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Risks for the long run: Estimation with time aggregation☆

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ABSTRACT

The discrepancy between the decision and data-sampling intervals, known as time aggregation, confounds the identification of long-, short-run growth, and volatility risks in asset prices. This paper develops a method to simultaneously estimate the model parameters and the decision interval of the agent by exploiting identifying restrictions of the Long Run Risk (LRR) model that account for time aggregation. The LRR model finds considerable empirical support in the data; the estimated decision interval of the agents is 33 days. Our estimation results establish that long-run growth and volatility risks are important for asset prices.

1. Introduction

The long-run risks (LRR) model developed by Bansal and Yaron (2004) has motivated a significant amount of research in macro and financial economics. The model captures the intuition that low-frequency fluctuations in consumption growth and its volatility are important for understanding asset prices. The implications of the LRR model are typically evaluated via calibrations because the estimation of the model is known to present a number of challenges. First, the return of the aggregate consumption claim that governs the dynamics of marginal utility in the LRR model is not observable. Second, time aggregation of consumption and dividend data may hinder the identification of the model risk components and parameters. This paper develops a method that addresses these challenges and estimates the model using consumption and financial market data. It shows that the LRR model is able to simultaneously account for the joint dynamics of aggregate consumption, asset cash flows and prices – the model's asset pricing restrictions find support both in a long sample of annual data and a shorter sample of post-war quarterly data. It further shows that the failure to account for time aggregation leads to a significant mismeasurement of risks, distorted parameter estimates, and false model rejections. Our paper, thus, underscores the importance of time aggregation for estimating the dynamics of the LRR model.

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Time aggregation arises from a mismatch between the decision interval of agents and the sampling frequency of the data. This impedes identification of the model parameters as discussed in Hansen and Sargent (1983). To appropriately identify various risks and structural parameters, we derive consumption, dividend and asset price dynamics of the time-aggregated LRR model by factoring in the unknown decision interval of the agent. By exploiting the derived moment restrictions we are able to extract the latent risk factors and estimate the decision frequency along with all other model parameters in a GMM framework of Hansen (1982). We show that the decision interval, a parameter that we estimate, has a significant effect on the models dynamics and, therefore, on the identification of the underlying risks.

Our empirical evidence provides considerable support for the LRR model and suggests that: (i) investors have a preference for early resolution of uncertainty, (ii) shocks to the expected growth component of consumption have a long-run effect that extends beyond typical business cycle frequencies, (iii) variation in consumption volatility, while relatively small, is very persistent, (iv) agents decision interval is roughly one month and, hence, is quite close to the value typically assumed in the calibrated versions of the LRR model. To be specific, based on annual data from 1930 to 2015, the estimates of the coefficient of risk aversion and the intertemporal elasticity of substitution (IES) are 9.7 and 2.2, respectively. Both have relatively tight standard errors – 1.4 for risk aversion and 0.2 for the IES. The long-run growth and volatility components of consumption are highly persistent, with implied annual autocorrelations of 0.75 and 0.98, respectively. The long-lasting nature of expected growth and volatility shocks manifests into high risk premia and high volatility of equity prices. The estimated model implies a market risk premium of 6.7% and a 17.3% volatility of stock market returns, and generates a low risk-free rate of about 1%. The number of decision periods within a year is estimated at 11, which corresponds to a decision interval of approximately 33 days. Importantly, this estimated model, referred to as the benchmark LRR model, is not rejected with a *p*-value associated with the *J*-test for overidentifying restrictions of 11%.[1]

The inference and the economic implications change significantly if in estimation the effect of time aggregation is ignored. To demonstrate this, we estimate an "annual" version of the LRR model that assumes that the decision interval of agents is equal to the annual sampling frequency of the data. First, the annual specification is strongly rejected. Second, while the estimates of the annual specification also imply relatively high persistence of the expected consumption growth, the contribution of long-run risks to the overall volatility of consumption growth in the annual specification is much smaller than in the benchmark model. This difference is driven by time aggregation. In the annual specification, most of the annual consumption shock is incorrectly identified as short-run risk. As the implied contribution of long-run risk in the annual specification is incorrectly estimated to be quite small, the model requires high risk aversion to match the equity premium, despite this it fails to account for the dynamics of asset prices and, therefore, is rejected.

Using simulations the paper documents that when time aggregation is ignored, the model is overly rejected, the risk aversion estimate is upward biased, and the contribution of long-run growth risks is severely understated, all of which is consistent with our empirical findings. When the restrictions of time aggregation are not imposed, a sizable portion of the low-frequency growth shock tends to be attributed to the short-run shock, which diminishes the role of long-run risks and thereby lowers the ability of the model to match volatility of asset returns and prices. Overall, our evidence suggests that accounting for temporal aggregation in estimating the model and measuring the contribution of different sources of risks is extremely important.

In addition to the time-series implications of the benchmark LRR model the paper also evaluates its implications for the cross-section of size and book-to-market sorted portfolios. We find that assets with large mean returns, such as value and small market capitalization portfolios, are more sensitive to long-run risks. Similarly to the implications for the market portfolio, low-frequency growth and volatility risks are the key source of risk premia in the cross section. The LRR model is able to replicate the failure of the CAPM; in particular, it generates low market betas and high CAPM alphas of the value-minus-growth and small-minus-large strategies, of the same magnitudes as in the data.

Earlier work by Epstein and Zin (1989) rely on the GMM technique of Hansen and Singleton (1982) to estimate a model based on recursive preferences by replacing the unobservable return on the consumption asset with the market equity return. Different from them, we infer the dynamics of the wealth return from the observed consumption data by exploiting the model's pricing restrictions. The ability of long-run growth risks to account for various features of asset market data has been explored in a series of papers. Bansal et al. (2005), and Hansen et al. (2008) show that long-run risks in cash flows are important for understanding the cross-sectional variation in risk premia. Recent contributions to the long-run risk literature also include Parker and Julliard (2005), Kiku (2006), Bansal et al. (2007), Bekaert et al. (2009), Malloy et al. (2009), Lettau and Ludvigson (2009), Constantinides and Gosh (2011), Colacito and Croce (2011), Ferson et al. (2013), Calvet and Czellar (2015), Colin-Dufresne et al. (2016), and Jagannathan and Liu (2016). Distinct from these papers, we estimate and evaluate the LRR model in a GMM framework while imposing the models restrictions on the joint dynamics of consumption, dividends, and prices that appropriately account for temporal aggregation and highlight the importance of time aggregation in identifying the underlying risks.

The paper continues as follows. Section 2 presents the model and its testable restrictions. Section 3 provides details of our estimation methodology. Section 4 describes the data. Section 5 presents and discusses results of our empirical analysis. Section 6 provides concluding remarks.

[1] As discussed below, the estimates and the implications of the model estimated using post-war quarterly data are quite similar.

2. Long-run risks model

This section specifies the long-run risks model based on Bansal and Yaron (2004). The underlying environment is one with complete markets and a representative agent that has Epstein and Zin (1989) type preferences, which allow for a separation of risk aversion and the elasticity of intertemporal substitution. Specifically, the agent maximizes her life-time utility, which is defined recursively as,

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is consumption at time t, $0<\delta<1$ reflects the agent's time preferences, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution (IES). Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t)R_{c,t+1}\ , \tag{2}$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the return on all invested wealth.

Consumption growth has the following dynamics:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1} \tag{3}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} \tag{4}$$

$$\sigma_{t+1}^2 = \sigma_0^2 + \nu(\sigma_t^2 - \sigma_0^2) + \sigma_w w_{t+1}\ , \tag{5}$$

where Δc_{t+1} is the growth rate of log consumption, and the three shocks, η, e, and w are assumed to be *i.i.d* Normal and uncorrelated. The conditional expectation of consumption growth is given by $\mu_c + x_t$, where x_t is a small but persistent component that captures long-run risks in consumption growth. The parameter ρ determines the persistence in the conditional mean of consumption growth. For parsimony, as in Bansal and Yaron (2004), time-varying volatility in realized and expected consumption growths is common. As shown in their paper, time-variation in variance leads to time-varying risk premia. The unconditional variance of consumption is σ_0^2 and ν governs the persistence of the volatility process.

2.1. Model solutions

The log of the intertemporal marginal rate of substitution (IMRS), $m_{t+1} = \log(M_{t+1})$, is given by

$$m_{t+1} = \theta \log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1}\ , \tag{6}$$

where $r_{c,t+1}$ is the continuous return on the consumption asset, which is endogenous to the model. Thus, in order to characterize the intertemporal marginal rate of substitution, one needs to solve for the unobservable return on the consumption claim. The solution for $r_{c,t+1}$ can be derived by exploiting the dynamics of consumption growth and the log-linear approximation of the continuous return, namely,

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} + \Delta c_{t+1} - z_t\ , \tag{7}$$

where $z_t = \log(P_t/C_t)$ is the log price-consumption ratio (i.e., the valuation ratio corresponding to a claim that pays aggregate consumption), and κ's are constants of log-linearization,

$$\kappa_1 = \frac{\exp(\bar{z})}{1+\exp(\bar{z})} \tag{8}$$

$$\kappa_0 = \log(1+\exp(\bar{z})) - \kappa_1 \bar{z}\ , \tag{9}$$

where $\bar{z}$ denotes the mean of the log price-consumption ratio.

To derive the time series for $r_{c,t+1}$, conjecture that the solution for log price-consumption ratio is affine,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2\ . \tag{10}$$

The solution coefficient A's depend on all the preference parameters and the parameters that govern the dynamics of consumption growth. For notational ease, let $z_t = \mathbf{A}'Y_t$, where $Y_t' = [1\ \ x_t\ \ \sigma_t^2]$ is the vector of state variables, and $\mathbf{A}' = [A_0\ \ A_1\ \ A_2]$, which is given by

$$\mathbf{A}' = \left[A_0 \quad \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} \quad -\frac{(\gamma-1)\left(1-\frac{1}{\psi}\right)}{2(1-\kappa_1\nu)}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right] \right] \tag{11}$$

be the corresponding vector of price-consumption elasticities.[2] As discussed in Bansal and Yaron (2004), the elasticities of

[2] The expressions for A_0 and Γ_0 in Eq. (15), as well as the derivations of all other expressions, are given in the online appendix.

the price-consumption ratio with respect to the expected growth component, x_t, and volatility, σ_t, depend on the preference configuration. In particular, for the elasticity A_1 to be positive, the IES parameter has to be greater than one. Also, for the price-consumption ratio to exhibit a negative response to an increase in economic uncertainty, the IES has to be larger than one, given that risk aversion is greater than one.

