Activist Trading Dynamics[a]

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Insead, Fontainebleau and Singapore
September 6 2023

[a]The views expressed in this paper are those of the authors and do not necessarily represent the position of the Federal Reserve Bank of New York or the Federal Reserve System.

background

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  - U.S., 1994-2018: +900 funds, targeting +3000 firms, +4600 events **(Brav et al. 2022)**

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This paper: **market-based mechanism** involving strategic block (de-)accumulation

- minimal elements to generate *dynamics*

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$$\rightsquigarrow \textbf{Total payoff: } \underbrace{(W^i + W^{-i})}_{\text{share value}} X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(W^i)^2$$

Main Finding and Roadmap

Dynamics + endogenous firm value $\rightsquigarrow$ **linear eqbm. w/manipulation motive**

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Plan

1. Kyle (1985) "101"
2. Equilibrium analysis: manipulation dynamics
3. Predictions: market outcomes & first-mover advantages
4. Hedge fund activism: *wolf packs*
5. Other linear equilibria & refinement

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$\therefore$ "Gap" strategy $\Rightarrow \mathbb{E}[\theta] = 0$: "unpredictable"

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- In our model, because of externalities, the leader does not have a gap strategy (we will return to why)

Back to our setting: incentives

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$$(X_T^i + X_T^{-i})X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(X_T^i)^2$$

- larger blocks $\rightarrow$ more effort (stronger intervention)
- public-goods problem is at play
- static incentives to trade: (i) stronger due to own effort ($X_T^i X_T^i$) and (ii) stronger/weaker depending on the other activist ($X_T^i X_T^{-i}$; linked to ρ)
- dynamic incentives: leader with larger blocks benefit more from follower's effort ($X_T^L X_T^F$) $\rightarrow$ applied to more shares

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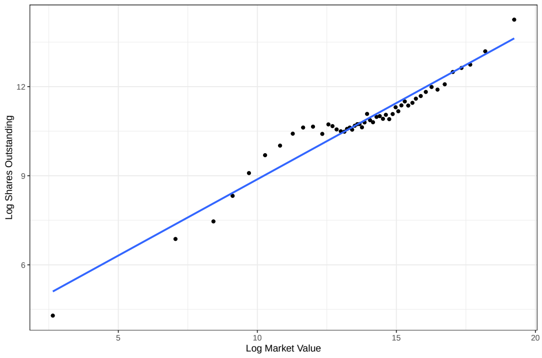

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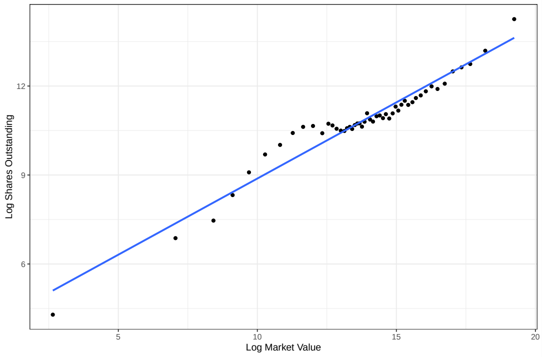

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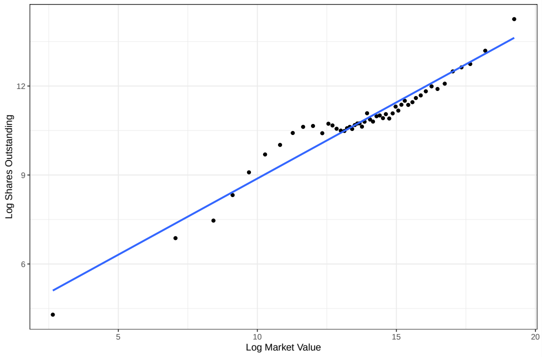

2. $\rho > 0$ more likely as market capitalization grows
3. Presence of large short and long positions indicative of $\rho < 0$

equilibrium analysis

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Focus on linear equilibria with *positive block sensitivity* (PBS): $\alpha_L > 0$ and $\alpha_F > 0$

- higher types accumulate more shares than low types (relatively)

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Note the price at time 1 is given by

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- We will see that $\beta_F \neq 1$.
- The numerator comes from the fact that we are predicting X_0^L and from this X_0^F.

