Building Supply Chain Resilience Through Vertical Control

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Abstract

We study the value of two types of vertical control—*vertical integration* and *direct sourcing*—for increasing resilience towards random raw materials shortages. We use a three-tier supply chain model in which a Tier 0 buyer sources a critical component from a Tier 1 supplier, who sources raw materials from disruption-prone Tier 2 suppliers. With vertical integration, the buyer purchases its Tier 1 supplier, taking control of its sourcing decisions. With direct sourcing, the buyer purchases raw materials from Tier 2 and sells to Tier 1. We find that either form of vertical control is most valuable when the likelihood of disruptions is moderate, with vertical integration preferred over direct sourcing for more severe disruptions. We next consider whether penalty contracts—in which the buyer charges the supplier a penalty for each unit the supplier fails to deliver—can replicate the outcome of vertical control, and find that while this is possible, the necessary non-delivery penalty may be too high for the supplier to accept, particularly when disruptions are of moderate likelihood, implying vertical control can be valuable even if penalty contracts are possible. Finally, we explore whether multi-sourcing in Tier 1 can eliminate the need for vertical control, and find that while it can reduce the need for vertical control, vertical control is still optimal when disruption risk is moderate. Together, our results show that, under specific circumstances, vertical control can be a valuable addition to the portfolio of strategies firms use to mitigate disruption risk.

Keywords: vertical integration, direct sourcing, supply chain disruptions, risk mitigation

1 Introduction

Supply chain disruptions have impacted nearly every industry, from automobiles to electronics to consumer packaged goods (Goodman, 2021; Goodman & Chokshi, 2021). Modern supply networks are complex and dispersed, exposing downstream (closer to end consumer) firms to an ever-expanding number of potential failure points. Because of this, firms have sought to insulate their supply chains against the risk of disruption via the use of mitigation strategies like holding inventory, multi-sourcing, and penalizing suppliers for non-delivery (Tomlin, 2006; Tomlin & Wang, 2011; Simchi-Levi *et al.*, 2015). In turn, a rich academic literature has developed that studies when and how to deploy disruption risk mitigation strategies in a variety of different contexts (Ang *et al.*, 2017; Bimpikis *et al.*, 2019).

Recently, however, facing growing supply chain shortages caused by natural disasters, climate risk, the COVID-19 pandemic, and geopolitical conflicts, firms have sought new ways to build resilience—the ability to mitigate the negative consequences of disruptions and limit financial

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losses—into their supply chains that go beyond the traditional methods (Williams-Alvarez, 2023). One such strategy for increasing resilience is to increase vertical control, i.e., for a downstream firm to take ownership of upstream activities in the supply chain that might normally be delegated to partners, with a goal of making more effective decisions that reduce the risk of disruptions due to materials or component shortages (Mashuda, 2021; Chow, 2022; Wellener *et al.*, 2022).

For example, Volkswagen has traditionally sourced electric vehicle batteries from Tier 1 suppliers such as LG Energy Solutions who, in turn, sourced raw materials from Tier 2 suppliers (Yang, 2021). However, facing "the worst supply chain shortages" in raw materials it has ever seen (Weldersee, 2022), Volkswagen has begun building its own battery manufacturing plants (Foldy, 2022), i.e., it has begun to vertically integrate battery production. This mimics the strategy of chief competitor Tesla, which has long made its own electric vehicle batteries. Other automakers, such as Ford and GM, have also considered vertical integration to manufacture their own semiconductors (Colias & Foldy, 2021).

Besides vertically integrating with Tier 1 suppliers, directly purchasing raw materials from Tier 2 and reselling them to Tier 1 suppliers is another means to gain control over the supply of critical raw materials and subcomponents that have been plagued by random shortages in recent years (Krauss & Ewing, 2023). Compared to vertical integration, this strategy has the advantage of lower fixed costs, and it typically leads to less regulatory scrutiny. Sourcing raw materials directly has seen a recent surge in popularity: for instance, GM has used it to secure critical raw materials such as lithium and nickel from Tier 2 suppliers, and there is speculation that Tesla, too, has relied on this approach to procure inputs for battery suppliers that it does not own (Colias & Patterson, 2023). As discussed by Krauss & Ewing (2023), both forms of vertical control have been observed in recent years (italicized portions added): "In the past, automakers let battery suppliers buy lithium and other raw material on their own. But lithium shortages have forced carmakers, which have deeper pockets, to directly acquire the essential metal and have it sent to battery factories, some owned by suppliers [*direct sourcing*] and others owned partly or fully by the automakers [*vertical integration*]."

These examples illustrate the implicit belief among firms that vertical control—in a variety of forms—can be a panacea for supply chain disruptions when the risk level is high. However, there is little academic research to validate this belief, or offer guidance on how—and when—vertical control should be used to increase supply chain resilience. In this paper, we seek to fill this gap by studying the use of vertical control as a supply chain disruption mitigation strategy. We accomplish this by analyzing a model of a three-tier supply chain with a Tier 0 buyer (e.g., Volkswagen), one or more Tier 1 suppliers that manufacture components (e.g., battery manufacturers), and a set of Tier 2 suppliers that provide raw materials to Tier 1. Tier 2 suppliers are prone to disruption due to, e.g., natural disasters or geopolitical events.

In the absence of vertical control, a Tier 1 supplier may mitigate Tier 2 disruption risk by diversifying suppliers (Tomlin, 2006; Ang *et al.*, 2017). However, the Tier 1 supplier's risk mitigation decisions may not align with the Tier 0 buyer's preferences. To remedy this, the Tier 0 buyer in our model can exert two types of vertical control: *vertical integration* (or VI) and *direct sourcing* (or DS). Vertical integration refers to the buyer purchasing a Tier 1 supplier (or replicating its functions in-house). This places the buyer in full control of Tier 1 production and sourcing decisions, allowing it to make more effective decisions to ensure a stable source of supply; however, it requires a significant amount of capital investment and increases the buyer's fixed costs (Buzzell, 1982; Power, 2013). Because of this, the buyer can consider another, potentially less costly, type of vertical control: direct sourcing. With direct sourcing, the buyer does not integrate with a Tier 1 supplier (and so cannot fully control its sourcing decisions), but instead, directly purchases raw materials from Tier 2 suppliers and resells them to Tier 1.

While there are clear potential benefits to vertical control, it is less clear whether and how vertical control possesses advantages relative to other mechanisms meant to influence supplier sourcing decisions or mitigate disruption risk. For example, if Tier 1 sourcing decisions are misaligned with buyer preferences due to, e.g., double marginalization in the supply chain, a natural remedy might be advanced contracts that align those preferences. On the other hand, instead of increasing ownership over Tier 1 activities, buyers could instead source from a larger number of Tier 1 suppliers, mitigating risk via diversification rather than costly forms of vertical control.

It is within this context that we seek to understand the role of vertical control as a disruption mitigation strategy, focusing on the following three research questions:

- **How does vertical control generate value as a disruption mitigation strategy for downstream buyers?** Using a base model consisting of a single buyer, a single Tier 1 supplier, and two Tier 2 suppliers, where the buyer purchases from Tier 1 using a standard wholesale price contract, we find that vertical control—via both vertical integration and direct sourcing—can be unnecessary at low or high disruption risk levels (i.e., low or high likelihood of a disruption). When the risk level is low, the buyer prefers not to mitigate disruptions. When the risk level is high, the incentive misalignment between the buyer and Tier 1 supplier is mild, meaning the buyer can induce the desired sourcing outcomes in Tier 1 at little cost. However, when the risk level is moderate, the buyer would prefer Tier 1 diversify its Tier 2 suppliers, but inducing this outcome via a wholesale price contract is costly due to double marginalization in the supply chain; consequently, vertical integration can offer the buyer a lower cost way to mitigate risk, either by taking over Tier 1's sourcing decisions (VI), or by having the buyer assume the role of a Tier 2 supplier to Tier 1 and lowering Tier 1's cost of diversifying Tier 2 suppliers (DS). Furthermore, we find that when choosing between the two forms of vertical control, VI is preferred by the buyer if disruptions are more severe in nature (result in minimal or no output from Tier 2) while DS is preferred is disruptions are less severe.

- **Can vertical control be replicated by advanced contracts that penalize Tier 1 suppliers for a failure to deliver units to the buyer?** Moving beyond the wholesale price contract in our base model, we next consider whether advanced contracts—specifically, penalty contracts—can "solve" the disruption risk problem for the buyer, replicating the outcome of vertical control at a lower cost. We find that while it is possible for penalty contracts to achieve this, particularly for disruptions that are very likely or very severe, there are practical obstacles to realizing this: the non-delivery penalty required to induce the buyer's desired outcome might be greater than what the Tier 1 supplier is willing to accept, something that is especially likely to happen if disruptions are of moderate likelihood and severity. Consequently, even with advanced contracts, both vertical integration and direct sourcing can be valuable risk mitigation strategies for the buyer at moderate risk levels.

- **Can the need for vertical control be eliminated by multi-sourcing in Tier 1?** Lastly, we extend our base model in which the buyer sources from a single Tier 1 supplier to one in which the buyer can source from two Tier 1 suppliers. We find that the ability to multi-source in Tier 1 gives the buyer diversification benefits and reduces the need for vertical control; however, vertical control may still be optimal at moderate disruption risk levels. The need for vertical control decreases as the correlation in risk between the two Tier 1 suppliers decreases, but it is not fully eliminated even if disruptions are independent.

Taken in sum, our results provide valuable guidance for firms about how and when vertical integration and direct sourcing should be used to help build resilience and mitigate disruption risk in supply chains. Vertical control is not a panacea for supply chain disruptions in all scenarios, and other strategies may be more effective in building supply chain resilience. However, vertical control can also not be fully replicated by alternative strategies such as advanced contracting or multi-sourcing in Tier 1, and hence may be preferred, particularly at moderate risk levels when the effects of incentive misalignment are most pernicious.

The remainder of this paper is organized as follows. §2 reviews the literature. §3 introduces our model and derives the optimal risk mitigation strategy in the absence of vertical control. §§4-5 then analyze, respectively, the value of vertical integration and the value of direct sourcing. §6 considers the impact of penalty contracts, §7 extends the model to allow for multi-sourcing in Tier 1, and §8 concludes the paper.

2 Related Literature

Our work builds on two primary streams of literature: the literature on disruption and risk management, and the literature on vertical control (e.g., vertical integration, direct sourcing, centralization, and insourcing). Supply chain disruptions (or, more generally, sourcing under random supply) have been extensively studied (Tang, 2006). Much of the early work on this topic focused on the case of two-tier supply chains, i.e., a buying firm sourcing from unreliable Tier 1 suppliers. For instance, in this context, Kleindorfer & Saad (2005) present a conceptual framework for risk assessment and mitigation when a firm faces supply chain disruptions. Tomlin (2006) and Tomlin & Wang (2011) analyze the value of several operational strategies, including inventory mitigation (holding safety stock) or sourcing mitigation (paying a higher price for more reliable supply and/or sourcing from multiple suppliers), in the presence of Tier 1 supplier disruptions. A number of papers enrich this classic setting, including Bakshi & Kleindorfer (2009), who examine joint investments in supply chain resilience by agents in a two-tier supply chain when supply is risky; Yang *et al.* (2009), who consider the impact of a perfectly reliable backup production option under information asymmetry about disruption risk; Hu & Kostamis (2015), who analyze optimal sourcing from an arbitrary number of Tier 1 suppliers; and Simchi-Levi *et al.* (2015), who build a risk-exposure model to evaluate the impact of supply chain disruptions and make better risk mitigation decisions at Ford.

None of these papers considers the types of vertical control that motivates our work. In addition, none consider supply disruptions in a multi-tier supply chain, as we do. There are a number of recent papers that do study risk mitigation in multi-tier supply chains, including Bimpikis *et al.* (2018), who examine multi-sourcing and supplier diversification in the presence of supply chain disruptions; Bimpikis *et al.* (2019), who investigate the optimal network structure for different stakeholders when there is disruption risk, and Bakshi & Mohan (2024), who investigate the information requirements to optimally mitigate disruption risk in a given multi-tier supply chain. The closest work to ours in this stream is Ang *et al.* (2017), who study optimal disruption mitigation in multi-tier supply chains. However, they consider only traditional risk mitigation strategies, such as holding inventory and supplier diversification. We build on the analysis in Ang *et al.* (2017) by modeling vertical integration and direct sourcing as additional risk mitigation strategies available to the buyer, and determining how these should complement or replace traditional mitigation strategies.

There is also a substantial literature on vertical integration (or insourcing) and direct sourcing in a variety of contexts. For example, Perry (1989) provides an overview of the determinants of vertical integration, Hart & Tirole (1990) examine the impact of vertical integration on a firm's competition, and Novak & Stern (2009) investigate the complementarity of vertical integration decisions in the automobile industry. More recently, Aid *et al.* (2011) study vertical integration in electricity markets, Lin *et al.* (2014) investigate under which conditions manufacturers would choose vertical integration or decentralization in a setting with two competing supply chains, Orsdemir *et al.* (2019) consider vertical integration as a strategy to ensure social and environmental responsibility, and Qu & Raff (2020) study the interaction of vertical integration and the bullwhip effect. Related to this, there is a stream of literature on delegation versus control of decisions in supply chains, such as Amaral *et al.* (2006) and Kayis *et al.* (2013), who analyze whether a firm should delegate Tier 2 component procurement to its Tier 1 suppliers or take control of this decision, under perfect and asymmetric information, respectively. Further, Chen *et al.* (2012) investigate an OEM's choice between direct sourcing and and delegated sourcing when its contract manufacturer also produces for a competing OEM, Belavina & Girotra (2012) discuss the advantages of sourcing through intermediaries as opposed to direct sourcing, Wang *et al.* (2020) investigate the impact of bargaining power on firm decisions to use direct sourcing and agent sourcing (or delegation), and Huang *et al.* (2022) consider when firms should delegate control of responsibility management in Tier 2 of the supply chain to a Tier 1 supplier. None of these papers, however, consider the value of vertical integration or direct sourcing under the threat of supply chain disruptions, as we do.

In sum, to the best of our knowledge, our work is the first that quantifies the impact of increased

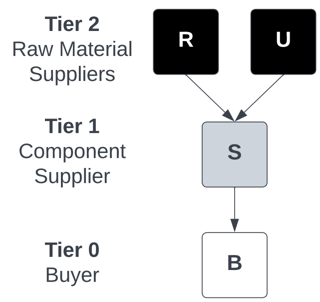


Figure 1. The base three-tier supply chain model, a "traditional" supply chain with no vertical control.

vertical control as a supply chain disruption risk mitigation strategy.

3 Model

3.1 Description

We consider a supply chain consisting of three tiers: Tier 0 (the "buyer"), Tier 1, and Tier 2. The Tier 0 buyer (B) sells a single product with deterministic demand D and selling price r over a single period. The Tier 1 supplier (S) makes a critical component for the buyer. In the base model, the buyer only has access to a single Tier 1 supplier, e.g., due to the specialized capabilities required to make the component; in §7, we extend the model to allow for multiple Tier 1 suppliers in order to study the interaction between sourcing diversification and vertical control.

To make the component the buyer needs, the Tier 1 supplier sources a critical raw material from Tier 2 suppliers.[1] As is common in the disruption literature (Tomlin, 2006; Ang *et al.*, 2017), we assume that there are two Tier 2 suppliers available to the Tier l supplier: an unreliable supplier (U) and a reliable supplier (R). Both Tier 2 suppliers nominally have infinite capacity; however, the unreliable supplier disrupts with probability $\lambda \in [0, 1]$ and has its capacity reduced to $K \geq 0$ (Aydin *et al.*, 2011; Ang *et al.*, 2017). The reliable supplier, on the other hand, never disrupts. Disruption risk in the model is therefore characterized by two parameters: the probability of disruptions (λ) and supplier capacity during disruptions (K), which is a proxy for the severity of disruptions. Regardless of their type (reliable or unreliable), Tier 2 suppliers are assumed to provide perfectly substitutable raw materials. To avoid trivial outcomes, we assume $K < D$, i.e., the buyer's market demand cannot be fulfilled by a disrupted unreliable Tier 2 supplier. Figure 1 provides a graphical depiction of the supply chain.

As in Tomlin (2006) and Ang *et al.* (2017), the unit sourcing cost that Tier 2 suppliers charge is exogenous; however, reliable Tier 2 suppliers charge c_r per unit, while unreliable suppliers charge c_u per unit.[2] To avoid trivial outcomes, we assume $c_u < c_r < r$. This means that, when sourcing

[1] While we refer to Tier 2 as supplying "raw materials" throughout the paper for simplicity and to align with several of our motivating examples, it could instead supply subcomponents or other inputs necessary for production of the component manufactured by the Tier 1 supplier.

[2] In particular, these prices are fixed *ex ante* and do not increase if a disruption occurs in Tier 2. Downstream

from Tier 2, the Tier 1 supplier faces a trade-off between reliability and sourcing cost: it can source from the unreliable Tier 2 supplier at a low cost, the reliable Tier 2 supplier at a high cost, or some combination of the two.

Disruptions occur only in Tier 2 of the supply chain (Ang *et al.*, 2017), i.e., the Tier 1 supplier is perfectly reliable. We make this assumption for three reasons. First, it is consistent with the examples described in the introduction in which buyers are seen to exert vertical control to help mitigate the risk of raw material (Tier 2) disruptions. Second, modeling disruptions in one tier at a time is helpful for tractability. And third, this allows us to sharply focus our model on our main question of interest, i.e., analyzing the value of increasing the buyer's control over its immediate supplier's disruption mitigation decisions, either by vertical integration (buying the Tier 1 supplier and taking control over sourcing decisions in Tier 2 directly) or direct sourcing (purchasing raw materials directly from Tier 2 and reselling to Tier 1).

In our base model, which is reflective of a "traditional" or "decentralized" supply chain, the Tier 0 buyer makes all Tier 1 sourcing decisions (i.e., decides contract terms and sourcing quantities with Tier 1) while the Tier 1 supplier makes all Tier 2 sourcing decisions (i.e., deciding which Tier 2 suppliers to use and how much to source from each). Thus, the sequence of events is as follows:

1. Consistent with our motivating examples in which the buyer is a powerful firm such as an auto manufacturer, the buyer leads by offering a contract consisting of a wholesale price (w) and purchase quantity (Q) to the Tier 1 supplier. We assume that the Tier 1 supplier accepts any contract that leaves it with non-negative expected profit.

2. Observing the contract terms (w, Q), S chooses how much to source from its Tier 2 suppliers, U and R.

3. A disruption at U randomly occurs, and S receives some or all of its raw materials order based on its Tier 2 sourcing decisions and whether U is disrupted. Specifically, if S orders more than K from U, and U is disrupted, then U only delivers K units to S.

4. The Tier 1 supplier makes as many components as it can and attempts to fill the buyer's order of Q units, and the buyer uses these components to make finished goods and satisfy as much of the demand D as possible.

The procurement quantity units are normalized so that one unit of raw materials can make one unit of components, which in turn can make one unit of finished goods. We assume that both the Tier 1 supplier and the Tier 0 buyer pay only for the items they receive (i.e., if there is a disruption and fewer units are delivered than were ordered, the buying firm only pays for the number of units delivered). In addition, if a Tier 2 supplier disrupts, we assume that no alternative sources of raw materials are available, i.e., all capacity in Tier 2 has already been committed to other purchasers.[3]

buyers are increasingly signing long-term contracts for critical raw materials rather than purchase purely on the spot market, e.g., lithium, cobalt, and nickel in the automotive supply chain, to insulate them against price volatility that may arise due to random supply disruptions (Azevedo *et al.*, 2018; Lambert, 2022; General Motors, 2022; Krauss & Ewing, 2023).