Note that the derived solutions depend on the approximating constants, κ_0 and κ_1, which, in turn, depend on the endogenous mean of the price-consumption ratio, $\bar{z}$. In order to solve for $\bar{z}$, we first substitute expressions for κ's (Eqs. (8) and (9)) into the expressions for A's and solve for the mean of the price-consumption ratio. Specifically, $\bar{z}$ can be found numerically by solving a fixed-point problem,

$$\bar{z} = \mathbf{A}(\bar{z})'\bar{Y}\,, \tag{12}$$

where the dependence of the A's on $\bar{z}$ is given above, and $\bar{Y}$ is the mean of the state vector Y.

Given the solution for z_t, the IMRS can be stated in terms of the state variables and innovations,

$$m_{t+1} = \mathbf{\Gamma}'Y_t - \mathbf{\Lambda}'\zeta_{t+1}\,, \tag{13}$$

where the three sources of risks are

$$\zeta'_{t+1} = \begin{bmatrix} \sigma_t\eta_{t+1} & \sigma_t e_{t+1} & \sigma_w w_{t+1} \end{bmatrix}, \tag{14}$$

and the three dimensional vectors $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ are given by,

$$\mathbf{\Gamma}' = \begin{bmatrix} \Gamma_0 & -\frac{1}{\psi} & -(\gamma-1)\left(\gamma-\frac{1}{\psi}\right)\frac{1}{2}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right] \end{bmatrix}, \tag{15}$$

$$\mathbf{\Lambda}' = \begin{bmatrix} \gamma & \left(\gamma-\frac{1}{\psi}\right)\frac{\kappa_1\varphi_e}{1-\kappa_1\rho} & -(\gamma-1)\left(\gamma-\frac{1}{\psi}\right)\frac{\kappa_1}{2\,(1-\kappa_1\nu)}\left[1+\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2\right] \end{bmatrix}. \tag{16}$$

Note that the stochastic discount factor in Eq. (13) is exact up to an approximation error emanating from the linearization around the theoretical value of the average price–consumption ratio. This approximation error is quite small and does not materially affect our empirical results that follow. The online appendix provides a detailed discussion of the magnitude of the approximation error and a comparison of the log-linear solution with a solution based on numerical methods.

Other assets can easily be priced using the IMRS given in Eq. (13). Dividend dynamics for any other asset j are assumed to follow

$$\Delta d_{j,t+1} = \mu_j + \phi_j x_t + \varphi_j \sigma_t u_{j,t+1} \tag{17}$$

where ϕ_j and φ_j determine asset j's exposure to the long-run and volatility risks, respectively. Exposure to short-run consumption risks is determined by the correlation between dividend and consumption innovations, $u_{j,t+1}$ and η_{t+1}, which is denoted by ϱ_j. We let Δd_{t+1} denote the dividend growth rate of the aggregate market portfolio, and reserve d-subscript for various quantities of the stock market index. Specifically, μ_d, ϕ_d, and ϱ_d correspond to the aggregate market dividend, and $z_{d,t}$ and $r_{d,t+1}$ denote the price-dividend ratio and the return on the aggregate market portfolio.

The first-order condition yields the following asset pricing Euler condition,

$$E_t\left[\exp(m_{t+1}+r_{j,t+1})\right] = 1\,, \tag{18}$$

where $r_{j,t+1}$ is the log of the gross return on asset j. Similar to the claim to consumption, the price-dividend ratio for any asset j, $z_{j,t} = \mathbf{A}_\mathbf{j}'Y_t$, with the solutions given in the online appendix. Furthermore, given the expression for the IMRS, it follows that the risk premium on asset j is,

$$E_t[r_{j,t+1} - r_{f,t} + 0.5\sigma^2_{t,r_j}] = \beta_{\eta,j}\lambda_\eta\sigma_t^2 + \beta_{e,j}\lambda_e\sigma_t^2 + \beta_{w,j}\lambda_w\sigma_w^2, \tag{19}$$

where $\beta_{i,j}$ is the return beta of asset j with respect to the ith risk source where $i = \{\eta, e, w\}$, and λ_i is the corresponding entry of the vector of market prices of risks, $\mathbf{\Lambda}$. Under the structure of the model, the return β's and market price of risks λ's will be functions of the preference parameters and the underlying parameters of consumption and dividend dynamics, details of which are given in the online appendix. Finally, it is easy to verify that the risk free rate can be represented as

$$r_{f,t} = F_0 + F_1 x_t + F_2\sigma_t^2 = \mathbf{F}'Y_t \tag{20}$$

where the loading coefficients are given in the online appendix.

The intertemporal elasticity of substitution is a critical parameter in the LRR model. Giovannini and Weil (1989), Tallarini (2000), Hansen et al. (2008), and Hansen and Sargent (2010) consider a special case where the IES parameter is one. Our estimation methodology nests this special case in a continuous fashion (details are given in the online appendix). Namely, the IMRS components as given in Eq. (13) adjust in a continuous way as one takes the limit of the IES parameter at one.[3] That

[3] Evaluating the pricing kernel in Eq. (13) under the above restrictions gives exactly the same solution as in Giovannini and Weil (1989), Tallarini (2000), and Hansen et al. (2008).

is,

$$\lim_{\psi\to 1}\kappa_1=\delta\,,\quad \lim_{\psi\to 1}\Gamma'=\Gamma'(\psi=1,\kappa_1=\delta)\,,\quad \lim_{\psi\to 1}\Lambda'=\Lambda'(\psi=1,\kappa_1=\delta)\,. \tag{21}$$

Thus, in estimation, the IES is treated as a free parameter that may take on any positive value, including one.

3. Estimation method

The estimation of the LRR model presents several challenges. First, the underlying state variables, long-run growth and volatility of consumption growth rates, are not observable. Second, reliable consumption data are available only at a quarterly and annual frequencies that may not correspond to the frequency at which agents make their decisions. A potential mismatch between the model and the data frequencies may hinder the identification of the underlying consumption risks and preference parameters and, therefore, may lead to distorted inference. Our estimation strategy, discussed below, is designed to address the two challenges and provide direct evidence of the importance of long-run growth and volatility risks in financial markets.

Our estimation is carried out using the Generalized Method of Moments (GMM). To account for a potential discrepancy between the sampling frequency of the data and the decision interval, the vector of model parameters, $\Theta=\{\gamma,\psi,\delta,\mu_c,\rho,\varphi_e,\sigma_0^2,\nu,\sigma_w,\mu_d,\phi_d,\varphi_d,\varrho_d\}$, is estimated simultaneously with the decision interval of the agent. That is, estimating the model, entails searching *jointly* for the best parameter set, Θ, and the decision frequency, h, that fit the data.[4] The estimation of the model is based on a rich set of moments that characterize the joint dynamics of consumption, dividends and asset prices.

This section first discusses the extraction of the state-variables, x_t and σ_t^2, from the observed data, and then provides analytical expressions for the unconditional moments of time-aggregated model that are exploited in estimation. It is worth noting that in the presence of time aggregation, the shocks to the IMRS (Eq. (13)) cannot be recovered and, therefore, the standard Euler equation-based estimation approach of Hansen and Singleton (1982) cannot be used.[5] Our approach allows for estimation even when the shocks and the IMRS are not available to an econometrician.

3.1. Recovering the state variables

The extraction of the state variables, x_t and σ_t^2, exploits the affine structure of the price-dividend ratio and the risk-free rate. Eqs. (10) and (20) satisfy,

$$\mathcal{S}_t=\begin{bmatrix} z_{d,t}\\ r_{f,t}\end{bmatrix}-\begin{bmatrix}\mathbf{A}_\mathbf{d}'\\ \mathbf{F}'\end{bmatrix}\left[\,Y_t\,\right]. \tag{23}$$

Thus, x_t and σ_t^2 can be recovered from the observed price-dividend ratio and the risk-free rate data. In particular, for each date t and a given set of model parameters, we solve for a pair of the state variables that minimizes the error in Eq. (23), while ensuring positivity of variance.[6] In addition, during the extraction step we impose autoregressive dynamics on the state variables as implied by the model. It is important to note that the extraction of the state variables is done simultaneously with the GMM estimation of the moment parameters, as discussed further below.

3.2. Time aggregation

In estimating the model, we account for the possibility that the decision frequency of the agent and the sampling frequency of the available data might be different. Note that under the assumption that the two frequencies coincide, the estimation can be simply carried out by exploiting a set of moment restrictions based on the dynamics presented in Section 2. However, if this assumption does not hold—for example, if the data are sampled at a coarser frequency—these restrictions are no longer satisfied because the dynamics of the observed time-aggregated data are different from the underlying model dynamics. Our estimates are not subject to such misspecification because we explicitly account for time aggregation of the available data by treating the decision frequency of the agent as unknown and estimating it jointly with other model

[4] Earlier papers that account for time aggregation in estimation in the asset-pricing context include Hansen and Sargent (1983) and Heaton (1995).

[5] Using Eq. (13), the log of the h-period time-aggregated IMRS follows:

$$m_{t+h,h}\equiv\sum_{j=1}^{h}m_{t+j}=\tilde{\Gamma}Y_t-\sum_{j=1}^{h}\left[\lambda_\eta\sigma_{t+j-1}\eta_{t+j}+\lambda_e\sigma_{t+j-1}e_{t+j}+\lambda_w\sigma_w w_{t+j}\right] \tag{22}$$

Long-run and volatility shocks, e_t and w_t, can be extracted from the available high-frequency financial data. However, short-run consumption innovations, η_t, cannot be recovered unless consumption data are observed at the (unknown) model frequency.

