Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2013

UNDERSTANDING ASSET PRICES

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UNDERSTANDING ASSET PRICES

1. Introduction

The behavior of asset prices is essential for many important decisions, not only for professional investors but also for most people in their daily life. The choice between saving in the form of cash, bank deposits or stocks, or perhaps a single-family house, depends on what one thinks of the risks and returns associated with these different forms of saving. Asset prices are also of fundamental importance for the macroeconomy because they provide crucial information for key economic decisions regarding physical investments and consumption. While prices of financial assets often seem to reflect fundamental values, history provides striking examples to the contrary, in events commonly labeled bubbles and crashes. Mispricing of assets may contribute to financial crises and, as the recent recession illustrates, such crises can damage the overall economy. Given the fundamental role of asset prices in many decisions, what can be said about their determinants?

This year's prize awards empirical work aimed at understanding how asset prices are determined. *Eugene Fama*, *Lars Peter Hansen* and *Robert Shiller* have developed methods toward this end and used these methods in their applied work. Although we do not yet have complete and generally accepted explanations for how financial markets function, the research of the Laureates has greatly improved our understanding of asset prices and revealed a number of important empirical regularities as well as plausible factors behind these regularities.

The question of whether asset prices are predictable is as central as it is old. If it is possible to predict with a high degree of certainty that one asset will increase more in value than another one, there is money to be made. More important, such a situation would reflect a rather basic malfunctioning of the market mechanism. In practice, however, investments in assets involve risk, and predictability becomes a statistical concept. A particular asset-trading strategy may give a high return on average, but is it possible to infer excess returns from a limited set of historical data? Furthermore, a high average return might come at the cost of high risk, so predictability need not be a sign of market malfunction at all, but instead just a fair compensation for risk-taking. Hence, studies of asset prices necessarily involve studying risk and its determinants.

Predictability can be approached in several ways. It may be investigated over different time horizons; arguably, compensation for risk may play less of a role over a short horizon, and thus looking at predictions days or weeks ahead simplifies the task. Another way to assess predictability is to examine whether prices have incorporated all publicly available information. In particular, researchers have studied instances when new information about assets becomes became known in the marketplace, i.e., so-called event studies. If new information is made public but asset prices react only slowly and sluggishly to the news, there is clearly predictability: even if the news itself was impossible to predict, any subsequent movements would be. In a seminal event study from 1969, and in many other studies, Fama and his colleagues studied short-term predictability from different angles. They found that the amount of short-run predictability in stock markets is very limited. This empirical result has had a profound impact on the academic literature as well as on market practices.

If prices are next to impossible to predict in the short run, would they not be even harder to predict over longer time horizons? Many believed so, but the empirical research would prove this conjecture incorrect. Shiller's 1981 paper on stock-price volatility and his later studies on longer-term predictability provided the key insights: stock prices are excessively volatile in the short run, and at a horizon of a few years the overall market is quite predictable. On average, the market tends to move downward following periods when prices (normalized, say, by firm earnings) are high and upward when prices are low.

In the longer run, compensation for risk should play a more important role for returns, and predictability might reflect attitudes toward risk and variation in market risk over time. Consequently, interpretations of findings of predictability need to be based on theories of the relationship between risk and asset prices. Here, Hansen made fundamental contributions first by developing an econometric method – the Generalized Method of Moments (GMM), presented in a paper in 1982 – designed to make it possible to deal with the particular features of asset-price data, and then by applying it in a sequence of studies. His findings broadly supported Shiller's preliminary conclusions: asset prices fluctuate too much to be reconciled with standard theory, as represented by the so-called Consumption Capital Asset Pricing Model (CCAPM). This result has generated a large wave of new theory in asset pricing. One strand extends the CCAPM in richer models that maintain the rational-investor assumption. Another strand, commonly referred to as behavioral finance – a new field inspired by Shiller's early writings – puts behavioral biases, market frictions, and mispricing at center stage.

A related issue is how to understand differences in returns across assets. Here, the classical Capital Asset Pricing Model (CAPM) – for which the 1990 prize was given to William Sharpe – for a long time provided a basic framework. It asserts that assets that correlate more strongly with the market as a whole carry more risk and thus require a higher return in compensation. In a large number of studies, researchers have attempted to test this proposition. Here, Fama provided seminal methodological insights and carried out a number of tests. It has been found that an extended model with three factors – adding a stock's market value and its ratio of book value to market value – greatly improves the explanatory power relative to the single-factor CAPM model. Other factors have been found to play a role as well in explaining return differences across assets. As in the case of studying the market as a whole, the cross-sectional literature has examined both rational-investor–based theory extensions and behavioral ones to interpret the new findings.

This document is organized in nine sections. Section 2 lays out some basic asset-pricing theory as a background and a roadmap for the remainder of the text. Sections 3 and 4 discuss short- and longer-term predictability of asset prices, respectively. The following two sections discuss theories for interpreting the findings about predictability and tests of these theories, covering rational-investor–based theory in Section 5 and behavioral finance in Section 6. Section 7 treats empirical work on cross-sectional asset returns. Section 8 briefly summarizes the key empirical findings and discusses their impact on market practices. Section 9 concludes this scientific background.

2. Theoretical background

In order to provide some background to the presentation of the Laureates' contributions, this section will review some basic asset-pricing theory.

2.1 Implications of competitive trading

A set of fundamental insights, which go back to the 19th century, derive from a basic implication of competitive trading: the absence of arbitrage opportunities. An arbitrage opportunity is a "money pump," which makes it possible to make arbitrary amounts of money without taking on any risk. To take a trivial example, suppose two assets pay safe rates of return R_a and R_b, where $R_a > R_b$. If each asset can be sold short, i.e., held in negative amounts, an arbitrage gain could be made by selling asset b short and investing the proceeds in asset a: the result would be a safe rate profit of $R_a - R_b$. Because this money pump could be operated at any scale, it would clearly not be consistent with equilibrium; in a competitive market, R_a and R_b must be equal. Any safe asset must bear the same return R_f (f for safe); the rate at which future payoffs of any safe asset are "discounted."

This simple reasoning can be generalized quite substantially and, in particular, can deal with uncertain asset payoffs. The absence of arbitrage opportunities can be shown to imply that the price of any traded asset can be written as a weighted, or discounted, sum of the payoffs of the asset in the different states of nature next period, with weights independent of the asset in question (see, e.g., Ross, 1978 and Harrison and Kreps, 1979). Thus, at any time *t*, the price of any given asset *i* is given by

$$P_{i,t} = \sum_s \pi_{t+1}(s) m_{t+1}(s) x_{i,t+1}(s).$$

Here, s denotes a state of nature, the πs the probabilities with which these states occur, and the ms non-negative discounting weights. The xs are the payoffs, which in the case of stocks are defined as next-period price plus dividends: $x_{i,t+1} = P_{i,t+1} + d_{i,t+1}$. In general, all these items depend on the state of nature. Note that the discounting weights *m* are the same for all assets.[1] They matter for the price of an individual asset *i* only because both *m* and x_i depend on *s*.

For a safe asset *f*, x does not depend on s, and the formula becomes

$$P_{f,t} = x_{f,t+1} \sum_s \pi_{t+1}(s) m_{t+1}(s).$$

Thus, we can now interpret $\sum_s \pi_{t+1}(s) m_{t+1}(s)$ as defining the time *t* risk-free discount rate $R_{f,t}$ for safe assets:

$$\sum_s \pi_{t+1}(s) m_{t+1}(s) \equiv 1/\left(1 + R_{f,t}\right).$$

More generally, though, the dependence of $m_{t+1}(s)$ on the state of nature s captures how the discounting may be stronger in some states of nature than in others: money is valued differently in different states. This allows us to capture how an asset's risk profile is valued by the market. If it pays off particularly well in states with low weights, it will command a lower price.

[1] In addition, if markets are complete (i.e., if there are as many independent assets as there are states of nature), the m that determines the prices for all assets is also unique.

The no-arbitrage pricing formula is often written more abstractly as

$$P_{i,t} = E_t(m_{t+1}x_{i,t+1}), \qquad (1)$$

where E now subsumes the summation and probabilities: it is the expected (probability-weighted) value. This formula can be viewed as an organizational tool for much of the empirical research on asset prices. With $x_{i,t+1} = P_{i,t+1} + d_{i,t+1}$, equation (1) can be iterated forward to yield the price of a stock as the expected discounted value of future dividends.[2]

Are asset prices predictable?

Suppose, first, that we consider two points in time very close to each other. In this case, the safe interest rate is approximately zero. Moreover, over a short horizon, m might be assumed not to vary much across states: risk is not an issue. These assumptions are tantamount to assuming that m equals 1. If the payoff is simply the asset's resale value P_{t+1}, then the absence of arbitrage implies that $P_t = E_t P_{t+1}$. In other words, the asset price may go up or down tomorrow, but any such movement is unpredictable: the price follows a martingale, which is a generalized form of a random walk. The unpredictability hypothesis has been the subject of an enormous empirical literature, to which Fama has been a key contributor. This research will be discussed in Section 3.

Risk and the longer run

In general, discounting and risk cannot be disregarded, so tests of the basic implications of competitive trading need to account for the properties of the discount factor m: how large it is on average, how much it fluctuates, and more generally what its time series properties are. Thus, a test of no-arbitrage theory also involves a test of a specific theory of how m evolves, a point first emphasized by Fama (1970).

Suppose we look at a riskless asset f and a risky asset i. Then equation (1) allows us to write the asset's price as

$$P_{i,t} = \frac{E_t(x_{i,t+1})}{1+R_{f,t}} + var_t(x_{i,t+1})\frac{cov_t(m_{t+1},x_{i,t+1})}{var_t(x_{i,t+1})}.$$

[2] This presumes the absence of a bubble, i.e., that the present value of dividends goes to zero as time goes to infinity. See Tirole (1985).

The discount factor $m_{t+1}(s)$ can be regarded as the value of money in state s. The above pricing equation thus says that the asset's value depends on the covariance with the value of money. If the covariance is negative, i.e., if the asset's payoff x is high when the value of money is low, and vice versa, then the asset is less valuable than the expected discounted value of the payoff. Moreover, the discrepancy term can be factorized into two parts: $var_t(x_{i,t+1})$, the "risk loading" (amount of risk), and $\frac{cov_t(m_{t+1}x_{i,t+1})}{var_t(x_{i,t+1})}$, the "risk exposure," of the asset.

The pricing formula can alternatively be expressed in terms of expected excess returns over the risk-free asset: $E_t[(R_{i,t+1} - R_{f,t})m_{t+1}] = 0$, where $1 + R_{i,t+1} = x_{i,t+1}/P_t$. This allows us to write

$$E_t R_{i,t+1} - R_{f,t} = -(1 + R_{f,t}) cov_t(m_{t+1} R_{i,t+1}).$$

An asset whose return is low in periods when the stochastic discount factor is high (i.e., in periods where investors value payoffs more) must command a higher "risk premium" or excess return over the risk-free rate. How large are excess returns on average? How do they vary over time? How do they vary across different kinds of assets? These fundamental questions have been explored from various angles by Fama, Hansen and Shiller. Their findings on price predictability and the determinants and properties of risk premia have deepened our understanding of how asset prices are formed for the stock market as a whole, for other specific markets such as the bond market and the foreign exchange market, and for the cross-section of individual stocks. In Section 4, we will discuss the predictability of asset prices over time, whereas cross-sectional differences across individual assets will be treated in Section 7.

2.2 Theories of the stochastic discount factor

The basic theory, described above, is based on the absence of arbitrage. The obvious next step is to discuss the determinants of the stochastic discount factor *m*. Broadly speaking, there are two approaches: one based on rational investor behavior, but possibly involving institutional complications, investor heterogeneity, etc., and an alternative approach based on psychological models of investor behavior, often called behavioral finance.

