ECE/CS 250
Computer Architecture

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Basics of Logic Design: Finite State Machines

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Slides are derived from work by
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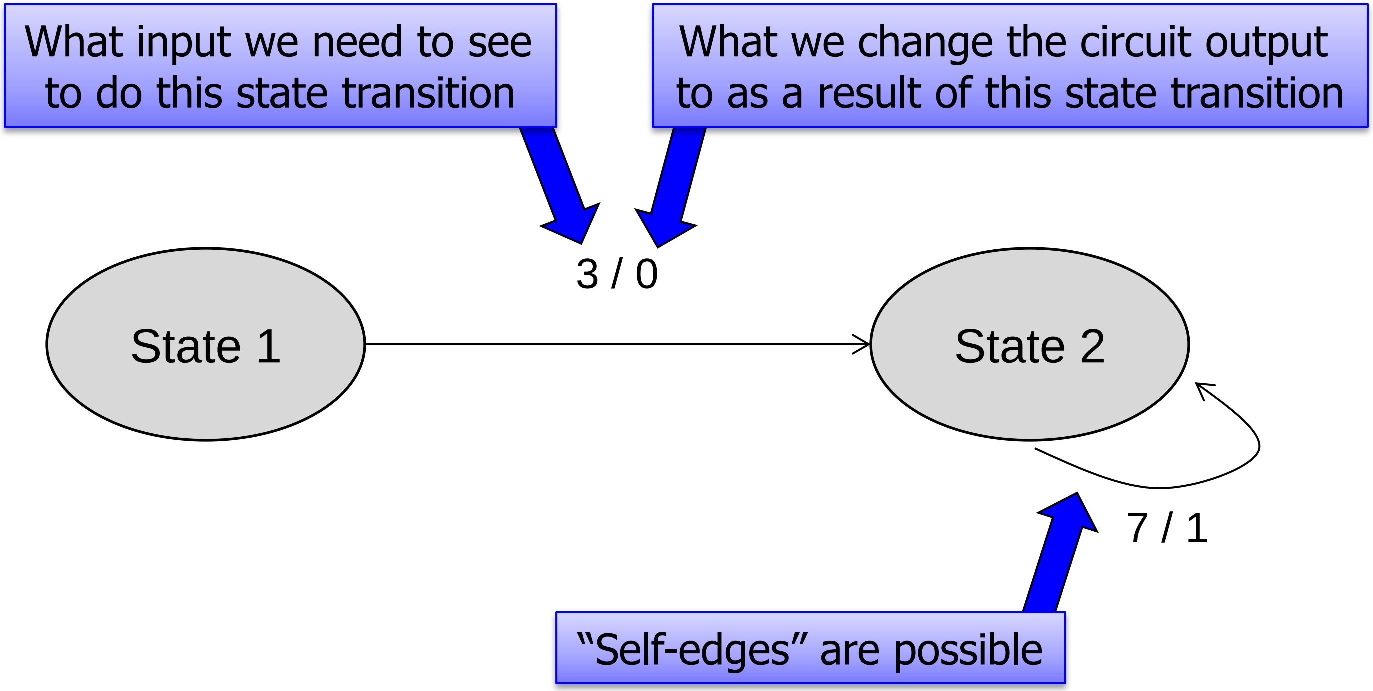
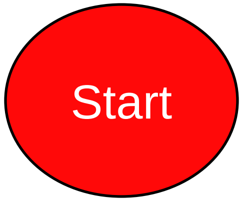
Finite State Machine (FSM)

- FSM = States + Transitions
  - Next state = function (current state, inputs)
  - Outputs = function (current state, inputs)
- What you do depends on what state you're in
  - Think of a calculator … if you type "+3=", the result depends on what you did before, i.e., the state of the calculator
- Canonical Example: Combination Lock
  - Must enter 3 8 4 to unlock

