ECE/CS 250
Computer Architecture

Fall 2021

Basics of Logic Design:
Boolean Algebra, Logic Gates, and the ALU
(Combinational Logic)

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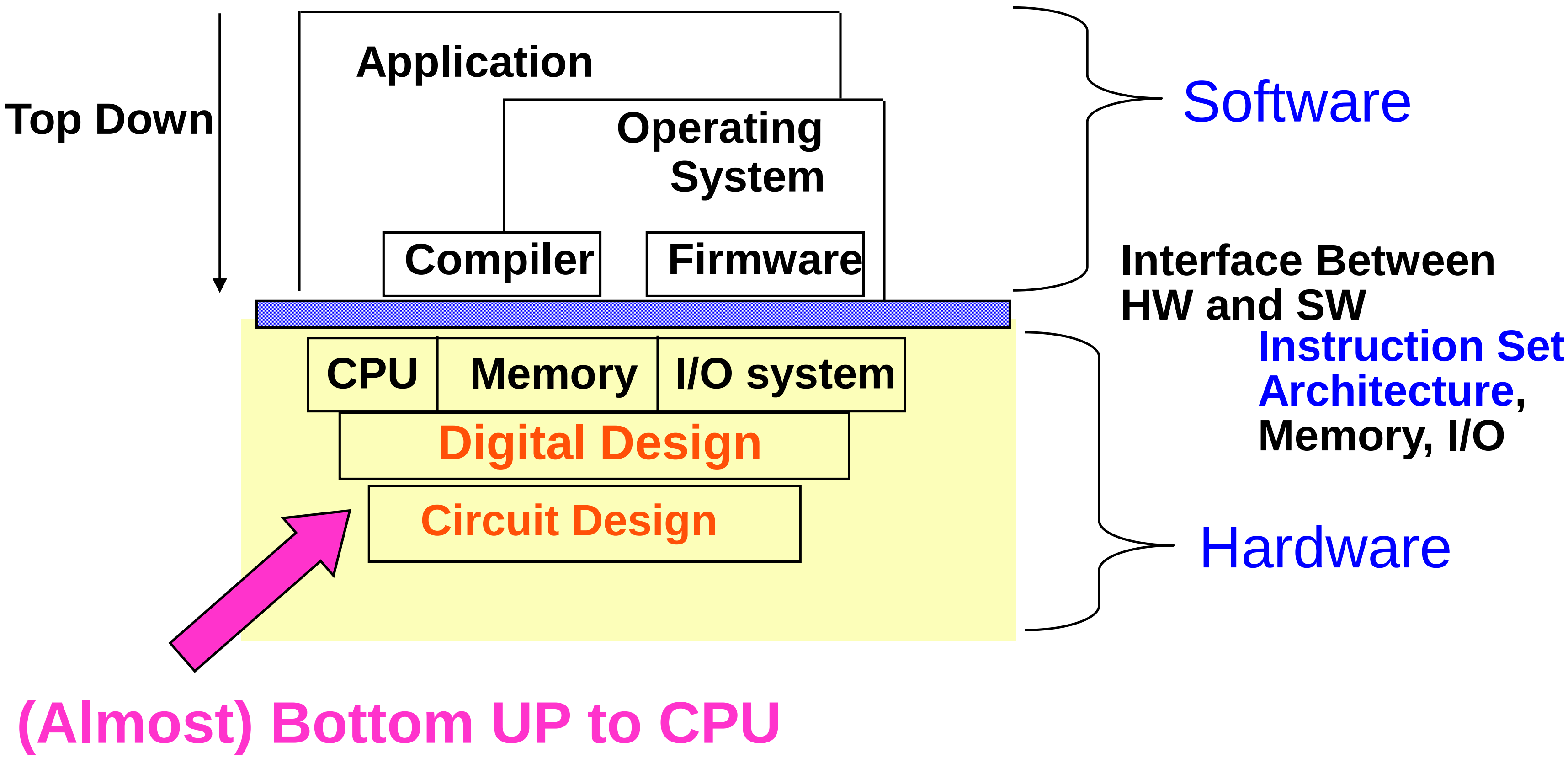
Slides are derived from work by
Daniel J. Sorin (Duke), Alvy Lebeck (Duke), and Drew Hilton (Duke)

Reading

- Appendix B (parts 1,2,3,5,6,7,8,9,10)
- This material is covered in MUCH greater depth in ECE/CS 350 – please take ECE/CS 350 if you want to learn enough digital design to build your own processor

What We've Done, Where We're Going



Computer = Machine That Manipulates Bits

- Everything is in binary (bunches of 0s and 1s)
  - Instructions, numbers, memory locations, etc.
- Computer is a machine that operates on bits
  - Executing instructions → operating on bits

- Computers physically made of transistors
  - Electrically controlled switches
- We can use transistors to build logic
  - E.g., if this bit is a 0 and that bit is a 1, then set some other bit to be a 1
  - E.g., if the first 5 bits of the instruction are 10010 then set this other bit to 1 (to tell the adder to subtract instead of add)

How Many Transistors Are We Talking About?

Pentium III

- Processor Core 9.5 Million Transistors
- Total: 28 Million Transistors

Pentium 4

- Total: 42 Million Transistors

Core2 Duo (two processor cores)

- Total: 290 Million Transistors

Core2 Duo Extreme (4 processor cores, 8MB cache)

- Total: 590 Million Transistors

Core i7 with 6-cores

- Total: 2.27 Billion Transistors

How do they design such a thing? Carefully!

Abstraction!

- Use of abstraction (key to design of any large system)
  - Put a few (2-8) transistors into a logic gate (or, and, xor, …)
  - Combine gates into logical functions (add, select,….)
  - Combine adders, shifters, etc., together into modules
    Units with well-defined interfaces for large tasks: e.g., decode
  - Combine a dozen of those into a core…
  - Stick 4 cores on a chip…

Boolean Algebra

- First step to logic: Boolean Algebra
  - Manipulation of True / False (1/0)
  - After all: everything is just 1s and 0s

- Given inputs (variables): A, B, C, P, Q…
  - Compute outputs using logical operators, such as:
- NOT: !A (= ~A = $\overline{A}$)
- AND: A&B (= A·B = A*B = AB = A∧B) = A&&B in C/C++
- OR: A | B (= A+B = A ∨ B) = A || B in C/C++
- XOR: A ^ B (= A ⊕ B)
- NAND, NOR, XNOR, Etc.

Truth Tables

- Can represent as truth table: shows outputs for all inputs

a	NOT(a)
0	1
1	0

a	b	AND(a,b)
0	0	0
0	1	0
1	0	0
1	1	1

a	b	OR(a,b)
0	0	0
0	1	1
1	0	1
1	1	1

a	b	XOR(a,b)
0	0	0
0	1	1
1	0	1
1	1	0

a	b	XNOR(a,b)
0	0	1
0	1	0
1	0	0
1	1	1

a	b	NOR(a,b)
0	0	1
0	1	0
1	0	0
1	1	0

Any Inputs, Any Outputs

- Can have any # of inputs, any # of outputs
- Can have arbitrary functions:

a	b	c	f_1	f_2
0	0	0	0	1
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	1

Let's Write a Truth Table for a Function…

- Example:

  (A & B) | !C

Start with Empty TT

Column Per Input

Column Per Output

A	B	C	Output

Let's write a Truth Table for a function…

- Example:

  (A & B) | !C

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Column Per Output

Fill in Inputs

Counting in Binary

A	B	C	Output
0	0	0	

Let’s write a Truth Table for a function…

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Let’s write a Truth Table for a function…

- Example:

    (A & B) | !C

Start with Empty TT

    Column Per Input

    Column Per Output

Fill in Inputs

    Counting in Binary

Compute Output

    (0 & 0) | !0 = 0 | 1 = 1

A	B	C	Output
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
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Let's write a Truth Table for a function…

- Example:

  (A & B) | !C

Start with Empty TT

Column Per Input

Column Per Output

Fill in Inputs

Counting in Binary

Compute Output

(0 & 1) | !0 = 0 | 1 = 1

A	B	C	Output
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	
1	0	0	
1	0	1	
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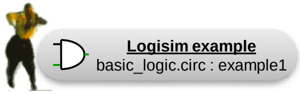

Suppose I turn it around…

- Given a Truth Table, find the formula?

Hmmm..

A	B	C	Output
0	0	0	1
0	0	1	1
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1	1	0	1
1	1	1	1

Suppose I turn it around…

- Given a Truth Table, find the formula?

Hmmm …

Could write down every "true" case

Then OR together:

```
(!A & !B & !C) |
(!A & !B & C)  |
(!A & B & !C) |
(A & B &!C) |
(A & B &C)
```

A	B	C	Output
0	**0**	**0**	**1**
0	**0**	**1**	**1**
0	**1**	**0**	**1**
0	1	1	0
1	0	0	0
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Suppose I turn it around…

- This approach: “sum of products”
  - Works every time
  - Result is right…
  - But really ugly

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```

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(!A & B & !C)  |
(A & B &!C)  |
(A & B &C)
```

Could just be (A & B) here ?

A	B	C	Output
0	**0**	**0**	**1**
0	**0**	**1**	**1**
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	**1**	**0**	**1**
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(!A & !B & !C) |
(!A & !B & C)  |
(!A & B & !C) |
(A&B)
```

A	B	C	Output
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0	**0**	**1**	**1**
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1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

```
(!A & !B) |
(!A & B & !C) |
(A&B)
```

Could just be (!A & !B) here

Suppose I turn it around…

- This approach: “sum of products”
  - Works every time
  - Result is right…
  - But really ugly

```
(!A & !B) |
(!A & B & !C) |
(A&B)
```

Looks nicer…

Can we do better?

A	B	C	Output
0	**0**	**0**	**1**
0	**0**	**1**	**1**
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Just did some of these by intuition.. but

- Somewhat intuitive approach to simplifying
- This is math, so there are formal rules
  - Just like “regular” algebra

Boolean Function Simplification

- Boolean expressions can be simplified by using the following rules (bitwise logical):
  - `A & A = A` `A | A = A`
  - `A & 0 = 0` `A | 0 = A`
  - `A & 1 = A` `A | 1 = 1`
  - `A & !A = 0` `A | !A = 1`

  - `!!A = A`

  - & and | are both commutative and associative
  - & and | can be distributed: `A & (B | C) = (A & B) | (A & C)`
  - & and | can be subsumed: `A | (A & B) = A`

DeMorgan's Laws

- Two (less obvious) Laws of Boolean Algebra:
  - Let's push negations inside, flipping & and |

```
!(A & B) = (!A) | (!B)
```

```
!(A | B) = (!A) & (!B)
```

  - You should try this at home – build truth tables for both the left and right sides and see that they're the same

Summary of all Boolean axioms

<table>
<tr><th>Name</th><th>AND form</th><th>OR form</th></tr>
<tr><td>Identity law</td><td>1 & A = A</td><td>0 | A = A</td></tr>
<tr><td>Null law</td><td>0 & A = 0</td><td>1 | A = 1</td></tr>
<tr><td>Idempotent law</td><td>A & A = A</td><td>A | A = A</td></tr>
<tr><td>Inverse law</td><td>A & !A = 0</td><td>A | !A = 1</td></tr>
<tr><td>Commutative law</td><td>A & B = B & A</td><td>A | B = B | A</td></tr>
<tr><td>Associative law</td><td>(A&B) & C = A & (B&C)</td><td>(A|B) | C = A | (B|C)</td></tr>
<tr><td>Distributive law</td><td>A | (B&C) = (A|B) & (A|C)</td><td>A & (B|C) = (A&B) | (A&C)</td></tr>
<tr><td>Absorption law</td><td>A & (A|B) = A</td><td>A | (A&B) = A</td></tr>
<tr><td>De Morgan’s law</td><td>!(A&B) = !A | !B</td><td>!(A|B) = !A & !B</td></tr>
<tr><td>Double negation law</td><td colspan="2">!!A = A</td></tr>
</table>

Adapted from http://studytronics.weebly.com/boolean-algebra.html

Simplification Example:

