Activist Trading Dynamics\textsuperscript{a}

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Insead, Fontainebleau and Singapore
September 6 2023

\textsuperscript{a}The views expressed in this paper are those of the authors and do not necessarily represent the position of the Federal Reserve Bank of New York or the Federal Reserve System.
background
Activist Shareholders

Blockholders that attempt to influence how firms are run.
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- In 2020, average market cap of a target was $21 billion
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- **Hedge funds**: trade and intervene
  - Trian, Third Point LLC, Elliott Mgmt, Pershing Square, Starboard...
  - U.S., 1994-2018: +900 funds, targeting +3000 firms, +4600 events (Brav et al. 2022)
Activist Shareholders

What?

- Capital structure (dividends, buybacks, debt/equity issuance)
- Business strategy (cut costs, sell divisions...)
- Corporate governance (exec comp., board composition, oust CEO...)

How?

- "Exit:" selling shares → threat (e.g., Admati & Pfleiderer 2009; Edmans, 2009; Edmans & Manso, 2011)
- "Voice:" interventions beyond voting (e.g., Shleifer and Vishny, 1986; Kahn & Winton (1998); Maug, 1998)
- Communication, formal proposals, proxy contests, lawsuits, etc.

"Voice" is a costly activity → effort

- Proxy advisors, research, consultants, legal fees → $	ext{MM}$ (Gantchev, 2013)

... that benefits all shareholders → public goods problem (e.g., Berle and Means, 1932)
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.arrowhead \Rightarrow \text{mechanisms by which activists steer others to add value?}

skin in the game: block size

This paper: market-based mechanism involving strategic block (de-)accumulation

- minimal elements to generate dynamics
Model: “Stackelberg model of trading and activism”

1) Leader (L) and follower (F) with initial positions $X_i^0$, $i = L, F$, on a firm's stock. Initial “blocks” are private information (types; Back et al, 2018).

2) • Period 1: L orders $\theta_L \in \mathbb{R}$ units, which generates a public order flow $\Psi_1 = \theta_L + \sigma Z_1$, where $\sigma > 0$ and $Z_1 \sim N(0, 1)$ ⊥ $X_i^0$. Executed at $P_1 = E[\text{firm's value}|F \Psi_1]$ (Kyle '85; “market-maker” (MM)).

• Period 2: Exactly as 1, but F ↔ L...

• Period 3: activists simultaneously exert costly effort $W_i \in \mathbb{R}$. ⇝ Total payoff: $(W_i + W - i) |\{z\} \text{share value } X_i^T - P_t(i) \theta_i - 1^2(W_i)$.
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$$X^i_0 \sim N(\mu, \phi) \text{ and } \text{Cov}[X^L_0, X^F_0] = \rho \in [-\phi, \phi]$$

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\[ \sim \text{ Total payoff: } \left( W_i + W^{-i} \right) X_T^i - P_{t(i)} \theta^i - \frac{1}{2} (W^i)^2 \]
Main Finding and Roadmap

Dynamics + endogenous firm value $\leadsto$ **linear eqbm. w/manipulation motive**

- $L$ steers $F$ to build stakes; departure from Kyle '85 and subsequent work
- We derive predictions about outcomes...
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Plan


2. Equilibrium analysis: manipulation dynamics

3. Predictions: market outcomes & first-mover advantages

4. Hedge fund activism: \textit{wolf packs}

5. Other linear equilibria & refinement
Firm’s value is exogenous $v \sim N(\mu, \phi)$; private information to an “insider”
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• MM observes \( \Psi = \theta + \sigma Z \) to set execution price \( P \).
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• MM observes $\Psi = \theta + \sigma Z$ to set execution price $P$. Insider’s objective is

$$\max_{\theta \in \mathbb{R}} \mathbb{E}[v(X_0 + \theta) - P\theta]$$
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Static Kyle ’85

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- Optimal trading + correct price/belief (i.e., $P = \mathbb{E}[v|\Psi]$)

$$\theta = \sqrt{\frac{\sigma^2}{\phi}} (v - \mu) \quad \text{and} \quad \Lambda = \frac{\alpha_K \phi}{\alpha_K^2 \phi + \sigma^2} = \frac{\text{Cov}[v, \Psi]}{\text{Var}[\Psi]}$$
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\]

\( \alpha^K := \)

\[ \therefore \text{"Gap" strategy } \Rightarrow \mathbb{E}[\theta] = 0: \text{“unpredictable”} \]
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- Back et. al. (2018) has single activist who has a “gap” strategy
- In our model, because of externalities, the leader does not have a gap strategy (we will return to why)
Back to our setting: incentives

\[ v\big(X^i_0 + \theta\big) - P_{t(i)}\theta^i \]
Back to our setting: incentives

$$(W^L + W^F)X^i_T - P_{t(i)}\theta^i - \frac{1}{2}(W^i)^2$$
Back to our setting: incentives

\[(X^i_T + X^T_i)X^i_T - P_{t(i)}\theta^i - \frac{1}{2}(X^i_T)^2\]