How FSMs are represented

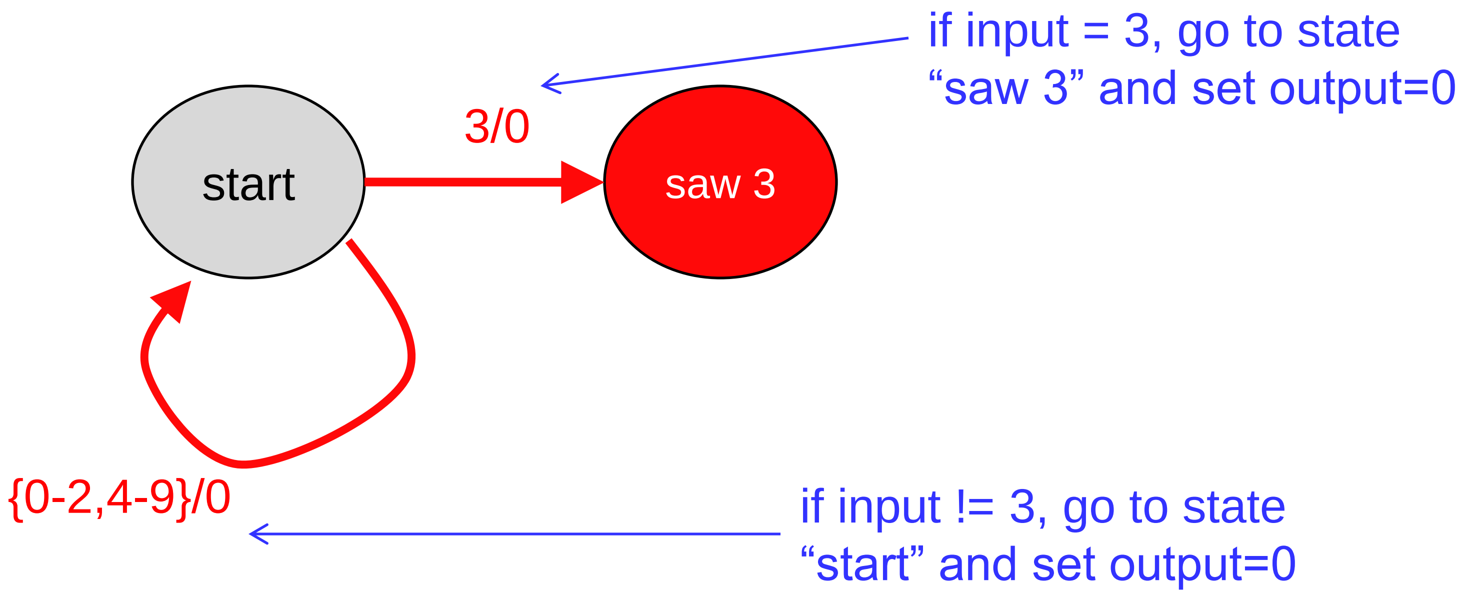


Finite State Machines: Example



- Combination Lock Example:
  - Need to enter 3 8 4 to unlock
- Initial State called “start”: no valid piece of combo seen
  - All FSMs get reset to their start state

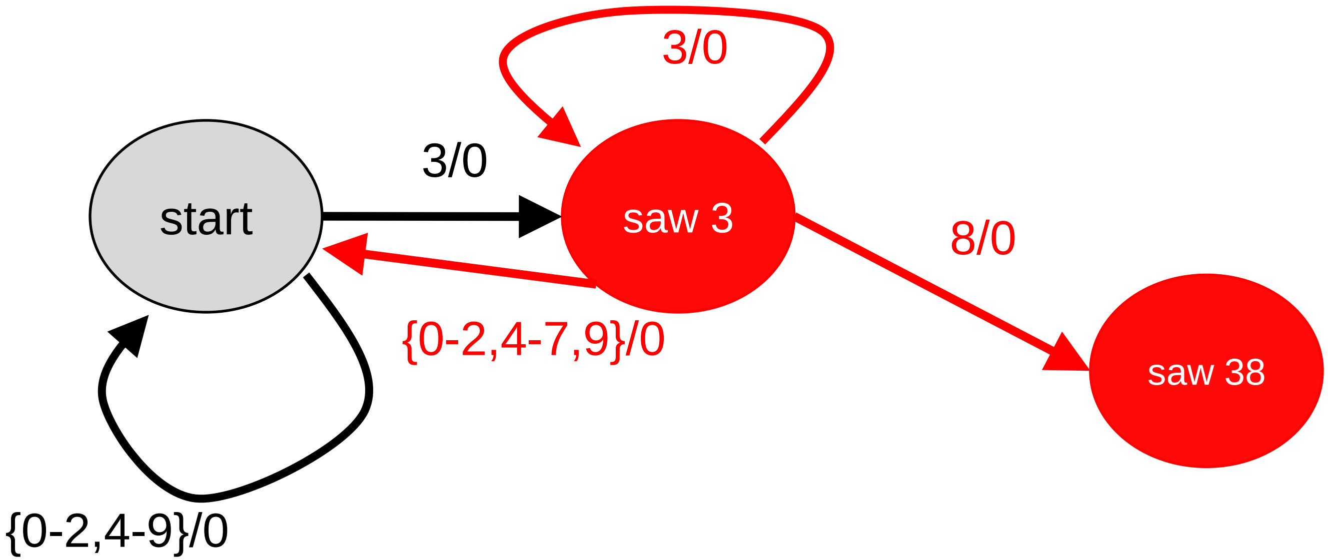
Finite State Machines: Example



- Combination Lock Example:
  - Need to enter **3** 8 4 to unlock

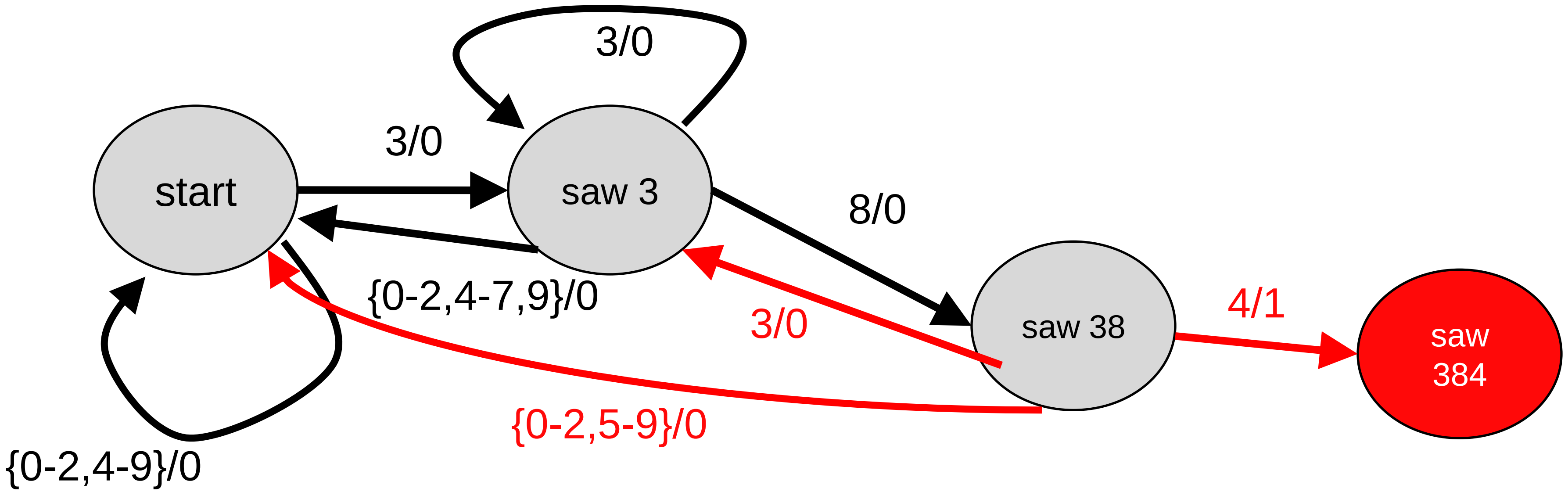
- Input of 3: transition to new state, output=0
- Any other input: stay in same state, output=0