Rational-investor theory

Theory based on the assumption of rational investor behavior has a long tradition in asset pricing, as in other fields of economics. In essence, it links the stochastic discount factor to investor behavior through assumptions about preferences. By assuming that investors make portfolio decisions to obtain a desired time and risk profile of consumption, the theory provides a link between the asset prices investors face in market equilibrium and investor well-being. This link is expressed through m, which captures the aspects of utility that turn out to matter for valuing the asset. Typically, the key link comes from the time profile of consumption. A basic model that derives this link is the CCAPM.[3] It extends the static CAPM theory of individual stock prices by providing a dynamic consumption-based theory of the determinants of the valuation of the market portfolio. CCAPM is based on crucial assumptions about investors' utility function and attitude toward risk, and much of the empirical work has aimed to make inferences about the properties of this utility function from asset prices.

The most basic version of CCAPM involves a "representative investor" with time-additive preferences acting in market settings that are complete, i.e., where there is at least one independent asset per state of nature. This theory thus derives m as a function of the consumption levels of the representative investor in periods $t+1$ and t. Crucially, this function is nonlinear, which has necessitated innovative steps forward in econometric theory in order to test CCAPM and related models. These steps were taken and first applied by Hansen.

In order to better conform with empirical findings, CCAPM has been extended to deal with more complex investor preferences (such as time non-separability, habit formation, ambiguity aversion and robustness), investor heterogeneity, incomplete markets and various forms of market constraints, such as borrowing restrictions and margin constraints. These extensions allow a more general view of how m depends on consumption and other variables. The progress in this line of research will be discussed in Section 5.

Behavioral finance

Another interpretation of the implied fluctuations of m observed in the data is based on the view that investors are not fully rational. Research along these lines has developed very rapidly over the last decades, following Shiller's original contributions beginning in the late

[3] The CCAPM has its origins inwork by Merton (1973), Lucas (1978) and Breeden (1979).

1970s. A number of specific departures from rationality have been explored. One type of departure involves replacing the traditional expected-utility representation with functions suggested in the literature on economic psychology. A prominent example is *prospect theory*, developed by the 2002 Laureate Daniel Kahneman and Amos Tversky. Another approach is based on market sentiment, i.e., consideration of the circumstances under which market expectations are irrationally optimistic or pessimistic. This opens up the possibility, however, for rational investors to take advantage of arbitrage opportunities created by the misperceptions of irrational investors. Rational arbitrage trading would push prices back toward the levels predicted by non-behavioral theories. Often, therefore, behavioral finance models also involve institutionally determined limits to arbitrage.

Combining behavioral elements with limits to arbitrage may lead to behaviorally based stochastic discount factors, with different determinants than those derived from traditional theory. For example, if the m is estimated from data using equation (1) and assuming rational expectations (incorrectly), a high m value may be due to optimism and may not reflect movements in consumption. In other words, an equation like (1) is satisfied in the data, but since the expectations operator assigns unduly high weights to good outcomes it makes the econometrician overestimate m. Behavioral-finance explanations will be further discussed in Section 6.

CAPM and the cross-section of asset returns

Turning to the cross-section of assets, recall from above that an individual stock price can be written as the present value of its payoff in the next period discounted by the riskless interest rate, plus a risk-premium term consisting of the amount of risk, $var_t(x_{i,t+1})$, of the asset times its risk exposure, $\frac{cov_t(m_{t+1}x_{i,t+1})}{var_t(x_{i,t+1})}$. The latter term is the "beta" of the particular asset, i.e., the slope coefficient from a regression that has the return on the asset as the dependent variable and m as the independent variable. This expresses a key feature of the CAPM. An asset with a high beta commands a lower price (equivalently, it gives a higher expected return) because it is more risky, as defined by the covariance with m. The CAPM specifically represents m by the return on the market portfolio. This model has been tested systematically by Fama and many others. More generally, several determinants of m can be identified and richer multi-factor models can be specified of the cross-section of asset returns, as stocks generally covary differently with different factors. This approach has been explored extensively by Fama and other researchers and will be discussed in Section 7.

3. Are returns predictable in the short term?

A long history lies behind the idea that asset returns should be impossible to predict if asset prices reflect all relevant information. Its origin goes back to Bachelier (1900), and the idea was formalized by Mandelbrot (1963) and Samuelson (1965), who showed that asset prices in well-functioning markets with rational expectations should follow a generalized form of a random walk known as a submartingale. Early empirical studies by Kendall (1953), Osborne (1959), Roberts (1959), Alexander (1961, 1964), Cootner (1962, 1964), Fama (1963, 1965), Fama and Blume (1966), and others provided supportive evidence for this hypothesis.

In an influential paper, Fama (1970) synthesized and interpreted the research that had been done so far, and outlined an agenda for future work. Fama emphasized a fundamental problem that had largely been ignored by the earlier literature: in order to test whether prices correctly incorporate all relevant available information, so that deviations from expected returns are unpredictable, the researcher needs to know what these expected returns are in the first place. In terms of the general pricing model outlined in section 2, the researcher has to know how the stochastic discount factor m is determined and how it varies over time. Postulating a specific model of asset prices as a maintained hypothesis allows further study of whether deviations from that model are random or systematic, i.e., whether the forecast errors implied by the model are predictable. Finding that deviations are systematic, however, does not necessarily mean that prices do not correctly incorporate all relevant information; the asset-pricing model (the maintained hypothesis) might just as well be incorrectly specified.[4] Thus, formulating and testing asset-pricing models becomes an integral part of the analysis.[5] Conversely, an asset-pricing model cannot be tested easily without making the assumption

[4] The joint-hypothesis problem has been generalized by Jarrow and Larsson (2012). They prove that the proposition that prices incorporate available information in an arbitrage-free market can be tested if the correct process for asset returns can be specified. Specifying an asset-pricing model can be viewed as a special case of this, since such a model implies an equilibrium process for asset returns.

[5] One exception is when two different assets have exactly identical payoffs. In such a case, an arbitrage-free market implies that these assets should trade at an identical price, regardless of any asset-pricing model. Hence, if we could find instances where two such assets trade at different prices, this would violate the assumption that no arbitrage is possible. Such violations have been documented in settings where market frictions limit arbitrage opportunities. Examples include documentation by Froot and Dabora (1999) of price deviations of the Royal Dutch Shell stock between the U.S. and Dutch stock market, and studies by Lamont and Thaler (2003) and Mitchell, Pulvino and Stafford (2002), who looked at partial spinoffs of internet subsidiaries, where the market value of a company was less than its subsidiary (implying that the nonsubsidiary assets have negative value).

that prices rationally incorporate all relevant available information and that forecast errors are unpredictable. Fama's survey provided the framework for a vast empirical literature that has confronted the joint-hypothesis problem and provided a body of relevant empirical evidence. Many of the most important early contributions to this literature were made by Fama himself. In Fama (1991) he assessed the state of the art two decades after the first survey.

In his 1970 paper, Fama also discussed what "available" information might mean. Following a suggestion by Harry Roberts, Fama launched the trichotomy of (i) weak-form informational efficiency, where it is impossible to systematically beat the market using historical asset prices; (ii) semi-strong–form informational efficiency, where it is impossible to systematically beat the market using publicly available information; and (iii) strong-form informational efficiency, where it is impossible to systematically beat the market using any information, public or private. The last concept would seem unrealistic *a priori* and also hard to test, as it would require access to the private information of all insiders. So researchers focused on testing the first two types of informational efficiency.

3.1 Short-term predictability

Earlier studies of the random-walk hypothesis had essentially tested the first of the three informational efficiency concepts: whether past returns can predict future returns. This work had addressed whether past returns had any power in predicting returns over the immediate future, days or weeks. If the stochastic discount factor were constant over time, then the absence of arbitrage would imply that immediate future returns cannot be predicted from past returns. In general, the early studies found very little predictability; the hypothesis that stock prices follow a random walk could not be rejected. Over short horizons (such as day by day), the joint-hypothesis problem should be negligible, since the effect of different expected returns should be very small. Accordingly, the early studies could not reject the hypothesis of weak-form informational efficiency.

In his PhD dissertation from 1963, Fama set out to test the random-walk hypothesis systematically by using three types of test: tests for serial correlation, runs tests (in other words, whether series of uninterrupted price increases or price decreases are more frequent than could be the result of chance), and filter tests. These methods had been used by earlier researchers, but Fama's approach was more systematic and comprehensive, and therefore had a strong impact on subsequent research. In 1965, Fama reported that daily, weekly and monthly returns were somewhat predictable from past returns for a sample of large U.S. companies. Returns tended to be positively auto-correlated. The relationship was quite weak, however, and the fraction of the return variance explained by the variation in expected returns was less than 1% for individual stocks. Later, Fama and Blume (1966) found that the deviations from random-walk pricing were so small that any attempt to exploit them would be unlikely to survive trading costs. Although not exactly accurate, the basic no-arbitrage view in combination with constant expected returns seemed like a reasonable working model. This was the consensus view in the 1970s.

3.2 Event studies

If stock prices incorporate all publicly available information (i.e., if the stock market is "semi-strong" informationally efficient, in the sense used by Fama, 1970), then relevant news should have an immediate price impact when announced, but beyond the announcement date returns should remain unpredictable. This hypothesis was tested in a seminal paper by Fama, Fisher, Jensen and Roll, published in 1969. The team was also the first to use the CRSP data set of U.S. stock prices and dividends, which had been recently assembled at the University of Chicago under the leadership of James Lorie and Lawrence Fisher. Fama and his colleagues introduced what is nowadays called an event study.[6] The particular event Fama and his co-authors considered was a stock split, but the methodology is applicable to any piece of new information that can be dated with reasonable precision, for example announcements of dividend changes, mergers and other corporate events.

The idea of an event study is to look closely at price behavior just before and just after new information about a particular asset has hit the market ("the event"). In an arbitrage-free market, where prices incorporate all relevant public information, there would be no tendency for systematically positive or negative risk-adjusted returns after a news announcement. In this case, the price reaction at the time of the news announcement (after controlling for other events occurring at the same time) would also be an unbiased estimate of the change in the fundamental value of the asset implied by the new information.

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[6] A note on precedence is warranted here. The basic idea of an event study may be traced at least back to James Dolley (1933), who studied the behavior of stock prices immediately after a split and provided a simple count of stocks that increased and stocks that decreased in price. A contemporaneous event study was presented by Ball and Brown (1968), and appeared in print a year before the 1969 paper by Fama et al. Ball and Brown acknowledge, however, that they build on Fama and his colleagues' methodology and include a working-paper version of that paper among their references. Rather than casting doubt on the priority of the 1969 paper, this illustrates how fast their idea spread in the research community.

Empirical event studies are hampered by the noise in stock prices; many things affect stock markets at the same time making the effects of a particular event difficult to isolate. In addition, due to the joint-hypothesis problem, there is a need to take a stand on the determinants of the expected returns of the stock, so that market reactions can be measured as deviations from this expected return. If the time period under study – "the event window" – is relatively short, the underlying risk exposures that affect the stock's expected return are unlikely to change much, and expected returns can be estimated using return data from before the event.

Fama and his colleagues handle the joint-hypothesis problem by using the so-called "market model" to capture the variation in expected returns. In this model, expected returns $R_{i,t}^{*}$ are given by

$$R_{i,t}^{*} = \alpha_i + \beta_i R_{m,t}$$

Here $R_{m,t}$ is the contemporaneous overall market return, and α_i and β_i are estimated coefficients from a regression of realized returns on stock i, $R_{i,t}$, on the overall market returns using data before the event.[7] Under the assumption that β_i captures differences in expected return across assets, this procedure deals with the joint-hypothesis problem as well as isolates the price development of stock *i* from the impact of general shocks to the market.

For a time interval before and after the event, Fama and his colleagues then traced the rate of return on stock *i* and calculated the residual $\varepsilon_{i,t} = R_{i,t} - R_{i,t}^{*}$. If an event contains relevant news, the accumulated residuals for the period around the event should be equal to the change in the stock's fundamental value due to these news, plus idiosyncratic noise with an expected value of zero. Since lack of predictability implies that the idiosyncratic noise should be uncorrelated across events, we can estimate the value impact by averaging the accumulated $\varepsilon_{i,t}$ values across events.