- larger blocks $\rightarrow$ more effort (stronger intervention)
- public-goods problem is at play
- static incentives to trade: (i) stronger due to own effort $(X^i_T X^i_T)$ and (ii) stronger/weaker depending on the other activist $(X^i_T X_{-i}^T$; linked to $\rho$)
- dynamic incentives: leader with larger blocks benefit more from follower’s effort $(X^{L}_T X^{F}_T) \rightarrow$ applied to more shares
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Back to our setting: correlation positions

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![Graph showing Log Market Value vs Log Shares Outstanding]

2. $\rho > 0$ more likely as market capitalization grows
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2. $\rho > 0$ more likely as market capitalization grows

3. Presence of large short and long positions indicative of $\rho < 0$
equilibrium analysis
An **equilibrium** is a pair of trading strategies \((\theta^L, \theta^F)\) and pricing rules \((P_1, P_2)\) s.t.
Linear equilibrium

An equilibrium is a pair of trading strategies \((\theta^L, \theta^F)\) and pricing rules \((P_1, P_2)\) s.t.

- \(\theta^i\) is optimal given \((\theta^{i^{-}}, P_1, P_2)\), \(i = L, F\)

\[ \hat{\theta}^i \text{ is optimal given } (\theta^{i^{-}}, P_1, P_2), i = L, F \]

Linear equilibrium:

\[ \hat{\theta}^L = \alpha^L X^L_0 + \delta^L \mu \]

\[ \hat{\theta}^F = \alpha^F X^F_0 + \beta^F P_1 + \delta^F \mu \]

\[ \hat{P}_t = E[W^L_t + W^F_t | \Psi_t], t = 1, 2 \] when \(\theta^i\) drives \(\Psi_t\) \((i)\)

Focus on linear equilibria with positive block sensitivity (PBS): \(\alpha^L > 0\) and \(\alpha^F > 0\)
An equilibrium is a pair of trading strategies \((\theta^L, \theta^F)\) and pricing rules \((P_1, P_2)\) s.t.

- \(\hat{\theta}^i\) is optimal given \((\theta^{-i}, P_1, P_2), i = L, F\)
- \(P_t = \mathbb{E}[W^L + W^F | \mathcal{F}_t^\Psi], t = 1, 2\) when \(\theta^i\) drives \(\Psi_t(i)\)
An **equilibrium** is a pair of trading strategies \((\theta^L, \theta^F)\) and pricing rules \((P_1, P_2)\) s.t.

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**Linear equilibrium:**

**Leader trades**
- \(\theta^L = \alpha^L X^L_0 + \delta^L \mu\)

**Follower trades**
- \(\theta^F = \alpha^F X^F_0 + \beta^F P_1 + \delta^F \mu\)

**Pricing rule**
- \(P_t\) is affine in \(\Psi_t\), \(t = 1, 2\) when \(\theta^i\) drives \(\Psi_{t(i)}\)
An \textbf{equilibrium} is a pair of trading strategies \((\theta_L, \theta^F)\) and pricing rules \((P_1, P_2)\) s.t.

- \(\hat{\theta}^i\) is optimal given \((\theta^{-i}, P_1, P_2)\), \(i = L, F\)
- \(P_t = \mathbb{E}[W^L + W^F | \mathcal{F}_t^\Psi], \ t = 1, 2\) when \(\theta^i\) drives \(\Psi_t(i)\)

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- Leader trades \(\theta^L = \alpha_L X^L_0 + \delta_L \mu\)
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Focus on linear equilibria with **positive block sensitivity (PBS):** \(\alpha_L > 0\) and \(\alpha_F > 0\)

- higher types accumulate more shares than low types (relatively)
Learning and pricing

$T = 0$: MM forms expectation $P_0$ of firm’s value, and activists form private beliefs
Learning and pricing

$T = 0$: MM forms expectation $P_0$ of firm’s value, and activists form private beliefs

$T = 1$: MM updates beliefs about both activists to set $P_1 = \mathbb{E}[X^L_T|\mathcal{F}_1^\Psi] + \mathbb{E}[X^F_T|\mathcal{F}_1^\Psi]$:

$\Lambda_1 = \alpha_L \phi_{\alpha_2 L} + \sigma^2$

Kyle’s $\Lambda_1 \times 1 + \alpha_L + \rho (1 + \alpha_F)/\phi_1 - \beta F$
Learning and pricing

$T = 0$: MM forms expectation $P_0$ of firm’s value, and activists form private beliefs

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$$P_1 = P_0 + \Lambda_1 \left\{ \psi_1 - \mu(\alpha_L + \delta_L) \right\}, \text{ with}$$

$$\Lambda_1 := \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \times \frac{1 + \alpha_L + \rho(1 + \alpha_F)/\phi}{1 - \beta_F}$$

Kyle’s $\Lambda$ endogenous firm value
Learning and pricing

\( T = 0 \): MM forms expectation \( P_0 \) of firm’s value, and activists form private beliefs

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\[
P_1 = P_0 + \Lambda_1 \{\Psi_1 - \mu(\alpha_L + \delta_L)\} \quad \text{with}
\]

\[
\Lambda_1 := \frac{\alpha_L \phi}{\alpha_L \phi + \sigma^2} \times \frac{1 + \alpha_L + \rho(1 + \alpha_F) / \phi}{1 - \beta_F}
\]

Kyle’s \( \Lambda \times 1 \) + \( \alpha_L + \rho(1 + \alpha_F) / \phi \) / endogenous firm value
Learning and pricing

