Bankruptcy Exemption of Repo Markets: Too Much Today for Too Little Tomorrow?

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ABSTRACT

We examine the desirability of granting "safe harbor" provisions to creditors of financial intermediaries in sale-and-repurchase (repo) contracts. A safe harbor for repo financiers in the form of an exemption from automatic stay in bankruptcy enables financial intermediaries to raise greater liquidity during normal times but also induces entry of intermediaries with higher leverage. Liquidity creation occurs, therefore, at the cost of ex-post distributive externalities when there are adverse aggregate shocks to the fundamental quality of collateral underlying repo contracts. When exempt from bankruptcy, repo financiers of highly-leveraged financial intermediaries respond to such shocks by engaging in collateral liquidations. Financial arbitrage by less-leveraged financial intermediaries equilibrates returns from acquiring collateral at fire-sale prices and making new loans to the real sector, inducing higher lending rates, a deterioration in endogenous asset quality, and possibly even, a credit crunch. Given this distributive externality – especially in case of illiquid collateral with high exposure to aggregate risk – taming the leverage cycle by not granting safe harbors, i.e., requiring an automatic stay on repo financiers in bankruptcy, can be not only ex-post optimal, but also ex-ante optimal.

Keywords: Automatic stay, safe harbor provisions, sale and repurchase contracts, fire sales, credit crunch, financial crises, systemic risk

JEL Classification: G01, G21, G28, G33, D62, K11, K12

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1. Introduction

A repurchase agreement – also known as a "sale and repurchase agreement" or more popularly as a repo – is a short-term transaction between two parties in which one party effectively borrows cash from the other by pledging a financial security as collateral. The balance-sheet growth of a range of financial intermediaries relies on financing raised via repo markets (Adrian and Shin [2010]). One important feature of the repo market in the United States is that a large fraction of transactions falling under the umbrella of repos are exempt from the automatic stay in bankruptcy of the counterparties and, therefore, can be settled with immediacy. For example, if the seller of the asset is unable to repurchase the asset, then the buyer can liquidate the underlying collateral and avoid being part of a bankruptcy filing of the seller. This exemption from bankruptcy, sometimes also called as a "safe harbor" provision, has been extended gradually to different repo markets, starting with Treasuries and Agency (Fannie Mae and Freddie Mac) securities in 1980s, and most recently in 2005, to non-agency mortgage-backed assets.[1] The failures of financial intermediaries exposed to mortgages or mortgage-backed securities, such as Countrywide, Bear Stearns and Lehman Brothers, all involved in some part a repo "run," that is, an inability of the borrower to roll over the repo contracts with the financiers. Indeed, since the global financial crisis, there has been stress in the form of fire sales and "repo rate spikes" even in the U.S. Treasuries market, notably during September 2019 and March 2020.[2]

We develop a model to understand the desirability of granting repo contracts such exemption from bankruptcy. Financial intermediaries (such as, broker dealers or their parent bank-holding companies) borrow funds from financiers (such as, money-market funds) to originate assets. These assets can suffer from funding illiquidity when the state of the economy becomes adverse, either due to frictions which restrict the pledgeability of financial assets to refinance short-term repo contracts, or simply due to a rise in illiquidity or risk premia.

Taking a purely partial equilibrium view of the bilateral contract, the ex-ante liquidity of intermediaries would seem to be greater if they grant liquidation rights on underlying assets to the financier (see Garbade [2006], Acharya and Viswanathan [2011], Infante [2013], and Lewis [2023]). The intuition is that if financiers are instead not granted liquidation rights (bankruptcy exemption),

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[1] See Acharya and Öncü [2014] for a chronology of these exemptions.

[2] See, in particular, Copeland et al. [2021] and d'Avernas and Vandeweyer [2020].

future economic shocks would expose the lender to greater risk without access to repo collateral. Financiers would anticipate this ex ante and provide less liquidity. The implication is that bankruptcy exemption of collateralized borrowing, as presently accorded to repo contracts, should enable financial intermediaries to raise greater liquidity and originate more assets.

Our key insight is that liquidity creation via extension of bankruptcy exemption occurs at potentially significant welfare costs due to distributive externalities when a general equilibrium view is considered. In particular, financial intermediaries can also originate assets in the future, say in the form of loans to the real sector. If adverse economic shocks lead to forced sale of repo collateral at such times, then the partial equilibrium desirability of the bankruptcy exemption status for repo contracts can get overturned as asset fire-sales may significantly raise lending rates to the real sector, and even induce a credit crunch. Therefore, there is an inherent conflict in the choice of bankruptcy exemption between supporting current and future asset origination; complete bankruptcy exemption amplifies this inter-temporal wedge, and can lead to too much origination and lending today for too little asset origination and lending tomorrow.

We consider a three date model in which an aggregate economic shock at the interim date affects the funding liquidity of financial intermediaries with varying levels of leverage (endogenously determined). Upon arrival of adverse news about underlying asset quality, highly-leveraged intermediaries face greater funding or rollover stress as their financiers factor in the funding illiquidity of collateral due to the economic shock. Therefore, the ability of these intermediaries to raise new financing to pay off earlier financiers is diminished, prompting them to sell some legacy financial assets. For an adverse enough shock, partial asset sales do not suffice to roll over existing contracts and all assets may have to be liquidated by financiers when given exemption from bankruptcy. Less-leveraged intermediaries, in contrast, have surplus capacity to raise financing and acquire the assets being liquidated. In the industry equilibrium, the market-clearing price of legacy financial assets reflects, in general, fire-sale discounts Shleifer and Vishny [1992], Gale and Allen [1994], Allen and Gale [1998].

Absent the consideration of new asset origination (e.g., loans to households or the real sector) at the interim date, such a market-based transfer of assets from highly-leveraged intermediaries to less-leveraged ones does not affect ex-post efficiency (in particular, fire-sale discounts may simply reflect welfare-neutral transfers of value). However, if there is a demand from the real sector for intermediation at the interim date, then this result is substantially overturned for the following reasons. Bankruptcy exemption facilitates a greater degree of ex-ante leverage, which we model as the marginal entry of intermediaries with higher leverage.[3] This, in turn, causes greater consequent liquidations in the event of an adverse economic shock, thereby providing opportunities to less-leveraged intermediaries to earn excess return from their surplus liquidity. Financial arbitrage implies that the expected return from originating new loans to the real sector must match the expected return from investing in the secondary market for legacy financial assets (as in Diamond and Rajan [2011], Hanson et al. [2011], Vayanos and Gromb [2012], and Stein [2012]). Therefore, in the new loan market, interest rates rise in tandem with the the extent of liquidation leading to a potential real inefficiency.

Furthermore, a moral hazard problem arises as borrowers in the new loan market (say, households) invest less effort (e.g., to maintain the property) when faced with higher interest rates, resulting in (an endogenously determined) lower loan quality. The drop in loan quality in turn affects the lender's (i.e., the surplus-liquidity intermediary's) expected profits. Thus, there is an upper bound on the interest rate that intermediaries can charge on new loans, or in other words, the marginal benefit of increasing the interest rate beyond this level is more than offset by the marginal reduction in loan quality. When bankruptcy exemption causes too much ex-post liquidation, fire-sale discounts can be large and the returns from investing in the financial asset market exceed the maximum return from the new loan market. Surplus-liquidity intermediaries are then no longer interested in deploying additional capital in the new loan market. Instead, they withdraw capital from the real sector and, in the extreme, the market for new loans shuts down.

Next, we show that bankruptcy exemption *can be* sub-optimal in our model, i.e., the negative distributive externality of bankruptcy exemption in the form of credit-crunch effects in future periods *can* overwhelm the positive effect of greater financial intermediation in the current period. The intuition for the result is as follows. While bankruptcy exemption induces more ex-ante asset creation, the *incremental* beneficiaries are intermediaries with larger investment requirements, i.e., a higher leverage, and these intermediaries would not have been financed if there was no safe harbor. These high-leverage intermediaries are more susceptible to adverse economic shocks, more likely to be liquidated by their financiers, and create financial arbitrage opportunities for less-leveraged intermediaries. This externality diverts the future surplus liquidity of less-leveraged intermediaries toward acquiring assets at fire-sale prices instead of financing real investment activity, thereby inducing adverse welfare consequences that can overwhelm the initial facilitation of financial inter-

[3] Acharya and Viswanathan [2011] provide motivating evidence that entry in shadow banking sector preceding the global financial crisis of 2007-09 featured progressively higher leverage. For historical evidence along these lines in underwriting of mortgages, see De Jong et al. [2023]. Finally, for theoretical modeling of why leverage booms feature greater leverage based on subjective beliefs, see Fostel and Geanakoplos [2008].

mediation by bankruptcy exemption.

In other words, it can be optimal to tame the leverage boom-bust cycle by not according bankruptcy exemption to repo financiers, subjecting them instead to an automatic stay (the polar opposite policy of bankruptcy exemption) in which repo financiers cannot seize the underlying collateral for immediate liquidations. Our model clarifies that an automatic stay on repo contracts in bankruptcy is optimal when fire-sale effects in underlying collateral are likely, for instance, in case of illiquid collateral, such as mortgages, which lose value when aggregate risk materializes. An automatic stay is also beneficial when the real sector funding needs are large and economic downturns are likely to be more severe. On the other hand, bankruptcy exemption of repo contracts is ex-ante optimal *only* when there are no fire-sale effects; such a situation arises when the magnitude of the adverse economic shock is mild, the collateral is of unimpeachable quality even under stress (potentially benefiting from flight-to-safety or flight-to-quality effects), and the real sector funding needs are small.

Section 2 relates our work to theoretical and empirical literature. Section 3 sets up the basic features of the model. Section 4 analyzes the model and presents the ex-post equilibrium outcomes, taking ex-ante leverage as given. Section 5 augments the model to study the ex-ante leverage of intermediaries. Section 6 derives results on ex-ante welfare analysis, which pins down the optimal level of bankruptcy exemption and its determinants. Section 7 examines the impact of capital requirements on optimal bankruptcy exemption level and Section 8 concludes. Key proofs are in the Appendix, with some additional details relegated to an Internet Appendix.

2. Related Literature

Our paper is motivated by the empirical literature on the role of repo market runs in exacerbating the financial crisis (Copeland et al. [2010, 2014], Gorton et al. [2010], Gorton and Metrick [2010, 2012], Gorton et al. [2020a], Gorton et al. [2020b], and Krishnamurthy et al. [2014]). By and large, this literature points out that the over-dependence of important financial institutions on repo financing in the period before 2008 exposed the financial system to systemic risk, which eventually led to an economic contraction. Our paper addresses a key design feature of repo markets, namely, the bankruptcy exemption of repo creditors, in exacerbating crisis-like situations. The specific model presented in our paper is closely related to four strands of academic literature.

The first strand deals with the role of financial frictions in exacerbating the impact of macroeconomic shocks. These frictions limit the ability of a highly-leveraged intermediary from continuing as a going-concern during an adverse economic shock unless it liquidates some of its assets, potentially at fire-sale prices. In addition to the seminal papers referred in the Introduction, our model is most closely related to the work of Acharya and Viswanathan [2011] and Lorenzoni [2008]. In Lorenzoni [2008], fire sales are generated by financial frictions that arise due to the limitation of agents to commit credibly to future loan repayments. In Acharya and Viswanathan [2011], funding liquidity is constrained by financial frictions that arise due to a risk-shifting problem; our model is agnostic to the type of financial friction and assumes that funding illiquidity limits the pledgeability of financial assets. More importantly, our model considers via cross-market arbitraging activity, the equilibrium interaction of fire sales generated by rollover risk in the financial sector with the loan market and attendant moral hazard problem in the real sector.

On this cross-market arbitrage dimension, our paper is related to a second strand of literature that deals with the welfare implications of leverage-induced fire sales. Such liquidations have been argued to cause inefficiencies in the economy (Bordo and Jeanne [2002], Diamond and Rajan [2001], Lorenzoni [2008], Acharya et al. [2010] and Stein [2012]). The central feature of these studies is that aggregate leverage and fire-sale effects are endogenously related. Bordo and Jeanne [2002] analyze the ex-post consequences of a sharp decline in asset prices (following an asset price boom) on real economic activity and study implications for optimal monetary policy. Diamond and Rajan [2001] show how a fear of fire sales in future can cause a credit freeze today as intermediaries hoard cash to capitalize on fire sales. Lorenzoni [2008] points out there is excess ex-ante borrowing that fails to internalize the ex-post inefficiency due to fire sales and a central planner can improve social welfare by limiting the amount of aggregate leverage in the economy. In Acharya et al. [2010] ex-post fire-sales affect ex-ante liquidity holdings, which can be excessive during crises and too low in economic booms. Stein [2012] examines the financial stability implications of short-term private money creation and how monetary policy and complementary tools such as open-market operations can be deployed to limit the negative externalities arising from fire sales on ex-ante origination.

More recently, in a third strand of literature, Dávila and Korinek [2018] show that financial frictions can lead to both *distributive externalities* (externalities between buyers and sellers of assets) and *collateral externalities* (externalities that depend on the effect of financial constraints on asset prices). Further, Lanteri and Rampini [2023] argue that distributive externalities are much larger than collateral externalities in a model with investment and collateral constraints. In our model, there is a large distributive externality in that the low price of repo collateral in the second period induces more capital allocation to the financial sector and less capital allocation to the real sector;

further the low price of collateral (high interest rate) reduces the value of the real sector asset due to moral hazard. This distributive externality leads to the result that bankruptcy exemption is welfare sub-optimal, except in special cases.[4]

The fourth strand of related literature discusses the implications of bankruptcy exemption on systemic risk. Duffie and Skeel [2012] recognize the role of bankruptcy exemption in increasing systemic risks and propose limiting the bankruptcy exemption to repos and (centrally cleared) derivative contracts that are backed with highly liquid collateral. Tuckman [2010], too, advocates restricting the safe harbor provision to only those derivatives that are centrally cleared to reduce the risk of fire sales in the event of an adverse shock and to also reduce the incentives of market participants to take up large position in complex, illiquid derivatives whose underlying assets are most susceptible to crashes. Bolton and Oehmke [2015] argue that according bankruptcy exemption to derivative contracts simply transfers default risk to other creditors, but it may be beneficial only in special circumstances when then the cross-netting benefits for derivative writers in derivative markets is sufficiently high. Finally, Acharya and Öncü [2014] recommend withdrawing the safe harbor exemption from all repo transactions other than those having government-backed claims as collateral. We confirm the intuition of this literature that stronger creditor rights accorded as safe-harbor provisions to repo contracts facilitate ex-ante credit availability, but cause ex-post inefficiencies in the event of an adverse aggregate shock to the economy. Our model also sheds lights on the debate among policy makers about the role of bankruptcy exemption – whether it reduces or exacerbates systemic risk (see for example, Federal Reserve Report [2011], written in the aftermath of the global financial crisis). We show that the view that bankruptcy exemption reduces systemic risk is overturned once we take an ex-ante as well as an economy-wide perspective by endogenizing the implications of safe harbor on leverage and of fire sales for the real economy.[5]

Finally, our model is also connected to the default literature in consumer bankruptcy exemption

[4] Recent studies have also explicitly modeled the bankruptcy exemption provision; however, these studies use fundamentally different modeling assumptions from our work. First, Antinolfi et al. [2015] show that fire-sale externalities arise due to bankruptcy exemption. However, as they themselves point out, this externality disappears in their model if the exchange of fire-sale assets arises in a competitive equilibrium. Second, Ma [2017] considers a structural model of the bankruptcy exemption provision to evaluate how it affects the coordination problem of creditors in a repo run and the strategic declaration of bankruptcy by the borrower; the model, however, does not consider the spillovers effects on the real sector. Finally, Zhong and Zhou [2021] endogenize ex-post bankruptcy payoffs to evaluate the ex-ante decision of creditors to stay invested in a firm. Thus, they are able to establish a time-consistent approach to ex-post and ex-ante creditor runs.

[5] The academic legal profession has also discussed the issue of bankruptcy exemption. Several articles in law journals have assessed the costs and benefits of the safe harbor provision. These articles also point out that collateral runs are an important factor in evaluating bankruptcy exemption (e.g., Edwards and Morrison [2005], Jackson [2009], Roe [2009], Skeel and Jackson [2011], Federal Reserve Report [2011], Mooney Jr [2014] and Morrison et al. [2014]).

and banking. While our model deals with secured credit, the consumer bankruptcy models consider personal unsecured credit. The tradeoffs in these bankruptcy models are of a similar nature in that increasing bankruptcy exemption increases ex-ante borrowing in both types of models. However, while increasing personal bankruptcy exemption hinders consumers' access to the unsecured credit markets in future (see Chatterjee et al. [2007] and Livshits et al. [2007]), increasing bankruptcy exemption in our model causes ex-post distributive externalities in the form of reduced real asset market lending.