`! (!A | !(A & (B | C)))`

DeMorgan's

`!!A & !! (A & (B | C))`

Double Negation Elimination

`A & (A & (B | C))`

Associativity of &

`(A & A) & (B | C)`

A & A = A

`A & (B | C)`

You try this:

Come up with a formula for this Truth Table

Simplify as much as possible

Simplify this part

Sum of Products:

```
(!A & !B & !C)   |
(!A & B & !C)    |
(A & !B & C)     |
(A & B & C)
```

A	B	C	Output
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

You try this:

Simplify:

```
(!A & !B & !C) | (!A & B & !C)
```

Regroup (associative/commutative):

```
((!A & !C) & !B) | ((!A & !C) & B)
```

Un-distribute (factor):

```
(!A & !C) & (!B | B)
```

OR identities:

```
(!A & !C) & true  = (!A & !C)
```

You try this:

Come up with a formula for this Truth Table

Simplify as much as possible

Sum of Products:

```
(!A & !C)  |
(A & !B & C)    |
(A & B & C)
```

Result of simplifying

You can simplify this part in the same way…

A	B	C	Output
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
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You try this:

Come up with a formula for this Truth Table

Simplify as much as possible

Sum of Products:

```
(!A & !C)  |
 (A & C)
```

A	B	C	Output
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

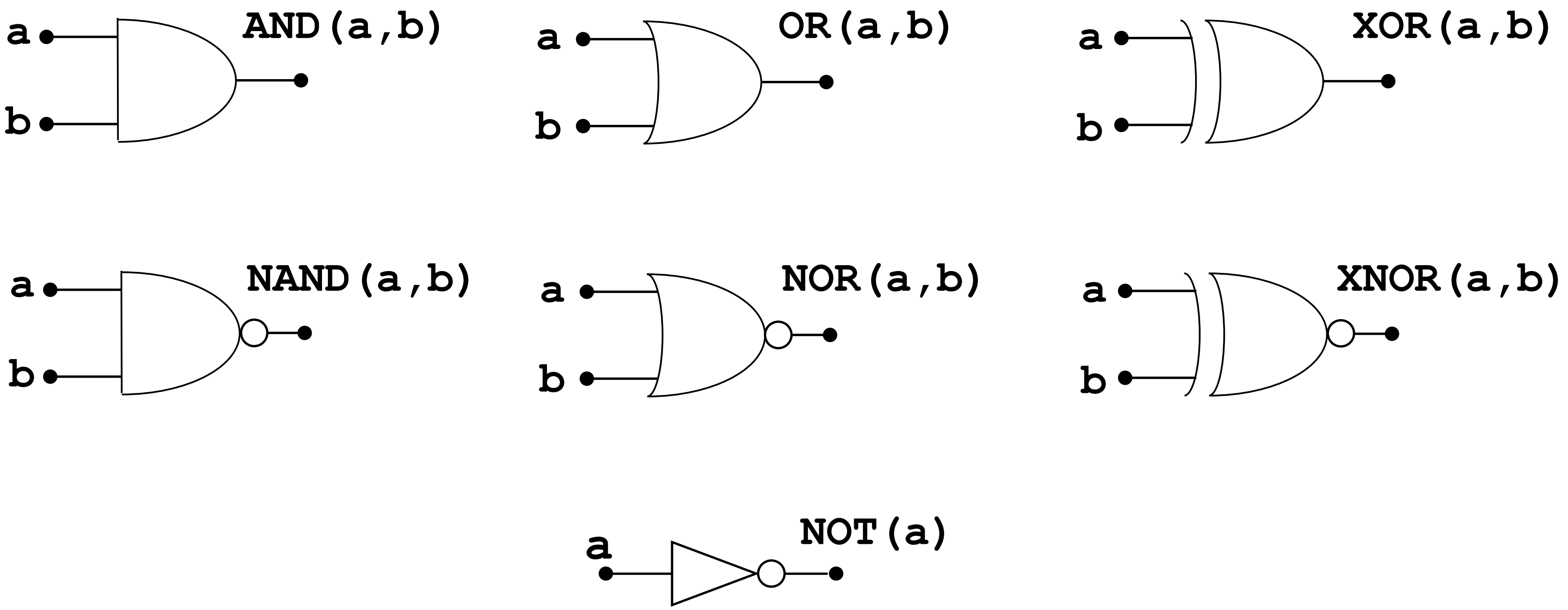
Logisim example
basic_logic.circ : example4

Applying the Theory

- Lots of good theory
- Can reason about complex Boolean expressions
- But why is this useful?

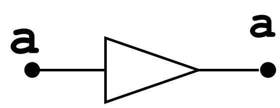
Boolean Gates

- Gates are electronic devices that implement simple Boolean functions (building blocks of hardware)

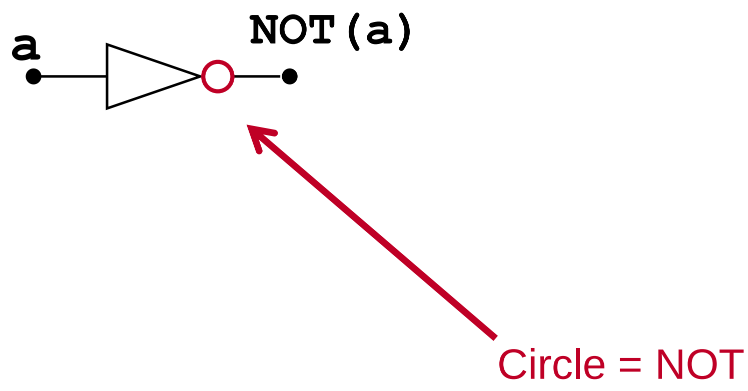


Guide to Remembering your Gates

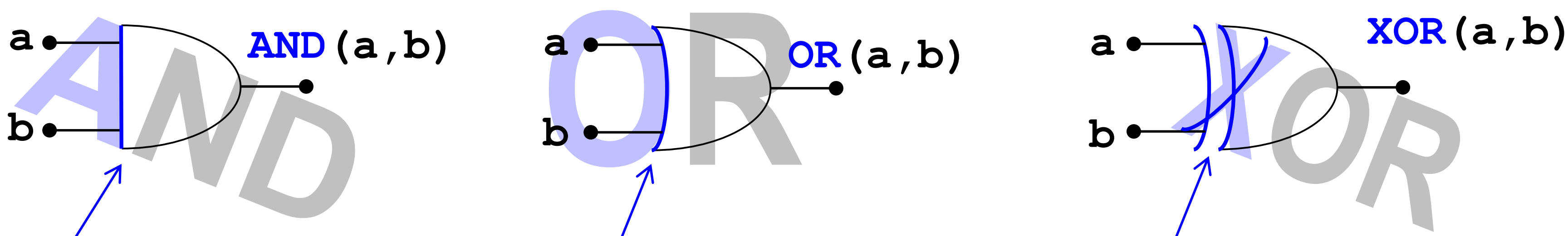
- This one looks like it just points its input where to go
  - It just produces its input as its output
  - Called a buffer



- A circle always means negate (invert)



Guide to Remembering your Gates

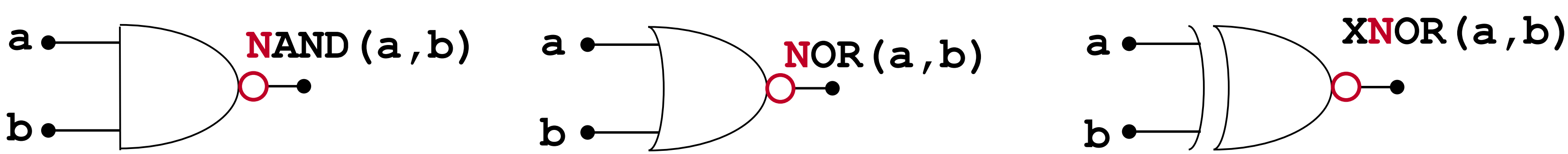


Straight like an A

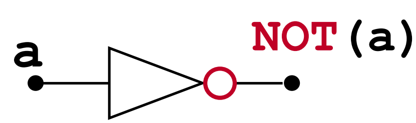
Curved, like an O

XOR looks like OR (curved line), but has two lines (like an X does)

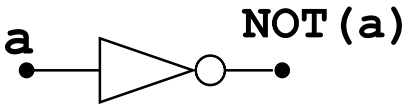
Circle means NOT

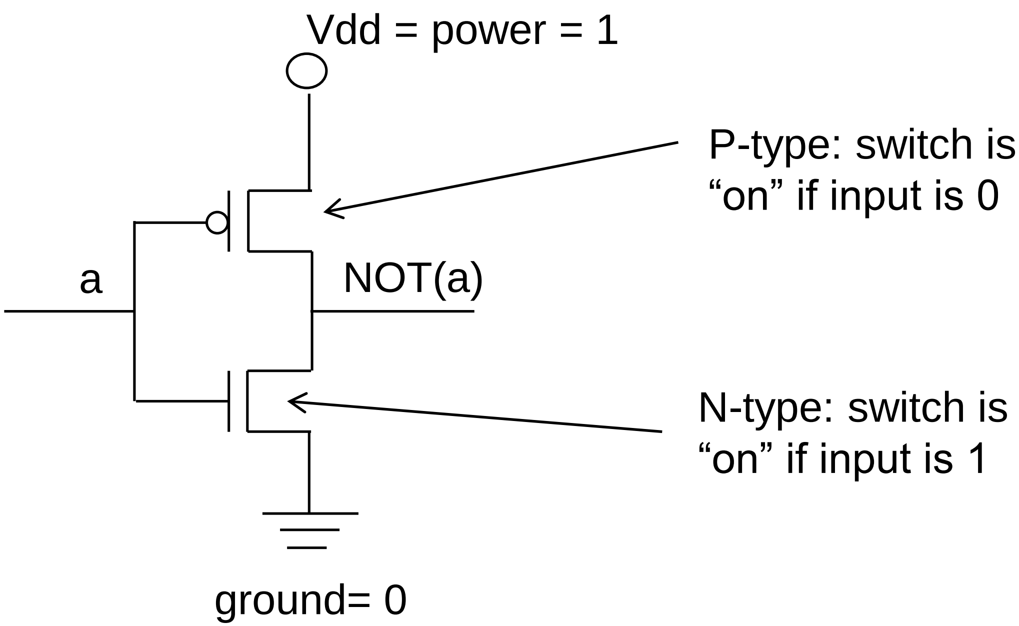


(XNOR is 1-bit “equals” by the way)



Brief Interlude: Building An Inverter

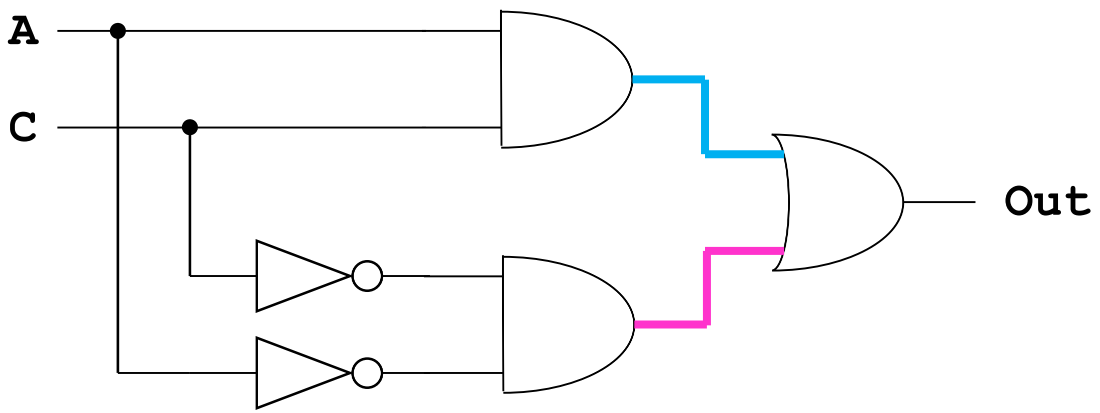


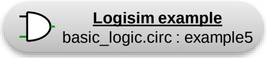


Boolean Functions, Gates and Circuits

- Circuits are made from a network of gates.