$T = 0$: MM forms expectation $P_0$ of firm’s value, and activists form private beliefs

$T = 1$: MM updates beliefs about both activists to set $P_1 = \mathbb{E}[X_T^L | \mathcal{F}_1^\Psi] + \mathbb{E}[X_T^F | \mathcal{F}_1^\Psi]$:

$$P_1 = P_0 + \Lambda_1 \{\Psi_1 - \mu(\alpha_L + \delta_L)\}, \text{ with}$$

$$= \Psi_1 - \mathbb{E}[\alpha_L X_0^L + \delta_L \mu]$$

$$\Lambda_1 := \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \times \frac{1 + \alpha_L + \rho(1 + \alpha_F)/\phi}{1 - \beta_F}$$

Kyle’s $\Lambda$  

endogenous firm value
Note the price at time 1 is given by

$$P_1 = \mathbb{E} \left[ (1 + \alpha_L)X_0^L + \delta_L \mu + (1 + \alpha_F)X_0^F + \beta_F P_1 + \delta_F \mu \mid \Psi_1 \right]$$

- Hence the price $P_1$ is on both sides of the equation, since the follower's trading strategy depends on $P_1$ leading to the denominator in the previous page.
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- Hence the price \( P_1 \) is on both sides of the equation, since the follower’s trading strategy depends on \( P_1 \) leading to the denominator in the previous page.

- With positive correlation, a high price in period 1 induces the follower to trade less and vice versa.
Explaining Pricing

Note the price at time 1 is given by

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Note the price at time 1 is given by

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- Hence the price \( P_1 \) is on both sides of the equation, since the follower’s trading strategy depends on \( P_1 \) leading to the denominator in the previous page.
- With positive correlation, a high price in period 1 induces the follower to trade less and vice versa.
- We will see that \( \beta_F \neq 1 \).
- The numerator comes from the fact that we are predicting \( X_0^L \) and from this \( X_0^F \).
Analysis: learning and pricing, continued

$T = 1$: Follower “inverts” $P_1$ to infer order flow $\Psi_1$ and update about the leader

- Forecast $W^F + W^L$; linear combination of $\Psi_1$ and $X_0^F$
Analysis: learning and pricing, continued

$T = 1$: Follower “inverts” $P_1$ to infer order flow $\Psi_1$ and update about the leader

- Forecast $W^F + W^L$; linear combination of $\Psi_1$ and $X^F_0$

$T = 2$: After observing $\Psi_2$, the MM updates again about both activists:
Analysis: learning and pricing, continued

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- Forecast $W^F + W^L$; linear combination of $\Psi_1$ and $X_0^F$

$T = 2$: After observing $\Psi_2$, the MM updates again about both activists:

$$P_2 = P_1 + \Lambda_2[\Psi_2 - (\alpha_F M_1^F + \beta_F P_1 + \delta_F \mu)],$$

with

$$\Lambda_2 = \frac{\alpha_F \gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} \times [1 + \alpha_F + \rho_1 / \gamma_1^F]$$

where $M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1^\Psi]$, $\gamma_1^F := \text{Var}(X_0^F | \mathcal{F}_1^\Psi)$, $\rho_1 := \text{Cov}(X_T^L, X_0^F | \mathcal{F}_1^\Psi)$
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Wedge admits analog interpretation. Leader doesn’t need to update using $P_2$
Trading: first-order conditions

The activists’ FOCs are

\[ F : \quad 0 = -\mathbb{E}_F[P_1 + \Lambda_2\{\Phi_2 - \mathbb{E}[\Phi_2|\mathcal{F}_1]\}|\theta] - \theta\Lambda_2 + \mathbb{E}_F[X^E_T + X^L_T|\theta] \]

\[ L : \quad 0 = -\mathbb{E}_L[P_0 + \Lambda_1\{\Phi_1 - \mathbb{E}[\Phi_1]\}|\theta] - \theta\Lambda_1 + \mathbb{E}_L[X^E_T + X^L_T|\theta] \]

\[ + X^L_T \frac{\partial\mathbb{E}_L[X^E_T|\theta]}{\partial \theta} \]
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F : \quad 0 = -\mathbb{E}_F[P_1 + \Lambda_2 \{ \Psi_2 - \mathbb{E}[\Psi_2 | F_1] \}]|\theta| - \theta \Lambda_2 + \mathbb{E}_F[X_T^F + X_T^L]|\theta|
\]

\[
L : \quad 0 = -\mathbb{E}_L[P_0 + \Lambda_1 \{ \Psi_1 - \mathbb{E}[\Psi_1] \}]|\theta| - \theta \Lambda_1 + \mathbb{E}_L[X_T^F + X_T^L]|\theta|
\]

\[
+ X_T^L \frac{\partial \mathbb{E}_L[X_T^F]|\theta]}{\partial \theta}
\]