3. Model Setup

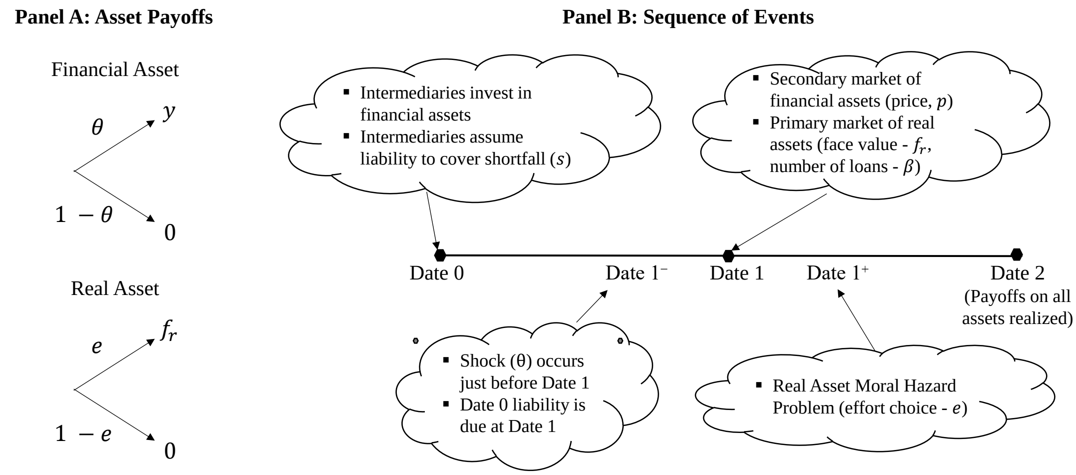
We build a model of financial intermediation using repo financing with the objective of determining the optimal extent of bankruptcy exemption for repo contracts. After laying out the model structure in this section, we partition our analysis into three sections: first, in Section 4 we examine the role of bankruptcy exemption on ex-post liquidation effects under an exogenous assumption about the ex-ante leverage in the economy; next, in Section 5 we endogenize the leverage decisions; and finally, we derive the ex-ante optimal level of bankruptcy exemption in Section 6. Our model follows the setup in Acharya and Viswanathan [2011]. The economy represents a two-period, three-date world – a start date (Date 0), an intermediate date (Date 1), and a terminal date (Date 2). We discuss below the role of financial intermediaries, the available assets for their investments and their Date 2 payoffs, followed by a summary of the sequence of key events in the model. Figure **1** shows the payoffs on the assets (Panel A) and the timeline (Panel B).

3.1. Financial Intermediaries

The economy consists of a continuum of financial intermediaries. They start out with differing levels of financial infrastructure and/or human capital required for participating in the intermediation sector. Depending on the accumulation of this capital, intermediaries require differing amounts of investment (henceforth, investment shortfall (s)) to start a business by acquiring a financial asset of unit scale. Similar to the approach followed by Anderson and Sundaresan [1996] in analyzing debt contract design, we assume that the investment shortfall is financed in the short-term debt market; more specifically, in the short-term repo market which provides financing with a "sale and repurchase" contract against the financial asset.[6] Effectively, at Date 0, financial intermediaries

[6] In earlier studies, Aghion and Bolton [1992] and Hart and Moore [1994] have used this approach in the context of security design.

Figure 1: **Description of the Model.** Panel A shows the Date 2 payoffs on the financial asset and the new loan. Panel B shows the sequence of events in the model.



operate at the same scale but vary in terms of the degree of leverage in their balance sheets.

3.2. Assets in the Economy

The financial asset originated at Date 0 by each intermediary could be a legacy loan or a commoditized pool of loans. At Date 2, the asset has a payoff of y with a probability of θ and a zero payoff (0) with a probability of $(1-\theta)$. The asset does not provide any intermediate cash flows at Date 1. At Date 1, the repo contracts issued by intermediaries against these assets to fund their investment shortfalls need to be refinanced; however, borrowers may face funding illiquidity if lenders demand a haircut when faced with uncertainty.

At Date 1, there is a demand in the the real sector of the economy for financial loans for real investments. Loans by intermediaries to the real sector consist of new mortgages or small business loans taken out by households. Each unit of the real asset requires a loan of 1 unit at Date 1. In return, the loan provides an uncertain binary payoff at Date 2 - with a probability e, the payoff is the loan face value (f_r); otherwise, it is 0. The probability e reflects the household effort choice based on a moral hazard problem. Both e and f_r will be endogenously determined below. Given that the loan amount is normalized to unity, the face value (f_r) effectively reflects the gross-of-interest repayment on household loans. These loans can be thought of as relatively illiquid loans – for simplicity, we assume that due to asset specificity, the cash flows from these loans are not pledgeable by intermediaries (at Date 0 or at Date 1) to raise finances. The size of the household sector demand at Date 1 is $\bar{\mathcal{B}} > 0$.

3.3. Summary of Sequence of Events/Decisions

At Date 0, intermediaries invest in a financial asset after borrowing the required financing (to cover the investment shortfall, (s) in the short-term repo market. At Date 1^- (an instant before Date 1), the economy experiences an observable but unverifiable shock (θ) that affects the payoff on the financial asset. Under these conditions, highly-leveraged intermediaries are unable to refinance their loans at Date 1, i.e., they are unable to repurchase their financial asset in entirety from the repo-financiers. In contrast to highly-leveraged intermediaries who are credit-constrained, less-leveraged intermediaries enjoy surplus liquidity. In light of this imbalance in liquidity within the set of intermediaries, a financial asset re-sale market emerges where surplus liquidity intermediaries acquire the assets of credit-constrained intermediaries, often at fire-sale prices. Note that, at Date 1, surplus liquidity intermediaries could also divert their liquidity towards the real sector to originate new loans. Thus, the seeds of cross-market parity in returns is endogenously built into our model.

At Date 1^+, households make an optimal choice on how much effort they will expend in maintaining the underlying property that backs the Date 1 loan. This effort choice influences the Date 2 payoffs on the underlying asset and in turn on the Date 1 loan.

While the model relies on the distinction in the sequence of events at Date 1^-, Date 1, and Date 1^+, for convenience we will often refer to the entire set of events as Date 1 events, e.g., a Date 1 economic shock. Intermediation decisions are thus made at Date 0 (raising repo financing to enter the financial sector) and Date 1 (repaying repo contracts, acquiring financial assets in the re-sale market, and extending illiquid loans to the real sector). The payoffs on both Date 0 and Date 1 investments of intermediaries are realized on Date 2.

3.4. Salient Features of the Model

Our model builds upon but differs from the Acharya and Viswanathan [2011] setup in three significant ways. First, in contrast to their work in which highly-leveraged intermediaries facing default must partially or wholly liquidate their financial asset at Date 1, in this model we recognize that not all intermediaries on the verge of bankruptcy are necessarily forced by lenders to liquidate their assets. In practice, we often observe strategic write-downs as a result of renegotiation between the borrower and its lenders. We define a parameter (q) that reflects the probability that a credit-constrained intermediary is unable to renegotiate successfully with its creditors, resulting in creditors repossessing the asset and liquidating it in the financial asset re-sale market. Conversely, $(1-q)$ is the probability that a credit-constrained intermediary is able to renegotiate with the lender and write-down its obligations. One could view q in the context of how the bankruptcy code treats repo contracts. If $q=1$, the asset is exempt from an automatic stay and the lender enjoys exclusive rights over the asset in the event of bankruptcy, a feature that allows the lender to seize the asset and concurrently liquidate the asset in the secondary market.

A more general approach could separate the acts of seizing the asset and liquidating the asset. We are implicitly assuming that both acts are simultaneously executed. It could be an interesting exercise to explore the implications of allowing the lender to optimally liquidate the asset after seizing the asset. However, note that money market funds (MMFs), which are a major lender in the short-term repo market, are constrained by regulation to maintain low average maturity of their holdings.[7] The empirical data regarding their portfolio holdings also confirms that they adhere to this requirement.[8] These observations suggest that MMFs are likely to prefer immediate liquidation of any collateral they repossess to meet the SEC requirements. While we recognize that separating the seizing decision and the liquidating decision could provide additional insights, for convenience, we collapse these decisions into a single contemporaneous decision in keeping with the empirical observations on MMF holdings.

We, therefore, refer to q as the bankruptcy exemption or the "safe harbor" parameter; it describes the likelihood of the lender retaining control of the asset in the event of a borrower default. For simplicity, we assume that q is independent across financial intermediaries, and in particular, within a continuum of intermediaries of each type (in our model, leverage), a fraction q has its collateral liquidated by repo financiers.

The second major point of departure from the Acharya and Viswanathan [2011] model is that we allow for the existence of a new loan market at Date 1. After the Date 1 shock has been realized, intermediaries that enjoy surplus liquidity can not only invest in the primary (origination) market for Date 1 loans but also in the secondary re-sale market for financial assets originated at Date 0. This characterization allows us to analyze the important interplay between the financial asset

[7] The most common collateral that money market funds lend against are Treasuries and Agency-Backed Securities, which have, on average, longer than one year maturity. While the repo contract is rolled over on a daily basis, the collateral in a repo when repossessed is treated as a long-term asset and its maturity is used in computing the maturity of the MMF's portfolio holdings. SEC Rule 2a-7 mandates that MMFs maintain an average portfolio maturity of less than 60 days. Furthermore, SEC also imposes a 120-day limit on the weighted average life of a MMF's portfolio holdings (Krishnamurthy et al. [2014]).

[8] Around 70 percent of MMF repos by volume are overnight, around 24 percent of MMF repos have maturities greater than one day but less than seven days, and the remaining 6 percent of MMF repos have other maturities (see Baklanova et al. [2021]).

re-sale market and the real economy, which is at the heart of our welfare analysis of bankruptcy exemption of repo contracts.

The third major point of departure is that we take into account moral hazard in the real economy. Fixed claims, such as debt, exacerbate moral hazard problems in the real sector when loan rates are too high, and our model captures this insight. For instance, in the case of mortgage loans, households being residual claimants on levered assets would have lower incentives to maintain the asset if the borrowing rate is too high (as we will show to be the case when an adverse shock occurs in the economy). This effect will also play a crucial role in our model in potentially shutting down the Date 1 loan market entirely when the shock (θ) is sufficiently adverse.

We refer to intermediary decisions/outcomes at Date 1 as coming from the *ex-post* model and decisions/outcomes at Date 0 as coming from the *ex-ante* model. The ex-ante model must take into account the optimal decision strategies and outcomes of the ex-post Date 1 equilibrium; at the same time, the ex-post equilibrium strategies and outcomes are affected by the strategies of ex-ante optimization, a key feature of the model, as in Acharya and Viswanathan [2011].

4. The Ex-Post Model

In this section, we lay out and solve the ex-post equilibrium at Date 1.

4.1. Funding Illiquidity in the Financial Sector

At Date 1^{-}, an economic shock arises due to which intermediaries may be unable to raise as much repo financing as they availed at Date 0. We assume that financial frictions cause the funding liquidity of the asset underlying the repo to be lower than the expected payoffs on the asset, θy. If the discount from the expected value is k, then the funding liquidity is denoted by $\rho^* \sim \theta y - k$, where $k > 0$ is the funding illiquidity of the asset. In general, one can think of k as a measure of the quality of collateral in the repo contract and the funding liquidity as the amount of rollover debt that can be raised by pledging this collateral. Funding illiquidity can arise due to a variety of financial frictions: (i) agency problems of the borrower (e.g., risk-shifting or other debt-overhang problems) and (ii) higher risk premiums demanded by lenders during uncertain times.[9]

[9] For illustration, see Appendix B1, which provides a micro-foundation of the funding illiquidity (k) based on a risk-shifting problem, as in Acharya and Viswanathan [2011].

4.2. Households' Moral Hazard Problem

Intermediaries that originate new loans at Date 1 provide one unit of financing at Date 1 to households in return for a promised payment of f_r at Date 2. Households use this financing to invest in a physical asset that provides a rental income of R at Date 2. Thus, households view their leveraged investment as paying a cash flow of $(R - f_r)$ in the high state (which occurs with a probability of e) and a cash flow of 0 in the low state (which occurs with a probability of $1-e$). The probability e, which is endogenously determined by the household, reflects its effort choice, and thus the asset quality. The expected benefit from renting is $e(R-f_r)$, and we assume that the pecuniary equivalent of expending effort is quadratic in the level of effort; more specifically, the cost is equal to $\frac{1}{2}\gamma e^2$, where $\gamma > 0$ captures the intensity of effort-aversion. Therefore, the household chooses an effort level e that trades off the benefits of asset-quality with effort-aversion, and maximizes its net expected payoffs of $e(R - f_r) - \frac{1}{2}\gamma e^2$. Given the bounds on the effort choice ($0 \leq e \leq 1$), the optimal solution is given by,

$$e^* = \min\left[\max[0, \frac{1}{\gamma}(R - f_r), 1]\right]. \tag{1}$$

Then, Lemma 1 implies that the moral hazard problem worsens when the interest rate (or the face value of the debt) increases:

LEMMA 1: *The optimal effort level of the representative household (e^*), and, thus, the asset quality, is negatively related to the face value (f_r) of the Date 1 loan.*

4.3. Optimizing Behaviour of Credit-Constrained Intermediaries

The continuum of intermediary firms differ from each other in terms of the investment shortfalls (s) required to enter the financial intermediation sector; equivalently, these intermediaries differ in terms of their outstanding liabilities (ρ) assumed at Date 0 and due at Date 1. Suppose the distribution of ρ is given by $\rho \sim G(\rho)$ over $[\rho_{\min}, \rho_{\max}]$; further down, $G(.)$ will be endogeneously determined based on an assumption of how the shortfall (s) is distributed.

At Date 1^-, when the economy-wide shock (θ) is realized, affecting the funding liquidity of the asset underlying the repo ($\rho^* = \theta y - k$), intermediaries will either be credit-constrained ($\rho \geq \rho^*$) or will enjoy surplus liquidity ($\rho < \rho^*$). In other words, the economic shock that arises at Date 1^- creates an imbalance in liquidity constraints of intermediaries. Thus, a re-sale market for the financial asset is created in which credit-constrained intermediaries liquidate the financial asset (supply-side) and surplus-liquidity intermediaries acquire it (demand-side). The market for financial assets clears at a price p, which will be derived keeping in mind that surplus-liquidity intermediaries can also participate in the household loan market at Date 1.[10] Note that, the market clearing price (p) must necessarily be greater than the funding liquidity of the asset (ρ^*) and no greater than the expected payoff of the asset (θy), i.e., $p \in (\rho^*, \theta y]$.

Surplus-liquidity intermediaries ($\rho \leq \rho^*$) will take long positions in the financial asset. On the other hand, credit-constrained (the highly leveraged ones with $\rho > \rho^*$) have a choice between (i.) partially liquidating the asset to reduce leverage and thereby avoiding default, and (ii.) triggering default and subjecting themselves to the bankruptcy regime. Lemma (2) presents the cut-off ($\bar{\rho}$) for leverage (ρ), below which intermediaries prefer partial liquidation to default.[11]

LEMMA 2: *The liability level (ρ) above which intermediaries would seek to strategically default is given by $\bar{\rho} = \rho^* + q(p - \rho^*)$. For $\rho < \bar{\rho}$, intermediaries liquidate a fraction $\frac{\rho-\rho^*}{p-\rho^*}$ of the financial asset and roll over the rest at the funding liquidity (ρ^*) in order to meet the the repayment ρ.*

Therefore, the aggregate supply of the financial asset in the re-sale market is determined as follows. Moderately credit-constrained intermediaries ($\rho^* < \rho \leq \bar{\rho}$), liquidate a fraction δ of their assets. At the same time, for severely credit-constrained intermediaries ($\rho > \bar{\rho}$), only a fraction q go into liquidation. The remaining fraction $(1 - q)$ of severely credit-constrained intermediaries obtain a strategic write-down by entering into negotiations with the financiers. We assume that the liability can be renegotiated downward to the asset's funding liquidity, ρ^*. Thus, given an adverse shock θ at Date 1, a fraction q of the severely credit-constrained intermediaries will be forced to liquidate some or part of their assets. If $g(\rho)$ denotes the p.d.f. of ρ, the aggregate supply of financial assets in the market is given by

$$S(\rho^*) = \int_{\rho^*}^{\bar{\rho}} \frac{\rho - \rho^*}{p - \rho^*} \, g(\rho) \, d\rho + \int_{\bar{\rho}}^{\rho_{\max}} q \, g(\rho) \, d\rho. \tag{2}$$

The first term represents a partial liquidation of $\frac{\rho-\rho^*}{p-\rho^*}$ by intermediaries with leverage $\rho \in (\rho^*, \bar{\rho})$, and the second term represents full liquidation by a fraction q of non-renegotiating intermediaries with leverage $\rho > \bar{\rho}$.

[10] The default decision takes into account the endogenously determined price of the asset in the spirit of the work of Amador and Bianchi [2024] on bank runs.

[11] Note that default is strategic only for intermediaries with moderately high leverage ($\bar{\rho} < \rho \leq p$). For intermediaries having extremely high leverage ($\rho > p$) default is involuntary.

4.4. Optimizing Behaviour of Surplus Liquidity Intermediaries

Suppose that an intermediary with surplus liquidity acquires α units of the financial asset in the asset sale market and lends β units in the new loan market at Date 1. Such intermediaries would optimally choose α and β, for a given p and f_r and a conjectured household effort choice (e).

Then for a given realization of the economic shock (θ) at Date 1, the optimizing behavior of agents with market-clearing results in an ex-post equilibrium which is determined as follows:

(i) Households maximize their effort given the face value (f_r) of the loan, as given by Equation (1), which is restated below:

$$e^* = \min\left[\max[0, \frac{1}{\gamma}(R - f_r), 1]\right]. \tag{3}$$

(ii) Surplus-liquidity intermediaries maximize the incremental benefits from acquiring α financial assets in the secondary market of legacy financial assets and providing β amount to households in the primary market of real sector loans; they have rational expectations over p and f_r and e^*; and solve

$$\max_{\alpha\geq 0,\beta\geq 0} \quad (1+\alpha)(\theta y - \rho^*) + \beta e f_r, \tag{4}$$

subject to the budget constraint

$$\alpha p + \beta \leq (1+\alpha)\rho^* - \rho. \tag{5}$$

(iii) Denoting the optimal choice for α and β for intermediaries with liquidity ρ be $\alpha^*(\rho)$ and $\beta^*(\rho)$, respectively, the aggregate demand for the financial asset is given by

$$\bar{\alpha} = \int_{\rho_{min}}^{\rho^*} \alpha^*(\rho) g(\rho) d\rho \leq S(p, \rho^*), \tag{6}$$

and the aggregate supply new loans is given by

$$\bar{\beta} = \int_{\rho_{min}}^{\rho^*} \beta^*(\rho) g(\rho) d\rho \leq \mathcal{B}, \tag{7}$$

where $\mathcal{B}$ denotes the size of the real sector in the economy at Date 1.