The event studied in the original paper was a stock split. The authors found that, indeed, stocks do not exhibit any abnormal returns after the announcement of a split once dividend changes are accounted for. This result is consistent with the price having fully adjusted to all available information. The result of an event study is typically presented in a pedagogical

[7] The market model is closely related to the Capital Asset Pricing Model (CAPM), according to which $R_i^{*} = R_f + \beta_i(R_m^{*} - R_f)$, where R_f is the risk-free rate and R_m^{*} is the expected market return. This is a sufficient but not necessary condition for the market model to be a correct description of asset returns, but CAPM puts the additional restriction on the coefficients that $\alpha_i = (1 - \beta_i)R_f$. See Sharpe (1964).

diagram. Here we reproduce the diagram from a study by Asquith and Mullins (1986) of the stock price reaction for 88 U.S. stocks around the time when the firms announced that they would start paying dividends. Time 0 marks the day the announcement of the dividend initiation was published in *The Wall Street Journal*, implying that the market learned the dividend news the day before, i.e., at time -1. The diagram plots the "cumulative abnormal returns," i.e., the accumulated residual return ε_t^i from 12 trading days before until 12 trading days after the publication of the announcement. As seen in the diagram, dividend news is quickly incorporated in stock prices, with a large stock price reaction of about 5% around the announcement day, and insignificant abnormal returns before or after the announcement. This pattern indicates that this type of news has no predictability.

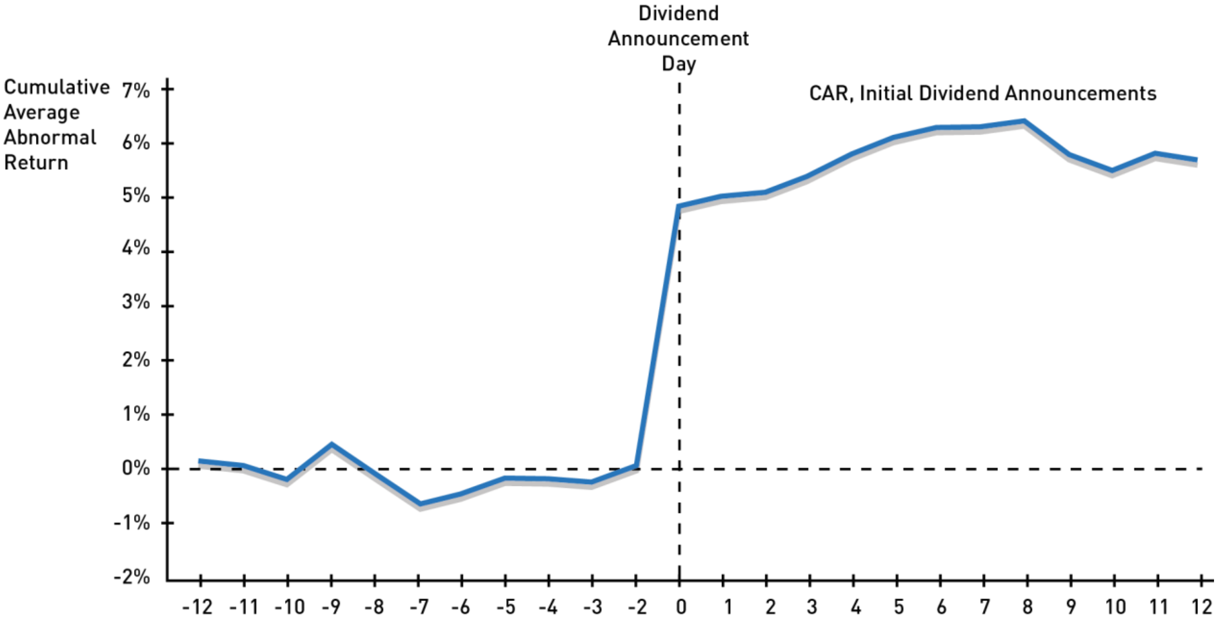

Figure 1: Abnormal stock returns for initial dividend announcements

The event-study methodology may seem simple, but the force of the original study by Fama, Fisher, Jensen and Roll and its results created a whole new subfield within empirical finance. An event study arguably offers the cleanest way of testing for whether new information is incorporated fully in prices, without generating predictable price movements. By and large, the vast majority of event studies have supported this hypothesis. Some exceptions have been found, however. The most notable and pervasive one probably is the so-called post-earnings announcement drift, first documented by Ball and Brown (1968).

One of the most common uses of event studies is to measure the value consequences of various events. If the market correctly incorporates the new information, the value effects of a particular event, such as a corporate decision, a macroeconomic announcement or a regulatory change, can be measured by averaging the abnormal returns across a large number of such events for different assets and time periods. This method has become commonly used to test predictions from various economic theories, in particular in corporate finance. See MacKinlay (1997) and Kothari and Warner (2007) for reviews of this extensive literature.

3.3 Subsequent studies of short-term predictability

A flood of empirical studies using longer time series and more refined econometric methods followed in the footsteps of the early work on predictability by Fama and others. Researchers found statistically significant short-term predictability in stock returns, but that such predictability is small in magnitude (e.g., French and Roll, 1986, Lo and MacKinlay, 1988, Conrad and Kaul, 1988). The autocorrelation turns out to be stronger for smaller and less frequently traded stocks, indicating that exploiting this predictability is very difficult, given trading costs. Focusing on the very short horizon, French and Roll (1986) compare the variance of per-hour returns between times when the market is open and weekends and nights when the market is closed. It turns out that prices vary significantly more when the market is open than they do over nights or weekends, measuring the price evolution per hour from closing to opening. This finding is intriguing, unless the intensity of the news is correspondingly much higher when the market is open. One interpretation is that uninformed "noise trading" causes short-term deviations of price from its fundamental value. Consistent with this, French and Roll found that higher-order autocorrelations of daily returns on individual stocks are negative. Although the interpretation of these findings is still subject to debate, a common explanation is that some of this predictability is due to liquidity effects, where the execution of large trades leads to short-term price pressure and subsequent reversals (Lehmann, 1990).

The research program outlined by Fama in his 1970 paper has by now yielded systematic evidence that returns on exchange-traded stocks are somewhat predictable over short horizons, but that the degree of predictability is so low that hardly any unexploited trading profits remain, once transaction costs are taken into account. In this specific sense, stock markets appear to be close to the no-arbitrage model with unpredictable forecasting errors. Lack of short-term predictability does not, however, preclude that longer-term stock market returns could display considerable predictability. Even if short-term returns are nearly unpredictable, returns could quite possibly be predictable over longer time horizons. In the next section we turn to the evidence on longer-term predictability.

4. Longer-term predictability

Studies of longer-term predictability have to confront the joint-hypothesis problem head on. To the extent that one is willing to maintain the hypothesis of arbitrage-free pricing, long-term return predictability would allow inference about the correct asset-pricing model. Conversely, finding long-term predictability may suggest the existence of arbitrage opportunities given a particular asset-pricing model.

Longer-term predictability of asset returns became a major research issue in the 1980s. The seminal contributions are attributable to Shiller. Important early contributions were also made by Fama; for example, Fama and Schwert (1977) showed that the short-term interest rate could be used to forecast the return on the stock market.

4.1Variance ratio tests

Are expected market returns constant over time or do they vary in a predictable way? Shiller addressed this question for bond markets (1979), as well as for stock markets (1981). He realized that the simple no-arbitrage hypothesis, with a constant expected return, could be tested by comparing the variance of asset returns in the short term and the long term. Until the early 1980s, most financial economists believed that cash-flow news was the most important factor driving stock market fluctuations. In the title of his 1981 paper, Shiller challenged this view by asking, “Do stock prices move too much to be justified by subsequent changes in dividends?”

To understand Shiller’s insight, recall that the basic pricing equation (1) implies that an asset price in an arbitrage-free market can be written as an expected present value of future “fundamentals”: the discounted value of future cash flows (dividends in the case of stocks), where discounting is represented by future values of m. As pointed out above, dividends as well as the discount factor are stochastic. Let $P_{i,t}^{*}$ denote the realization of the fundamental value of a stock i at time t, i.e., the discounted sum of future realized dividends from time $t+1$ and onwards. This value is not known at time t but has to be predicted by investors. Any unexpected movement in stock prices must come from a surprise change to $P^*_{i,t}$, either due to a dividend movement or a movement in the stochastic discount factor. The theory thus says that $P_{i,t} = E_t[P^*_{i,t}]$, so that the forecast error, $P_{i,t} - P^*_{i,t}$, must be uncorrelated with any information available today, in particular the current price. Otherwise the expectations would not make rational use of the available information. Because by definition $P^*_{i,t} \equiv P_{i,t} + \left(P^*_{i,t} - P_{i,t}\right)$ and the price and the forecast error are uncorrelated, it follows that $Var(P^*_{i,t}) = Var(P_{i,t}) + Var\left(P^*_{i,t} - P_{i,t}\right)$, i.e., the variance of the realized fundamental value P^* in a no-arbitrage market equals the sum of the variance of the price P and the variance of the forecast error. This implies that $Var\left(P^*_{i,t}\right) > Var\left(P_{i,t}\right)$. In other words, the variance of the price must be smaller than the variance of the realized discounted value of future dividends.

To investigate this relation empirically, Shiller (1981a) assumed a *constant* discount factor, which implies that (realized) fundamentals are given by

$$P^*_{i,t} = \sum_{j=1}^{\infty} m^j \, d_{i,t+j}.$$

The resulting time series, based on dividends in the New York Stock Exchange, is displayed in the figure below together with the stock index itself. The contrast in volatility between the two series is striking. Contrary to the implication of the present-value model with constant discount rates, the price variance is much larger than the variance of the discounted sum of future dividends.[8]

The early excess-volatility findings were challenged on econometric grounds by Marsh and Merton (1986) and Kleidon (1986), who noted that the test statistics used by Shiller (1979, 1981) are only valid if the time series are stationary. This issue was addressed by Campbell and Shiller (1987). They used the theory of cointegrated processes, which had been recently developed by the 2002 Laureates Clive Granger and Robert Engle[9], to design new tests of the present-value model that allow the processes generating prices and dividends to be nonstationary. The model was again rejected, even under these more general and realistic conditions. The paper by Campbell and Shiller (1987) also was important in showing how

[8] At about the same time, and independently, LeRoy and Porter (1981) also studied the excess volatility of stock prices using a different methodology, where they constructed a joint test of price volatility and payoff volatility from a bivariate model for dividends and prices. They found evidence of excess volatility, but it appeared to be of borderline statistical significance.
[9] See Engle and Granger (1987).

cointegration methods can be used as a natural extension of Fama's (1970) notion of "weak form" tests.[10]

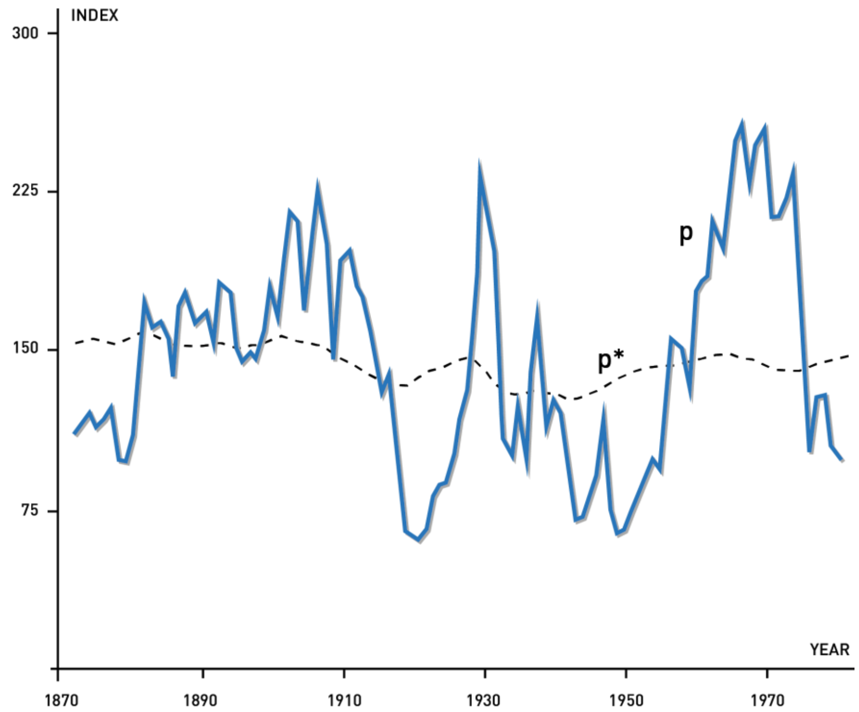

Figure 2: Real Standard and Poor's Composite Stock Price Index (solid line *p*) and *ex post* rational price (dotted line *p**), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable *p** is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter.