Finite State Machines: Example



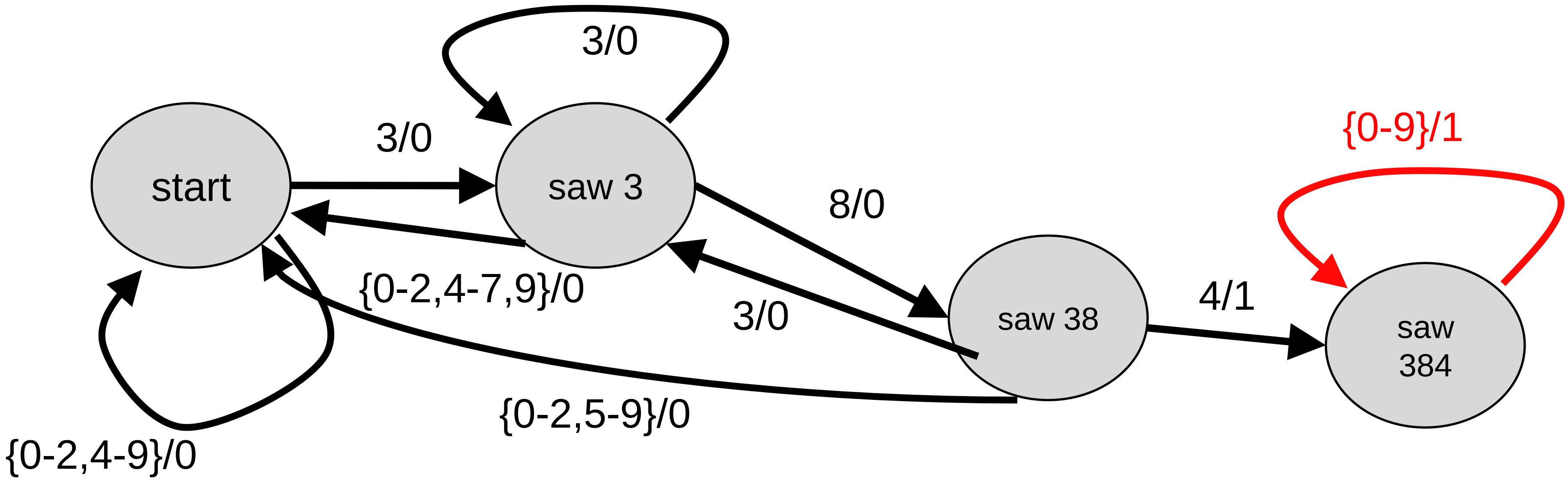
- Combination Lock Example:
  - Need to enter **3 8** 4 to unlock
- If in state "saw 3":
  - Input = 8? Goto state "saw 38" and output=0

Finite State Machines: Example



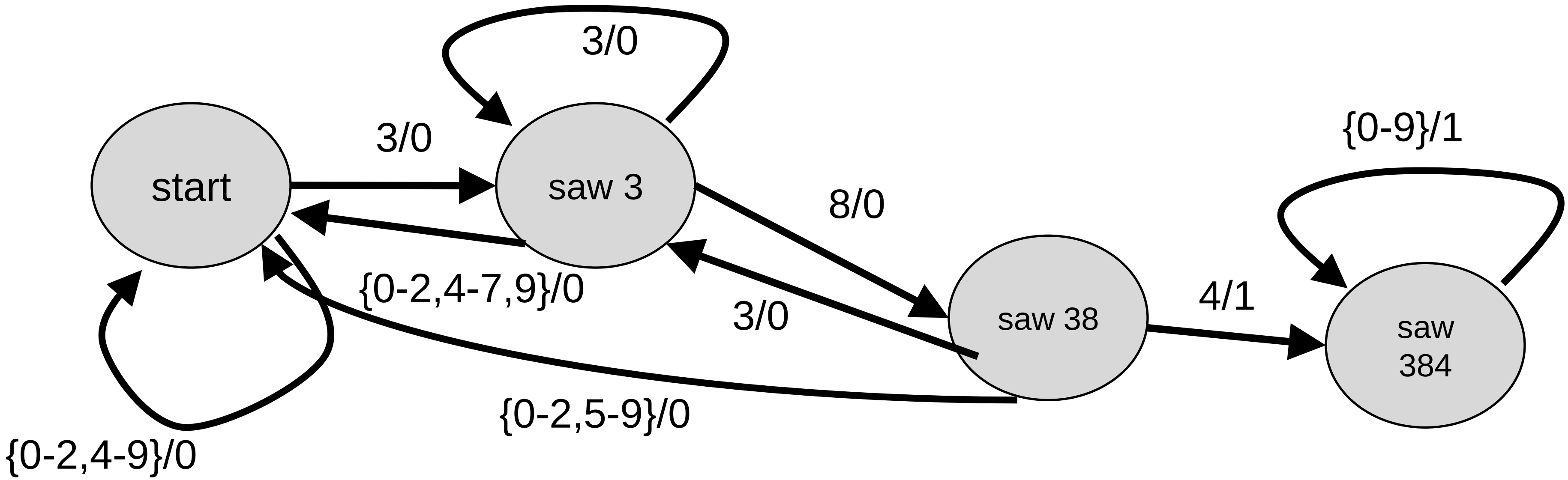
- Combination Lock Example:
  - Need to enter **3 8 4** to unlock
- If in state "saw 38":
  - Input = 4? Goto state "saw 384" and set output=1 → Unlock!