The objective function in (4) captures the incremental benefits associated with acquiring assets up for resale and making new loans. Acquiring one unit of the financial asset yields an expected payoff of θy, which implies that the marginal benefit over and above the funding liquidity of the financial asset is $(\theta y - \rho^*)$. Since its cash flows cannot be pledged, the marginal benefit of making one unit of the new loan is the same as its expected payoff, i.e., ef_r.

The constraint in (5) is the budget constraint of a surplus-liquidity intermediary. The right hand side reflects the available surplus liquidity. The left hand side represents the allocation of liquidity toward acquiring α financial assets in the re-sale market and making β household loans to the real sector. The other two constraints are that there is a non-negative demand for the financial asset and the supply of new loans. Finally, some technical restrictions on the loan face value (f_r), the effort aversion parameter (γ), and the financial asset price (p) must be satisfied in equilibrium to ensure that they are bounded within feasible ranges, which are derived in Section A4 of the Appendix.

4.5. Implications of Cross-Market Equilibrium

The market for financial assets clears at a price p, which will be derived keeping in mind that surplus-liquidity intermediaries can also participate in the household loan market at Date 1.

The surplus-liquidity intermediaries' optimization problem yields an equilibrium relation between the marginal expected return from investing in the financial asset ($r_{fin} = \frac{k}{p-\rho^*}$) and making a new loan ($r_l = ef_r$), as stated in the lemma below:

LEMMA 3: *(i) When both the financial asset market and the loan market are open:*

$$\bar{\beta} > 0 \implies \frac{k}{p-\rho^*} = ef_r. \tag{8}$$

(ii) When only the financial asset market is open:

$$\bar{\beta} = 0 \implies \frac{k}{p-\rho^*} > ef_r. \tag{9}$$

Note that the financial market must necessarily clear (i.e., α is strictly greater than 0) because it is a secondary market of legacy assets. In contrast, the new loan market is a primary market that will manifest only if there is ample spare liquidity in the economy, and therefore, could remain closed when there is a sufficiently adverse economic shock. Equation (8) states that the incremental expected return from investing in the two markets must be equal. If they are unequal, all surplus liquidity will flow to the market offering higher return, thereby causing a shutdown of the other market. Thus, when both markets are open, it must be the case the returns are equal across the two markets. Equation (9) states that when only the financial asset market is open, the return from investing in the financial asset must necessarily be strictly greater than the return from investing in the loan market.

4.6. Financial Asset Resale Market Clearing Price (p)

Integrating Equation (5) for intermediaries that are surplus-liquidity, i.e., $\rho < \rho^*$, and using Equation (7) we obtain the following aggregate budget constraint.

$$\bar{\alpha}(p - \rho^*) + \bar{\beta} = \int_{\rho_{\min}}^{\rho^*} (\rho^* - \rho) g(\rho) d\rho, \tag{10}$$

which can be solved using Equation (6) to yield financial asset market-clearing, as given below:

$$\int_{\rho^*}^{\bar{\rho}} \frac{\rho - \rho^*}{p - \rho^*}\, g(\rho)\; d\rho + \int_{\bar{\rho}}^{\rho_{\max}} q\; g(\rho)\; d\rho + \bar{\beta}\frac{1}{p - \rho^*} = \int_{\rho_{\min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*}\, g(\rho) d\rho. \tag{11}$$

Equation (11) can be solved to determine the market clearing price of the financial asset (p):

LEMMA 4: *The financial asset market clears at an equilibrium price ($p(\bar{\beta};\theta)$) given by*

$$p = min\left[\rho^* + \frac{1}{q\; G(\rho_{max})}\left[\int_{\rho_{min}}^{\bar{\rho}} G(\rho) d\rho - \mathcal{B}\right],\; \theta y\right]. \tag{12}$$

The first term on the right hand side of Equation (12) represents the funding liquidity of the financial asset, $\rho^* = \theta y - k$. The combination of the second and the third terms reflects the spare liquidity in the economy. If the spare liquidity in the economy is sufficiently high and exceeds the funding illiquidity of the asset (k), the financial asset will trade at its fair value of θy. This situation would arise when the economic shock (θ) is too mild. When the spare liquidity in the economy is lower than k, fire sales arise and the financial asset trades at a discount to its fair value.

PROPOSITION 1: *Conditional on the economic shock (θ), the economy lies in either one of two mutually exclusive regions: the Fair Pricing region, where both the financial asset and the new loans are fairly price, or the Fire Sale region, where both the financial asset and the new loans are priced at a discount to the fair value. In the Fair Pricing region, the equilibrium characteristics are given by*

$$p = \theta y, \tag{13}$$

$$\bar{f}_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma} < \frac{R}{2}, \tag{14}$$

$$\bar{\beta} = \mathcal{B}. \tag{15}$$

We characterize the Fire Sale equilibrium below. The critical factor driving the type of region is the amount of spare liquidity in the economy. For a given economic shock (θ), the spare liquidity depends on the bankruptcy exemption parameter (q).[12] At lower values of q, bankruptcy exemption is rarely applicable and most credit-constrained intermediaries are able to renegotiate their debt to a lower face value and roll over their obligations. There is minimal liquidation in such an economy and the spare liquidity of surplus-liquidity intermediaries is sufficiently high to cause the market-clearing price of the financial asset to hit the fair value of θy (Fair Pricing region). For higher values of q, there is greater liquidation of the financial asset subsequent to the economic shock, and the spare liquidity of surplus-liquidity intermediaries is stretched, resulting in a market-clearing price lower than the fair value, i.e., fire sales arise (Fire Sale region). We can show further that

PROPOSITION 2: *The Fire Sale region ($p < \theta y$) consists of three types of equilbria, depending on the value of the bankruptcy exemption parameter (q), as discussed below.*

(i) The Price Discrimination Equilibrium: Both the financial asset market and the new loan market are open and loans exhibit price discrimination:

$$\bar{\beta} = \mathcal{B}, \tag{16}$$

$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - \frac{4\gamma k}{p - \rho^*}} > \bar{f}_r, \tag{17}$$

$$p = \rho^* + \frac{1}{q\ G(\rho_{max})}\left[\int_{\rho_{min}}^{\bar{\rho}} G(\rho)d\rho - \mathcal{B}\right]. \tag{18}$$

(ii) The Liquidity Crunch Equilibrium: Both the financial asset market and the new loan market are open, and the loan market experiences a quantity constraint:

$$\bar{\beta} = -q(p - \rho^*)\ G(\rho_{max}) + \int_{\rho_{min}}^{\bar{\rho}} G(\rho)d\rho < \mathcal{B}, \tag{19}$$

$$f_r = \frac{R}{2}, \tag{20}$$

$$p = \rho^* + \frac{4\gamma k}{R^2}. \tag{21}$$

(iii) The Credit Crunch Equilibrium: The new loan market shuts down. Only the financial asset market is open. The equilibrium price (p) is given as below (note that $\bar{\beta} = 0$, although $f_r = \frac{R}{2}$):

$$p = \rho^* + \frac{1}{q\ G(\rho_{max})}\int_{\rho_{min}}^{\bar{\rho}} G(\rho)d\rho. \tag{22}$$

[12] In the ex-post equilibrium, we take the economic shock (θ) as given on Date 1, but in general, the combination of (θ, q) determines the aggregate liquidation of financial assets by credit-constrained intermediaries, as described in Equation (2), which in turn, causes the market price to trade at or below the fair value.

For a given economic shock (θ), if q increases from 0 toward 1, the economy transitions from the Fair Pricing region to the Price Discrimination region, then to the Liquidity Crunch region, and finally to the Credit Crunch region. The three fire-sale regions are discussed in greater detail below.

4.7. Price Discrimination Equilibrium

If q is higher than that implied by the border of the Fair Pricing and Fire Sale regions, there is enough liquidation of assets to cause the financial asset market clearing price to be lower than the fair value of θy. In this region, there is a fire-sale "price" effect in that as q increases, the price discount to fair value increases. This pricing feature is similar to the "cash-in-the-market" pricing in Gale and Allen [1994] and Allen and Gale [1998].

The fire-sale "price" effect causes the gross return from investing in the financial asset to exceed 1. Cross-market arbitraging activity would then imply that the expected return from investing in new loans must match that from investing in the financial asset. Consequently, the face value (equivalently, the effective interest rate) on loans would increase to offer the same return as on the financial asset. We refer to this equilibrium as Price Discrimination because surplus-liquidity intermediaries will divert their resources to the new loan market only if they can earn supra-normal rents, i.e., discriminate on loan rate to ensure that they get the same return as on the financial asset.

At a sufficiently high value of q, the economy transitions to the Liquidity Crunch region, as discussed next.

4.8. Liquidity Crunch Equilibrium

There is a limit up to which surplus-liquidity intermediaries can engage in price discrimination, by increasing the face value on new loans. There is an upper bound on the face value because of the moral hazard problem. Borrowers, being residual cash flow claimants, expend less effort as the face value increases, as shown in Equation (1), and the asset quality suffers. The expected profit from lending in the real sector is, therefore, concave in the face value of the loan. The profit-maximizing face value is $\frac{R}{2}$, and surplus-liquidity intermediaries would never find it incentive compatible to post a higher face value than $\frac{R}{2}$ because the marginal benefit from a higher face value will be lower than the marginal cost in the form of loans with lower asset quality.[13] When this upper bound on the

[13]The expected profit from lending to households (ef_r) is concave in f_r and is maximized at f_r equal to $\frac{R}{2}$. It is worth highlighting that the competitive equilibrium face value (f_r) is the same as the profit-maximizing value for lenders. Thus, the equilibrium is stable to off-equilibrium offers because surplus-liquidity intermediaries would make

loan face value is hit due to an increase in q, the economy transitions from Price Discrimination region to Liquidity Crunch.

In the Liquidity Crunch region, the financial asset reflects a fire-sale "price" effect but remains invariant to q because the new loan return has hit an upper bound and cannot increase any further even when q increases. Cross-market arbitraging activity implies that the financial asset return is also fully arrested, and the price of the financial asset stays at the same level for all values of q in this region. Financial market-clearing is now ensured by sucking out liquidity from the real sector, i.e., by a reduction in $\bar{\beta}$. This diversion of surplus-liquidity intermediaries' resources is required to clear the financial asset market, and new lending to the real sector shrinks with an increase in q in this region. This phenomenon is a fire-sale effect; however, it appears as a quantity discrimination effect in the new loan market, and we refer to it as the fire-sale "quantity" constraint.

The process of shrinking loans to the real sector continues as q increases in this region. At a sufficiently high value of q, the new loan market completely collapses. The economy now transitions to Credit Crunch, which is discussed next.

4.9. Credit Crunch Equilibrium

In this region, the cross-market equilibrium return condition is irrelevant because the value of q is high enough to cause a breakdown of the new loan market. Only the financial asset market is open and now the price of the financial asset can adjust freely to ensure its market-clearing. As in the Price Discrimination Equilibrium, there is a fire-sale "price" effect in this region. The return on the financial asset is no longer bounded by the return on the new loan; in fact, the return on the financial asset always exceeds the potential return on the new loan.

To summarize, an interaction between the funding illiquidity problem in the financial asset and the moral hazard problem in the loan market (which affects the underlying asset quality) drives the underlying economics of the model. First, funding illiquidity triggers fire sales in the financial sector when an adverse economic shock arises. Cross-market arbitraging activity (which ensures that the expected returns in the two markets are the same) implies that the moral hazard problem in loans to the real sector (effort-aversion) is in sync with funding illiquidity problem in the financial sector.

We now move to the ex-ante equilibrium, so that we can evaluate the ex-ante optimal bankruptcy parameter (q) after taking into account the ex-post fire-sale effects.

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lower profits at any other value of f_r.

5. The Ex-Ante Model

In this section, we endogenize the debt obligations assumed by intermediaries who face varying levels of investment shortfall (s) at Date 0. We assume that the investment shortfall (s) is uniformly distributed across intermediaries as U[s_{min}, s_{max}].[14] Intermediaries finance this investment shortfall in the short-term repo market, which is subject to rollover risk at Date 1. Let the outstanding liability at Date 1 to finance shortfall (s) be denoted as $\rho(s)$. Financiers can refuse to roll over debt at Date 1 if they calculate that the state of the economy (θ) at Date 1 will make it impossible for the intermediary to honor its outstanding liability ($\rho(s)$). In such an event, as discussed in Section 4, intermediaries either liquidate a fraction of their asset to overcome the funding deficit, or declare bankruptcy leading to either a liquidation of their asset by the financier with a probability q or a negotiated write-down of their liability to ρ^* with a probability of $(1-q)$.

The key to analyzing the ex-ante model is the observation that the financial asset market-clearing price at Date 1 (i.e., the liquidation price, $p(\theta)$), and the liabilities ($\rho(s)$) assumed at Date 0 are endogenously related. The initial liability structure of intermediaries affects the extent of financial asset liquidation at Date 1, and therefore, its price. Financiers anticipate the implied distribution of the liquidation price (p) over θ and accordingly determine the face value of repo financing to be disbursed at Date 0, i.e., the initial liability structure of financial intermediaries.

Formally, while solving the ex-post model, we assumed an exogenous distribution of ρ and derived the ex-post equilibrium outcomes ($\bar{\beta}, f_r, p$). In the ex-ante model, we begin with a distribution of investment shortfalls (s) at Date 0 which translates into a corresponding distribution of Date 1 liabilities ($\rho(s)$). We denote the resulting distribution of liabilities as $\hat{G}(\rho(s))$. The liquidation price at Date 1 depends on the distribution of ρ across intermediaries. In other words, $\hat{G}(\rho)$ and $p(\theta)$ are determined jointly in equilibrium.

We solve for this equilibrium next and eventually explore the role of the bankruptcy exemption parameter (q) in trading off ex-ante financing against ex-post real outcomes.

5.1. The Ex-Ante Model Set-up

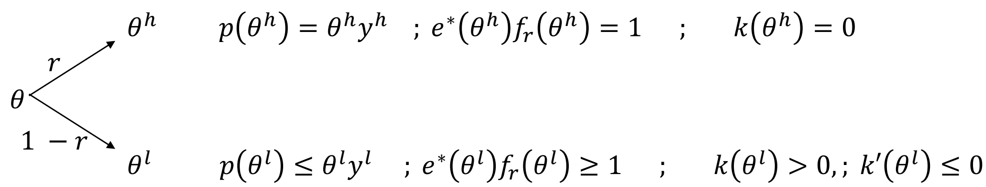
Figure (**2**) provides the basic set-up for the ex-ante model. As of Date 0, the Date 1 shock, θ, is unknown. For tractability, we consider a discrete two-state distribution for θ: with a probability, r, the state of the economy is described by θ^h (which we refer to as the high state), and with a

[14] s_{max} is the maximum shortfall at which the asset is still NPV positive.

probability, $(1-r)$, the state of the economy is described by θ^l (which we refer to as the low state). The asset has a payoff of y^h in the high state and y^l in the low state such that $y^h > y^l$.

We recognize that the role of financial frictions in repo markets is contingent on the state of the economy. Gorton and Metrick [2010] build on insights from Gorton and Pennacchi [1990] and Dang et al. [2010] to argue that repo securities are "information insensitive" securities during normal times (resulting in high liquidity), but are highly "information sensitive" when the economic shock is severe (resulting in liquidity drying up). To incorporate these ideas, we assume that the financial frictions that lead to funding illiquidity (as discussed in Section 4.1) arise only in the low state of the economy, i.e., (i) $k(\theta^h) = 0$ and (ii) $k(\theta^l) > 0$. Further, as the financial frictions driving the illiquidity discount are likely to exacerbate with the severity of the economic shock, we assume k is a non-increasing function of θ^l (i.e., $k'(\theta) <= 0$).[15]

Figure **2**: **Ex-ante view of the states of the economy (θ).** The economy is in the high state (θ^h) with a probability r and in the low state (θ^l) with a probability $1-r$. In the high state of the economy, both the financial asset and the new loans are fairly priced. However, in the low state of the economy, both assets *could* exhibit fire-sale effects.



Secondly, we also assume that moral-hazard (effort-aversion) in loans to the real sector market is relevant only in the low state (i.e., $\gamma = 0$ in the high state).[16] Thus, the funding liquidity of the financial asset in the high state is equal to its fair value ($p = \theta^h y^h$), and due to arbitrage, new loans at Date 1 would also be fairly priced, i.e., $ef_r = 1$. Further, since household borrowers exhibit no effort aversion ($\gamma = 0$), the effort (e) in the high state hits the highest feasible value of 1. It follows that the face value of Date 1 loans (f_r) equals 1 in the high state.

Finally, we assume that the market for Date 1 loans is fully satiated in the high state, i.e., there is no unmet credit demand of household borrowers in the high state.[17] In the high state, all

[15] In Appendix B, we showcase an agency-theoretic motivation for these reduced-form assumptions. We endogenously derive the funding illiquidity (k) and show that $k(\theta^l) > 0$ and $k'(\theta^l) < 0$.