[6] To guarantee positivity of the variance component, the minimization problem is solved by searching over a two-dimensional grid in the $\{x_t,\sigma_t^2\}$-space, with $\sigma_t^2>0$. Because our grid is very fine, our approach is equivalent to solving a constrained least-squares problem.

parameters. Specifically, the model is estimated by exploiting the moment conditions of the time-aggregated LRR dynamics. These dynamics are derived below for a general frequency of time aggregation.

Let τ denote the time index of the sampling frequency of the data and let h denote the (integer) number of decision intervals within a sampling period. For example, if an econometrician uses annual data and the agent's decision interval is monthly (weekly), then a τ-increment is equal to one year and $h=12(52)$. To summarize, the available sample of *observable* data is indexed by $\tau=1,2,\ldots\mathcal{T}$, the underlying *unobservable* series are indexed by t, and the calendar time t corresponds to the sampling frequency τ via $t=\tau\cdot h$. Note that if the sampling frequency of the data corresponds to the decision frequency of the agent, then $\tau=t$ and $h=1$.

Given out notation, let C_t denote the unobservable consumption series and let $C_\tau^a=\sum_{i=1}^h C_{t-h+i}$ denote the available time-aggregated data. It can be shown that the growth of the observed time-aggregated consumption growth, $\Delta c_\tau^a=\log(C_\tau^a/C_{\tau-1}^a)$, can be approximated by:

$$\Delta c_\tau^a\equiv\log\frac{\sum_{i=1}^h C_{t-h+i}}{\sum_{i=1}^h C_{t-2h+i}}\approx\sum_{j=2}^h\frac{j-1}{h}\Delta c_{t-2h+j}+\sum_{j=1}^h\frac{h-j+1}{h}\Delta c_{t-h+j}\,,\forall t=\tau\cdot h \tag{24}$$

Given the dynamics of the agent's consumption in Eq. (3), the growth rate of time-aggregated consumption can be further expressed in terms of the state variables and a sequence of conditionally mean-zero innovations:

$$\begin{aligned}\Delta c_\tau^a\approx{}& h\mu_c+\frac{\rho(1-\rho^h)^2}{h(1-\rho)^2}x_{t-2h}+\sum_{j=1}^{h-1}a_j\varphi_e\sigma_{t-2h-1+j}e_{t-2h+j}+\sum_{j=1}^{h}b_j\varphi_e\sigma_{t-1-j}e_{t-j}\\&+\sum_{j=0}^{h-1}\frac{j+1}{h}\sigma_{t-1-j}\eta_{t-j}+\sum_{j=0}^{h-2}\frac{h-j-1}{h}\sigma_{t-h-1-j}\eta_{t-h-j}\,,\quad\forall t=\tau\cdot h\,,\end{aligned}\tag{25}$$

where $a_j=\frac{1}{h\rho^{j-1}}\left[\left(\frac{1-\rho^h}{1-\rho}\right)-\frac{1}{1-\rho}\left(\frac{1-\rho^{j-1}}{1-\rho}-(j-1)\rho^{j-1}\right)\right]$ and $b_j=\frac{1}{h\rho^{j-1}}\left[j-\rho\frac{1-\rho^j}{1-\rho}\right]$.

The dynamics of time-aggregated dividend growth, asset prices and returns can be derived similarly. For concreteness suppose that the observed data are annual and consider the annual price-dividend ratio: $z_{d,\tau}^a\equiv\log\frac{P_t}{\sum_{j=0}^{h-1}D_{t-j}}$, defined as the log of the end-of-year price over the sum of dividends within a year. Recall that the solution to the model's price-dividend ratio takes the form $z_{d,t}=A_{0,d}+A_{1,d}x_t+A_{2,d}\sigma_t^2$ with the coefficients, A_d's, given in the online appendix. It can be shown that the dynamics of the annual price-dividend ratio follow:

$$\begin{aligned}z_{d,\tau}^a\approx{}&A_{0,d}+A_{2,d}\sigma_0^2\left(1-\nu^h\right)-\log(h)+0.5\mu_d(h-1)\\&+\left[\pi+A_{1,d}\rho\right]x_{t-h}+A_{2,d}\nu^h\sigma_{t-h}^2+\sum_{j=1}^h\left[q_j+A_{1,d}\rho^j\right]\varphi_e\sigma_{t-h-1+j}e_{t-h+j}\\&+\sum_{j=1}^{h-1}\frac{h-j}{h}\varphi_d\sigma_{t-j}u_{t-j+1}+\sum_{j=1}^{h}A_{2,d}\sigma_w\nu^{h-j}w_{t-h+j}\,,\quad\forall t=\tau\cdot h\,,\end{aligned}\tag{26}$$

where π and q_j are given in the online appendix.

3.3. Moments

Our estimation is based on moments that capture key features of aggregate consumption, dividends, and asset price data. To be specific, three sets of moments are exploited: (i) moments that characterize the joint dynamics of consumption and dividend growth rates, (ii) the level of expected returns, and return volatility (iii) predictability of asset returns and consumption. The first set of moments comprises the mean, volatility and autocorrelation of consumption and dividend growth rates as well as their correlation. The second set of moments consists of the mean and volatility of the equity returns, the risk-free rate, and the price-dividend ratio and its autocorrelation, thus confronting the model with both the equity premium and the volatility puzzles. To account for predictability of consumption growth and equity returns, we use the correlations of the price-dividend ratio with future consumption growth and with future market returns. Additional moments comprise the orthogonality restrictions implied by the model and moments that provide information about volatility dynamics. A full list of moment conditions is presented in Table 3.

The computation of several moments is discussed below and further details are provided in the online appendix. Using Eq. (25), it follows that the conditional mean of time-aggregated consumption growth is $h\mu_c+\frac{\rho(1-\rho^h)^2}{h(1-\rho)^2}x_{t-2h}$ and, consequently, the unconditional mean and variance are given by:

$$E[\Delta c_\tau^a]=h\mu_c, \tag{27}$$

$$Var\left[\Delta c_{\tau}^{a}\right]=\left[\frac{\rho(1-\rho^{h})^{2}}{h(1-\rho)^{2}}\right]^{2}var(x_{t-2\,h})+\sum_{j=1}^{h-1}\left[(a_{j}\varphi_{e})^{2}+\left(\frac{h-j}{h}\right)^{2}\right]\sigma_{0}^{2}+\sum_{j=1}^{h}\left[(b_{j}\varphi_{e})^{2}+\left(\frac{j}{h}\right)^{2}\right]\sigma_{0}^{2}. \tag{28}$$

Note that in the case of annual data and a monthly decision interval, the mean of observed consumption growth is equal to $12\mu_c$ and μ_c is interpreted as a monthly quantity. In the case of an annual sampling frequency and an annual decision interval, μ_c corresponds to an annual growth.

Similarly, the mean and variance of the aggregated price-dividend ratio are:

$$E[z_{d,\tau}^{a}]=A_{0,d}+A_{2,d}\sigma_{0}^{2}-\log(h)-0.5\mu_{d}(h-1), \tag{29}$$

$$Var[z_{d,\tau}^{a}]=\left[\pi+A_{1,d}\rho^{h}\right]^{2}var(x_{t})+\left[A_{2,d}\nu^{h}\right]^{2}var(\sigma_{t}^{2})+\sum_{j=1}^{h}[q_{j}+A_{1,d}\rho^{h-j}]^{2}(\varphi_{e}\sigma_{0})^{2}+\sum_{j=1}^{h-1}\left[\frac{h-j}{h}\varphi_{d}\right]^{2}\sigma_{0}^{2}+\sum_{j=1}^{h}\left[A_{2,d}\sigma_{w}\nu^{h-j}\right]^{2}. \tag{30}$$

All other moments can be computed similarly (see the online appendix). Note that the moment conditions exploited in estimation are functions of the structural model parameters as well as the decision interval, h, and therefore, are informative about the unknown decision frequency of the agent.

3.4. Estimation

Let $\mathcal{M}(\Theta;h,\{Data_{\tau}\})$ denote the difference between the model based moment conditions (evaluated at Θ) and their data counterparts when there are h decision periods within a sampling interval. The parameter vector Θ is chosen by evaluating time-aggregated moment conditions $\mathcal{M}(\Theta;h,\{Data_{\tau}\})$ while *simultaneously* choosing h and the state variables, x_t and σ_t^2, as described above. The parameter vector Θ and h are estimated by minimizing the standard GMM criteria,

$$\{\hat{\Theta};\hat{h}\}=\underset{\{\Theta;h\}}{arg\ min}\ \mathcal{M}(\Theta;h,\{Data_{\tau}\})'W(\Theta;h)\mathcal{M}(\Theta;h,\{Data_{\tau}\}) \tag{31}$$

where $Data_{\tau}$ pertains to observed annual or quarterly data used in evaluating the moment conditions. The weighting matrix $W(\Theta;h)$ used in estimation is the diagonal inverse of the variance-covariance matrix of the moment conditions and is updated continuously, motivated by Hansen et al. (1996). To construct the chi-squared test for over-identifying restrictions, we compute *J*-statistic using Lemma 4.2 in Hansen (1982), which holds for a general weighting matrix. The variance–covariance matrix is computed using the Newey and West (1987) estimator. Standard errors of the parameter estimates are constructed via a block bootstrap by resampling the data in eight-year blocks and re-estimating the model parameters using bootstrap samples.

4. Data

Data on consumption and asset prices span the time period from 1930 till 2015. Our sample is an extension of the dataset used in Bansal et al. (2012) and Beeler and Campbell (2012) that discuss the implications of the calibrated versions of the long-run risks model. Hence, our dataset provides a proper benchmark for the comparison of our estimates with prior calibrations of the LRR model. We use the longest available sample, starting from 1930, in order to capture the overall variation in asset and macro-economic data and to ensure identification of low-frequency risks.

In addition, the long span of the data helps us achieve more reliable statistical inference. The data sampled on an annual frequency as they are less prone to errors that arise from seasonalities and other measurement problems highlighted in Wilcox (1992).