Finite State Machines: Example



- Combination Lock Example:
  - Need to enter **3 8 4** to unlock
- If in state "saw 384":
  - Stay in this state forever and output=1

Finite State Machines: Example



In this picture, the circles are states.
The arcs between the states are transitions.

The figure is a state transition diagram, and it's the first thing you make when designing a finite state machine (FSM).

Finite State Machines: Caveats

Do NOT assume all FSMs are like this one!

•A finite state machine (FSM) has at least two states, but can have many, many more. There's nothing sacred about 4 states (as in this example). Design your FSMs to have the appropriate number of states for the problem they're solving.

- Question: how many states would we need to detect sequence 384384?

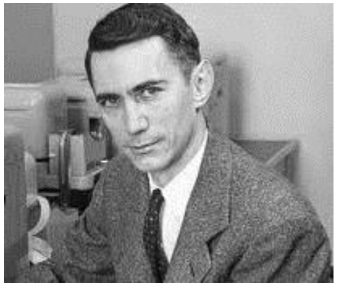
•Most FSMs don't have state from which they can't escape.

FSM Types: Moore and Mealy

- Recall: FSM = States + Transitions
  - Next state = function (current state, inputs)
  - Outputs = function (current state, inputs)
  - *Write the output on the edges*
  - This is the most general case
    - Called a "Mealy Machine"
    - We will assume Mealy Machines in this lecture

"Mealy Machine" developed in 1955 by George H. Mealy

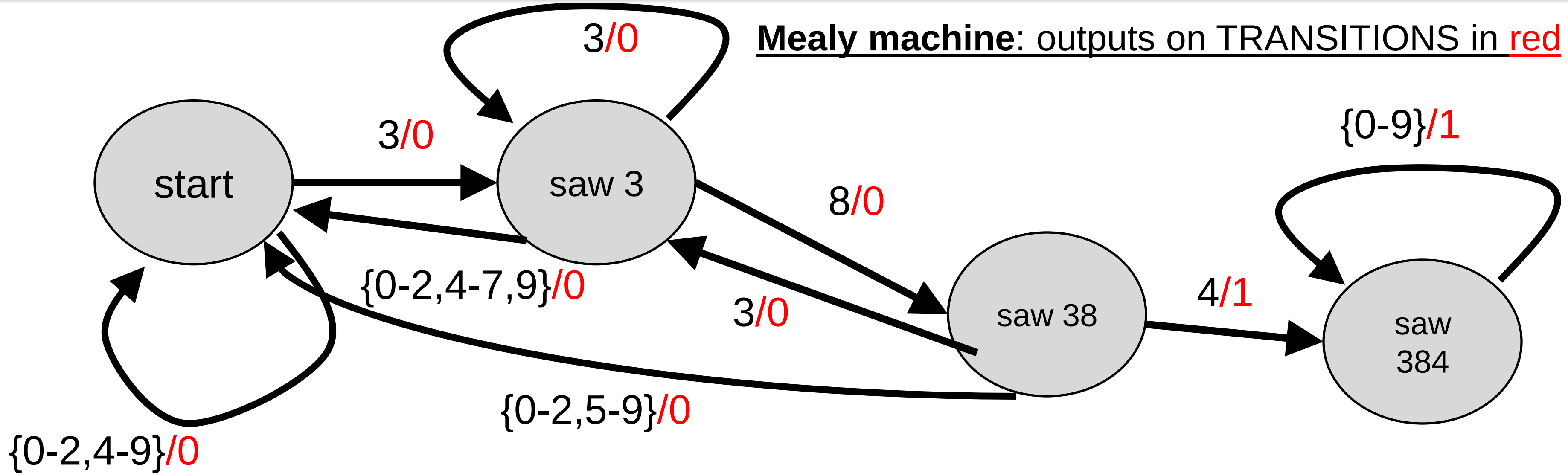
- A more restrictive FSM type is a "Moore Machine"
  - Outputs = function (current state)
  - *Write the output in the states*
  - More often seen in software implementations



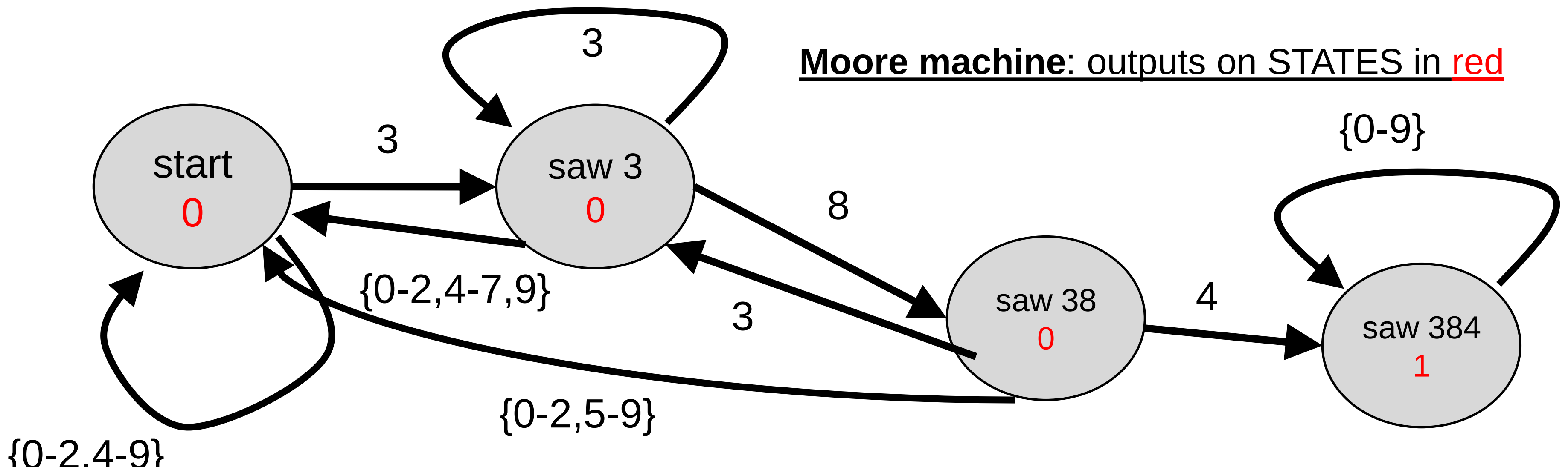
"Moore Machine" developed in 1956 by Edward F. Moore