4.2 Predictability in stock returns

The finding that stock and bond returns are more volatile in the short term than in the long term implies that returns are "mean reverting," i.e., above-average returns tend to be followed by below-average returns and vice versa. This also implies that future returns can be predicted from past returns. Evidence that stock returns may be predictable in the medium and long term had started to emerge already in the 1970s. Basu (1977, 1983) documented that stocks with high earnings-to-price or dividend-to-price ratios outperform stocks with low ratios. Fama and Schwert's (1977) investigation of the relationship between stock returns and inflation showed that periods of high short-term interest rates tend to be followed by lower subsequent stock-market returns.

[10] Campbell and Shiller were also inspired by the work of Hansen and Sargent (1980), who showed how the concept of Granger causality could be used in testing rational-expectation models.

The finding that stock prices are excessively volatile relative to dividends made it natural to focus on current dividend levels as a predictor of future returns. Shiller (1984) studied U.S. stock market data going back to the 1870s. By regressing the one-year-ahead rate of return on the current dividend-price ratio, he found a positive relationship: high dividends relative to price predict above-normal returns. Apparently, an investor could earn higher returns by going against the market, buying when prices are low relative to dividends and selling when prices are high. In a later paper, Campbell and Shiller (1988a) studied the predictive power of a long moving average of real earnings. They found that this variable has a strong power in predicting future dividends and that the ratio of this earnings variable to the current stock price is a powerful predictor of future stock returns. Other early studies of stock-return predictability include Keim and Stambaugh (1986) and Campbell (1987). These and other studies identified a variety of variables that forecast future stock returns. Typically these variables are correlated with key macroeconomic indicators, suggesting that the discount factor varies with the state of the business cycle.

Consistent with the limited predictability over very short horizons discussed in section 3, Fama and French (1988a) documented that predictability increases with the horizon. This finding is illustrated in the table below, taken from Cochrane (2001). Over a one-year horizon, the dividend/price ratio explains 15% of the variation in excess returns, but over a five-year horizon, the explanatory power is as high as 60%.[11]

Coefficients from regressing excess returns over different horizons on the ratio of dividend to price

Horizon (years)	*Coefficient (standard error)*	R^2
1	5.3 (2.0)	0.15
2	10 (3.1)	0.23
3	15 (4.0)	0.37
5	33 (5.8)	0.60

[11] These regressions are associated with some econometric problems. The dividend-yield series is very persistent, and return shocks are negatively correlated with dividend-yield shocks. As a result, the return-forecast regression inherits the near-unit-root properties of the dividend yield. For such time series, standard test statistics may suffer from small sample biases. Nelson and Kim (1993) and Stambaugh (1999) have proposed methods for dealing with this problem. See also Cochrane (2007).

In a related contribution, Campbell and Shiller (1988b) explore the determinants of the dividend-price ratio, d_t/P_t. Basic pricing theory implies that this ratio should reflect expectations of future dividend growth and discount rates. In the simplest case of no uncertainty, constant dividend growth at rate g, and a constant discount rate R, the pricing expression simplifies to $d_t/P_t = R - g$, the so-called Gordon formula. In general, however, given its nonlinearity, implementing an asset-pricing equation for empirical studies is not straighforward. The methodology developed by Campbell and Shiller allows an analyst to gauge to what extent variations in *d/P* can be explained by variations in expected dividends and discount rates, respectively. It builds on a linearization that decomposes the logarithm of *d/P* into a weighted sum of future expected log discount rates and log dividend changes. To generate expectations, Campbell and Shiller estimated a vector-autoregression system based on alternative measures of discount rates, e.g., interest rates and consumption growth. They found some evidence that *d/P* is positively affected by future dividend growth. None of the discount rate measures used, however, helped to explain the dividend-price ratio, and overall, most of the variation in this ratio remained unexplained. The Campbell-Shiller decomposition has become very influential both by providing an empirical challenge for understanding what drives asset prices and by providing a methodology for addressing this challenge.

4.3 Predictability in other asset markets

The findings of excess volatility and predictability by Shiller and others turned out to be a pervasive phenomenon, not only in the stock market but also in other asset markets. As a precursor to his work in 1980, Shiller (1979) already found evidence of excess volatility for government bonds. Under the assumption of a constant risk premium (the so-called expectations hypothesis), long-term interest rates should equal weighted averages of expected future short-term rates, and consequently the volatility of long-term rates should be smaller than the volatility of short-term rates. Shiller found just the opposite. The volatility of long-term rates turned out to be many times larger than the volatility of short-term rates. Similar to stock prices, the excess volatility of long-term bond prices implies that bond returns are predictable. Subsequently, Shiller, Campbell and Schoenholtz (1983), Fama and Bliss (1987), and Campbell and Shiller (1991) all found that the slope of the U.S. Treasury yield curve predicts bond returns at all maturities. Moreover, Campbell (1987) and Fama and French

(1989) showed that the term structure of interest rates predict stock returns as well, and that excess returns on long-term bonds and stocks move together.

Similar results were found in foreign exchange markets. According to the expectations hypothesis, forward exchange rates should be equal to expected spot rates. The expectations hypothesis implies that the so-called carry trade, which involves borrowing in a low-interest currency and investing in a high-interest currency, should not yield positive excess returns, as the higher interest rate should be offset by currency depreciation. Hansen and Hodrick (1980) developed econometric tests using multiple forward rates of different maturities, and were able to reject the expectations hypothesis in foreign exchange markets.[12] Similarly, Fama (1984) showed that the coefficient of the forward rate in a regression on future spot rates is actually negative, rather than plus one as the expectations hypothesis would predict. These studies, as well as many others that followed, indicated that foreign exchange markets exhibit significant return predictability as well.

The upshot from these results is that the volatility and predictability of stock, bond and foreign exchange returns can only be consistent with arbitrage-free markets if the expected return, i.e., the discount factor, is highly variable over time. The question then is whether theoretical models are able to generate such high variability in the discount factor.

5. Risk premia and volatility in rational-agent models

The findings of excess volatility and predictability – and related findings, such as high return premia on stocks – by Shiller and other researchers illustrate the need for a deeper understanding of what drives the variation in expected returns over time. A major line of research, initiated in the 1970s, continues to strive to construct dynamic asset-pricing models that build on optimizing behavior, implying arbitrage-free prices. In a dynamic model, risk preferences of investors can vary over time, e.g., as a result of consumption or wealth shocks, thus generating fluctuations in risk premia and predictability of returns.

[12] The econometric approach taken by Hansen and Hodrick (1980) can be viewed as a precursor to Hansen's (1982) GMM, discussed in section 5.3 below.

5.1 The consumption capital-asset–pricing model (CCAPM)

The most basic dynamic pricing model, the CCAPM, starts from the assumption that the economy can be described by a representative agent who maximizes expected utility given by

$$E\left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}) | I_t\right],$$

where u is a utility function of consumption c and β is the subjective discount factor. Here, we write the conditional expectation $E_t(\cdot)$ as $E(\cdot\,|I_t)$ in order to specify explicitly the information set I_t on which the expectation is based. The agent faces a simple budget constraint

$$\Sigma_i w_{i,t} P_{i,t} + c_t \leq \Sigma_i w_{i,t-1}(P_{i,t} + d_{i,t}) + y_t,$$

where $w_{i,t}$ is the number of units invested in the risky asset i at time t, $d_{i,t}$ is the dividend generated by that asset, and y_t is labor income at time t. The key equation of CCAPM is the first-order condition for utility maximum:

$$u'(c_t) = \beta E\left[\frac{u'(c_{t+1}) \cdot x_{i,t+1}}{P_{i,t}} | I_t\right].$$

Here, as before, $x_{i,t+1} \equiv P_{i,t+1} + d_{i,t+1}$ is the asset's payment at time $t + 1$. An optimizing agent is indifferent between consuming a unit at time t, thus receiving the marginal utility of one unit at that time (the left-hand side), and investing it to earn a rate of return x_{t+1}/P_t and obtaining the discounted marginal utility from consuming that at $t + 1$ (the right-hand side). This so-called Euler equation can be rewritten as an asset-pricing equation:

$$P_{i,t} = E\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \cdot x_{i,t+1} | I_t\right]. \qquad (2)$$

Equation (2) is thus a present-value equation of the same type as the one derived from the absence of arbitrage, equation (1), with the stochastic discount factor m_{t+1} now given by $\beta u'(c_{t+1})/u'(c_t)$, which is the marginal rate of substitution between consumption today and tomorrow. This equation shows why the discount rate would be low during recessions: in bad times, when c_t is low, the marginal utility $u'(c_t)$ is high, and thus the ratio of marginal utilities $u'(c_{t+1})/u'(c_t)$ is correspondingly low (conversely, the discount factor should be high during booms).

The CCAPM would thus seem to give a possible qualitative explanation for the findings of predictability and excess volatility based on rational behavior. What about the quantitative content of the theory?

5.2 Testing the consumption capital-asset–pricing model (CCAPM)

Confronting economic theory with data is a methodological challenge, especially when the theory gives rise to nonlinear dynamic equations. For that reason, researchers often evaluate models informally, for example, by using calibration, where model parameters are selected based on non-statistical criteria and the model is solved and simulated. By comparing the resulting model-generated time series to actual data, calibration can be useful in assessing whether a model may be capable of quantitatively matching the actual data at all. A more rigorous approach, of course, would be to use formal statistical methods. But before the 1980s, the methodological challenges were daunting, and not until Hansen's development of the GMM did formal tests of the CCAPM become commonplace. Thus, empirical evaluation of the CCAPM began with informal methods.

…using calibration and informal statistics

Grossman and Shiller (1981) were the first to evaluate the CCAPM quantitatively. They assumed utility to be given by a power function (implying constant relative risk aversion). The discount factor $\beta u'(c_{t+1})/u'(c_t)$ can then be calculated from consumption data for any given value of relative risk aversion. Using U.S. consumption data, Grossman and Shiller found, however, that the observed stock-price volatility could only be consistent with CCAPM if the marginal utility of consumption was extremely sensitive to variations in consumption, i.e., if the representative consumer was extremely risk averse. This finding left excess volatility as a challenge for future asset-pricing research. Subsequently, Shiller (1982) showed that the model implied a lower bound on the marginal rate of intertemporal substitution. This insight is a precursor to the influential contribution by Hansen and Jagannathan (1991), which is further discussed in Section 5.3.

In passing, Grossman and Shiller also noted that the CCAPM implied a much lower level of equity returns than observed in data, hence providing an early illustration of what Mehra and Prescott (1985) subsequently came to term the *equity-premium puzzle*. In their paper, Mehra and Prescott highlighted the extreme difficulty that traditional models have in matching an observed excess return of stocks relative to a risk-free asset of over 5% per year, a magnitude that had been observed in data for the U.S. and many other countries. To match the data, coefficients of relative risk aversion of around 50 were needed, and such levels of risk aversion were viewed to be unrealistic from an applied microeconomic perspective, at least for the average investor.

… and using formal statistical methods

As discussed above, the CCAPM implies that returns are predictable as long as agents are risk averse and variations in consumption can be predicted. However, in order to test this theory researchers face several difficulties. One difficulty is the inherent nonlinearity of the main estimating equation. Another is the need to specify a full stochastic process for consumption. In fact, these difficulties, along with serial correlation of any errors in the dynamic system, are shared by a large set of models used in economics. In the early 1980s, the only way to handle these difficulties was by making a range of very specific assumptions – assumptions that were not even perceived to be central to the main issue at hand. Thus, any statistical rejection would be a rejection of the joint hypothesis of the main asset-pricing equation and all the specific assumptions to which the researcher was not necessarily wed.

