

# Long-Run Risks Explanation of Forward Premium Puzzle

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Comments Welcome

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## Long Run Risks Explanation of Forward Premium Puzzle

### Abstract

A conventional intuition that a high-interest rate bearing currency should be expected to depreciate in the future is violated in the data. Using a rational-expectation, general equilibrium model featuring long-run risks, stochastic market volatility and generalized utility function, we present an explanation for the failure of the uncovered interest rate hypothesis for a risk-averse agent whose intertemporal elasticity of substitution is high enough. Numerical calibrations show that our framework can match the salient features of consumption data within and across the countries, as well as a number of properties of domestic and international financial markets.

# 1 Introduction

The hypothesis of uncovered risk premium parity assumes that the risk premium in foreign exchange markets is constant, which implies that the high interest rate bearing currencies are expected to depreciate by the difference in interest rates at home and abroad. Quantitatively, the projection of the change in exchange rate on the interest rate differential should reveal the regression coefficient of one. The numerous empirical studies, e.g. Fama (1984), Bansal (1997), Bansal & Dahlquist (2000), Backus, Foresi & Telmer (2001), Verdelhan (2005) present the opposite evidence of negative and statistically significant slope coefficients and dub the anomaly the "forward premium puzzle."

The rejection of the uncovered interest rate parity hypothesis in the data and its direct implication that the foreign exchange risk premia are time-varying spurred an extensive research in economics and finance. A wide array of economic models have been proposed to account for the puzzle. A partial list of these attempts includes consumption-based asset-pricing theories of Bansal, Gallant, Hussey & Tauchen (1995), Verdelhan (2005) and Lustig & Verdelhan (2005), term-structure models of Backus, Foresi & Telmer (2001) and Bansal (1997), and statistical models of Fama (1984), Baillie & Bollerslev (2000) and Maynard & Phillips (2001). Evidence contained in these articles suggest that the successful explanation of the forward premium puzzle is a formidable challenge for any rational expectations economic model.

Following Bansal & Yaron (2004), we propose a two-country general equilibrium framework where the endowment growth in each country contains a small and persistent component in the mean, and is driven by conditionally heteroscedastic news which exhibit time-varying stochastic volatility. The preferences of the representative agent are characterized by a non-expected recursive utility of Epstein & Zin (1989) and Epstein & Zin (1991). Relative to a nested CRRA case, the generalized pricing kernel separates the risk-aversion coefficient from the inter-temporal elasticity of substitution and allows for the long-term risks and volatility risks to be priced. These new channels prove highly effective to interpret the asset markets within the countries. Indeed, Bansal & Yaron (2004) are able to solve the risk-premium, risk-free rate and volatility puzzles, while Bansal, Dittmar & Lundblad (2005) and Kiku (2005) apply the framework to provide a risk-based explanation for the cross-section of returns and value versus growth in par-

ticular. Long-term properties of the economy stemming from the permanent risks in consumption are addressed in Bansal, Dittmar & Kiku (2005) and Hansen, Heaton & Li (2005).

The extension of Bansal & Yaron (2004) model to a multi-country case has a potential to interpret foreign markets as well. Colacito & Croce (2005) show that a high correlation between the persistent components across countries can resolve the puzzle of excessive volatility of exchange rates relative to a small correlation of consumption growth rates noted among others by Brandt, Cochrane & Santa-Clara (2005). We further develop the model by focusing on stochastic and persistent market volatilities which are assumed to correlate across the countries. While the time-varying volatility is clearly necessary to generate a time variation in foreign exchange risk premia, it is not obvious a priori that it has the ability to address the forward premium anomaly for reasonable values of the parameters of the model. We show that we can indeed meet the additional necessary conditions of Fama (1984) or Backus, Foresi & Telmer (2001) for a wide range of parameters as long as the risk-averse agents have a high enough level of substitution (higher than one). For this configuration, a positive shock to domestic uncertainty leads to a decrease in short-term bond returns at home, and an appreciation of the domestic currency now and in the future. However, as the latter effect is strongest for contemporaneous spot currency prices, relative to today future value of the domestic currency is expected to go down. Thus, in the model the stochastic volatility provides a channel to obtain a negative slope coefficient in foreign exchange projections. We also find and document some support for this mechanism in the data.

The rest of the paper is organized as follows. In the next section we setup the long-run risks model where we review the preferences of the agents and structure of the economies and discuss the asset-pricing implications for domestic and international asset markets. In Section 2 we assess the ability of our model to capture the key financial and macroeconomic features of the data. Conclusion follows.

## 2 Long Run Risks Model

### 2.1 Preferences and the Environment

We consider a representative agent in each country with the generalized preferences developed in Epstein & Zin (1989). The logarithm of the Intertemporal Marginal Rate of Substitution (IMRS) for these preferences, as shown in the Epstein and Zin paper, is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)r_{a,t+1}, \quad (1)$$

for a domestic investor and

$$m_{t+1}^* = \theta^* \log \delta^* - \frac{\theta^*}{\psi^*} g_{t+1}^* + (\theta^* - 1)r_{a,t+1}^* \quad (2)$$

for a foreign one, where  $g_{t+1}$  equals  $\log(C_{t+1}/C_t)$  — the log growth rate of aggregate consumption. The return,  $r_{a,t+1}$ , is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period. The time discount factor is  $\delta$  and the parameter  $\theta \equiv \frac{1-\gamma}{1-\frac{\gamma}{\psi}}$ , where  $\gamma \geq 0$  is the risk-aversion (sensitivity) parameter, and  $\psi \geq 0$  is the intertemporal elasticity of substitution. The sign of  $\theta$  is determined by the magnitudes of the risk aversion and the elasticity of substitution.<sup>1</sup> Note, when  $\theta$  equals one, the above IMRS and the asset pricing implications collapse to the usual case of power utility considered in Mehra & Prescott (1985).

If  $R_{t+1}$  be the gross return on the asset, then  $\log(R_{t+1}) \equiv r_{t+1}$  is the continuous return on the asset. Using the standard asset pricing restriction for any continuous return  $r_{i,t+1}$  it follows, that <sup>2</sup>

$$E_t \left[ \exp\left(\theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)r_{a,t+1} + r_{i,t+1}\right) \right] = 1, \quad (3)$$

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<sup>1</sup>In particular, if  $\psi > 1$  and  $\gamma > 1$ , then  $\theta$  will be negative. Note that when  $\theta = 1$ , that is,  $\gamma = (1/\psi)$ , the above recursive preferences collapse to the standard case of expected utility. Further, when  $\theta = 1$  and in addition  $\gamma = 1$ , we get the standard case of log utility.

<sup>2</sup>Note that the standard asset pricing condition in frictionless markets is

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1,$$

where the inter-temporal marginal rate of substitution is  $M_{t+1}$  and  $\log(M_{t+1}) \equiv m_{t+1}$ . The logarithm of the gross return for an asset equals  $r_{t+1}$ .

The return to the aggregate consumption claim,  $r_{a,t+1}$ , is not observed in the data while the return on the dividend claim corresponds to the observed return on the market portfolio  $r_{m,t+1}$ . The levels of market dividends and consumption are not equal; aggregate consumption is much larger than aggregate dividends. The difference is financed by labor income. In the model, aggregate consumption and aggregate dividends are treated as two separate processes and the difference between them implicitly defines the agent's labor income process.

The key ideas of the model are developed and the intuition is provided via approximate analytical solutions. However, all the quantitative results reported in the paper are based on numerical solutions of the model. To derive the approximate-analytical solutions for the model we use the standard first-order Taylor series approximation developed in Campbell & Shiller (1988)<sup>3</sup>,

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \quad (4)$$

where lowercase letters refer to variables in logs, in particular,  $r_{a,t+1} = \log(R_{a,t+1})$  is the continuous return on the consumption claim, and  $z_t \equiv \log(P_t/C_t)$  is the log price-consumption ratio. Analogously,  $r_{m,t+1}$  and  $z_{m,t}$  correspond to the continuous return on the dividend claim and its log price-dividend ratio. As  $\frac{P_t+C_t}{C_t}$  is the agent's wealth to consumption ratio, fluctuations in the price to consumption ratio, consequently, also correspond to movements in the wealth to consumption ratio. Parameters  $\kappa_0$  and  $\kappa_1$  are approximating constants that both depend only on the average level of  $z$ .<sup>4</sup>

From equation (1) it follows that the innovation in IMRS,  $m_{t+1}$ , is driven by the innovations in  $g_{t+1}$  and  $r_{a,t+1}$ . Covariation with the innovation in  $m_{t+1}$  determines the

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<sup>3</sup>Any equity return can be written as

$$R_{t+1} = \frac{1 + \frac{P_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t}$$

The approximate return expression follows from taking the log of the gross return and then taking a first-order Taylor series approximation of  $\log(1 + \frac{P_{t+1}}{D_{t+1}})$  around the average value of the log of price-dividend ratio, which is referenced as  $z$  in equation (4). The approximating constants  $\kappa_0$  and  $\kappa_1$  are solely determined by the average value of  $z$ .