[16] Lack of effort-aversion for household borrowers in the high state is assumed to mirror the lack of frictions in the financial asset market. However, the results of the paper follow even in the absence of this assumption.

[17] In general, one can put an explicit restriction on $\mathcal{B}$ to be strictly less than an endogenously determined $\bar{\beta}$ in the high state, thereby ensuring that there will be no unmet demand. This restriction would essentially result in a constraint on θ^h. To avoid clutter, we express this constraint as a simple assumption, which states that there is no unmet demand for new loans in the high state.

intermediaries will be able to roll over their debt because funding liquidity is equal to the fair value of the asset. Thus, the system is in the Fair Pricing region:[18]

$$p(\theta^h) = \theta^h y^h \; ; \qquad f_r(\theta^h) = 1 \; ; \qquad \bar{\beta}(\theta^h) = \mathcal{B} \tag{23}$$

In contrast, in the low state, some intermediaries are always credit-constrained and unable to roll over their debt without liquidating some or all of their assets. Furthermore, the Date 1 loan market is not always satiated in the low state. Thus, any of the four equilibrium types described in Section (4.6) could exist in the low state depending on the severity of the economic shock (θ^l).[19] The equilibrium characteristics in the low state are as specified in Propositions (1) & (2). For simplicity of notation, we omit explicit reference of the state when referring to the equilibrium characteristics of the low state in the following sections (i.e., p refers to $p(\theta^l)$, f_r refers to $f_r(\theta^l)$, $\bar{\beta}$ refers to $\bar{\beta}(\theta^l)$, ρ^* refers to $\rho^*(\theta^l)$, k refers to $k(\theta^l)$ and $\bar{p}$ refers to $\bar{p}(\theta^l)$). We continue to use explicit references to the high state while discussing its equilibrium characteristics, as in Equation (23).

5.2. *Payoffs and Investment Shortfall Financing*

As shown in Figure (**2**), the high state occurs with a probability of r and the low state with a probability of $1 - r$. Financiers take into account the payoff potential in both states of the world. In the high state (θ^h), the payoff potential is $p(\theta^h) = \theta^h y^h$. In the low state, the possible payoffs are determined as follows.

Financiers are repaid in full by surplus-liquidity intermediaries ($\rho \leq \rho^*(\theta^l)$) and moderately credit-constrained intermediaries ($\rho^*(\theta^l) < \rho \leq \bar{p}(\theta^l, q)$).

For severely credit-constrained intermediaries ($\rho > \bar{p}(\theta^l, q)$), with a probability q, financiers take control and liquidate the asset at the market-clearing price of $p(\theta^l)$, while with a probability of $(1 - q)$, the liability is renegotiated downward to the asset's funding liquidity, ρ^*; thus, given an adverse shock θ^l at Date 1, financiers can expect a maximum payoff of $\bar{p}$, given by:

$$\bar{p}(\theta^l, q) = qp(\theta^l, q) + (1 - q)\rho^*(\theta^l). \tag{24}$$

[18] The results for $p(\theta^h)$ and $\bar{\beta}(\theta^h)$ follow from the equilibrium characteristics of the system in the fair-pricing region as obtained in Proposition (1). However, in the absence of effort-aversion in households, households exert maximal effort ($e^* = 1$); implying that a fairly priced Date 1 loan ($e^* f_r = 1$) would have unit face value ($f_r = 1$).

[19] Note that fair pricing in the high state is not the same as fair pricing in the low state. First, as $\rho^*(\theta^h) = p(\theta^h) = \theta^h y^h$, all intermediaries can roll over their debt in the high state; in the low state, $\rho^*(\theta^l) = \theta^l y^l - k$ and intermediaries having $\rho > \rho^*(\theta^l)$ will be unable to roll over their debt without partially (or fully) liquidating their financial asset even in the fair-pricing equilibrium. Second, due to the absence of effort-aversion by households in the high state, $f_r(\theta^h) = 1$; whereas in the low state due to non-zero effort-aversion, $f_r(\theta^l) = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma}$ in the fair-pricing equilibrium, as given by Proposition (1).

Note that $\bar{p}(\theta^l, q)$, in the above equation, is equal to $\bar{\rho}(\theta^l, q)$, where $\bar{\rho}$ is the cutoff above which credit-constrained intermediaries strategically default, as elaborated in Lemma (2). In other words, $\bar{p}$, the expected proceeds (under bankruptcy) of financiers in the low state, is exactly equal to the leverage level above which intermediaries would strategically default. This is intuitively reasonable: credit-constrained intermediaries would rather strategically default and incur an expected payoff of $\bar{p}$ rather than pay $\rho > \bar{p}$, even if they can liquidate assets and pay ρ.

Figure **3**: **Ex-ante Payoff Tree.** The financier's Date 1 payoff for a given state (θ^l) in different cases is shown along with the probability of the case.

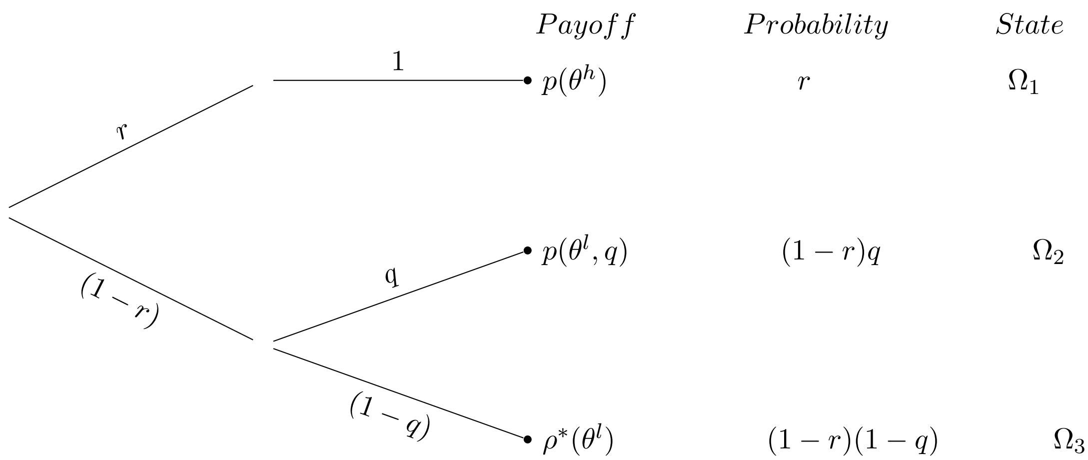


Figure (3) summarizes the payoffs. From the financier's perspective, the maximum shortfall that can be financed based on the asset's payoffs is given by $\hat{s}(q) = rp(\theta^h) + (1-r)\bar{p}(\theta^l, q)$, which is always less than or equal to s_{max}. Consequently, the range of shortfalls that get financed at Date 0 is given by $[s_{min}, \hat{s}]$, i.e., intermediaries with shortfalls $(\hat{s}, s_{max}]$ are rationed at Date 0. The next lemma below discusses the endogenous distribution of leverage in the economy at Date 0.

LEMMA 5: *Given a uniform distribution of investment shortfalls in the economy (i.e., $H(\tilde{s})$ is $U[s_{min}, s_{max}]$), the endogenous distribution of leverage ($\rho : \rho \in [\rho_{min}, \rho_{max}]$) at Date 0 – that takes into account the expected payoff to the financiers at Date 2 – is specified by $\hat{G}(\rho)$, as follows:*

$$\hat{G}(\rho) = \frac{\tilde{s}(\rho) - s_{min}}{s_{max} - s_{min}},$$

$$\text{where } \tilde{s}(\rho) = \begin{cases} \rho & \textit{if} \quad \rho_{min} \leq \rho \leq \bar{p}(\theta^l), \\ r\rho + (1-r)\bar{p}(\theta^l) & \textit{if} \quad \bar{\rho}(\theta^l) < \rho \leq p(\theta^h), \end{cases} \tag{25}$$

where we have suppressed in notation the dependence of $\bar{p}(\theta^l)$ and $\bar{\rho}(\theta^l)$ on q.

5.3. *Ex-ante Dynamic Equilibrium*

The ex-ante dynamic equilibrium is (i) a pair of functions $\rho(s)$ and $p(\theta^l)$, which respectively give the promised face value ($\rho(s)$) for raising short-term repo financing of s units at Date 0 and the equilibrium price ($p(\theta^l)$) at Date 1 given the interim signal of asset quality of θ^l; and (ii) a truncation point $\hat{s}$, such that $\rho(s)$, $p(\theta^l)$ and $\hat{s}$ satisfy the following fixed-point recursion:

1. For a given θ^l, the asset's price ($p(\theta^l)$) is given by the market-clearing and cross-market arbitrage determined price function in Proposition (1) and Proposition (2).

2. Individual rationality of financiers: Given the price function $p(\theta^l)$, for every shortfall $s \in [s_{min}, \hat{s}]$, the promised face value $\rho(s)$ is determined by the requirement that financiers receive in expectation the amount being lent, i.e., $\tilde{s}(\rho(s)) = s$, where $\tilde{s}(\rho(s))$ is given by Equation 25.

3. The derived distribution of leverage, $\hat{G}(\rho)$, depends on $\tilde{s}(\rho) \in [s_{min}, \hat{s}]$ where $\hat{s}$ is the maximal investment shortfall that is financed (Equation 25).[20]

The ex-ante equilibrium is defined for a given θ^h and θ^l. In the high state, the endogenous distribution of leverage has no impact on the equilibrium characteristics. In the low state, the equilibrium characteristics will mirror the solution provided in Proposition (1) and Proposition (2), except that the specified distribution of leverage ($G(\rho)$) in Equations (18), (19), and (22) must now be substituted by the endogenously derived distribution ($\hat{G}(\rho)$), as described in Equation (25).[21] The bankruptcy exemption parameter (q) affects the equilibrium characteristics both through its ex-post impact on liquidation price ($\bar{p}(\theta^l)$) and its ex-ante impact on distribution of leverage ($\tilde{s}(\rho)$ and $\hat{s}$).

5.4. *Equilibrium regions*

Keeping θ^h fixed, we vary θ^l and analyze the relation between the equilibrium characteristics in the low state and the bankruptcy exemption parameter (q).[22]

Figure (4) shows the typical demarcation of the feasible (q, θ^l) space into the Fair Pricing (FP) region, as shown in white, and the Fire Sale (FS) region, as shown by the gray shade. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC), and the Credit Crunch

[20]Because $\tilde{s}(\rho)$ depends on the asset's price ($p(\theta^l)$), the derived distribution, $\hat{G}(\rho)$, depends on the asset price.

[21]In the ex-ante setup, $\bar{\beta}$ in the Liquidity Crunch Equilibrium as well as p in the Price Discrimination Equilibrium and the Credit Crunch Equilibrium are functions of the distribution of leverage; consequently, the specification of these terms vary from that obtained for the ex-post equilibrium (see the Appendix B for the closed-form equilibrium solutions of $\bar{\beta}$ and p).

[22]The interval $[\theta^l_{min}, \theta^l_{max})]$ over which we vary θ^l is determined by feasibility constraints. The lower bound θ^l_{min} ensures financial market clearing for all θ^l, while the upper bound θ^l_{max} ensures that $\theta^l < \theta^h$.

(CC) equilibria; we use increasingly darker shades of gray to represent greater fire-sale effects. For different magnitudes of the economic shock (θ^l), we see how the type of equilibrium changes with the bankruptcy parameter (q). The solid $\bar{\bar{q}}(\theta^l)$ curve represents the boundary between the FP and PD regions. The long dashed $\bar{q}(\theta^l)$ curve represents the boundary between the PD and LC regions. The dotted $\hat{q}(\theta^l)$ curve represents the boundary between the LC and CC regions. Consider, for example, the case with $\theta^l = 0.35$. The vertical dotted line emanating from this level of θ captures how the system transitions across different types of equilibrium regions, as q increases from 0 to 1 along the dotted vertical line.

5.5. Equilibrium Characteristics for a Given Economic Shock

Figure (5) shows the evolution of equilibrium values of p (Panel A), $\bar{p}$ (Panel B), $\bar{\beta}$ (Panel C), and f_r (Panel D) as we vary q from 0 to 1. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines. Although difficult to detect by observing the figures, the relation between $\bar{p}$ with q is non-monotonic.[23] Furthermore, it can be seen in Panel C that $\bar{\beta}$ is (weakly) decreasing q, and in Panel D that f_r is (weakly) increasing in q. Thus, the real sector characteristics are monotonic in q.

Panel E shows the equilibrium return on the financial asset market and the new loan market. The returns in both these markets are the same in the FP, PD, and LC regions, but diverge in the CC region, where the financial asset market returns exceeds that of the new loan market which shuts down. Panel F shows the decreasing relation between effort and bankruptcy exemption; it implies that the loan quality worsens as bankruptcy exemption parameter (q) increases.

6. Welfare Analysis

In this section, we examine the welfare implications of bankruptcy exemption for a given $\theta^l \in (\theta^l_{min}, \theta^l_{max})$. We evaluate the economic surplus created due to lending at Date 0 and lending at Date 1 as a function of q. We show that surplus due to Date 0 lending surplus is weakly increasing in bankruptcy exemption, while surplus due to Date 1 lending is weakly decreasing in bankruptcy exemption. Thus, from an ex-ante welfare perspective, bankruptcy exemption may create a tension between surplus created due to Date 0 lending and Date 1 lending, and bankruptcy exemption can

[23] In the ex-ante setup, q has the following impact on equilibrium characteristics (Proofs in the Internet Appendix B): $\frac{dp}{dq}\Big|_{PD,CC} < 0$, $\frac{dp}{dq}\Big|_{LC} = 0$, $\frac{d\bar{p}}{dq}\Big|_{PD,CC} = 0$, $\frac{d\bar{p}}{dq}\Big|_{LC} > 0$, $\frac{df_r}{dq}\Big|_{PD} > 0$, $\frac{df_r}{dq}\Big|_{LC,CC} = 0$, $\frac{d\bar{\beta}}{dq}\Big|_{PD,CC} = 0$, and $\frac{d\bar{\beta}}{dq}\Big|_{LC} < 0$.

Figure 4: **Equilibrium regions.** Typical demarcation of the feasible $q - \theta^l$ space into the Fair Pricing (FP), as shown by the white region and the Fire Sale (FS) region, as shown by the gray shaded region. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC) and the Credit Crunch (CC) equilibria. The solid $\bar{\bar{q}}(\theta^l)$ curve is the boundary between the FP and PD equilibrium regions. The long dashed $\bar{q}(\theta^l)$ curve is the boundary between the PD and LC equilibrium regions. The dotted $\hat{q}(\theta^l)$ curve is the boundary between the LC and CC equilibrium regions. The PD, LC and CC equilibrium regions jointly constitute the Fire Sale Equilibrium Region which is indicated by the differing shades of gray (the darker shades indicate greater fire-sale effects). For a strong economic shock, indicated by $\theta^l = 0.35$, as q is increased from 0, the system transitions from FP equilibrium to PD equilibrium at $q = 0.15$, then from PD equilibrium to LC equilibrium at $q = 0.31$ and finally from LC equilibrium to CC equilibrium at $q = 0.61$. For a mild economic shock indicated by $\theta^l = 0.5$, the system remains in FP equilibrium for any q. For a severe economic shock, indicated by $\theta^l = 0.25$, the system starts in LC equilibrium at $q = 0$ and transitions to CC equilibrium at $q = 0.1$. $\theta^l = 0.35$ is indicated by a thin vertical dashed line. Parameter Configuration used: $\theta^l_{min} = 0.15$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$, $k = 1.5$, and $\mathcal{B} = 0.15$.