Mealy vs Moore

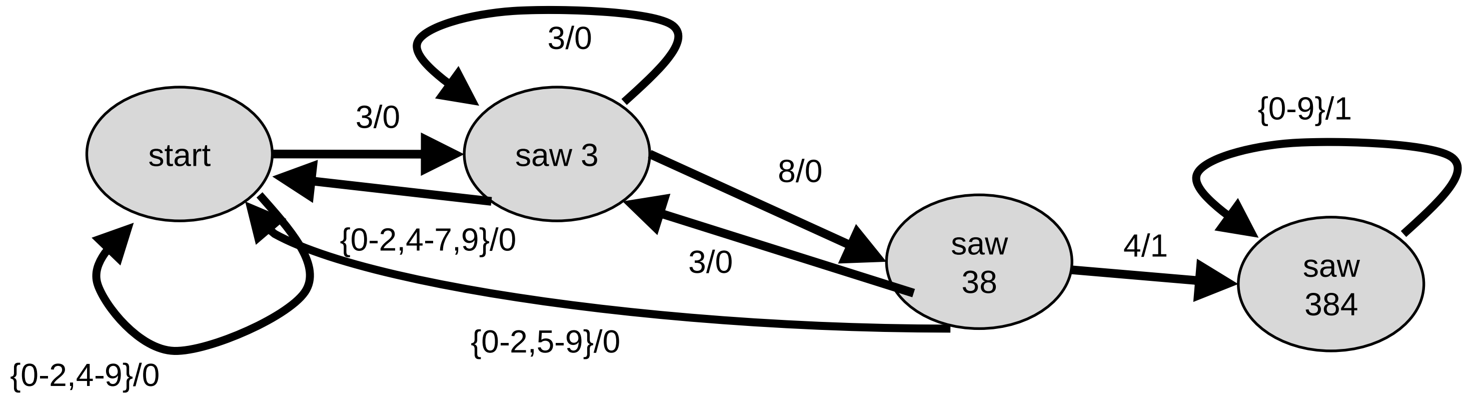
Mealy machine: outputs on TRANSITIONS in red



Moore machine: outputs on STATES in red

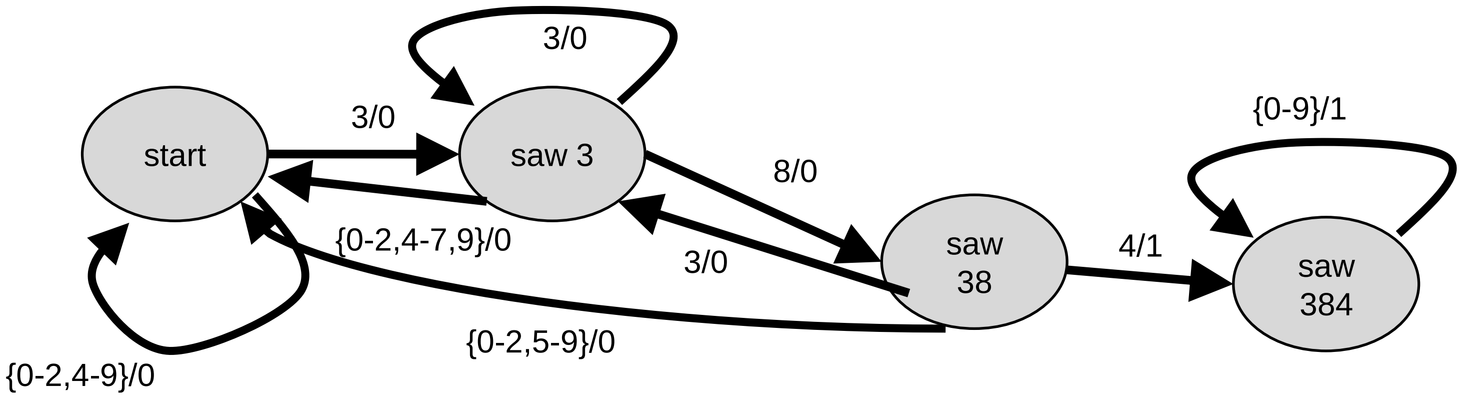


State Transition Diagram → Truth Table



Current State	Input	Next state	Output
Start	3	Saw 3	0 (closed)
Start	Not 3	Start	0
Saw 3	8	Saw 38	0
Saw 3	3	Saw 3	0
Saw 3	Not 8 or 3	Start	0
Saw 38	4	Saw 384	1 (open)
Saw 38	3	Saw 3	0
Saw 38	Not 4 or 3	Start	0
Saw 384	Any	Saw 384	1

State Transition Diagram → Truth Table



Digital logic → must represent everything in binary, including state names.
But mapping is arbitrary!