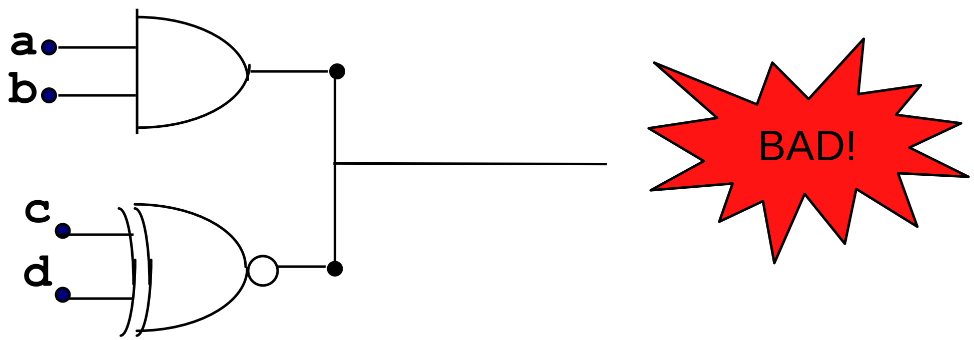
`(!A & !C) | (A & C)`





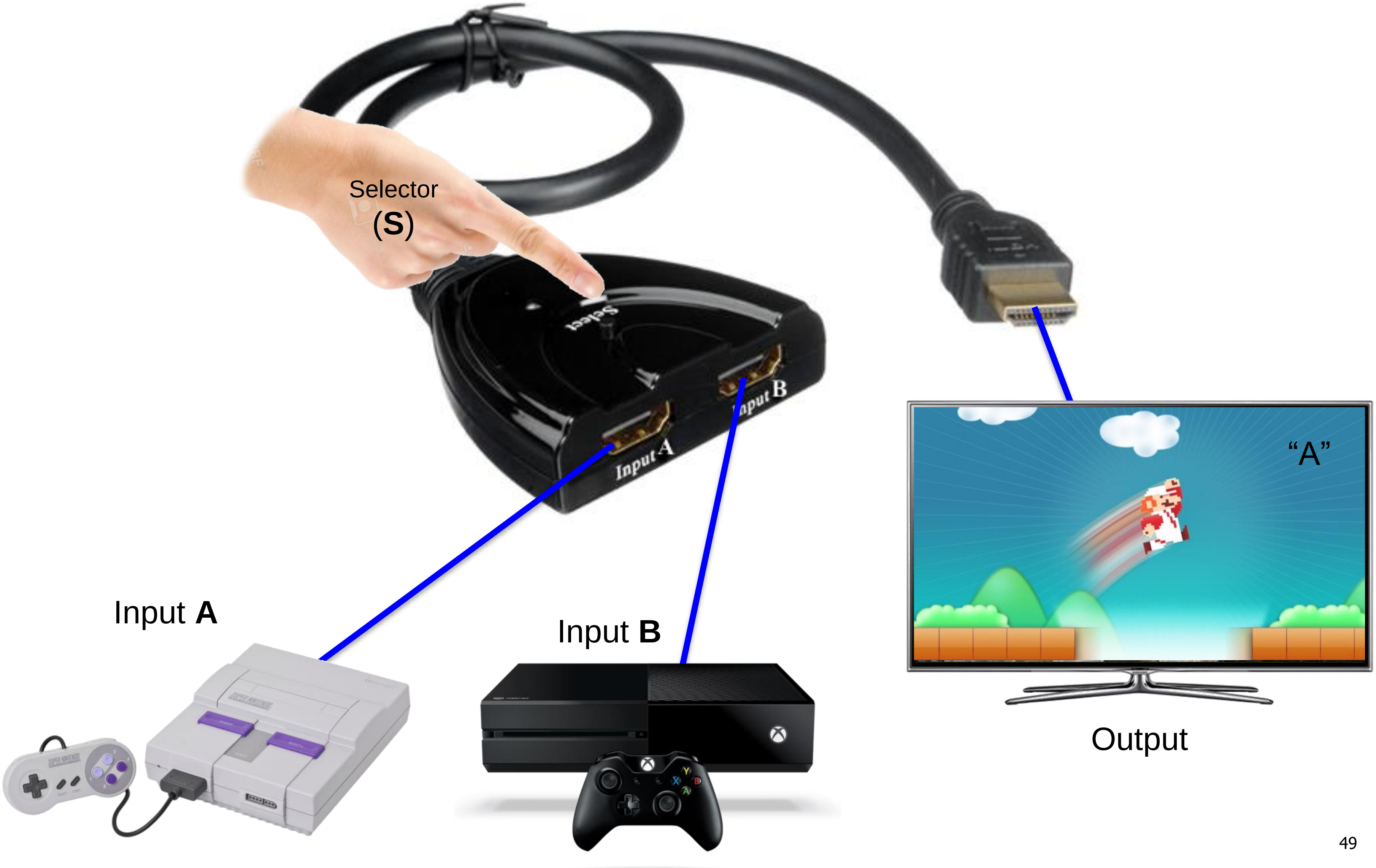
A few more words about gates

- Gates have inputs and outputs
  - If you try to hook up two outputs, bad things happen (your processor catches fire)



  - If you don't hook up an input, it behaves kind of randomly (also not good, but not set-your-chip-on-fire bad)

Introducing the Multiplexer ("mux")



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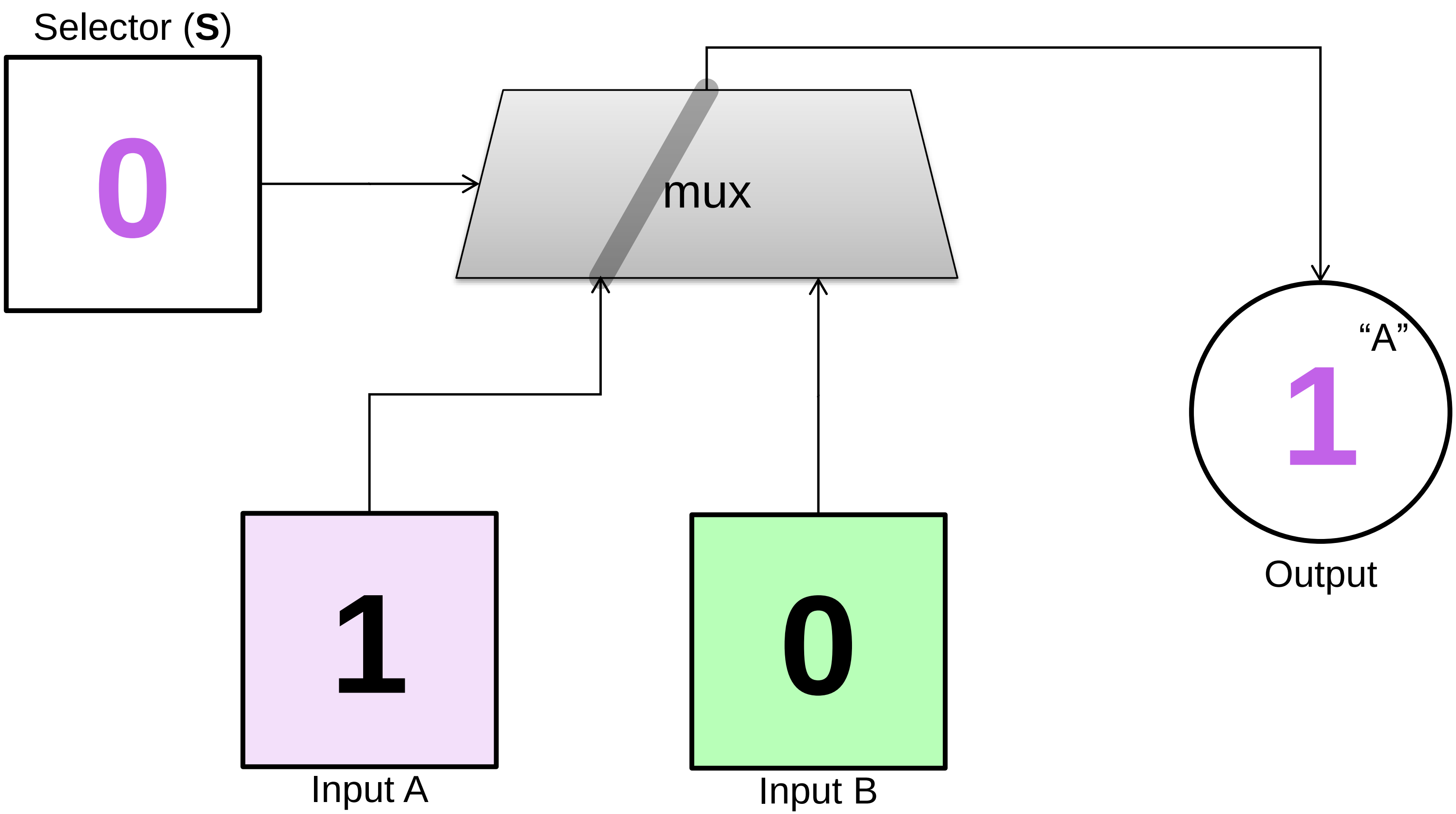

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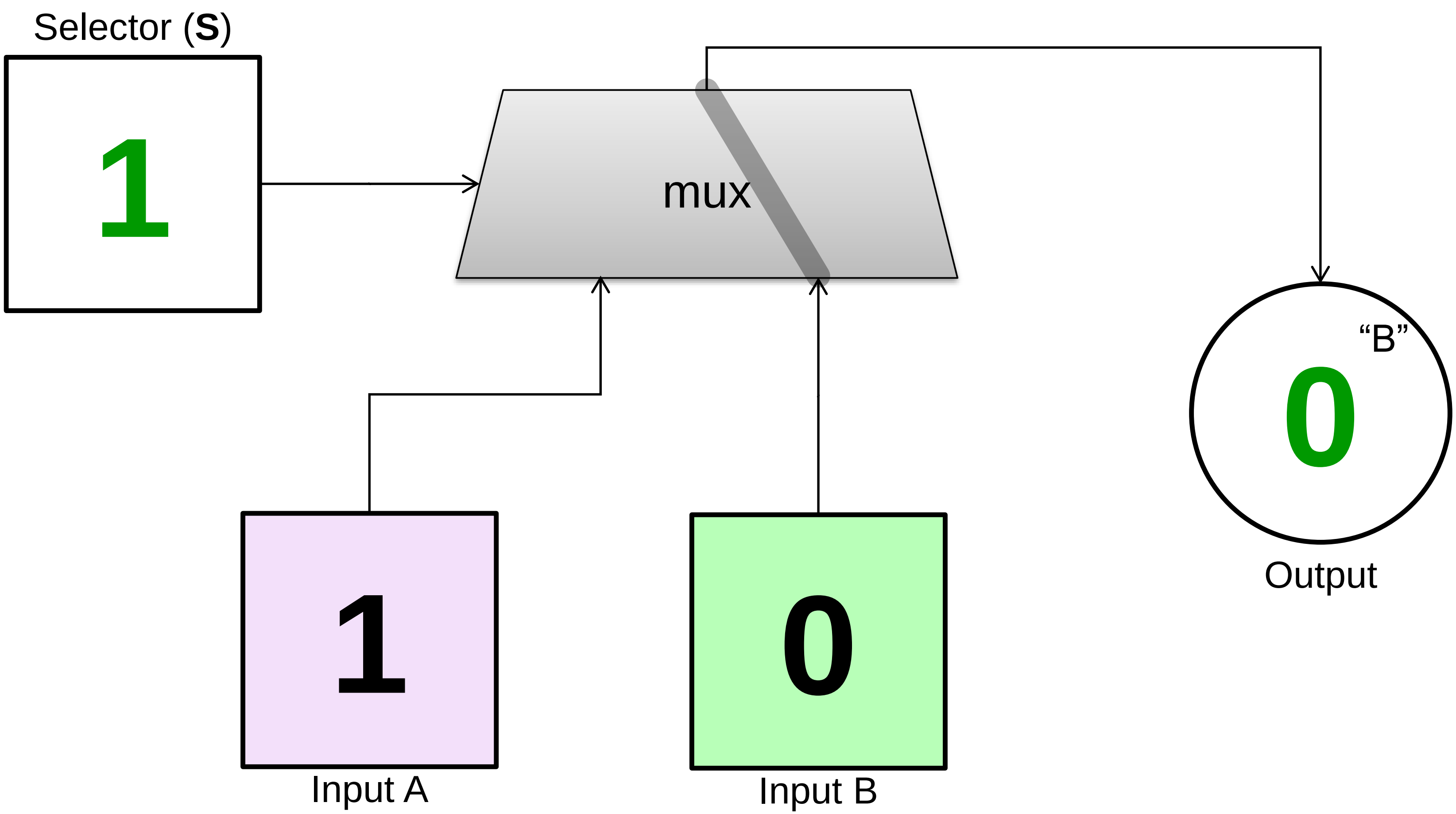