[3]In practice, Tier 2 raw materials might be available from a spot market, potentially at a higher price (due to

3.2 Disruption Mitigation without Vertical Control

To serve as a baseline for comparison to the case of vertical control, we first derive the buyer's optimal sourcing without vertical control, which we refer to as a "traditional" supply chain. The traditional supply chain is similar to the model of Ang *et al.* (2017), except with a single Tier 1 supplier rather than two. The analysis proceeds via backward induction, starting with the decisions of the Tier 1 supplier after receiving a contract (w, Q) from the buyer. Let q_u, q_r be the Tier 1 supplier's sourcing quantities from U and R, respectively. Then, the Tier 1 supplier's optimal expected profit is given by:

$$\Pi_S^* = \max_{q_u, q_r \geq 0} (1-\lambda)\Big[w \min\{q_u + q_r, Q\} - c_u q_u - c_r q_r\Big] + \\ \lambda\Big[w \min\big\{\min\{q_u, K\} + q_r, Q\big\} - c_u \min\{q_u, K\} - c_r q_r\Big]. \quad (1)$$

The first term represents profit if there is no disruption at the unreliable Tier 2 supplier, while the second term is profit if the unreliable Tier 2 suppliers disrupts. The optimal Tier 1 sourcing quantities are found by solving this maximization problem. Defining $(x)^+ \equiv \max(x, 0)$, we have the following result:

Lemma 1. *The optimal sourcing quantities of the Tier 1 supplier are given by:*

$$(q_u, q_r)^* = \begin{cases} (Q, 0) & \text{if } c_u \leq w < c_u + \frac{c_r - c_u}{\lambda} \\ \big(\min(K, Q), (Q-K)^+\big) & \text{if } w \geq c_u + \frac{c_r - c_u}{\lambda} \end{cases}.$$

Proof. All proofs from §§3-5 appear in Appendix A. □

Note that Lemma 1 implies that since $\lambda \leq 1$, to induce the Tier 1 supplier to use reliable sourcing, the buyer has to offer a wholesale price weakly greater than c_r ($c_u + (c_r - c_u)/\lambda$ is decreasing in λ and equals c_r when $\lambda = 1$); this results in double marginalization in the supply chain. Also note that, if $Q > K$, the Tier 1 supplier always sources at least K from its unreliable supplier, since this quantity will always be delivered at unit cost c_u.

We next move to the optimal sourcing strategy of the Tier 0 buyer. The buyer's expected profit is given by:

$$\Pi_t^B = \max_{w \geq c_u, Q \geq 0} f(w)\big[r \min\{\min\{Q, K\}, D\} - w \min\{Q, K\}\big] + \\ (1 - f(w))\big[r \min\{Q, D\} - wQ\big]. \quad (2)$$

limited quantities available) in the wake of a disruption. For simplicity, we assume that such a spot market does not exist, or equivalently, the spot price following a disruption is so high that it is unprofitable to purchase. Allowing the purchase of raw materials from the spot market following a disruption—i.e., a costly emergency source of supply—will not significantly impact our insights, provided that such purchases are more costly than sourcing *ex ante* from any of the Tier 2 suppliers in our model; see Yang *et al.* (2009) for a detailed exploration of backup sources of supply.

Strategy	Wholesale Price	Prob. of Disruption
Passive Acceptance (PA)	c_u	λ
Supplier Mitigation (SM)	$c_u + (c_r - c_u)/\lambda$	0

Table 1. The buyer's Tier 1 sourcing strategies in a traditional supply chain.

In (2), $f(w)$ is the probability that the Tier 1 supplier experiences a disruption, and $1 - f(w)$ is the probability that the Tier 1 supplier does not experience a disruption. The buyer can influence these probabilities by the wholesale price it offers to the Tier 1 supplier. From Lemma 1 and equation (2), we make the following observations. First, with a single Tier 1 supplier, the buyer will always choose a sourcing quantity D (the same will not be true with multiple Tier 1 suppliers in §7). Second, the buyer has two sourcing strategies available. It can offer a low wholesale price ($w = c_u$), which induces the Tier 1 supplier to single source from the unreliable Tier 2 supplier, yielding $f(c_u) = \lambda$ and $1 - f(c_u) = 1 - \lambda$. We refer to this as a *passive acceptance* (PA) strategy (Tomlin, 2006), as the buyer pays a low price for components but accepts that it will experience a supply disruption with probability λ. On the other hand, the buyer can offer a high wholesale price ($w = c_u + (c_r - c_u)/\lambda$), which induces the Tier 1 supplier to dual source from both Tier 2 suppliers. In this case, $f(c_u + (c_r - c_u)/\lambda) = 0$ and $1 - f(c_u + (c_r - c_u)/\lambda) = 1$, i.e., the risk of a disruption is eliminated. We refer to this as a *supplier mitigation* (SM) strategy (Ang *et al.*, 2017), as the buyer pays the Tier 1 supplier to mitigate disruption risk. These strategies are summarized in Table 1.

We next define the cost threshold $\bar{c}_r$ as follows:

$$\bar{c}_r \equiv \frac{r(D - K) + c_u K}{D}. \tag{3}$$

The buyer's optimal sourcing strategy without vertical control is given by the following lemma:

Lemma 2. *Without vertical control:*
(i) If $c_r \geq \bar{c}_r$, the buyer's optimal sourcing strategy is passive acceptance.
(ii) If $c_r < \bar{c}_r$, the buyer's optimal sourcing strategy is passive acceptance if $\lambda \leq \sqrt{\frac{(c_r - c_u)D}{(D-K)(r-c_u)}}$, and supplier mitigation otherwise.

If the cost of reliable supply is high (case (i) of Lemma 2), inducing reliable sourcing from Tier 1 (using strategy SM) would require a very high wholesale price (Lemma 1), so the buyer never finds this optimal, instead following a PA strategy and accepting the risk of a disruption in return for paying a low wholesale price.

On the other hand, if reliable supply is less costly (case (ii)), the buyer may employ an SM strategy, provided disruptions are sufficiently likely (λ is above a threshold or, equivalently, K is below a threshold). The latter condition ensures that disruptions are a large enough threat to the buyer to justify investing in SM, and also ensures that the wholesale price necessary to induce SM is not too high (from Lemma 1, the wholesale price that induces SM is decreasing in λ).

These insights are illustrated in Figure 3. The figure shows that, when the disrupted capacity (K) is not too high, the buyer prefers PA if the disruption probability (λ) is low and uses SM

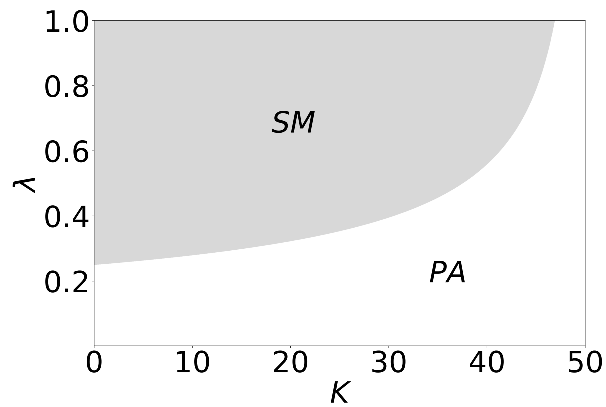


Figure 2. The buyer's optimal strategy without vertical control. In the example, $D = 50$, $c_u = 4$, $c_r = 5$, $r = 20$.

otherwise. Further, if K is high, the buyer uses PA for all feasible λ. In this example, when $K \gtrapprox 45$, $\bar{c}_r$ exceeds c_r, and case (i) of Lemma 2 applies.

An alternative way to express Lemma 2 is in terms of the following threshold:

$$\lambda_t^* = \begin{cases} 1 & \text{if } c_r \geq \bar{c}_r \\ \sqrt{\frac{(c_r - c_u)D}{(D-K)(r-c_u)}} & \text{otherwise} \end{cases}. \tag{4}$$

If $\lambda > \lambda_t^*$, the buyer uses supplier mitigation, while it uses passive acceptance otherwise. λ_t^* will be useful in our analysis to follow.

4 Vertical Integration

With vertical integration, the buyer purchases its Tier 1 supplier,[4] placing the buyer in full control of raw materials sourcing decisions and potentially allowing the buyer to make more effective Tier 2 sourcing decisions than an independent Tier 1 supplier would, i.e., in accordance with Lemma 1. To reflect the fact that vertical integration requires a significant capital investment, we assume that integrating results in a strictly positive fixed cost, $F_v > 0$.[5]

Once the buyer has purchased the Tier 1 supplier, it can access its Tier 2 suppliers and source from the unreliable supplier at unit cost c_u and the reliable supplier at unit cost c_r. Note that c_r is lower than the wholesale price required to induce SM (see Lemma 1). Consequently, integrating with Tier 1 allows the buyer to source from the reliable Tier 2 supplier at a lower marginal cost than in a traditional supply chain.

After vertically integrating and incurring the fixed cost F_v, the buyer chooses quantities q_u and q_r (how much to source from Tier 2 suppliers U and R, respectively) to maximize its expected

[4] Equivalently, the buyer could replicate the functions of the Tier 1 supplier in-house, leading to similar insights. Throughout the paper we refer to vertical integration as the purchase of the buyer's existing supplier for consistency.

[5] Throughout the paper, the subscript v refers to the vertical integration case.

Strategy	Quantity from U	Quantity from R	Prob. of Disruption
Single-Sourcing (SS)	D	0	λ
Multi-Sourcing (MS)	K	$D-K$	0

Table 2. The buyer's Tier 2 sourcing strategies in a vertically integrated supply chain.

profit, which is given by:

$$\Pi_v^B = \max_{q_u, q_r \geq 0} \lambda\big[r \min\{\min\{q_u, K\} + q_r, D\} - c_u \min\{q_u, K\}\big] + (1-\lambda)\big[r \min\{q_u + q_r, D\} - c_u q_u\big] - c_r q_r - F_v. \tag{5}$$

Similar to the Tier 1 supplier in Lemma 1, the buyer can always obtain K units from U at a low cost with no risk; therefore, the buyer will never single source from R. Because of this, the buyer has two sourcing strategies available under vertical integration: *single-sourcing* (SS) a quantity of D from U, and *multi-sourcing* (MS) a quantity of K from U and $D-K$ from R. These strategies are summarized in Table 2. Defining the following threshold on the disruption probability

$$\lambda_v^* \equiv \frac{c_r - c_u}{r - c_u} < \lambda_t^*, \tag{6}$$

the next lemma describes when the buyer follows each strategy.

Lemma 3. *With a vertically integrated supply chain,*
(i) If $\lambda \leq \lambda_v^*$, the buyer's optimal Tier 2 sourcing strategy is single-sourcing.
(ii) Otherwise, the buyer's optimal Tier 2 sourcing strategy is multi-sourcing.

With vertical integration, if the chance of a disruption is low ($\lambda \leq \lambda_v^*$), the buyer single sources from the unreliable Tier 2 supplier, replicating the PA strategy of a traditional supply chain (Table 1). If the chance of a disruption is high ($\lambda > \lambda_v^*$), the buyer multi-sources in Tier 2, replicating the risk profile of the SM strategy in a traditional supply chain but at a lower marginal cost. Note that with vertical integration, because the buyer can source from R at a lower cost than in a traditional supply chain, for sufficiently high disruption probabilities, the buyer finds it optimal to source from R even if c_r is very high (as long as $c_r < r$). This is not the case in a traditional supply chain (Lemma 2), where the buyer avoids SM if c_r is above a threshold. Because of this, the risk level that causes the buyer to use the reliable Tier 2 supplier is lower with a vertically integrated supply chain than in a traditional supply chain (i.e., $\lambda_v^* \leq \lambda_t^*$).

Next, we consider whether vertical integration increases the buyer's profit, defining the "value of vertical integration" to be

$$\Delta_{vt} \equiv \Pi_v^B - \Pi_t^B, \tag{7}$$

where Π_v^B and Π_t^B are the buyer's optimal expected profit in vertically integrated and traditional supply chains, respectively, as defined in (2) and (5), using the optimal sourcing strategies in Lemmas 2 and 3. We first discuss the value of vertical integration when the reliable Tier 2 supplier is expensive ($c_r \geq \bar{c}_r$):

Theorem 1. *If the cost of reliable supply is high ($c_r \geq \bar{c}_r$), the value of vertical integration is weakly increasing in λ. Furthermore, there exist thresholds $\bar{F}_1 > 0$ and $\bar{\lambda}^1_{vt} \in [0,1]$ such that:*
(i) If $F_v < \bar{F}_1$, the value of vertical integration is positive if and only if $\lambda > \bar{\lambda}^1_{vt}$.
(ii) If $F_v \geq \bar{F}_1$, the value of vertical integration is non-positive for all $\lambda \in [0,1]$.

Recall that when the buyer follows strategy PA in a traditional supply chain and strategy SS in a vertically integrated supply chain, the risk profile and marginal sourcing costs are identical, but the vertically integrated supply chain incurs a higher fixed cost (due to F_v), meaning the value of integration is negative. When $c_r \geq \bar{c}_r$, the buyer always uses a PA strategy in a traditional supply chain (case (i) of Lemma 2). Because of this, the buyer's optimal expected profit is strictly decreasing in λ for all $\lambda \in [0,1]$. From Lemma 3, with vertical integration the supplier follows a SS strategy if the disruption probability is low ($\lambda \leq \lambda^*_v$), and follows a MS strategy—which eliminates disruption risk—if the disruption probability is high ($\lambda > \lambda^*_v$). Thus, with vertical integration, the buyer's expected profit is strictly decreasing in λ when $\lambda \leq \lambda^*_v$ and constant in λ when $\lambda > \lambda^*_v$. Consequently, the value of vertical integration is negative for sufficiently low λ and weakly increasing in λ. Depending on the fixed integration cost, the value of vertical integration is either negative for all λ (if F_v is large) or becomes positive for sufficiently high λ (if F_v is smaller).

Theorem 1 thus shows that vertical integration can be profitable as a disruption mitigation strategy for the buyer when λ is high and F_v is low. Vertical integration generates value because of the incentive misalignment that exists in a traditional supply chain: the buyer feels the cost of a disruption more than the Tier 1 supplier, because the buyer loses a large margin $(r - w)$ while the Tier 1 supplier loses a comparatively smaller margin $(w - c_u)$ on each unit of demand that is not met. Inducing SM therefore requires the buyer to offer a high wholesale price to increase Tier 1's incentive to source reliably (i.e., its lost margin on a sale); however, doing so is costly to the buyer and reduces the *buyer's* margin. When $c_r \geq \bar{c}_r$, this incentive misalignment leads to a complete breakdown of reliable sourcing in a traditional supply chain; with vertical integration, this incentive misalignment is removed and the buyer can profitably source from the reliable Tier 2 supplier and eliminate disruption risk.

Next, we consider the case when the reliable Tier 2 supplier is less expensive ($c_r < \bar{c}_r$):

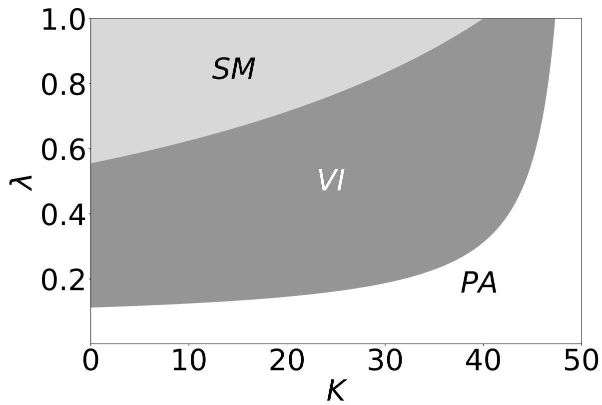
Theorem 2. *If cost of reliable supply is low ($c_r < \bar{c}_r$), the value of vertical integration is weakly increasing in λ if $\lambda < \lambda^*_t$ and decreasing in λ otherwise. Furthermore, there exist thresholds $0 < \bar{F}_2 < \bar{F}_3$ and $\bar{\lambda}^1_{vt}, \bar{\lambda}^2_{vt} \in [0,1]$ such that:*
(i) If $F_v < \bar{F}_2$, the value of vertical integration is positive if and only if $\lambda > \bar{\lambda}^1_{vt}$.
(ii) If $\bar{F}_2 \leq F_v < \bar{F}_3$, the value of vertical integration is positive if and only if $\bar{\lambda}^1_{vt} < \lambda < \bar{\lambda}^2_{vt}$.
(iii) If $F_v \geq \bar{F}_3$, the value of vertical integration is non-positive for all $\lambda \in [0,1]$.

When $c_r < \bar{c}_r$, the value of vertical integration is not monotonic in λ: it is quasi-concave, increasing for $\lambda < \lambda^*_t$ and decreasing for $\lambda \geq \lambda^*_t$. This happens because, when $\lambda \geq \lambda^*_t$, in both the vertically integrated and traditional supply chains, the buyer sources (or induces Tier 1 to source) from a reliable Tier 2 supplier, eliminating the risk of raw materials disruption. The unit cost to achieve this in a vertically integrated supply chain is c_r, while the unit cost in a traditional supply chain is, per Lemma 1, $c_u + (c_r - c_u)/\lambda$. The latter cost is decreasing in λ. Put differently, the incentive misalignment between the buyer and Tier 1 supplier lessens as λ increases, because the Tier 1 supplier is more likely to face a disruption and therefore requires a smaller incentive (in the form of a higher wholesale price) to source reliably. Thus, the SM strategy in a traditional supply chain becomes relatively less costly as λ increases due to reduced incentive misalignment, and hence, when $c_r < \bar{c}_r$—and, as a result, the buyer finds it optimal to engage in SM at high enough λ—the value of vertical integration is first increasing, then *decreasing*, in the disruption probability.

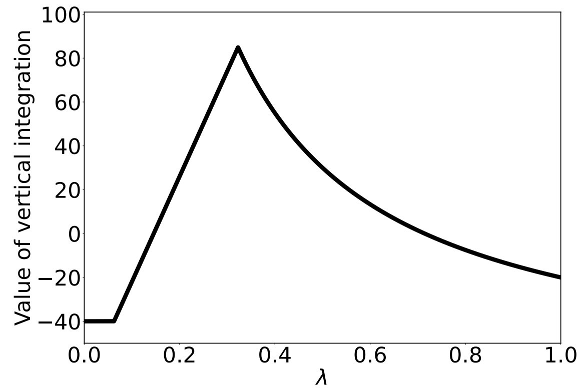
This further implies three possible outcomes. If the fixed integration cost is very low (case (i) of the theorem), the decreasing segment of Δ_{vt} never becomes negative as $\lambda \to 1$. In this case, as in case (i) of Theorem 1, vertical integration is beneficial to the buyer for all $\lambda > \bar{\lambda}^1_{vt}$. On the other extreme, if the fixed integration cost is very high (case (iii) of the theorem), Δ_{vt} never becomes positive even at its peak, λ^*_t. In this case, vertical integration is never beneficial to the buyer for any λ, as in case (iii) of Theorem 1.

The most interesting part of Theorem 2 is case (ii): when the vertical integration cost is moderate, the quasi-concave shape of Δ_{vt} implies it is positive only for moderate disruption probabilities, i.e., for $\bar{\lambda}^1_{vt} < \lambda < \bar{\lambda}^2_{vt}$. When the risk level is low ($\lambda \leq \bar{\lambda}^1_{vt}$), the incentive misalignment between the buyer and Tier 1 supplier is minimal: neither the buyer nor supplier wishes to source reliably, and vertical integration is not worth its fixed cost. When the risk level is high ($\lambda \geq \bar{\lambda}^2_{vt}$), the incentive misalignment between is also minimal: both the buyer and Tier 1 supplier face strong natural incentives to source reliably, and as a result, the buyer can overcome the misalignment with a slightly inflated wholesale price, a preferable strategy to costly vertical integration. When the risk level is moderate ($\bar{\lambda}^1_{vt} < \lambda < \bar{\lambda}^2_{vt}$), incentive misalignment is most severe: the buyer would benefit from reliable sourcing in a traditional supply chain, but inducing this outcome is very costly to the buyer since the Tier 1 supplier faces low natural incentives to source reliably, and hence vertical integration possesses the most value.