Consumption data represent per capita series of real consumption expenditure on non-durables and services from the NIPA tables available from the Bureau of Economic Analysis. Aggregate stock market data consist of annual observations of returns, dividends, and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. Price and dividend series are constructed on the per-share basis as in Campbell and Shiller (1988), Bansal et al. (2005), and Hansen et al. (2008). Market data are converted to real using the consumer price index (CPI) from the Bureau of Labor Statistics. Growth rates of consumption and dividends are constructed by taking the first difference of the corresponding log series. The ex ante real risk-free rate is constructed following the approach in Bansal et al. (2012). Table 1 provides key sample statistics for aggregate consumption growth, the stock market index, and the risk-free rate.

5. Empirical findings

This section presents the estimates of the LRR model and discusses model's implications for the joint dynamics of aggregate consumption, and dividends and prices of the market portfolio. It also highlights the effect of time aggregation on parameter estimates and inference, and evaluates the cross-sectional predictions of the model.

Table 1
Summary statistics. Table 1 presents descriptive statistics for aggregate consumption growth, returns, dividend growth and the logarithm of the price-dividend ratio of the stock market portfolio and the risk-free rate. Returns are value-weighted, dividends and price-dividend ratios are constructed on the per-share basis, growth rates are measured by taking the first difference of the logarithm of the corresponding series. All data are real, sampled on an annual frequency and cover the period from 1930 to 2015.

Variable	Mean	Std. Dev.
Consumption growth	0.018	0.02
Dividend growth	0.014	0.11
Market return	0.081	0.19
Log (P/D)	3.404	0.45
Risk-free rate	0.005	0.03

5.1. Estimation evidence

Using the methodology outlined above, we estimate two model specifications: the LRR model and a nested specification, labeled "No-Vol", that restricts the conditional volatility process to be constant. The "No-Vol" specification is estimated by exploiting the same set of moments as in the LRR case, except for the conditional moment of the volatility dynamics, which is not defined when time-variation in the conditional second moment is ruled out. We use data sampled on the annual frequency and take into account potential time-aggregation effects by estimating the decision frequency (h) along with the structural parameters of preferences and the dynamics of consumption and dividends. Table 2 presents the GMM estimates of the two models, their standard errors and the χ^2-test of overidentifying restrictions. The sample moments, their model-implied counterparts and the t-statistics for the difference between the two are provided in Table 3.

The parameter estimates of the LRR model, reported in the left panel of Table 2, provide strong evidence of (i) time aggregation, and (ii) a persistent predictable component in growth rates and persistent time-varying uncertainty. The estimate of the decision frequency ($\hat{h}$) is 11 which corresponds to a decision interval of approximately 33 days (365/11). Note that the estimate of h is significantly greater that one, i.e., the decision interval is much shorter than a year.

The estimate of ρ, which governs the autocorrelation of the conditional mean of consumption growth, is about 0.98 and is significantly different from zero. The magnitude of long-run risks is quite small, $\hat{\varphi}_e = 0.032(\text{SE} = 0.005)$, suggesting that the predictable growth component contributes relatively little to the overall variation of consumption growth, which explains the difficulty in detecting long-run risks from consumption data alone. The estimated long-run risk component, along with realized values of annual consumption growth, are plotted in Fig. 1. As the figure shows, the extracted long-run risk component captures well low-frequency variations in consumption growth.

As shown in Table 2, dividends of the market portfolio are significantly exposed to long-run consumption risks with $\hat{\phi}_d = 4.51(\text{SE} = 0.45)$. Similar to the conditional mean, the conditional volatility process is highly persistent, $\hat{\nu} = 0.998(\text{SE} = 0.0007)$, but is driven by quantitatively small shocks. The extracted volatility component exhibits a pronounced variation across time, a considerable decline in the 1990s and an increase in the Great Recession. Consistent with the model, the volatility component is a strong predictor of future returns. The regression of 5-year ahead excess returns of the market portfolio on the extracted variance yields an R^2 of 27%.

The estimate of risk aversion in the LRR specification is about 9.7, which is relatively low from the perspective of the asset pricing literature. The IES estimate is above one ($\hat{\psi} = 2.2, \text{SE} = 0.21$), which is essential for a negative price of volatility risks and a positive relationship between the price-consumption ratio and expected growth.[7] Our estimate of the IES is driven by the negative correlation between the price-dividend ratio and consumption volatility, as well as by the low level of the real risk-free rate, an observation also underscored in Hansen et al. (2008). Hall (1988) and Beeler and Campbell (2012) report a smaller magnitude of the IES, however, they do not impose a broad set of equilibrium asset pricing restrictions in their estimation of the IES parameter.[8]

The estimates of the parameters governing the dynamics of expected consumption growth and volatility capture the economic mechanism highlighted in the LRR literature, first discussed in Bansal and Yaron (2004). Because the extracted expected growth and volatility shocks are long-lasting, they have an economically significant impact on growth expectations and future uncertainty, and therefore on assets' valuations. As Table 3 shows, the LRR model is indeed able to account for the level and volatility of the market returns, while matching the low mean of the observed risk-free rate. Consistent with the data, the model generates a persistent and volatile price-dividend ratio with a first-order autocorrelation of 0.93 and a standard deviation of 0.42. As argued above, the magnitude of long-run consumption risks is relatively small, which allows the model to match the time-series dynamics of consumption growth. The standard deviation and the first-order autocorrelation of annual consumption growth at the point estimates are 2.2% and 0.4, respectively. As further shown in the

[7] It is worth noting that the parameter estimates presented in Table 2, including the estimate of the decision frequency, are largely within one standard error of the values commonly used in calibrations of the LRR model and, in particular, are close to those in Bansal et al. (2012). This is not surprising as calibrations are informal estimation.

[8] Bansal and Yaron (2004) and Bansal et al. (2012) show that the approach of estimating the IES solely based on the risk-free rate (e.g., Hall, 1988) can yield sizably downward biased estimates when the underlying shocks exhibit stochastic volatility.

Table 2
Model estimates: Parameters. Table 2 presents parameter estimates and χ^2-test of overidentifying restrictions for two models. The "LRR Model" is the benchmark long-run risk model that incorporates both persistent expected growth and time-varying volatility in cash flows. The "No-Vol Model" is a specification that allows for time-variation in conditional means but rules out variation in conditional second moments of consumption and dividend growth rates. The description of the model parameters is provided in Section 2. The models are estimated via GMM using annual data from 1930 till 2015 and taking into account the effect of time aggregation. Standard errors of the parameter estimates are constructed via a block bootstrap by resampling the data in eight-year blocks. The set of moment conditions used in estimation is given in Table 3.

Parameters	**LRR model**		**No-Vol model**	
	Estimate	SE	Estimate	SE
Preferences				
γ	9.67	1.44	7.98	1.94
ψ	2.18	0.21	2.28	0.85
δ	0.9990	0.0001	0.9987	0.0004
Cash flows				
μ_c	0.0016	0.0005	0.0016	0.0006
ρ	0.9762	0.0035	0.9796	0.0091
φ_e	0.0318	0.0053	0.0399	0.0113
σ_0	0.0070	0.0009	0.0081	0.0014
ν	0.9984	0.0007		
σ_w	2.12e–6	5.32e–7		
μ_d	0.0027	0.0010	0.0025	0.0010
ϕ_d	4.51	0.45	4.77	1.74
φ_d	4.65	0.48	4.44	1.04
ϱ_d	0.51	0.10	0.41	0.18
Aggregation				
h	11	2.16	9	2.69
χ^2-test	10.4		78.5	
p-Value	0.11		0.00	

table, the model is successful in explaining other key moments of the joint distribution of consumption, dividends and asset returns. This success manifests itself formally through the χ^2-test of overidentifying restrictions that indicates that the LRR model is not rejected at the conventional 5% significance level.

In contrast to the LRR model, the restricted "No-Vol" specification is strongly rejected. The estimation results for the constant-volatility specification are presented in the right panel of Table 2. Overall, the estimates of the (unrestricted) parameters of the "No-Vol" specification are similar in magnitude to the corresponding estimates of the fully specified LRR model. The estimate of the decision frequency is 9 (SE = 2.69), which remains statistically different from one; the estimates of risk aversion and IES are around 8 and 2.3, respectively. Notice, however, that the preference parameters in this case are estimated less precisely, especially the time-discount factor and the IES. The larger standard errors point out difficulties in separately identifying the rate of time preferences and IES when time-variation in risk premia is ruled out. The "No-Vol" specification confirms the presence of a small persistent component in consumption growth. Moreover, since volatility risks are now shut off, the contribution of long-run (and short-run) consumption risks under the "No-Vol" specification is amplified relative to the estimated dynamics of the LRR model. This allows the constant-volatility set-up to generate a sizable equity premium and high variation in asset returns. However, as Table 3 shows, the "No-Vol" specification fails to match several empirical moments and, in particular, predictability of consumption growth rates and returns. With no variation in uncertainty, the price–dividend ratio is driven solely by the long-run risk component generating excessive predictability of future growth rates. This specification is also unable to account for predictability of asset returns as it implies a constant risk premium. The rejection of the constant-volatility specification reveals the importance of time-variation in consumption volatility and the ensuing variation in risk premia.

One essential feature of the LRR framework is predictability of consumption growth rates and returns. Our estimation incorporates the correlations of the price-dividend ratio with one-year ahead consumption growth and market returns. Beeler and Campbell (2012) argue that the LRR model may imply high (low) long-horizon predictability of consumption growth (returns) relative to the data. However, as shown in Bansal et al. (2012), long-horizon predictability moments are estimated quite imprecisely. It is, therefore, unlikely that they receive a significant weight in estimation and that they may alter the parameter estimates in a significant way. Indeed, our estimates, the model implications and inference are all robust to the inclusion of the additional moment restrictions of long-horizon predictability of consumption growth rates and market returns.[9]

[9] The evidence based on the extended set of moments is reported in the online appendix.