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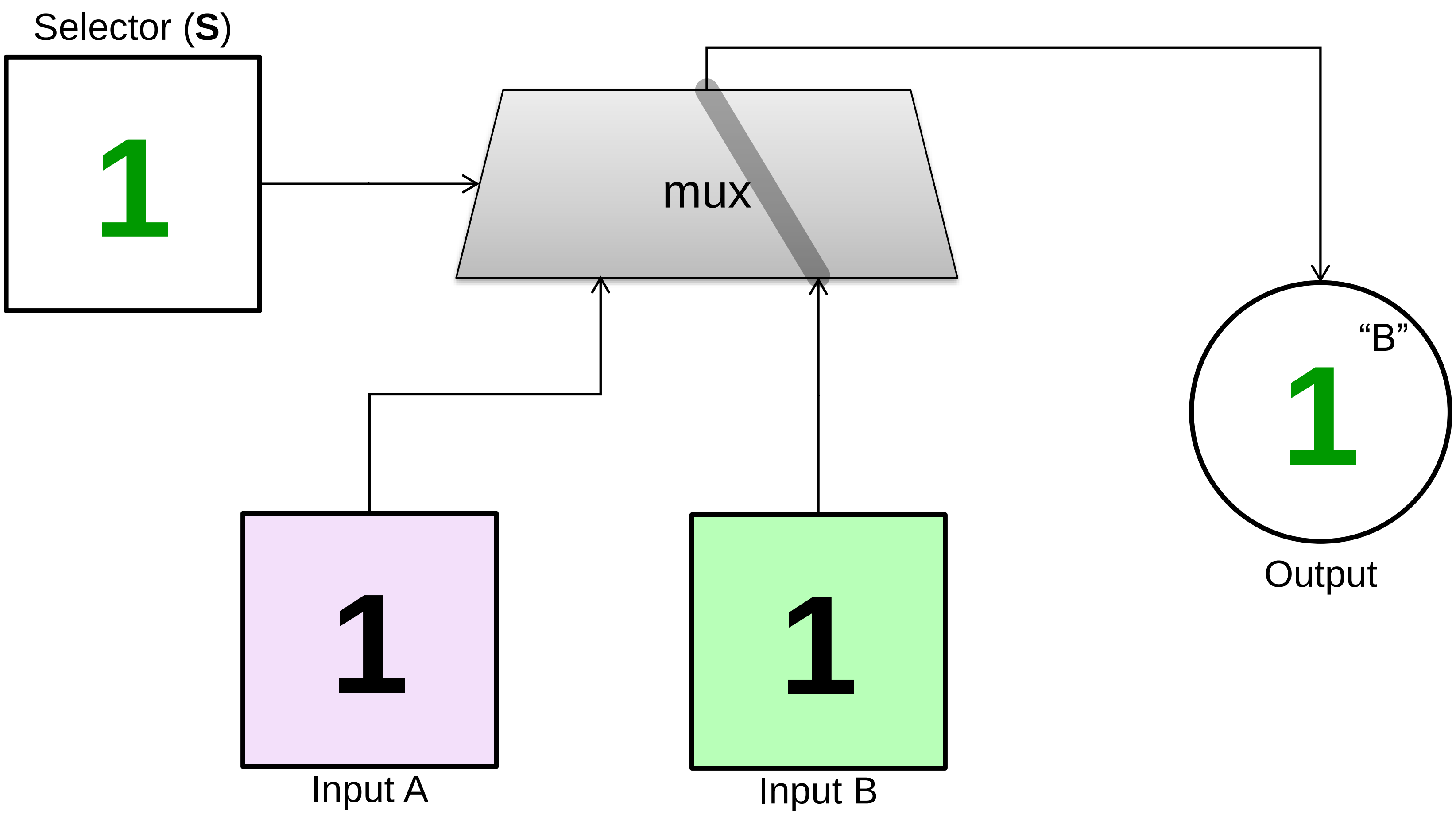


Let's Make a Useful Circuit

- Pick between 2 inputs (called 2-to-1 MUX)
  - Short for multiplexor

- What might we do first?
  - Make a truth table?
    - S is selector:
      - S=0, pick A
      - S=1, pick B
- Next: sum-of-products

```
(!A & B & S) |
(A & !B & !S) |
(A & B & !S ) |
(A & B & S)
```

- Simplify

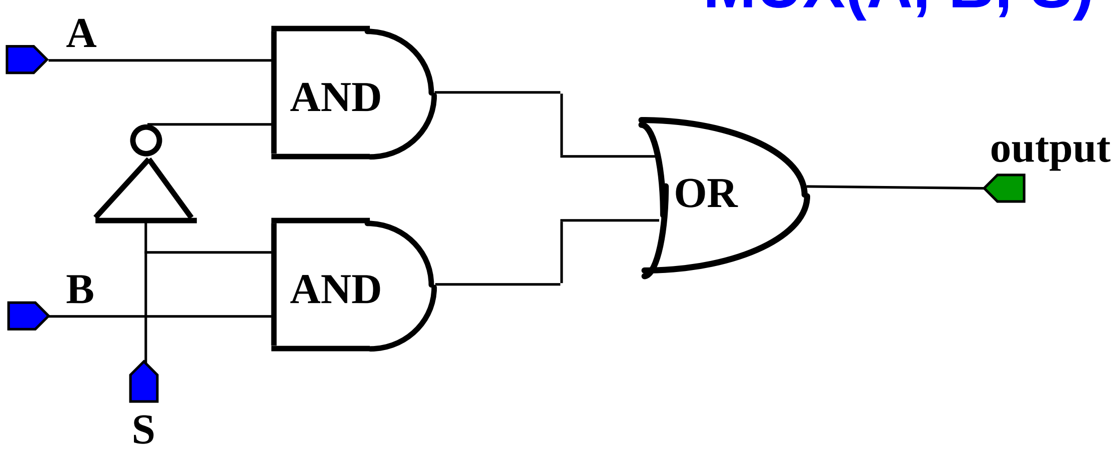
```
(A & !S) | (B &  S)
```

A	B	S	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

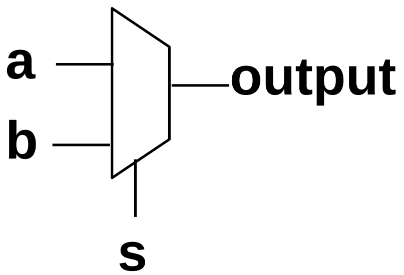
Circuit Example: 2x1 MUX

Draw it in gates:

MUX(A, B, S) = (A & !S) | (B & S)

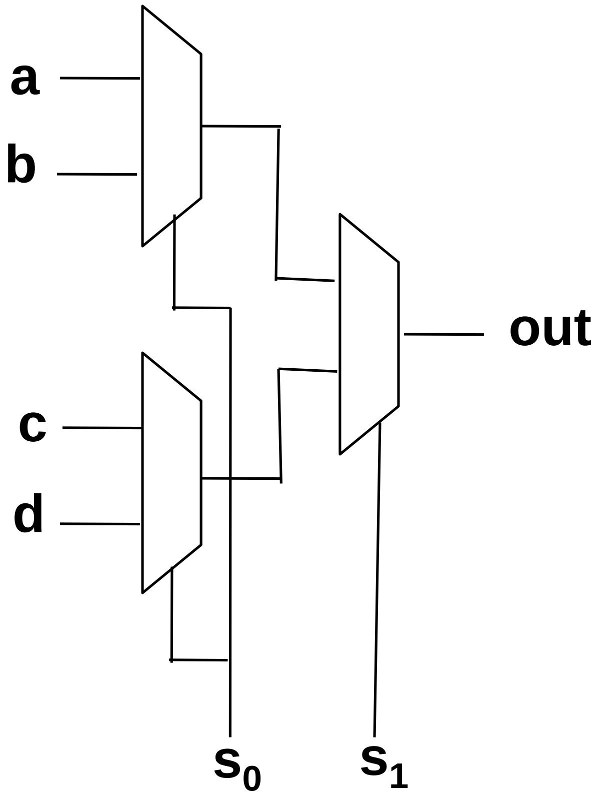


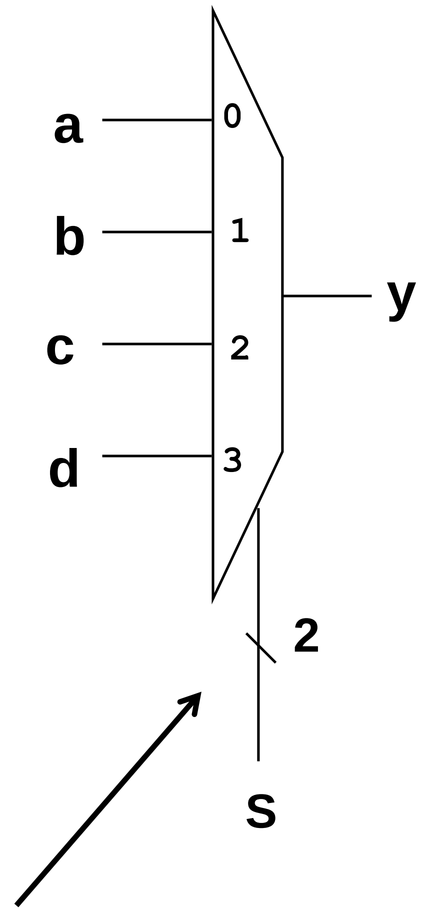
So common, we give it
its own symbol:



Logisim example
basic_logic.circ : mux-2x1

Example 4x1 MUX





The / 2 on the wire means “2 bits”

Logisim example
basic_logic.circ : mux-4x1

Arithmetic and Logical Operations in ISA

- What operations are there?
- How do we implement them?
  - Consider a 1-bit Adder

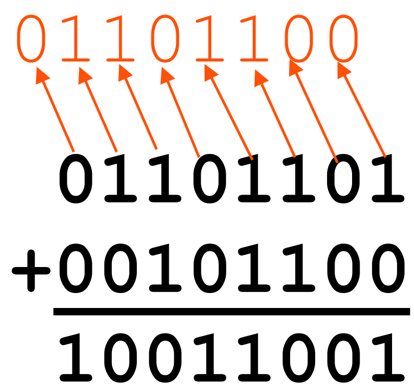
Designing a 1-bit adder

- What boolean function describes the low bit?
  - XOR
- What boolean function describes the high bit?
  - AND

```
0 + 0 = 00
0 + 1 = 01
1 + 0 = 01
1 + 1 = 10
```

Designing a 1-bit adder

- Remember how we did binary addition:
  - Add the **two bits**
  - Do we have a **carry-in** for this bit?
  - Do we have to **carry-out** to the next bit?



