Hence, $\lambda_b m_u = \lambda_n - \lambda_u$. But this implies that $\lambda_u = 0$, $\forall e \in \mathcal{E}$, since λ_n and λ_u can not both be strictly positive. Moreover, $\lambda_n = 0$ if and only if $\lambda_b = 0$.

Suppose $\lambda_b > 0$ and hence $b = \beta\theta p_u i_u(1-\delta)$ and $i_n = 0$. Then, i_u solves equation (7) in the paper and, totally differentiating, we have $\frac{di_u}{de} > 0$ as in the paper.

[31] The reason is that in the case where $p_u = 1 - \beta m_u$ the return on saving, $R = \beta^{-1}$, equals the return on substituting new capital for used capital, $m_u/(1 - p_u) = \beta^{-1}$.

Suppose $\lambda_b = 0$, and hence $\lambda_n = 0$ and $\mu_0 = \mu_1\beta^{-1}$. Equation (5) in the paper then implies that $1 = \beta(\alpha k^{\alpha-1} + p_u(1-\delta))$ which defines $\bar{k}$. Now agents in this range are indifferent between investing in new and used capital at the margin. However, we can determine the minimum used capital investment that is required for given e. At the margin, investing in new capital instead of used capital is equivalent to investing in used capital and reducing borrowing by the difference $1 - p_u$. Thus, the way to obtain a capital stock of $\bar{k}$ while saving the minimum amount is by investing in used capital only. At the lower boundary of the region, the agent invests in used capital only and borrows $b^{max} = \beta\theta p_u \bar{k}(1-\delta)$. Moreover, since $\lambda_b = 0$, we have

$$u'(e - p_u\bar{k}(1 - \beta\theta(1-\delta))) = u'(\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta) - m_u\bar{k}) \tag{1}$$

which defines $\bar{e}_u$. Thus, the minimum used capital investment at $\bar{e}_u$ is $i_u^{min} = \bar{k}$. Above $\bar{e}_u$, the minimum used capital investment, which implies $b^{max} = \beta\theta p_u\bar{k}(1-\delta)$, decreases since c_1 must be increasing in e and $\frac{dc_1}{de} = -m_u \frac{di_u^{min}}{de}$. At the upper boundary of this region, the agent invests in new capital only and $b^{max} = \beta\theta p_u\bar{k}(1-\delta)$, and

$$u'(e - \bar{k}(1 - \beta\theta p_u(1-\delta))) = u'(\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta)) \tag{2}$$

which defines $\bar{e}_n$, and, comparing (1) and (2), $\bar{e}_n > \bar{e}_u$. Above $\bar{e}_n$, $i_u^{min} = 0$, $i_n^{max} = \bar{k}$, and $b^{max} < \beta\theta p_u\bar{k}(1-\delta)$. □

The numerical results are similar to those of the example in the paper. The parameters and equilibrium level of capital are described in the caption of Figure 1. The only difference in the primitives of the economy from the example in the paper is the distribution over endowments. In the example in the paper, the distribution over endowments is exponential on the state space, so that there are more agents with low endowments than high endowments, whereas endowments are distributed uniformly in the example computed here. Still, as before, agents with low endowments invest exclusively in used capital, are constrained, and start smaller firms. Moreover, above $\bar{e}_u$ the minimum investment in used capital is monotonically decreasing in agents' endowment, and reaches zero at $\bar{e}_n$, where maximum borrowing no longer exhausts agents' borrowing capacity. Analogous to Figure 1 in the paper, Figure 1 presents the investment and borrowing decisions, along with the shadow price of used capital, and the multiplier on the borrowing constraint, as a function of agents' initial endowment of internal funds. Notice that $\bar{e}_u$ is higher than in the example in the paper so agents with higher endowment levels invest exclusively in used capital. This is because the price of used capital is not inflated by the effect of credit constraints. Moreover, unconstrained agents invest in less capital here than in the case where credit constraints affect pricing, since they can sell the used capital at a premium in that case.

Figure 1: Investment in New and Used Capital in an Economy where Credit Constraints do not Affect Pricing

Top Left Panel: Maximum investment in new capital (dash dotted), minimum investment in used capital (solid), and total investment (dotted) as a function of the amount of internal funds. Middle Left Panel: Minimum investment in used capital as percentage of total investment. Bottom Left Panel: Agent specific shadow price of new capital (dotted) and used capital (solid). Top Right Panel: Maximum borrowing. Middle Right: Multiplier on the borrowing constraint $\lambda_b(e)$ (normalized by the marginal utility of consumption at time 0) as a function of the amount of internal funds.

Parameter values: Preferences: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\beta = 0.96$, $\gamma = 2$; technology: $\alpha = 0.33$, $\delta = 0.12$, $m_u = 0.5$; collateralization rate: $\theta = 0.2$; discretized state space: $i_u, i_n \in [0 : 0.002 : 0.5]$, $b \in [-0.2 : 0.002 : 0,\ 0.001 : 0.001 : 0.05]$; distribution of internal funds: $\mathcal{E} = [0.05 : 0.05 : 1.75]$, $\pi(e) = [1/35, \ldots, 1/35]$.

Equilibrium implications: price of used capital: $p_u = 0.52 = 1 - \beta m_u$; level of capital: $\bar{k} = 0.4265$.