- Cost of last unit (expected price)
The activists’ FOCs are

\[ F : \quad 0 = -\mathbb{E}_F[P_1 + \Lambda_2 \{ \Psi_2 - \mathbb{E}[\Psi_2|\mathcal{F}_1]\}|\theta] - \theta \Lambda_2 + \mathbb{E}_F[X_F^T + X_L^T|\theta] \]

\[ L : \quad 0 = -\mathbb{E}_L[P_0 + \Lambda_1 \{ \Psi_1 - \mathbb{E}[\Psi_1]\}|\theta] - \theta \Lambda_1 + \mathbb{E}_L[X_F^T + X_L^T|\theta] \]

\[ + X_L^T \frac{\partial \mathbb{E}_L[X_F^T|\theta]}{\partial \theta} \]

- Cost of last unit (expected price)
- **Price impact** on all inframarginal units
Trading: first-order conditions

The activists’ FOCs are

\[ F : \quad 0 = -\mathbb{E}_F[P_1 + \Lambda_2 \{ \Psi_2 - \mathbb{E}[\Psi_2 | \mathcal{F}_1]\}] | \theta \] 
\[ - \theta \Lambda_2 + \mathbb{E}_F[X^F_T + X^L_T | \theta] \]

\[ L : \quad 0 = -\mathbb{E}_L[P_0 + \Lambda_1 \{ \Psi_1 - \mathbb{E}[\Psi_1]\}] | \theta \] 
\[ - \theta \Lambda_1 + \mathbb{E}_L[X^E_T + X^L_T | \theta] \]

\[ + X^L_T \frac{\partial \mathbb{E}_L[X^E_T | \theta]}{\partial \theta} \]

- Cost of last unit (expected price)
- Price impact on all inframarginal units
- Expected value of marginally higher block (effort is at an optimum)
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\[
F : \quad 0 = -E_F[P_1 + \Lambda_2 \{ \Psi_2 - E[\Psi_2 | F_1^\psi] \}]|\theta] - \theta \Lambda_2 + E_F[X_F^T + X_L^T | \theta] \\
L : \quad 0 = -E_L[P_0 + \Lambda_1 \{ \Psi_1 - E[\Psi_1] \}]|\theta] - \theta \Lambda_1 + E_L[X_F^T + X_L^T | \theta] \\
+ X_L^T \frac{\partial E_L[X_F^T | \theta]}{\partial \theta}
\]

- Cost of last unit (expected price)
- Price *impact* on all inframarginal units
- Expected value of marginally higher block (effort is at an optimum)
- Leader’s *value of manipulation*
  - discrepancy wrt Kyle stems from endogeneity + non-trivial cont. value
Second-order conditions:

\[ 1 - 2\Lambda_2 < 0 \quad \text{for follower} \]
\[ 1 - 2\Lambda_1 (1 - \beta_F) < 0 \quad \text{for leader} \]
Trading: second-order conditions

Second-order conditions:

\[
\begin{align*}
1 - 2\Lambda_2 &< 0 \quad \text{for follower} \\
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\end{align*}
\]

- 1: endogenous fundamentals (extra convexity)
Second-order conditions:

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- \( \Lambda_1 \): endogenous fundamentals (extra convexity)
- \( \Lambda_1(1 - \beta_F) \): leader’s effective price impact due to manipulation
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- 1: endogenous fundamentals (extra convexity)
- \( \Lambda_1 (1 - \beta_F) \): leader’s effective price impact due to manipulation
- find candidate equilibrium and check \( \beta_F \neq 1 \) ex post
Follower's equilibrium trading

Recall that $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$ and $M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1^\psi]$, $\gamma_1^F := \text{Var}(X_0^F | \mathcal{F}_1^\psi)$.
Recall that \( \theta^F = \alpha^F X_0^F + \beta^F P_1 + \delta^F \mu \) and \( M_1^F := \mathbb{E}[X_0^F | \mathcal{F}^\psi_1], \gamma_1^F := \text{Var}(X_0^F | \mathcal{F}^\psi_1) \)

### Proposition

In any PBS equilibrium:

1. \( \alpha^F = \sqrt{\frac{\sigma^2}{\gamma_1^F}} \),
2. \( \beta^F < 1 \) with \( \text{sign}(\beta^F) = -\text{sign}(\rho) \) and
3. \( \delta^F < 0 \).

Further, \( \theta^F \) admits the “gap” representation \( \theta^F = \alpha^F (X_0^F - M_1^F) \).
Follower's equilibrium trading

Recall that \( \theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu \) and \( M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1^\psi], \gamma_1^F := \text{Var}(X_0^F | \mathcal{F}_1^\psi) \)

**Proposition**

In any PBS equilibrium:

(i) \( \alpha_F = \sqrt{\frac{\sigma^2}{\gamma_1^F}} \),
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(iii) \( \delta_F < 0 \).

Further, \( \theta^F \) admits the “gap” representation \( \theta^F = \alpha^F (X_0^F - M_1^F) \).