We'll use this mapping:
start = 00
saw 3 = 01
saw 38 = 10
saw 384 = 11

State Transition Diagram → Truth Table

Current State	Input	Next state	Output
00 (start)	3	01	0 (closed)
00	Not 3	00	0
01	8	10	0
01	3	01	0
01	Not 8 or 3	00	0
10	4	11	1 (open)
10	3	01	0
10	Not 4 or 3	00	0
11	Any	11	1

4 states → 2 flip-flops to hold the current state of the FSM
inputs to flip-flops are D_1D_0
outputs of flip-flops are Q_1Q_0

State Transition Diagram → Truth Table

Q1	Q0	Input	D1	D0	Output
0	0	3	0	1	0 (closed)
0	0	Not 3	0	0	0
0	1	8	1	0	0
0	1	3	0	1	0
0	1	Not 8 or 3	0	0	0
1	0	4	1	1	1 (open)
1	0	3	0	1	0
1	0	Not 4 or 3	0	0	0
1	1	Any	1	1	1

Input can be 0-9 → requires 4 bits
input bits are in3, in2, in1, in0

State Transition Diagram → Truth Table

Q1	Q0	In3	In2	In1	In0	D1	D0	Output
0	0	0	0	1	1	0	1	0
0	0	Not 3 (all binary combos other than 0011)				0	0	0
0	1	1	0	0	0	1	0	0
0	1	0	0	1	1	0	1	0
0	1	Not 8 or 3 (all binary combos other than 1000 & 0011)				0	0	0
1	0	0	1	0	0	1	1	1
1	0	0	0	1	1	0	1	0
1	0	Not 4 or 3 (all binary combos other than 0100 & 0011)				0	0	0
1	1	Any				1	1	1

From here, it's just like combinational logic design!
Write out product-of-sums equations, optimize, and build.

State Transition Diagram → Truth Table

Q1	Q0	In3	In2	In1	In0	D1	D0	Output
0	0	0	0	1	1	0	1	0
0	0	Not 3				0	0	0
0	1	1	0	0	0	1	0	0
0	1	0	0	1	1	0	1	0
0	1	Not 8 or 3				0	0	0
1	0	0	1	0	0	1	1	1
1	0	0	0	1	1	0	1	0
1	0	Not 4 or 3				0	0	0
1	1	Any				1	1	1

Output = (Q1 & !Q0 & !In3 & In2 & !In1 & !In0) | (Q1 & Q0)

D1 = (!Q1 & Q0 & In3 & !In2 & !In1 & !In0) | (Q1 & !Q0 & !In3 & In2 & !In1 & !In0) | (Q1 & Q0)

D0 = do the same thing

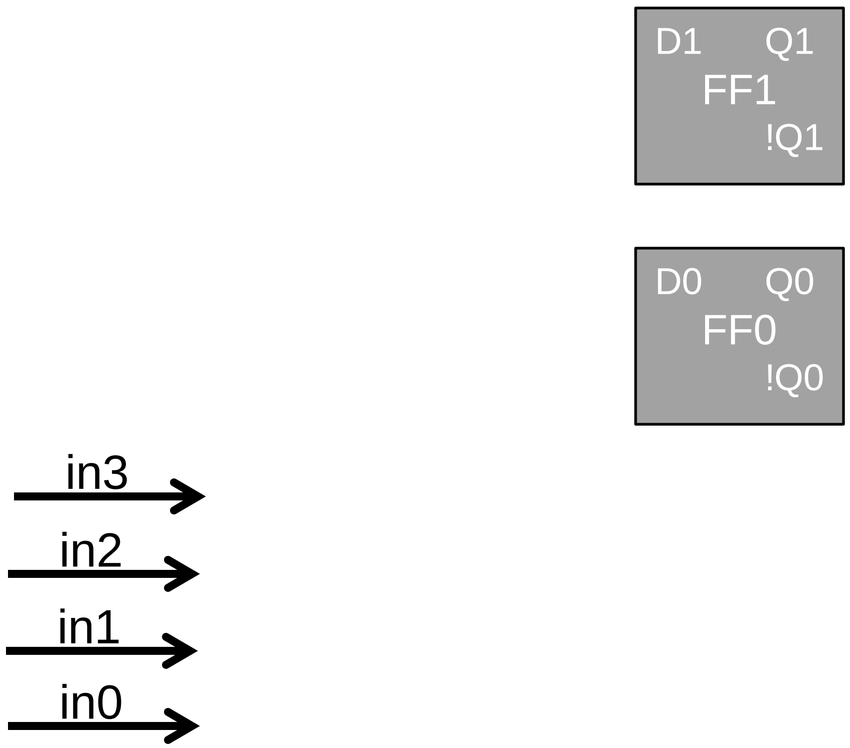
State Transition Diagram → Truth Table

Q1	Q0	In3	In2	In1	In0	D1	D0	Output
0	0	0	0	1	1	0	1	0
0	0	Not 3				0	0	0
0	1	1	0	0	0	1	0	0
0	1	0	0	1	1	0	1	0
0	1	Not 8 or 3				0	0	0
1	0	0	1	0	0	1	1	1
1	0	0	0	1	1	0	1	0
1	0	Not 4 or 3				0	0	0
1	1	Any				1	1	1