An influential illustration of this point was given by Hansen and Singleton (1983), who dealt with the difficulties by a combination of approximations and specific assumptions. Assuming jointly normally distributed error terms, they developed the following log-linear version of CCAPM:

$$E_t\left[\ln\left(1 + R_{i,t+1}\right)\right] = -\ln\beta - \gamma E_t[\Delta \ln \mathrm{c}_{\mathrm{t+1}}] + [\sigma_i + \gamma^2\sigma_c - 2\gamma\sigma_{ic}]/2.$$

This equation expresses expected log returns as the sum of three terms: the log rate of time preference β, a term that is multiplicative in the rate of risk aversion γ and the expected rate of consumption change, and a term that depends on variances and covariances. Hansen and Singleton then estimated this linearized model for monthly stock returns, using a maximum-likelihood estimator. Based on a value-weighted stock index, the model worked relatively well, giving estimates of relative risk aversion between zero and two and showing little evidence against the parameter restrictions. When estimated based on returns of individual stocks and bonds, however, the model was strongly rejected. This failure is an early indication of a serious challenge to the rational-agent–based asset-pricing model. At the time, however, it was unclear how much of the rejection was due to the linearization and error process assumptions and how much was an inherent limitation of the theory. The GMM provided a way to address these problems.

5.3 The Generalized Method of Moments (GMM)

The asset-pricing context

Consider again the main equation of the CCAPM model, equation (2). Defining $R_{i,t+1} \equiv x_{i,t+1}/P_{it}$, it can be rewritten as

$$1 = E\left[\beta\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{i,t+1} | I_t\right]. \qquad (3)$$

This is a nonlinear function of the stochastic processes for consumption and returns and any relevant additional variables in the conditioning set I_t. The expression $\beta\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{i,t+1} - 1$ can be regarded as a one-period-ahead forecast error. Under rational expectations, this error must be independent of any information I_t available at time t. Let us use z_{jt} to denote a variable in the information set I_t, e.g., a historical asset price. This implies, for any asset i and conditioning variable or "instrument" z_j, that

$$E\left[\left(\beta\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{i,t+1} - 1\right) \cdot z_{jt} | I_t\right] = 0. \qquad (4)$$

This equation, which is an implication of equation (3), is the basis for GMM estimation of an asset-pricing model.

Some econometric theory

Equation (4) can be viewed as an element in the following vector equation

$$E\boldsymbol{g}(\boldsymbol{x}_t, \boldsymbol{\theta}) = \mathbf{0} \qquad (5)$$

where $\boldsymbol{x}_t$ is a vector stochastic process (sequence of random variables) and $\boldsymbol{\theta}$ is a parameter vector to be estimated. The vector-valued function $\boldsymbol{g}$ expresses the key orthogonality condition – one equation for each asset i and instrument z_{jt}. In the example, $\boldsymbol{x}_t$ consists of c, R_i (for all assets i), and z_{jt} (for at least one instrument j), and $\boldsymbol{\theta}$ consists of β and the other parameters in (4). The (i,j)th element of the $\boldsymbol{g}$ vector would thus be $\left(\beta\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{i,t+1} - 1\right) \cdot z_{jt}$, for asset i and a particular instrument z_{jt}. This element has expectation zero and can be interpreted as a form of forecast error.

In a paper that has turned out to be one of the most influential papers in econometrics, Hansen (1982) suggested the GMM as an attractive approach for estimating nonlinear systems like equation (5). A main reason why this estimator has become so popular is that it places only very weak restrictions on the stochastic process $\boldsymbol{x}_t$, which is allowed to be any weakly stationary, ergodic process, and on $\boldsymbol{g}$, which is allowed to be nonlinear. This generality is particularly important in panel-data and time-series applications, such as asset pricing ones, where the stochastic process is correlated and the key relationships are nonlinear. Moment conditions such as (5) had been used in parameter estimation since Pearson (1894, 1900), see also Neyman and Pearson (1928), but their use had been confined to cases where the components of $\boldsymbol{x}_t$ are independent over time, e.g., as in the case of repeated independent experiments. Hansen's contribution was to generalize the previous theory of moment estimation to the case where $\boldsymbol{x}_t$ is a stationary and ergodic process.

The GMM estimator can be defined using the sample moment function

$$\boldsymbol{g}_T(\boldsymbol{\theta}) \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{g}(\boldsymbol{x}_t, \boldsymbol{\theta})$$

and the quadratic form

$$S_T(\boldsymbol{\theta}) \equiv T\boldsymbol{g}_T(\boldsymbol{\theta})'\boldsymbol{W}\boldsymbol{g}_T(\boldsymbol{\theta}),$$

where $\boldsymbol{W}$ is a positive definite weight matrix. The GMM estimator $\widehat{\boldsymbol{\theta}}_T$ minimizes $S_T(\boldsymbol{\theta})$. Hansen (1982) showed that this estimator is consistent for the true parameter vector under certain regularity conditions and that it is asymptotically normal given some mild restrictions on $\boldsymbol{g}(\boldsymbol{x}_t, \boldsymbol{\theta})$. As already indicated, the proof allows rather general stochastic temporal dependence for the stochastic process $\boldsymbol{x}_t$.

Furthermore, Hansen defined the asymptotic covariance matrix

$$\boldsymbol{\Omega} \equiv \sum_{\boldsymbol{j}=-\infty}^{\infty} E\boldsymbol{g}_t(\boldsymbol{\theta})\boldsymbol{g}_{t-j}(\boldsymbol{\theta})'.$$

Hansen showed that the selection $\boldsymbol{W} = \boldsymbol{\Omega}^{-1}$ ensures that the resulting estimator $\widehat{\boldsymbol{\theta}}_T$ minimizes (in the matrix sense) the asymptotic covariance matrix of the estimator. This result provides an asymptotic efficiency bound for the GMM estimator – a bound because the true $\boldsymbol{\Omega}$ is not known.

Hansen also showed how to estimate the asymptotic covariance matrix and, using its inverse as a weighting matrix, derived the resulting asymptotic normal distribution. Hansen's construction of the estimate of $\boldsymbol{\Omega}$ is based on a consistent estimate of $\boldsymbol{\theta}$ for the sample at hand, but at the same time, the estimated $\boldsymbol{\Omega}$ is needed to construct an efficient estimate of $\boldsymbol{\theta}$ This conundrum means that there is no straightforward way of obtaining the efficient estimate. Hansen therefore proposed a two-stage procedure: start with an arbitrary weighting matrix and use it to construct a consistent estimator and use that estimator to estimate the asymptotic covariance matrix; then use that matrix to obtain the efficient estimator of $\boldsymbol{\theta}$. Alternative procedures were proposed later to improve on this two-stage approach.

Finally, Hansen demonstrated how to construct a test of over-identifying restrictions, based on a method proposed by Sargan (1958). Under the null hypothesis this test statistic has an asymptotic χ^2 distribution with $k - r$ degrees of freedom, where k is the number of moment conditions and r the number of linear combinations of these conditions (to find r parameters of interest).[13]

In summary, Hansen provided the necessary statistical tools for dealing with estimating dynamic economic models using panel data, where serially correlated variables are commonplace and where specifying a full model is not always desirable or even possible; GMM can be applied to a subset of the model equations. GMM has made a huge impact in many fields of economics where dynamic panel data are used, e.g., to study consumption, labor supply or firm pricing. It is now one of the most commonly used tools in econometrics, both for structural estimation and forecasting and in microeconomic as well as macroeconomic applications.[14]

The asset-pricing application

Equipped with GMM, researchers analyzing asset prices could now go to work. The first direct application of Hansen's GMM procedure is reported in the paper on asset-pricing by Hansen and Singleton (1982). But an earlier use of the essential idea behind GMM can be found in work by Hansen and Hodrick (1980), who looked at currencies and asked whether forward exchange rates are unbiased predictors of future spot rates. Serial correlation in errors and nonlinearities make traditional approaches invalid for this issue, and the authors derived asymptotic properties based on methods that turned out to be a special case of GMM.

The main purpose of Hansen and Singleton (1982) was to test the CCAPM. To operationalize the model, the authors assumed utility, as did Grossman and Shiller (1981), to display

[13] Hansen has followed up his seminal piece with a number of important extensions, including alternative estimators (Hansen, Heaton and Yaron, 1996), the choice of instruments (Hansen, 1985, and Hansen, Heaton and Ogaki, 1988), continuous-time models (Hansen and Scheinkman, 1995), and GMM with non-optimal weighting matrices (Hansen and Jagannathan, 1997).

[14] See the review articles by Hansen and West (2002) and Jagannathan, Skoulakis and Wang (2002) for illustrations of the use of GMM in macroeconomics and finance. In microeconometrics, GMM has also been a commonly used model for estimation with panel data – see Arellano and Bond (1991) and Blundell and Bond (1998).

constant relative risk aversion: $u(c) = c^{(1-\gamma)}/(1-\gamma)$. With this specification, an element of $\boldsymbol{g}$, representing a certain asset i and an instrument z_j, takes the form $\left(\beta\left(\frac{c_t}{c_{t+1}}\right)^{\gamma} \cdot R_{i,t+1} - 1\right) \cdot z_{jt}$, with β and γ as the parameters to be estimated. The corresponding moment condition thus becomes

$$\frac{1}{T}\sum_{t=1}^{T}\left[\left(\beta\left(\frac{c_t}{c_{t+1}}\right)^{\gamma} \cdot R_{i,t+1} - 1\right) \cdot z_{jt}\right] = 0.$$

With *n* assets and *m* instruments, there are *nm* such moment conditions. With fewer than *nm* parameters to estimate, the model may be tested for over-identifying restrictions. This instrumental-variables formulation illustrates the point made by Fama (1970): testing an asset-pricing model amounts to a joint test of the model-generated hypothesis and the lack of predictable forecast errors. If the over-identifying restrictions are rejected, this means either that the model is incorrect – i.e., that the no-arbitrage condition is violated – or that the orthogonality condition of the instruments is violated, or both.

Hansen and Singleton (1982) estimated this model by GMM, using lagged values of R_i as instruments. The data are aggregate indexes for the New York Stock Exchange as well as indexes for different industries, and the model is estimated both on single and multiple return series. All versions of the model yield economically meaningful estimates with γ close to unity (although with a large standard error) and β slightly smaller than unity. When applied to more than one stock index, the over-identifying restrictions are generally rejected, however.[15] Hence, in line with the excess-volatility findings of Grossman and Shiller (1981), this simple version of CCAPM does not fit the data very well. This result has led to a vast amount of research aimed at understanding the shortcomings of the basic model.

In the search for a model that better fits the data, a diagnostic tool that states the properties that the stochastic discount factor must possess would be useful. Hansen and Jagannathan (1991) showed that the so-called Sharpe ratio[16] – expressed by the ratio of the expected excess return of an asset over the risk-free rate to the standard deviation of the excess return – gives a lower bound to the volatility of the discount factor. Specifically,

$$\frac{\sigma(m_{t+1})}{E(m_{t+1})} \geq \frac{E(R_{t+1}^{e})}{\sigma(R_{t+1}^{e})},$$

[15] In a later paper, Hansen and Singleton (1984) corrected an error in their original data series. Using the revised data, the CCAPM was more strongly rejected.

[16] Sharpe (1966).

where the left-hand side is the ratio of the standard deviation of the discount factor to its expected value and the right-hand side is the Sharpe ratio. This relation was originally stated by Shiller (1982) for a single risky asset and generalized by Hansen and Jagannathan to cover many assets and no risk-free asset. In subsequent work, Hansen and Jagannathan (1997) extended their analysis and derived formal tests of the performance of different stochastic discount factor proxies.

Hansen-Jagannathan bounds have become widely used in practical applications. Many assets and investment strategies, such as momentum (going long in stocks with high past returns and shorting stocks with low past returns; see Section 7) or carry trade (borrowing in low-interest-rate currencies and investing in high-interest-rate currencies) have very high Sharpe ratios. For the postwar U.S. stock market, the Sharpe ratio in annual data is around one half, which implies that the annualized standard deviation of the discount factor has to be at least 50%, which is very high considering that the mean of the discount factor should be close to one. This discrepancy poses a serious problem for consumption-based models such as the CCAPM, since the low volatility of observed consumption, along with a realistic level of risk aversion, implies too low a volatility of the stochastic discount factor according to CCAPM.