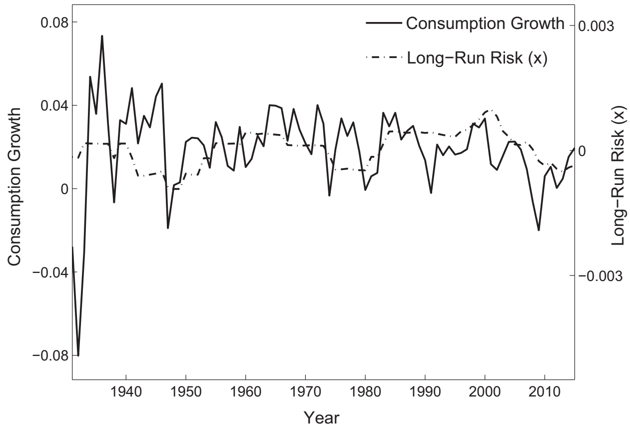

Fig. 1. Realized and expected growth of consumption. Fig. 1 plots time series of realized (solid line) and expected (dash line) growth in consumption. Consumption is defined as the per-capita expenditure on non-durables and services. The data are real, sampled on an annual frequency and cover the period from 1930 to 2015.

Table 3
Model estimates: moments. Table 3 presents sample- and model-based moments, and t-statistics for their differences. The "LRR Model" is the benchmark long-run risk model that incorporates both persistent expected growth and time-varying volatility in cash flows. The "No-Vol Model" is a specification that allows for time-variation in conditional means but rules out variation in conditional second moments of consumption and dividend growth rates. $E(\cdot)$, $vol(\cdot)$, $AC1(\cdot)$, $AC2(\cdot)$, $corr(\cdot,\cdot)$ denote the mean, standard deviation, first- and second-order autocorrelations, and correlation respectively. Δc_τ^a and Δd_τ^a denote time-aggregated annual consumption and dividend growth rates. η_τ^a and u_τ^a correspond to innovations into annual consumption and dividend growth, respectively which depend on the model-specification under consideration. The annual price-dividend ratio, $z_{d,\tau}^a$, is defined as the log of the end of year price over the twelve-month trailing sum of dividends. $r_{d,\tau}^a \equiv \log(R_{d,\tau}^a)$ is the continuously compounded annual return of the aggregate market, and $r_{f,\tau}^a \equiv \log(R_{f,\tau}^a)$ is the logarithm of the annual risk-free rate. x_τ and σ_τ are end of year-τ expected growth and conditional volatility of consumption growth, respectively.

Moments	**LRR model**			**No-Vol model**		
	Sample	Model	t(diff)	Sample	Model	t(diff)
Consumption and Dividends						
$vol(\Delta c_\tau^a)$	0.021	0.022	−0.52	0.021	0.024	−0.85
$AC1(\Delta c_\tau^a)$	0.472	0.395	0.06	0.472	0.461	−0.26
$AC2(\Delta c_\tau^a)$	0.184	0.159	0.15	0.184	0.247	−0.30
$vol(\Delta d_\tau^a)$	0.111	0.101	0.26	0.111	0.110	0.02
$AC1(\Delta d_\tau^a)$	0.189	0.388	−1.27	0.189	0.482	−1.81
$corr(\Delta c_\tau^a, \Delta d_\tau^a)$	0.508	0.625	−0.49	0.508	0.614	−0.46
$E(\eta_\tau^a)$	0.001	0.000	0.51	0.006	0.000	1.52
$E(u_\tau^a)$	−0.014	0.000	−1.14	−0.003	0.000	−0.18
$E(\eta_\tau^a x_{\tau-2})$	−4.1e−07	0.000	−0.48	−2.6e−05	0.000	−3.02
$E([\eta_\tau^{a^2} - E_{\tau-2}\eta_\tau^{a^2}]\sigma_{\tau-2}^2)$	−7.5e−09	0.000	−0.74			
$vol(\eta_\tau^{a^2})$	0.001	0.001	0.36	0.001	1.8e−04	1.49
$AC1(\eta_\tau^{a^2})$	0.209	0.152	0.35	0.240	0.082	1.59
Asset prices						
$E(z_{d,\tau}^a)$	3.404	3.409	0.18	3.404	3.366	0.74
$vol(z_{d,\tau}^a)$	0.450	0.423	−0.03	0.450	0.320	1.08
$AC1(z_{d,\tau}^a)$	0.859	0.929	−0.44	0.859	0.814	0.13
$E(R_{d,\tau}^a - R_{f,\tau}^a)$	0.075	0.067	1.03	0.075	0.067	1.04
$vol(r_{d,\tau}^a)$	0.194	0.173	0.04	0.194	0.204	−0.96
$E(r_{f,\tau}^a)$	0.005	0.010	−1.31	0.005	0.010	−1.44
Predictability						
$corr(r_{d,\tau}^a, z_{d,\tau-1}^a)$	−0.165	−0.088	−1.24	−0.165	0.028	−2.20
$corr(\Delta c_\tau^a, z_{d,\tau-1}^a)$	0.176	0.240	−0.55	0.176	0.593	−2.29

5.1.1. *The role of permanent and cyclical components*

Our analysis help shed light on the role of the permanent component in the stochastic discount factor (e.g., Bansal and Lehman, 1997; Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Bansal et al., 2010; Alvarez and Jermann, 2005) show that to account for the dynamics of asset prices, the permanent component of marginal utility has to be large. In our model, the log level of marginal utility, which is just the accumulation of the IMRS in Eq. (13), can be decomposed into permanent and transitory components as follows (suppressing the deterministic term):

$$\sum_{j=0}^{\infty} m_{t+1-j} = \Upsilon_1 \sum_{j=0}^{\infty} \sigma_{t-j}\left(\eta_{t+1-j} + \frac{\kappa_1 \varphi_e}{1-\kappa_1\rho} e_{t+1-j}\right) + \Upsilon_2 \sum_{j=0}^{\infty}\left[\kappa_1 \frac{1-\rho^{j+1}}{1-\rho} - \frac{1-\rho^j}{1-\rho}\right]\sigma_{t-j} e_{t+1-j}$$
$$+\Upsilon_3 \sum_{j=0}^{\infty} \sigma_w w_{t+1-j} + \Upsilon_4 \sum_{j=0}^{\infty} \nu^j \sigma_w w_{t+1-j} \tag{32}$$

where $\Upsilon_1 = -\gamma, \Upsilon_2 = \frac{1}{\psi}\frac{\varphi_e}{1-\kappa_1\rho}, \Upsilon_3 = (\theta-1)\frac{\kappa_1 - 1}{1-\nu}A_2, \Upsilon_4 = -(\theta-1)\frac{\kappa_1\nu - 1}{1-\nu}A_2$

The first two terms correspond to permanent and transitory growth components while the third and fourth terms are the respective contributions of permanent and transitory components of volatility shocks. The model-implied variation in marginal utility and the contribution of each component to the overall variation can be evaluated using the point estimates in Table 2. We find that at all horizons, the model-implied volatility of marginal utility that correspond to the maximal Sharpe ratio is quantitatively large because of large permanent components. For example, at the one-month horizon, volatilities of marginal utility, and permanent growth and volatility components are 0.15, 0.11 and 0.04, respectively. Volatility of the five-year ahead innovation into marginal utility is 1.11 out of which 0.86 and 0.31 are due to permanent growth and volatility risks, respectively. The sizable contribution of permanent growth and volatility shocks to the variation in marginal utility is what allows our model to account for the observed large equity premium. Transitory risks contribute little to the overall variation in marginal utility and, therefore, play a relatively small role in asset pricing. In all, our evidence is consistent with the prior literature that argues that a large permanent component in the SDF is a required feature to explain the dynamics of asset prices.

5.2. *The effect of time aggregation*

Our GMM estimates presented above suggest that the frequency of the model dynamics (roughly monthly) is significantly shorter than the sampling frequency of the data (annual). It has been recognized in the time-series literature that temporal aggregation may cause a substantial loss of information about the underlying dynamics and, if not appropriately taken into account, may systematically bias inference (e.g., Working, 1960; Hansen and Sargent, 1983; Christiano et al., 1991; Marcet, 1991; Drost and Nijman, 1993; Heaton, 1995). This issue may be particularly relevant in the context of the LRR model, in which the conditional distribution of consumption and cash-flow growth rates is time-varying and is driven by small persistent risks.

To assess biases resulting from a misspecification of the model's frequency, we re-estimate the model imposing the restriction that the decision interval is annual. In our notation, we set $h=1$, and run estimation using the same set of annual data. In other words, we consider an econometrician who is entirely ignorant of the issue of time aggregation and assumes that the frequency of the model dynamics coincides with the frequency of the data. The left panel of Table 4 provides the parameter estimates of this "Annual" specification along with their standard errors and the χ^2-test statistic. The first apparent difference between the estimates of the "Annual" specification and those of the LRR model, presented in Table 2, is in the estimate of risk aversion. Ignoring time aggregation results in a high estimate of risk aversion of about 14, which is about 40% larger than that of the time-aggregated LRR model. Further, as Table 4 shows, the "Annual" specification is strongly rejected by the data.

To understand the rejection of this specification, Table 5 reports population moments of the joint distribution of consumption, dividends and asset prices implied by the "Annual" estimates and the corresponding statistics in the data. As the table shows, the rejection of the "Annual" specification comes primarily from its failure to account for the dynamics of equity prices and returns. It significantly underestimates variation in the price-dividend ratio, generates only 13% volatility in equity returns, and despite the large estimate of risk aversion, is unable to explain the high level of risk premia.