Analysis: learning and pricing, continued

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Wedge admits analog interpretation. Leader doesn't need to update using P_2

Trading: first-order conditions

The activists' FOCs are

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- Leader's *value of manipulation*
  - discrepancy wrt Kyle stems from endogeneity + non-trivial cont. value

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Second-order conditions:

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1 - 2\Lambda_2 &< 0 \qquad \text{for follower} \\
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- find candidate equilibrium and check $\beta_F \neq 1$ ex post

Follower's equilibrium trading

Recall that $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$ and $M_1^F := \mathbb{E}[X_0^F | \mathcal{F}_1^\Psi]$, $\gamma_1^F := \mathrm{Var}(X_0^F | \mathcal{F}_1^\Psi)$

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In any PBS equilibrium:

$$(i) \quad \alpha_F = \sqrt{\frac{\sigma^2}{\gamma_1^F}}, \quad (ii) \quad \beta_F < 1 \text{ with } sign(\beta_F) = -sign(\rho) \quad \text{and} \quad (iii) \quad \delta_F < 0.$$

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- high P_1 indicative of high X_0^L $\rightarrow$ high M_1^F if $\rho > 0$ $\rightarrow$ F trades less $\rightarrow$ $\beta_F < 0$
- ... $\rightarrow$ low M_1^F if $\rho < 0$ $\rightarrow$ F trades more $\rightarrow$ $\beta_F > 0$

Interpretation of $\theta^F = \sqrt{\frac{\sigma^2}{\gamma_1^F}}(\theta - M_1^F)$

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2) Why Kyle's weight? Recall Λ_2 is scaled up by $1 + \alpha_F + \rho_1/\gamma_1^F$...

- Follower's expectation of firm value is also scaled by the same factor...
- Price impact wedge reflects a change in marginal incentives of the same size

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$\therefore$ with Stackelberg structure, need non-trivial ρ to get dynamics...

PBS equilibrium: general case

Proposition

Fix $\sigma, \phi > 0$. *There exists* $\underline{\rho} \in (-\phi, 0)$ *s.t. for all* $\rho \in [\underline{\rho}, \phi]$, *there is a unique PBS equilibrium. There,* $\theta^L = \alpha_L X_0^L + \delta_L \mu$, *where* $\alpha_L > 0$ *and* $\delta_L < 0$ *and*

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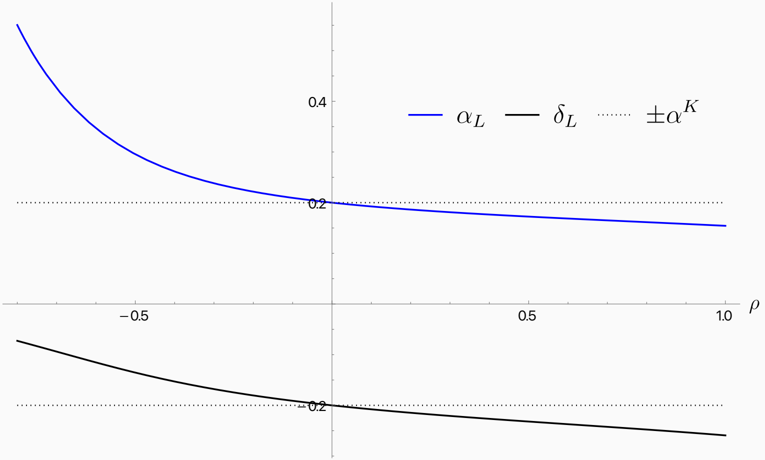
$$\alpha_L \neq |\delta_L|$$

is a generic property

PBS equilibrium: general case



$\alpha_L \neq \alpha_K$ and the Value of Manipulation

Recall the leader's value of manipulation:

$$X_T^L \frac{\partial \mathbb{E}_L[X_T^F | \theta^L]}{\partial \theta^L} = X_T^L \beta_F \frac{\partial \mathbb{E}_L[P_1]}{\partial \Psi_1} = X_T^L \beta_F \Lambda_1$$

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  - $\alpha_L > 0$: higher types end up with higher terminal blocks
  - $\alpha_L < \alpha^K$: high types scale back more on absolute terms

$\delta_L \neq \delta_K$ and Price Impact Λ_1

$$\delta_L = \frac{1}{(1-\beta_F)\Lambda_1} \times \underbrace{\frac{\partial}{\partial \mu}\left(\mathbb{E}_L[W_L + W_F] - P_1\right)}_{\text{sensitivity of arbitrage wrt } \mu}$$

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1) $\delta_L < 0$: all types scale back as μ grows; trivial when v exogenous

- Here: leader is less sensitive than MM to an increase in μ
- she uses both μ and private information to forecast firm's value

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all types scale back along both types of info $\Rightarrow$ *symmetry breaks*

$\rho < 0$: **trade more aggressively**

predictions

Predictions: market outcomes

Proposition

In the unique PBS equilibrium,

1. *Order flow*: $\mathbb{E}[\Psi_1] = \mathbb{E}[\theta^L] < 0$ *iff* $\rho > 0$, *while* $\mathbb{E}[\Psi_2] = 0$