- high \( P_1 \) indicative of high \( X_0^L \) → high \( M_1^F \) if \( \rho > 0 \) → \( F \) trades less → \( \beta_F < 0 \)
- ... → low \( M_1^F \) if \( \rho < 0 \) → \( F \) trades more → \( \beta_F > 0 \)
1) Why a gap on initial information?
Interpretation of $\theta^F = \sqrt{\frac{\sigma^2_F}{\gamma_1}} (\theta - M^F_1)$

1) Why a gap on initial information? Traditionally

$$\text{trade} \propto \text{mispricing} = \text{[private belief about firm value]} - \text{[public belief]}$$
Interpretation of $\theta^F = \sqrt{\frac{\sigma^2}{\gamma_1}} (\theta - M_1^F)$

1) Why a gap on initial information? Traditionally

\[ \text{trade} \propto \text{mispricing} = [\text{private belief about firm value}] - [\text{public belief}] \]

- Linear strategies + Gaussian learning: $\mathbb{E}_F[W^i] - \mathbb{E}[W^i | F_1^\Psi] \propto X_0^F - M_1^F$
Interpretation of $\theta^F = \sqrt{\frac{\sigma^2}{\gamma_1^F}} (\theta - M_1^F)$

1) Why a gap on initial information? Traditionally

\[ \text{trade} \propto \text{mispicing} = [\text{private belief about firm value}] - [\text{public belief}] \]

- Linear strategies + Gaussian learning: $\mathbb{E}_F[W^i] - \mathbb{E}[W^i | \mathcal{F}^\Psi_1] \propto X_0^F - M_1^F$

2) Why Kyle's weight? Recall $\Lambda_2$ is scaled up by $1 + \alpha_F + \rho_1/\gamma_1^F$...

- Follower's expectation of firm value is also scaled by the same factor...
- Price impact wedge reflects a change in marginal incentives of the same size
Corollary: i.i.d. case

If $\rho = 0$, MM learns nothing about the follower from $\Psi_1$: $M_1^F = \mu$ and $\gamma_1^F = \phi$. 
Corollary: i.i.d. case

If $\rho = 0$, MM learns nothing about the follower from $\Psi_1$: $M_1^F = \mu$ and $\gamma_1^F = \phi$

- $\theta^F = \sqrt{\frac{\sigma^2}{\phi}} (X_0^F - \mu)$
If $\rho = 0$, MM learns nothing about the follower from $\Psi_1$: $M^F_1 = \mu$ and $\gamma^F_1 = \phi$

- $\theta^F = \sqrt{\frac{\sigma^2}{\phi}} (X^F_0 - \mu)$

- $F'$s trade is independent of the history of the game...
If $\rho = 0$, MM learns nothing about the follower from $\Psi_1$: $M_1^F = \mu$ and $\gamma_1^F = \phi$

- $\theta^F = \sqrt{\frac{\sigma^2}{\phi}} (X^F_0 - \mu)$

- $F$'s trade is independent of the history of the game...

- So the leader’s problem is identical: $\theta_L = \sqrt{\frac{\sigma^2}{\phi}} (X^L_0 - \mu)$
Corollary: i.i.d. case

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This equilibrium is unique within the linear class
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This equilibrium is unique within the linear class

$\therefore$ with Stackelberg structure, need non-trivial $\rho$ to get dynamics...
Proposition

Fix $\sigma, \phi > 0$. There exists $\rho \in (-\phi, 0)$ s.t. for all $\rho \in [\rho, \phi]$, there is a unique PBS equilibrium. There, $\theta^L = \alpha^L X^L_0 + \delta^L \mu$, where $\alpha^L > 0$ and $\delta^L < 0$ and

- if $\rho > 0$, then $\alpha^L < \alpha^K$ and $\delta^L < -\alpha^K$
- if $\rho < 0$, then $\alpha^L > \alpha^K$ and $\delta^L > -\alpha^K$

Further, both $\alpha^L$ and $\delta^L$ are decreasing in $\rho$. 
Proposition

Fix $\sigma, \phi > 0$. There exists $\underline{\rho} \in (-\phi, 0)$ s.t. for all $\rho \in [\underline{\rho}, \phi]$, there is a unique PBS equilibrium. There, $\theta^L = \alpha_L X_0^L + \delta_L \mu$, where $\alpha_L > 0$ and $\delta_L < 0$ and

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Further, both $\alpha_L$ and $\delta_L$ are decreasing in $\rho$. 

$$\alpha_L \neq |\delta_L|$$

is a generic property
PBS equilibrium: general case

\[ \begin{align*}
\alpha_L & \quad \delta_L & \quad \pm \alpha^K
\end{align*} \]
Recall the leader’s value of manipulation:

\[ X_L^T \frac{\partial \mathbb{E}_L[X^F_T|\theta^L]}{\partial \theta^L} = X_T^L \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \psi_1} = X_T^L \beta_F \Lambda_1 \]
Recall the leader’s value of manipulation:

\[ X^L_T \frac{\partial \mathbb{E}_L[X^E_T | \theta^L]}{\partial \theta^L} = X^L_T \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \psi_1} = X^L_T \beta_F \Lambda_1 \]

Consider \( \rho > 0 \):
\( \alpha_L \neq \alpha_K \) and the Value of Manipulation

Recall the leader’s value of manipulation:

\[
X_T^L \frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L} = X_T^L \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \psi_1} = X_T^L \beta_F \Lambda_1
\]

Consider \( \rho > 0 \):

- \( \beta_F < 0 \): all leader types deviate downward relative to Kyle
  - Drive \( P_1 \) (or \( M_1^F \)) downwards \( \rightarrow F \) acquires a larger position
  - \( F \)'s arbitrage opportunity: \( X_0^F - M_1^F \)