The failure of the "Annual" specification is driven by its inability to identify the magnitude and the contribution of persistent, long-run and volatility, risks. Conceptually, if the true decision interval of the agent is shorter than annual, the "Annual"-based moment restrictions that ignore time aggregation are severely misspecified. This misspecification shifts the emphasis of the model from long-run risks to short-run innovations in consumption. Eq. (25) that describes the dynamics of the time-aggregated consumption growth helps understand the intuition. For concreteness, assume that the true model is monthly while the data are sampled annually. Note that the innovation in annual consumption growth is a mixture of the underlying long- and short-run monthly shocks. While the LRR model appropriately accounts for this composite innovation structure by allowing for temporal aggregation in estimation, the restricted "Annual" specification has no means to separate out the two shocks and tends to attribute the whole innovation to short-run fluctuations in consumption growth. In other words, the "Annual" specification amplifies the contribution of short-run risks at the expense of low-frequency movements in consumption. Consequently, the misspecified "Annual" set-up appears similar to a specification based on the i.i.d. dynamics. Quantitatively, the variance decomposition of annual consumption growth reveals that the contribution of long-run risks implied by the estimates of the LRR model is around 24%, whereas under the "Annual" specification, long-run risks

Table 4
No time aggregation: parameter estimates of the annual specification. Table 4 presents the estimated parameters and the χ^2-test of overidentifying restrictions of the annual specification of the long-run risks model. The estimation does *not* account for time aggregation of the data, and both the sampling frequency and the decision interval are assumed to be annual. The first set of columns, under the heading of "Empirical data", provides the 1930–2015 sample estimates and their standard errors. Standard errors of the parameter estimates are constructed via a block bootstrap by resampling the data in eight-year blocks. The second set of columns reports the corresponding estimates in time-aggregated simulated data that have been generated at the parameter estimates of the LRR model in Table 2. Population estimates are based on a long sample of simulated data, the 5, 50 and 95 percentiles characterize the finite-sample distribution of the annual estimates. The description of the model parameters is provided in Section 2.

Parameters	**Empirical data**		**Simulated data**			
	Estimate	SE	Population	5%	50%	95%
Preferences						
γ	13.83	3.42	12.93	11.01	14.78	19.64
ψ	1.05	0.61	1.22	0.86	1.17	1.62
δ	0.9944	0.0018	0.994	0.987	0.996	0.999
Cash flows						
μ_c	0.0141	0.0025	0.015	0.010	0.016	0.021
ρ	0.8741	0.0646	0.859	0.832	0.893	0.952
φ_e	0.1661	0.0345	0.172	0.133	0.177	0.288
σ_0	0.0241	0.0054	0.026	0.001	0.015	0.023
ν	0.9220	0.0415	0.860	0.728	0.901	0.954
σ_w	3.49e–06	4.57e–06	3.5e-06	2.1e–06	3.5e–06	5.3e–06
μ_d	0.0107	0.0052	0.009	0.000	0.009	0.013
ϕ_d	2.51	0.54	2.72	1.872	2.647	3.601
φ_d	5.12	0.56	5.27	2.958	5.162	7.294
ϱ_d	0.60	0.02	0.60	0.560	0.601	0.651
χ^2-test	231.5			18.4	137.9	792.4
p-Value	0.00			0.00	0.00	0.01

account for only 10% of the overall variation of consumption growth. Further, consistent with Drost and Nijman (1993), our analysis reveals that it is much harder to detect time-variation in the conditional volatility of growth rates using low-frequency data and disregarding restrictions of temporal aggregation.

To corroborate the above argument, Fig. 2 compares impulse responses of consumption growth and its conditional variance implied by the LRR model and the "Annual" specification. The impulse response functions are constructed by fitting an ARMA model to the data simulated at the point estimates of the two specifications.[10] To make the model comparison meaningful, annual consumption growth rates and variance observations are sampled at the end of each simulation year. Note that for the LRR model, consumption observations are simulated at a frequency of 33 days (i.e., 11 decision periods per year), then they are aggregated to the annual frequency by summing-up consumption levels within a year to compute annual growth rates. The cumulative response of annual consumption growth for each specification is presented in Panel (a) of Fig. 2. While the contemporaneous responses in the two cases are the same, the subsequent response of consumption growth in the "Annual" specification is much lower than that in the LRR model. In the limit, a one-percent ARMA-shock raises the level of annual consumption by about 2% in the LRR model and by only 1.5% in the "Annual" specification. The implications of the two specifications for the variance dynamics are significantly different as well. Panel (b) of Fig. 2 reveals that variance risks in the "Annual" specification, although persistent, taper off much more rapidly than those in the LRR model. These differences allow us to better understand the limitations of the "Annual" specification and its ultimate rejection in the data. To be able to fit the dynamics of observed consumption and dividends when temporal aggregation is ruled out, this specification has to suppress the contribution of low-frequency and volatility risks and, instead, magnify short-run fluctuations in cash-flow growth rates. However, short-run risks provide little help in explaining the dynamics of asset prices.

To further highlight the consequences of ignoring time aggregation, we simulate the LRR model using the parameter estimates reported in Table 2. We aggregate the simulated data to construct annual consumption, dividends and prices and use them to estimate the "Annual" specification. This experiment is designed to illustrate what happens if the true model frequency is shorter than annual but an econometrician, equipped with annual data, assumes a yearly decision interval and, therefore, ignores restrictions of temporal aggregation. The right panel of Table 4 presents the finite-sample distribution of the parameter estimates as well as population values estimated using a long sample of simulated data in this experiment. Overall, the estimated parameters in these simulations are quite close to the "Annual" estimates based on the observed data.

[10] To highlight differences in population, this analysis is performed using a long sample of simulated data. The ARMA specification comprises eight autoregressive and eight moving-average terms for both consumption growth and its variance. The impulse responses are robust to changes in the ARMA specification as long as there are enough terms to account for predictable variations.

Table 5
No time aggregation: moments of the annual specification. Table 5 presents sample- and model-based moments of the annual specification, and t-statistics for their differences. The estimation does *not* account for time aggregation, and both the sampling frequency and the decision interval are assumed to be annual. E(·), vol(·), AC1(·), AC2(·), corr(·, ·) denote the mean, standard deviation, first- and second-order autocorrelations, and correlation respectively. Δc^a_τ and Δd^a_τ denote annual consumption and dividend growth rates. η^a_τ and u^a_τ correspond to innovations into annual consumption and dividend growth, respectively. $z^a_{d,\tau}$ is the log of the annual price-dividend ratio, $r^a_{d,\tau} \equiv \log(R^a_{d,\tau})$ is the continuously compounded annual return of the aggregate market, and $r^a_{f,\tau} \equiv \log(R^a_{f,\tau})$ is the logarithm of the annual risk-free rate. x_τ and σ_τ are end of year-τ expected growth and conditional volatility of consumption growth, respectively.

Moments	Annual specification		
	Sample	Model	t(diff)
Consumption and dividends			
vol(Δc^a_τ)	0.021	0.025	−1.11
AC1(Δc^a_τ)	0.472	0.092	1.48
AC2(Δc^a_τ)	0.184	0.080	0.49
vol(Δd^a_τ)	0.111	0.125	−0.42
AC1(Δd^a_τ)	0.189	0.024	1.07
corr($\Delta c^a_\tau, \Delta d^a_\tau$)	0.508	0.613	−0.49
E(η^a_τ)	−0.004	0.000	−0.41
E(u^a_τ)	−0.004	0.000	−0.73
E($\eta^a_\tau x_{\tau-2}$)	−0.002	0.000	−1.86
E($[\eta^{a^2}_\tau - E_{\tau-2}\eta^{a^2}_\tau]\sigma^2_{\tau-2}$)	9.7e−06	0.000	1.09
vol($\eta^{a^2}_\tau$)	0.009	0.024	−1.87
AC1($\eta^{a^2}_\tau$)	0.370	1.3e−07	1.05
Asset prices			
E ($z^a_{d,\tau}$)	3.404	3.421	0.04
vol($z^a_{d,\tau}$)	0.450	0.083	3.61
AC1($z^a_{d,\tau}$)	0.859	0.874	−0.20
E($R^a_{d,\tau} - R^a_{f,\tau}$)	0.075	0.040	2.24
vol($r^a_{d,\tau}$)	0.194	0.130	1.38
E($r^a_{f,\tau}$)	0.005	0.011	−1.88
Predictability			
corr($r^a_{d,\tau}, z^a_{d,\tau-1}$)	−0.165	0.061	−2.49
corr($\Delta c^a_\tau, z^a_{d,\tau-1}$)	0.176	0.323	−1.06

In particular, the misspecification of the model frequency results in a considerably weaker contribution of long-run and volatility risks, and consequently, in a significantly biased estimate of risk aversion. When time aggregation is ignored, long-run risks account for only 15% of the variation in consumption growth in finite samples on average, and for only 10% in population (which is considerably low compared with the true contribution of 24%). The estimate of risk aversion under the "Annual" specification increases in half (from its true value of 9.7 to about 14.8, on average) and this bias, although reduced, does not vanish asymptotically. In simulations, as in the data, the "Annual" specification fails to account for high volatility of prices and returns. As Table 4 also shows, although cash-flow and price dynamics are generated under the LRR model, the misspecification caused by ignoring time aggregation alone is likely to lead to a statistical rejection of the model. This evidence confirms difficulties in detecting long-run and volatility risks if one neglects temporal aggregation and ignores how the underlying model dynamics are integrated to lower-frequency data employed in estimation.

