University of Oslo
Department of Economics

Professor Rampini
Duke University

Financial Contracting and the Business Cycle
August 20-23, 2007

Practice Problem: Financing with Costly State Verification

This is a suggested practice problem. Solving this problem should help you prepare for the course.

Problem. This problem studies the optimal contract between a risk neutral entrepreneur and a risk neutral (representative) lender when there is private information about the project outcome but the lender has access to a costly state verification (CSV) technology as in Townsend (1979). We assume that the lender can commit to stochastic monitoring as in Mookherjee and Png (1989).

There are two dates, 0 and 1, and n possible outcomes of the project, i.e., cash flows y_i, $i = 1, ..., n$, where $0 \leq y_1 < y_2 < \cdots < y_n$ and probability of cash flow i is π_i ($\pi_i > 0$, $\forall i$). The project requires an investment $I > 0$ and the entrepreneur has assets (or internal funds) $A \geq 0$, where $I > A$ so that there is a financing need. Assume w.l.o.g. that the entrepreneur contributes all his assets to the project. The entrepreneur consumes at date 1 only and is subject to limited liability, i.e., the entrepreneur's consumption has to be non-negative. The entrepreneur observes the cash flow y_i, i.e., state i, but the lender observes it only at a cost of $\kappa > 0$. By the revelation principle, we can restrict attention to direct truth-telling mechanisms, in which the entrepreneur announces the state, say j. We can assume that only announcements of the type $j \leq i$ are feasible in state i, e.g., because the entrepreneur has to show the cash to the investor and can only hide cash (which turns out to be w.l.o.g.).

Consider the optimal incentive-compatible contract which maximizes the utility of the entrepreneur. Notation: Let c_i be the entrepreneur's consumption when he announces i (truthfully) and is not monitored. Let c_i^m be his consumption when he announces i (truthfully) and is monitored. When the entrepreneur is monitored and found to be lying his consumption is 0 (why?). Let the probability that the agent is monitored in state i be p_i. Finally, assume the interest rate is r so that the lender needs to be repaid $(1 + r)(I - A)$.

[Remark: To prepare for the course, solve the questions below for $n = 2$. Of course, if you would like, you should feel free to (and be able to) solve the questions below for general n as well.]

(a) Write down the problem of maximizing the entrepreneur's expected utility, subject to (i) the participation constraint for the lender (with Kuhn-Tucker multiplier μ), (ii) incentive compatibility constraints (with multipliers λ_{ij}, $j < i$), (iii) non-negativity constraints on the entrepreneur's consumption in all states (with multipliers ν_i and ν_i^m, respectively), and subject to (iv) the constraints that p_i are between zero and one (with multipliers η_i^0 and η_i^1, respectively). Write down the first order conditions.

(b) Prove that the optimal contract minimizes expected auditing costs subject to (i)-(iv).

(c) Prove that expected auditing costs under the optimal contract are nonincreasing in the entrepreneur's assets A.

(d) Note that the multiplier on the lender's participation constraint $\mu \geq 1$. (The multiplier μ can be interpreted as the shadow-value of internal funds. Why?) Thus, there are two cases either $\mu = 1$ or $\mu > 1$. Prove that the optimal contract involves no auditing if and only if the lender's required return is less than the value of the worst possible outcome of the project, i.e., $p_i = 0, \forall i, \Leftrightarrow y_1 \geq (1+r)(I-A)$.

(e) For this part and all following parts, consider the case where $(1+r)(I-A) > y_1$ and hence $\mu > 1$. Prove that in any state in which there is a positive probability of monitoring, the entrepreneur receives positive consumption only if he is audited, i.e., $p_i > 0 \Rightarrow c_i = 0$.

(f) Prove that the entrepreneur receives no consumption in the worst state $c_1^m = c_1 = 0$.

(g) Let $\bar{c}_i = p_i c_i^m + (1-p_i)c_i$ be the entrepreneur's expected consumption in state i. Prove that $\bar{c}_i$ is nondecreasing in i, i.e., the entrepreneur does better in better states.

(h) Prove that there is never any auditing in the highest state $p_n = 0$.

(k) Prove that the probability of auditing is nonincreasing in the announced state, i.e., p_i is nonincreasing in i.