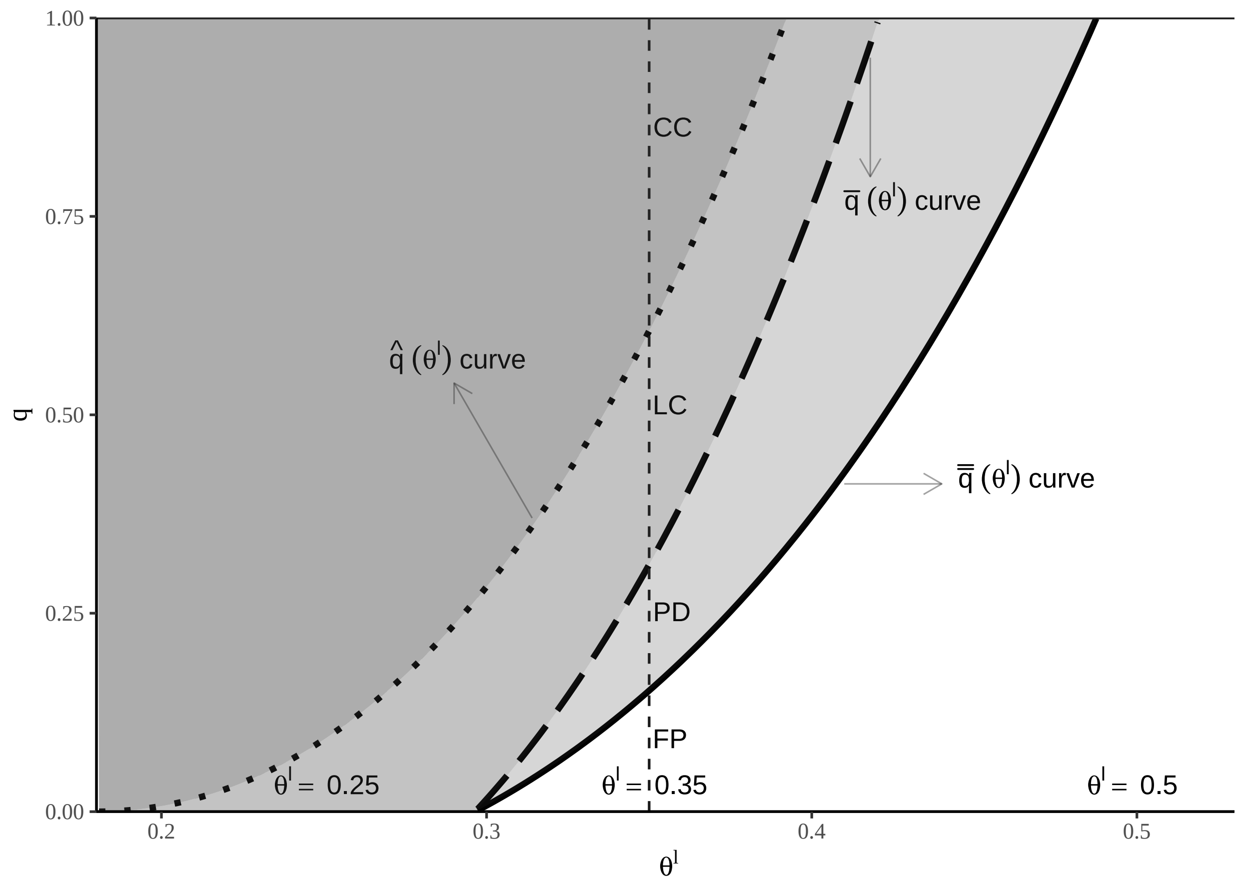
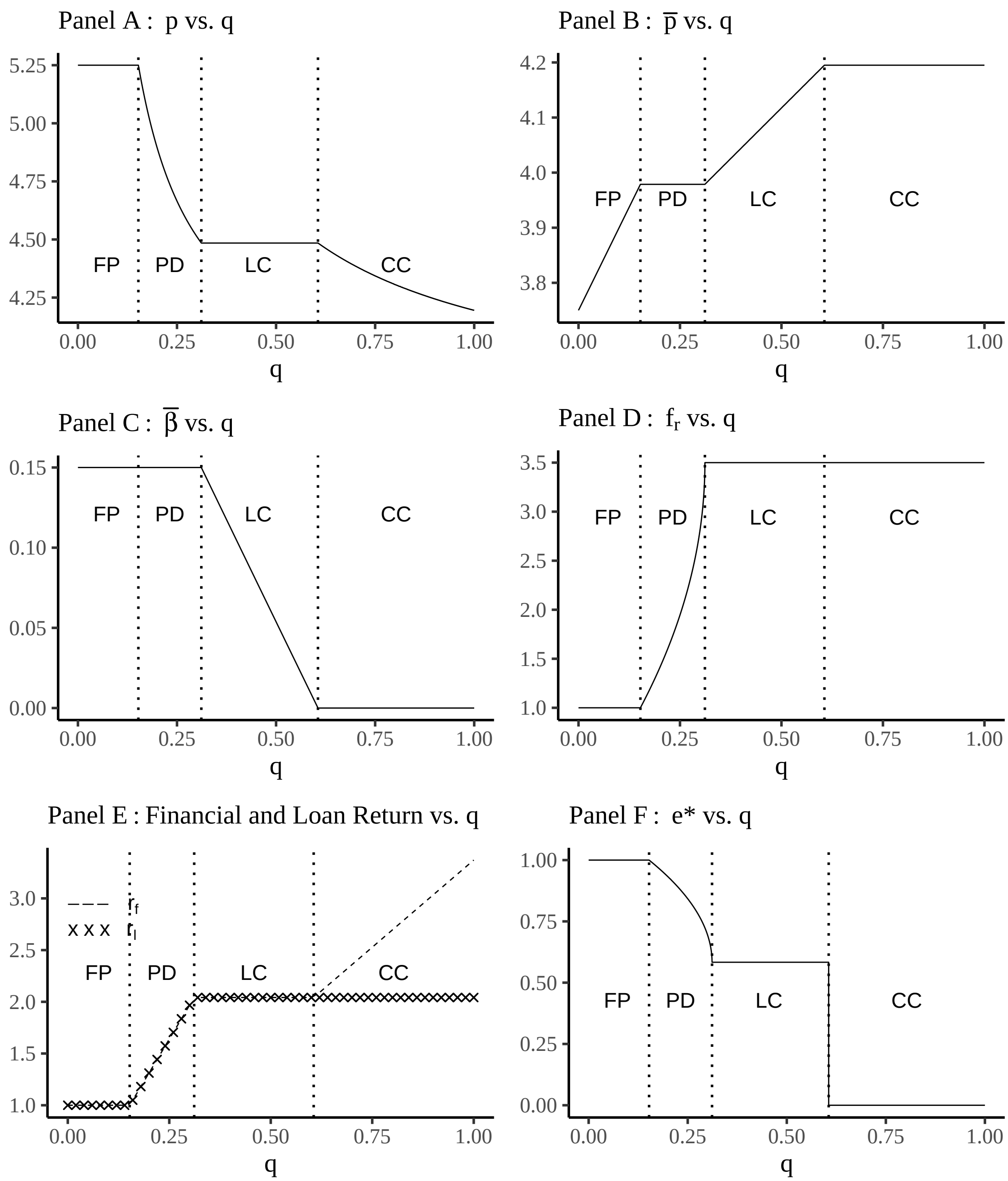


Figure 5: **Evolution of equilibrium p, $\bar{p}$, $\bar{\beta}$, f_r, r_f, $_r$ and e^* with q for a given θ^l.** Panel A depicts the price of the financial asset (p), Panel B depicts the financiers' expected payoff from the financial asset ($\bar{p}$), Panel C depicts the level of Date 1 loan loans made ($\bar{\beta}$), Panel D depicts the face value of new loans (f_r), Panel E depicts the returns from the financial (r_f) and loan (r_l) asset and Panel F depicts the optimal effort (e^*) exerted by a borrower in the Date 1 loan market. The evolution of the equilibrium level of these variables is shown as q is increased from 0 to 1 at $\theta^l = 0.35$. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines. Transition points: FP to PD at $\bar{\bar{q}} = 0.15$, PD to LC at $\bar{q} = 0.31$ and LC to CC at $\hat{q} = 0.61$. Parameter Configuration used: $\theta^l = 0.35$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$, $k = 1.5$, and $\mathcal{B} = 0.15$.



be set at an optimal tradeoff. We begin the analysis with surplus creation due to Date 1 lending.

6.1. Surplus Creation Due to Date 1 *Lending*

The Date 1 surplus, conditional on θ (θ^h or θ^l) depends in general on q through the number of Date 1 loans supplied ($\bar{\beta}(q;\theta)$) and the surplus created per real asset loan ($S_r(q;\theta)$), which is given by expected payoff of the real asset created at Date 1, net of pecuniary equivalent of effort (e) expended by households. More specifically, in the high state, $S_r(q;\theta^h) = e^*(\theta^h)R = R$ as there is no effort aversion. In the low state $S_r(q;\theta^l) = e^*(\theta^l)R - \frac{1}{2}\gamma[e^*(\theta^l)]^2$, where effort, $e^*(\theta^l) = \frac{1}{\gamma}[R - f_r(\theta^l)]$, is endogenously determined because the equilibrium face value (f_r) depends on q. Using these results for the high state (θ^h) and the low state (θ^l), the expected Date 1 surplus is

$$S_{D1}(q) \; = r\mathcal{B}R + (1-r)\bar{\beta}(q;\theta^l)S_r(q;\theta^l). \tag{26}$$

In the high state (θ^h), the face value is equal to 1 and there is no unmet demand in the new loan market, i.e., $\mathcal{B}$ loans are originated. Thus, Date 1 surplus created in the high state is equal to $\mathcal{B}R$, which is independent of q, and the high state occurs with probability r, giving the first term. The second term in Equation (26) reflects the Date 1 surplus, conditional on the low state (θ^l), after factoring in the probability of the low state $(1-r)$. This term depends on q through the aggregate loan amount $\bar{\beta}(q;\theta^l)$ as well as the surplus created per unit loan $S_r(q;\theta^l)$. Furthermore, the dependence on q varies across different types of equilibrium that may arise in the low state.

We rely on the comparative statics (Footnote 23) to show that, for a given θ^l and θ^h, S_{D1} is invariant to q in the Fair Pricing and Credit Crunch regions but strictly decreasing in q in the Price Discrimination and Liquidity Crunch regions. The relationship of S_{D1} with q can thus be summarized as weakly decreasing. The first set of rows in Table 1 provides specific insights for understanding this relation across all the different types of equilibrium. In essence, fire-sale "price" effects, which affect f_r, and fire-sale "quantity" effects, which affect $\bar{\beta}$, cause S_{D1} to be (weakly) decreasing in q. Interestingly, an important implication arising from this result is that the expected Date 1 surplus is *never* increasing in the bankruptcy exemption parameter q.

6.2. Surplus Creation Due to Date 0 *Lending*

The expected surplus created by Date 0 lending (S_{D0}) is calculated as follows. Recall our modeling assumption that financial intermediaries face investment shortfalls ($\tilde{s}$) that arise from a Uniform distribution $H(\tilde{s}) \sim U(s_{min}, s_{max})$. By investing an amount s, a financial intermediary creates an asset with an expected payoff of $E_\theta[\theta y]$; thus, the surplus created by a financial intermediary is the NPV of the financial asset, i.e., $E_\theta[\theta y - s]$. Then, the expression for S_{D0} is given by aggregating the expected surplus across all financial intermediaries that have NPV positive projects at Date 0 (i.e., those intermediaries that have investment shortfall, $\tilde{s}$, less than $s_{max} = E_\theta(\theta y)$). Therefore, the expected Date 0 surplus is

$$S_{D0}(q) = \int_{s_{min}}^{\hat{s}} E_\theta[\theta y - s] dH(s) \tag{27}$$

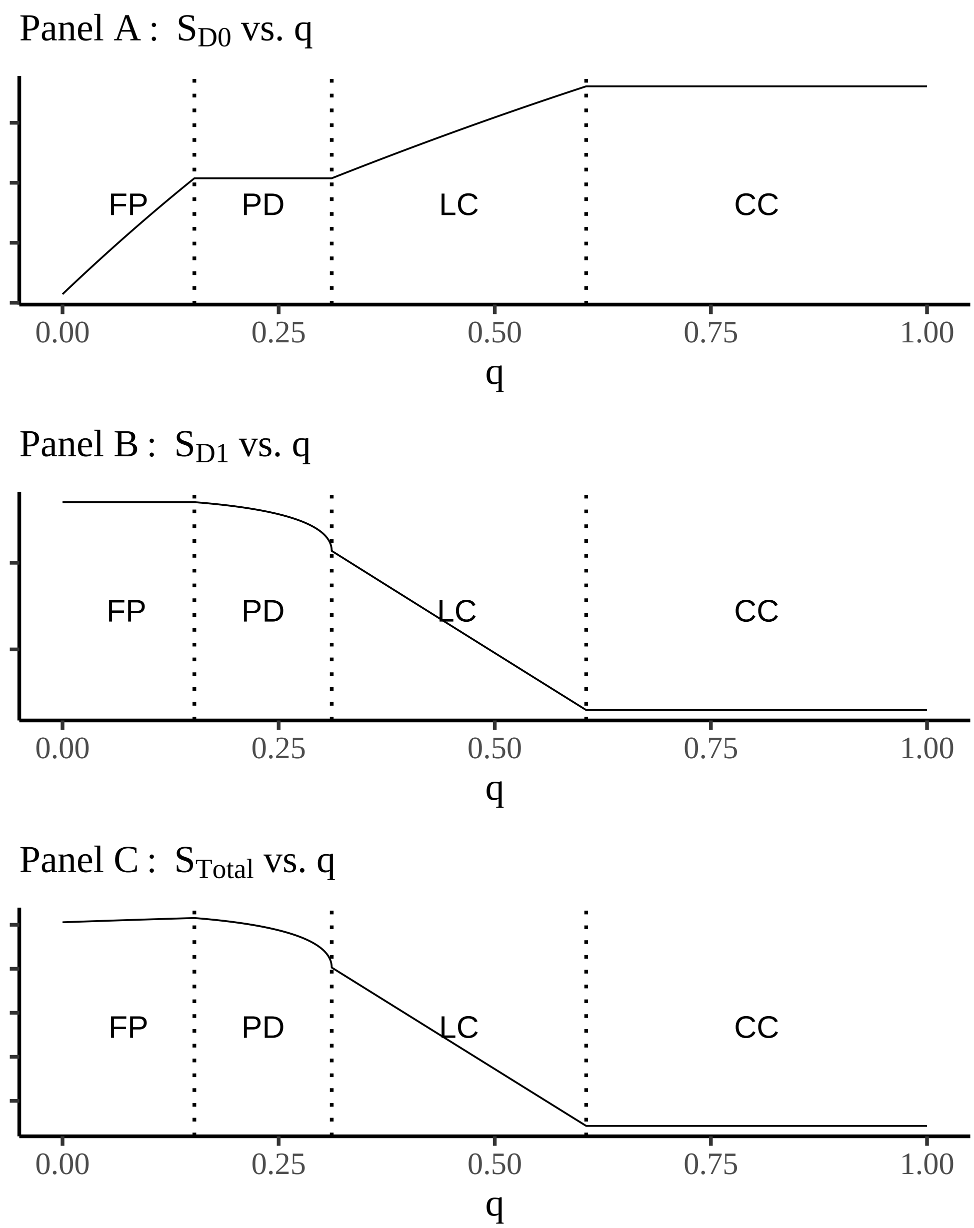
$$= \hat{s} - s_{min} - \frac{1}{2}\frac{(\hat{s} - s_{min})^2}{(s_{max} - s_{min})}. \tag{28}$$

It can be shown that $S_{D0}(q)$ is increasing in $\hat{s}$. Furthermore, since $\hat{s}(q)$ is equal to $rp(\theta^h) + (1 - r)\bar{p}(\theta^l, q)$, it is increasing in $\bar{p}$, and it follows that S_{D0} is increasing in $\bar{p}$. Thus, the relation between S_{D0} and q depends on the relation between $\bar{p}$ and q.

As discussed earlier, the expected financial asset price ($\bar{p}$) could be increasing or invariant in q depending on the type of equilibrium. In the Fair Pricing and the Liquidity Crunch regions, $\bar{p}$ is increasing in q, but in the Price Discrimination and the Credit Crunch regions, $\bar{p}$ is invariant in q. The second set of rows in Table 1 provides specific insights for understanding this relation across the different types of equilibrium.

Figure (**6**) illustrates the evolution of the expected Date 0 surplus (S_{D0}), the expected Date 1 surplus (S_{D1}), and the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q), conditional on a strong Date 1 shock ($\theta^l = 0.35$), as indicated by the marker in Figure (**4**). We see that the system transitions from the Fair Pricing to the Price Discrimination to the Liquidity Crunch and finally to the Credit Crunch region as q increases. In the Fair Pricing region, Date 0 surplus (S_{D0}) increases with q while Date 1 surplus (S_{D1}) is invariant in q causing the total surplus (S_{Total}) to increase in q. However, when the system transitions to the Price Discrimination region at $q = 0.15$, both S_{D0} and S_{D1} decrease with q causing S_{Total} to decrease as well. As q is further increased the system transitions into the Liquidity Crunch region at $q = 0.31$. While S_{D0} increases with q here, this increase is swamped by the reduction in S_{D1}, leading to an overall reduction in S_{Total} with q in the Liquidity Crunch region. Finally, the system transitions to the Credit Crunch region at $q = 0.61$, the new loan market shuts down, i.e., S_{D1} is again invariant in q, but S_{D0} decreases with q in this region. Consequently, S_{Total} is decreasing in the Credit Crunch region, as well. Therefore, as can be seen in Panel C, expected total surplus (S_{Total}) is maximized at the boundary of the Fair Pricing and Price Discrimination regions ($q = 0.15$).

Figure 6: **Ex-ante Equilibrium Total Surplus Evolution.** Panel A shows the evolution of the expected Date 0 surplus (S_{D0}), Panel B shows the evolution of the expected Date 1 surplus (S_{D1}) and Panel C shows the evolution of the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q) for a strong Date 1 shock ($\theta^l = 0.35$). As q increases, the system transitions from Fair Pricing (FP) equilibrium to Price Discrimination (PD) equilibrium at $q = 0.15$, then from PD equilibrium to Liquidity Crunch (LC) equilibrium at $q = 0.31$ and finally from LC equilibrium to Credit Crunch (CC) equilibrium at $q = 0.61$. The dotted lines represent the boundaries between the equilibrium regions. The dynamics are obtained for the same parameter configuration for which the demarcation of the feasible $q - \theta^l$ space is shown in Figure 4 (i.e., $\theta^l = 0.35$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$, $k = 1.5$, and $\mathcal{B} = 0.15$.)



6.3. *Total Surplus Creation*

We can now assess the optimal choice of the bankruptcy exemption parameter (q) by maximizing the sum of the expected surplus created at Date 0 and the expected surplus created at Date 1, i.e., the expected total surplus $S_{Total} = S_{D0} + S_{D1}$. Given that S_{D0} is (weakly) increasing in q and that S_{D1} is (weakly) decreasing in q, it seems reasonable to expect that there is an optimal q that maximizes the S_{Total}.