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   - Prediction on average prices: "abnormally" low iff $\rho > 0$

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Suppose ρ not too negative. There exists a unique symmetric PBS equilibrium

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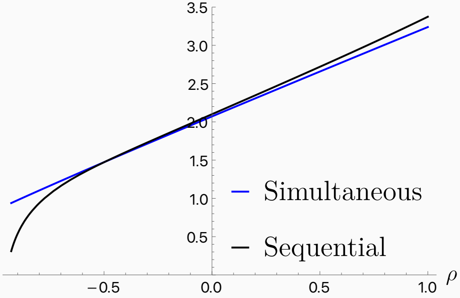
- Competition effect: $2\sqrt{\frac{\sigma^2}{2\phi}} > \sqrt{\frac{\sigma^2}{\phi}}$

Predictions: first-mover advantages

L's ex ante payoffs as ρ varies:



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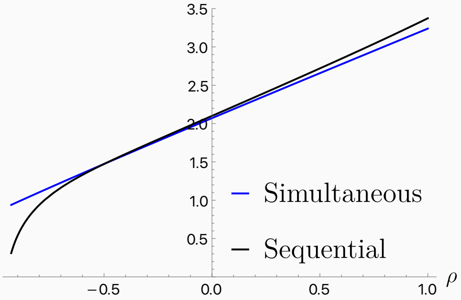

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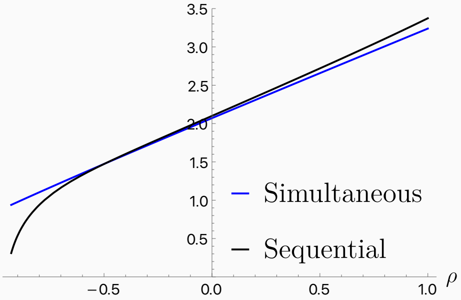

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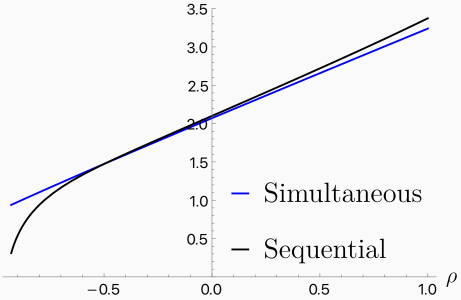

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Similarity: our mechanism is plausible if ρ is not too negative

Predictions: number of followers

N "normalized" followers: μ/N, ϕ/N^2 and $\mathrm{Cov}[X_0^F, X_0^L] = \rho/N \rightarrow$ fixed uncertainty

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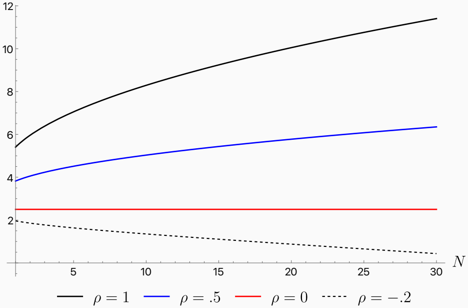
smaller fraction of the total $\rightarrow$ followers are more aggressive $\rightarrow$ higher value of manipulation $\rightarrow$ higher payoffs

Complementarities: $\rho > 0$ and N

L's expected payoff $(N; \rho)$:



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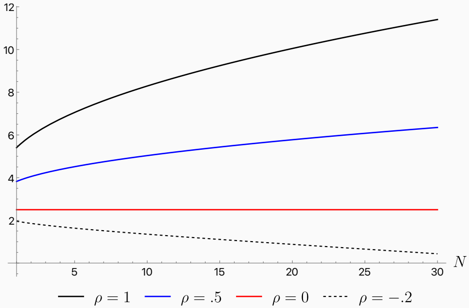

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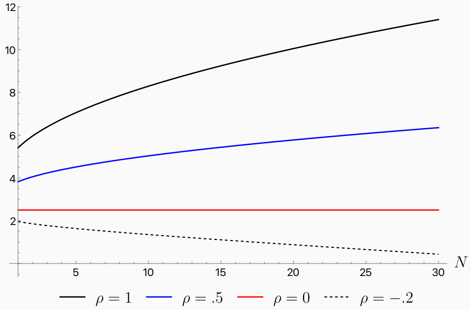

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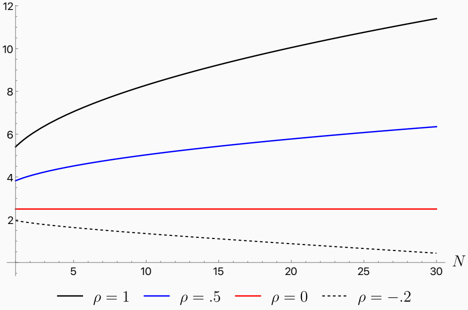