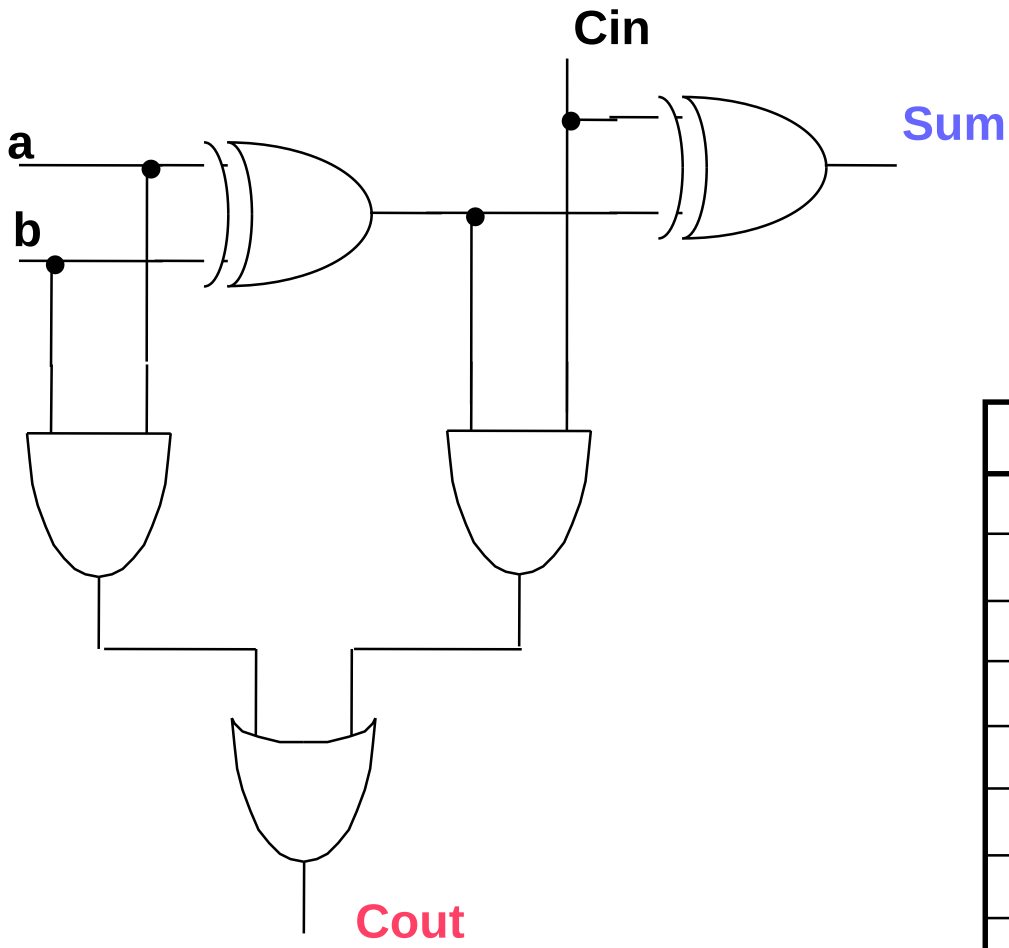
Designing a 1-bit adder

- So we'll need to add three bits (including carry-in)
- Two-bit output is the **carry-out** and the **sum**

a		b		C_{in}		carry-out, sum
0	+	0	+	0	=	00
0	+	0	+	1	=	01
0	+	1	+	0	=	01
0	+	1	+	1	=	10
1	+	0	+	0	=	01
1	+	0	+	1	=	10
1	+	1	+	0	=	10
1	+	1	+	1	=	11

Turn into expression,
simplify,
circuit-ify,
yadda yadda yadda…

A 1-bit Full Adder

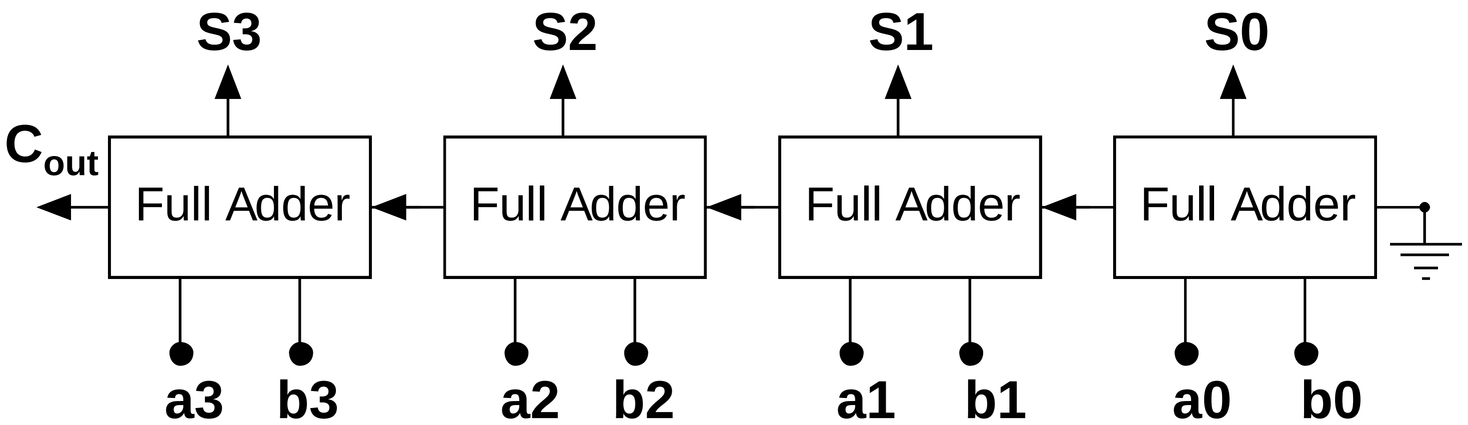



```
 01101100
 01101101
+00101100
---------
 10011001
```

a	b	C_{in}	Sum	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Logisim example
basic_logic.circ : full-adder

Example: 4-bit adder



Logisim example
basic_logic.circ : 4bit-adder

Subtraction

- How do we perform integer subtraction?
- What is the hardware?
  - Recall: hardware was why 2's complement was good idea

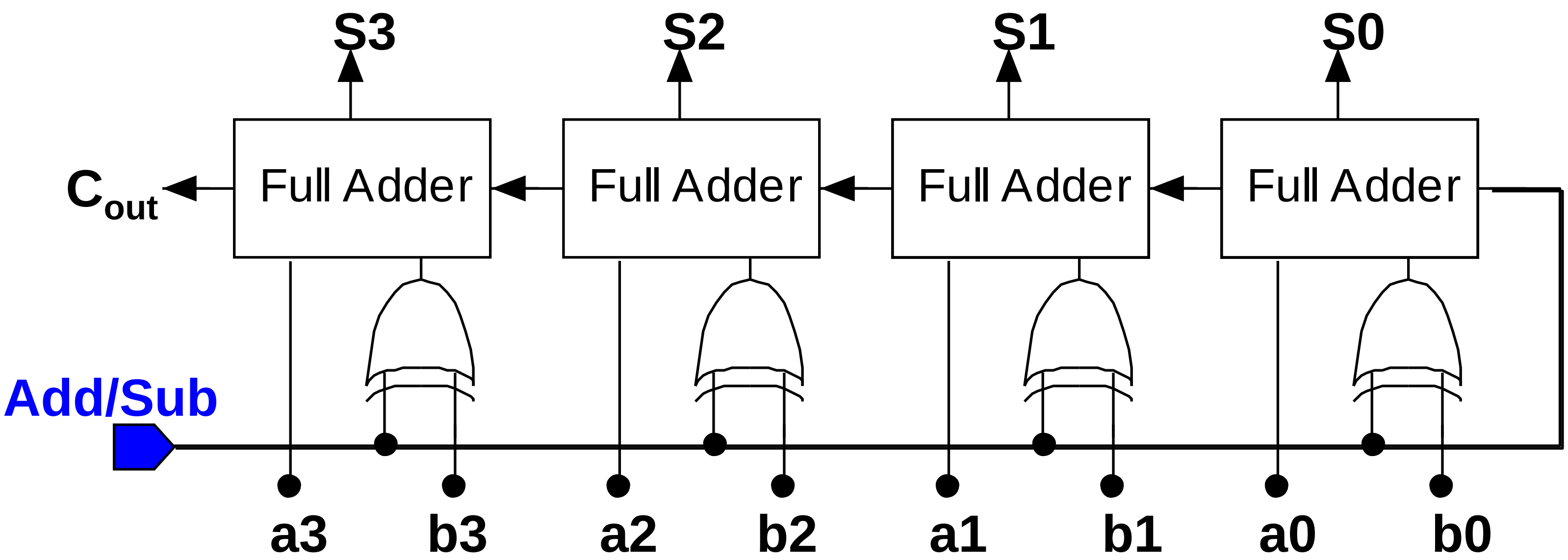
- Remember: Subtraction is just addition

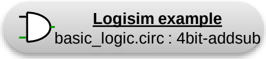
  X – Y =

  X + (-Y) =

  X + (~Y +1)