Table 6 further explores the implications of time aggregation. By varying the time-aggregation parameter h, it shows how different assumptions about the length of the decision interval affect the model's estimates and inference. In addition to the already discussed "Annual" set-up, it presents three specifications: "Bi-weekly" (with $h=26$), "Monthly" ($h=12$), and "Quarterly" ($h=4$). As the table shows, the farther the decision frequency deviates from its optimal value of 11, the more the model fit deteriorates. At the monthly frequency, both the estimates and the pricing implications are quite similar to those of the unrestricted LRR model, and the p-value of the overidentifying restrictions test is marginally significant. The "Bi-weekly" specification features a further decrease in the p-value. As the frequency changes to quarterly, the model's ability to account for the level and variation in asset returns and prices diminishes, and the model is strongly rejected. These implications, including the increase in the estimate of risk aversion, are similar to the predictions of the "Annual" specification and follow from the failure to properly identify long-run and volatility risks. It is important to emphasize that identification of the latent state dynamics deteriorates not only when the frequency of time aggregation is understated (as in the "Quarterly" and "Annual" specifications) but also when temporal aggregation is overstated (as in the "Bi-weekly" setting). In particular, under all of these specifications, the contribution of long-run growth risks is reduced while the contribution of short-run risks is amplified relative to our baseline LRR model. Long-run risks account for 21% of the

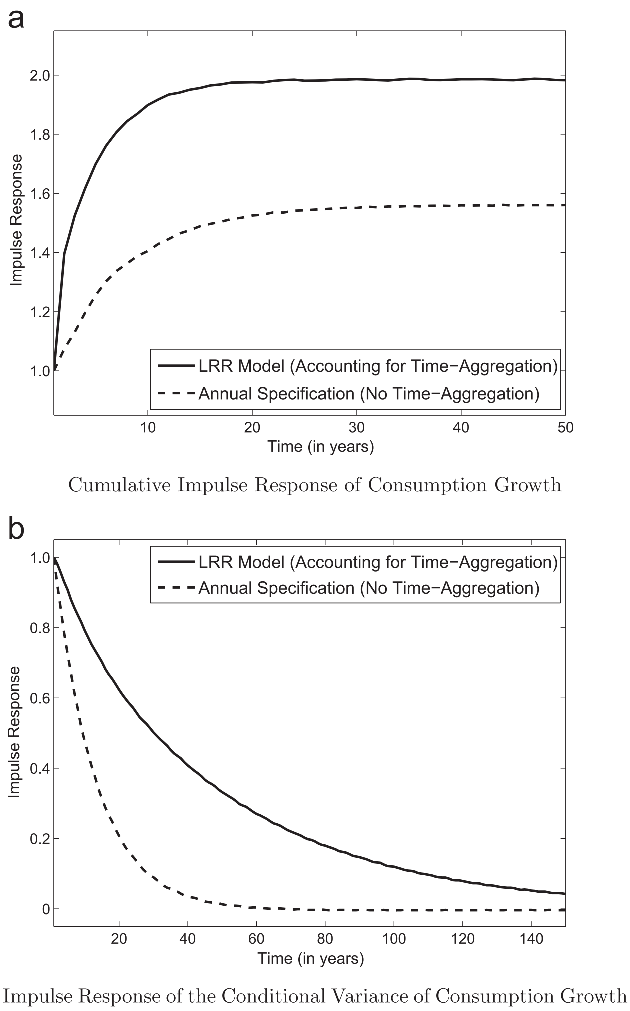

Fig. 2. Impulse Response Functions. Fig. 2 plots impulse response functions of annual consumption growth (Panel (a)) and the conditional variance of consumption growth (Panel (b)) to unit growth-rate and variance shocks, respectively. Shocks are constructed by fitting an ARMA model to a long sample of simulated annual data. The data are simulated at the point estimates of the LRR model (solid line) and those of the annual specification (dashed line).

variation in annual consumption growth under the "Bi-weekly" specification, and for about 12% and 10% under the "Quarterly" and "Annual" specifications, respectively, compared with 24% under our baseline LRR model. These results underscore the importance of properly incorporating the restrictions of temporal aggregation for drawing inferences about the underlying risk dynamics and investors' preferences.

5.3. Cross-sectional implications

One of the important dimensions of financial data is the cross-sectional heterogeneity in mean returns, in particular along size and book-to-market dimensions. As shown in Table 7, the average return of the top book-to-market quintile portfolio is higher than that of the bottom book-to-market quintile by about 5.4% per annum. This is the well-known value premium. Similarly, the portfolio of small market-capitalization firms outperforms the large-firm portfolio by about 6.5%, on average.[11] The observed dispersion in mean returns on size and book-to-market sorted portfolios is known to present a challenge for the standard CAPM. In the data, the market betas for the value-minus-growth and small-minus-large portfolios are quite small, while the market-adjusted returns (i.e., CAPM α's) are large and significant.

This section evaluates the implications of the LRR model for the cross-section of size and book-to-market portfolios. We estimate the dynamics of dividend growth rates for four portfolios: small, large, value and growth, and assess the ability of

[11] The construction of book-to-market and size sorted portfolios is standard (see Fama and French, 1993). Value-weighted monthly returns, as well as per-share price and dividend series are constructed for each portfolio. Monthly data are time aggregated to an annual frequency and converted to real using the consumer price index.

Table 6
Model estimates with fixed decision frequency Table 6 presents parameter estimates and χ^2-test of overidentifying restrictions for alternative frequency specifications of the long-run risks model. In "Bi-weekly", "Monthly", "Quarterly" and "Annual" specifications the time-aggregation parameter, *h*, is set at 26, 12, 4 and 1 respectively. The model parameters are estimated via GMM using annual data from 1930 till 2015 and, save for the annual specification, taking into account the effect of time aggregation. The description of the model parameters is provided in Section 2.

Parameters	Bi-weekly	Monthly	Quarterly	Annual
Preferences				
γ	6.54	8.12	11.66	13.83
ψ	2.44	2.45	1.82	1.05
δ	0.9996	0.9990	0.9965	0.9944
Cash flows				
μ_c	0.0005	0.0016	0.0024	0.0141
ρ	0.9946	0.9753	0.9682	0.8741
φ_e	0.0090	0.0340	0.0395	0.1661
σ_0	0.0055	0.0064	0.0116	0.0241
ν	0.9989	0.9981	0.9978	0.9220
σ_w	1.21e–06	3.56e–06	6.07e–06	3.49e–06
μ_d	0.0008	0.0026	0.0059	0.0107
ϕ_d	4.32	4.23	4.89	2.51
φ_d	5.00	5.55	4.80	5.12
ϱ_d	0.49	0.54	0.46	0.60
χ^2-test	34.4	15.2	119.5	231.5
p-Value	0.00	0.03	0.00	0.00

the LRR model to simultaneously account for the value and size premia, and the magnitudes on the CAPM betas and alphas. To keep the estimation problem manageable, the cross-sectional parameters are estimated using the state variables extracted in our benchmark estimation of the model, and holding preferences, consumption, and market-dividend parameters fixed at the point estimates reported in Table 2. For each portfolio, the following set of moment conditions is exploited in estimation: the mean and volatility of dividend growth rates, their correlation with consumption growth, the risk premium, the volatility of returns, the mean and volatility of the price-dividend ratio, and the market beta. The cross-sectional moments are evaluated using annual data and incorporating restrictions of temporal aggregation.

Panel A of Table 7 presents the cross-sectional estimates and, in particular, dividends' exposure to consumption risks for the four portfolios. Consistent with empirical evidence in Bansal et al. (2005) and Hansen et al. (2008), the value portfolio features much higher exposure to low-frequency risks in consumption relative to the growth portfolio (7.3 versus 5.3). Similarly, long-run risk exposure of the small-size portfolio exceeds that of the large portfolio (10.7 versus 4.7). Small and value portfolios are also characterized by a higher short-run correlation of their dividends with consumption relative to portfolios with opposite size and book-to-market characteristics, although short-run risk dynamics are estimated with larger standard errors. The estimate of φ_j, which governs dividend exposure to volatility risks, is higher for the small portfolio compared to the large portfolio, and is quite similar across book-to-market sorted portfolios. The bottom line of Panel A presents the model-implied risk premium for each of the four portfolios. Comparable to the premia observed in the data, the model predicts a sizeable value premium of about 4.8% and a large size premium of 7.4%. The risk-premium decomposition reveals that, on average across portfolios, about 55% of the premium comes as a compensation for long-run risks, about 30% is accounted for by volatility risks, and the remaining fraction is due to asset exposure to short-run consumption risks.

Panel B of Table 7 shows the CAPM implications of the long-run risks model. It presents the implied market betas and alphas for the small-minus-large and value-minus-growth portfolios. As in the data, the model-implied CAPM betas of the spread portfolios are quite low: 0.86 and 0.45 for the small–large and value–growth strategies, respectively. Consequently, the LRR model is able to replicate the failure of the CAPM by generating quantitatively sizable alphas of the arbitrage portfolios. The model-implied alphas of the small–large and value–growth portfolios are about 1.6% and 1.8%, respectively. The ability of the LRR model to account for a significant portion of the value- and size-premium puzzles comes from the fact that in the model, the market beta is not a sufficient risk statistics (i.e., the market return is not perfectly correlated with the SDF). In particular, the market exposure to long-run risks is significantly lower than that of the underlying pricing kernel. Therefore, the model features high market-adjusted alphas of the small–large and value–growth portfolios as those are highly exposed to long-run risks in consumption.

5.4. Estimates based on quarterly post-war data

This section shows that our empirical findings are robust and are not simply driven by the Great Depression period. To this end, the model is estimated using post-war quarterly data starting from 1948 till the end of 2015.[12] The left set of columns in Table 8 provides the point estimates of the structural parameters estimated using the post-war sample and

Table 7
Model implications for the cross-section of returns. Panel A of Table 7 presents estimated parameters of dividend dynamics for the top and bottom quintile portfolios sorted by size (large and small) and book-to-market characteristic (value and growth), and their risk premia in the data and implied by the model. The description of the model parameters is provided in Section 2. Standard errors of the parameter estimates are constructed via a block bootstrap by resampling the data in eight-year blocks. Panel B provides the CAPM betas and alphas for the small-minus-large and value-minus-growth strategies.

Panel A: Cross-sectional estimates and risk premia

	Small	**Large**	**Growth**	**Value**
Parameters				
$\mu_j(\%)$	0.48 (0.08)	0.21 (0.09)	0.27 (0.07)	0.50 (0.09)
ϕ_j	10.69 (1.45)	4.70 (1.85)	5.33 (0.98)	7.29 (1.27)
φ_j	10.42 (2.04)	5.83 (2.14)	6.09 (1.81)	7.51 (1.75)
ϱ_j	0.41 (0.18)	0.40 (0.15)	0.20 (0.16)	0.61 (0.10)
Risk premia (%)				
Data	13.61	7.12	7.03	12.38
Model	13.93	6.52	6.65	11.46

Panel B: CAPM Implications

	Small–large		**Value–growth**	
	Data	Model	Data	Model
β^{CAPM}	0.59	0.86	0.30	0.45
$\alpha^{CAPM}(\%)$	2.07	1.63	3.13	1.78

taking into account time aggregation. As the table shows, preference and time-series estimates are overall quite similar to those obtained using the full sample of annual data. In particular, risk aversion is estimated at 7.5 and the IES is around 2. Consumption growth features a persistent predictable component and persistent time-varying volatility dynamics. The estimated decision interval within a quarter is 2 corresponding to approximately 45 days (a somewhat longer decision interval than the 33 days estimated in Table 2). The test for overidentifying restrictions in the post-war sample yields a p-value of 4%.