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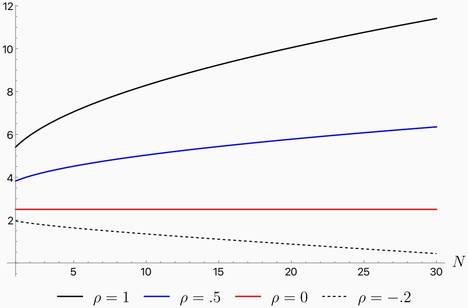

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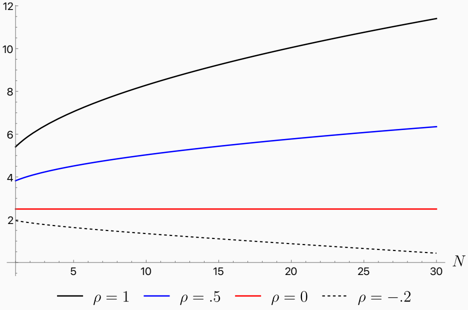

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if a positive statistical link, when there are more followers acting non-cooperatively

Hedge fund activism and wolf packs

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Important area of research in finance and law

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"[...] institutional investors such as activist hedge funds engage via so-called "wolf packs" in which multiple funds with small to moderate stakes (who do not act as a formal group) each engage in costly efforts to change firm policies"

(Brav et al, 2021, p.1)

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- SEC regulation circa 1992 permits **limited communication**
- 2000-2010: +400 engagements involving multiple hedge funds **(Becht et al 2017)**

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Multiplayer-engagements: 1) filings and/or 2) *abnormality* (vol, returns) within disclosure window

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- Bebchuk et al (2013): completion on day that 5% is crossed,or $+1$;
  Collin-Dufresne and Fos (2015): leader purchases $\sim 1\%$ largely on crossing date

Empirical Evidence: Abnormal Returns & Market Capitalization

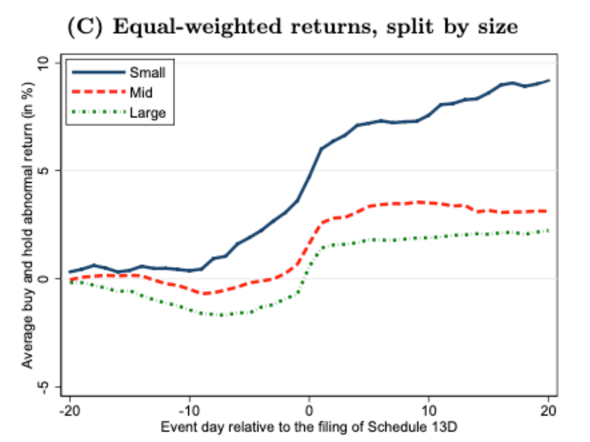

Figure 1: Source: Brav et al. (2022)

Model: prices lower than in “normal” (i.e., no-activism) times when $\rho > 0$

Empirical Evidence: Abnormal Returns & "Short" Activists

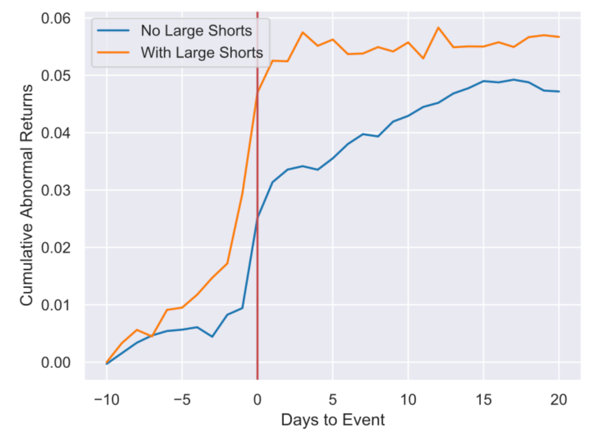

Figure 2: Source: Li et al. (2022)

Model: prices higher than in "normal" (i.e., no-activism) times when $\rho < 0$ ▸ Conclusions

other linear equilibria

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Remaining case: $\alpha_L < 0$ and/or $\alpha_F < 0$.

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market illiquidity refines the equilibrium under study

Concluding remarks

Disclosure $> 5\%$ is a key institutional feature. Model still relevant:

- block completion < 10 days; ways to circumvent filing; campaigns $< 5\% \nearrow$
- Forced to disclose at all levels? Randomization(?) **(Huddart et al, ECMA 2001)**

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Full dynamics: multiple activists in all trading rounds

- effect present even with i.i.d. initial positions
- over time: neg. corr (MM's learning) but evolving positions $\rightarrow$ pos. corr