Example: Adder/Subtractor

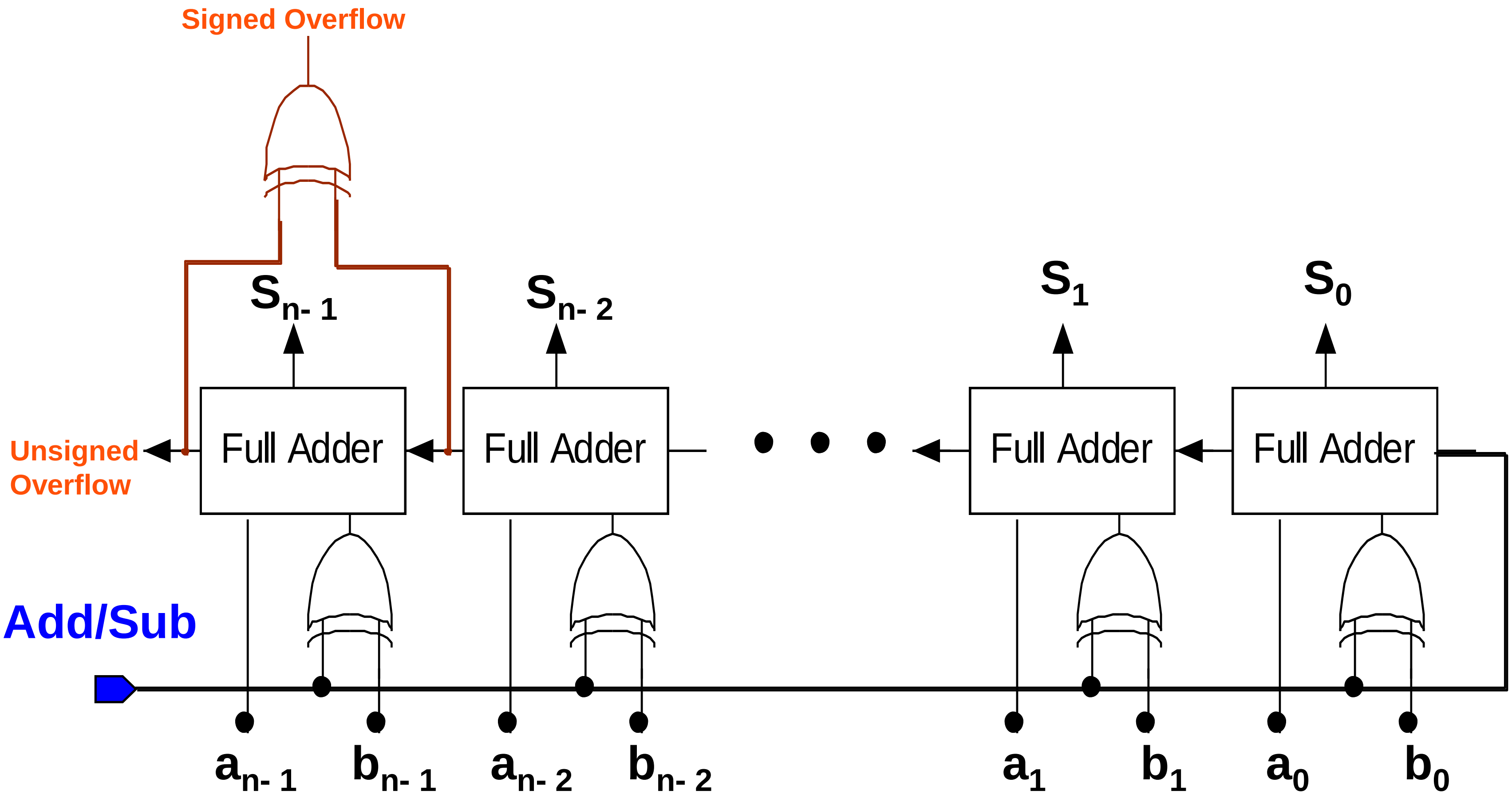




Overflow

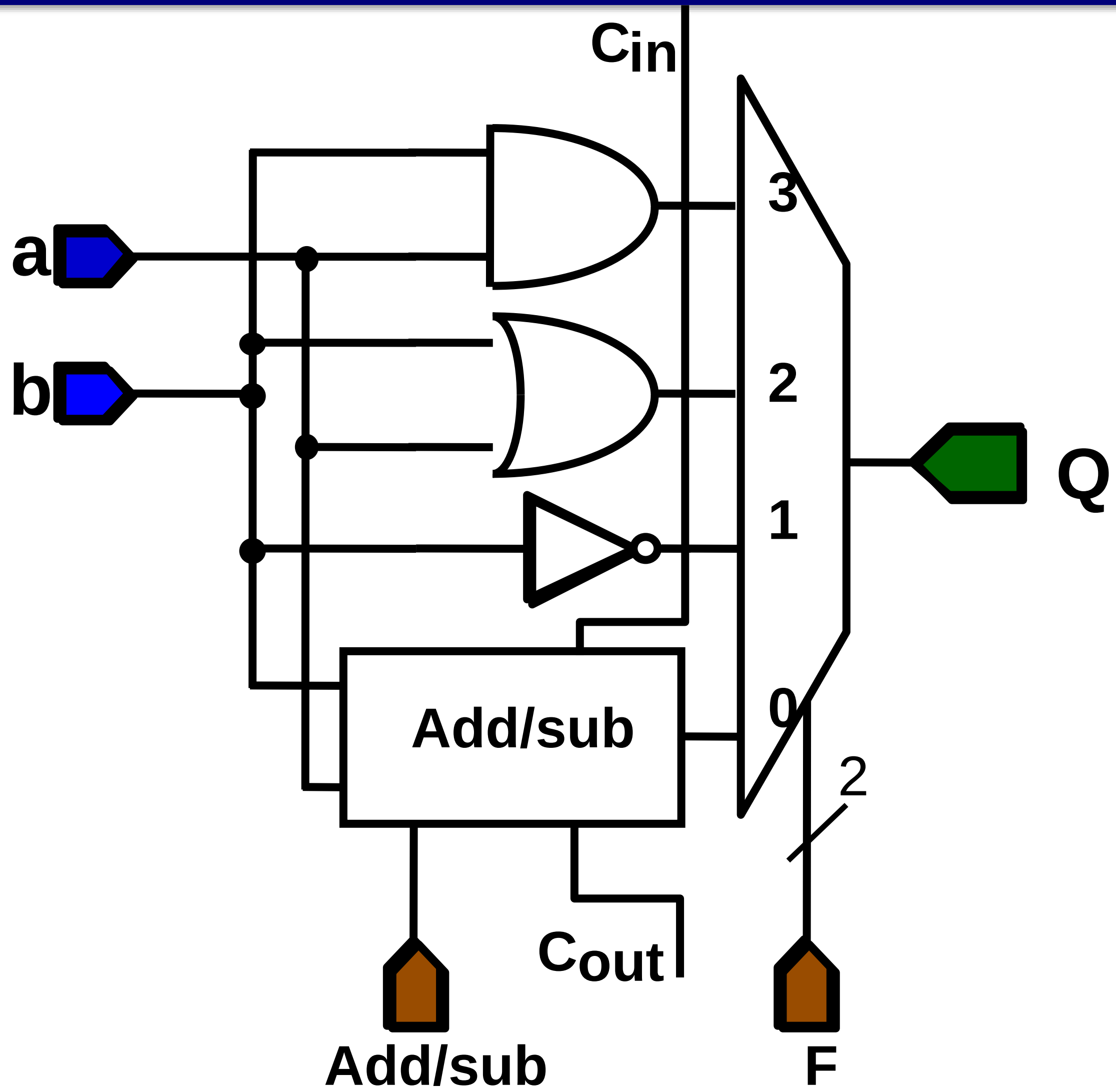
- We can detect **unsigned overflow** by looking at CO
- How would we detect **signed overflow**?
  - If adding positive numbers and result “is” negative
  - If adding negative numbers and result “is” positive
  - At most significant bit of adder, check if CI != CO
  - Can check with XOR gate

Add/Subtract With Overflow Detection



Logisim example
basic_logic.circ : 4bit-addsub2

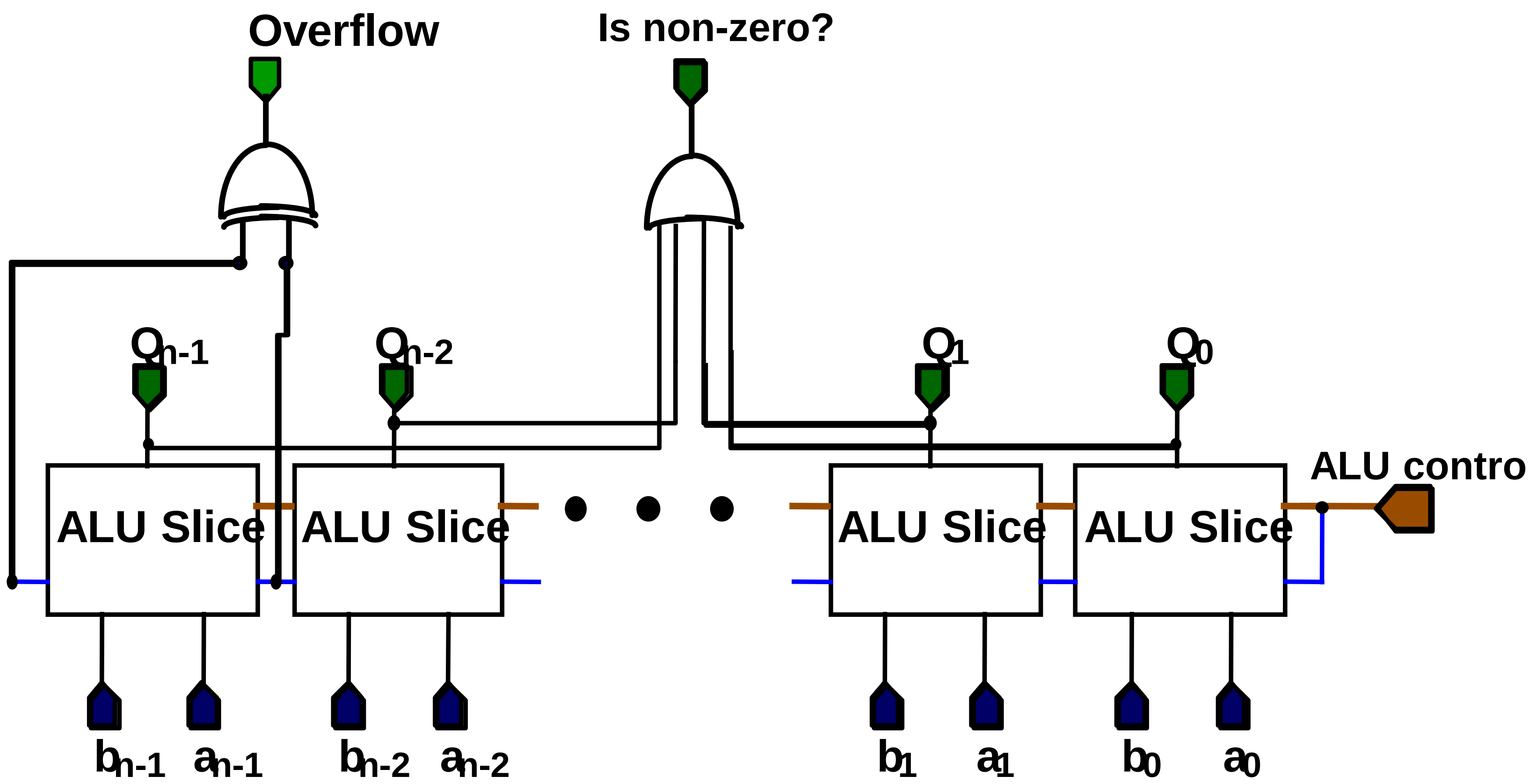
ALU Slice



A	F	Q
0	0	a + b
1	0	a - b
-	1	NOT b
-	2	a OR b
-	3	a AND b

Logisim example
basic_logic.circ : alu-slice

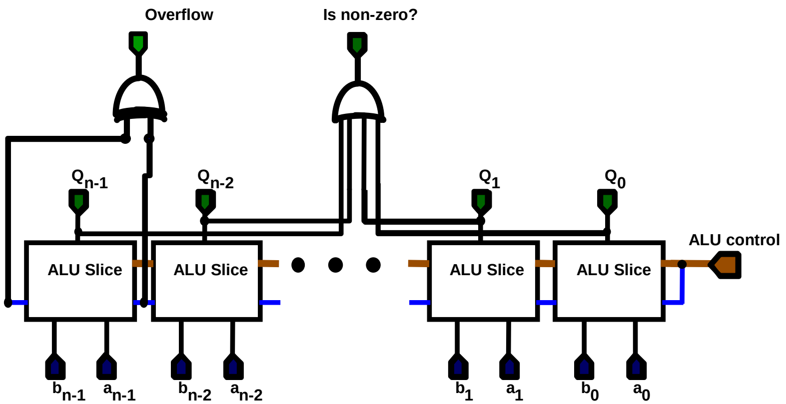
The ALU



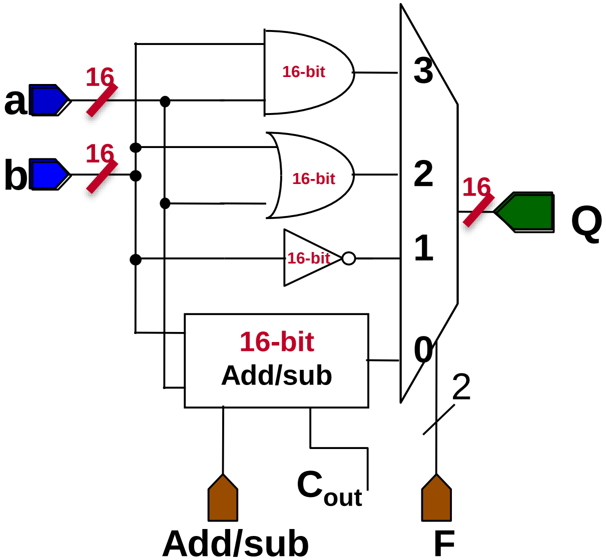
Logisim example
basic_logic.circ : alu

Alternate ALU design

- Previous design did ALU stuff for each bit, then chained them.

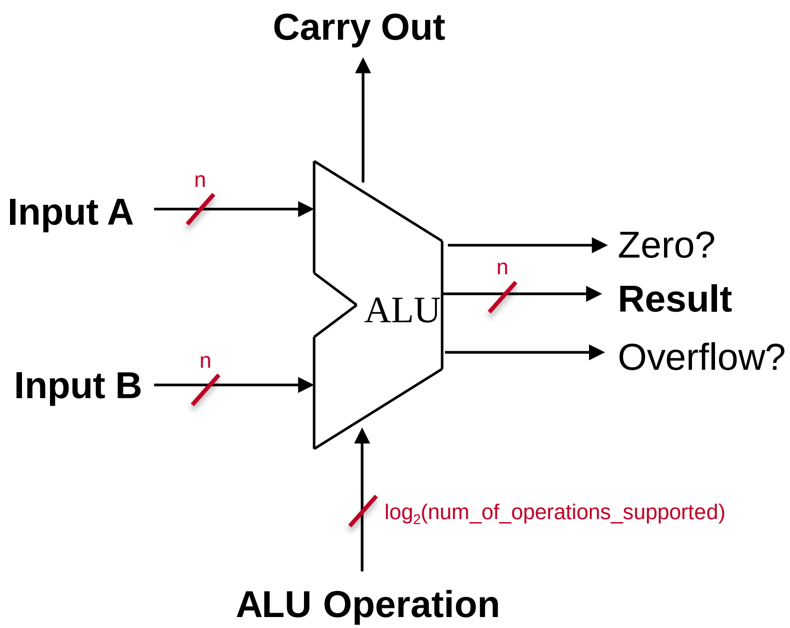


- Can also do each word-size operation and mux the resulting words.