5.4 Extensions of the CCAPM

Rejection of the CCAPM with standard preferences does not necessarily reject the basic intuition of the model, i.e., that the expected return on equity is higher in "bad times" when current consumption is low. Starting with the study by Fama and French (1989), several studies have related predictability to business cycle conditions, showing that expected returns are lower at the peak of the business cycle and higher in the trough. Fama and French (1989) also showed that expected returns in equity markets and bond markets move together, and that the term premium (the difference in yields between long- and short-term bonds) has additional predictive power for stock returns, in addition to dividend yields. Similarly, macro variables such as the consumption-wealth ratio (Lettau and Ludvigson, 2001) have been shown to predict equity returns (in addition to dividend yields and term premia).[17] Rather, the problem is that the covariation between asset returns and consumption is not large enough to generate high enough expected returns and volatility using standard expected utility preferences.

[17] See Cochrane (2011) for an overview.

These results have led many researchers to explore alternative model specifications, changing assumptions about investor utility, market completeness, the stochastic process for consumption or these assumptions in combination. Several of these approaches have had some success in explaining equity premia, volatility and predictability within a modified CCAPM framework, although it is fair to say that currently no widely accepted "consensus model" exists.

One approach has been to address one of the main shortcomings of the standard von Neumann-Morgenstern expected utility model, namely that the same parameter determines both risk aversion and intertemporal substitution, even though there is no compelling economic or behavioral reason for this to be the case. Building on Kreps and Porteus (1978), Epstein and Zin (1989) developed a class of recursive preferences that allow preferences for risk and intertemporal substitution to be separated and argued that these preferences could help resolve the consumption-based model.[18] Hansen contributed to this line of research (Eichenbaum, Hansen and Singleton, 1988, and Eichenbaum and Hansen, 1990). This research program is still very active, with some success in improving the models' fit with data. Using Epstein-Zin preferences, Bansal and Yaron (2004) proposed a model where consumption and dividend growth contain a small predictable long-run component, and consumption volatility is time-varying. Given these preferences and dynamics, Bansal and Yaron were able to generate a stochastic discount factor *m* that can justify the observed equity premium, the risk-free rate and the return volatility and that also generates dividend-yield predictability. This approach has been quite influential and has led to a number of follow-up studies, including one from Hansen, Heaton and Li (2008).

A second approach to modify preferences has been to introduce habits into the utility function (Deaton, 1992) and make consumer utility not just dependent on the *absolute level* of consumption, but also sensitive to *changes* in consumption levels. Thus, Sundaresan (1989), Constantinides (1990) and Abel (1990) included habit formation in the CCAPM framework, and showed that habits can increase the volatility of the stochastic discount factor. In a highly cited study, Campbell and Cochrane (1999) were able to explain the equity premium puzzle in a model where an "external" habit (which makes agents care about changes in aggregate, and not only individual consumption) is added to the standard power-utility framework.

A third approach that has also met with some success, is to consider heterogeneity in investor preferences. In particular, if investors have different attitudes toward risk, the stochastic

[18] See also Weil (1989).

discount factor *m* that appears as a result of market trading will be influenced not just by aggregate consumption but also by its distribution. To understand the equity premium, for example, it might be more relevant to test the CCAPM implications using data from the sub-group of investors actually owning significant amounts of stock. Indeed, it turns out that the consumption of individual stockholders fluctuates more than does aggregate consumption, a difference that at least goes part of the way toward explaining the pricing puzzles and confirms investor heterogeneity as a fruitful hypothesis (see, e.g., Malloy, Moskowitz and Vissing-Jorgensen, 2009). A large number of studies of market incompleteness against individual risks show that wealth heterogeneity that is a result of individual wage shocks generates heterogeneity in risk attitudes (for early contributions, see Mankiw, 1986, Heaton and Lucas, 1992, Huggett, 1993, Telmer, 1993, and Constantinides and Duffie, 1996). An important finding reported in this literature is that individual wage risk is procyclical, which helps explain the pricing puzzles further.

A common feature of most of the models discussed here is the assumption that the consumer does not only process information in a rational and efficient manner, but also knows the true data generating process. In joint work with Thomas Sargent (e.g., Hansen and Sargent, 2001, and Cagetti *et al*., 2002), Hansen has investigated the consequences of assuming that the representative agent is uncertain about the true model and follows a policy of robust control across a set of alternative models. Hansen and Sargent showed that model uncertainty can be seen as an extra risk factor; the fear of a worst outcome makes the risk aversion of agents effectively larger and can account for a higher price of risk than in an equivalent standard model.

6. Excess volatility and predictability: behavioral-finance approaches

6.1 Robert Shiller and behavioral finance

The findings of excess volatility and predictability are challenging for the notion that prices incorporate all available information or for standard asset-pricing theory – or for both. Based on his early findings, Shiller (1981b) argued that the excess volatility he documented seemed difficult to reconcile with the basic theory and instead could be indicative of “fads” and overreaction to changes in fundamentals. In his 1984 paper, entitled “Stock Prices and Social Dynamics,” he developed these arguments further. This paper became an important starting point for a growing research literature in "behavioral finance," for which Shiller became one of the most influential proponents.[19]

In his paper, Shiller outlined a number of arguments that were followed and developed by subsequent researchers. First, he argued that the lack of (risk-adjusted) price predictability does not preclude the existence of irrational investors. The trading of such investors could make prices excessively volatile and noisy, which should make deviations from a random walk very hard to detect over short horizons (especially if rational investors would tend to eliminate the most obvious mispricings). In subsequent work, Shiller and Perron (1985) and Summers (1986) argued more formally that the power of short-run predictability tests is likely to be very low.[20]

Second, Shiller reviewed some of the psychology literature showing that individuals are subject to decision biases, such as Tversky and Kahneman's (1974) finding of people overreacting to "superficially plausible evidence" without any statistical basis. Shiller argued that stock prices are particularly vulnerable to psychological biases because of the ambiguity in the true value of a stock, due to the lack of an accepted valuation model (i.e., investors face "Knightian uncertainty" rather than risk). These psychological biases are reinforced and exacerbated by "social movements" because investors are subject to group psychology dynamics, such as peer pressure. Hence, one investor's opinion of the value of a stock is likely to be affected by the opinions of others. As a result, as opinions diffuse throughout the population, stock prices will fluctuate in a way similar to what would be caused by fads or fashions. Shiller also reviewed informal evidence that supported the idea that fads and fashions had contributed to past market booms and busts.

Finally, to illustrate his argument more formally, Shiller posited a simple model economy populated by "ordinary" investors, whose demand does not respond to expected returns, and "smart money" investors who respond rationally to expected returns but are limited by their wealth. In such a model, the trades of ordinary investors will lead to temporary deviations of

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[19] Other early work that met similar challenges with behavioral explanations was done by Slovic (1972), Miller (1977), Harrison and Kreps (1978), and Modigliani and Cohn (1979). Slovic (1972) argued that the (then recent) psychological evidence on heuristics and biases can be applied to finance. Miller (1977) argued that differences of opinion among investors, together with the fact that stocks are difficult to sell short, will lead to over-optimistic stock prices. Harrison and Kreps (1978) made a similar argument in a dynamic setting, and showed that differences in opinion and short-sales constraints can lead to speculative bubbles. Modigliani and Cohn (1979) argued that the negative relation between expected inflation and stock prices (documented by Fama and Schwert, 1977) could be explained by investors suffering from "money illusion," i.e., an inability to distinguish real from nominal returns.

[20] Shiller and Perron (1985) derived power functions for tests aiming to reject the random walk hypothesis in a runs test (e.g., Fama, 1965) when the true asset return process is mean reverting. Apart from showing that these tests have low power when the time span of the data is short, they also showed that increasing the sampling frequency for a given time span does not increase power – only increasing the length of the span does.

stock prices from fundamental values, and these deviations can generate overreaction to dividend news, excess volatility, and mean reverting stock prices, consistent with the finding that high dividend yields predict lower stock prices.

In subsequent work, Shiller has continued to argue for the importance of social psychology, using evidence from investor surveys (Shiller, 1987, 1988, 1989, Shiller and Pound, 1989). He has also extended his analysis of fads and bubbles to housing markets (Case and Shiller, 1987, 1989, 2003).

Following Shiller's original work, many researchers turned to psychological evidence on individual behavior and biases, including prospect theory (Kahneman and Tversky, 1979), overconfidence (Oskamp, 1965) and mental accounting (Kahneman and Tversky, 1984, and Thaler, 1985). This psychology-based work has derived new asset-pricing models that could explain the documented asset-pricing anomalies.[21] Some models attribute under- and overreaction to information due to overconfidence and/or bounded rationality, which leads to excess volatility, momentum and mean reversion in asset prices (Barberis *et al.*, 1998, Daniel *et al.*, 1998, Hong and Stein 1999). Other models modify preferences based on psychological evidence such as prospect theory (Benartzi and Thaler, 1995, Barberis *et al*., 2001), ambiguity aversion (Epstein and Wang, 1994, Epstein and Schneider, 2008, Cagetti *et al.*, 2002) or disappointment aversion (Routledge and Zin, 2010) to explain excess volatility, predictability and the equity premium. Many of these papers can be thought of as modifying the preference assumptions in rational-agent models, such as the CCAPM, which illustrates a convergence between rational and behavioral models in recent research.

In his 1984 paper, Shiller also addressed a serious criticism against behavioral explanations (often attributed to Friedman, 1953), namely that even if some (or most) investors are irrational, the (perhaps few) rational investors in the market could make money as long as there were arbitrage opportunities. Such arbitrage trades would lead the irrational investors to lose money and be forced out of the market, ultimately eliminating any mispricing. Shiller argued that rational investors control too little wealth for this to work in practice. Another early argument for limits to arbitrage is the difficulty of short-selling overpriced stocks (Miller, 1977). Subsequently, a number of researchers provided more rigorous theoretical models explaining the limited ability of rational investors to make markets informationally efficient. A common approach is to model rational arbitrageurs as financial intermediaries (e.g., hedge funds), whose capital is withdrawn by investors in case they experience persistent

[21] See Shleifer (2000) and Barberis and Thaler (2003) for overviews of this literature.

losses (DeLong *et al*., 1990a, Shleifer and Vishny, 1997). Because of such withdrawals, the rational arbitrageurs may not be able to trade against substantial market mispricing, or they may even find it optimal to trade in the opposite direction if the mispricing is expected to increase in the short-run, thus increasing the mispricing rather than decreasing it (DeLong *et al*., 1990b, Abreu and Brunnermeier, 2002, 2003).

6.2. Further work in behavioral finance

Shiller's early studies stimulated a large body of empirical research aimed at backing up the behavioral-finance arguments with empirical evidence.[22] Many of these studies have focused on apparent anomalies in the cross-section of stock returns, rather than mispricing in the market as a whole, and are reviewed in section 7.

An early study that found the evidence hard to reconcile with informational efficiency was Roll's (1984) investigation of the orange juice futures market. Even though weather is the most obvious and significant influence on orange crops, Roll found that weather surprises explain only a small fraction of the variability in futures prices.

A number of studies have documented deviations from the "law of one price" in financial markets and argue that these deviations are indicative of irrational market sentiment. Froot and Dabora (1999) studied "twin stock" companies with stocks traded in more than one location. They found that the prices of these twins frequently differ across trading locations, and that a twin's relative price rises when the market on which it is traded more intensively rises. This suggests that stock prices are driven to some extent by local investor sentiment, rather than simply by changes in fundamental values.

Another group of studies analyze the so-called "closed-end fund puzzle" originally discovered by Zweig (1973), i.e., the finding that closed-end equity funds typically trade at values different from the market value of their underlying stock portfolio, and frequently at a

[22] This discussion focuses on the behavioral finance literature, which seeks to explain asset-pricing patterns and anomalies. Parallel to this literature, a vast behavioral literature has also emerged that analyzes the impact of psychology on financial decisions of individuals. See Barberis and Thaler (2003), section 7, for an overview. This literature has been influential in providing practical policy advice, e.g., in the area of individual pension saving schemes (see Thaler and Benartzi, 2004).

discount to their net asset value.[23] Lee, Shleifer and Thaler (1991) argued that the discounts on closed-end funds can be interpreted as a measure of irrational investor sentiment. They showed that the discounts across closed-end funds with very different asset portfolios exhibit significant co-movement over time, and also co-move with returns on small stocks, a market segment where individual investors are also dominant. Baker and Wurgler (2007) reviewed the investor sentiment literature and showed that a "sentiment index" (including closed-end fund discounts and other variables) is highly correlated with aggregate stock returns.