Theorem 2 thus shows that it is not always true that vertical integration benefits the buyer most in high risk environments: in general, vertical integration is most valuable at *moderate* disruption probabilities. These insights are depicted graphically in Figure 3. In Figure 3a, we plot the buyer's optimal strategy as a function of λ and K. In Figure 3b, we plot the value of vertical integration as a function of λ. In this example, for $\lambda \lessapprox 0.05$, the buyer uses SS with vertical integration and PA in a traditional supply chain, resulting in an identical risk profile and marginal sourcing cost. Therefore, in this region, the value of vertical integration is constant and is equal to $-F_v = -40$. At $\lambda \approx 0.05$, in a vertically integrated supply chain, the buyer begins to source raw materials from R using strategy MS, fully eliminating disruption risk. Hence, Π^B_v is constant for $\lambda \gtrapprox 0.05$. However, in a traditional supply chain, the buyer does not use SM until $\lambda \gtrapprox 0.35$. This shows how vertical integration allows the buyer to pursue reliable sourcing at a much lower risk level ($\lambda \approx 0.05$ vs $\lambda \approx 0.35$) and at a much lower price per unit (meaning $\Delta_{vt} > 0$) than a traditional supply chain.



(a) The buyer's optimal strategy



(b) The value of vertical integration with $K = 20$

Figure 3. The buyer's optimal strategy with the option of vertical integration (a) and the value of vertical integration (b) as a function of λ. In the example, $D = 50$, $c_u = 4$, $c_r = 5$, $r = 20$, and $F_v = 40$.

However, for $\lambda \gtrapprox 0.7$, the downward pressure on the wholesale price from increasing λ (evident in Lemma 1) begins to outweigh the higher fixed costs of vertical integration, meaning the traditional supply chain is preferred as λ continues to increase.

5 Direct Sourcing

While vertical integration can be a valuable strategy for mitigating disruption risk, its high fixed cost may make it infeasible in practice. Besides cost, vertically integrating with a Tier 1 supplier may not be possible for regulatory reasons or because buyers choose to follow a narrowly focused operational strategy, outsourcing secondary functions like component production. In such cases, there is another type of vertical control that firms might use to mitigate the risk of supply chain disruptions: direct sourcing, which allows the buyer to purchase raw materials directly from Tier 2 suppliers and resell them to Tier 1 suppliers (Amaral *et al.*, 2006; Bimpikis *et al.*, 2018; Wang *et al.*, 2020; Chen *et al.*, 2012; Kayis *et al.*, 2013). In this section, we consider this type of vertical control and assume that the buyer can purchase raw materials directly from the reliable Tier 2 supplier, R.

With direct sourcing, the model is similar to the traditional supply chain analyzed in §3, with some modifications. In step 1 of the sequence of events, the buyer determines a unit price p and the maximum quantity of raw materials m it is willing to offer the Tier 1 supplier along with the other contract parameters (quantity and wholesale price). Hence, the contract offered to the supplier in step 1 is of the form (w, Q, p, m).

In step 2 of the sequence of events, the Tier 1 supplier still has access to its Tier 2 suppliers (U at unit cost c_u and R at unit cost c_r) and may choose to source from these suppliers rather than (or in addition to) the buyer, depending on the offered price p. Let q_u, q_r, and q_m be the quantity S sources from U, R, and the buyer, respectively. We assume that supplier will prioritize purchasing raw materials from the buyer if it is indifferent between the buyer and sourcing from one of the Tier 2 suppliers. After the Tier 1 supplier determines q_m, the buyer orders q_m units of raw materials from the reliable Tier 2 supplier and, upon delivery, provides them to the Tier 1 supplier.

In practice, either the buyer or its Tier 1 supplier might be capable of negotiating better deals with Tier 2 suppliers depending on a variety of factors (Chen *et al.*, 2012; Amaral *et al.*, 2006). In our analysis, we assume that the Tier 1 supplier can procure at a lower price from the reliable Tier 2 supplier than the buyer, e.g., because the Tier 1 supplier has closer relationships with Tier 2 suppliers, or the scale to place considerably larger orders to its Tier 2 suppliers compared to the buyer (since the Tier 1 supplier may produce components for multiple buyers; Amaral *et al.*, 2006). Consequently, the marginal cost to the buyer of procuring Tier 2 raw materials is c_d, where $c_r < c_d < r$. (At the end of this section, we discuss the implications of the buyer having a lower Tier 2 sourcing cost than the supplier, i.e., $c_d \leq c_r$.) We also assume that, unlike vertical integration, direct sourcing incurs no fixed investment cost.[6] The combination of these assumptions implies that direct sourcing is a low fixed cost/high variable cost type of vertical control, while vertical integration is high fixed cost/low variable cost pathway to vertical control.

While it may appear that the buyer serves as a redundant source of reliable supply for the Tier 1 supplier, different from in the traditional supply chain, the buyer can influence Tier 1's sourcing decisions by adjusting p (the cost of this new source of reliable supply to the Tier 1 supplier) in addition to adjusting w (the wholesale price paid to the Tier 1 supplier). The optimal expected profit of S under direct sourcing is given by:

$$\begin{aligned}\Pi_S^d = \max_{q_u,q_r \geq 0, m \geq q_m \geq 0} (1-\lambda)\Big[w \min\big(q_u + q_r + q_m, Q\big) - c_u q_u\Big] + \\ \lambda\Big[w \min\big(\min(q_u, K) + q_r + q_m, Q\big) - c_u \min(q_u, K)\Big] - c_r q_r - p q_m. \qquad (8)\end{aligned}$$

The optimal sourcing decisions of the Tier 1 supplier are given by the following lemma:

Lemma 4. *With direct sourcing, the optimal sourcing quantities of the Tier 1 supplier are given by:*
(i) If $p \leq c_u$*,*

$$(q_u, q_r, q_m)^* = \begin{cases} \big(0, 0, \min(Q, m)\big) & \text{if } p \leq w < c_u \\ \big((Q-m)^+, 0, \min(Q, m)\big) & \text{if } c_u \leq w < c_u + \frac{c_r - c_u}{\lambda} \\ \Big(\min\big(K, (Q-m)^+\big), (Q-m-K)^+, \min(Q, m)\Big) & \text{if } w \geq c_u + \frac{c_r - c_u}{\lambda} \end{cases}.$$

(ii) If $c_u < p \leq c_r$*,*

$$(q_u, q_r, q_m)^* = \begin{cases} (Q, 0, 0) & \text{if } c_u \leq w < c_u + \frac{p - c_u}{\lambda} \\ \Big(\min(Q, K) + (Q-m-K)^+, 0, \min\big(m, (Q-K)^+\big)\Big) & \text{if } c_u + \frac{p-c_u}{\lambda} \leq w < c_u + \frac{c_r - c_u}{\lambda} \\ \Big(\min(Q, K), (Q-m-K)^+, \min\big(m, (Q-K)^+\big)\Big) & \text{if } w \geq c_u + \frac{c_r - c_u}{\lambda} \end{cases}.$$

[6] It is straightforward to incorporate a fixed cost of direct sourcing, $F_d \geq 0$, reflecting additional managerial effort such as establishing a warehouse for inventory or hiring dedicated procurement specialists (Kayis *et al.*, 2013). As long as the fixed cost of direct sourcing is less than the fixed cost of vertical integration, $F_d \leq F_v$, our insights continue to qualitatively hold. On the other hand, if $F_d > F_v$, vertical integration dominates direct sourcing, since it would result in lower fixed and variable costs of reliable supply.

Strategy	Wholesale Price	Direct Sourcing Quantity	Prob. of Disruption
Passive Acceptance (PA)	c_u	0	λ
Supplier Mitigation (SM)	$c_u + (c_r - c_u)/\lambda$	0	0
Direct Sourcing (DS)	c_u	$D - K$	0

Table 3. The buyer's sourcing strategies with the option of direct sourcing.

(iii) If $p > c_r$, then optimal sourcing quantities are given by Lemma 1, with $q_m^* = 0$.

Note that, in order to sell raw materials to Tier 1, the buyer must set $p \leq c_r$, as otherwise the Tier 1 supplier would purchase from the reliable Tier 2 supplier itself. Since the buyer's cost satisfies $c_d > c_r$, this implies the buyer loses money on each unit of raw materials sold to Tier 1. However, the loss on each unit sold to Tier 1 $(c_d - p)$ may be less than the cost of inducing SM (inflating w to $c_u + (c_r - c_u)/\lambda$). Importantly, it will never be optimal for the buyer to generate reliable supply simultaneously via direct sourcing and inducing SM, since one of these two strategies is less costly than the other (depending on the parameter values).

Thus, broadly speaking, under direct sourcing, the buyer has two options: set $m = 0$ (do not source any raw materials from Tier 2) and follow the same sourcing strategy as in a traditional supply chain, or set $m > 0$ and serve as the reliable supplier to Tier 1. We refer to the latter case as "using the direct sourcing option" and abbreviate it by DS. If DS is optimal for the buyer, as shown by Lemma A.1 in §A.1 of the appendix, it will always procure exactly $D - K$ units of raw materials from Tier 2: any more would not be purchased by Tier 1 (and, implicitly, we have assumed zero salvage value for excess raw materials purchased by the buyer) and any less would result in a potential shortage of raw materials. Hence, the optimal contract parameters are $(w, Q, p, m)^* = (c_u, D, c_u, D - K)$, yielding profit $\Pi_d^B = rD - c_u K - c_d(D - K)$. Under these parameters, the buyer induces the Tier 1 supplier to source the risk-free quantity from the unreliable Tier 2 supplier (K) while sourcing the remainder from the buyer $(D - K)$. Conversely, under strategies PA and SM, the buyer's expected profit Π_d^B is identical to the corresponding cases in the traditional supply chain. The strategies available to the buyer under direct sourcing are summarized in Table 3.

Note that the buyer's profit under direct sourcing is always weakly higher than in the traditional supply chain, since there is no fixed cost associated with direct sourcing and the option to source directly in Tier 2 is only used if it increases the buyer's profit. Furthermore, note that if $c_d > r - c_u$, PA dominates DS, i.e., PA yields higher profit even if disruptions are assured $(\lambda = 1)$. In this case, direct sourcing has such a high variable cost that it is never used by the buyer. For the remainder of this section, we focus on the more interesting case where $c_d \leq r - c_u$ and DS may be optimal.

In the next two theorems, we describe the buyer's optimal sourcing strategy under direct sourcing. In both theorems, if the buyer *does not* use the direct sourcing option, its optimal strategy is given by Lemma 2; whenever the buyer does use the direct sourcing option, it results in a strictly positive increase in the buyer's profit, i.e., the value of direct sourcing is positive. We begin with the case of expensive reliable supply:

Theorem 3. *With direct sourcing, if the cost of reliable supply is high ($c_r \geq \bar{c}_r$), there exists a threshold $\bar{\lambda}_d^1 \in [0,1]$ such that the buyer uses the direct sourcing option if and only if $\lambda > \bar{\lambda}_d^1$.*

Similar to Lemma 3, vertical control (in the form of direct sourcing) only affects the buyer's sourcing strategy if the risk level is sufficiently high. If $c_r > \bar{c}_r$, in a traditional supply chain, the buyer never induces reliable sourcing. Therefore, with the option of direct sourcing, the buyer effectively chooses between PA and DS. Profit in PA is decreasing in λ while profit in DS is independent of λ (since risk is fully mitigated); hence, using the direct sourcing option can only be optimal if the chance of a disruption is sufficiently high.

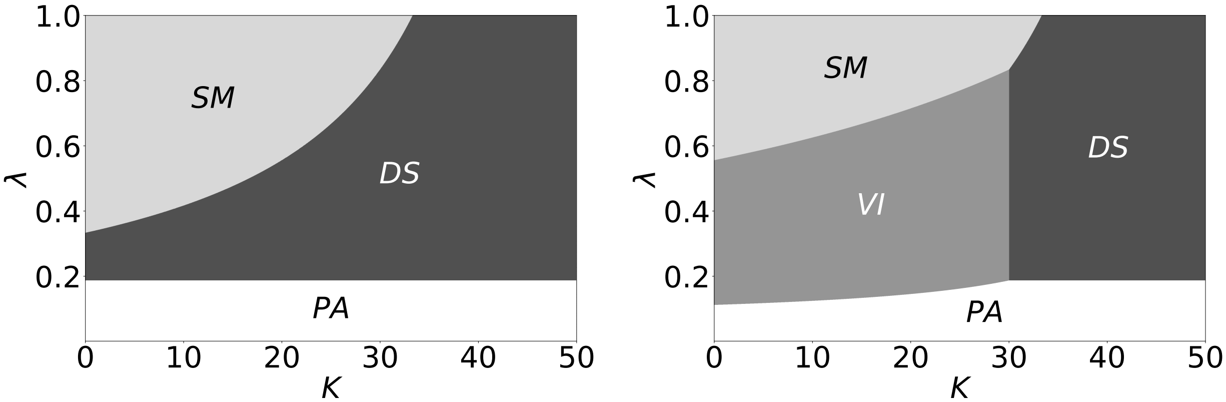
Next, we consider the case when the cost of reliable supply is low:

Theorem 4. *With direct sourcing, if the cost of reliable supply is low ($c_r < \bar{c}_r$), there exist thresholds $c_1^d < c_2^d$ such that:*
(i) If $c_d < c_1^d$, there exists a threshold $\bar{\lambda}_d^1 \in [0,1]$ such that the buyer uses the direct sourcing option if and only if $\lambda > \bar{\lambda}_d^1$.
(ii) If $c_1^d \leq c_d < c_2^d$, there exist two thresholds $\bar{\lambda}_d^1, \bar{\lambda}_d^2 \in [0,1]$ such that the buyer uses the direct sourcing option if and only if $\bar{\lambda}_d^1 < \lambda < \bar{\lambda}_d^2$.
(iii) Otherwise, the buyer does not use the direct sourcing option for any λ.

Case (i) of Theorem 4 is similar to Theorem 3: the buyer uses the direct sourcing option if the risk level is high. Conversely, in case (iii), the buyer never finds the direct sourcing option profitable, as inducing SM is a cheaper pathway to reliable supply at high λ. In case (ii), the direct sourcing option is optimal at moderate disruption probabilities. Similar to the vertical integration case, for moderate λ, the incentive misalignment between the buyer and Tier 1 supplier is greatest: in this regime, the buyer would prefer to source from the reliable Tier 2 supplier, but inducing the Tier 1 supplier to do this via the wholesale price is very costly. It is precisely in this region where the buyer finds it optimal to source directly from Tier 2 and provide a lower cost source of reliable supply to the Tier 1 supplier, hence the DS strategy is optimal.

Together, Theorems 3 and 4 show that direct sourcing behaves in a similar fashion to vertical integration: it is most valuable to the buyer (and hence the direct sourcing option is most likely to be used) at moderate disruption risk levels. Direct sourcing provides a different, but analogous, mechanism for overcoming the incentive misalignment problems of a traditional supply chain: rather than directly controlling Tier 1's sourcing decision, the buyer can influence that decision by providing low-cost (and reliable) supply. This is most valuable to the buyer when the incentive misalignment problem is greatest, i.e., at moderate disruption probabilities.

These insights are illustrated in Figure 4a, which plots the buyer's optimal strategy with the direct sourcing option (but without the option to vertically integrate) for different levels of disruption probability (λ) and severity (K). Note that Figure 4a appears qualitatively similar to Figure 3a, which confirms that direct sourcing is most valuable to buyers—and therefore optimal—under similar conditions to vertical integration. Also note that due to the absence of a fixed cost for direct sourcing, it may be optimal when K is high (close to D, which is equal to 50 in the example in the figure), unlike vertical integration.



(a) With direct sourcing

(b) With direct sourcing and vertical integration

Figure 4. The buyer's optimal strategy with the option of direct souring, without (a) and with (b) the option of vertical integration, as a function of λ and K. In the example, $D = 50$, $c_u = 4$, $c_r = 5$, $c_d = 7$, $r = 20$, and $F_v = 40$.

We conclude this section by comparing the two types of vertical control to one another. As noted previously, a key difference between direct sourcing and vertical integration is cost: direct sourcing is high variable cost/low fixed cost, while vertical integration is the opposite. In fact, the choice between direct sourcing and vertical integration depends critically on these differences in variable and fixed costs. It what follows, we define the critical cost threshold:

$$\bar{F}_{vd} \equiv (c_d - c_r)(D - K). \tag{9}$$

This represents the total increase in variable costs resulting from a direct sourcing strategy compared to a vertical integration strategy. Given this, we have the following result:

Corollary 1. *Vertical integration is preferred to direct sourcing if and only if* $F_v \leq \bar{F}_{vd}$.

Corollary 1 yields a surprising insight: the choice between the two forms of vertical control depends on fixed and variable costs but is independent of the likelihood of a disruption. If the additional fixed cost of vertical integration (F_v) is less than the additional variable cost incurred with direct sourcing ($\bar{F}_{vd}$), vertical integration dominates direct sourcing, and the buyer's optimal strategy follows in accordance with the analysis in §4. On the other hand, if the additional fixed cost of vertical integration exceeds the additional variable cost from direct sourcing, the opposite is true: direct sourcing dominates vertical integration, and the buyer's optimal strategy is as described in §5.

This is not to say that the choice between direct sourcing and vertical integration is completely independent of disruptions. In particular, the threshold $\bar{F}_{vd}$ increases if disruptions are more severe (K is smaller). This leads to an important observation: vertical integration is preferred to mitigate the risk of more severe disruptions, while direct sourcing is preferred to mitigate the risk of milder disruptions. In the latter case, to mitigate the risk of milder disruptions, the buyer has to direct source a smaller amount of raw materials to ensure resilience in its supply chain, meaning it pays the higher variable costs of direct sourcing on a smaller quantity. Combined with the lower fixed cost of direct sourcing, this can make direct sourcing more profitable than vertical integration for mild disruptions.

The implications of Corollary 1 are depicted in Figure 4b, which shows the buyer's optimal risk mitigation strategy when given the option to use *both* firms of vertical control. In this example, we use the same parameters as in Figures 3a and 4a. Figure 4b illustrates two important findings. First, the buyer's choice between the two vertical control strategies does not depend on λ. Second, the buyer's optimal form of vertical control transitions from vertical integration to direct sourcing as K increases above a threshold (in the example, direct sourcing is optimal for $K > 30$).

Corollary 1 also suggests what happens if $c_d \leq c_r$, which might occur if the buyer can negotiate a better price with its Tier 2 suppliers than Tier 1 supplier. If $c_d \leq c_r$, direct sourcing becomes a low fixed cost *and* low variable cost alternative to vertical integration. Thus, $\bar{F}_{vd}$ becomes negative and the condition in Corollary 1 is never satisfied. In this case, direct sourcing weakly dominates vertical integration and is the optimal form of vertical control for all K.

6 Penalty Contracts

Given that vertical control generates value by offering a way to overcome incentive misalignment between the buyer and Tier 1 supplier, an important question is whether that misalignment can be addressed via other mechanisms such as advanced contracting. Clearly, if the buyer could contract with the Tier 1 supplier on *which* Tier 2 suppliers are used, the buyer could dictate the exact sourcing strategy it prefers and replicate the outcome of vertical control at zero cost. However, in practice, it can be challenging for buyers to enforce such restrictions, and non-compliance (i.e., using unauthorized suppliers) to reduce costs is a common problem (see Caro *et al.*, 2020 and references therein). Hence, in this section, we focus on a form of advanced contracts that seeks to align buyer and supplier preferences by manipulating the Tier 1 supplier's compensation rather than dictating its sourcing strategy: penalty contracts.

Under a penalty contract, the buyer offers the supplier a contract of the form (w, Q, z), where z is a penalty paid by the supplier for each unit ordered by the buyer but not delivered. In practice, penalty contracts may or may not be feasible: some buyers have shown hesitance to penalize suppliers for exogenous events occurring in their supply chain, often deriving from natural disasters like extreme weather or earthquakes, and in some cases buyers have even helped subsidize (rather than penalize) suppliers that experience disruptions by financially supporting their recovery or helping them identify alternative Tier 2 sources (Ang *et al.*, 2017). Despite this, such contracts have the potential to coordinate the supply chain and align the interests of the buyer and supplier in the presence of disruptions, so long as the penalty for undelivered items is sufficiently high. However, the unit penalty required to achieve coordination might be so high that it is legally unenforceable (Wegener & Caplan, 2021) or poses an existential risk to the supplier's business, making suppliers unwilling to accept such terms (Hwang *et al.* 2018). To capture this practical element, we assume that the buyer can charge the supplier at most $\bar{z}$ per unit for undelivered items.[7]

In addition, we assume the buyer charges the minimum possible penalty if a range of penalty values achieve the same expected profit for the buyer and supplier. Note that penalty contracts include wholesale price contracts as a special case, since the buyer can set $z = 0$; hence, penalty contracts are generalization of wholesale price contracts.