Abstraction: The ALU

- General structure
- Two operand inputs
- Control inputs



- We can build circuits for
  - Multiplication
  - Division
  - They are more complex

Another Operations We Might Want: Shift

- Remember the << and >> operations?
  - Shift left/shift right?
  - How would we implement these?
- Suppose you have an 8-bit number

  $b_7b_6b_5b_4b_3b_2b_1b_0$
- And you can shift it left by a 3-bit number

  $s_2s_1s_0$

- Option 1: Truth Table?
  - 2^{11} = 2048 rows? Yuck.

    …but you can do it. Truth table gives this expression for output bit 0:

(b0 & !b1 & !b2 & !b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & !b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & b5 & !b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & !b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & b5 & b6 & !b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & 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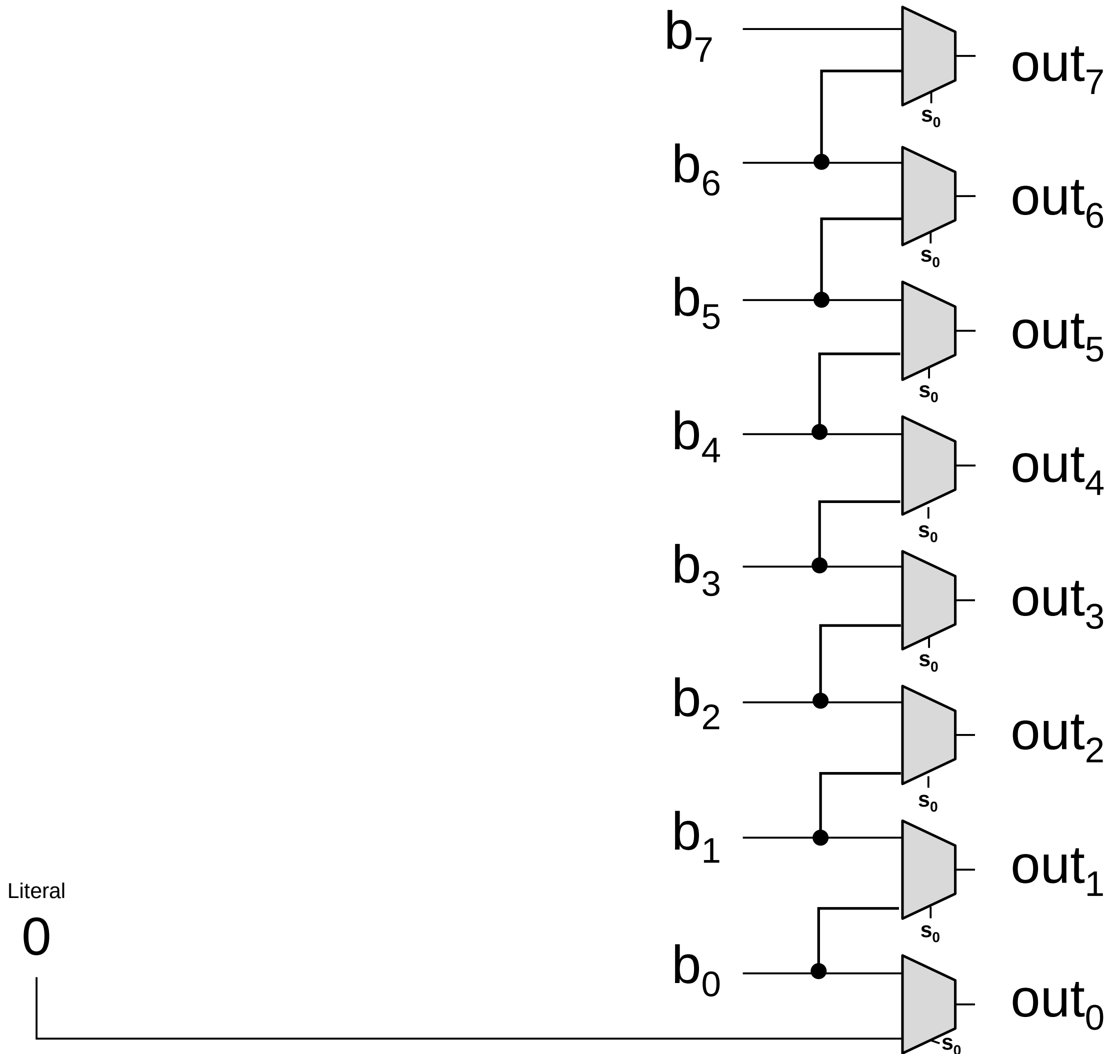
!s2) | (b0 & !b1 & b2 & b3 & b4 & b5 & !b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & b5 & !b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & !b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & !b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & !b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & !b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & !b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & !b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & !b2 & b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & !b2 & b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & !b1 & b2 & b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2) | (b0 & b1 & b2 & b3 & b4 & b5 & b6 & b7 & !s0 & !s1 & !s2)

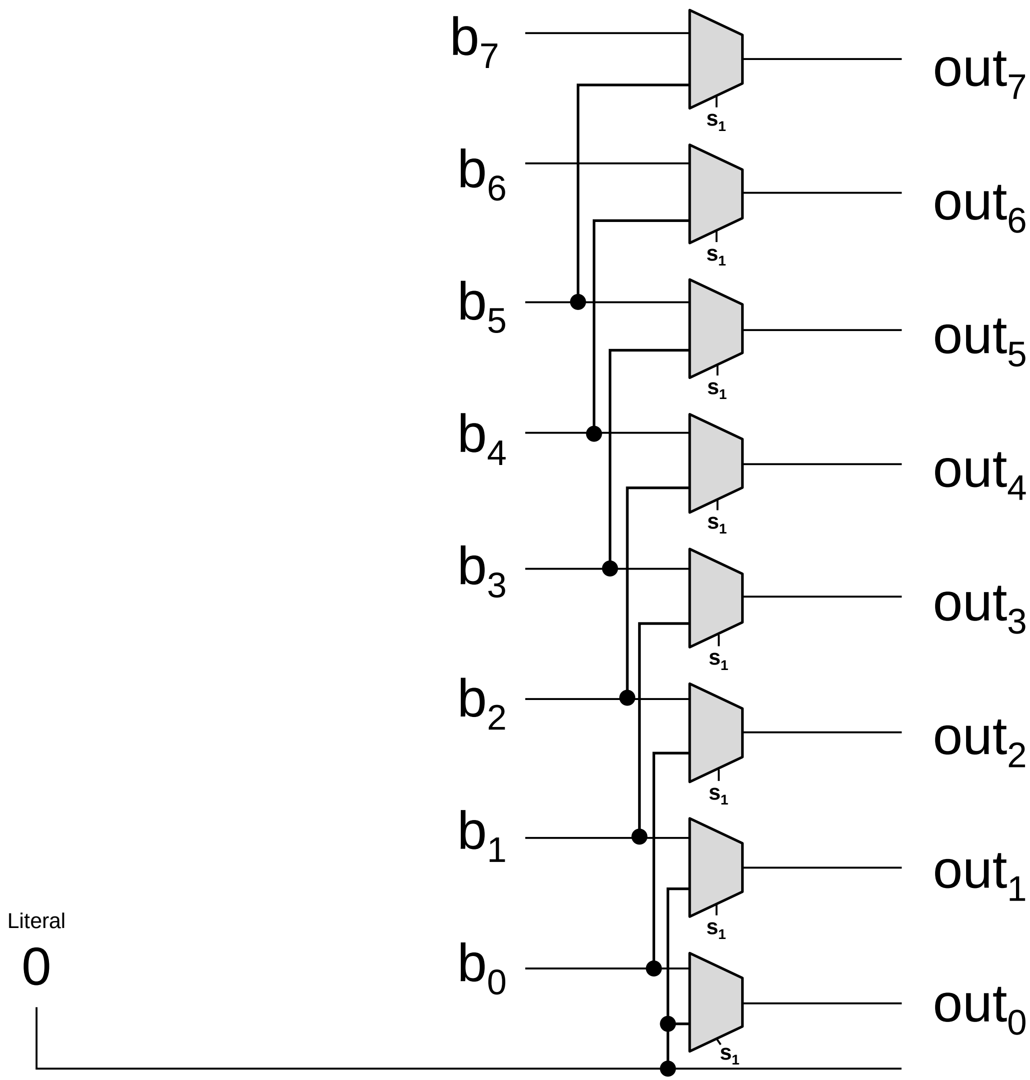
Building a bit shifter

- Simpler problem: A shift-by-one circuit, all controlled by the same 1 bit input (s_0)



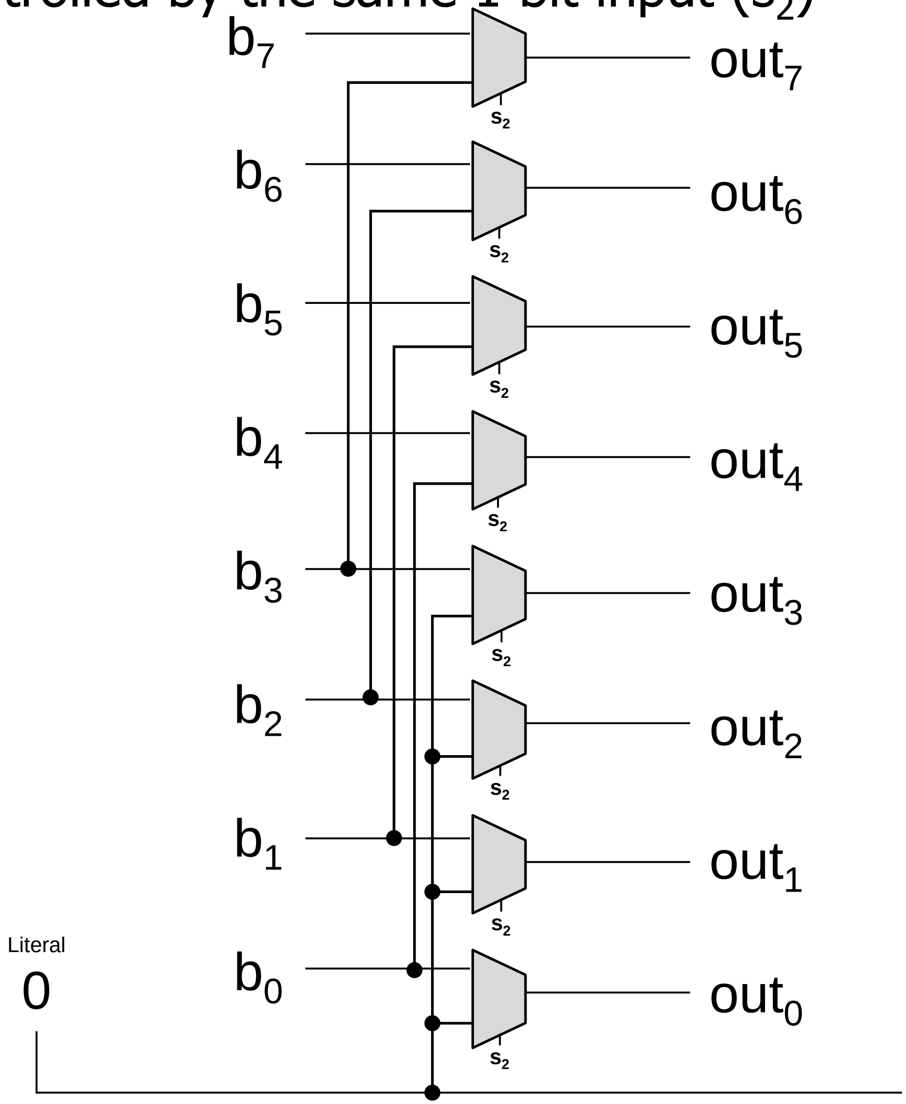
Building a bit shifter

- Simpler problem: A shift-by-two circuit, all controlled by the same 1 bit input (s_1)