<sup>4</sup>The Campbell-Shiller approximate return follows from using a first order Taylor series expansion of the continuous return. Note that  $\kappa_1 = \exp(\bar{z})/(1 + \exp(\bar{z}))$ . The value of  $\kappa_1$  is set at 0.997, which is consistent with the magnitude of  $\bar{z}$  in our sample and with magnitudes used in Campbell & Shiller (1988).

risk premium for any asset. The full model that incorporates long run growth rate and economic uncertainty risks is presented in the next section.

## 2.2 Foreign Exchange Puzzle

Following Brandt, Cochrane & Santa-Clara (2005), Backus, Foresi & Telmer (2001) and Bansal (1997), we assume the case of the complete markets and thus the uniqueness of the generalized preferences discount factor in each country. This assumption is not particularly restrictive, as all the results continue to hold in incomplete markets with a particular choice of a pricing kernel. Let  $S_t = e^{s_t}$  stand for a spot exchange rate, per unit of foreign currency (dollars spot price of one pound) and denote by  $F_t = e^{f_t}$  the current dollar price (forward rate) of a contract to deliver one pound tomorrow.

Then Euler equation and the assumption of complete markets imply that the change in the exchange rate is equal to the ratio of the discount factors in the two countries:

$$\frac{S_{t+1}}{S_t} = \exp(m_{t+1}^* - m_{t+1}). \quad (5)$$

The *covered parity condition* implies that the forward premium is equal to the interest rate differential between the two countries,

$$f_t - s_t = r_{ft} - r_{ft}^*. \quad (6)$$

Therefore, an investor can borrow money domestically, exchange them to the foreign currency using a current spot price, invest into foreign risk-free bonds and perfectly hedge all the risks

To obtain the *uncovered interest rate parity* we follow Fama (1984) to decompose the forward risk premium into the expected excess return ("risk premium")  $p_t$  and the expected rate of the exchange rate depreciation  $q_t$  :

$$\begin{aligned} f_t - s_t &= (f_t - E_t s_{t+1}) + (E_t s_{t+1} - s_t) \\ &\equiv p_t + q_t \end{aligned} \quad (7)$$

The hypothesis of uncovered risk premium parity assumes that the variance of the risk premium  $p_t$  is zero, which implies that the currencies of high interest rates should be

expected to depreciate. Quantitatively, the projection of the change in exchange rate on the interest rate differential should reveal the regression coefficient of 1.

Assuming rational expectations, we can express the OLS slope coefficient in the following way:

$$\begin{aligned}\beta_{UIP} &= \frac{Cov(s_{t+1} - s_t, f_t - s_t)}{Var(f_t - s_t)} \\ &= \frac{Var(p_t) + Cov(p_t, q_t)}{Var(p_t - q_t)}.\end{aligned}$$

With this decomposition Fama (1984) shows that any theoretical model which wishes to explain a negative slope in uncovered interest parity regressions should be able to produce a negative covariance between  $p$  and  $q$  and a greater variance of  $p$  than  $q$ .

In the next section we use our solution to the IMRS to characterize explicitly the forward risk premium, the expected depreciation rate and the population projection coefficient  $\beta_{UIP}$ , and relate our results to the two Fama's conditions above.

## 2.3 Long Run Growth and Uncertainty Risks

Bansal & Yaron (2004) model fluctuating economic uncertainty as time-varying volatility of consumption growth. The consumption and dividends dynamics that incorporate stochastic volatility are:

$$\begin{aligned}x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\ g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1}\end{aligned}\tag{8}$$

$$\begin{aligned}g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}\end{aligned}\tag{9}$$

where  $\sigma_{t+1}$ , the conditional volatility of consumption growth, represents the time-varying economic uncertainty incorporated in consumption growth rate. The unconditional mean of the time-varying variance of consumption growth is  $\sigma^2$  and  $\sigma_w$  determines the volatility of shocks to consumption uncertainty. The parameter  $\nu$  determines the persistence of shocks to consumption variance.



Similarly, the dynamics for the foreign consumption satisfy

$$x_{t+1}^* = \rho^* x_t^* + \varphi^* \sigma_t^* e_{t+1}^*, \quad (10)$$

$$g_{t+1}^* = \mu^* + x_t^* + \sigma_t^* \eta_{t+1}^*, \quad (11)$$

$$\begin{aligned} g_{d,t+1}^* &= \mu_d^* + \phi^* x_t^* + \varphi_d^* \sigma_t^* u_{t+1}^* \\ \sigma_{t+1}^{*2} &= \sigma^{*2} + \nu^* (\sigma_t^{*2} - \sigma^{*2}) + \sigma_w^* w_{t+1}^*. \end{aligned} \quad (12)$$

In other words, we assume that the general laws of evolution of the economies are the same, while the particular parameter values can differ across countries. In an empirical investigation we will further simplify these dynamics by equating the key parameter values across the countries.

We allow for a covariation between the dividend shocks  $u_t$  and short-run shocks to consumption growth  $\eta_t$ , and correlate the respective innovations between the countries, so that

$$\begin{aligned} Cov(u, \eta) &= \alpha, \\ Cov(e, e^*) &= \tau_e, \\ Corr(\eta, \eta^*) &= \tau_\eta, \\ Corr(w, w^*) &= \tau_w. \end{aligned}$$

In the interest of space, the next set of results are provided for the home country only; their extension to the foreign one is straightforward. The relevant state variables in solving for the equilibrium price-consumption (and price-dividend) ratio are  $x_t$  and  $\sigma_t^2$ . Thus, the approximate solution for the price-consumption ratio is  $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$ ; an analogous expression holds for the log price-dividend ratio  $z_{m,t}$ .<sup>5</sup> The solutions for  $A_1$  and  $A_{1,m}$  are

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \quad A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho}. \quad (13)$$

It follows that  $A_1$  is positive if the IES,  $\psi$ , is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises.

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<sup>5</sup>The expression for the intercept terms  $A_0$  for the valuation ratio for the consumption claim, and  $A_{0,m}$  for the dividend claim are not important for our qualitative results.

The level of consumption rises due to a rise in expected growth, however, wealth rises more than consumption. In the standard power utility model with risk aversion larger than one, the IES is less than one, and hence  $A_1$  is negative — a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.<sup>6</sup> Corporate payouts (i.e., dividends), with  $\phi > 1$ , are more sensitive to long run risks (i.e.,  $A_{1,m} > A_1$ ) and changes in expected growth rate lead to a larger reaction in the price of the dividend claim than in the price of the consumption claim.

The solution coefficient  $A_2$  for measuring the sensitivity of price-consumption ratio to volatility fluctuations is

$$A_2 = \frac{0.5[(\theta - \frac{\theta}{\psi})^2 + (\theta A_1 \kappa_1 \varphi_e)^2]}{\theta(1 - \kappa_1 \nu_1)}. \quad (14)$$

An analogous coefficient for the market price-dividend ratio,  $A_{2,m}$ , is provided in Bansal and Yaron (2004).

The expression for  $A_2$  provides two valuable insights. First, if the IES and risk aversion are larger than one, then  $\theta$ , and consequently  $A_2$  are negative. In this case a rise in consumption volatility lowers asset valuations and increases the risk premia on all assets. To capture the intuition that a rise in economic uncertainty lowers asset valuations requires that the IES be larger than one. Bansal, Khatchatrian & Yaron (2005) present robust empirical evidence that asset markets dislike economic uncertainty — that is,  $A_2$  is negative. This empirical evidence, given the expression for  $A_2$ , has direct bearing on the plausible magnitude for the IES. Second, an increase in the permanence of volatility shocks, that is  $\nu$ , magnifies the effects of volatility shocks on valuation ratios as changes in economic uncertainty are perceived by investors as being long lasting.