\( \alpha_L > 0 \): higher types end up with higher terminal blocks

\( \alpha_L < \alpha_K \): high types scale back more on absolute terms
Recall the leader’s value of manipulation:

\[ X_T^L \frac{\partial \mathbb{E}_L[X_T^F | \theta^L]}{\partial \theta^L} = X_T^L \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \Psi_1} = X_T^L \beta_F \Lambda_1 \]

Consider \( \rho > 0 \):

- \( \beta_F < 0 \): all leader types **deviate downward** relative to Kyle
  - Drive \( P_1 \) (or \( M_1^F \)) downwards \( \rightarrow F \) acquires a larger position
  - \( F \)'s arbitrage opportunity: \( X_0^F - M_1^F \)
- Higher types benefit more: they own more shares
Recall the leader’s value of manipulation:

\[ X_L^T \frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L} = X_L^T \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \Psi_1} = X_L^T \beta_F \Lambda_1 \]

Consider \( \rho > 0 \):

- \( \beta_F < 0 \): all leader types deviate downward relative to Kyle
  - Drive \( P_1 \) (or \( M_1^F \)) downwards \( \rightarrow F \) acquires a larger position
  - \( F \)'s arbitrage opportunity: \( X_0^F - M_1^F \)
- Higher types benefit more: they own more shares
  - \( \alpha_L > 0 \): higher types end up with higher terminal blocks
  - \( \alpha_L < \alpha^K \): high types scale back more on absolute terms
\( \delta_L \neq \delta_K \) and Price Impact \( \Lambda_1 \)

\[
\delta_L = \frac{1}{(1 - \beta_F)\Lambda_1} \times \left. \frac{\partial}{\partial \mu} \left( \mathbb{E}_L[W_L + W_F] - P_1 \right) \right|_{\text{sensitivity of arbitrage wrt } \mu}
\]
\( \delta L \neq \delta K \) and Price Impact \( \Lambda_1 \)

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sensitivity of arbitrage wrt \( \mu \)

1) \( \delta_L < 0 \): all types scale back as \( \mu \) grows; trivial when \( \nu \) exogenous
\( \delta_L \neq \delta_K \) and Price Impact \( \Lambda_1 \)

\[
\delta_L = \frac{1}{(1 - \beta_F)\Lambda_1} \times \left( \frac{\partial}{\partial \mu} (E[L[W_L + W_F] - P_1]) \right)
\]

sensitivity of arbitrage wrt \( \mu \)

1) \( \delta_L < 0 \): all types scale back as \( \mu \) grows; trivial when \( \nu \) exogenous

- Here: leader is less sensitive than MM to an increase in \( \mu \)
$\delta_L \neq \delta_K$ and Price Impact $\Lambda_1$

$$\delta_L = \frac{1}{(1 - \beta_F)\Lambda_1} \times \frac{\partial}{\partial \mu} \left( \mathbb{E}_L[W_L + W_F] - P_1 \right)$$

sensitivity of arbitrage wrt $\mu$

1) $\delta_L < 0$: all types scale back as $\mu$ grows; trivial when $\nu$ exogenous

- Here: leader is less sensitive than MM to an increase in $\mu$

- she uses both $\mu$ and private information to forecast firm’s value
$\delta_L \neq \delta_K$ and Price Impact $\Lambda_1$

\[
\delta_L = \frac{1}{(1 - \beta_F)\Lambda_1} \times \frac{\partial}{\partial \mu} \left( \mathbb{E}_L [W_L + W_F] - P_1 \right)
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2) $\delta_L < -\alpha_K$: Fixing $\rho > 0$, lower signaling ($\alpha$) $\Rightarrow$ lower $\Lambda_1 \Rightarrow$ less costly to scale back
\( \delta_L \neq \delta_K \) and Price Impact \( \Lambda_1 \)

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\delta_L = \frac{1}{(1 - \beta_F)\Lambda_1} \times \frac{\partial}{\partial \mu} \left( \mathcal{E}_L[W_L + W_F] - P_1 \right)
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2) \( \delta_L < -\alpha_K \): Fixing \( \rho > 0 \), lower signaling (\( \alpha \)) \( \Rightarrow \) lower \( \Lambda_1 \) \( \Rightarrow \) less costly to scale back

all types scale back along both types of info \( \Rightarrow \) symmetry breaks

\( \rho < 0 \): trade more aggressively
predictions
Proposition

*In the unique PBS equilibrium,*

1. **Order flow:** \( \mathbb{E}[\Psi_1] = \mathbb{E}[\theta^L] < 0 \) *iff* \( \rho > 0 \), while \( \mathbb{E}[\Psi_2] = 0 \)
Proposition

In the unique PBS equilibrium,

1. **Order flow**: \( E[\psi_1] = E[\theta^L] < 0 \) iff \( \rho > 0 \), while \( E[\psi_2] = 0 \)
   - Selling (buying) pressure when \( \rho > 0 \) (\( \rho < 0 \))
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In the unique PBS equilibrium,

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2. **Price impact**: $\partial \Lambda_1 / \partial \rho > 0$ in a neighborhood of $\rho = 0$
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3. **Firm value:** $\mathbb{E}[W^L + W^F] = (2 + \alpha_L + \delta_L) \mu < 2\mu$ iff $\rho > 0$; but always $> \mu$. 