The right panel of Table 8, titled "No Time Aggregation", presents quarterly estimates that are obtained by ignoring time aggregation (i.e., when an econometrician uses quarterly data and assumes a quarterly decision interval). As in our benchmark annual sample, when time aggregation is ignored, the resulting risk aversion estimate is amplified, and the variation in the market return and equity premia decline compared with the time-aggregated model. Formally, as Table 8 shows, the "No Time Aggregation" quarterly specification is strongly rejected by the data. In all, the evidence based on the quarterly data reinforces our benchmark results, providing further support for the risk mechanisms featured in the LRR model and confirming the ramifications of ignoring time aggregation.

6. Conclusions

This paper analyzes the importance of persistent variations in growth rates and economic uncertainty in financial markets. It shows that time aggregation that emanates from a discrepancy between the decision interval of the agents and the sampling frequency of the data confounds the identification of long- and short-run risk components and the underlying structural parameters. To address this issue, it develops a method for estimating asset pricing models with recursive preferences and latent state dynamics that can be implemented when the sampling frequency of the data differs from the decision frequency of the model. Exploiting the identification restrictions of the long-run risks model while factoring in the implications of time aggregation, it estimates latent risk components and the model parameters jointly with the decision interval of the agents.

Empirically, the paper shows that the long-run risks model is able to successfully capture both the time-series and the cross-sectional variation in returns. It finds a strong evidence of persistent variations in consumption and dividend growth rates and low-frequency variations in growth rate volatility. Our empirical evidence shows that investors have a preference for early resolution of uncertainty and make their decisions at an approximately monthly frequency. The estimated contribution of long-run growth and volatility risks to the overall risk premium is 55% and 30%, respectively. The estimated model accounts for the observed low level of the risk free rate, the market, value, and size premia, as well as the observed

[12] While, as shown below, the model estimates and inference based on the post-war data are similar to our baseline results, we argue against throwing away data and consider the annual sample a proper benchmark (as the recent experience of the Great Recession suggests, deep recessions provide important and valuable data and, therefore, should not be treated as outliers).

Table 8
Model estimates using post-war quarterly data. Table 8 presents parameter estimates and the χ^2-test of overidentifying restrictions of the long-run risk model based on the post-war quarterly data. The model is estimated via GMM using quarterly data from 1948 till 2015 with and without taking into account time aggregation. The description of the model parameters is provided in Section 2. Standard errors of the parameter estimates are constructed via a block bootstrap by resampling the data in eight-year blocks.

Parameters	**Time-aggregation**		**No time aggregation**	
	Estimate	SE	Estimate	SE
Preferences				
γ	7.45	0.75	8.66	2.00
ψ	2.02	0.23	2.83	0.66
δ	0.9987	0.0003	0.9981	0.0006
Cash flows				
μ_c	0.0013	0.0003	0.0041	0.0008
ρ	0.9861	0.0034	0.9766	0.0249
φ_e	0.0504	0.0076	0.0785	0.0244
σ_0	0.0041	0.0005	0.0034	0.0007
ν	0.9909	0.0030	0.9945	0.0061
σ_w	2.92e–06	6.70e07	1.68e–06	1.67e–06
μ_d	0.0027	0.0006	0.0020	0.0011
ϕ_d	4.29	0.30	6.50	1.19
φ_d	5.11	0.26	5.39	0.55
ϱ_d	0.03	0.06	0.01	0.12
Aggregation				
h	2	0.50	1	nan
χ^2-test	13.3		22.2	
p-Value	0.04		0.00	

variation of the market return, the risk free rate, and the price-dividend ratio. In all, our evidence provides empirical support for the long-run and volatility risk channels featured in the long-run risks model.

Finally, the paper shows that ignoring time aggregation in estimation carries a substantial cost. When time aggregation of the model dynamics is ignored, the underlying risks are mis-measured, the model is overly rejected and the estimates of preference parameters are significantly biased. In particular, a model specification that ignores time aggregation heavily understates the contribution of low-frequency variations in growth rates and volatility and, therefore, fails to explain financial market data. Thus, our analysis emphasizes the importance of accounting for time aggregation in understanding the dynamics of asset prices.

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2016.07.003.

References

Alvarez, F., Jermann, U., 2005. Using asset prices to measure the persistence of the marginal utility of wealth. Econometrica 73 (6), 1977–2016.
Bansal, R., Dittmar, R.F., Lundblad, C., 2005. Consumption, dividends, and the cross-section of equity returns. J. Financ. 60 (4), 1639–1672.
Bansal, R., Gallant, R., Tauchen, G., 2007. Rational pessimism, rational exuberance, and asset pricing models. Rev. Econ. Stud. 74 (4), 1005–1033.
Bansal, R., Kiku, D., Yaron, A., 2010. Long-run risks, the macroeconomy, and asset prices. Am. Econ. Rev. Pap. Proc. 100 (2), 542–546.
Bansal, R., Kiku, D., Yaron, A., 2012. An empirical evaluation of the long run risks model for asset pricing. Crit. Financ. Rev. 1, 183–221.
Bansal, R., Lehman, B., 1997. Growth-optimal portfolio restrictions on asset pricing models. Macro Dyn. 1 (2), 333–354.
Bansal, R., Yaron, A., 2004. Risks for the long run: a potential resolution of asset pricing puzzles. J. Financ. 59 (4), 1481–1509.
Beeler, J., Campbell, J., 2012. The long run risks model and aggegate asset prices: an empirical evaluation. Crit. Financ. Rev. 1, 141–182.
Bekaert, G., Engstrom, E., Xing, Y., 2009. Risk, uncertainty and asset prices. J. Financ. Econ. 91 (1), 59–82.
Calvet, L., Czellar, V., 2015. Through the looking glass: indirect inference via simple equilibria. J. Econ. 185 (2), 343–358.
Campbell, J., Shiller, R., 1988. Stock prices, earnings, and expected dividends. J. Financ. 43 (3), 661–676.
Christiano, L., Eichenbaum, M., Marshall, D., 1991. The permanent income hypthoesis revisited. Econometrica 59 (2), 397–423.
Colacito, R., Croce, M., 2011. Risks for the long run and the real exchange rate. J. Polit. Econ. 119 (1), 153–181.
Colin-Dufresne, P., Johannes, M., Lochstoer, L., 2016. Parameter learning in general equilibrium: the asset pricing implications. Am. Econ. Rev. 106 (3), 664–698.
Constantinides, G., Gosh, A., 2011. Asset pricing tests with long-run risks in consumption growth. Rev. Asset Pricing Stud. 1 (1), 96–136.
Drost, F.C., Nijman, T., 1993. Temporal aggregation of garch processes. Econometrica 61 (4), 909–927.

Epstein, L.G., Zin, S.E., 1989. Substitution, risk aversion, and the intertemporal behavior of consumption and asset returns: a theoretical framework. Econometrica 57 (4), 937–969.
Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33 (1), 3–56.
Ferson, W., Nallareddy, S., Zxie, B., 2013. The out-of-sample performance of the long-run risk model. J. Financ. Econ. 107 (3), 537–556.
Giovannini, A., Weil, P., 1989. Risk aversion and intertemporal substitution in the capital asset pricing model. NBER Working Paper 2824.
Hall, R., 1988. Intertemporal substitution in consumption. J. Polit. Econ. 96 (2), 339–357.
Hansen, L., 1982. Large sample properties of generalized methods of moments estimators. Econometrica 50 (4), 1009–1054.
Hansen, L., Heaton, J., Li, N., 2008. Consumption strikes back? J. Polit. Econ. 116 (2), 260–302.
Hansen, L., Heaton, J., Yaron, A., 1996. Finite-sample properties of some alternative GMM estimators. J. Bus. Econ. Stat. 14 (3), 262–280.
Hansen, L., Sargent, T., 2010. Fragile beliefs and the price of model uncertainty. Quant. Econ. 1 (1), 129–162.
Hansen, L.P., Sargent, T., 1983. Aggregation over time and the inverse optimal predictor problem for adaptive expectations in continuous time. Int. Econ. Rev. 24 (1), 1–20.
Hansen, L.P., Scheinkman, J., 2009. Long term operator: an operator approach. Econometrica 77 (1), 177–234.
Hansen, L.P., Singleton, K., 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica 50 (5), 1269–1286.
Heaton, J., 1995. An empirical investigation of asset pricing with temporally dependent preference specifications. Econometrica 63 (3), 681–717.
Jagannathan, R., Liu, B., 2016. Dividend dynamics, learning, and expected stock index returns. Working Paper.
Kiku, D., 2006. Long-run risks and the value premium puzzle. Working Paper.
Lettau, M., Ludvigson, S., 2009. Euler equation errors. Rev. Econ. Dyn. 12 (2), 255–283.
Malloy, C., Moskowitz, T., Vissing-Jorgensen, A., 2009. Long-run stockholder consumption risk and asset returns. J. Financ. 64 (6), 2427–2479.
Marcet, A., 1991. Time Aggregation of Economic Time Series. Westview Press, pp. 237–281, rational Expectations Econometrics, Lars P. Hansen and Thomas Sargent, editors.
Newey, W., West, K., 1987. A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. Econometrica 55 (3), 703–708.
Parker, J.A., Julliard, C., 2005. Consumption risk and the cross-section of expected returns. J. Polit. Econ. 113 (1), 185–222.
Tallarini, T., 2000. Risk sensitive real business cycles. J. Monet. Econ. 45 (3), 507–532.
Wilcox, D., 1992. The construction of the u.s. consumption data: some facts and their implications for empirical work. Am. Econ. Rev. 82 (4), 922–941.
Working, H., 1960. Note on the correlation of first differences of averages in a random chain. Econometrica 28 (4), 916–918.