### 2.3.1 Marginal Rate of Substitution

Using the approximate solutions for the price-consumption ratio we can show that the first two conditional moments of the marginal rate of substitution are linear in the state

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<sup>6</sup>An alternative interpretation with the power utility model is that higher expected growth rates increase the risk-free rate to an extent that discounting dominates the effects of higher expected growth rates. This leads to a fall in asset prices.

variables  $x_t$  and  $\sigma_t^2$ . Indeed, the drift is equal to,

$$E_t m_{t+1} = m_0 - \frac{1}{\psi} x_t + \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] \sigma_t^2, \quad (15)$$

for some constant  $m_0$ , while the innovations into  $m_{t+1}$  are given by

$$m_{t+1} - E_t(m_{t+1}) = -\lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,e} \sigma_t e_{t+1} - \lambda_{m,w} \sigma_w w_{t+1}, \quad (16)$$

so that its conditional variance becomes

$$Var_t m_{t+1} = \lambda_{m,w}^2 \sigma_w^2 + (\lambda_{m,\eta}^2 + \lambda_{m,e}^2) \sigma_t^2, \quad (17)$$

where  $\lambda_{m,\eta}$ ,  $\lambda_{m,e}$ , and  $\lambda_{m,w}$  are the market price of risks for the short run, long run, and volatility risks. The market prices of systematic risks, including the compensation for stochastic volatility risk in consumption, can be expressed in terms of underlying preferences and parameters that govern the evolution of consumption growth as,

$$\begin{aligned} \lambda_{m,\eta} &= \gamma \\ \lambda_{m,e} &= \left( \gamma - \frac{1}{\psi} \right) \left[ \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right] \\ \lambda_{m,w} &= \left( \gamma - \frac{1}{\psi} \right) (1 - \gamma) \left[ \frac{\kappa_1 (1 + (\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho})^2)}{2 (1 - \kappa_1 \nu_1)} \right] \end{aligned} \quad (18)$$

In the special case of power utility, when  $\theta = 1$  or more specifically, where  $\gamma = \frac{1}{\psi}$  the risk compensation parameters  $\lambda_{m,e}$  and  $\lambda_{m,w}$  are zero, and the IMRS collapses to the standard

$$m_{t+1} = \delta - \gamma g_{c,t+1}. \quad (19)$$

The long run risks and volatility risks are not reflected in the innovation of the pricing kernel. With power utility there is no separate risk compensation for long run growth rate risks and volatility risks — with generalized preferences both risks are priced. The pricing of long run and volatility risks is an important and novel feature of the Bansal and Yaron model.

### 2.3.2 Foreign Exchange Markets in Long-Run Risks Economy

We can use the Euler condition and the solution for the pricing kernel in (15) and (16) to compute the return on the riskless security in the following way:

$$\begin{aligned}
r_{ft} &\equiv -\log E_t e^{m_{t+1}} \\
&= -\left(E_t(m_{t+1}) + \frac{1}{2}\text{Var}_t(m_{t+1})\right) \\
&= r_{0,f} + \frac{1}{\psi}x_t + r_\sigma\sigma_t^2,
\end{aligned} \tag{20}$$

for

$$r_\sigma = -\frac{1}{2}\left(\lambda_{m,\eta}^2 + \lambda_{m,e}^2 + \left(\frac{1}{\psi} - \gamma\right)(\gamma - 1)\left[1 + \left(\frac{\kappa_1\varphi_e}{1 - \kappa_1\rho}\right)^2\right]\right). \tag{21}$$

As the first two moments of the pricing kernel are linear in the state variables, so is the risk-free rate. In particular, it increases with the expected consumption growth with a coefficient of proportionality  $\frac{1}{\psi}$  equal to  $\gamma$  in a CRRA setup. The first component  $\lambda_{m,\eta}^2 = \gamma^2$  in the loading on the variance of consumption growth captures a standard negative effect of the precautionary motive of saving. The next two terms highlight the novel dimension of the generalized preferences IMRS to account for long-run and volatility risks, as they disappear in a time-separable power-utility specification of  $\gamma = 1/\psi$ .

Using the expressions for the market prices of risks, it is easy to show that for  $\gamma > 1$  and  $\psi > 1$  the coefficient  $r_\sigma$  is negative, so that market uncertainty increases the demand for safe assets, driving bond prices up and their returns down. The effect is qualitatively the same in the CRRA setup; however, with generalized preferences its sensitivity depends on both the risk aversion and the IES, and the dynamics of the long-run risk factor.

Imposing the symmetry between the two countries, the forward premium can be obtained from the solutions to the riskfree rates shown above:

$$\begin{aligned}
f_t - s_t &\equiv r_{ft} - r_{ft}^* \\
&= (p_0 + q_0) + \frac{1}{\psi}(x_t - x_t^*) + r_\sigma(\sigma_t^2 - \sigma_t^{*2}).
\end{aligned} \tag{22}$$

When  $r_\sigma$  is negative, positive shocks to domestic uncertainty decrease the domestic risk-free rate relative to its foreign counterpart.

From (15) and (17), the expected depreciation rate of domestic currency can be calculated in the following way:

$$q_t = q_0 + \frac{1}{\psi}(x_t - x_t^*) + \frac{(\gamma - \frac{1}{\psi})(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] (\sigma_t^2 - \sigma_t^{*2}), \quad (23)$$

for some constant  $q_0$ . As can be seen from the above expression, standard power utility case implies that market uncertainty bears no effect on the expected changes in the currency valuation, as they solely depend on the difference in expected consumption growths next period. On the contrary, separation of the risk aversion level from the intertemporal elasticity of substitution opens a channel for the persistent conditional volatility of the endowment streams which will go a long way to explain the behavior of the foreign exchange markets. Indeed, if the IES is high enough, (22) and (23) suggest that higher domestic uncertainty is associated with an expected *depreciation* of the domestic currency and *low* domestic risk-free rates, which is at the heart of the uncovered interest rate puzzle.

It follows that the foreign exchange risk premium reflects the compensation for bearing short and long run risks:

$$\begin{aligned} p_t &\equiv f_t - s_t - E_t(s_{t+1} - s_t) \\ &= p_0 - \frac{1}{2}(\lambda_{m,\eta}^2 + \lambda_{m,e}^2)(\sigma_t^2 - \sigma_t^{*2}), \end{aligned} \quad (24)$$

for some  $p_0$ . The negative of  $p_t$  corresponds to the ex-ante dollar excess return on the uncovered position in one-period foreign bonds. High domestic market uncertainty relative to the foreign one corresponds to a lower conditional volatility of the foreign IMRS when the returns abroad are high, so that the foreign currency is risky and thus demands a high risk premium. This mechanism is identical to the one used by Lustig & Verdelhan (2005) for the cross-section of currency portfolio returns.

Now we can use the solutions to the FX market to determine explicitly the population slope in the uncovered interest rate parity regression. To simplify the notations, denote by  $v_{x\sigma}$  the ratio of the unconditional variances in long-run risks and market volatility differentials:

$$v_{x\sigma} = \frac{Var(x_t - x_t^*)}{Var(\sigma_t^2 - \sigma_t^{*2})}.$$

Then, given the dynamics for the forward premium in (22) and the expected depreciation rate in (23), the unconditional uncovered interest rate parity regression coefficient

can be calculated in the following way:

$$\beta_{UIP} = 1 + \frac{r_\sigma}{2} \left( \frac{\gamma^2 + (\gamma - \frac{1}{\psi})^2 \left( \frac{\kappa_1 \varphi e}{1 - \kappa_1 \rho} \right)^2}{r_\sigma^2 + \frac{1}{\psi^2} v_{x\sigma}} \right). \quad (25)$$

In the power utility case the slope coefficient simplifies to

$$\beta_{UIP}^{CRRA} = \frac{v_{x\sigma}}{\gamma^2 + v_{x\sigma}} > 0.$$

Indeed, both the expected currency depreciation and the forward premium load positively on the long-run risk differential  $x_t - x_t^*$ , which in a CRRA case is the only factor which drives the expected change in spot exchange rate. Naturally then, the regression coefficient is positive. When the variation in long-run risk differential shrinks and  $v_{x\sigma}$  is close to 0, the expected currency depreciation is constant through time, and the uncovered interest rate parity regression coefficient is effectively zero.

In a general generalized preferences case the contribution of the time-varying volatility of consumption growth and the separation of the risk aversion from the IES allow for a negative  $\beta_{UIP}$  under quite general empirically plausible scenarios. Recall that the two Fama's necessary conditions for  $\beta_{UIP} < 0$  are  $Cov(p_t, q_t) < 0$  and  $Var(p_t) > Var(q_t)$ . Using the expressions for  $p_t$  and  $q_t$  above it can be easily shown that the sign of the covariance between  $p_t$  and  $q_t$  in our model is the same as the sign of the market price of volatility risk  $\lambda_{m,w}$ . When  $\gamma > 1$ , they are negative for  $\psi > 1/\gamma$ .