The detailed analysis of penalty contracts is contained in §B of the appendix. Here, we briefly describe the insights obtained from that analysis. We will focus our discussion on the ability of penalty contracts to replicate the outcome achieved with vertical control; as both forms of vertical control are most valuable to the buyer under similar conditions, we restrict our attention to vertical integration as the sole form of vertical control available to the buyer.

A critical outcome of the analysis of penalty contracts is that there exists a threshold penalty level defined as follows:

$$z^*_{\min} \equiv (c_r - c_u)\left(\frac{1}{\lambda} - 1 + \frac{K}{D}\right).$$

To replicate the sourcing decisions of a vertically integrated supply chain and fully extract the Tier 1 supplier's surplus, the buyer must set the penalty $z \geq z^*_{\min}$. If this is feasible (i.e., if $z^*_{\min} \leq \bar{z}$), the buyer can achieve the same outcome as vertical integration at zero cost with a penalty contract: by setting a high non-delivery penalty, the buyer induces the Tier 1 supplier to source from the reliable Tier 2 supplier, and moreover the buyer can extract all surplus from Tier 1 by charging a wholesale price that leaves the supplier with zero expected profit. On the other hand, if $z^*_{\min} > \bar{z}$, the supplier will not accept a penalty high enough to achieve this outcome.

From the definition of $z^*_{\min}$, we see that this threshold is increasing in K and decreasing in λ. Moreover, when $\lambda \to 1$ and $K \to 0$, $z^*_{\min} \to 0$. Hence, for disruptions that are very severe (K is low) and very likely (λ is high), buyers can create contracts with relatively small penalties that can replicate vertical integration—inducing reliable sourcing while extracting all surplus from the supplier—at significantly lower fixed cost to the buyer. However, note that when disruptions are very severe and very likely, vertical integration is not especially valuable to the buyer (see Figure 3) due to the strong incentive of the Tier 1 supplier to mitigate risk by sourcing reliably. Hence, penalty contracts may be easier for the supplier to accept in these circumstances, but their incremental value to the buyer can be limited.

On the other hand, when disruptions are less severe (K is higher) and less likely (λ is lower), the minimum penalty necessary to replicate vertical integration, $z^*_{\min}$, will be higher, and might exceed the maximum penalty the Tier 1 supplier is willing to accept, $\bar{z}$ (note that $z^*_{\min}$ is unbounded as $\lambda \to 0$). When this occurs, penalty contracts can help reduce the cost of inducing supplier

[7] In our model, the supplier can completely eliminate disruption risk by sourcing from a reliable Tier 2 supplier. While this might suggest that very high penalties should be acceptable to the supplier provided the equilibrium outcome is reliable sourcing (and hence, there is no chance that the supplier ever pays the high penalty), in practice, it is impossible to fully eliminate the risk of disruptions, and hence from a practical perspective buyers cannot set arbitrarily high penalties; moreover, contractual clauses specifying penalties that are excessively high compared to actual losses are considered punishments and are legally unenforceable (Cornell Legal Information Institute, 2020). Our assumption of an upper bound $\bar{z}$ on the penalty is intended to reflect these facts.

mitigation, as the threat of a penalty allows the buyer to pay Tier 1 a lower wholesale price and still induce SM, but they cannot extract all surplus from the Tier 1 supplier. Hence, penalty contracts are less effective if disruptions are of moderate to low severity, or moderate to low likelihood, as they require a very high penalty that may be unacceptable to the supplier.

This suggests that there is still a place for vertical control as a disruption mitigation strategy, especially when λ is moderate: in this scenario, vertical integration has the highest value to the buyer, but replicating vertical integration via contracts may require a penalty that is too high for the supplier to accept. Consequently, penalty contracts do not fully replace the need for vertical control at moderate risk levels. The following theorem formalizes these insights:

Theorem 5. *(i) If the maximum possible penalty exceeds* $z^*_{\min}$*, the ability to charge a non-delivery penalty eliminates the need for vertical integration.*
(ii) If the maximum possible penalty is less than $z^*_{\min}$*, the ability to charge a non-delivery penalty increases the region of optimality of supplier mitigation in a traditional supply chain, but vertical integration can still be optimal at moderate levels of* λ*.*

The theorem is illustrated graphically in Figure 5, which replicates Figure 4b with the additional option of using penalty contracts. In panel (a), $\bar{z} = 0$ (i.e., penalties are not allowed). In panels (b), (c), and (d), $\bar{z} = 0.5$, 2, and 20, respectively, corresponding to increasing maximum penalties. As the figure shows, increasing $\bar{z}$ effectively increases the region of optimality of the SM strategy and decreases the region of optimality for all other strategies (PA, VI, and DS). However, even when penalties are allowed, vertical control—both direct sourcing and vertical integration—may be optimal for the buyer at moderate risk levels.

7 Multi-sourcing in Tier 1

In our base model, the buyer sources from a single Tier 1 supplier, who in turn sources from one or two Tier 2 suppliers. To help reduce disruption risk, the buyer could increase vertical control over Tier 1, as we have considered; however, another strategy the buyer could pursue is to diversify its Tier 1 suppliers, thereby reducing the buyer's exposure to a disruption arising in the supply chain of any particular Tier 1 supplier (Ang *et al.*, 2017). In this section, we consider precisely this strategy, and explore whether the buyer's ability to multi-source in Tier 1 can obviate the need for vertical control. The detailed analysis of multi-sourcing is contained in §C of the appendix. In what follows, we focus on the key insights that emerge from that analysis.

To maintain consistency with the existing literature, many of our assumptions in this section mirror those of Ang *et al.* (2017), who study a similar problem (with multiple Tier 1 and Tier 2 suppliers) but without considering vertical control. Specifically, we assume that the buyer has access to two *ex ante* identical Tier 1 suppliers, S_i, $i = 1, 2$. Each Tier 1 supplier, in turn, has access to one reliable and one unreliable Tier 2 supplier, R_i and U_i, respectively, for Tier 1 supplier i. The supply chain is depicted in Figure 6. We assume that Tier 2 suppliers of the same type are identical: that is, R_1 and R_2 are both perfectly reliable and both sell to their Tier 1 suppliers at unit cost

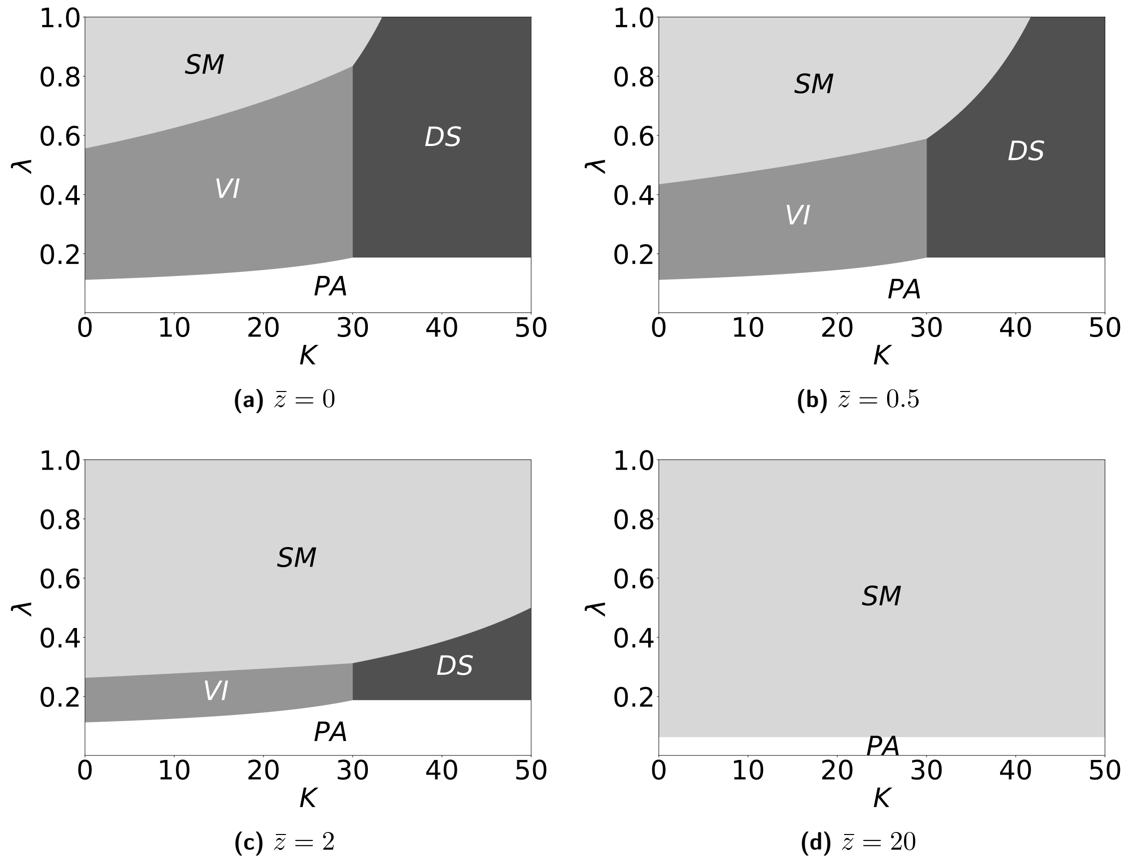


Figure 5. The buyer's optimal strategy—allowing for both forms of vertical control and penalty contracts—as a function of λ and K for different $\bar{z}$ values. In the example, $D = 50$, $c_u = 4$, $c_r = 5$, $c_d = 7$, $r = 20$, and $F_v = 40$.

c_r, while U_1 and U_2 both disrupt with probability λ (in which case their capacity is reduced to K) and sell to their Tier 1 suppliers at unit cost c_u. (Our analysis in §C of the appendix allows for asymmetric disrupted capacity at the two Tier 2 unreliable suppliers; for ease of exposition, we begin our discussion in this section with the symmetric case, and comment on the impact of asymmetry at the end of the section.)

Disruptions at the unreliable Tier 2 suppliers can be arbitrarily positively correlated. For instance, if U_1 and U_2 are physically close to one another, disruptions caused by natural disasters (e.g., earthquakes or hurricanes) are likely to be highly positively correlated. On the other hand, if U_1 and U_2 are located in different countries, disruptions caused by geopolitical conflict are likely to be independent. To flexibly model positive correlation between the disruptive events, we define two conditional probabilities:

$$P(U_1 \text{ disrupts}|U_2 \text{ disrupts}) = P(U_2 \text{ disrupts}|U_1 \text{ disrupts}) = (1-\alpha)\lambda + \alpha,\ \alpha \in [0,1].$$

If $\alpha = 0$, the disruptions are independent; if $\alpha = 1$, the disruptions are perfectly positively correlated. Any intermediate positive correlation can be achieved by setting α in the interval $(0, 1)$. Given this,

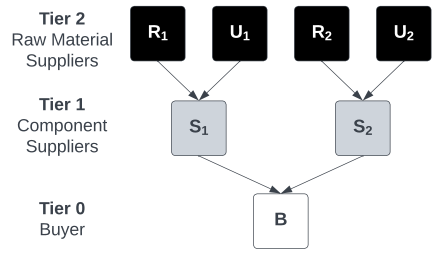


Figure 6. The three-tier supply chain model with two Tier 1 suppliers.

we may express the probabilities of all possible combinations of disruptions in Tier 2 in terms of λ and α as follows:

$$\begin{aligned} P(\text{Both } U_1 \text{ and } U_2 \text{ disrupt}) &= \lambda(\lambda - \alpha\lambda + \alpha) \\ P(\text{Only } U_1 \text{ disrupts}) &= (1-\alpha)(1-\lambda)\lambda \\ P(\text{Only } U_2 \text{ disrupts}) &= (1-\alpha)(1-\lambda)\lambda \\ P(\text{Neither } U_1 \text{ nor } U_2 \text{ disrupt}) &= (1-\lambda)(1-\lambda+\alpha\lambda). \end{aligned}$$

The sequence of events is identical to the base model, except instead of the buyer interacting with a single Tier 1 supplier at each step, the buyer can choose to interact (i.e., source from) one or both Tier 1 suppliers. Let $(w_i, Q_i), i = 1, 2$, be the contract offered to S_i. We assume that sourcing and production leadtimes are sufficiently long that the buyer has no ability to shift production from one supplier to the other if a disruption occurs; hence, as in the base model, the consequence of a disruption is that the buyer may experience lost sales.

The sourcing decisions of any one supplier are given by Lemma 1, just as in the base model, because once a supplier has been offered a contract by the buyer, the existence of other suppliers does not affect its decisions. Thus, the buyer can choose to pay each supplier a low wholesale price (c_u) that induces single-sourcing from an unreliable Tier 2 supplier, or a high wholesale price ($c_u + (c_r - c_u)/\lambda$) that induces dual-sourcing from both Tier 2 suppliers.

When the buyer has access to multiple Tier 1 suppliers, it is always optimal for the buyer to multi-source in Tier 1 to both diversify risk and reduce costs. The latter benefit arises because the buyer can always obtain K units risk free from a Tier 1 supplier at a low wholesale price; hence, even if the buyer induces S_1 to source reliably by paying a high wholesale price, it will still source K units from S_2 at a low wholesale price to reduce costs.

Consequently, *multi-sourcing in Tier 1* (MS1) is always a part of the buyer's optimal sourcing strategy with two Tier 1 suppliers. In addition, the buyer may find it optimal to source a larger quantity from Tier 1 than its demand D. This excess inventory helps insulate the buyer against a disruption in any one Tier 1 supplier's supply chain. We call this strategy of ordering excess *inventory mitigation* (IM). Besides MS1 and IM, the buyer can engage in supplier mitigation (SM) by paying a high wholesale price to Tier 1, as in the base model. Thus, the strategies available to the buyer with two Tier 1 suppliers are summarized in Table 4.

Strategy	Description
Multi-sourcing in Tier 1 (MS1)	Source from multiple Tier 1 suppliers simultaneously
Inventory Mitigation (IM)	Source excess inventory from Tier 1 suppliers
Supplier Mitigation (SM)	Pay Tier 1 suppliers a high price to induce Tier 2 multi-sourcing

Table 4. Traditional risk mitigation strategies for the buyer with two Tier 1 suppliers.

Clearly the buyer can do no worse with multiple Tier 1 suppliers than with a single Tier 1 supplier: every outcome that can occur with a single Tier 1 supplier can be replicated with multiple Tier 1 suppliers by simply ignoring all but one of the Tier 1 suppliers. Moreover, the ability to multi-source in Tier 1 (and to engage in inventory mitigation) serves to increase the buyer's profit in the absence of vertical control, lessening the need to use vertical integration or direct sourcing. Our first result illustrates that with "mild" disruptions, this ability to multi-source in Tier 1 fully eliminates the need for vertical control. In the following theorems, when we say vertical integration or direct sourcing are "optimal," we mean that they strictly increase the buyer's profit assuming the other form of vertical control is not possible.

Theorem 6. *With two Tier 1 suppliers, if disruptions are mild ($K \geq D/2$), neither neither form of vertical control is optimal.*

With two Tier 1 suppliers and mild disruptions (specifically, disruptions where an unreliable Tier 2 supplier's output is more than half the buyer's demand), the buyer can completely eliminate disruption risk by multi-sourcing in Tier 1 and paying Tier 1 suppliers a low wholesale price (c_u). This allows the buyer to extract all surplus from Tier 1 while simultaneously fully mitigating disruptions. As a result, vertical control is not beneficial to the buyer: vertical integration strictly reduces the buyer's profit due to its fixed cost, and it is never optimal for the buyer to use the direct sourcing option due to its marginal cost.

The same is not true with severe disruptions, however. In what follows, we define the threshold

$$\widehat{F}_{vd} \equiv (c_d - c_r)(D - 2K),$$

which is analogous to (9) from the base model.

Theorem 7. *With two Tier 1 suppliers, if disruptions are severe ($K < D/2$):*
(i) Vertical integration is optimal for disruption probabilities in an intermediate range (i.e., $\bar{\lambda}^1_{vt} < \lambda < \bar{\lambda}^2_{vt}$, where $0 \leq \bar{\lambda}^1_{vt} \leq \bar{\lambda}^2_{vt} \leq 1$).
(ii) Direct sourcing is optimal for disruptions probabilities in an intermediate range (i.e., $\bar{\lambda}^1_{dt} < \lambda < \bar{\lambda}^2_{dt}$, where $0 \leq \bar{\lambda}^1_{dt} \leq \bar{\lambda}^2_{dt} \leq 1$).
(iii) Vertical integration is preferred to direct sourcing if and only if $F_v \leq \widehat{F}_{vd}$.

Theorem 7 confirms the insights of the base model hold with multiple Tier 1 suppliers, provided disruptions are severe enough that the buyer cannot fully mitigate them by multi-sourcing in Tier 1. Specifically, vertical control is most valuable for moderate disruption probabilities, and the

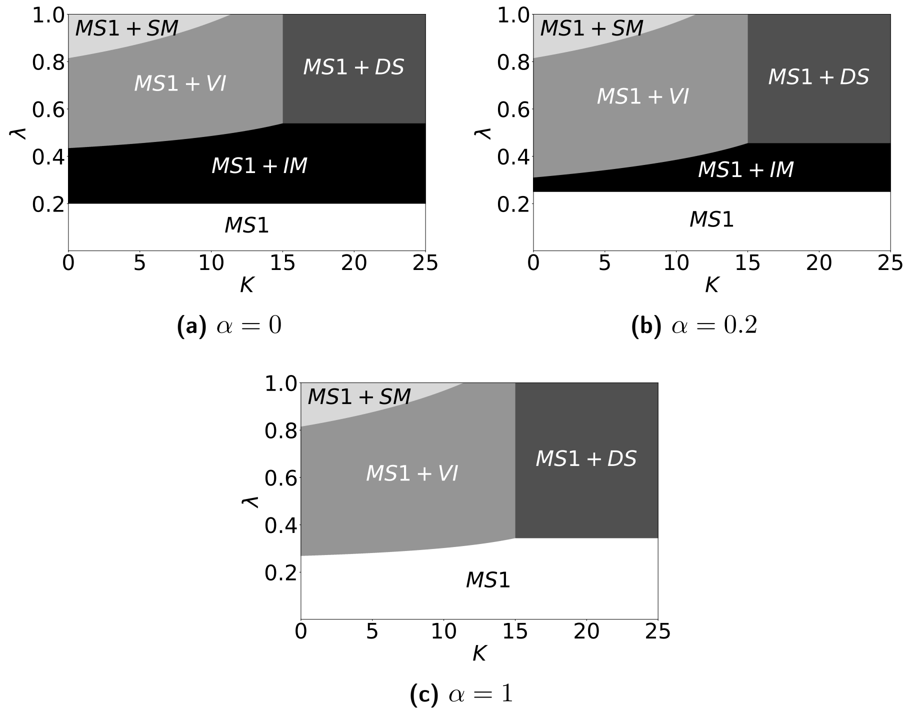


Figure 7. The buyer's optimal strategy with multi-sourcing in Tier 1 as a function of λ and K for different values of α. In this example, $D = 50$, $c_u = 4$, $c_r = 7.5$, $c_d = 9.5$, $r = 20$, and $F_v = 40$.

choice between direct sourcing and vertical control depends only on the severity of disruptions, the incremental cost of direct sourcing, and the buyer's demand (i.e., it does not depend on the probability of a disruption or on the correlation of Tier 2 disruptions). Thus, we can conclude from Theorems 6 and 7 that multi-sourcing in Tier 1 can eliminate the need for vertical control if disruptions are mild, but not if they are severe, and in the latter case, vertical control (as well as the buyer's preferences between different forms of vertical control) behaves in a similar manner to the base model.