Remember, these represent **DFF outputs** …and these are the **DFF inputs**

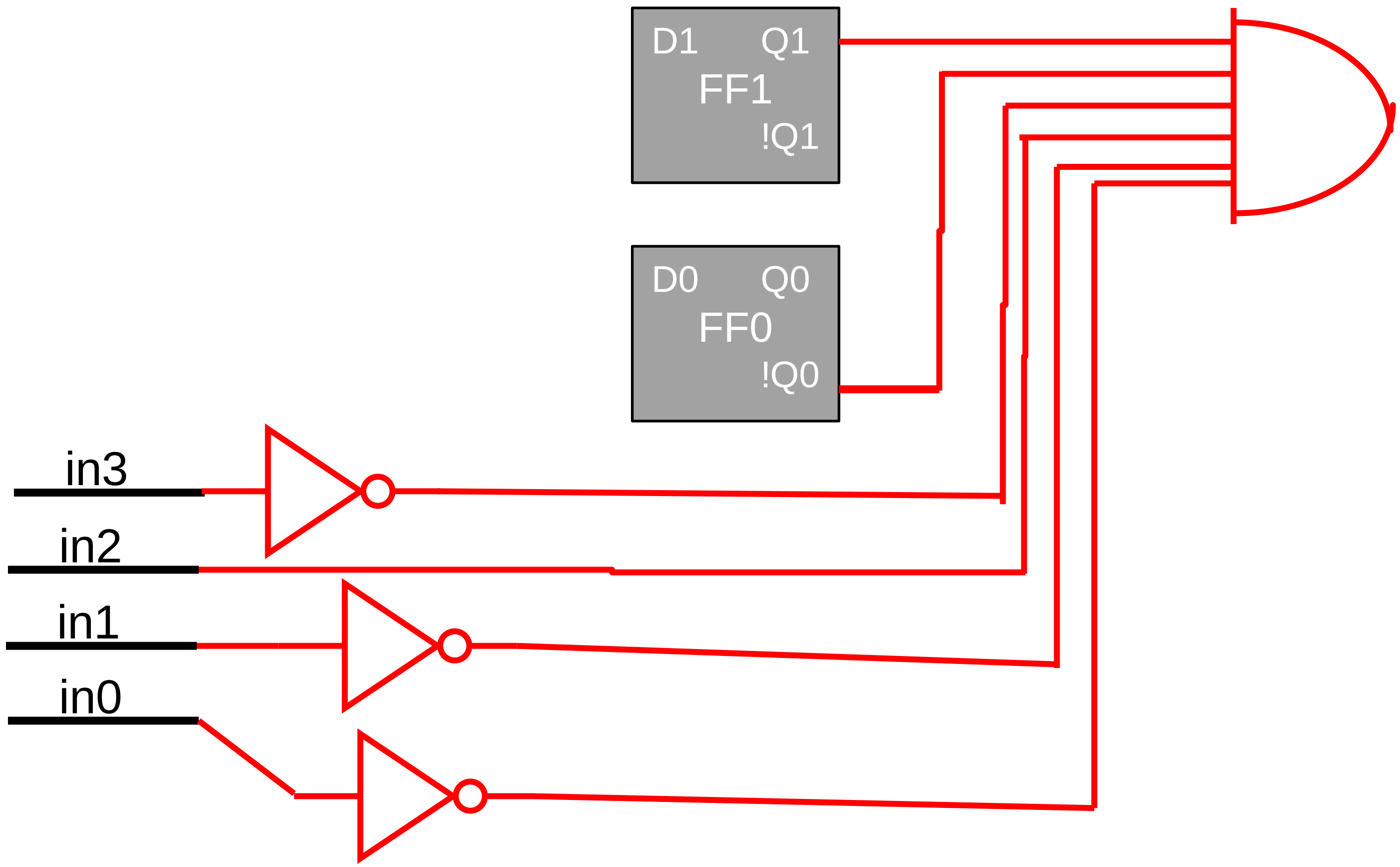
The DFFs are how we store the **state**.

Truth Table → Sequential Circuit



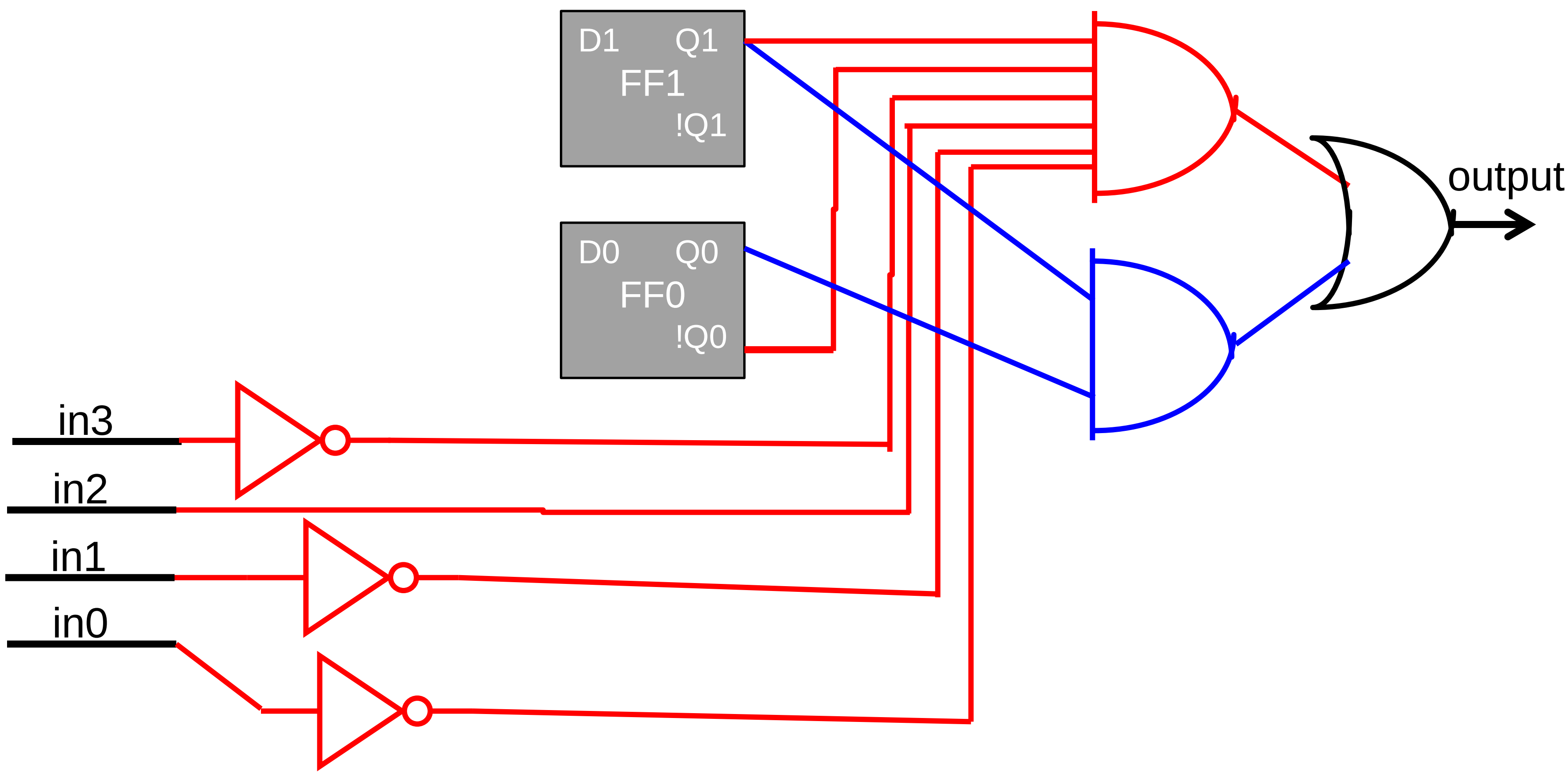
Start with 2 FFs and 4 input bits. FFs hold current state of FSM.
(not showing clock/enable inputs on flip flops)