As for the expected currency depreciation rate itself, for the calibrated parameter values discussed in the next section more than 95% of its variation is explained by the the volatilities of endowment streams. Thus, ignoring the contribution of the long-run risks differential,

$$Std(q_t) - Std(p_t) \approx r_\sigma Std(\sigma_t^2 - \sigma_t^{*2}),$$

which is negative for  $r_\sigma < 0$ . Therefore, the two Fama's conditions can be easily satisfied in our long-run risks and stochastic market volatility framework for an investor whose generalized preferences exhibit more risk-aversion than log utility and the intertemporal elasticity of substitution above unity.

## 2.4 Data and Model Implications

### 2.4.1 Data and the Growth Rate Dynamics

Following Bansal & Yaron (2004), we calibrate the model described in (8) and (10) at the monthly frequency. From this monthly model we derive time-aggregated annual growth rates of consumption and dividends to match key aspects of annual aggregate consumption and foreign exchange market data. Further, as in Campbell & Cochrane (1999) and Kandel & Stambaugh (1991), we assume that the decision interval of the agent is monthly but the targeted data to match are annual.<sup>7</sup>

We use five countries for our empirical analysis, such as United Kingdom, Germany, Japan and United States. US is chosen to be a "domestic" country, so that spot and forward exchange rates are expressed as dollars per unit of foreign currency. We extend the sample on foreign exchange and interest rates in Bansal & Dahlquist (2000) with monthly quotes from Datastream; the data from both sources agree very well for the overlapping periods with correlations of returns in excess of 0.95. The full sample contains spot and forward rates and Euro-Currency (LDN:FT) Middle Rates of 1 month to maturity for the period of January 1976 to November 2005 for Germany, UK and US and from July 1978 to November 2005 for Japan.

To construct CPI indices and real consumption growth for all the countries except the US we extend the data set from John Campbell using the IMF: International Financial Series CD-ROM.<sup>8</sup> For US consumption, BEA data on real per capita annual consumption growth of non-durables and services for the period 1929 to 1998 are utilized. This is the longest single data source of consumption data. Datastream is also used to construct monthly equity returns for the same time period on Morgan Stanley International Index for each country other than US. Dividends and the value-weighted market return data for the United States are taken from CRSP. Nominal asset returns are deflated using the expected inflation from ARMA(1,1) filter, while all the other nominal quantities are

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<sup>7</sup>The evidence regarding the model is based on numerical solutions using standard polynomial-based projection methods discussed in Judd (1998). The numerical results are quite close to those based on the approximate analytical solutions.

<sup>8</sup>The data is available at John Campbell's website at <http://kuznets.fas.harvard.edu/campbell/data.html>. The extrapolation techniques we use are consistent with earlier revisions of the data set and are also discussed on the web-site.

adjusted with the realized CPI. To facilitate comparisons between the model, which is calibrated to a monthly decision interval, and the annual data, the monthly model is time aggregated to the annual frequency to derive annual statistics.

Based on the longest and most accurate US sample, the annual real per-capita consumption growth mean is 1.96 percent and its standard deviation is about 2.17 percent. This volatility is somewhat lower for our sample than for the period considered in Mehra & Prescott (1985), Kandel & Stambaugh (1991) and Abel (1999). Tables 3 and 7 show that in the data, consumption growth has a large first-order autocorrelation coefficient, a small second-order one, and is positively correlated across the countries. The standard errors in the data for these auto and cross-correlations are sizeable. An alternative way to view the long horizon properties of the consumption and dividend growth rates is to use variance ratios which are themselves determined by the autocorrelations (see Cochrane (1988)). In the data the variance ratios first rise significantly and at about 7 years start to decline. The standard errors on these variance ratios, not surprisingly, are quite substantial. There is considerable evidence of small sample biases in estimating autoregression coefficients and variance ratios (see Hurwicz (1950) and Ansley & Newbold (1980)). To account for any small sample biases, we also report statistics based on 10,000 Monte-Carlo experiments each with 900 monthly observations corresponding to the 75 annual observations available in the annual data set.

The base-line calibration parameters are reported in Table 3. Specifically, as in Bansal & Yaron (2004) we set  $\rho$  at 0.979, which determines the persistence in the long run component in growth rates. Our choice of  $\varphi_e$  and  $\sigma$  ensures that the model matches the unconditional variance and the autocorrelation function of annual consumption growth. The standard deviation of the one-step ahead innovation in consumption, that is  $\sigma$ , equals 0.0075. This parameter configuration implies that the predictable variation in monthly consumption growth is very small. The exposure of the corporate sector to long run risks is governed by  $\phi$ , and its magnitude is similar to that in Abel (1999). The standard deviation of the monthly innovation in dividends,  $\varphi_d\sigma$ , is 0.03375.

As in Bansal & Yaron (2004) we consider the consumption and dividend dynamics that incorporate time varying volatility (see equation 8). The parameters of the volatility process are chosen to capture the persistence in consumption volatility. Based on the evidence of slow decay in volatility shocks, we calibrate  $\nu$ , the parameter governing the persistence of conditional volatility at 0.84. The shocks to the volatility process have very



small volatility,  $\sigma_w$  is calibrated at  $0.233 \times 10^{-5}$ . With this configuration the assumed consumption and dividend growth rates very closely match the key consumption and dividends data features reported in Table 7.

It is often argued that consumption and dividend growth, in the data, is close to being *i.i.d.* Bansal and Yaron show that their model of consumption and dividends is also consistent with the observed data on consumption and dividends growth rates. However, while the financial market data are hard to interpret from the perspective of the *i.i.d.* growth rate dynamics, Bansal and Yaron show that it is interpretable from the perspective of the growth rate dynamics that incorporate long run risks.

This issue is further considered in Shephard & Harvey (1990), Barsky & DeLong (1993) and Bansal & Lundblad (2002), who show that discrimination across the *i.i.d.* growth rate specification and the one that incorporates long run components is extremely difficult in finite samples. Given these difficulties in discrimination across models, Anderson, Hansen & Sargent (2002) utilize features of the long run growth rate dynamics developed in Bansal and Yaron for motivating economic models that incorporate robust control.

To match the international dimension of the data, we also consider the correlations between the short-run, long-run and volatility shocks across the countries. In particular, as in Colacito & Croce (2005) we assume a perfect covariance between the long-run shocks  $e_t$  and  $e_t^*$ , which implies a very high covariation between the long-run risks factors  $x_t$  and  $x_t^*$  themselves. Shocks to conditionally heteroscedastic market variances are assumed to co-vary with  $\eta_w = 0.9$ , and we also set the covariance between the short-run risks to consumption growth at  $\eta_w = 0.5$ . As shown in Table 8, the population correlation coefficient between the consumption growths is 0.7, which is somewhat higher than the sample counterparts presented in Table 4. In a standard power-utility setup, the correlation between the discount factors is equal to that between the consumption growths, as can be easily seen from expression (19). However, as discussed in Brandt, Cochrane & Santa-Clara (2005), this value is insufficient to explain the low volatility of exchange rates observed in the data. The generalized preferences break the perfect link between the consumption growth and the discount factor. The model implied correlation between the discount rates is 0.88, which brings the volatility of exchange rates to less than 16%, close to the values reported in the data.

### 2.4.2 Foreign Exchange Market Dynamics

Table 1 reports nominal statistics for the monthly depreciation rates of US dollar, forward premia, inflation and interest rates for the five countries, while Table 2 shows the real quantities. Average forward premia agree with the differences in reported means in interest rates; a more detailed analysis (not shown here) confirms that the covered interest rate parity holds reasonably well in our data set. The equity risk premium is slightly below 6% per annum and the real risk free rate is below 3%. The annual foreign exchange volatility is 10 - 12%, and that of the real risk free is quite small, of less than 1% per annum.

As Table 8 suggests, we can match the above values quite well. The model implied risk premium on the stock market portfolio is 4.9%, and the risk free rate is 1.5%. The volatility of the exchange rate is 16%, while those of the dividend asset and risk-free rate are 20.1% and 1.3%, respectively.