These insights are illustrated in Figure 7, which shows how the level of positive correlation between disruptions in Tier 2 affects the buyer's optimal strategy with two Tier 1 suppliers in the presence of severe disruptions ($K < D/2$). Specifically, the figure compares the buyer's optimal strategy with two Tier 1 suppliers who experience independent Tier 2 disruptions ($\alpha = 0$, panel (a)), moderately positively correlated Tier 2 disruptions ($\alpha = 0.2$, panel (b)), and perfectly positively correlated disruptions ($\alpha = 1$, panel (c)). As the figure shows, as Tier 2 disruptions become more positively correlated, vertical control becomes more beneficial to the buyer.

Finally, although we have assumed homogeneous suppliers in each tier, our framework can be easily adapted to study the impact of supplier heterogeneity. For instance, one might consider the impact on the buyer's optimal sourcing strategy of allowing the two unreliable suppliers in Tier 2 to differ in their resilience to disruptions. In the context of our model, such asymmetry would imply that, in case of a disruption, U_1's capacity is reduced to K_1 and U_2's capacity is reduced for

K_2. Assuming without loss of generality that $K_1 \geq K_2 \geq 0$, this suggests that U_1 is more resilient (faces a smaller capacity reduction) when a disruption occurs. Our analysis in §C of the appendix allows for such asymmetry, and shows that it does not have a qualitative impact on the insights presented in Theorems 6 and 7, as these results do not depend on the assumption $K_1 = K_2 = K$ in any way, and the buyer still prefers vertical control for severe disruptions (i.e., $K_1 + K_2 < D$) and for moderate levels of disruption probability.

Further, even if Tier 2 suppliers are heterogeneous in their resilience level, the buyer is indifferent about which Tier 1 supplier to vertically integrate with. This arises from the observation that, if vertical integration is optimal, it must mean that the buyer should find it optimal to source as little as possible from the unreliable suppliers (directly and indirectly) and as much as possible from the reliable supplier that it has direct access to. Thus, the buyer sources K_i from $U_i, i = 1, 2$, and $D - K_1 - K_2$ from the reliable supplier it has direct access to. These sourcing quantities are optimal regardless of which Tier 1 supplier the buyer vertically integrates with, and hence the buyer is indifferent between choosing to integrate with a Tier 1 supplier that has a more or less resilient Tier 2 supplier. We make a similar observation regarding the direct sourcing strategy. As the buyer sets the contract terms such that S_i orders K_i units from $U_i, i = 1, 2$ and the supplier which is offered the direct sourcing option orders $D - K_1 - K_2$ from the buyer, the buyer's profit does not depend on which supplier it offers the direct sourcing option to. Hence, asymmetry in unreliable Tier 2 supplier resilience does not qualitatively impact our key insights.

8 Conclusion

Recent decades have seen multinational firms expand their supply chains for many reasons, including seeking lower costs, greater capabilities, and responding to market changes (Cohen *et al.*, 2018). While generating many benefits, the expansion of supply chains has also exposed firms to more potential points of failure. Facing an increasingly volatile landscape due to viral pandemics, natural disasters, extreme weather events, and geopolitical risk, firms have sought selective ways to mitigate the risk of materials shortages while maintaining the benefits they have gained from the globalization of supply chains (Cohen *et al.*, 2022). In this paper, motivated by recent examples in practice (Foldy, 2022; Chow, 2022), we have analyzed the value of a class of such risk mitigation measures, which we call vertical control, and have determined when these strategies benefit firms the most, how they should be integrated with other commonly studied risk mitigation strategies, and how firms should choose between them.

Our analysis indicates that one important way vertical control can help firms mitigate disruption risk is by enabling them to overcome incentive misalignment in the supply chain and make more effective Tier 2 sourcing decisions. Generally speaking, vertical control is most valuable in this regard when Tier 2 disruptions are of moderate likelihood. Moreover, the choice between the two types of vertical control—vertical integration and direct sourcing—is unaffected by the disruption probability, but depends critically on the severity of disruptions, with vertical integration preferred for more severe disruptions.

We also considered whether alternative risk mitigation strategies can eliminate the need for vertical control. One obvious solution to the incentive misalignment problem in the supply chain is a penalty contract, in which the buyer penalizes the Tier 1 supplier for a failure to deliver components. While penalty contracts can coordinate the supply chain in the sense of replicating the outcome of vertical integration, they require a sufficiently high penalty to achieve this, and—unless disruptions are very likely and very severe—this may exceed the penalty level the Tier 1 supplier is willing to accept. Hence, vertical control can still be valuable to buyers even if penalty contracts are possible, particularly when the disruption probability is moderate and incentive misalignment in the supply chain is most severe.

Another possible solution to the buyer's disruption risk problem is to diversify Tier 1 suppliers, rather than exert greater vertical control over a single Tier 1 supplier. Having access to multiple Tier 1 suppliers allows the buyer to both multi-source in Tier 1 and engage in inventory mitigation, i.e., ordering more inventory than its demand to insulate against disruptions. While Tier 1 diversification can completely replace vertical control for mild disruptions, if disruptions are severe, vertical control is still optimal for moderate disruption probabilities. Thus, while multi-sourcing in Tier 1 reduces the need for vertical control, it does not eliminate it.

Collectively, our results show that vertical control can be a valuable tool for buyers to mitigate the risk of disruptions, complementing other more traditional risk mitigation strategies like supplier mitigation, inventory mitigation, multi-sourcing, and penalty contracts. Of course, there are many aspects of vertical control that we do not consider in our analysis. For one, firms might consider vertical control for numerous other reasons besides risk mitigation, such as its potential to give them competitive advantage, and we do not model these reasons. Further, even in settings where vertical control is used to mitigate disruption risk, it might result in other risk mitigation benefits besides increasing the buying firm's control over Tier 2 sourcing decisions. Most notably, a vertical integration strategy can eliminate information asymmetry in a setting where the Tier 1 suppliers have better information than the buyer about the reliability or production cost of Tier 2 suppliers. Therefore, future work could use our results as building blocks to study other possible advantages of vertical control in risk mitigation settings.

In sum, our results have both theoretical and practical value for firms considering vertical integration or direct sourcing to attenuate the adverse effects of supply chain disruptions. It is important for such firms to understand the costs and benefits of vertical control, as well as how it performs relative to other, more well-studied, risk mitigation strategies. As our results show, vertical control does increase a firm's control over its supply chain and help it mitigate disruption risk; however, other, less costly, risk mitigation strategies may perform better, implying vertical control should be used judiciously and under specific conditions to help build more resilient supply chains.

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Online Supplement to "Building Supply Chain Resilience Through Vertical Control"

A Proofs from the Main Model

This section contains proofs of the results in §§3-5 of the main text.

Proof of Lemma 1. From equation (1), two cases arise: $Q \leq K$ and $Q > K$. Suppose first that $Q \leq K$. Then, sourcing from the unreliable supplier is risk-free for S, and as long as $w \geq c_u$, $q_u = Q$ and $q_r = 0$. If $Q > K$, we can make three simplifying observations. First, it will be optimal for the supplier to choose $q_u + q_r \geq Q$, which can be seen from (1). Second, it must be true that $q_r + K \leq Q$, since otherwise S would have unsold inventory even during a disruption and S could increase its expected profit by reducing q_r. Finally, since source K units risk-free from U at a unit cost of c_u, it must be true that $q_u \geq K$. Given these observations, Π_S^* simplifies to:

$$\Pi_S^* = \max_{q_u, q_r \geq 0} (1-\lambda)\Big[wQ - c_u q_u - c_r q_r\Big] + \lambda\Big[w(q_r + K) - c_u K - c_r q_r\Big].$$

Note that $\frac{\partial \Pi_S}{\partial q_u} = -(1-\lambda)c_u$ and $\frac{\partial \Pi_S}{\partial q_r} = \lambda w - c_r$, and due to linearity, there are two possible solutions: $(q_u, q_r) = (Q, 0)$ or $(q_u, q_r) = (K, Q-K)$. If $\frac{\partial \Pi_S}{\partial q_u} > \frac{\partial \Pi_S}{\partial q_r} \iff w < c_u + \frac{c_r - c_u}{\lambda}$, $(q_u, q_r)^* = (Q, 0)$. Otherwise, $(q_u, q_r)^* = (K, Q-K)$. Combining these cases yields the result.

□

Proof of Lemma 2. According to Equation (2), the buyer's profit maximization problem is given by:

$$\Pi_t^B = \max_{w \geq c_u, Q \geq 0} f(w)\big[r \min\{\min\{Q,K\}, D\} - w \min\{Q,K\}\big] + \\ (1 - f(w))\big[r \min\{Q, D\} - wQ\big],$$

and regardless of the strategy the buyer chooses, it is always optimal to set $Q^* = D$.

With the passive acceptance strategy, the buyer sets $w = c_u$, which leads to the expected profit $\Pi_t^B(PA) = (r - c_u)\big[(1-\lambda)D + \lambda K\big]$, and with the supplier mitigation strategy, the buyer sets $w = c_u + (c_r - c_u)/\lambda$, which leads to expected profit $\Pi_t^B(SM) = \big(r - c_u - (c_r - c_u)/\lambda\big)D$. Note that $\Pi_t^B(PA)$ is strictly decreasing in λ and $\Pi_t^B(SM)$ is strictly increasing in λ. Further, it is clear that $\Pi_t^B(PA) > \Pi_t^B(SM)$ as $\lambda \to 0$, and as $\lambda \to 1$, $\Pi_t^B(PA) = (r - c_u)K$ and $\Pi_t^B(SM) = (r - c_r)D$. Then, we have the following two cases:

Case (i): $\Pi_t^B(PA) \geq \Pi_t^B(SM)$ as $\lambda \to 1$, which holds if $c_r \geq \bar{c}_r \equiv \frac{r(D-K)+c_u K}{D}$. In this case, the buyer's optimal strategy is always passive acceptance.

Case (ii): Otherwise, $\Pi_t^B(PA)$ intersects with $\Pi_t^B(SM)$ from above for some $\lambda^* < 1$. In this case, for $\lambda \leq \lambda^*$, passive acceptance is optimal and for $\lambda > \lambda^*$, supplier mitigation is optimal for the buyer. Setting $\Pi_t^B(PA) = \Pi_t^B(SM)$ and solving for λ yields $\lambda^* = \sqrt{\frac{(c_r-c_u)D}{(D-K)(r-c_u)}}$. It can be readily checked that $c_r < \bar{c}_r$ is precisely the condition required for $\lambda^* < 1$.

$\square$

Proof of Lemma 3. From Equation (5), with single sourcing, the buyer's expected profit is $\Pi_v^B(SS) = (r-c_u)\big[(1-\lambda)D + \lambda K\big] - F_v$, and with multi-sourcing, the buyer's expected profit is $\Pi_v^B(MS) = rD - c_u K - c_r(D-K) - F_v$.

Note that $\Pi_v^B(SS)$ is strictly decreasing in λ, and $\Pi_v^B(MS)$ is constant in λ. Further, it is clear that $\Pi_v^B(SS) > \Pi_v^B(MS)$ as $\lambda \to 0$, and as $\lambda \to 1$, $\Pi_v^B(SS) = (r-c_u)K - F_v$ and $\Pi_v^B(MS) = rD - c_u K - c_r(D-K) - F_v$, which implies that $\Pi_v^B(MS) - \Pi_v^B(SS) = (r-c_r)(D-K) > 0$ as $\lambda \to 1$. Thus, $\Pi_v^B(SS)$ intersects $\Pi_v^B(MS)$ once from above as λ increases. Setting $\Pi_v^B(SS) = \Pi_v^B(MS)$ and solving for λ yields $\lambda^* = \frac{c_r-c_u}{r-c_u}$, and it is clear that $\lambda^* < 1$. Hence, we conclude that $\Pi_v^B(MS) > \Pi_v^B(SS)$ if and only if $\lambda > \lambda^*$.

$\square$

Proof of Theorem 1. Suppose $c_r \geq \bar{c}_r$, which implies that the buyer always prefers PA in a traditional supply chain per Lemma 2. Let Π_t^B be the buyer's optimal profit in a traditional supply chain (i.e., the expected profit under the PA strategy). In this case, $\Pi_t^B = (r-c_u)\big[(1-\lambda)D + \lambda K\big]$ for all feasible λ, and it is clear that Π_t^B is strictly decreasing in λ.

Further, let Π_v^B be the buyer's optimal profit in a vertically integrated supply chain, where the buyer uses MS if $\lambda > \lambda_v^* \equiv \frac{(c_r-c_u)}{(r-c_u)}$ and uses SS otherwise. Recall that the buyer's profit under SS is $\Pi_v^B(SS) = (r-c_u)\big[(1-\lambda)D + \lambda K\big] - F_v$ and under MS is $\Pi_v^B(MS) = rD - c_u K - c_r(D-K) - F_v$. Hence, Π_v^B is decreasing in λ for $\lambda \leq \lambda_v^*$, and this decrease is with the same rate as Π_t^B, since the buyer uses PA in a traditional supply chain for all feasible λ. Thus, $\Delta_{vt} = \Pi_v^B - \Pi_t^B$ is constant in λ for $\lambda \leq \lambda_v^*$. On the other hand, for $\lambda > \lambda_v^*$, Π_v^B is constant, which immediately implies that Δ_{vt} is strictly increasing in λ for $\lambda > \lambda_v^*$. In sum, Δ_{vt} is weakly increasing in λ.

As $\lambda \to 0$, $\Delta_{vt} = -F_v$, and as $\lambda \to 1$, $\Delta_{vt} = (r-c_r)(D-K) - F_v$. Clearly, if $F_v \geq \bar{F}_1 \equiv (r-c_r)(D-K) > 0$, $\Pi_t^B \geq \Pi_v^B$ even as $\lambda \to 1$, which proves (ii). On the other hand, if $F_v < \bar{F}_1$, there is a threshold for λ, $\bar{\lambda}_{vt}^1$, such that $\Pi_v^B > \Pi_t^B$ if and only if $\lambda > \bar{\lambda}_{vt}^1$, which proves (i).

$\square$

Proof of Theorem 2. Suppose $c_r < \bar{c}_r$, which implies, according to Lemma 2, that the buyer prefers PA in a traditional supply chain if and only if $\lambda \leq \lambda_t^* \equiv \sqrt{\frac{(c_r-c_u)D}{(D-K)(r-c_u)}}$. Let Π_t^B be the buyer's optimal profit in a traditional supply chain, i.e., $\Pi_t^B = \Pi_t^B(PA) = (r-c_u)\big[(1-\lambda)D + \lambda K\big]$ if $\lambda \leq \lambda_t^*$ and $\Pi_t^B = \Pi_t^B(SM) = \big(r - c_u - (c_r - c_u)/\lambda\big)D$ otherwise. Hence, Π_t^B is strictly decreasing in λ for $\lambda \leq \lambda_t^*$ and strictly increasing in λ for $\lambda > \lambda_t^*$.

Further, let Π_v^B be the buyer's optimal profit in a vertically integrated supply chain, where the buyer uses MS if $\lambda > \lambda_v^* \equiv \frac{(c_r-c_u)}{(r-c_u)}$ and uses SS otherwise. Recall that the buyer's profit under SS is $\Pi_v^B(SS) = (r-c_u)\big[(1-\lambda)D + \lambda K\big] - F_v$ and under MS is $\Pi_v^B(MS) = rD - c_uK - c_r(D-K) - F_v$. Hence, Π_v^B is decreasing in λ for $\lambda \leq \lambda_v^*$ and it is constant in λ otherwise. Note that $\lambda_v^* \equiv \frac{(c_r-c_u)}{(r-c_u)} < \sqrt{\frac{(c_r-c_u)D}{(D-K)(r-c_u)}} \equiv \lambda_t^*$. Then, we have:

1. For $\lambda \leq \lambda_v^*$, $\Delta_{vt} = -F_v$. In this region, Δ_{vt} is constant and strictly negative in λ.
2. For $\lambda_v^* < \lambda \leq \lambda_t^*$, $\Delta_{vt} = (\lambda r + (1-\lambda)c_u - c_r)(D-K) - F_v$ and is increasing in λ.
3. For $\lambda > \lambda_t^*$, $\Delta_{vt} = \frac{(c_r-c_u)((1-\lambda)D+\lambda K)}{\lambda} - F_v$ and is decreasing in λ.

Based on this, three possibilities arise:

Case (i): $\Pi_v^B > \Pi_t^B$ as $\lambda \to 1$, which holds if $F_v < \bar{F}_2 \equiv (c_r - c_u)K$. In this case, Δ_{vt} crosses the x-axis from below once and stays above the x-axis. This implies that there is a λ value, $\bar{\lambda}_{vt}^1$, such that $\Pi_v^B > \Pi_t^B$ if and only if $\lambda > \bar{\lambda}_{vt}^1$.

Case (ii): $\Pi_v^B \leq \Pi_t^B$ at $\lambda = 1$, which holds if $F_v \geq \bar{F}_2$ and $\Pi_v^B > \min_\lambda \Pi_t^B$ (i.e., Π_t^B at $\lambda = \lambda_t^*$). Regarding the latter, there is a threshold for F_v, such that if $F_v < \bar{F}_3$, $\Pi_v^B > \min_\lambda \Pi_t^B$. In this case, there is a λ value, $\lambda_t^* > \bar{\lambda}_{vt}^1 > \lambda_v^*$, at which Π_t^B intersects the constant segment of Π_v^B from above, and another λ value, $\bar{\lambda}_{vt}^2 > \lambda_t^*$, at which the increasing segment of Π_t^B intersects with the constant portion Π_v^B from below. This implies that $\Pi_v^B > \Pi_t^B$ if and only if $\bar{\lambda}_{vt}^1 < \lambda < \bar{\lambda}_{vt}^2$. The threshold $\bar{F}_3$ can be found by setting $\Pi_v^B = \min_\lambda \Pi_t^B = \Pi_t^{\min}$, where $\Pi_t^{\min} = \Pi_t^B(\lambda_t^*) = rD - \sqrt{(c_r-c_u)D(D-K)(r-c_u)} - c_uD$. Doing this yields $\bar{F}_3 \equiv \sqrt{(c_r-c_u)(r-c_u)D(D-K)} - (c_r-c_u)(D-K)$.

Case (iii): $\Pi_v^B \leq \Pi_t^{\min}$, which happens if $F_v \geq \bar{F}_3$, which implies that $\Delta_{vt} \leq 0$ and a traditional supply chain yields a higher profit for any feasible λ.

We finally show$\bar{F}_2 < \bar{F}_3$ if $c_r < \bar{c}_r$. Let $\Delta F = \bar{F}_3 - \bar{F}_2$. Then,

$$\begin{aligned}\Delta F &= \sqrt{(c_r-c_u)(r-c_u)D(D-K)} - (c_r-c_u)D \\ &= \sqrt{(c_r-c_u)D}\big(\sqrt{(r-c_u)(D-K)} - \sqrt{(c_r-c_u)D}\big).\end{aligned}$$

Hence, $\Delta F > 0$ if $\sqrt{(r-c_u)(D-K)} - \sqrt{(c_r-c_u)D} > 0$, which holds if $(r-c_u)(D-K) \geq (c_r-c_u)D \iff c_r < \frac{r(D-K)+c_uK}{D} \equiv \bar{c}_r$.

□

Proof of Lemma 4. Depending on how p compares to c_u and c_r, three possibilities arise:

Case (i): If $p \le c_u$, S always sources $\min(m, Q)$ from the buyer as long as $p \le w$, and therefore, if $Q \le m$, S single sources from the buyer regardless of the wholesale price. If $Q > m$, and $p \le w < c_u$, S does not find sourcing from the unreliable (or reliable) Tier 2 supplier profitable, leading to the first optimal strategy in case (i). If $Q > m$ and $w \ge c_u$, setting q_u and q_r is reduced to the original problem of S studied in Lemma 1, with the change that the supplier now needs to source $Q - m$ from U and R (leading to the second or third strategy in case (i)).