Fair Pricing region	Price Discrimination region	Liquidity Crunch region	Credit Crunch region
$\bar{\beta} \leftrightarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \leftrightarrow$ with q	$\bar{\beta} \leftrightarrow$ with q $f_r \uparrow$ with q $\Rightarrow S_{D1} \downarrow$ with q	$\bar{\beta} \downarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \downarrow$ with q	$\bar{\beta} \leftrightarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \leftrightarrow$ with q
$\bar{p} \uparrow$ with q $\hat{s} \uparrow$ with q $\Rightarrow S_{D0} \uparrow$ with q	$\bar{p} \leftrightarrow$ with q $\hat{s} \leftrightarrow$ with q $\Rightarrow S_{D0} \leftrightarrow$ with q	$\bar{p} \uparrow$ with q $\hat{s} \uparrow$ with q $\Rightarrow S_{D0} \uparrow$ with q	$\bar{p} \leftrightarrow$ with q $\hat{s} \leftrightarrow$ with q $\Rightarrow S_{D0} \leftrightarrow$ with q
$S_{Total} \uparrow$ with q	$S_{Total} \downarrow$ with q	$S_{Total} \downarrow\uparrow$ with q	$S_{Total} \leftrightarrow$ with q

Table 1: **Equilibrium Characteristics in each region in the low state of the economy**. Behavior of the price of the financial asset (p), the expected Date 1 payoff to financiers from the financial asset ($\bar{p}$), the equilibrium face value of loans (f_r) and the number of loans ($\bar{\beta}$), as a function of the bankruptcy exemption parameter (q) in each of the equilibrium regions in the low state of the economy.

Table (1) summarizes the welfare trade-offs under each equilibrium type in the low state, conditional on a given value of (θ^l). At low q, the system is in the Fair Pricing region, as shown in the first column of Table (1). As q increases the system transitions into the Fire Sale regions, as shown in the second, third, and fourth columns of Table (1).

As elaborated in Table 1 (bottom row), it is only in the Liquidity Crunch region that there exists a trade-off between Date 0 surplus and Date 1 surplus. We show that under a reasonable condition (to be discussed shortly), S_{Total} is decreasing with q in the Liquidity Crunch region as well. Furthermore, since the expected total surplus (S_{Total}) is invariant to q in the Credit Crunch region, it follows that the optimal q is always at the boundary of the curve demarcating the Fair Pricing region and the Fire Sale regions (i.e., $q^{opt} = \bar{\bar{q}}$, see Figure 4).

PROPOSITION 3: *For payoff structures of financial assets underlying repo and real assets underlying Date* 1 *loans that satisfy* $E_\theta[\rho^*(\theta)] \geq \theta^l y^l$, *the optimal* q (q^{opt}) *that maximizes total surplus* (S_{Total})

is at the border of the Fair Pricing region and the Fire Sale region:

$$q^{opt} = \frac{-r(\theta^h y^h - \rho^*) + \sqrt{[r(\theta^h y^h - \rho^*)]^2 + (1-2r)\left[(\rho^* - s_{min})^2 - 2\mathcal{B}(s_{max} - s_{min})\right]}}{(1-2r)k} \tag{29}$$

The intuition behind this result can be stated as follows. A marginal increase in q results in incremental lending at Date 0; these additional loans are made to those intermediaries who face high investment shortfalls. Two implications follow: (i) the NPV of the assets originated by these intermediaries is low because of the high investment requirements, and (ii) these intermediaries are also the most leveraged intermediaries because of the large investment requirements that they have to finance with repo financing. As a consequence, Date 0 lending, *at the margin*, results in *low NPV* asset origination by highly leveraged intermediaries, who will face adverse fire-sale effects at Date 1 when an economic shock occurs. Thus, the loss in Date 1 surplus dominates the low NPV gain from *incremental* assets created at Date 0, provided the condition on asset payoffs in Proposition 3 holds. Intuitively, the ex-ante welfare gain from incremental lending at Date 0 is swamped by the large ex-post distributive externality imposed by the incremental financing of highly leveraged intermediaries.

The condition on asset payoffs in Proposition 3 states that the ex-ante expected funding liquidity should be at least as high as the ex-post payoffs in the adverse state of the economy. Violation of this condition implies that repo-financing would be unattractive for highly leveraged intermediaries. If the ex-ante expectation of funding liquidity is too low, highly leveraged intermediaries realize that they would be unable to roll over their loans at Date 1; this deters all these intermediaries from participating in the economy, and the overall leverage in the economy would be low. As a consequence, fire-sale effects would be negligible, and it may be optimal to increase q beyond the border of the Fair Pricing and Fire Sale region.

The condition in Proposition (3) is likely to be satisfied when k is low and r is high, i.e., when the collateral quality is high and when the probability of shock that leads to fire sales is low; in such a case the repo has sufficiently high funding liquidity and the safe-harbor provision can be optimal even in a region where there are fire sales.[24] Note that the typical extension of the safe-harbor provisions is to highly liquid and low-risk assets such as Treasuries and Agencies (i.e., low k), so that from a practical standpoint of a normal economic environment (high r), the relevant region of parameters is likely to be the one where complete safe-harbor provision and entailing of fire sales is

[24] Upon simplification of the condition (see Appendix B, Section (A13.1)), it can be shown that it is equivalent to $\frac{r}{1-r} \geq \frac{k}{(\theta^h y^h - \theta^l y^l)}$.

optimal. Appendix (B12) lays out details of the optimal q when the condition in Proposition 3 is violated.

In numerical analysis of the model, we observe that feasible parameter spaces that violate the condition stated in Proposition (3) rarely occur. This assumption, which also helps in model tractability, is employed for the remainder of the paper.

To summarize, our results demonstrate that providing bankruptcy exemption in repo markets (i.e., setting $q = 1$) while creating "too much today" may also provide "too little tomorrow". There is a trade-off between these two effects that determines the socially optimal bankruptcy exemption parameter (q^{opt}).

The intuition behind our results can be stated as follows. An increase in the bankruptcy exemption parameter (q) enhances social welfare in the Fair Pricing region because it facilitates greater ex-ante creation of financial assets. Any further increase in q beyond the Fair Pricing region (and thus in the Fire sale region) attracts highly-leveraged marginal borrowers who are prone to ex-post fire sales in the event of an adverse economic shock. Less leveraged borrowers with surplus liquidity take advantage of this fire sale opportunity and reduce their supply of capital to the Date 1 real asset loan market, thereby causing a significant distributive externality in the real economy. Furthermore, the value addition due to the ex-ante increase in origination of financial assets is low because the marginal borrowers are highly leveraged and the surplus created by these intermediaries is mostly consumed in servicing their debt. Taken together, any further increase in the bankruptcy exemption parameter (q) beyond the border of Fair Pricing and the Fire Sale region is welfare-reducing, and the optimal q ensures that the Fire Sale regions are avoided.

We can derive the intuitive relationship of the level of the optimal bankruptcy exemption parameter (q) to three key parameters of the model.

PROPOSITION 4: *The optimal bankruptcy exemption (q^{opt}) is weakly decreasing in the severity of the economic shock (θ^l), inverse collateral quality (k), and size of the real economy's loan demand at Date* 1 *($\mathcal{B}$).*

The implication is that bankruptcy exemption is costlier during adverse economic times and when the demand of loans for the real sector is large. Thus, the socially optimal choice in these cases could be to provide an automatic stay on repos.

Lewis [2023] empirically studied credit supply around the introduction of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA), which accorded safe harbor to risky mortgage collateral. Lewis [2023] found that this resulted in an increase in credit supply of loans tied to low-quality collateral. Our model suggests that according safe harbor to low-quality collateral might not be desirable from an aggregate/systemic standpoint as it triggers subsequent liquidation to meet refinancing constraints, thereby adversely affecting subsequent credit supply. Consistent with this argument, the Federal Reserve Report [2011], presented in the aftermath of the global financial crisis of 2008, and the studies of Edwards and Morrison [2005], Jackson [2009], Skeel and Jackson [2011], and Duffie and Skeel [2012] point out that full repeal of the safe harbor provisions is not desirable. These authors argue that bankruptcy exemption should be continued for Qualified Financial Contracts (QFCs) in which collateral is in the form of cash or cash-equivalent assets but should be removed for QFCs with less liquid assets. Consistent with this argument, our model shows that full bankruptcy exemption can be optimal when the quality of collateral is good (Appendix B).

7. Capital Requirements and Optimal Bankruptcy Exemption

Finally, we explore the role of capital requirements in the presence of bankruptcy exemption. Intuitively, one would expect that imposing capital requirements may further constrain leverage in the economy and thereby reduce the ex-post adverse effects of excess liquidation by over-leveraged firms. On the other hand, capital requirements would also cause an ex-ante contraction in the financial sector. There could thus be a tradeoff between these two effects and our model allows us to evaluate this tradeoff.[25]

We model capital requirements as the maximum shortfall (s) that could be financed by a financial firm.[26] We refer to this maximum amount as $\bar{s}$. Recall that intermediaries with shortfall greater than $\hat{s}$ are not financed in equilibrium. Therefore, the only relevant case is if $\bar{s} < \hat{s}$. Our analysis in the appendix establishes that imposing external capital constraints beyond that what is imposed by the equilibrium truncation ($\hat{s}$) is never optimal. This result is not surprising because $\hat{s}$ internalizes the fire-sale effects and so long as capital constraints are imposed to eliminate the problem of excessive leveraging by financial firms, this objective is fully attained through $\hat{s}$.

PROPOSITION 5: *A social planner aiming to maximize total surplus by imposing external capital constraints can never improve upon the total surplus achieved by setting the bankruptcy exemption*

[25] Aldasoro et al. [2023] also argue that bank regulation in the form of balance sheet constraints could be effective in mitigating ex-post fire sales.

[26] As the financial asset being considered in the model is same for all the firms, capital requirements that specify a percentage of equity to be set aside for acquiring this risky asset would translate into a restriction on the amount of borrowing (s) that can be undertaken to finance the asset.

parameter at the border of the Fair Pricing region and the Price Discrimination region.

Proposition (5) implies that optimizing on the bankruptcy exemption parameter in our model never compromises on the total surplus that can be achieved by imposing capital constraints. This is a useful result in that capital constraints are prone to leakages from "regulatory arbitrage" and the system can be gamed by individual financial firms which can indulge in masking the extent of their leverage. On the other hand, the bankruptcy exemption parameter is a macro-level constraint that is uniformly imposed across all intermediaries and is thus shielded more from manipulation.

8. Conclusion

We examine the role of bankruptcy exemption for short-term financing such as repo (sale-and-repurchase contracts) in determining the extent of leverage in the economy, and thereby its consequent impact on financial stability. While bankruptcy exemption is usually seen as facilitating financial sector growth in the hope of priming real sector growth, our model highlights that such a prescription must be viewed with caution. We show that bankruptcy exemption creates upfront leverage-inducing growth, which can cause financial instability via distributive externalities arising from fire sales - credit to the real sector is reduced, which in extremis, can lead to a credit crunch. We conclude that bankruptcy exemption may require a re-think for repo collateral whose quality is highly sensitive to economic shocks.

The Treasury repo rate spikes and fire sales observed during September 2019 ("repo rate spike") and March 2020 ("dash for cash") suggest that our conclusions, while derived in the context of risky underlying collateral, may carry over to relatively safe collateral such as Treasuries too. As Barth et al. [2021] note, some of this stress, especially in 2020, can be attributed to a liquidation of speculative positions in the cash-futures basis trades held by hedge funds and the growing build-up of such positions in the first place. To the extent that bankruptcy exemption in repo markets encourages leverage in these speculative positions, without (at least direct) attendant real benefits, there might be a possible case for revisiting safe harbor provisions in Treasury (and Agency) repo markets as well. Indeed, one favorable interpretation of the recent SEC proposal to require Treasury repo contracts to clear via a central counterparty (CCP)[27] is that this would reduce the ex-post fire-sale externality. By transferring and managing defaulted contracts via the CCP, clearing would effectively not allow repo financiers to simply seize and liquidate the underlying collateral. If this

[27]See, e.g., "In the Market: Treasury market braces for seismic SEC rule," by Paritosh Bansal (Reuters), October 30, 2023

limits ex-ante liquidity, then it may also ration ex-ante entry by leveraged hedge funds, which together with capital constraints on CCP dealers and counterparties could further reduce the risk of ex-post fire sales and be overall desirable ex ante.

Finally, our study offers a couple of interesting research avenues for future exploration. First, Davila [2020] provides a road map for quantifying the welfare gains of bankruptcy exemption in terms of a set of sufficient-statistics-style variables that can be empirically estimated. Using this approach, policy makers can make a direct empirical assessment of potential regulatory changes related to bankruptcy exemption. Another interesting avenue is to consider the role of the central bank as a lender of last resort in averting a financial crisis. Expectations about central bank interventions may influence ex-ante leveraging behavior; in particular, while the lender of last resort might be able to diminish the ex-post fire-sale induced spillovers to the real economy, its expectation might raise even greater ex-ante leverage in intermediaries aggravating the fire-sale problem. How such moral hazard would interact with safe harbor provisions in repo financing is a fruitful area for future inquiry.

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Appendix A: Key Results

A1. List of Symbols is provided in Table 2 with respective definitions.

Symbol	Definition	Expansion / Reference
ρ^*	Funding liquidity of the asset	$\theta y - k$
r	Probability of the high (good) state	$---$
λ	Value of $p-\rho^*$ in the LC equilibrium	$\frac{4\gamma k}{R^2}$
ω	Ratio of λ to k	$\frac{4\gamma}{R^2}$
ϕ	Surplus liquidity of the least leveraged firm	$\rho^* - s_{min}$
π	Intermediate term used for simplicity	$r(\theta^h y^h - p)$
m	Probability of states with non-zero payoff to creditors	$r + (1-r)q$
s_{max}	Maximum shortfall at which asset is NPV positive	$r\theta^h y^h + (1-r)\theta^l y^l$
$\hat{s}$	Maximum shortfall that is financed	$\pi + \rho^* + m(p-\rho^*)$
Δs_{max}	Diff. between max. & min. shortfalls for positive NPV projects ($s_{max} - s_{min}$)	$r\theta^h y^h + (1-r)\theta^l y^l - s_{min}$
$\Delta\hat{s}$	Diff. between max. & min. shortfalls for projects that are financed ($\hat{s} - s_{min}$)	$\pi + \phi + m(p-\rho^*)$
$\bar{\bar{q}}$	Value of q when the system transitions from FP to PD equilibrium	See Figure (4)
$\bar{q}$	Value of q when the system transitions from PD to LC equilibrium	See Figure (4)
$\hat{q}$	Value of q when the system transitions from LC to CC equilibrium	See Figure (4)

Table 2: **List of Symbols**.

A2. Proof of Lemma (1)

Given the result in (1), it follows that optimal effort is decreasing in f_r. The expected profits of the lender ef_r is equal to $\frac{1}{\gamma}(R - f_r)f_r$ is quadratic in f_r with a negative coefficient on $(f_r)^2$, implying a concave relationship. The first order condition yields $\frac{1}{\gamma}(R - f_r - f_r) = 0$, i.e., $f_r = \frac{R}{2}$, i.e, the expected profit function is maximized at $f_r = \frac{R}{2}$.

A3. Proof of Lemma (2)

Essentially, $\rho = \bar{\rho}$ is the level of leverage at which the intermediary is indifferent between liquidating δ fraction of the asset to reduce its liability to ρ^* or filing for a strategic default (i.e., $\delta(p,\bar{\rho})\theta y + (1 - \delta(p,\bar{\rho}))p = (1-q)(\theta y - \rho^*)$). When $\rho > \bar{\rho}$ the intermediary is better off defaulting on its liability, while for $\rho < \bar{\rho}$, it is optimal to liquidate a fraction of the asset to meet the demands of the creditors. Table (3) summarizes the payoffs for intermediaries and repo financiers based on the level of leverage ρ. Note that, when there is full exemption from automatic stay (i.e., $q = 1$), $\bar{\rho} = p$, implying that there is no strategic default. On the other hand, when there is no exemption (i.e., $q = 0$), $\bar{\rho} = \rho^*$, it is optimal for all credit-constrained intermediaries to do a strategic default.[28]

[28] The bankruptcy exemption parameter (q) can be thought of as an average value that captures the average "style" of heterogeneous judges who interpret the bankruptcy code in their individual style. From a cross-sectional

Intermediary Leverage (ρ)	Intermediary Strategy	Financier Payoff	Intermediary Payoff	Asset Fraction Liquidated
$\rho \leq \rho^*$	Use surplus liquidity to acquire new assets.	ρ	$\theta y - \rho$	0
$\rho^* < \rho \leq \bar{\rho}$	Liquidate δ asset to pay back creditor in full.	ρ	$(1-\delta)\theta y + \delta p - \rho$	δ
$\bar{\rho} < \rho \leq p$	Strategic default.	$qp + (1-q)\rho^*$	$(1-q)(\theta y - \rho^*)$	q
$p < \rho$	Involuntary default.	$qp + (1-q)\rho^*$	$(1-q)(\theta y - \rho^*)$	q

Table 3: **Intermediary and Financier Payoffs**.