Table 5 reports the centered uncovered interest rate parity regression of the form,

$$s_{t+1} - s_t - (r_t - r_t^*) = \alpha + \beta_{CUIP}(r_t - r_t^*) + \xi_t. \quad (26)$$

The left-hand side represents the realized dollar excess return on an uncovered position in foreign bonds; its conditional expectation corresponds to the negative of the risk premium,  $-p_t$  defined in (24). The projection coefficient above is equal to the standard slope coefficient in uncovered interest rate parity regression minus 1:

$$\beta_{CUIP} = \beta_{UIP} - 1,$$

so that it should be equal to 0 under the null of the uncovered interest rate parity hypothesis. As can be seen from Table 5, in the data this coefficient is negative and significant for all the countries, both in real and nominal regressions. The results are similar on quarterly frequency. From Table 8 the model implied regression coefficient is  $-2.91$  which is very similar to the values in the sample. Baillie & Bollerslev (2000) and Maynard & Phillips (2001) suggest the principal failure of the uncovered interest rate parity hypothesis could be statistical in nature, caused by the difference in persistence between the two series. Maynard (2006) revisits the arguments using the robust econometric methods and finds that while the statistical theory could play the role in the anomaly, the successful resolution of the puzzle belongs to economics.

The theoretical arguments presented in the previous section show that a rise in consumption volatility differential increases the ex-ante dollar return on foreign bond (see expression (24)). Figure 4 uses our international data to highlight the volatility channel. The expected excess returns on foreign bonds are obtained from the projection of the realized returns on the forward premium above, while the consumption volatility measures are constructed by averaging up 4.5 years of the absolute value of consumption residuals, see Bansal, Khatchatrian & Yaron (2005) for additional details. It is evident from the graphs that a rise in the domestic consumption volatility increases the ex-ante dollar returns for all countries under consideration, and the fit substantially improves in the second part of the sample —this highlights the volatility channel provided by the generalized preference structure.

In Appendix we present a solution for the *level* of the spot exchange rate implied by the model. In particular, the impulse response of the spot rate at horizon  $j \geq 0$  to the one-deviation shock to the volatility differential  $w_{t+1} - w_{t+1}^*$  can be expressed in the following way:

$$\Phi_j(w) = -\frac{(\gamma - \frac{1}{\psi})(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \sigma_w \left( \frac{\nu^j}{1 - \nu} + \frac{\kappa_1}{1 - \kappa_1 \nu} - \frac{1}{1 - \nu} \right)$$

Thus, an increase in domestic uncertainty leads to an *appreciation* of domestic currency now and in the future. However, because the effect of the volatility is strongest for contemporaneous spot rate and then dies off, relative to the today's rate future spot prices of the domestic currency is expected to go down.

This tendency of the dollar to be strong when the domestic uncertainty exceeds the foreign one is supported by the data. We plot the spot prices of the foreign currency against the volatility differential measure in Figures 5 - 7. This effect is especially pronounced for the United Kingdom, with contemporaneous correlation of  $-0.5$ , but the negative covariation also obtains for Germany ( $-0.2$ ) and Japan ( $-0.1$ ).

### 2.4.3 Preference Parameters

The preference parameters take account of economic considerations. The time preference parameter  $\delta < 1$ , and the risk aversion parameter  $\gamma$  is set at 6. Mehra & Prescott (1985) do not entertain risk aversion values larger than 10. As in Bansal and Yaron, we focus on

an IES of 1.5 — an IES value larger than one is important for our quantitative results.

There is considerable debate about the magnitude of the IES. Hansen & Singleton (1982) and Attanasio & Weber (1989) estimate the IES to be well in excess of 1. More recently, Guvenen (N.d.) and Attanasio & Vissing-Jorgensen (2003) also estimate the IES over one — they show that their estimates are close to that used in Bansal and Yaron. However, Hall (1988) and Campbell (1999) estimate the IES to be well below one. Bansal & Yaron (2004) argue that the low IES estimates of Hall and Campbell are based on a model without time varying volatility. They show that ignoring the effects of time-varying consumption volatility leads to a serious downward bias in the estimates of the IES. If the population value of the IES in the Bansal and Yaron model is 1.5, then the estimated value of the IES using Hall estimation methods, will be less than 0.3. Bansal and Yaron show that with fluctuating consumption volatility the projection of consumption growth on the level of the risk free rate does not equal the IES, leading to the downward bias. This suggests that Hall and Campbell’s estimates are not a robust guide for calibrating the IES.

In addition to the above arguments, we showed that  $\psi > 1$  plays an important role in satisfying Fama’s conditions. Indeed, Figure 3 examines the sensitivity of the theoretical slope coefficient  $\beta_{UIP}$  to  $\psi$  when all the other parameters of the model are set at their baseline calibration values. As the graph suggests, only for  $\psi > 1$  can the model produce a negative slope coefficient —this highlights the volatility channel and motivates the specification of IES larger than one.

#### 2.4.4 Long-Horizon Projection

Using the solution to the discount factor in (15) and (17), we can derive the slope coefficient in the multi-period uncovered interest rate parity regression of the form,

$$s_{t+n} - s_t = \alpha + \beta_{UIP,n}(r_{t,t+n} - r_{t,t+n}^*) + \xi_{t,t+n},$$

where  $r_{t,t+n}$  stands for the return on a domestic discount bond with maturity  $n$ , and similarly for  $r_{t,t+n}^*$ . While the details of the derivation are presented in the Appendix, the term structure of the slope coefficient for the assumed calibration values is shown in Figure 2. Recall that the variation in the expected depreciation is driven by the long-run risks factors which correlate positively with the risk-free rate differentials, and volatility

factors which have a negative correlation with them. At shorter horizons, the second effect dominates, so that the uncovered interest rate parity coefficient is negative. When the horizon increases beyond 16 months, the long-run risks contribution outweighs that of the volatility differential, and the regression coefficient becomes positive.

## Conclusion

We extend a discrete-time general equilibrium model developed by Bansal & Yaron (2004), which uses long-run risks in endowment streams, stochastic volatility of the market and the generalized utility of the representative agent, to explain the joint behavior of exchange rates and interest rates between the countries. Separation of the risk aversion from the inter-temporal elasticity of substitution and the ability to price long-run and volatility risks open up a channel to rationalize the key features of the domestic and international financial markets.

The numerical calibrations show that we can match the salient properties of the consumption data within and across the countries, and account for high ex-ante excess returns on stocks and low risk free rate, high volatility of the stock market and low variation in exchange rate, and negative coefficients in the foreign exchange projections. As for the last one, due to a positive shock in domestic uncertainty, domestic short-term bond rates decrease and the domestic currency is expected to depreciate, contrary to the conventional logic of the uncovered interest rate hypothesis. As documented elsewhere, the relationship between interest rates and exchange rates at longer maturities better conforms with the uncovered interest rate hypothesis, and we are able to reproduce this result in our model, too.

We also present an empirical support for the role of the volatilities of the endowment streams across the countries in explaining the foreign exchange markets. In the data, the volatility differential is positively related to the foreign exchange risk premium, and negatively to the spot prices of the foreign currencies, as predicted by the theory.

# Appendix

## A1 Spot Exchange Rate

We solve for the level of the spot exchange rate by iterating on a complete-markets condition,

$$s_t - s_{t-1} = m_t^* - m_t,$$

to obtain that

$$s_t = \sum_{i=0}^{\infty} (m_{t-i}^* - m_{t-i}).$$

In particular the resulting solution for the spot rate is,

$$\begin{aligned} s_t = & -\frac{1}{\psi} \frac{\rho}{1-\rho} (x_{t-1} - x_{t-1}^*) + \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \frac{\nu}{1 - \nu} (\sigma_{t-1}^2 - \sigma_{t-1}^{*2}) \\ & + (\gamma - \frac{1}{\psi}) \xi (\sigma_{t-1} e_t - \sigma_{t-1}^* e_t^*) + \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \frac{\nu}{1 - \nu} \frac{\kappa_1 \sigma_w}{1 - \kappa_1 \nu} (w_t - w_t^*) \\ & + \sum_{i=0}^{\infty} \gamma (\sigma_{t-i-1} \eta_{t-i} - \sigma_{t-i-1}^* \eta_{t-i}^*) \\ & + \sum_{i=1}^{\infty} \left( (\gamma - \frac{1}{\psi}) \frac{\kappa_1 \sigma_w}{1 - \kappa_1 \nu} + \frac{1}{\psi} \frac{\varphi}{1 - \rho} \right) (\sigma_{t-i-1} e_{t-i} - \sigma_{t-i-1}^* e_{t-i}^*) \\ & + \sum_{i=1}^{\infty} \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \frac{\nu}{1 - \nu} \frac{\kappa_1 \sigma_w}{1 - \kappa_1 \nu} \left( \frac{\kappa_1}{1 - \kappa_1 \nu} - \frac{1}{1 - \nu} \right) (w_{t-i} - w_{t-i}^*). \end{aligned}$$

Therefore, a one standard deviation shock to volatility differential  $w_t$  implies that the response of the spot rate at horizon  $j \geq 0$  is equal to,

$$\Phi_j(w) = -\frac{(\gamma - \frac{1}{\psi})(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \sigma_w \left( \frac{\nu^j}{1 - \nu} + \frac{\kappa_1}{1 - \kappa_1 \nu} - \frac{1}{1 - \nu} \right)$$

## A2 Long-Horizon Projections

Let  $m_{t+1}$  stand for a log discount factor which gives today's price of the payoffs tomorrow.