Case (ii): If $c_u < p \le c_r$, S sourcing from U is cheaper (but riskier) than sourcing from the buyer. However, sourcing from the buyer is weakly cheaper than sourcing from R. Thus, if $Q \le K$, S single sources from U as long as $w \ge c_u$. Otherwise, S sources at least K from U and sources the remaining $Q - K$ from U, the buyer, or R depending on the wholesale price. Specifically, if w is low, S single sources from U, which is the first strategy in case (ii). If w is moderate (with the lower and upper bounds for moderate w determined as in the proof of Lemma 1), S sources as much as possible from the buyer, which amounts to $\min(m, Q - K)$. Since the wholesale price in this case is not sufficiently high for S to source from R, if $m < Q - K$, S sources the remaining $Q - K - m$ from U, which is the second strategy in case (ii). Finally, by using the same arguments as in the proof of Lemma 1, if the wholesale price is very high, S sources K from U and $\min(m, Q - K)$ from the buyer. If $m < Q - K$, S sources the remaining $Q - K - m$ from R, leading to the third strategy in case (iii).

Case (iii): If $p > c_r$, S prefers sourcing from R to sourcing from the buyer. Thus, S's problem reverts to its original problem, studied in Lemma 1.

When $c_r \ge \bar{c}_r$, PA is always preferred to SM; hence, under direct sourcing, the buyer chooses between PA and DS. The buyer's expected profit under each strategy is:

$$\Pi_d^B(PA) = (r - c_u)D - \lambda(r - c_u)(D - K)$$
$$\Pi_d^B(DS) = (r - c_u)D - (c_d - c_u)(D - K)$$

The first terms in each expression are identical and hence may be ignored when determining the optimal strategy. PA is preferred to DS if and only if $\lambda \le (c_d - c_u)/(r - c_u)$, which leads to the result.

□

Proof of Theorem 4. When $c_r < \bar{c}_r$, under direct sourcing, the buyer chooses between PA, SM, and DS. The buyer's expected profit under each strategy is:

$$\Pi_d^B(PA) = (r - c_u)D - \lambda(r - c_u)(D - K)$$
$$\Pi_d^B(SM) = (r - c_u)D - D(c_r - c_u)/\lambda$$
$$\Pi_d^B(DS) = (r - c_u)D - (c_d - c_u)(D - K).$$

Recall that $\Pi_d^B(PA)$ is strictly decreasing in λ and $\Pi_d^B(SM)$ is strictly increasing in λ, and as $\lambda \to 0$, $\Pi_d^B(PA) > \Pi_d^B(SM)$ and $\Pi_d^B(PA) > \Pi_d^B(DS)$. Also, as $\lambda \to 1$, $\Pi_d^B(PA) \le \Pi_d^B(SM)$. Further, PA is preferred to SM iff $\lambda < \sqrt{\frac{(c_r-c_u)D}{(r-c_u)(D-K)}} \le 1$, which implies that the buyer's optimal profit without vertical control attains its minimum at $\lambda = \sqrt{\frac{(c_r-c_u)D}{(r-c_u)(D-K)}} = \lambda_t^*$. Let Π_t^B be the buyer's optimal profit without vertical control.

For the buyer to use the direct sourcing option as λ increases, we need $\Pi_d^B(DS) > \Pi_d^B(SM)$ and $\Pi_d^B(DS) > \Pi_d^B(PA)$ for some λ. In other words, if $\Pi_d^B(DS) \le \Pi_d^B(SM)$ and $\Pi_d^B(DS) \le \Pi_d^B(PA)$ for all λ, the buyer never prefers DS. Note that this happens if $\Pi_d^B(DS) \le \min_\lambda(\Pi_t^B) = rD - \sqrt{(c_r-c_u)D(D-K)(r-c_u)} - c_uD$, or $c_d \ge c_u + \sqrt{\frac{(c_r-c_u)(r-c_u)D}{D-K}} = c_2^d$, which shows case (iii).

If $c_d < c_u + \sqrt{\frac{(c_r-c_u)(r-c_u)D}{D-K}}$, $\Pi_d^B(DS) > \min_\lambda(\Pi_t^B)$, and therefore, the buyer prefers DS for some λ values. Since it is clear that the buyer prefers PA for small λ, there are two possibilities:

Case (i): $\Pi_d^B(DS) > \Pi_t^B$ as $\lambda \to 1$, or equivalently, $\Pi_d^B(DS) > \Pi_d^B(SM)$ as $\lambda \to 1$. This holds if $\Pi_d^B(DS) = rD - c_uK - c_d(D-K) > (r-c_r)D$, or $c_d < \frac{c_rD-c_uK}{D-K} = c_1^d$. In this case, $\Pi_d^B(PA)$ intersects $\Pi_d^B(DS)$ from above at some λ value and stays below it thereafter. This implies that there is a λ value, $\bar{\lambda}_d^1$, such that $\Pi_d^B(DS) > \Pi_d^B(PA)$ if and only if $\lambda > \bar{\lambda}_d^1$.

Case (ii): $\Pi_d^B(DS) \le \Pi_t^B$ as $\lambda \to 1$, or equivalently, $\Pi_d^B(DS) \le \Pi_d^B(SM)$ as $\lambda \to 1$, which happens if $c_d \ge c_1^d$. In this case, $\Pi_d^B(PA)$ intersects $\Pi_d^B(DS)$ from above at some λ value and $\Pi_d^B(SM)$ intersects $\Pi_d^B(DS)$ from below at another, larger λ value. This implies that there are λ values, $\bar{\lambda}_d^1$ and $\bar{\lambda}_d^2$, such that $\Pi_d^B(DS) > \Pi_d^B(PA)$ and $\Pi_d^B(DS) > \Pi_d^B(SM)$ if and only if $\bar{\lambda}_d^1 < \lambda < \bar{\lambda}_d^2$.

We finally show that $c_1^d = \frac{c_rD-c_uK}{D-K} < c_u + \sqrt{\frac{(c_r-c_u)(r-c_u)D}{D-K}} = c_u + \frac{\sqrt{(c_r-c_u)(r-c_u)D(D-K)}}{D-K} = c_2^d$. Consider their difference:

$$c_u + \frac{\sqrt{(c_r-c_u)(r-c_u)D(D-K)}}{D-K} - \frac{c_rD-c_uK}{D-K} = \frac{\sqrt{(c_r-c_u)D}\big(\sqrt{(r-c_u)(D-K)} - \sqrt{(c_r-c_u)D}\big)}{D-K}.$$

We have already shown that the numerator of this expression is positive if $c_r < \bar{c}_r$ in the proof of Theorem 2.

□

Proof of Corollary 1. Let Π_t^B be the buyer's optimal profit without vertical control, $\Pi_v^B(MS)$ be its expected profit under the multi-sourcing strategy in a vertically integrated supply chain, and $\Pi_d^B(DS)$ be its expected profit under direct sourcing. Then, we have:

$$\Pi_v^B(MS) = (r-c_u)D - (c_r-c_u)(D-K) - F_v, \text{ and}$$
$$\Pi_d^B(DS) = (r-c_u)D - (c_d-c_u)(D-K).$$

The buyer's optimal profit with both forms of vertical control allowed can be written as $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS), \Pi_d^B(DS)\big)$. To show sufficiency, suppose $F_v \le \bar{F}_{vd} \equiv (c_d-c_r)(D-K)$. This immediately implies $\Pi_v^B(MS) \geq \Pi_d^B(DS)$, which means $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS), \Pi_d^B(DS)\big) = \max\big(\Pi_t^B, \Pi_v^B(MS)\big)$. This in turn implies that the buyer prefers vertical integration to direct sourcing if $F_v \leq \bar{F}_{vd}$.

We prove necessity by contraposition. Suppose $F_v > \bar{F_{vd}}$, which immediately implies $\Pi_v^B(MS) < \Pi_d^B(DS)$ and therefore, $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS), \Pi_d^B(DS)\big) = \max\big(\Pi_t^B, \Pi_d^B(DS)\big)$. This in turn implies that the buyer prefers direct sourcing to vertical integration if $F_v > \bar{F}_{vd}$. Taken together, the buyer prefers vertical integration to direct sourcing if and only if $F_v \leq \bar{F}_{vd}$.

□

A.1 Additional Supporting Results

Lemma A.1. *When using the direct sourcing option is optimal for the buyer, the optimal contract is $(w, Q, p, m)^* = \big(c_u, D, c_u, D - K\big)$, and the buyer's expected profit is $\Pi_d^B(DS) = rD - c_uK - c_d(D - K)$.*

Proof of Lemma A.1. If the buyer sets $p > c_r$, S would never order raw materials from the buyer, which would correspond to a traditional supply chain. Thus, we can start by restricting our attention to $p \leq c_r$.Then, we can make further simplifying observations: (i) $Q^* = D$, as there is still a single Tier 1 supplier in this case; (ii) we can restrict our attention to the case $m \leq Q^* = D$, as the supplier would never set $q_m^* > D$ even if $m > D$, and (iii) $w^* \geq p^*$, as otherwise, S would never order raw materials from the buyer, which would again correspond to a traditional supply chain. Then, depending on how the buyer sets p and w, several possibilities arise:

Case (i): The buyer sets $p \leq c_u$. In this case, the buyer has three further options:

Case (i-a): $p \leq w < c_u$. In this case, S sets $q_m^* = m$, and the buyer's profit is simply:

$$\Pi_d^{1a} = (r - w + p - c_d)m.$$

It is also clear that the buyer would offer S the lowest possible wholesale price, which would imply that $w^* = p^*$, leading to the expected profit $\Pi_{d,1} = (r - c_d)m$. Since $r > c_d$, $m^* = D$, and $\Pi_d^{1a} = (r - c_d)D$. Thus, one potential contract for the direct sourcing strategy is given by $(w, Q, p, m) = (p, D, p, D)$ for any $0 \leq p < c_u$, which leads to the expected profit $\Pi_d^{1a} = (r - c_d)D$.

Case (i-b): $p \leq c_u \leq w < c_u + \frac{c_r - c_u}{\lambda}$. In this case, S sets $q_u^* = D - m$ and $q_m^* = m$. If the buyer sets $D - m < K \iff m > D - K$,the buyer is guaranteed to get D units from S in which case its expected profit is given by:

$$\Pi_d^{1b} = \Pi_d^{1b} = (r - w)D + (p - c_d)m, .$$

which is clearly decreasing in m since $p \leq c_u < c_d$, and therefore, the optimal solution cannot lie in this region. Conversely, if the buyer sets $D - m \geq K \iff m \leq D - K$, it receives $m + K$ units from S if U disrupts in which case its expected profit is:

$$\Pi_d^{1b} = (r - w)\Bigg[\lambda(m + K) + (1 - \lambda)D\Bigg] + (p - c_d)m.$$

The expression above is clearly decreasing in w, which implies that $w^* = c_u$. Due to linearity in m, we either have $m^* = 0$, which simply leads to the passive acceptance strategy, or $m^* = D - K$, which leads to a potential direct sourcing strategy. Finally, if the buyer sets $m = D - K$, it is clear that $p = c_u$, as the expression above increases in p for any $m > 0$. In sum, another potential contract for the direct sourcing strategy is $(w, Q, p, m) = (c_u, D, c_u, D - K)$, which leads to the expected profit $\Pi_d^{1b} = rD - c_u K - c_d(D - K)$.

Case (i-c): $c_u + \frac{c_r - c_u}{\lambda} \leq w$. The buyer induces S to multi-source from itself, U, and R by offering a high wholesale price, clearly eliminating any disruption risk. In this case, the buyer's profit is:

$$\Pi_d^{1c} = (r - w)D + (p - c_d)m,$$

which is clearly decreasing in m since $p \leq c_u < c_d$. Thus, $m^* = 0$, which leads to the supplier mitigation strategy.

Case (ii): The buyer sets $c_u < p \leq c_r$. In this case, the buyer has three further options:

Case (ii-a): $c_u \leq w < c_u + \frac{p - c_u}{\lambda}$. In this case, the buyer induces S to single source from U, which leads to the passive acceptance strategy.

Case (ii-b): $c_u + \frac{p - c_u}{\lambda} \leq w < c_u + \frac{c_r - c_u}{\lambda}$. If the buyer sets $m > D - K$, S sets $q_u^* = K$ and $q_m^* = D - K$, and the buyer's expected profit is:

$$\Pi_d^{2b} = (r - w)D + (p - c_d)m,$$

which is clearly decreasing in m, and therefore, the optimal solution cannot lie in the region where $m > D - K$. Conversely, if the buyer sets $m \leq D - K$,S sets $q_u^* = D - m$ and $q_m^* = m$, and the buyer receives $m + K$ units from S if U disrupts in which case its expected profit is:

$$\Pi_d^{2b} = (r - w)\Bigg[\lambda(m + K) + (1 - \lambda)D\Bigg] + (p - c_d)m.$$

The expression above is clearly decreasing in w, which implies that $w^* = c_u + \frac{p - c_u}{\lambda}$. Due to linearity in m, we either have $m^* = 0$, which cannot lead to a direct sourcing strategy, or $m^* = D - K$. If the buyer sets $m = D - K$, its expected profit is simplified to:

$$\Pi_d^{2b} = \left(r - c_u - \frac{p - c_u}{\lambda}\right)D + (p - c_d)(D - K) = \left(r - c_u + \frac{c_u}{\lambda}\right)D + \left(1 - \frac{1}{\lambda}\right)pD - c_d(D - K),$$

which is clearly decreasing in p (since $\lambda < 1$), and therefore the optimal solution cannot lie in the region where $c_u < p \leq c_r$ either.

Case (ii-c): $c_u + \frac{c_r - c_u}{\lambda} \leq w$. The buyer induces S to multi-source from itself, U, and R by offering a high wholesale price, again, clearly eliminating any disruption risk. In this case, the buyer's profit is:

$$\Pi_d^{2c} = (r - w)D + (p - c_d)m,$$

which is clearly decreasing in m since $p \leq c_u < c_d$. Thus, $m^* = 0$, which leads to the supplier mitigation strategy.

To conclude, we have two potential direct sourcing strategies: (i) $(w, Q, p, m) = (p, D, p, D)$, for any $0 \leq p < c_u$, which leads to the expected profit $\Pi_d^{1a} = (r - c_d)D$, and (ii) $(w, Q, p, m) = (c_u, D, c_u, D - K)$, which leads to the expected profit $\Pi_d^{1b} = rD - c_u K - c_d(D - K)$. As $c_d > c_u$, it is clear that $\Pi_d^{1b} > \Pi_d^{1a}$. Therefore, it must be true that when strategy DS is optimal, $(w, Q, p, m)^* = (c_u, D, c_u, D - K)$, and $\Pi_d^B(DS) = rD - c_u K - c_d(D - K) = (r - c_u)D - (c_d - c_u)(D - K)$.

□

B Analysis of Penalty Contracts

This section contains the analysis and supporting results for our investigation of penalty contracts (§6 of the main text). Our first lemma derives the optimal sourcing strategy of S under a penalty contract:

Lemma B.1. *With a penalty contract (w, Q, z), the optimal sourcing quantities of the Tier 1 supplier are given by:*

$$(q_u, q_r)^* = \begin{cases} (Q, 0) & \text{if } c_u \leq w + z < c_u + \frac{c_r - c_u}{\lambda} \\ \left(\min(K, Q), (Q - K)^+\right) & \text{if } w + z \geq c_u + \frac{c_r - c_u}{\lambda} \end{cases}.$$

To ensure supplier participation (i.e., non-negative expected profit), the following should hold: to induce single-sourcing in Tier 2, $w \geq c_u + z\frac{(Q-K)\lambda}{(1-\lambda)Q + \lambda K}$, and to induce dual-sourcing in Tier 2, $w \geq \frac{c_u K + c_r(Q-K)}{Q}$.

Proof. Throughout the proof, we assume that $Q \geq K$; it is straightforward to see that this will always be optimal for the buyer, since $D \geq K$. In addition, we assume $w \geq c_u$ (otherwise, S would not participate). The supplier's profit maximization problem can be written:

$$\begin{aligned}\Pi_S^* = \max_{q_u,q_r\geq 0}(1-\lambda)\Big[w\min\{q_u+q_r,Q\} - c_u q_u - z\big(Q-q_u-q_r\big)^+\Big]+ \\ \lambda\Big[w\min\big\{\min\{q_u,K\}+q_r,Q\big\} - c_u\min\{q_u,K\} - z\big(Q-\min\{q_u,K\}-q_r\big)^+\Big] \\ -c_r q_r.\end{aligned} \tag{B.1}$$

As with simple wholesale price contracts, at optimality $q_u \geq K$. Similarly, at optimality it must be true that $q_u + q_r \geq Q$, as otherwise the supplier would incur a penalty even without a disruption. Finally, it must also be true that $q_r + K \leq Q$, since otherwise the supplier would have unsold inventory even during a disruption. Based on these observations, Π_S^* simplifies to:

$$\Pi_S^* = \max_{q_u,q_r\geq 0}(1-\lambda)\Big[wQ - c_u q_u\Big] + \lambda\Big[w(q_r+K) - c_u K - z(Q-K-q_r)\Big] - c_r q_r.$$

Note that $\frac{\partial \Pi_S^*}{\partial q_u} = -(1-\lambda)c_u$ and $\frac{\partial \Pi_S^*}{\partial q_r} = \lambda(w+z)-c_r$, and thus, due to linearity, there are two possible optimal solutions: $(q_u,q_r) = (Q,0)$ or $(q_u,q_r) = (K,Q-K)$. If $\frac{\partial \Pi_S^*}{\partial q_u} > \frac{\partial \Pi_S^*}{\partial q_r} \iff w+z < c_u + \frac{c_r-c_u}{\lambda}$, $(q_u,q_r)^* = (Q,0)$. Otherwise, $(q_u,q_r)^* = (K,Q-K)$. Lastly, we must ensure the supplier has non-negative expected profit. If $(q_u,q_r)^* = (Q,0)$, the optimal profit of S is:

$$\Pi_S^* = (1-\lambda)\Big[wQ - c_u Q\Big] + \lambda\Big[wK - c_u K - z(Q-K)\Big].$$

Setting this greater than or equal to 0 yields the participation condition $w \geq c_u + z\frac{(Q-K)\lambda}{(1-\lambda)Q+\lambda K}$. Finally, if $(q_u,q_r)^* = (K,Q-K)$, the optimal profit of S is:

$$\Pi_S^* = wQ - c_u K - c_r(Q-K).$$

Setting this greater than or equal to 0 yields the participation condition $w \geq \frac{c_u K + c_r(Q-K)}{Q}$.

□

Based on Lemma B.1, the buyer has two options:

- **The buyer can follow a PA strategy, inducing S to single source from U.** This can be achieved by setting $c_u \leq w+z$ and $w \geq c_u + z\frac{(Q-K)\lambda}{(1-\lambda)Q+\lambda K}$. It is straightforward to show that $z^* = 0$ and $w^* = c_u$ yields an optimal solution for the buyer (there can be multiple optima, in general, but all of them extract all supplier surplus and yield identical profit; recall that we assume the buyer chooses the lowest penalty value when there are multiple optima). In other words, if the buyer uses PA, wholesale price contracts and penalty contracts are equivalent, since a penalty of zero achieves the optimal outcome.

- **The buyer can follow an SM strategy, inducing S to dual source in Tier 2.** This can be achieved by setting $w \geq c_u + (c_r - c_u)/\lambda - z$ and $w \geq c_u K/Q + c_r(1 - K/Q)$. Since $Q = D$ will clearly be optimal for the buyer, the buyer's profit in this case is $\Pi_B^* = (r - w)D$. Hence, the minimum w that satisfies the two constraints is optimal for the buyer. One of two cases is thus possible:

  - The buyer could set a large enough z so that $c_u + (c_r - c_u)/\lambda - z = c_u K/D + c_r(1 - K/D)$. Any larger z would not affect the wholesale price (because the participation constraint becomes binding). Let

$$z^*_{\min} \equiv c_u + (c_r - c_u)/\lambda - c_u K/D - c_r(1 - K/D) \tag{B.2}$$
$$= (c_r - c_u)\left(\frac{1}{\lambda} - 1 + \frac{K}{D}\right)$$

    be the penalty that causes the two constraints to be equal. If $z^*_{\min} \leq \bar{z}$, this is the optimal penalty for the buyer, i.e., $z^* = z^*_{\min}$ and $w^* = c_u K/D + c_r(1 - K/D)$.
  - The buyer may be unable to set a large enough z to set the two constraints equal to one another, i.e., it could be the case that $z^*_{\min} > \bar{z}$. In that scenario, $c_u + (c_r - c_u)/\lambda - z > c_u K/D + c_r(1 - K/D)$ for all feasible z, and so to minimize the wholesale price the buyer will set the largest penalty possible, $z^* = \bar{z}$, and set a wholesale price $w^* = c_u + (c_r - c_u)/\lambda - \bar{z}$.