Truth Table → Sequential Circuit



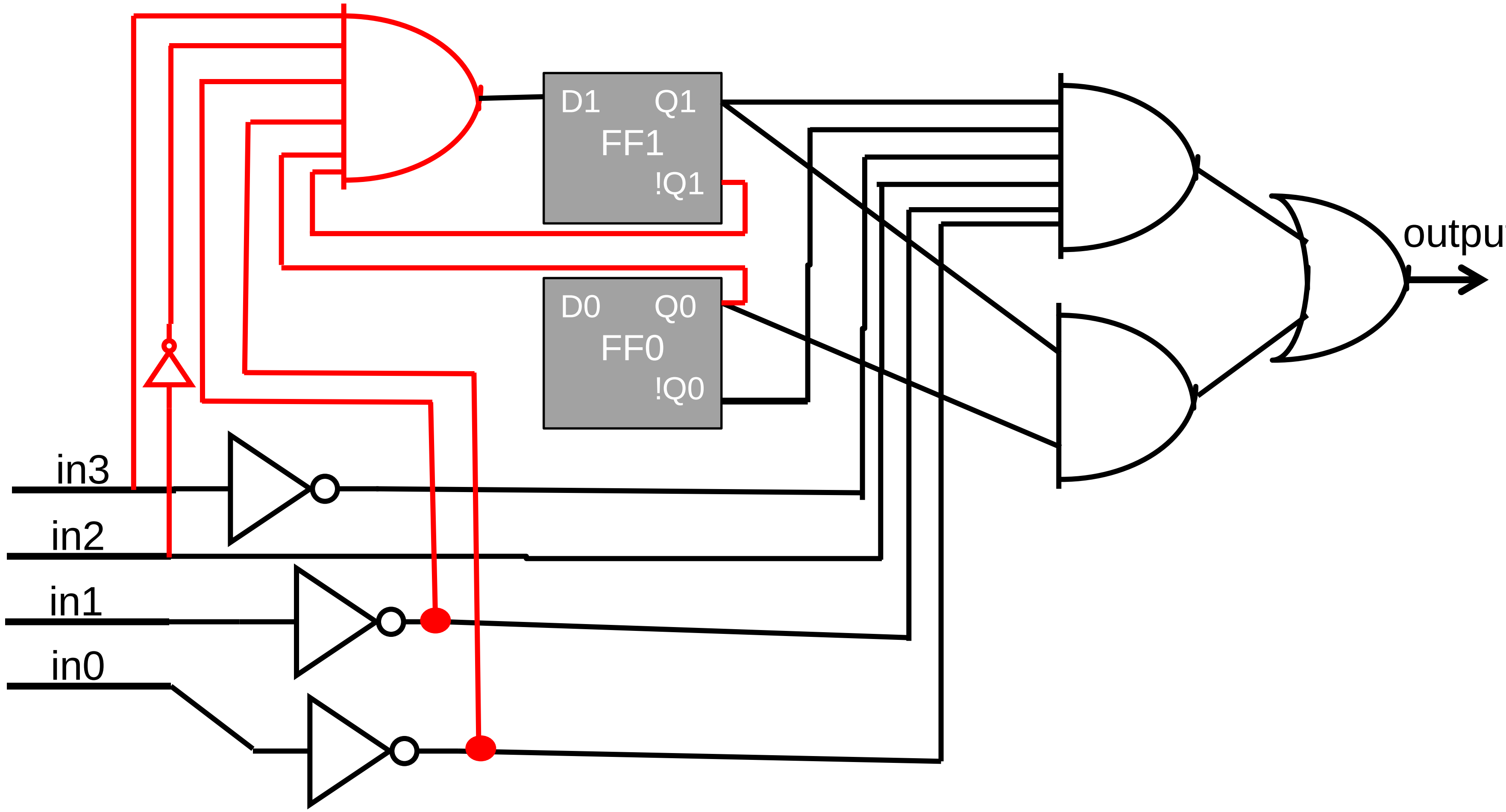
output = (Q1 & !Q0 & !In3 & In2 & !In1 & !In0) | (Q1 & Q0)

Truth Table → Sequential Circuit



output = (Q1 & !Q0 & !In3 & In2 & !In1 & !In0) | (Q1 & Q0)

Truth Table → Sequential Circuit



D1 = (!Q1 & Q0 & In3 & !In2 & !In1 & !In0) | (Q1 & !Q0 & !In3 & In2 & !In1 & !In0) | (Q1 & Q0)

Not pictured

Follow a similar procedure for D0…

FSM Design Principles

- Systematic approach that always works:
  - Start with state transition diagram
  - Make truth table
  - Write out sum-of-products logic equations
  - Optimize logic equations (optional)
  - Implement logic in circuit
- Sometimes can do something non-systematic
  - Requires cleverness, but tough to do in general

- <u>Do not do any of the following!</u>
  - Use clock as an input (D input of FF)
  - Perform logic on clock signal

    (except maybe a NOT gate to go from rising to falling edge triggered)