To discount payoffs  $n \geq 1$  periods from now we use

$$m_{t,t+n} \equiv \sum_{k=1}^n m_{t+k}.$$

Using the solution to the one-period discount factor and the dynamics of the consumption growth, we can extend our results for the multi-period discount factor in the following way:

$$E_t m_{t,t+n} = a_{0,n} + a_{x,n} x_t + a_{\sigma,n} \sigma_t^2, \quad (27)$$

$$Var_t m_{t,t+n} = 2(b_{0,n}^2 + b_{\sigma,n}^2 \sigma_t^2), \quad (28)$$

for

$$\begin{aligned} a_{x,n} &= -\frac{1}{\psi} \frac{1 - \rho^n}{1 - \rho}, \quad (29) \\ a_{\sigma,n} &= \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2 \right] \frac{1 - \nu^n}{1 - \nu}, \\ b_{\sigma,n}^2 &= \frac{\gamma^2 + (\gamma - \frac{1}{\psi})^2 \left( \frac{\kappa_1 \varphi}{1 - \kappa_1 \rho} \right)^2}{2} \frac{1 - \nu^n}{1 - \nu} + \frac{\varphi^2}{2\psi^2(1 - \rho)^2} \left( \frac{1 - \nu^n}{1 - \nu} - 2\frac{\rho^n - \nu^n}{\rho - \nu} + \frac{\rho^{2n} - \nu^n}{\rho^2 - \nu} \right) \\ &\quad + \frac{\varphi \xi (\gamma - \frac{1}{\psi})}{\psi(1 - \rho)} \left( \frac{1 - \nu^n}{1 - \nu} - \frac{\rho^n - \nu^n}{\rho - \nu} \right). \quad (30) \end{aligned}$$

Now, when the countries are symmetric, the interest differential on  $n$ -months-to-maturity bonds and the expected depreciation rate over this horizon can be expressed as,

$$\begin{aligned} r_{t,t+n} - r_{t,t+n}^* &= -(E_t m_{t,t+n} + \frac{1}{2} Var_t m_{t,t+n}) + (E_t m_{t,t+n}^* + \frac{1}{2} Var_t m_{t,t+n}^*) \\ &= (p_{0,n} + q_{0,n}) + a_{x,n}(x_t^* - x_t) + (a_{\sigma,n} + b_{\sigma,n}^2)(\sigma_t^{*2} - \sigma_t^2); \end{aligned}$$

$$\begin{aligned} q_{t,t+n} &= E_t m_{t,t+n}^* - E_t m_{t,t+n} \\ &= q_{0,n} + a_{x,n}(x_t^* - x_t) + a_{\sigma,n}(\sigma_t^{*2} - \sigma_t^2) \end{aligned}$$

Therefore, the expression for the slope coefficient in a multi-period UIP regression is similar to the one in a single-period case:

$$\beta_{UIP,n} = \frac{a_{\sigma,n}(a_{\sigma,n} + b_{\sigma,n}^2) + a_{x,n}^2 v_{x\sigma}}{(a_{\sigma,n} + b_{\sigma,n}^2)^2 + a_{x,n}^2 v_{x\sigma}},$$

for  $v_{x\sigma}$  defined as in the main body of the paper,

$$v_{x\sigma} = \frac{\text{Var}(x_t^* - x_t)}{\text{Var}(\sigma_t^{*2} - \sigma_t^2)}.$$



### 3 Tables and Figures

Nominal Series		UK	Germany	Japan	USA
<i>FX Changes:</i>					
	Mean	-0.53	1.47	3.10	
	Std. Dev.	10.69	11.18	12.09	
<i>Forward Premia:</i>					
	Mean	-2.33	1.58	3.25	
	Std. Dev.	0.88	0.89	0.86	
<i>Inflation Rates:</i>					
	Mean	5.53	2.53	1.85	4.24
	Std. Dev.	2.04	0.96	1.86	1.15
<i>Interest Rates:</i>					
	Mean	9.15	5.27	3.57	6.82
	Std. Dev.	1.13	0.73	0.94	1.08

Table 1: Summary Statistics for the one month Nominal values for the change in spot exchange rate, forward premium, inflation rate and interest rate for UK, Germany, Japan and US. Spot and forward exchange rates are defined per unit of foreign currency. The data is collected at the end of month, from Jan 1976 (July 1978 for Japanese interest rate) to Nov 2005. Means and standard deviations are expressed in per cent per annum.

Real Series		UK	Germany	Japan	USA
<i>FX Changes:</i>					
	Mean	0.72	-0.25	0.65	
	Std. Dev.	10.99	11.27	12.32	
<i>Interest Rates:</i>					
	Mean	3.38	2.69	1.78	2.57
	Std. Dev.	0.76	0.53	0.96	0.96
<i>Interest Differential:</i>					
	Mean	-0.80	-0.11	0.98	
	Std. Dev.	1.00	0.85	0.96	
<i>Equity Premium:</i>					
	Mean	5.46	4.86	2.83	5.50
	Std. Dev.	17.30	20.78	18.56	14.74

Table 2: Summary Statistics for the one month Real values for the change in spot exchange rate, interest rate, interest rate differential and equity returns for UK, Germany, Japan and US. Real interest rates are obtained deflating the respective nominal ones by the expected inflation; real change in FX is obtained from the nominal one subtracting the realized inflation rate differential. Spot and forward exchange rates are defined per unit of foreign currency; interest rate differential is equal to US minus foreign. The data on exchange, interest and inflation rates and equity returns is collected at the end of month, from Jan 1976 (July 1978 for Japanese interest rate) to Nov 2005 (Dec 2004 for US Equity returns). Means and standard deviations are expressed in per cent per annum.

Consumption Growth	UK	Germany	Japan	US	US 29
Mean	2.08	1.76	1.97	1.95	1.96
Std. Dev	2.55	1.66	1.90	1.11	2.17
AR(1)	0.32	0.54	0.23	0.31	0.47

Table 3: Summary Statistics based on the annual data on real consumption growth in UK, Germany, Japan and US from 1972 to 2004, and from 1929 to 2004 for US 29.

Consumption Growth	Germany	Japan	USA
UK	-0.04	0.36	0.53
Germany		0.42	0.29
Japan			0.39

Table 4: Correlation coefficients between the real consumption growths in Germany, Japan, UK and US based on the annual data from 1972 to 2004.

Uncovered Excess Return	Constant	Int Differential	$R^2, \%$
<i>Nominal Regression:</i>			
UK	-4.36 (2.55)	-2.63 (0.94)	3.75
Germany	2.73 (2.36)	-1.81 (0.77)	1.9
Japan	10.84 (2.94)	-3.77 (0.66)	5.78
<i>Real Regression:</i>			
UK	-0.25 (2.10)	-2.20 (0.80)	3.89
Germany	-0.43 (2.20)	-1.74 (0.72)	1.68
Japan	0.28 (2.38)	-2.35 (0.51)	3.19

Table 5: Centered Uncovered Interest Rate Parity Regressions for Germany, Japan and UK versus USA, for the differentials in Euro-Currency (LDN:FT) Middle Rates of 1 Month to Maturity. The regressions are performed at Monthly Frequency, from Jan 1976 to Nov 2005 for Germany, UK and US, and from July 1978 to Nov 2005 for Japan. Nominal returns are adjusted by the expected inflation rates; nominal changes in FX by realized ones. Standard Errors are Newey-West with 10 lags.