Another group of papers documented apparent limits to arbitrage and its effect on stock prices. Starting with Shleifer (1986), a number of papers have shown that the price of a stock tends to increase when it is included in a market index (such as the S&P 500), consistent with buying pressure from index funds. Wurgler and Zhuravskaya (2002) showed that the index inclusion effect is stronger among stocks without close substitutes, which makes mispricings harder to arbitrage away.

Shiller developed his arguments further in his book *Irrational Exuberance* (2000), which had a considerable impact on the popular debate. In this book, he used a combination of statistical evidence, survey data, and a review of psychological and sociological studies to argue that fads and feedback loops have contributed to past stock market booms. Notably, he argued that the dramatic rise in stock prices, technology stocks in particular, in the late 1990s was driven by fads – an argument made only a few months before the significant stock market decline in 2000–2001.[24]

A number of studies following Shiller's book documented several anomalies in the pricing of technology stocks during the recent stock market boom. For example, Mitchell, Pulvino and Stafford (2002) and Lamont and Thaler (2003) used partial spin-offs (or "carve-outs") of tech companies to show that the valuation of these spinoff stocks was irrationally high. In particular, comparing the value of the spin-off with the value of the parent company, which still held a partial stake in the spun-off company, the valuation of the spin-off implied that the parent's remaining assets had negative value.[25] They also reported evidence that short-sales constraints, together with the fact that mispricings often increased before they eventually

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[23] Closed-end funds are typically owned by individual investors and are similar to mutual funds, with the exception that investors cannot redeem their fund shares for cash but have to sell their shares in the market to get their money back.

[24] In the second edition of his book, published in 2005, Shiller extended his analysis to real estate, arguing that the real estate market was similarly irrationally overvalued, and he predicted large problems for financial institutions with the eventual burst of the real estate market "bubble."

[25] Also, Ofek and Richardson (2003) linked the high prices of internet stocks to short-sale constraints and "lock-ups" preventing insiders from selling their shares after an IPO.

disappeared, made it difficult and risky for an arbitrageur to profit from these mispricings. Along the same lines, Brunnermeier and Nagel (2004) showed that sophisticated investors, such as hedge funds, preferred to acquire tech stocks and "ride the bubble" in the late 1990s rather than shorting these stocks. More recently, Xiong and Yu (2011) documented a bubble in the pricing of Chinese warrants in the late 2000s, which they showed traded far above their fundamental value, and they argued that short-sales constraints again prevented the mispricing from being arbitraged away.[26]

In recent years, Shiller has continued to explore the impact of psychological factors on financial markets in popular books such as Shiller (2008) and Akerlof and Shiller (2010).

7. What determines differences in expected returns across assets?

The research reviewed so far primarily has been focused on the time-series pattern of asset prices. A related question concerns the cross-sectional pattern of prices, in particular for stocks. Why is a particular stock more highly valued than another one at the same point in time? According to equation (1), the answer depends on expected future cash flows, $\{x_{i,t+j}\}$, and the discount rates (return requirements), $\{m_{t+j}\}$. The discount rates should reflect time preferences in the economy as well as risk premia. Since investors are generally risk-averse, they should demand a higher expected return for more risky assets. A central insight, dating back to the portfolio model of Markowitz (1959), is that investors should only demand compensation for systematic risk, i.e., risk that cannot be eliminated by holding a well-diversified portfolio. But which systematic risks drive stock returns, and to what extent are investors compensated for them in terms of higher expected returns? Alternatively, to the extent that fads and investor irrationality affect stock prices, as Shiller (1984) suggested, how would this affect differences in expected returns across stocks?

7.1 Early tests of the Capital Asset Pricing Model (CAPM)

The CAPM was developed by Sharpe (1964), Lintner (1965) and Mossin (1966). William Sharpe was awarded the 1991 Prize for his contribution to developing the CAPM, which still

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[26] A body of literature also focuses on the asset-pricing bubbles in an experimental setting. See, for example, Smith, Suchanek and Wiliams (1988).

remains a fundamental asset-pricing model taught to students. According to the static CAPM, the expected return R_i^* of a given financial asset i is given by

$$R_i^* = R_f + \beta_i(R_m^* - R_f),$$

where R_f is the risk-free rate, R_m^* is the expected return on the market portfolio (i.e., a portfolio of all assets in the economy), and β_i is the key measure of systematic risk – which should be compensated by a higher rate of return – equal to the covariance of asset i with the market portfolio (the "beta" of the stock). In the mid-1960s, this model provided a promising explanation of asset prices, but it had not yet been empirically tested in a rigorous way.

How good is the CAPM at explaining the cross-section of asset prices? After its development in the mid-1960s, economists set out to test the model empirically. These tests started from time-series regressions of stock returns on index returns to generate estimates of stock-specific beta coefficients, $\hat{\beta}_i$. Assuming that market expectations are rational – so that observed returns $R_{i,t}$ are equal to expected returns plus a random error $\varepsilon_{i,t}$ – CAPM can be tested based on this equation:

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t} \cdot \hat{\beta}_i + \varepsilon_{i,t}.$$

If CAPM is correct, then $\gamma_{0t} = R_f$, the risk-free rate, and $E(\gamma_{1,t}) = R_m^* - R_f$, the expected return on the market in excess of the risk-free rate. Early tests by Douglas (1969), Black and Scholes (1973), and Black, Jensen and Scholes (1972) used time-series stock data to estimate $\hat{\beta}_i$ in a first step, and then cross-sectional data to regress the return on beta in a second step. Results typically yielded a positive relation in accordance with theory, but the estimated coefficient implied an implausibly high value for the riskless rate of return. In addition, these studies did not account for the strong cross-sectional correlation in stock returns that is caused by common shocks that affect groups of stocks at the same time. Not accounting for such correlation leads to biased inference. Typically, the estimated standard errors are downward biased.[27]

Fama and MacBeth (1973) suggested an alternative approach to test the CAPM, and their approach has become a standard method for testing cross-sectional asset-pricing models. Their simple but powerful insight was that lack of predictability, with constant expected returns over time, implies that stock returns are uncorrelated over time, even though they are

[27] For example, Black, Jensen and Scholes (1972) noticed that their CAPM tests gave "unreasonably high *t* values."

correlated across stocks at a given time. Based on this insight, Fama and MacBeth (1973) presented a two-step approach for dealing with the problem of cross-sectional correlation.

The first step estimates a sequence of cross-sectional regressions – say, month by month – of stock returns on the characteristics that should determine expected returns according to the asset-pricing model. In the case of the CAPM, each cross-sectional test regresses stock returns on an estimated beta (which in turn had been estimated using data from, say, the previous five years). The second step calculates the time-series average of the coefficients from the cross-sectional regressions, and tests whether these averages deviate significantly from the expected values according to theory. The coefficients from each cross-sectional regression can be interpreted as returns on portfolios weighted by these characteristics (an interpretation developed by Fama, 1976, chapter 9). These returns should be serially uncorrelated under the joint hypothesis that forecast errors are unpredictable and that the first-stage regressors include all relevant determinants of expected returns. The correct standard error for the test can be calculated from the time-series variability of the coefficients from the cross-sectional regressions.

Using their methodology, Fama and MacBeth found that CAPM betas seemed to explain differences in expected returns across stocks. They found, however, that the intercept ($\gamma_{0,t}$) in the regression was larger than the risk-free rate, which is inconsistent with the Sharpe-Lintner-Mossin CAPM, but possibly consistent with the "zero-beta" version of the CAPM due to Black (1972).

The Fama-MacBeth two-step approach quickly became widely used in empirical asset-pricing research, due to both its simplicity in implementation and its robustness. Even though CAPM was refuted in later tests, as discussed below, the Fama-MacBeth procedure is still a standard method for testing multi-factor cross-sectional asset-pricing models, and has been used in thousands of applications.[28]

[28] An econometric issue not explicitly addressed in the original Fama-MacBeth study is that the betas used in the second step suffer from estimation error from the first step. Shanken (1992) and Jagannathan and Wang (1998) accounted for this and derived correct asymptotic standard errors.

7.2 CAPM anomalies

Although the early CAPM tests seemed promising, the empirical support for the model was increasingly questioned towards the end of the 1970s.[29]

First, in an influential paper, Roll (1977) criticized tests of the CAPM, showing that any valid CAPM test presupposed complete knowledge of the market portfolio. According to the CAPM theory, the market portfolio contains every individual asset in the economy, including human capital, and is therefore inherently unobservable. Using a stock market index as a proxy for the market portfolio, which previous tests had done, would therefore lead to biased and misleading results.[30]

Second, numerous studies tested for the determinants of cross-sectional differences in returns, using the methodologies developed in the earlier tests. These tests led to the discovery of a number of CAPM "anomalies," where stock-specific characteristics seemed related to differences in returns. A consistent finding was that various versions of (the inverse of) "scaled stock price," such as the earnings/price (E/P) ratio (Basu, 1977, 1983), the "book-to-market" ratio (i.e., the book equity value divided by the market equity value; Statman, 1980, Rosenberg, Reid and Landstein, 1985), and the debt/equity ratio (Bhandari, 1988), were positively related to expected returns, even after the CAPM beta had been controlled for.

Furthermore, DeBondt and Thaler (1985) showed that stocks that had overperformed over longer horizons, such as the last three to five years, tended to underperform over subsequent years (and vice versa). Finally, stocks of firms with a smaller market value of equity were shown to have higher expected returns (Banz, 1981) than stocks of larger firms, the so-called "size effect."[31] To make matters worse for the CAPM, several studies indicated that CAPM beta did not seem very successful in explaining returns as the sample period of the earlier tests was extended (Reinganum, 1981, and Lakonishok and Shapiro, 1986) or when controlling for other macroeconomic factors (Chen, Roll and Ross, 1986).[32]

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[29] Fama (1991) provided an extensive review of this research.

[30] Later, Gibbons, Ross and Shanken (1989) developed a test of the CAPM that addressed Roll's critique. Using the insight that the CAPM implies that the market portfolio is mean-variance efficient, they derived a test of the ex-ante mean-variance efficiency of a given portfolio.

[31] A number of papers also documented seasonality in stock returns, most prominently the so-called "January effect," indicating that stocks outperform in the month of January (Rozeff and Kinney, 1976). Keim (1983) showed that the January effect is essentially also a small-firm or size effect.

[32] As an alternative to the standard CAPM, the CCAPM predicts that a stock's return covariation with aggregate consumption, its "consumption beta," should be related to expected return differences across assets. The CCAPM tests did not fare better in explaining anomalies, however, and the consumption beta was even shown to be dominated by the standard CAPM beta (Mankiw and Shapiro, 1986, Breeden, Gibbons and Litzenberger, 1989).

Most of these results were integrated in the widely cited paper by Fama and French (1992), which convincingly established that the CAPM beta has practically no additional explanatory power once book-to-market and size have been accounted for.[33]

7.3 The Fama-French three-factor model

The body of work discussed above was synthesized into the three-factor model of Fama and French (1993). Building on the rejection of the simple version of CAPM in their earlier paper (Fama and French, 1992), the paper presented a model which added two new factors to CAPM and suggested a methodology for constructing and testing such factors, building on Fama and Macbeth (1973). The two factors, "small-minus-big" market value (SMB) and "high-minus-low" book-to-market ratio (HML), are based on portfolios of stocks sorted according to the two characteristics that had been found to correlate with expected returns, size and book-to-market value. Each factor is equivalent to a zero-cost arbitrage portfolio that takes a long position in high book-to-market (small-size) stocks and finances this with a short position in low book-to-market (large-size) stocks. Fama and French showed that the SMB and HML factors, apart from explaining differences in expected returns across stocks, also explain a significant amount of variation in the time-series, i.e., stocks with a similar exposure to these factors move together. Hence, they argued, SMB and HML are priced risk factors and the three-factor model should be interpreted as a multi-factor model in the sense of Merton (1973) and Ross (1976).[34]

Over the following years, Fama and French extended this work in a number of papers. For example, in a study published in 1996, they found that the three-factor model captures the return differences from other anomalies, including E/P, leverage and the return reversal of DeBondt and Thaler (1985). They also showed that HML in particular has similar explanatory power for international stock returns (Fama and French, 1998) and is present in U.S. data earlier than that used for their original study (Davis, Fama and French, 2000).