When the leverage of a borrower is equal to $\bar{\rho}$, the borrower is indifferent between liquidating δ fraction of the asset to roll over the debt and exercising strategic default. Therefore, we have:

$$\delta p + (1-\delta)\theta y - \bar{\rho} = (1-q)(\theta y - \rho^*)$$

Noting that $\delta(\bar{\rho}, p) = \frac{\bar{\rho}-\rho^*}{p-\rho^*}$ and $\theta y - \rho^* = k$, we obtain:

$$\begin{aligned} & (\bar{\rho}-\rho^*)p + (p-\bar{\rho})(\rho^*+k) - (p-\rho^*)\bar{\rho} = \ (1-q)(k)(p-\rho^*) \\ \Rightarrow \ & (p-\bar{\rho})k = \ (1-q)(k)(p-\rho^*) \\ \Rightarrow \ & \bar{\rho} = \ \rho^* + q(p-\rho^*) \quad as \ \ k > 0 \qquad \text{(A1)} \end{aligned}$$

A4. Equilibrium Restrictions on face value (f_r), effort aversion parameter (γ) and price p

Some basic restrictions on the loan face value (f_r), effort aversion parameter (γ) and the financial asset price (p) must be satisfied in equilibrium:

(i) For non-trivial effort choice, we require $e^* > 0$, i.e., $\frac{1}{\gamma}(R - f_r) > 0$, i.e, $f_r < R$.

(ii) We require $f_r \leq f_r^m = \frac{R}{2}$, where f_r^m denotes the surplus-liquidity intermediary's profit-maximizing face value. Note that f_r^m can be solved as $\text{argmax}_{f_r} e f_r$ s.t. $e = \frac{1}{\gamma}(R - f_r)$; it follows that $f_r^m = \frac{R}{2}$. Since expected profits are concave in f_r, lenders have no incentive to post a higher face value than f_r^m.

(iii) $ef_r \geq 1$, otherwise there is no investment in real sector, i.e., $ef_r = \frac{1}{\gamma}(R - f_r)f_r \geq 1$.

perspective, q can also be thought of as capturing judge fixed effects.

(iv) $\frac{R}{2} - \sqrt{R^2 - 4\gamma} \leq f_r \leq \frac{R}{2}$. The additional restrictions on γ can be derived as follows. Under fair pricing of household loans (i.e., when $ef_r = 1$), the face value f_r is equal to $\frac{R}{2} - \frac{\sqrt{(R^2-4\gamma)}}{2}$, which is the lower root of the quadratic equation in f_r. To ensure that $e \leq 1$, we require $f_r \geq 1$, i.e., we require $(R-2)^2 \geq (R^2 - 4\gamma)$ which implies $\gamma \geq R - 1$. Furthermore, we also require $\gamma \leq \frac{R^2}{4}$; a greater value of γ would result in an imaginary solution for f_r. Combining these restrictions, we require $R - 1 \leq \gamma \leq \frac{R^2}{4}$.

(v) Combining all the above constraints, we get: $\frac{R}{2} - \sqrt{R^2 - 4\gamma} \leq f_r \leq \frac{R}{2}$.

(vi) The financial asset price (p) must lie in the interval $(\rho^*, \theta y)$.

The last restriction on the price of the financial asset (p) follows because (i) it cannot exceed the expected payoffs on the asset (θy) and (ii) it must be strictly higher than the funding liquidity (ρ^*), otherwise the demand for the asset would be infinite.

A5. Proof of Lemma (3)

Using the results in Lemma (1), namely, $e = \frac{1}{\gamma}(R - f_r)$, and noting that $\theta y - \rho^* = k$, we can re-formulate the optimization problem in (4) - (5) as a Lagrangian optimization problem with μ, η, and ν as Lagrangian parameters. μ is the Lagrangian parameter for the budget constraint, whereas η and ν are the Lagrangian parameters employed for the non-negativity constraints, $\alpha > 0$ and $\beta \geq 0$, respectively.

$$\max_{\alpha>0,\beta\geq 0} (1+\alpha)k + \beta e f_r - \mu\left[\alpha(p-\rho^*) + \beta - (\rho^* - \rho)\right] + \eta\alpha + \nu\beta \tag{A2}$$

The solution depends on the following first order condition for α and β, along with the complementary slack conditions on μ, η, and ν, respectively.

$$k - \mu(p - \rho^*) + \eta = 0 \tag{A3}$$

$$ef_r - \mu + \nu = 0 \tag{A4}$$

$$\mu\big[\alpha(p-\rho^*) + \beta - (\rho^* - \rho)\big] = 0 \tag{A5}$$

$$\eta\alpha = 0 \tag{A6}$$

$$\nu\beta = 0 \tag{A7}$$

Firstly, we note that $\mu > 0$ as the budget constraint in (5) is always binding due to non-satiation, i.e., surplus-liquidity intermediaries will always have incentive to deploy their spare liquidity fully in either of the two markets.

Since the secondary market for legacy financial assets must necessarily clear, we impose the condition that $\alpha > 0$, which implies that the Lagrangian parameter $\eta = 0$. It follows from Equation (A3) that

$$\mu = \frac{k}{p - \rho^*} > 0. \tag{A8}$$

The Date 1 loan market is a primary market and we must account for the possibility of the market being closed ($\beta = 0$) and the market being open ($\beta > 0$); these cases correspond to the Lagrangian parameter, ν, being strictly greater than 0 or equal to 0, respectively. From Equation (A4), we get $\nu = \mu - ef_r$. Thus, after incorporating the result in Equation (A8), we can conclude that:

$$\frac{k}{p - \rho^*} = ef_r, \qquad if \;\; \nu = 0, \tag{A9}$$

and

$$\frac{k}{p - \rho^*} > efr, \qquad if \;\; \nu > 0. \tag{A10}$$

A6. Proof of Lemma (4):

We start with the aggregate budget constraint, which equates aggregate supply and demand as shown in Equation (11), restated below:

$$q \int_{\bar{\rho}}^{\rho_{\max}} g(\rho)\, d\rho + \bar{\beta}\frac{1}{p - \rho^*} = \int_{\rho_{\min}}^{\bar{\rho}} \frac{\rho^* - \rho}{p - \rho^*}\, g(\rho) d\rho \tag{A11}$$

Integrating the RHS by parts while noting that $G(\rho_{min}) = 0$, we obtain:

$$q(p - \rho^*)\left[G(\rho_{max}) - G(\bar{\rho})\right] + \bar{\beta} = (\rho^* - \bar{\rho})G(\bar{\rho}) - \int_{\rho_{min}}^{\bar{\rho}} (-1)G(\rho)d\rho \tag{A12}$$

Substituting for $\bar{\rho}$ from Lemma (2) and rearranging, we obtain:

$$\bar{\beta} = -q(p - \rho^*)G(\rho_{max}) + \int_{\rho_{min}}^{\bar{\rho}} G(\rho)d\rho \tag{A13}$$

A7. Proof of Proposition (1)

For parsimony, we characterize the equilibrium in terms of the triplet $(p,\, \bar{\beta},\, f_r)$. In the Fair Pricing (FP) Equilibrium both financial and Date 1 loans are fairly priced, i.e., price of an asset is equal to the expected payoff from the asset ($p = E(y)$ and $ef_r = 1$) and expected return on investment for surplus-liquidity is 0. This outcome results when the supply of liquidity exceeds the demand for liquidity leading to the satiation of the Date 1 loan market even when the price of the financial asset (p) is at its highest possible value of θy. Consequently, in the FP equilibrium, we have:

$$p = E(y) = \theta y \tag{A14}$$

$$ef_r = 1 \Rightarrow \frac{1}{\gamma}(R - f_r)f_r = 1 \Rightarrow f_r = \frac{R}{2} - \frac{\sqrt{R^2 - 4\gamma}}{2} \tag{A15}$$

$$\bar{\beta} = \mathcal{B} \tag{A16}$$

A8. Proof of Proposition (2):

A8.1. Price Discrimination Equilibrium (PD)

Conditional on a given θ, the system transitions from the Fair Pricing region to the Fire Sale region as q increases and there is too much liquidation of assets at Date 1. In this situation, the market clearing price (p) falls below the fair value (θy). The Date 1 loan market continues to remain fully satiated ($\bar{\beta} = \mathcal{B}$), as in the Fair Pricing region. The price of the financial asset is obtained by substituting for $\bar{\beta} = \mathcal{B}$ in Equation (A13).

The cross-market equilibrium return condition implies that the face value of the Date 1 loan (f_r) increases to ensure that the returns on both assets are equal. Equation (8) reflects the cross-market equilibrium return condition, yielding:

$$\bar{\beta} > 0 \implies \frac{k}{p - \rho^*} = ef_r > \mu > 0. \tag{A17}$$

(A17) can be simplified into a quadratic equation in f_r, after recognizing that $e^* = \frac{1}{\gamma}(R - f_r)$ and $\rho^* = \theta y - k$. Note that, in equilibrium, the larger root greater than $\frac{R}{2}$ can be ignored due to constraints expressed in Section (A4), yielding:[29]

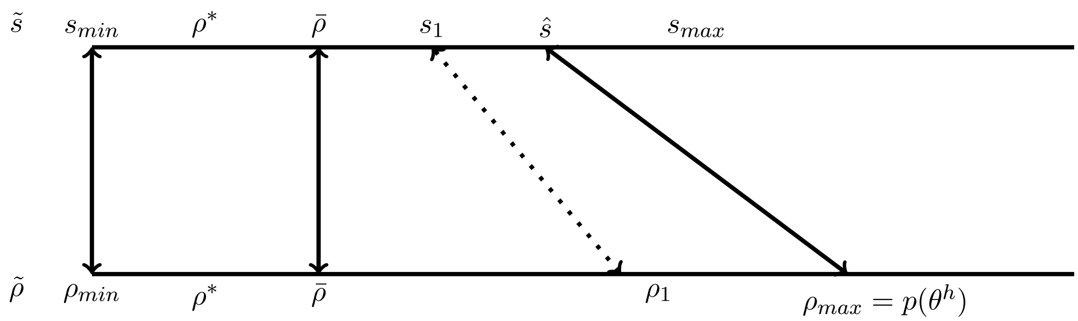
$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - \frac{4\gamma k}{p - \rho^*}} \tag{A18}$$

A8.2. Liquidity Crunch Equilibrium (LC)

As q increases in the Price Discrimination region, the face value (f_r) increases in equilibrium (a result that will be shown further down). The maximum value of f_r is equal to $\frac{R}{2}$, as discussed in

[29] $f_r^m = R/2$ is the face value of the loan at which the lender's profit is maximized when effort level of the households is endogenously determined. Consequently, it is never in the interest of lenders to charge a face value higher than f_r^m, implying $f_r < f_r^m = R/2$.

Figure **7**: **Derived Distribution of Debt.** The figure below shows a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ. The full double arrow lines indicate borders around which the ρ function changes and the dotted double arrow lines are specific values of ρ and s used to derive the distribution of ρ given $s \leq \hat{s}$.



Section (A4). If the demand for liquidity exceeds supply when f_r is at its highest possible value of $R/2$, supply-demand equilibrium is achieved through the rationing of the Date 1 loan market with the aggregate number of Date 1 loan loans extended ($\bar{\beta}$) falling below $\mathcal{B}$. In the LC equilibrium, $f_r = R/2$, $\bar{\beta}$ is given by Equation (A13). p can be obtained as follows from the cross-market equilibrium return condition in Equation (A17) while noting that when $f_r = R/2$, $ef_r = \frac{R^2}{4\gamma}$:

$$\begin{aligned} p - \rho^* =& \frac{k}{ef_r} = \frac{4\gamma k}{R^2} \\ \Rightarrow p =& \rho^* + \lambda \quad where \;\; \lambda = \frac{4\gamma k}{R^2} \end{aligned} \tag{A19}$$

A8.3. Credit Crunch Equilibrium (CC)

Note that $\bar{\beta}$ is decreasing in q in the Liquidity Crunch region (this result is derived in Section B6.2). Thus, as q increases, $\bar{\beta}$ will decrease and at a sufficiently high value of q, $\bar{\beta}$ will be equal to 0, and the system will transition to the Credit Crunch region. In this case, the equilibrium price, p, is given by the solution of Equation (12), in which $\bar{\beta}$ is set equal to 0. Furthermore, the cross-market equilibrium return condition is irrelevant. The equilibrium should satisfy (9) and (12) evaluated at $\bar{\beta} = 0$. The equilibrium triplet (p, $\bar{\beta}$, f_r) will now be reduced to singleton, (p), because $\bar{\beta}$ and f_r are irrelevant when the Date 1 loan market is closed.

A9. Proof of Lemma (5): Derived Distribution of Debt

Figure (**7**) presents a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ. To obtain the derived distribution of $\hat{G}(.) = G(\rho | s \leq s_{max})$, we first note that for

$\rho_{min} \leq \rho \leq \bar{\rho}$, $\tilde{\rho} = \tilde{s}$ is uniform over $[\rho_{min}, \bar{\rho}]$ because $\tilde{s}$ is uniformly distributed over $[s_{min}, \bar{\rho}]$ with $\rho_{min} = s_{min}$. Then, as shown in the adjoining figure, consider $\rho_1 \in (\bar{\rho}, \rho_{max}]$, where ρ_1 is the face value that finances an investment shortfall of s_1, and $\rho_{max} = p(\theta^h)$. We obtain:

$$\begin{aligned}\hat{G}(\rho_1) &= G(\tilde{\rho} \leq \rho_1 | \tilde{s}(\rho_1) \leq s_{max}) \\ &= \frac{P(\tilde{s}(\rho_1) \leq s_1 \cap \tilde{s}(\rho_1) \leq s_{max})}{P(\tilde{s}(\rho_1) \leq s_{max})} \\ &= \frac{P(\tilde{s}(\rho_1) \leq s_1)}{P(\tilde{s}(\rho_1) \leq s_{max})} \\ &= \frac{s_1 - s_{min}}{s_{max} - s_{min}}\end{aligned}$$

Therefore, we have $\hat{G}(\rho)$ specified as follows for $\rho_{min} \leq \rho \leq p$ (where $\bar{p} = \bar{\rho}$):

$$\hat{G}(\rho) = \frac{s(\rho) - s_{min}}{s_{max} - s_{min}} \text{ where } s(\rho) = \begin{cases} \rho, & \text{if } \rho_{min} \leq \rho \leq \bar{\rho} \\ r\rho + (1-r)\bar{p}, & \text{if } \bar{\rho} < \rho \leq \rho_{max} = p(\theta^h) \end{cases} \quad \text{(A20)}$$

A10. Model Parameter Space Restrictions

A well defined model parameter space should satisfy the following constraints.

$$\theta^l_{min} = (s_{min} + k)/y^l \quad \text{(A21)}$$

$$\theta^l_{max} < \theta^h \quad \text{(A22)}$$

Equation (A21) ensures financial market clearing for any $\theta^l \in [\theta^l_{min}, \theta^l_{max}]$ by ensuring that the surplus liquidity in the system is non-negative (i.e., $\rho^*(\theta^l_{min}) \geq s_{min}$) even for the most severe shock.

A11. Variation of expected Surplus at Date 1 (S_{D1}) with q

First, some notation to simplify the expression for $\Delta\hat{s}$:

$$\begin{aligned}\Delta\hat{s} &= r\theta^h y^h + (1-r)\rho^* + (1-r)q(p - \rho^*) - s_{min} \\ &= r(\theta^h y^h - \rho^* - (p - \rho^*)) + \big(r + (1-r)q\big)(p - \rho^*) + \rho^* - s_{min} \\ &= \pi + m(p - \rho^*) + \phi,\end{aligned}$$

where $\pi = r(r\theta^h y^h - p), \phi = \rho^* - s_{min}$, and $m = r + (1-r)q$.

In Equation (26), the first term is a constant while both $\bar{\beta}$ and $S_r(\theta^l)$ new loan potentially vary with q. By noting that $e^* = \frac{1}{\gamma}(R - f_r)$, we obtain $S_r(\theta^l) = \frac{1}{2\gamma}(R^2 - f_r^2)$. Therefore, we have:

$$\frac{dS_{D1}}{dq} = (1-r)\left[S_r(\theta^l)\frac{d\bar{\beta}}{dq} + \bar{\beta}\frac{dS_r(\theta^l)}{dq}\right] = (1-r)\left[S_r(\theta^l)\frac{d\bar{\beta}}{dq} - \bar{\beta}f_r\frac{df_r}{dq}\right] \quad \text{(A23)}$$

As both $\bar{\beta}$ and $S_r(\theta^l)$ are always positive, using results from Proposition (1) & Footnote (23), we obtain:

(i) FP equilibrium: $\frac{dS_{D1}}{dq}\Big|_{FP} = 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{FP} = 0$ and $\frac{df_r}{dq}\Big|_{FP} = 0$.

(ii) PD equilibrium: $\frac{dS_{D1}}{dq}\Big|_{PD} = -(1-r)\bar{\beta} f_r \frac{df_r}{dq}\Big|_{PD} < 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{PD} = 0$ and $\frac{df_r}{dq}\Big|_{PD} > 0$.

(iii) LC equilibrium: $\frac{dS_{D1}}{dq}\Big|_{LC} = (1-r)S_r(\theta^l)\frac{d\bar{\beta}}{dq}\Big|_{LC} < 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{LC} < 0$ and $\frac{df_r}{dq}\Big|_{LC} = 0$.

(iv) CC equilibrium: $\frac{dS_{D1}}{dq}\Big|_{CC} = 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{CC} = 0$ and $\frac{df_r}{dq}\Big|_{CC} = 0$.

A12. Variation of expected Surplus at Date 0 (S_{D0}) with q

We differentiate Equation (27) with respect to q, to obtain:

$$\frac{dS_{D0}}{dq} = \frac{[(1-r)\theta^l y^l + r\theta^h y^h] - \hat{s}}{s_{max} - s_{min}}\frac{d\hat{s}}{dq} = \frac{(1-r)[k - q(p-\rho^*)]}{s_{max} - s_{min}}\frac{d\hat{s}}{dq} \tag{A24}$$

As the first term on the RHS in above expression is positive, the sign of $\frac{dS_{D0}}{dq}$ depends only on the sign of $\frac{d\hat{s}}{dq}$. Therefore, using results from Proposition (1) & Footnote (23), we obtain:

(i) FP equilibrium: $\frac{dS_{D0}}{dq}\Big|_{FP} > 0$; as $\frac{d\bar{p}}{dq}\Big|_{FP} > 0$.