Parameter		Value
Subjective discount factor	$\delta$	0.9993
Inter-temporal elasticity of substitution	$\psi$	1.5
Risk aversion coefficient	$\gamma$	6.00
Persistence in long-run risks	$\rho$	0.9792
Volatility loading of long-run risks	$\varphi$	0.044
Long-run volatility of consumption growth	$\sigma$	7.5e-03
Mean of consumption growth	$\mu_c$	0.0015
Persistence in volatility	$\nu$	0.84
Volatility of market variance	$\sigma_w$	2.33e-06
Correlation of volatility shocks	$\tau_w$	0.90
Correlation of long-run shocks	$\tau_e$	1.00
Correlation of short-run shocks	$\tau_\eta$	0.50
Dividend leverage	$\phi$	3.5
Volatility loading of dividend growths	$\varphi_d$	4.5
Correlation of consumption and dividend innovations	$\alpha$	0.3

Table 6: Baseline Calibration Parameters.

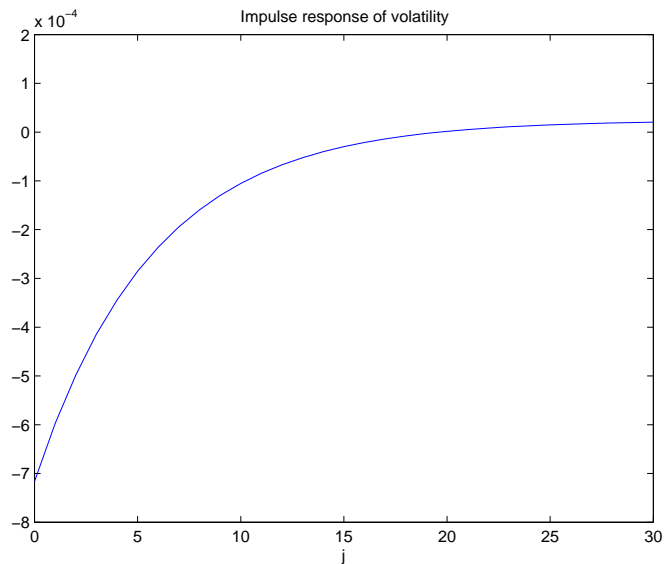


Figure 1: Response of the spot rate to the volatility shock  $j$  periods before.

Variable	Data		Model				
	Estimate	S.E.	Mean	95%	5%	P-val	Pop
$\sigma(g)$	2.17	(0.60)	2.72	3.25	2.26	0.98	2.81
AR(1)	0.44	(0.11)	0.47	0.64	0.27	0.61	0.52
AR(2)	0.16	(0.14)	0.23	0.47	-0.02	0.68	0.29
AR(5)	-0.01	(0.09)	0.07	0.32	-0.18	0.69	0.14
AR(10)	0.04	(0.06)	-0.03	0.23	-0.28	0.34	0.04
VR(2)	1.58	(0.15)	1.46	1.64	1.27	0.16	1.52
VR(5)	2.23	(0.83)	2.18	3.00	1.41	0.46	2.43
VR(10)	1.74	(0.52)	2.68	4.54	1.19	0.81	3.23
$\sigma(g_d)$	10.89	(2.69)	11.23	13.14	9.48	0.64	11.53
AR(1)	0.24	(0.13)	0.40	0.57	0.21	0.92	0.44
$Corr(g, g_d)$	0.54	(0.20)	0.51	0.68	0.33	0.43	0.55

Table 7: Data and Model output. The statistics for the data are based on the annual series from 1930 to 2005. Standard errors are Newey-West corrected using 10 lags. Model output is based on 10,000 simulations of 900 months aggregated to yearly horizon.

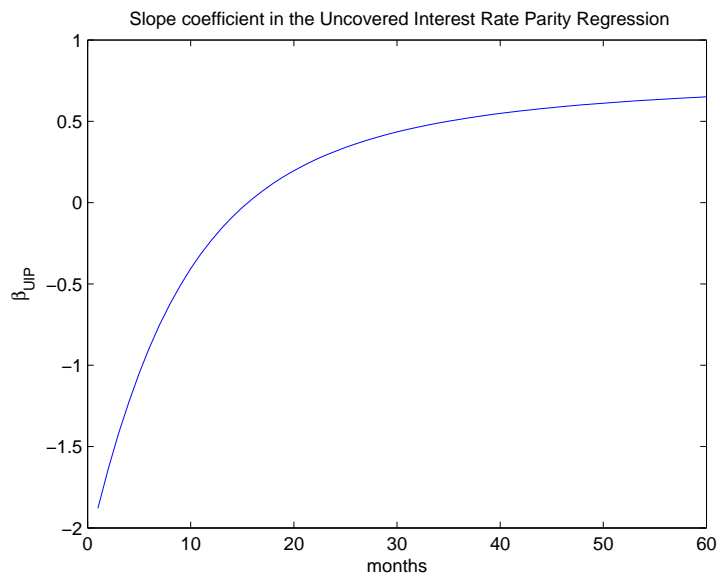


Figure 2: Term-structure of  $\beta_{ucp}$ .

Parameter	Sim Mean	Sim Std Dev.
<i>Equity Market:</i>		
$E_t R_{m,t+1} - R_{f,t}$	4.72	0.25
$\sigma(R_{m,t+1})$	18.67	0.45
$E(R_{ft})$	1.47	0.03
$\sigma(R_{ft})$	0.38	0.01
$AR(1) R_{ft}$	0.98	0.00
<i>FX Market:</i>		
$\beta_{FX}$	-3.08	4.51
$\sigma(s_{t+1} - s_t)$	15.59	0.02
$AR(1) s_{t+1} - s_t$	0.00	0.00
$Corr(g, g^*)$	0.71	0.01
$Corr(m, m^*)$	0.89	0.01

Table 8: Model Implications.  $g_{t+1}$  is the consumption growth,  $s_{t+1}$  is the spot exchange rate.  $R_m$  represent the level return on the market portfolio.  $\beta_{CUIP}$  is the population coefficient in the Centered Uncovered Interest Parity regression. The model is simulated 1000 times on monthly frequency (250,000 months) which is then aggregated to produce annual observations and compute the statistics. Slope coefficient  $\beta_{CUIP}$  is computed on monthly frequency. Means and volatilities are annualized and multiplied by 100.

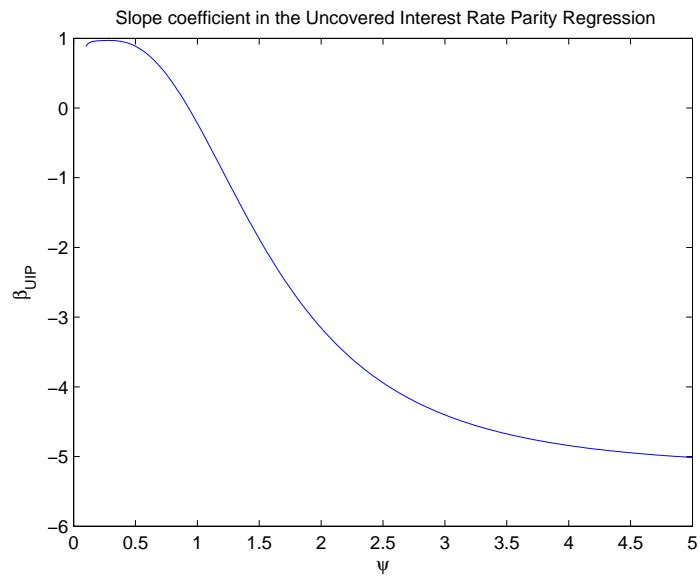


Figure 3:  $\beta_{ucp}$ . against the inter-temporal elasticity of substitution  $\psi$ .