[33] The fact that the beta on the stock market does not relate to expected returns in the *cross-section* is particularly striking because the stock market factor explains substantial variation in the *time series* for individual stocks. Partly because of this time series significance, the stock market beta is typically included in multi-factor models, despite its weak explanatory power in the cross-section.

[34] In addition, Fama and French (1993) showed that two other factors, related to maturity and default risk, capture much of the time-series and cross-sectional variation in bond returns, and that stock and bond market returns are linked through shared variation in the stock and bond factors.

7.4 Rational and behavioral explanations for the cross section of stock returns

Empirically, the Fama-French approach has provided an effective way to simplify and unify the vast literature on the cross section of stock returns, and their method has been widely used both as a reference model for academic research and as a practical guide for professional investors.[35] A weakness of the three-factor model is that it is primarily an empirical model that describes stock returns, but it is silent on the underlying economic reasons for why these risk factors have nonzero prices.

As shown by Ball (1978), characteristics such as book-to-market and size are essentially the inverse of a scaled stock price, and can thus be thought of as proxies for the stochastic discount factor for the stock, as evident from the simple present-value relationship. Consequently, scaled price variables should be related to expected returns when added to any misspecified asset-pricing model.[36]

In their original work, Fama and French developed a rational, multi-factor interpretation of their results, arguing that HML and SMB are capturing fundamental risk factors for which investors demand compensation. In support of this argument, Fama and French (1995) showed that high book-to-market predicts lower earnings, and argued that the excess return to HML therefore should be interpreted as compensation for distress risk.

In contrast, other researchers interpreted the significance of HML and SMB as capturing the effects of market mispricing and investor irrationality along the lines of Shiller (1984). Lakonishok, Shleifer and Vishny (1994) argued that excess return to high book-to-market stocks, or "value stocks," is due to the fact that they are underpriced by investors, while low book-to-market stocks are overpriced "glamor" stocks that subsequently underperform the market.

Although the size and book-to-market effects could be consistent with models of investor mispricing and psychological biases, recent research has found considerable co-movement among stocks with similar book-to-market ratios, i.e., value (low book-to-market) versus growth (high book-to-market) stocks (see, e.g., Campbell and Vuolteenaho, 2004), and that value stocks co-move with "value strategies" in other asset classes, such as fixed income and currencies (see Asness *et al.*, 2013), which is consistent with attributing the higher excess

[35] Later, Carhart (1997) suggested adding momentum as a fourth factor, which is now commonly added to the Fama-French benchmark model.
[36] Also see Berk (1995).

return to value stocks to a common risk factor.[37] Campbell and Vuolteenaho (2004), Campbell. Polk, and Vuloteenaho (2009), and Campbell et al. (2012) argue that the book-to-market effect can be explained by an intertemporal CAPM model (in the sense of Merton, 1973) in which investors care more about permanent cash-flow-driven movements than about temporary discount-rate-driven movements in the aggregate stock market. In their model, the required return on a stock is determined not by its overall beta with the market, but by two separate betas, one with permanent cash-flow shocks to the market (to which high book-to-market "value" stocks are more sensitive), and the other with temporary shocks to market discount rates (to which low book-to-market "growth" stocks are more sensitive). Recently, Campbell *et al*. (2012) found that the same argument can explain a large part of cross-sectional returns for other assets as well, such as equity index options and corporate bonds.

A more serious challenge to informational efficiency is Jegadeesh and Titman's (1993) discovery of "momentum" in stock prices.[38] They found that an investment strategy that buys stocks that have performed well and sells stocks that have performed poorly over the past 3- to 12-month period generates significant excess returns over the following year. The fact that "winner stocks keep on winning and losers keep losing" is consistent with a story where relevant information only gradually disseminates into prices, and this pattern seems unlikely to be explained by changes in risk, given the relatively short horizon. Moreover, unlike many of the other anomalies, momentum is not captured by the Fama-French three-factor model.

Based on these findings, a number of behavioral-finance papers have built theories based on investor psychology to explain both the book-to-market and momentum effects, e.g., based on investor underreaction to news in the short-run (leading to momentum) and overreaction in the longer run (leading to reversals, or book-to-market effects). Examples have been presented by Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1998), and Hong and Stein (1999). Moreover, consistent with limits to arbitrage, momentum has been shown to be particularly pronounced among smaller, more illiquid stocks, and momentum strategies entail substantial risk due to the high correlation within industries (Moskowitz and Grinblatt, 1999).

Another strand of the literature has retained (or remained agnostic to) the standard assumption of rational investors with standard preferences, but instead introduced financial market

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[37] Although Campbell and Vuolteenaho (2004) also found similar co-movement with respect to size, the excess returns to small stocks seem to be much weaker, or even have largely disappeared in recent data.

[38] In the words of Eugene Fama: "Of all the potential embarrassments to market efficiency, momentum is the primary one."(Fama and Litterman, 2012).

frictions to explain asset-pricing patterns. One group of papers has introduced market segmentation, which implies limited risk-sharing among investors. This in turn leads to downward-sloping demand curves for assets in the short term, and mean reversion due to slow-moving capital across markets (Duffie, 2010). Another group of papers has focused on frictions due to financial intermediaries that are restricted by regulation and/or agency problems in their trading of financial assets. These frictions can lead to fire sales of assets when the capital or liquidity of intermediaries becomes scarce (Brunnermeier and Pedersen, 2009, and Coval and Stafford, 2007). Finally, a number of studies have focused on liquidity and its impact on asset pricing. In these models, investors demand an additional risk premium for holding illiquid assets that cannot be easily sold when investors need liquidity, e.g., for consumption (Amihud and Mendelson, 1986, Pastor and Stambaugh, 2003, and Acharya and Pedersen, 2005). Models based on financial frictions and liquidity have been shown to have explanatory power during the recent financial crisis (Brunnermeier, 2009).

8. Influences on market practice

Asset pricing is one of the fields in economics where academic research has had the most impact on non-academic practice. Even though there is still no broad consensus regarding the interpretation of some results, the research initiated by Fama, Shiller and Hansen has produced a body of robust empirical findings, which have important practical implications:

1. In the short term, predictability in stock returns is very limited, which is consistent with stock prices quickly reflecting new public information about future cash flows. To the extent that short-term return predictability can be found, it is too small to profit from because of transaction costs.
2. In the longer term, there is economically significant predictability in stock returns, indicative of variations in expected returns or discount rates. In particular, expected returns in “good” times (at the peak of the business cycle, when measures of relative valuation such as price/dividend ratios are high) are lower than expected returns in “bad” times.
3. In the cross-section of stocks, a number of factors such as book-to-market predict differences in expected returns. Stocks with a similar exposure to these factors co-move, implying that the higher returns come with higher risk.

The early findings on the lack of short-term predictability in stock prices had considerable practical impact. One implication is that it should be extremely hard for asset managers to generate excess risk-adjusted returns. In one of the first studies on this issue, Jensen (1968) evaluated mutual fund performance and found that the majority of funds did not generate any excess risk-adjusted returns. Subsequent studies of mutual-fund performance generally have failed to find positive excess performance (and often found negative excess performance) after fund fees. Recently, Fama and French (2010) have documented that only the extreme tails of active mutual funds generate significant (negative and positive, respectively) risk-adjusted excess returns before fees, and that the aggregate portfolio of active mutual funds in fact is close to the market portfolio. The latter means that the sector as a whole gives negative excess returns to investors.

Inspired by the work of Fama, Jensen and others, so-called index funds started to emerge in the early 1970s.[39] Today, passively managed funds, such as index funds and Exchange Traded Funds (ETFs), exist for a large variety of indexes and asset classes, including size and book-to-market. In 2012, these funds had over $3.6 trillion (U.S.) under management and accounted for 41% of the worldwide flows into mutual funds.

The research on market predictability and on cross-sectional return differences across financial assets has also had considerable practical impact and has contributed to the growth of "quantitative investment management," where investors use quantitative factors and statistical modeling to make investment decisions. For example, many professional investors use factor models such as the Fama-French model to guide their portfolio decisions, and long-term institutional investors commonly use variables that have been shown to predict medium-run stock market returns to adjust the fraction of equity relative to bonds in their portfolios.

The academic work on the determinants of cross-sectional returns has also had a large impact on the practice of portfolio performance measurement. Given that a portfolio manager can get a higher rate or return on her portfolio simply by investing in assets with higher risk, she needs to be evaluated based on risk-adjusted returns. Jensen (1968) introduced a measure of risk-adjusted performance, the so-called "Jensen's alpha," which is essentially the intercept of a regression of excess returns on risk factors, such as the Fama-French three factors.[40] Intuitively, since an investor can achieve a high return simply by investing in assets with high

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[39] The world's first index fund was started at the U.S. bank Wells Fargo in 1971, managed on behalf of Samsonite Corporation ("the Samsonite Luggage Fund"). A few years later in 1975, Vanguard launched the first index fund directed towards retail investors. See Bernstein (2005)

[40] In Jensen's original study, he measured alpha relative to the "market model," i.e., controlling for the CAPM beta.

loadings on the Fama-French factors, a portfolio manager should only be rewarded on excess performance relative to these factors, i.e., on alpha. Alpha has become a standard tool for evaluating portfolio managers and mutual funds (used, e.g., by Morningstar). Moreover, following the work of Fama and French, it has become standard to evaluate performance relative to “size” and “value” benchmarks, rather than simply controlling for overall market returns.

Research findings from empirical asset pricing have also had practical impact outside the investment management industry. The event-study methodology of Fama, Fisher, Jensen and Roll (1969) has become an important tool in legal practice for assessing damages in lawsuits, for example in securities-fraud cases (Mitchell and Netter, 1994). Event studies have also been used by competition authorities, to evaluate the competitive effect of mergers by looking at the stock-price reaction of a merger on the other firms in the industry (e.g., Beverley, 2007).

Another area of practical impact is the measurement of asset returns and price indexes. The CRSP data set created at the University of Chicago was the first comprehensive stock market database in existence. It has had a profound impact not only on academic research but also on quantitative investment strategies used by the industry.

Beyond stock prices, Case and Shiller (1987) constructed the first systematic, high-quality indexes of U.S. house prices. The S&P Case-Shiller index is now the standard real estate price index in the U.S., widely used by practitioners and policymakers. Shiller’s interest in index construction was motivated by the insight that the volatility of house prices constitutes a major risk for many households. In his 1991 book *Macro Markets*, Shiller highlighted the fact that major risks in society, like house-price risks, are uninsurable despite their importance. He argued that developing markets for derivative contracts based on price indexes would help households to hedge against such risks. In particular, such contracts would allow households to go short in the housing market. But it would also allow households to speculate against an overvalued housing market. Shiller has also translated these insights into practice and has helped setting up a market in cash-settled house-price futures at the Chicago Mercantile Exchange based on the S&P Case-Shiller indexes.

9. Conclusions

Eugene Fama, *Lars Peter Hansen*, and *Robert Shiller* have developed empirical methods and used these methods to reach important and lasting insights about the determination of asset prices. Their methods have shaped subsequent research in the field and their findings have been highly influential both academically and practically. The waves of research following the original contributions of the Laureates constitute a landmark example of highly fruitful interplay between theoretical and empirical work.

We now know that asset prices are very hard to predict over short time horizons, but that they follow movements over longer horizons that, on average, can be forecasted. We also know more about the determinants of the cross-section of returns on different assets. New factors – in particular the book-to-market value and the price-earnings ratio – have been demonstrated to add significantly to the prior understanding of returns based on the standard CAPM. Building on these findings, subsequent research has further investigated how asset prices are fundamentally determined by risk and attitudes toward risk, as well as behavioral factors.

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