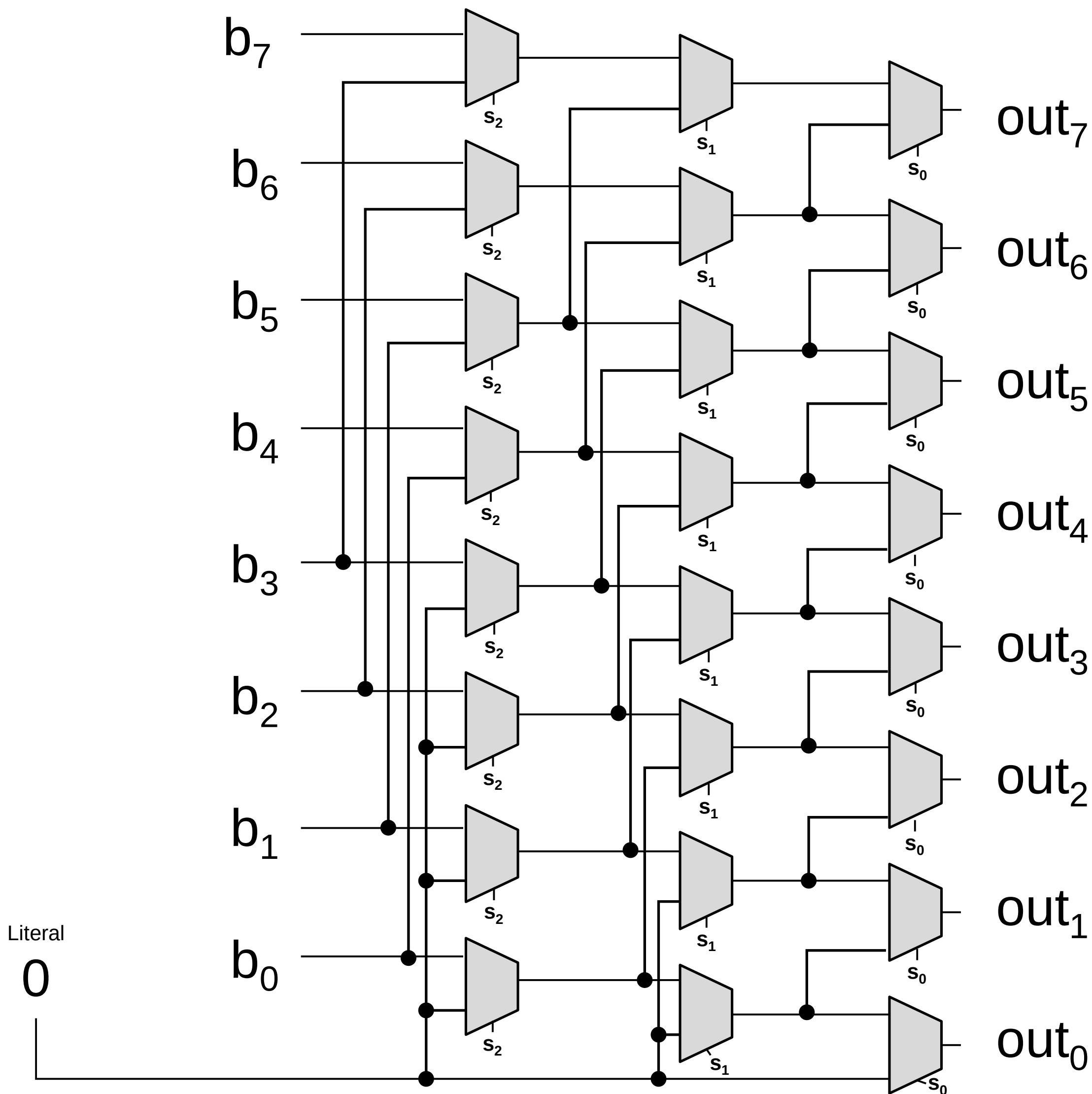
Building a bit shifter

- Simpler problem: A shift-by-four circuit, all controlled by the same 1 bit input (s_2)



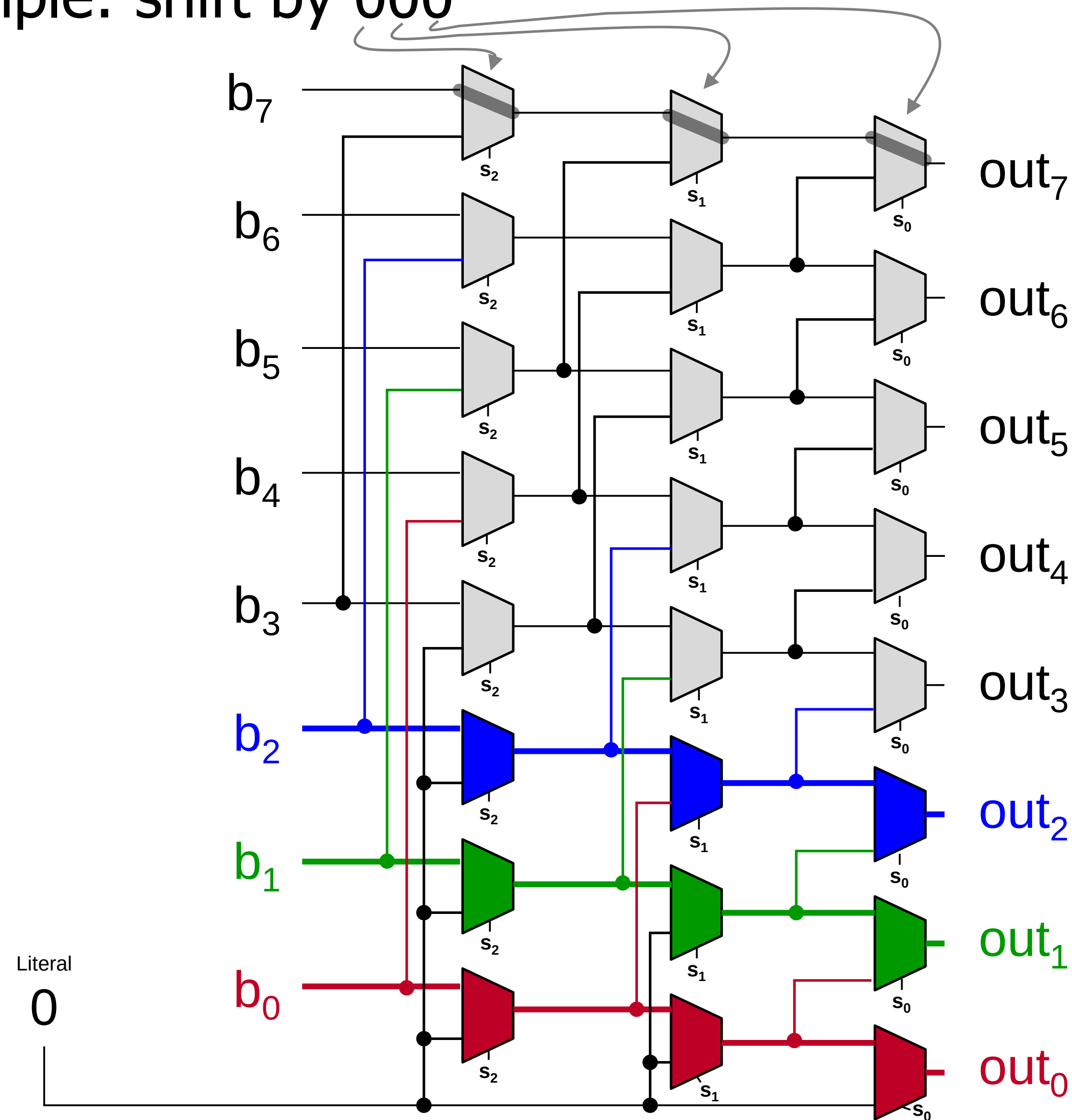
Now shifted by 3-bit number

- Full problem: stick them all together, controlled by 3-bit value $s_{2:0}$



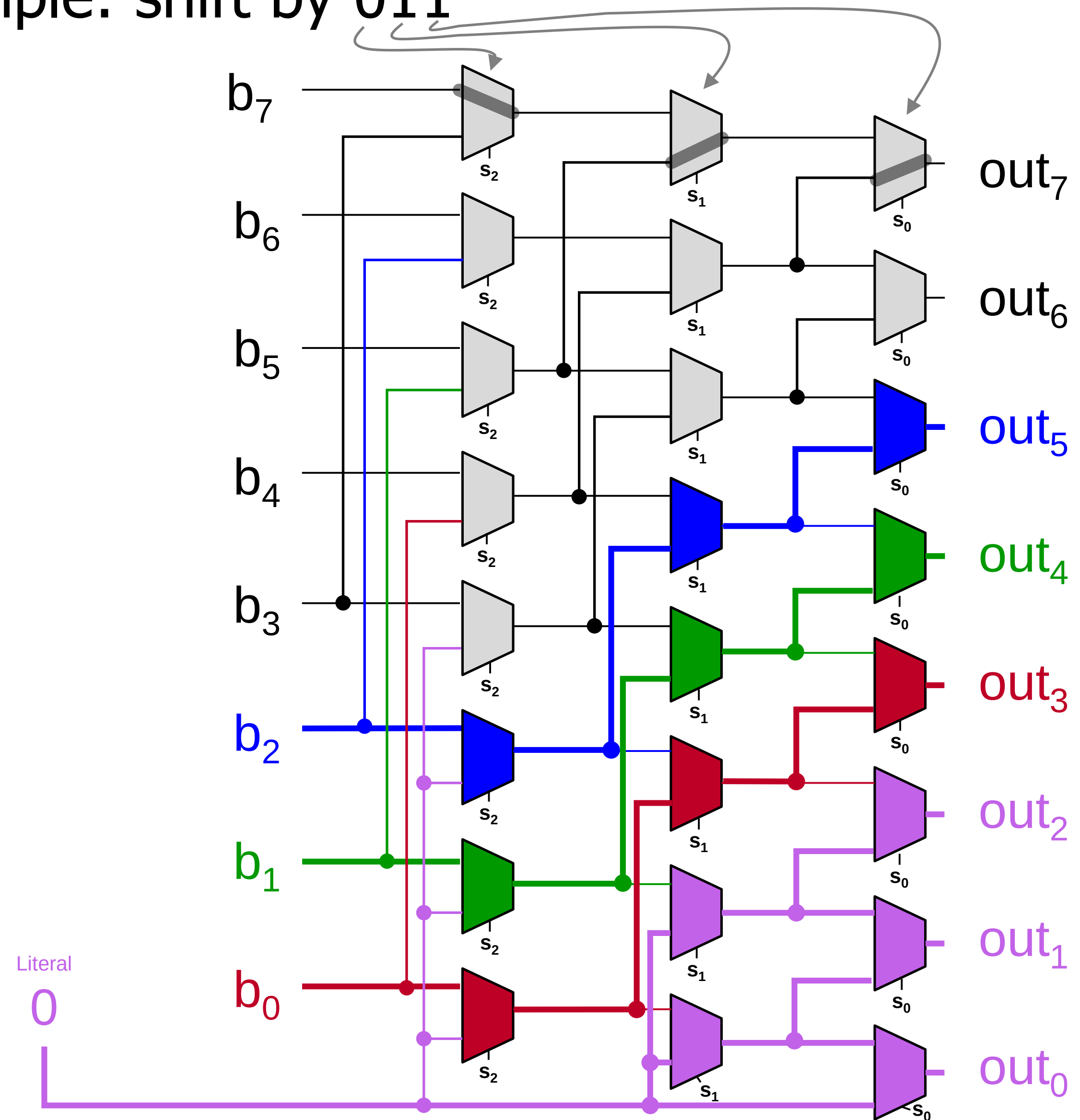
Now shifted by 3-bit number

- Example: shift by 000



Now shifted by 3-bit number

- Example: shift by 011



Summary

- Boolean Algebra & functions
- Logic gates (AND, OR, NOT, etc)
- Multiplexors
- Adder
- Arithmetic Logic Unit (ALU)
- Bit shifting