Prediction on average prices: "abnormally" low iff $\rho > 0$.
Proposition

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   - if \( \rho > 0 \) (\( \rho < 0 \)), value is lower (higher) relative to no activism/trading world
   - leader brings additional value relative to lone activist (Becht et al., 2017)

Predictions: market outcomes
Proposition

In the unique PBS equilibrium,

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   - Prediction on average prices: “abnormally” low iff \( \rho > 0 \)
Incentive to become a leader? Examine $L$ and $F$ in one simultaneous trading round.
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Proposition

Suppose $\rho$ not too negative. There exists a unique symmetric PBS equilibrium

$$\theta^i = \sqrt{\frac{\sigma^2}{2\phi}}(X^i_0 - \mu).$$

If $\rho$ is near 0, the leader gets a higher expected payoff if she moves first.
Incentive to become a leader? Examine $L$ and $F$ in one simultaneous trading round.

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Suppose $\rho$ not too negative. There exists a unique symmetric PBS equilibrium

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If $\rho$ is near 0, the leader gets a higher expected payoff if she moves first.

- Competition effect: $2\sqrt{\frac{\sigma^2}{2\phi}} > \sqrt{\frac{\sigma^2}{\phi}}$
Predictions: first-mover advantages

$L$’s ex ante payoffs as $\rho$ varies:
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$\rho > 0$: moving first dominates; skip competition \& manipulation cost effective
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$\rho > 0$: moving first dominates; skip competition $\land$ manipulation cost effective

$\rho \ll 0$: simultaneous dominates: follower effectively a “supplier”

**Similarity:** our mechanism is plausible if $\rho$ is not too negative
Predictions: number of followers

\( N \) “normalized” followers: \( \mu/N, \phi/N^2 \) and \( \operatorname{Cov}[X_0^F, X_0^L] = \rho/N \rightarrow \) fixed uncertainty
Predictions: number of followers

$N$ “normalized” followers: $\mu/N$, $\phi/N^2$ and $\text{Cov}[X_0^F, X_0^L] = \rho/N \rightarrow$ fixed uncertainty

$\vdash$ isolate strategic effects (direct: less manipulation due to followers shrinking)
Predictions: number of followers

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$\therefore$ isolate **strategic effects** (direct: less manipulation due to followers shrinking)

**Proposition**

*Fix any $\rho \in (0, \phi]$. In the unique (symmetric) PBS equilibrium*

$$\theta^F = \alpha_F (X_0^F - M_{1}^F), \quad \text{where} \quad \alpha_F = \sqrt{\frac{\sigma^2}{N \gamma_1^F}}$$

Also: (i) $\alpha_F$ is increasing in $N$; (ii) $\alpha_L$ and the firm’s ex ante value are decreasing in $N$; and (iii) the leader’s ex ante payoff $\uparrow \sim \sqrt{N}$ asymptotically.
Predictions: number of followers

\( N \) “normalized” followers: \( \mu/N, \phi/N^2 \) and \( \text{Cov}[X_0^F, X_0^L] = \rho/N \) → fixed uncertainty

∴ isolate strategic effects (direct: less manipulation due to followers shrinking)

Proposition

Fix any \( \rho \in (0, \phi] \). In the unique (symmetric) PBS equilibrium

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Also: (i) \( \alpha_F \) is increasing in \( N \); (ii) \( \alpha_L \) and the firm’s ex ante value are decreasing in \( N \); and (iii) the leader’s ex ante payoff \( \uparrow \sim \sqrt{N} \) asymptotically.

smaller fraction of the total \( \rightarrow \) followers are more aggressive \( \rightarrow \) higher value of manipulation \( \rightarrow \) higher payoffs
Complementarities: $\rho > 0$ and $N$

$L$’s expected payoff $(N; \rho)$:

![Graph showing the expected payoff for different values of $\rho$.]
Complementarities: $\rho > 0$ and $N$

$L$’s expected payoff $(N; \rho)$:

mechanism is more likely when activists:
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respond to arbitrage opportunities, have similar stakes
Complementarities: $\rho > 0$ and $N$

$L$’s expected payoff $(N; \rho)$:

mechanism is more likely when activists:
respond to arbitrage opportunities, have similar stakes of small/moderate size
Complementarities: $\rho > 0$ and $N$

$L$’s expected payoff $(N; \rho)$:

mechanism is more likely when activists:

respond to arbitrage opportunities, have similar stakes of small/moderate size
if a positive statistical link, when there are more followers acting non-cooperatively
Hedge fund activism and wolf packs
Important area of research in finance and law
Wolf Packs

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“The term “wolf pack” [...] will mean a loose network of activist investors that act in a parallel fashion, but deliberately avoid forming a group

(Coffee and Palia, 2016, pp.561-562)
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Important area of research in finance and law

“The term “wolf pack” [...] will mean a loose network of activist investors that act in a parallel fashion, but deliberately avoid forming a group

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“[...] institutional investors such as activist hedge funds engage via so-called “wolf packs” in which multiple funds with small to moderate stakes (who do not act as a formal group) each engage in costly efforts to change firm policies”