Note that regardless of whether $z^*_{\min} \leq \bar{z}$ or not, the ability to charge a penalty allows the buyer to achieve SM at a lower unit cost than with a wholesale price only contract. In fact, if $z^*_{\min} \leq \bar{z}$, the buyer can extract all surplus from the Tier 1 supplier (leaving it with zero expected profit), allowing the buyer to replicate the outcome of a vertically integrated supply chain, as the following lemma shows:

Lemma B.2. *Suppose $z^*_{\min} \leq \bar{z}$. Then the buyer's profit when inducing SM using a penalty contract with non-delivery penalty $z^* = z^*_{\min}$ and wholesale price $w^* = c_u K/D + c_r(1 - K/D)$ strictly exceeds the buyer's profit under vertical integration when using a MS strategy.*

Proof. This follows immediately, since both strategies fully eliminate disruption risk at the same average unit sourcing cost (note that $c_u K/D + c_r(1 - K/D)$ is precisely the average unit sourcing cost under vertical integration in strategy MS), but vertical integration incurs a strictly positive fixed cost $F_v > 0$.

□

In other words, penalty contracts dominate vertical integration when $z^*_{\min} \leq \bar{z}$. However, when $z^*_{\min} > \bar{z}$, the performance of penalty contracts degrades: they cannot extract all surplus from the Tier 1 supplier and so cannot perfectly replicate the sourcing outcome of a vertically integrated supply chain. Because of this, the buyer's profit under vertical integration can exceed its profit under an optimal penalty contract when $z^*_{\min} > \bar{z}$, as the following lemma shows:

Lemma B.3. *Suppose $z^*_{\min} > \bar{z}$. Then the buyer's profit when inducing SM using a penalty contract with non-delivery penalty $z^* = \bar{z}$ and wholesale price $w^* = c_u + (c_r - c_u)/\lambda - \bar{z}$ strictly exceeds the buyer's profit under vertical integration when using a MS strategy if and only if $F_v > D(z^*_{\min} - \bar{z})$.*

Proof. The buyer's profit with a penalty contract is higher if

$$(r - c_u - (c_r - c_u)/\lambda + \bar{z})D > (r - c_u K/D - c_r(1 - K/D))D - F_v$$

Rewriting this inequality and using the definition $z^*_{\min} \equiv c_u + (c_r - c_u)/\lambda - c_u K/D - c_r(1 - K/D)$, penalty contracts are preferred to vertical integration if

$$(r + \bar{z})D > (r + z^*_{\min})D - F_v$$

Simplifying further yields the outcome in the lemma.

□

The lemma shows that if the buyer is forced to charge a penalty less than $z^*_{\min}$, vertical integration can still be optimal, provided the fixed integration cost is not too high, i.e., $F_v \leq D(z^*_{\min} - \bar{z})$. Put differently, penalties increase the buyer's profit when using SM in a traditional supply chain. When $z^*_{\min} \leq \bar{z}$, they increase the buyer's profit so much that vertical integration is never optimal. When $z^*_{\min} > \bar{z}$, vertical integration can still be optimal, but the region of optimality for SM will be larger than without penalties.

Given this analysis, we may formally prove Theorem 5 from the main text:

Proof of Theorem 5. (i) This follows immediately from Lemma B.2.

(ii) Suppose $\bar{z} < z^*_{\min}$, which holds if and only if:

$$\lambda < \bar{\lambda}^1_p \equiv \min\left(1, \frac{(c_r - c_u)D}{(c_r - c_u)(D - K) + D\bar{z}}\right).$$

In this case, with a penalty contract, the buyer has two options. It can either set $(z, w) = \left(\bar{z}, c_u K/D + c_r(1 - K/D) - \bar{z}\right)$ and use SM or set $(z, w) = \left(0, c_u\right)$ and use PA. With these strategies, the buyer's profit expressions are (with p denoting a traditional supply chain with a penalty contract):

$$\Pi_p^B(PA) = (r - c_u)\big[(1-\lambda)D + \lambda K\big],$$
$$\Pi_p^B(SM) = (r - c_u - (c_r - c_u)/\lambda + \bar{z})D$$

First, it is clear that $\Pi_p^B(SM) \geq (r - c_u - (c_r - c_u)/\lambda)D = \Pi_t^B(SM)$ for any $\bar{z} \geq 0$, which proves the first claim in case (ii) of the theorem. Second, the buyer's profit with vertical integration when using a MS strategy (VI-MS for short) is $\Pi_v^B(MS) = rD - c_uK - c_r(D-K) - F_v$, and the buyer prefers VI-MS to PA if $\lambda > (c_r - c_u)/(r - c_u) \equiv \lambda_v^*$ and, as shown by Lemma B.3, prefers VI-MS to SM if:

$$(r+\bar{z})D > (r + z_{\min})D - F_v \iff \lambda < \min\left(1, \frac{(c_r - c_u)D}{F_v + (c_r - c_u)(D-K) + D\bar{z}}\right) \equiv \bar{\lambda}_p^2 \leq \bar{\lambda}_p^1.$$

Thus, if $\lambda_v^* < \lambda < \bar{\lambda}_p^2$, the buyer still uses VI even if it can charge a non-delivery penalty, which proves the second claim in case (ii) of the theorem.

□

C Analysis of Two Tier 1 Suppliers

This section contains the analysis and supporting results for our investigation of multisourcing in Tier 1 (§7 of the main text). In the analysis that follows we assume that the disrupted capacities of unreliable Tier 2 suppliers R_1 and R_2 are K_1 and K_2, respectively, where $K_1 \geq K_2 \geq 0$, to study the potential heterogeneity in Tier 2 resilience faced by S_1 and S_2. Note that this includes the main case discussed in §7, where $K_1 = K_2$, as a special case. We start with a traditional supply chain, and discuss the buyer's optimal sourcing strategy when it has access to two Tier 1 suppliers.

C.1 Disruption Mitigation without Vertical Control

Note that, in this case, the buyer's expected profit is given by:

$$\begin{aligned}\Pi_t^B = \max_{w_1,w_2,Q_1,Q_2\geq 0} & f_1(w_1,w_2)\big[r\min\{\min\{Q_1,K_1\}+\min\{Q_2,K_2\},D\} - w_1\min\{Q_1,K_1\} - w_2\min\{Q_2,K_2\}\big]+\\ & f_2(w_1,w_2)\big[r\min\{\min\{Q_1,K_1\}+Q_2,D\} - w_1\min\{Q_1,K_1\} - w_2Q_2\big]+\\ & f_3(w_1,w_2)\big[r\min\{Q_1+\min\{Q_2,K_2\},D\} - w_1Q_1 - w_2\min\{Q_2,K_2\}\big]+\\ & f_4(w_1,w_2)\big[r\min\{Q_1+Q_2,D\} - w_1Q_1 - w_2Q_2\big]. \end{aligned} \tag{C.1}$$

In (C.1), f_1 is the probability that both Tier 1 suppliers experience a disruption, f_2 is the probability that only S_1 experiences a disruption, f_3 is the probability that only S_2 experiences a disruption, and f_4 is the probability that neither supplier experiences a disruption. As before, the buyer can influence the values of these probabilities by the wholesales prices it offers to the Tier 1 suppliers. Based on Equation (C.1), we now examine the optimal profit under each strategy or a combination of strategies described in Table (4).

As discussed in the main text, MS1 is always part of the buyer's optimal sourcing strategy.

Further, as the reliable suppliers are uncapacitated, it is also suboptimal for the buyer to induce one of the Tier 1 suppliers to source reliably, which eliminates the disruption risk faced by that Tier 1 supplier, and order more than a total of D units. Hence the buyer never combines IM and SM, and the only three possible combinations of strategies the buyer can use are (i) MS1, (ii) MS1+IM, and (iii) MS1+SM.

Lemma C.1. *Let $\{(w_1, Q_1), (w_2, Q_2)\}$ be the set of contracts offered by the buyer to S_1 and S_2. In a traditional supply chain, we have the following:*
(i) If disruptions are mild (i.e., $K_1 + K_2 \geq D$), with MS1, a continuum of optimal contracts exist. In this case, the optimal contracts are $\Big\{\big(c_u, \theta D\big), \big(c_u, (1-\theta)D\big)\Big\}$ for any $\theta \in [(D-K_2)/D, K_1/D]$, and the buyer's optimal expected profit is $\Pi_t^B(MS1) = \big(r - c_u\big)D$, which is the highest possible profit the buyer can achieve in this setting.
(ii) If disruptions are severe (i.e., $K_1 + K_2 < D$), with MS1, a continuum of optimal contracts exist. In this case, the optimal contracts are $\Big\{\big(c_u, \theta D\big), \big(c_u, (1-\theta)D\big)\Big\}$ for any $\theta \in [K_1/D, (D-K_2)/D]$, and the buyer's optimal expected profit is $\Pi_t^B(MS1) = (r - c_u)[(1-\lambda)D + \lambda(K_1 + K_2)]$.
(iii) With MS1+IM, the optimal contract is $\Big\{\big(c_u, D - K_2\big), \big(c_u, D - K_1\big)\Big\}$, and the buyer's optimal expected profit is $\Pi_t^B(MS1 + IM) = (1-\lambda)(1-\lambda+\alpha\lambda)(rD - c_u(2D - K_1 - K_2)) + 2(1-\alpha)(1-\lambda)\lambda(r - c_u)D + \lambda(\lambda - \alpha\lambda + \alpha)(r - c_u)(K_1 + K_2)$. In this case, for any $\lambda < 1$, $\Pi_t^B(MS1 + IM)$ is strictly decreasing in α.
(iv) With MS1+SM, the optimal contract is $\Big\{\big(c_u, K_1\big), \big(c_u + \frac{c_r - c_u}{\lambda}, D - K_1\big)\Big\}$, and the buyer's optimal expected profit is $\Pi_t^B(MS1 + SM) = (r - c_u)D - \frac{c_r - c_u}{\lambda}(D - K_1)$.

Proof of Lemma C.1. (i) Consider the buyer's profit maximization problem given by Equation (C.1) when the optimal wholesale prices are $w_1 = w_2 = c_u$. In this case, $f_1(c_u, c_u) = \lambda(\lambda - \alpha\lambda + \alpha)$, $f_2(c_u, c_u) = f_3(c_u, c_u) = (1-\alpha)(1-\lambda)\lambda$, $f_4(c_u, c_u) = (1-\lambda)(1-\lambda+\alpha\lambda)$. From Equation (C.1), it must be true that, at optimality, $Q_1 + Q_2 \geq D$. This implies that, if $K_1 + K_2 \geq D$, the buyer can eliminate all risk by allocating its order quantities such that $Q_i \leq K_i, i = 1, 2$ and $Q_1 + Q_2 \geq D$ are both satisfied simultaneously. This implies that the buyer is guaranteed to get a total D units at a unit cost of c_u, leading to the optimal profit $\Pi_t^B(MS1) = \big(r - c_u\big)D$.

(ii) On the other hand, if $K_1 + K_2 < D$, it must be true at optimality that $Q_i + K_{-i} \leq D, i = 1, 2$ and $Q_i \geq K_i, i = 1, 2$, since, in both cases, $K_i, i = 1, 2$ can be obtained with certainty from each supplier. Π_t^B thus simplifies to:

$$\begin{aligned}\Pi_t^B = \max_{Q_1,Q_2\geq 0} &\lambda(\lambda-\alpha\lambda+\alpha)(r-c_u)(K_1+K_2)+\\ &(1-\alpha)(1-\lambda)\lambda(r-c_u)\big(K_1+Q_2)+\\ &(1-\alpha)(1-\lambda)\lambda(r-c_u)(K_2+Q_1)+\\ &(1-\lambda)(1-\lambda+\alpha\lambda)(rD-c_uQ_1-c_uQ_2).\end{aligned}$$

Note $\frac{\partial \Pi_t^B}{\partial Q_1} = \frac{\partial \Pi_t^B}{\partial Q_2} = (1-\lambda)[(1-\alpha)\lambda r - c_u]$. If $\frac{\partial \Pi_t^B}{\partial Q_i} \leq 0, i=1,2 \iff \lambda \leq \min\left(1, \frac{c_u}{(1-\alpha)r}\right)$, then the buyer's profit is maximized by sourcing as little as possible from both suppliers, i.e., by setting $(Q_1,Q_2)^* = (K_1, D-K_1)$, or $(Q_1,Q_2)^* = (D-K_2, K_2)$, or any linear combination of the two (due to $w_1 = w_2 = c_u$), which leads to the MS1 strategy.

(iii) Otherwise, i.e., if $\frac{\partial \Pi_t^B}{\partial Q_i} > 0, i=1,2 \iff \lambda > \min\left(1, \frac{c_u}{(1-\alpha)r}\right)$, the buyer wants to source as much as possible from both suppliers while satisfying all constraints, which implies $(Q_1,Q_2)^* = (D-K_2, D-K_1)$, leading to the MS1+IM strategy. Note, in addition, that:

$$\frac{\partial \Pi_t^B}{\partial \alpha} = -(D-K_1-K_2)(1-\lambda)\lambda r < 0 \text{ for any } \lambda < 1.$$

(iv) For MS1+SM, as the reliable suppliers are uncapacitated, it is straightforward to see that the buyer never induces both Tier 1 suppliers to source reliably by paying both of them a high wholesale price. Thus, to employ MS1+SM, the buyer decides which supplier to mitigate and then selects two quantities: how much to source from the supplier that is not induced to use reliable sourcing and how much to source from the supplier that is induced to use reliable sourcing. First, suppose the buyer induces S_1 to source reliably. Then, in this case, $f_1(w_1,w_2) = 0$, $f_2(w_1,w_2) = 0$, $f_3(w_1,w_2) = \lambda$, and $f_4(w_1,w_2) = (1-\lambda)$. As before, we make the following observations to simplify the expression above: at optimality, $Q_1 + Q_2 \geq D$, $Q_1 + K_2 \leq D$, and $Q_2 \geq K_2$. Then, Π_t^B simplifies as follows:

$$\begin{aligned}\Pi_t^B = \max_{Q_1,Q_2\geq 0} &\lambda\Big[r(Q_1+K_2) - (c_u + (c_r-c_u)/\lambda)Q_1 - c_uK_2\Big]+\\ &(1-\lambda)\Big[rD - (c_u + (c_r-c_u)/\lambda)Q_1 - c_uQ_2\Big].\end{aligned}$$

By linearity, there are two possible solutions: $(Q_1,Q_2) = (0,D)$ or $(Q_1,Q_2) = (D-K_2, K_2)$. Evaluating the profit under each solution shows that the optimal sourcing quantities under MS1+SM are $(Q_1,Q_2)^* = (D-K_2, K_2)$, and $\Pi_t^B = (r-c_u)D - \frac{c_r-c_u}{\lambda}(D-K_2)$ if the buyer chooses to induce S_1 to source reliably. Similarly, one can see that if the buyer chooses to induce S_2 to source reliably, the optimal sourcing quantities are $(Q_1,Q_2)^* = (K_1, D-K_1)$, and $\Pi_t^B = (r-c_u)D - \frac{c_r-c_u}{\lambda}(D-K_1)$. Since $K_1 \geq K_2$, the latter is larger, implying that for MS1+SM, $(Q_1,Q_2)^* = (K_1, D-K_1)$, and $\Pi_t^B = (r-c_u)D - \frac{c_r-c_u}{\lambda}(D-K_1)$. This implies that the buyer finds it optimal to mitigate the Tier 1 supplier whose unreliable supplier is less resilient to disruptions.

□

Next, we present the buyer's optimal sourcing strategy in a traditional supply chain in the case of severe disruptions:

Theorem C.1. *Suppose disruptions are severe (i.e., $K_1+K_2 < D$). Then, there exist two thresholds for λ, $0 \le \bar{\lambda}_t^1 \le \bar{\lambda}_t^2 \le 1$, such that the buyer prefers $MS1$ if $\lambda \le \bar{\lambda}_t^1$, $MS1+IM$ if $\bar{\lambda}_t^1 < \lambda \le \bar{\lambda}_t^2$, and $MS1+SM$ if $\lambda > \bar{\lambda}_t^2$.*

Proof. Consider the buyer's expected profit under each strategy:

$$\begin{aligned}
\Pi_t^B(MS1) =& (r-c_u)[(1-\lambda)D + \lambda(K_1+K_2)],\\
\Pi_t^B(MS1+IM) =& (1-\lambda)(1-\lambda+\alpha\lambda)(rD - c_u(2D-K_1-K_2))+\\
& 2(1-\alpha)(1-\lambda)\lambda(r-c_u)D + \lambda(\lambda-\alpha\lambda+\alpha)(r-c_u)(K_1+K_2),\\
\Pi_t^B(MS1+SM) =& (r-c_u)D - \frac{c_r-c_u}{\lambda}(D-K_1).
\end{aligned}$$

Clearly, $\Pi_t^B(MS1)$ is strictly decreasing in λ, and $\Pi_t^B(MS1+SM)$ is strictly increasing in λ. In the proof of Lemma C.1, we have shown that the buyer prefers MS1 to MS1+IM if $\lambda \le \min\Big(1, \frac{c_u}{(1-\alpha)r}\Big)$.

First, suppose $\alpha < 1 - c_u/r \implies \frac{c_u}{(1-\alpha)r} < 1$, which implies that the buyer prefers MS1 to MS1+IM if $\lambda < \frac{c_u}{(1-\alpha)r} < 1$. Consider:

$$\frac{\partial(MS1+IM)}{\partial\lambda} = (D-K_1-K_2)(2c_u - \alpha r - 2(1-\alpha)\lambda r),$$

which is negative if $\lambda > \frac{2c_u-\alpha r}{2r(1-\alpha)}$. It can be readily checked that $\frac{2c_u-\alpha r}{2r(1-\alpha)} < \frac{c_u}{(1-\alpha)r}$ for any $\alpha < 1$. This implies that $\Pi_t^B(MS1+IM)$ is decreasing in λ for $1 \ge \lambda > \frac{c_u}{(1-\alpha)r}$, which is the region where $\Pi_t^B(MS1+IM) > \Pi_t^B(MS1)$. Therefore, we conclude that $\Pi_t^1 \equiv \max\Big(\Pi_t^B(MS1), \Pi_t^B(MS1+IM)\Big)$ is strictly decreasing in λ if $\alpha < 1 - c_u/r$. Next, suppose, $\alpha \ge 1 - c_u/r$ in which case the buyer always prefers MS1 to MS1+IM, and $\Pi_t^1 \equiv \max\Big(\Pi_t^B(MS1), \Pi_t^B(MS1+IM)\Big) = \Pi_t^B(MS1)$, which again is strictly decreasing in λ.

Thus, the buyer's expected optimal expected profit between MS1 and MS1+IM (Π_t^1) is decreasing in λ, and as λ increases, we have three possibilities depending on whether and where Π_t^1 intersects with the buyer's expected profit under MS1+SM, $\Pi_t^B(MS1+SM)$:

(i) $\Pi_t^B(MS1+SM)$ intersects Π_t^1 from below at some $\lambda < \min\Big(1, \frac{c_u}{(1-\alpha)r}\Big)$. In this case, the buyer's optimal strategy transitions from MS1 to MS1+SM. Setting $\Pi_t^B(MS1) = \Pi_t^B(MS1+SM)$ and solving for λ yields that the buyer prefers MS1 to MS1+SM if $\lambda \le \min\Big(1, \sqrt{\frac{(c_r-c_u)(D-K_1)}{(D-K_1-K_2)(r-c_u)}}\Big)$.