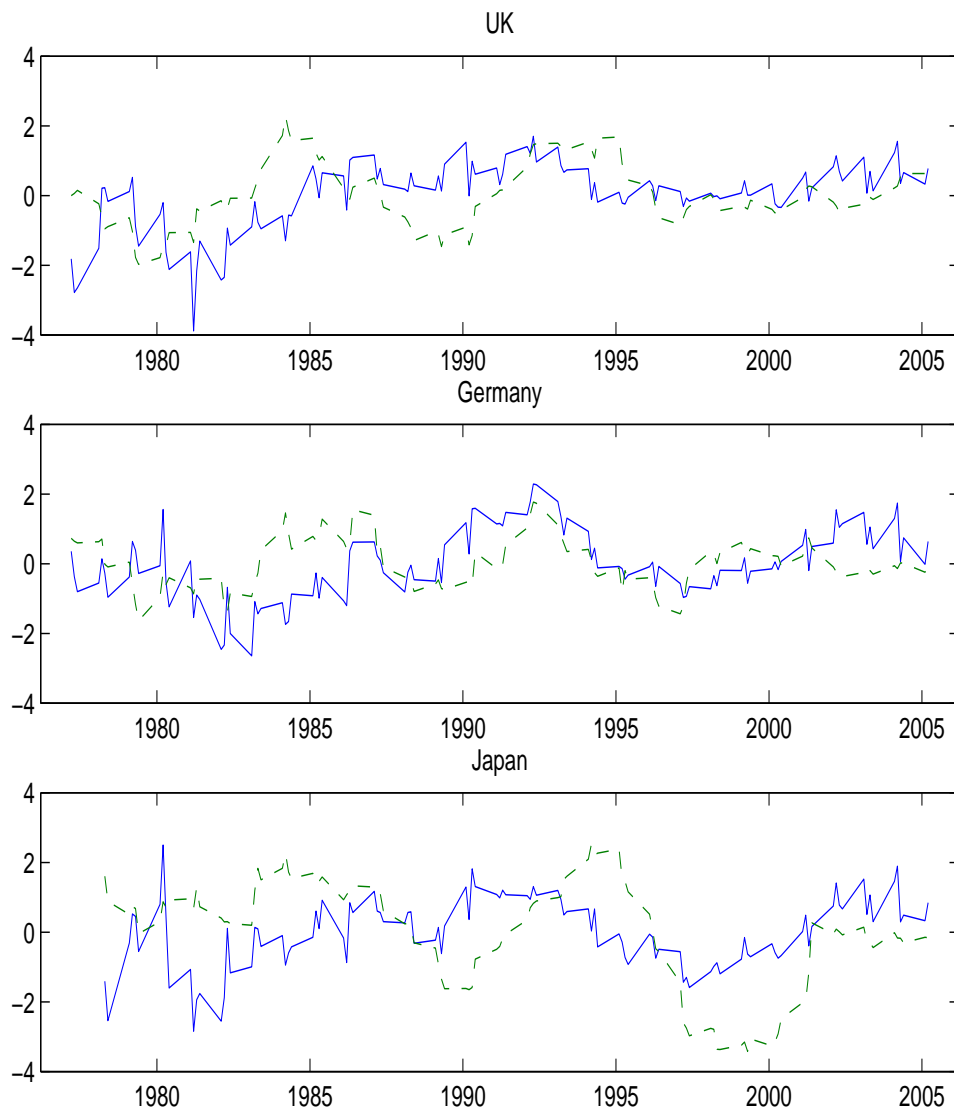


Figure 4: Fitted FX projections (solid line) against Volatility differentials (dashed line). Volatility proxies are based on 4.5 year sum of absolute residuals from AR(3) regression of consumption growths. Volatilities and FX projections are normalized.



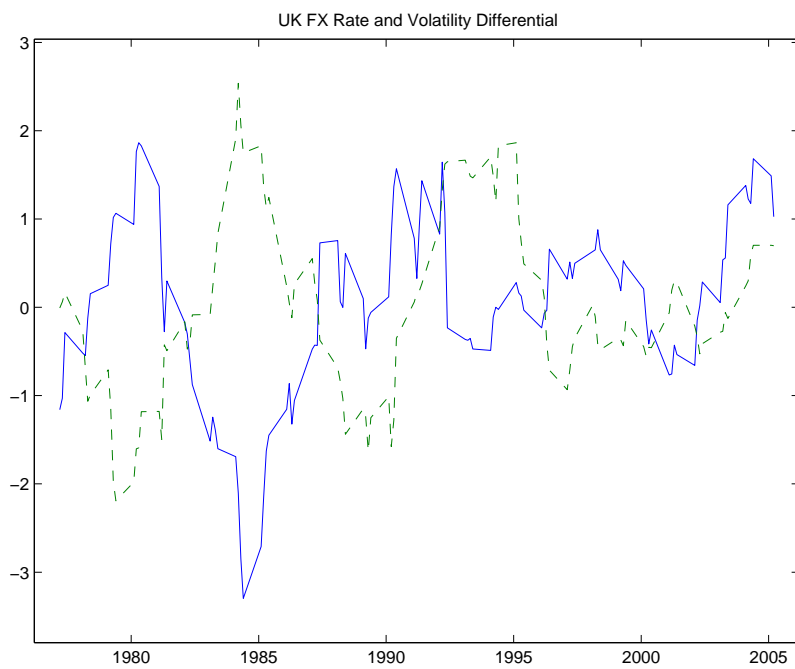


Figure 5: UK spot exchange rate (solid line) and the volatility differential between the US and UK (dashed line). Volatility proxies are based on 4.5 year sum of absolute residuals from AR(3) regression of consumption growths. Volatilities and spot rates are normalized for better visibility. Contemporaneous correlation is  $-0.47$ .

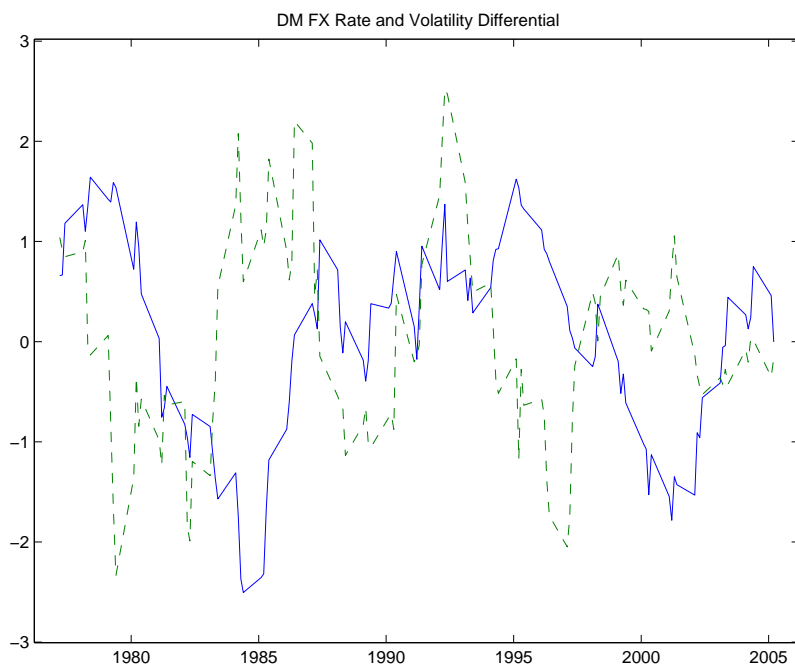


Figure 6: German spot exchange rate (solid line) and the volatility differential between the US and Germany (dashed line). Volatility proxies are based on 4.5 year sum of absolute residuals from AR(3) regression of consumption growths. Volatilities and spot rates are normalized for better visibility. Contemporaneous correlation is  $-0.18$ .

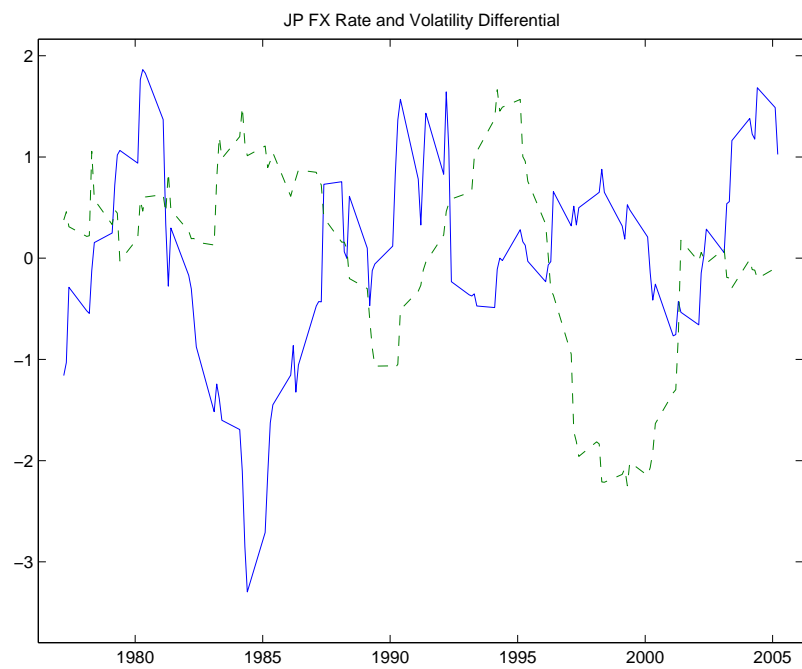


Figure 7: Japanese spot exchange rate (solid line) and the volatility differential between the US and Japan (dashed line). Volatility proxies are based on 4.5 year sum of absolute residuals from AR(3) regression of consumption growths. Volatilities and spot rates are normalized for better visibility. Contemporaneous correlation is  $-0.10$ .

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