(Brav et al, 2021, p.1)
arbitrage opportunities: highly sensitive to mispricing
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non-cooperative behavior: substantial costs when acting as a formal group
  • must disclose stake exceeding 5% within 10 days
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- group = single entity
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Hedge Fund Activism: Institutional Details

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- nec. smaller stakes, and potential reaction by target (block acquisitions, litigation,...n/a if anonymous)

similarity: business model & median stakes 6.3%-6.6% (Bebchuk et al. 2013, Brav et al, 2022)

awareness: informed of the presence of others and of potential targets

SEC regulation circa 1992 permits limited communication

2000-2010: +400 engagements involving multiple hedge funds (Becht et al 2017)
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**Multiplayer engagements:** 1) filings and/or 2) *abnormality* (vol, returns) within disclosure window

Not all the abnormality is attributed to the disclosing HF (Wong, 2022)

Sequentiality: incentives argument

⇒ important costs above 10% ⇒ <50% of the terminal position acquired in the 10-day window

HFs prefer less competition → complete position fast

Bebchuk et al (2013): completion on day that 5% is crossed, or +1; Collin-Dufresne and Fos (2015): leader purchases ∼1% largely on crossing date
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  Collin-Dufresne and Fos (2015): leader purchases $\sim 1\%$ largely on crossing date
Empirical Evidence: Abnormal Returns & Market Capitalization

Figure 1: Source: Brav et al. (2022)

Model: prices lower than in “normal” (i.e., no-activism) times when $\rho > 0$
Empirical Evidence: Abnormal Returns & “Short” Activists

Figure 2: Source: Li et al. (2022)

Model: prices higher than in “normal” (i.e., no-activism) times when $\rho < 0$
other linear equilibria
Trading against initial position/private information

Remaining case: $\alpha_L < 0$ and/or $\alpha_F < 0$. 

ˆ More of a “coordination” flavor

ˆ Negative firm values matter; but not implausible: a negative position can be profitable if it lowers firm value (e.g., Goldstein and Guembel, 2008)

Proposition (i) $\rho > 0$: for $\sigma > 0$ “large”, there exists a linear eqbm with $\alpha_L < 0$ and $\alpha_F < 0$ 

ˆ “large” $\sigma \to$ manipulation is difficult $\to$ coordination emerges

(ii) $\rho = -\phi$: no linear eqbm. with sign ($\alpha_L$) = sign ($\alpha_F$). A linear eqbm. with $\alpha_L < 0 < \alpha_F$ exists for all $\sigma > 0$.

ˆ want to meet on the same side
Remaining case: $\alpha_L < 0$ and/or $\alpha_F < 0$. More of a “coordination” flavor.
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- If we are both “long,” and I expect you to go short... self-fulfilling
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Existence of PBS and Refinement

PBS lost if $\rho$ too negative.
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PBS lost if $\rho$ too negative. **Manipulation affects limits to arbitrage**
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PBS lost if $\rho$ too negative. **Manipulation affects limits to arbitrage**

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Existence of PBS and Refinement

PBS lost if $\rho$ too negative. Manipulation affects limits to arbitrage

- $\rho > 0$: buying is costly for $L$ due to price impact and manipulation
- $\rho < 0$: $L$ buys more, more effort by $F \rightarrow$ against price impact $\rightarrow$ convexity
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Dual role of $\sigma$. 
Existence of PBS and Refinement

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- $\rho > 0$: buying is costly for $L$ due to price impact and manipulation
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Dual role of $\sigma$. As $\sigma$ falls (price impact grows)
Existence of PBS and Refinement

PBS lost if \( \rho \) too negative. Manipulation affects limits to arbitrage

- \( \rho > 0 \): buying is costly for \( L \) due to price impact and manipulation
- \( \rho < 0 \): \( L \) buys more, more effort by \( F \rightarrow \) against price impact \( \rightarrow \) convexity

Dual role of \( \sigma \). As \( \sigma \) falls (price impact grows)

- manipulation is easier, so coordination less plausible (e.g., \( \rho > 0 \) region)
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**Dual role** of $\sigma$. As $\sigma$ falls (price impact grows)

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- restores concavity (e.g., $\rho < \underline{\rho} < 0$)
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Proposition

Fix $\rho \in (-\phi, \phi)$. If $\sigma > 0$ is sufficiently small, the PBS equilibrium exists and is the unique eqbm. within the linear class
Existence of PBS and Refinement

PBS lost if $\rho$ too negative. **Manipulation affects limits to arbitrage**

- $\rho > 0$: buying is costly for $L$ due to price impact and manipulation
- $\rho < 0$: $L$ buys more, more effort by $F \to$ against price impact $\to$ **convexity**

Dual role of $\sigma$. As $\sigma$ falls (price impact grows)

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**Proposition**

*Fix $\rho \in (−\phi, \phi)$. If $\sigma > 0$ is sufficiently small, the PBS equilibrium exists and is the unique eqbm. within the linear class*

market illiquidity refines the equilibrium under study
Concluding remarks

**Disclosure** > 5% is a key institutional feature. Model still relevant:

- block completion < 10 days; ways to circumvent filing; campaigns < 5% →
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**Full dynamics**: multiple activists in all trading rounds
- effect present even with i.i.d. initial positions
- over time: neg. corr (MM’s learning) but evolving positions \( \rightarrow \) pos. corr