(ii) $\Pi_t^B(MS1+SM)$ intersects Π_t^1 from below at some λ, $\bar{\lambda}_{is}$, such that $\min\Big(1, \frac{c_u}{(1-\alpha)r}\Big) \le \bar{\lambda}_{is} < 1$. Note that this case is only possible if $\min\Big(1, \frac{c_u}{(1-\alpha)r}\Big) < 1 \iff \alpha < 1 - c_u/r$. In this case, the buyer's optimal strategy transitions from MS1 to MS1+IM to MS1+SM, and the buyer prefers MS1 if $\lambda \leq \min\left(1, \frac{c_u}{(1-\alpha)r}\right)$, MS1+SM if $\lambda > \bar{\lambda}_{is}$ and MS1+IM otherwise.

(iii)$\Pi_t^B(MS1 + SM)$ does not intersect Π_t^1 from below at any feasible λ. In this case, the buyer's optimal strategy can only transition from MS1 to MS1+IM, and the buyer prefers MS1 if $\lambda \leq \min\left(1, \frac{c_u}{(1-\alpha)r}\right)$, MS1+SM if $\lambda > 1$ and MS1+IM otherwise.

Combining the three cases, the buyer's optimal strategy is MS1 if $\lambda \leq \min\left(1, \frac{c_u}{(1-\alpha)r}, \sqrt{\frac{(c_r-c_u)(D-K_1)}{(D-K_1-K_2)(r-c_u)}}\right) \equiv \bar{\lambda}_t^1$, MS1+SM if $\lambda > \min\left(1, \max\left(\sqrt{\frac{(c_r-c_u)(D-K_1)}{(D-K_1-K_2)(r-c_u)}}, \bar{\lambda}_{is}\right)\right) \equiv \bar{\lambda}_t^2$, and MS1+IM otherwise. As $\bar{\lambda}_{is} \geq \min\left(1, \frac{c_u}{(1-\alpha)r}\right)$ by definition, $\bar{\lambda}_t^2 \geq \bar{\lambda}_t^1$. Also, clearly, $\bar{\lambda}_t^2 \leq 1$, and $0 \leq \bar{\lambda}_t^1$. Note the buyer's profit is decreasing in λ for $\lambda \leq \bar{\lambda}_t^2$ (when MS1 or MS1+IM are optimal) and increasing in λ otherwise (when MS1+SM is optimal).

□

C.2 Disruption Mitigation with Vertical Integration

With vertical integration, as before, the buyer gains direct access to the Tier 2 suppliers of the Tier 1 supplier it vertically integrates with. In this case, as in a traditional supply chain, MS1 (which, in the case of vertical integration, is interpreted to mean sourcing through the vertically integrated Tier 1 supplier as well as the non-vertically integrated Tier 1 supplier) is always part of the buyer's optimal sourcing strategy since the buyer can source (either directly or through a Tier 1 supplier) risk-free quantities of K_1 and K_2 units from U_1 and U_2, respectively, at a unit cost of c_u. Further, as the reliable suppliers are uncapacitated, it is also suboptimal for the buyer to vertically integrated with two Tier 1 suppliers and incur the fixed cost of vertical integration twice. However, the MS1 strategy implies that the buyer still does business with the Tier 1 supplier it does not vertically integrate with.

Similarly, it is suboptimal for the buyer to mitigate the other Tier 1 supplier by paying it a high wholesale price, as it can always source from the reliable supplier it has direct access to at a cost of c_r. Further, due to the same reason, the buyer never sources from the reliable supplier it has access to and orders more than a combined total of D units from its supplier. Finally, it is straightforward to see that if the buyer uses a sourcing configuration that does not source from the reliable supplier it has direct access to, it can achieve the exact same configuration in a traditional supply chain (i.e., without vertical integration) without incurring the fixed cost of vertical integration, F_v. Thus, for vertical integration to have the potential to increase the buyer's profit, it must be true that the buyer finds it optimal to source from the reliable supplier it can source from at cost.

Taken in sum, in addition to the traditional strategies, vertical integration adds one additional strategy to the buyer's potential list of strategies, which we denote by MS1+RS to reflect the combination of MS1 and reliable sourcing in Tier 2 from the vertically integrated Tier 1 supplier. To employ this strategy, the buyer has to determine which Tier 1 supplier it vertically integrates with (call it S_i); q_u^i, how much to source from U_i; q_u^{-i}, how much to source from U_{-i} through S_{-i}, and q_r, how much to source from R_i.

Lemma C.2. *In a vertically integrated supply chain, with MS1+RS, the buyer's optimal strategy is* $(q_u^1, q_u^2, q_r)^* = (K_1, K_2, D - K_1 - K_2)$, *and the buyer's optimal expected profit is* $\Pi_v^B(MS1 + RS) = rD - c_u(K_1 + K_2) - c_r(D - K_1 - K_2) - F_v$.

Proof. For MS1+RS, there are two possibilities depending on which Tier 1 supplier the buyer vertically integrates it. First, consider the buyer's profit maximization problem if the buyer vertically integrates with S_1. Now, the buyer chooses q_u^1, how much to source directly from U_1; q_u^2, how much to source from U_2 through S_2; and q_r, how much to source directly from R_1:

$$
\begin{aligned}
\Pi_v^B = \max_{q_u^1, q_u^2, q_r \geq 0} & \lambda(\lambda - \alpha\lambda + \alpha)\Big[r\big(\min\{\min\{q_u^1, K_1\} + \min\{q_u^2, K_2\} + q_r^1, D\}\big) - c_u \min\{q_u^1, K_1\} - c_u \min\{q_u^2, K_2\} - c_r q_r\Big] + \\
& (1-\alpha)(1-\lambda)\lambda\Big[r\big(\min\{\min\{q_u^1, K_1\} + q_u^2 + q_r, D\}\big) - c_u \min\{q_u^1, K_1\} - c_u q_u^2 - c_r q_r\Big] + \\
& (1-\alpha)(1-\lambda)\lambda\Big[r\big(\min\{q_u^1 + \min\{q_u^2, K_2\} + q_r, D\}\big) - c_u q_u^1 - c_u \min\{q_u^2, K_2\} - c_r q_r\Big] + \\
& (1-\lambda)(1-\lambda+\alpha\lambda)\Big[r\big(\min\{q_u^1 + q_u^2 + q_r, D\}\big) - c_u q_u^1 - c_u q_u^2 - c_r q_r\Big] - F_v.
\end{aligned}
$$

Again, we make several simplifying observations: (a) $q_u^1 + q_u^2 + q_r \geq D$, (b) $q_u^1 \geq K_1$, (c) $q_u^2 \geq K_2$, (d) $q_r + K_1 + K_2 \leq D$, (e) $q_r + q_u^1 \leq D - K_2$, and (f) $q_r + q_u^2 \leq D - K_1$, under which Π_v^B simplifies to:

$$
\begin{aligned}
\Pi_v^B = \max_{q_u^1, q_u^2, q_r \geq 0} & \lambda(\lambda - \alpha\lambda + \alpha)\Big[r\big(q_r + K_1 + K_2\big) - c_u(K_1 + K_2) - c_r q_r\Big] + \\
& (1-\alpha)(1-\lambda)\lambda\Big[r(q_u^1 + q_r + K_2) - c_u(q_u^1 + K) - c_r q_r\Big] + \\
& (1-\alpha)(1-\lambda)\lambda\Big[r(q_u^2 + q_r + K_1) - c_u(q_u^2 + K) - c_r q_r\Big] + \\
& (1-\lambda)(1-\lambda+\alpha\lambda)\Big[rD - c_u(q_u^1 + q_u^2) - c_r q_r\Big] - F_v.
\end{aligned}
$$

Due to linearity, there are two possibilities: $q_r^* = 0$ or $q_r^* = D - K_1 - K_2$. Clearly, the former cannot be a MS1+RS strategy, and thus, the optimal sourcing strategy under MS1+RS when the buyer vertically integrates with S_1 is $(q_u^1, q_u^2, q_r^1)^* = (K_1, K_2, D - K_1 - K_2)$, and $\Pi_v^B(MS1 + RS) = rD - c_u(K_1 + K_2) - c_r(D - K_1 - K_2) - F_v$. Note that the buyer's profit maximization problem is exactly the same if the buyer vertically integrates with S_2. Thus, the buyer's optimal sourcing strategy and resulting expecting profit under MS1+RS are exactly the same, and therefore, the buyer is indifferent between vertically integrating with S_1 and with S_2.

□

C.3 Disruption Mitigation with Direct Sourcing

With direct sourcing, the buyer can now act as a reliable supplier for both of its Tier 1 suppliers. In this case, the buyer offers S_i a contract of the form (w_i, Q_i, p_i, m_i) with all the parameters are analogous to their counterparts discussed in Section 5 of the main text. Further, the optimal sourcing strategy of the Tier 1 suppliers are exactly the same as that presented by Lemma 4.

As before, the buyer loses money on each unit of raw materials it sells to Tier 1, since the required selling price $p_i \leq c_r$ while the cost is $c_d > c_r$. This implies that, even in the presence of two Tier 1 suppliers, the buyer will never find it optimal to sell raw materials to Tier 1 while also inducing MS1+SM, since both are risk-free sources of supply but direct sourcing results in a loss on each unit. However, the buyer may find direct sourcing optimal when not inducing MS1+SM, since this means the buyer can pay Tier 1 a significantly lower wholesale price. In addition, because raw materials sold to Tier 1 using direct sourcing are risk-free and the buyer can freely set all the contract parameters, the buyer has no additional benefit from selling to more than one Tier 1 supplier at a time. The combination of these facts means that, when evaluating the buyer's optimal sourcing strategy with direct sourcing, it suffices to restrict attention to the strategies where the buyer sets $m_i > 0$ and $m_{-i} = 0$ for $i = 1, 2$. In this case, for a strategy to be a direct sourcing strategy, it must be that $Q_i > 0$. Also, since S_{-i} can provide K_{-i} risk-free units at a cost of $c_u < c_d$, the buyer finds it optimal to set $Q_{-i} \geq K_{-i}$. Therefore, MS1 is always part of the buyer's optimal direct sourcing strategy. Finally, since the buyer never combines direct sourcing with MS1+SM, $w^*_{-i} = c_u$.

We discuss the buyer's optimal direct sourcing strategy the buyer can use if it offers the direct sourcing option only to $S_i, i = 1, 2$ (i.e., $m_i > 0$ and $m_{-i} = 0$) with the following lemma:

Lemma C.3. *(i) With direct sourcing offered only to S_1, the optimal contracts are* $\Big\{\big(c_u, D - K_2, c_u, D - K_1 - K_2\big), \big(c_u, K_2, t, 0\big)\Big\}$ *for any $t \geq 0$, and the buyer's optimal expected profit is* $\Pi^B_{d,1}(MS1 + DS) = rD - c_d(D - K_1 - K_2) - c_u(K_1 + K_2)$.

(ii) With direct sourcing offered only to S_2, the optimal contracts are $\Big\{\big(c_u, K_1, t, 0\big), \big(c_u, D - K_1, c_u, D - K_1 - K_2\big)\Big\}$ *for any $t \geq 0$, and the buyer's optimal expected profit is* $\Pi^B_{d,2}(MS1 + DS) = rD - c_d(D - K_1 - K_2) - c_u(K_1 + K_2)$.

Proof. (i) With direct sourcing offered only to S_1, per our discussion in the main text, we have $Q_2 \geq K_2$, $w_2 = c_u$, and $m_2 = 0$. Since it is clear that $K_2 \leq Q_2 \leq D$ (and the buyer's profit is linear in Q_2 since none of the cost or revenue parameters depend on Q_2) and we are seeking a solution with $Q_1 > 0$, we must have that $Q^*_2 = K_2$ and $Q^*_1 \geq D - K_2$. Therefore, we have already determined the optimal contract that the buyer should offer S_2: $\big(w_2, Q_2, p_2, m_2\big)^* = \big(c_u, K_2, t, 0\big)$ for any $t \geq 0$, since p_2 can take on any positive value due to $m_2 = 0$.

Thus, since $K_1 < D - K_2$, the buyer's profit maximization problem is simplified to its profit maximization problem (presented in A.1) under direct with Tier 1 supplier (S_1 in this case) and some $D \geq Q^*_1 \geq D - K_2$. Then, by going through very similar arguments as in the proof presented in A.1, we conclude that $(w_1, Q_1, p_1, m_1) = (c_u, Q_1, c_u, Q_1 - K_2)$. Since the buyer is guaranteed to get a total of $Q_1^* + K_2 \geq D$ units with these contracts, this leads to the expected profit for $\Pi_{d,1}^B = rD - c_d(Q_1^* - K_2) - c_u(K_1 + K_2)$, and as this expression is clearly decreasing in Q_1^*, we must have $Q_1^* = D - K_2$. Hence, $(w_1, Q_1, p_1, m_1)^* = (c_u, D - K_2, c_u, D - K_1 - K_2)$, leading to the optimal expected profit $\Pi_{d,1}^B(MS1 + DS) = rD - c_d(Q_1^* - K_2) - c_u(K_1 + K_2)$.

(ii) With direct sourcing offered only to S_2, per our discussion in the main text, we have $Q_1 \geq K_1$, $w_1 = c_u$, and $m_1 = 0$. Since it is clear that $K_1 \leq Q_1 \leq D$ (and the buyer's profit is linear in Q_1 since none of the cost or revenue parameters depend on Q_1) and we are seeking a solution with $Q_2 > 0$, we must have that $Q_1^* = K_1$ and $Q_2^* \geq D - K_1$. By going through very similar arguments as in (i), we can conclude that $\big(w_1, Q_1, p_1, m_1\big)^* = \big(c_u, K_1, t, 0\big)$ and $\big(w_2, Q_2, p_2, m_2\big)^* = \big(c_u, D - K_1, c_u, D - K_1 - K_2\big)$, for any $t \geq 0$, and the optimal expected profit is $\Pi_{d,2}^B(MS1 + DS) = rD - c_d(D - K_1 - K_2) - c_u(K_1 + K_2)$.

□

C.4 Proofs of Theorems 6 and 7

Given the analysis in the preceding subsections, we may now prove Theorems 6 and 7 from the main text:

Proof of Theorem 6. This result immediately follows from Lemma C.1(i).

□

Proof of Theorem 7. With two Tier 1 suppliers, suppose $K_1 + K_2 < D$. Then:

(i) Theorem C.1 shows that there is a threshold for λ, $0 \leq \bar{\lambda}_t^2 \leq 1$, such that the buyer's optimal expected profit in a traditional supply chain (Π_t^B) is decreasing in λ for $\lambda \leq \bar{\lambda}_t^2$ and increasing in λ otherwise. This further implies that Π_t^B attains its minimum precisely at $\lambda = \bar{\lambda}_t^2$, and we denote this by $\Pi_t^{\min}$. Finally, note that as $\lambda \to 0$, $\Pi_t^B = (r - c_u)D$, and $\lambda \to 1$, $\Pi_t^B = \max\big((r - c_u)(K_1 + K_2), (r - c_r)D\big)$. It is clear that $\Pi_t^B(\lambda \to 0) > \Pi_t^B(\lambda \to 1)$.

Note that the buyer's expected profit in a vertically integrated supply chain when using MS1+RS is constant in λ. Further note that as $\lambda \to 0$, $\Pi_t^B > \Pi_v^B(MS + RS)$. Thus, there are three possibilities:

(a) $\Pi_t^B \geq \Pi_v^B(MS1 + RS)$ for all $\lambda \iff \Pi_t^{\min} \geq \Pi_v^B(MS1 + RS)$. This implies that the buyer does not prefer vertical integration for $0 \leq \lambda \leq 1$.

(b) Π_t^B intersects $\Pi_v^B(MS1 + RS)$ from above once at some $\lambda \leq \bar{\lambda}_t^2 \leq 1$ and stays below $\Pi_v^B(MS1 + RS)$ thereafter. This implies that there exists a threshold for λ, $0 \leq \bar{\lambda}_v^1(\alpha) \leq \bar{\lambda}_t^2 \leq 1$, such that buyer prefers vertical integration for $\bar{\lambda}_v^1(\alpha) < \lambda \leq 1$ and prefers a traditional supply chain otherwise. As Π_t^B is weakly decreasing in α and $\Pi_v^B(MS1 + RS)$ is not a function of α, $\bar{\lambda}_v^1(\alpha)$ is weakly decreasing in α as well.

(c) Π_t^B intersects $\Pi_v^B(MS1 + RS)$ from above once at some $\lambda \leq \bar{\lambda}_t^2 \leq 1$ and then intersects $\Pi_v^B(MS1 + RS)$ from below once at some $\bar{\lambda}_t^2 < \lambda \leq 1$. This implies that there exists two threshold for λ, $0 \leq \bar{\lambda}_v^1(\alpha) \leq \bar{\lambda}_t^2 \leq \bar{\lambda}_v^2 \leq 1$, such that buyer prefers vertical integration if $\bar{\lambda}_v^1(\alpha) < \lambda < \bar{\lambda}_v^2$. As $\bar{\lambda}_v^2$ is the λ value such that $\Pi_v^B(MS1 + RS) = \Pi_t^B = \Pi_t^B(MS1 + SM)$ and neither of these profit functions depend on α, $\bar{\lambda}_v^2$ is not a function of α.

Combining everything, we conclude that the buyer prefers vertical integration if $\bar{\lambda}_{vt}^1 \equiv \min\Big(\bar{\lambda}_v^1(\alpha), 1\Big) < \lambda < \bar{\lambda}_{vt}^2 \equiv \min\Big(\bar{\lambda}_v^2, 1\Big)$. Finally, note that $\bar{\lambda}_{vt}^1$ is weakly decreasing in α and $\bar{\lambda}_{vt}^2$ does not depend on α.

(ii) Note that the buyer's expected profit with direct sourcing is constant λ. Further note that as $\lambda \to 0$, $\Pi_t^B > \Pi_d^B(MS1 + DS)$. By going through very similar arguments as in (i), one can show that there exist two thresholds for λ, $\bar{\lambda}_{dt}^1$ and $\bar{\lambda}_{dt}^1$, such that the buyer prefers direct sourcing if $\bar{\lambda}_{dt}^1 < \lambda < \bar{\lambda}_{dt}^2$ and that $\bar{\lambda}_{dt}^1$ is weakly decreasing in α and $\bar{\lambda}_{dt}^2$ does not depend on α.

(iii) Let Π_t^B be the buyer's optimal profit in a traditional supply chain (with no vertical control), $\Pi_v^B(MS1 + RS)$ be its expected profit under the MS1+RS in a vertically integrated supply chain, and $\Pi_d^B(MS1+DS)$ be its expected profit under the MS1+DS strategy with direct sourcing. Then, we have:

$$\begin{aligned}\Pi_v^B(MS1 + RS) =& rD - c_u(K_1 + K_2) - c_r(D - K_1 - K_2) - F_v \\ \Pi_d^B(MS1 + DS) =& rD - c_u(K_1 + K_2) - c_d(D - K_1 - K_2)\end{aligned}$$

The buyer's optimal profit with both forms of vertical control allowed can be written as $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS1 + RS), \Pi_d^B(MS1 + DS)\big)$. To show sufficiency, suppose $F_v \leq \widehat{F}_{vd} \equiv (c_d - c_r)(D - 2K)$. This immediately implies $\Pi_v^B(MS1 + RS) \geq \Pi_d^B(MS1 + DS)$, which means $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS1 + RS), \Pi_d^B(MS1 + DS)\big) = \max\big(\Pi_t^B, \Pi_v^B(MS1 + RS)\big)$. This in turn implies that the buyer prefers vertical integration to direct sourcing if $F_v \leq \bar{F}_{vd}$.

We prove necessity by contraposition. Suppose $F_v > \widehat{F}_{vd}$, which immediately implies $\Pi_v^B(MS1+ RS) < \Pi_d^B(MS1 + DS)$ and therefore, $\Pi^B = \max\big(\Pi_t^B, \Pi_v^B(MS1 + RS), \Pi_d^B(MS1 + DS)\big) = \max\big(\Pi_t^B, \Pi_d^B(MS1 + DS)\big)$. This in turn implies that the buyer prefers direct sourcing to vertical integration if $F_v > \widehat{F}_{vd}$. Taken together, the buyer prefers vertical integration to direct sourcing if and only if $F_v \leq \widehat{F}_{vd}$.

□