(ii) PD equilibrium: $\frac{dS_{D0}}{dq}\Big|_{PD} = 0$; as $\frac{d\bar{p}}{dq}\Big|_{PD} = 0$.

(iii) LC equilibrium: $\frac{dS_{D0}}{dq}\Big|_{LC} > 0$; as $\frac{d\bar{p}}{dq}\Big|_{LC} > 0$.

(iv) CC equilibrium: $\frac{dS_{D0}}{dq}\Big|_{CC} = 0$; as $\frac{d\bar{p}}{dq}\Big|_{CC} = 0$.

A13. Proof of Proposition (3): q^{opt} is on the boundary of FP and PD equilibrium

Noting that $S_{Total} = S_{D0} + S_{D1}$, using results from Sub Sections (A11) and (A12) we easily obtain that $\frac{dS_{Total}}{dq}\Big|_{FP} > 0$, $\frac{dS_{Total}}{dq}\Big|_{PD} < 0$ and $\frac{dS_{Total}}{dq}\Big|_{CC} = 0$. In the LC equilibrium, we use $p - \rho^* = \lambda$, $\frac{d\hat{s}}{dq} = (1-r)\lambda$, $\frac{d\bar{\beta}}{dq}$ given by Equation (B15), $f_r = R/2$ and $S_r(\theta^l) = \frac{3R^2}{8\gamma} = \frac{3k}{2\lambda}$, to obtain:

$$\begin{aligned}\frac{dS_{Total}}{dq} &= \frac{(1-r)[k - q(p-\rho^*)]}{\Delta s_{max}}\frac{d\hat{s}}{dq} + \frac{3(1-r)k}{2\lambda}\frac{d\bar{\beta}}{dq} \\ &= \frac{(1-r)(k - q\lambda)}{\Delta s_{max}}(1-r)\lambda - \frac{3(1-r)k}{2\lambda}\frac{\Big(\pi + (m - rq)\lambda\Big)\lambda}{\Delta s_{max}}\end{aligned}$$

Notating $\omega = \frac{4\gamma}{R^2} = \frac{\lambda}{k}$ and noting that $\pi = r(\theta^h y^h - p) = r(\theta^h y^h - \theta^l y^l) + r(k - \lambda)$, we have:

$$\begin{aligned}\frac{dS_{Total}}{dq} &= -\frac{(1-r)\lambda}{2\Delta s_{max}}\left[\frac{3k\pi}{\lambda} + 3(m - rq)k - 2(1-r)(k - q\lambda)\right] \\ &= -\frac{(1-r)k}{2\Delta s_{max}}\left[3\pi + 3r\lambda + 3q\lambda - 6rq\lambda - 2\lambda + 2q\omega\lambda + 2r\lambda - 2rq\omega\lambda\right] \\ &= -\frac{(1-r)k}{2\Delta s_{max}}\left[\Big(3\pi + 5r\lambda - 2\lambda\Big) + q\lambda\Big(3 + 2\omega - r(6 + 2\omega)\Big)\right] \end{aligned} \tag{A25}$$

We first consider the case where $0 < r \leq \frac{3+2\omega}{6+2\omega}$. The restriction on collateral quality in Proposition (3) can be restated to obtain $r(\theta^h y^h - \rho^*) \geq k \Rightarrow \pi \geq k - r\lambda$. Therefore, Equation (A25) can be restated to obtain:

$$\frac{dS_{Total}}{dq} \leq -\frac{(1-r)k^2}{2\Delta s_{max}}\left[\Big(3 - 2\omega(1-r)\Big) + q\omega\Big(3 + 2\omega - r(6 + 2\omega)\Big)\right] < 0 \quad \forall r \leq \frac{3+2\omega}{6+2\omega} \tag{A26}$$

Next, consider the case where $\frac{3+2\omega}{6+2\omega} \leq r < 1$: In this case, $\frac{dS_{Total}}{dq}$ is increasing in q and therefore, its maximum value is attained at $q = 1$. Therefore, evaluating Equation (A25) at $q = 1$, we obtain the following condition:

$$\frac{dS_{Total}}{dq} \leq -\frac{(1-r)k}{2\Delta s_{max}}\Big[3\pi + (1-r)(1+2\omega)\lambda\Big] < 0 \quad \forall r > \frac{3+2\omega}{6+2\omega} \tag{A27}$$

Combining the results from Equations (A26) and (A27), we have $\frac{dS_{Total}}{dq} < 0$ in the LC equilibrium. As $\frac{dS_{Total}}{dq}$ is strictly increasing in q in the FP equilibrium, strictly decreasing in q in the PD and LC equilibria and invariant with q in the CC equilibrium, it follows that S_{Total} is maximized at the boundary between FP and PD equilibrium (i.e., $q^{opt} = \bar{\bar{q}}$).

For a given set of parameters, we denote the value of q at which the system transitions from FP to PD equilibrium as $\bar{\bar{q}}$. $\bar{\bar{q}}$ can be obtained by solving for q in Equation (18) after setting $\lambda_{PD} = p - \rho^* = k$ on the FP-PD boundary. Therefore, we obtain:

$$\begin{aligned} &2\mathcal{B}\Delta s_{max} + 2qk\Big(r\theta^h y^h + (1-r)\rho^* + (1-r)qk - s_{min}\Big) - \phi^2 - q^2k^2 - 2q\phi k = 0 \\ \Rightarrow &(1-2r)k^2q^2 + 2rk(\theta^h y^h - \rho^*)q + \big[2\mathcal{B}\Delta s_{max} - \phi^2\big] = 0 \end{aligned} \tag{A28}$$

Solving the above quadratic for q^{opt}, we obtain:[30]

$$q^{opt} = \frac{-r(\theta^h y^h - \rho^*) + \sqrt{\left[r(\theta^h y^h - \rho^*)\right]^2 + (1-2r)\left[\phi^2 - 2\mathcal{B}\Delta s_{max}\right]}}{(1-2r)k} \tag{A29}$$

[30]The other root of the quadratic in Equation (A28) can be ignored as for that root we get $q^{opt} < 0$ when $r < 1/2$ and $q^{opt} > 1$ for $r > 1/2$. When $r = 1/2$, Equation (A28) is linear and $q^{opt} = \frac{\phi^2 - 2\mathcal{B}\Delta s_{max}}{(\theta^h y^h - \theta^l y^l)k}$.

A13.1. Alternative form of the Condition in Proposition

Finally, the condition stated in the Proposition can be stated in terms of the fundamental parameters of the model as follows:

$$\begin{aligned}
&E_\theta[\rho^*(\theta)] \geq \theta^l y^l \\
\Rightarrow & r\theta^h y^h + (1-r)(\theta^l y^l - k) \geq \theta^l y^l \\
\Rightarrow & r(\theta^h y^h - \theta^l y^l + k) + \theta^l y^l - k \geq \theta^l y^l \\
\Rightarrow & r(\theta^h y^h - \theta^l y^l) + k(r-1) \geq 0 \\
\Rightarrow & r(\theta^h y^h - \theta^l y^l) \geq k(1-r) \\
\Rightarrow & \frac{r}{1-r} \geq \frac{k}{\theta^h y^h - \theta^l y^l} \qquad \text{(A30)}
\end{aligned}$$

A14. Proof of Proposition (4)

A14.1. Impact of θ^l on q^{opt}

Denoting the FP-PD boundary in the $\theta^l - q$ space as $\bar{\bar{q}}(\theta^l)$, we write the boundary as $\lambda_{PD}(\theta^l, \bar{\bar{q}}(\theta^l)) = k$ and differentiate this expression with respect to θ^l to obtain:

$$\begin{aligned}
&\frac{\partial \lambda_{PD}}{\partial \theta^l} + \frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\theta^l)} \frac{d\bar{\bar{q}}(\theta^l)}{d\theta^l} = \frac{dk}{d\theta^l} \\
\Rightarrow \quad & \frac{d\bar{\bar{q}}(\theta^l)}{d\theta^l} = \frac{\frac{dk}{d\theta^l} - \frac{\partial \lambda_{PD}}{\partial \theta^l}}{\frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\theta^l)}} > 0 \qquad \text{(A31)}
\end{aligned}$$

For the fraction in the RHS of Equation (A31), both the numerator (as $\frac{dk}{d\theta^l} < 0$ and $\frac{\partial \lambda_{PD}}{\partial \theta^l} > 0$ from Equation (B27)) and the denominator are negative (as $\frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}} = \frac{\partial p}{\partial q}\Big|_{PD} < 0$ from Footnote 23). Thus, the FP-PD boundary is positively sloped in the $\theta^l - q$ space implying that q^{opt} decreases with the severity of the economic shock.

A14.2. Impact of k on q^{opt}

Denoting the PD-FP boundary in the $k - q$ space as $\bar{\bar{q}}(k)$, we write the boundary as $\lambda_{PD}(k, \bar{\bar{q}}(k)) = k$ and differentiate this expression with respect to k to obtain:

$$\frac{\partial \lambda_{PD}}{\partial k} + \frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(k)} \frac{d\bar{\bar{q}}(k)}{dk} = 1$$

$$\Rightarrow \frac{d\bar{\bar{q}}(k)}{dk} = \frac{1 - \frac{\partial \lambda_{PD}}{\partial k}}{\frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(k)}} < 0 \tag{A32}$$

For the fraction in the RHS of Equation (A32), the numerator is positive (see Equation (B28)) and the denominator is negative (see Footnote (23)). Thus, the PD-FP boundary is negatively sloped in the $k - q$ space implying that q^{opt} is decreasing in collateral quality.

A14.3. Impact of $\mathcal{B}$ on q^{opt}

Denoting the FP-PD boundary in the $\mathcal{B} - q$ space as $\bar{\bar{q}}(\mathcal{B})$, we write the boundary as $\lambda_{PD}(\mathcal{B}, \bar{\bar{q}}(\mathcal{B})) = k$ and differentiate this expression with respect to $\mathcal{B}$ to obtain:

$$\frac{\partial \lambda_{PD}}{\partial \mathcal{B}} + \frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\mathcal{B})} \frac{d\bar{\bar{q}}(\mathcal{B})}{d\mathcal{B}} = 0$$

$$\Rightarrow \frac{d\bar{\bar{q}}(\mathcal{B})}{d\mathcal{B}} = - \frac{\frac{\partial \lambda_{PD}}{\partial \mathcal{B}}}{\frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\mathcal{B})}} < 0 \tag{A33}$$

For the fraction in the RHS of Equation (A33), both the numerator and the denominator are negative (see Equation B29 and Footnote 23)). Thus, the FP-PD boundary is negatively sloped in the $\mathcal{B} - q$ space implying that the optimal q^{opt} is decreasing in the size of the real sector.

A15. Proof of Proposition (5)

To establish Proposition (5), we first evaluate q^{opt} in the presence of binding capital requirements (i.e., $\bar{s} < \hat{s}$) in Section (A15.1), then we evaluate the dynamics of q^{opt} in the $\bar{s} - q$ space in Section (A15.2), and finally identify the optimal operating point in the $\bar{s} - q$ space that maximizes S_{Total} in Section (A15.2).

A15.1. q^{opt} in the presence of Binding Capital Requirements

Capital requirements are binding when $\bar{p} < \bar{s} \leq \hat{s}$.[31] When capital requirements are not binding (i.e., $\bar{s} > \hat{s}$), we note from Footnote (23) that $\bar{p}$ is weakly increasing in q. Consequently, $\hat{s} = r\theta^h y^h + (1-r)\bar{p}$ is weakly increasing in q and for any given set of system parameters, $\hat{s}(q = 0) \leq \hat{s}(q = 1)$. Therefore, as capital requirements are imposed it will always become binding at higher values of q before it becomes binding at lower values of q. Thus, two possible cases of binding capital requirements can

[31] As the objective of capital requirements is to deter liquidation of assets resulting from default, the leverage level ($\rho = \bar{p}$ when $s(\rho) = \bar{p}$) beyond which default becomes viable for firms forms the lower bound for the tightest capital requirement (i.e., $\bar{s} > \bar{p}$).

arise – i) Capital requirements are binding at all q (i.e., $\bar{s} < \hat{s}(q=0) \leq \hat{s}(q=1)$), and ii) Capital requirements are non-binding for $q < q_b$ and binding for $q \geq q_b$ (i.e., $\hat{s}(q=0) < \bar{s} = \hat{s}(q=q_b) \leq \hat{s}(q=1)$).[32]

To evaluate the impact of q on S_{Total} when $\bar{s}$ is binding, we first establish dynamics of the equilibrium regions when $\bar{s}$ is binding. In the FP region, $p = \theta^l y^l$, $\bar{\beta} = \mathcal{B}$ and $f_r = \frac{R}{2}\left[1-\sqrt{1-\omega}\right]$.[33]
In the PD region, $p = \rho^* + \lambda_{PD}$, $\bar{\beta} = \mathcal{B}$, and $f_r = \frac{R}{2}\left[1-\sqrt{1-\frac{\lambda}{\lambda_{PD}}}\right]$ where λ_{PD} is obtained by solving Equation (A34).[34]

$$2\mathcal{B}\Delta s_{max} = -2q\lambda_{PD}\Delta\bar{s} + (\phi + q\lambda_{PD})^2 \tag{A34}$$

Differentiating Equation (A34) with respect to q, we obtain:[35]

$$\begin{aligned}
&-2\Delta\bar{s}\left[\lambda_{PD} + q\frac{d\lambda_{PD}}{dq}\right] + 2(\phi + q\lambda_{PD})\left[\lambda_{PD} + q\frac{d\lambda_{PD}}{dq}\right] = 0 \\
&\Rightarrow [\Delta\bar{s} - \phi - q\lambda_{PD}]\left[\lambda_{PD} + q\frac{d\lambda_{PD}}{dq}\right] = 0 \\
&\Rightarrow \frac{d\lambda_{PD}}{dq} = -\frac{\lambda_{PD}}{q} < 0 \quad as \quad \bar{s} > \rho^* + q\lambda_{PD}
\end{aligned} \tag{A35}$$

By extension, $\left.\frac{dp}{dq}\right|_{PD} = \frac{d\lambda_{PD}}{dq} < 0$ and $\left.\frac{df_r}{dq}\right|_{PD} = -\frac{\lambda R}{4\lambda_{PD}^2}\left[1-\frac{\lambda}{\lambda_{PD}}\right]^{-\frac{1}{2}}\frac{d\lambda_{PD}}{dq} > 0$.[36]
In the LC equilibrium, $p = \rho^* + \lambda$, $\bar{\beta}$ is obtained from Equation (A36) and $f_r = \frac{R}{2}$.

$$2\bar{\beta}\Delta s_{max} = -2q\lambda\Delta\bar{s} + (\phi + q\lambda)^2 \tag{A36}$$

Differentiating Equation (A36) with respect to q, we obtain:

$$\frac{d\bar{\beta}}{dq} = -\frac{\lambda\left[\Delta\bar{s} - \phi - q\lambda\right)]}{\Delta s_{max}} < 0 \tag{A37}$$

Next, we consider the two components of S_{Total}. Evaluating, S_{D0} when $\bar{s}$ is binding, while using the notations of $\Delta\bar{s} = \bar{s} - s_{min}$ and $\Delta s_{max} = s_{max} - s_{min}$, we obtain:

$$S_{D0} = \int_{s_{min}}^{\bar{s}} E_\theta[\theta y - s]dH(s)$$

[32] Even when capital controls are binding, $\bar{p}$ is weakly increasing in q as we shall see in Equation (A35) and Footnote (36). Consequently, if capital controls are binding at a given value of q, they never become non-binding as q increases.
[33] $\omega = \frac{4\gamma}{R^2}$.
[34] Results in Equations (A34 and A36) and Footnote (36) are obtained by solving the fundamental demand-supply relationship for the system in Equation (12) after limiting the maximum leverage in the economy to $\bar{s}$ to obtain $G(\rho_{max}) = G(\rho(\bar{s})) = \frac{\Delta\bar{s}}{\Delta s_{max}}$. Equation (12) gets modified to $q(p-\rho^*)\Delta\bar{s} = \Delta s_{max}\left[\int_{\rho_{min}}^{\bar{\rho}} G(\rho)d\rho - \bar{\beta}\right]$.
[35] The final result of Equation (A35) follows from noting that $\Delta\bar{s} - \phi - q\lambda_{PD} = \bar{s} - \bar{p} > 0$ based on the rational lower bound on $\bar{s}$.
[36] Using a similar approach, we can show that when capital requirements are binding, $\frac{d\lambda_{CC}}{dq} = -\frac{\lambda_{CC}}{q} < 0$.

$$=\Delta\bar{s}-\frac{1}{2}\frac{(\Delta\bar{s})^2}{\Delta s_{max}} \quad \text{(A38)}$$

As S_{D0} is not a function of q when capital requirements are binding (irrespective of the equilibrium region), S_{D0} remains a constant as q varies from 0 to 1.

Now, evaluating S_{D1}, we have from Equation (26) that $S_{D1} = r\mathcal{B}R + (1-r)\bar{\beta}S_r$. As in the case when $\bar{s}$ was not binding, in the FP equilibrium, both $\bar{\beta} = \mathcal{B}$ and $S_r = \frac{2+\omega+2\sqrt{1-\omega}}{2\omega}$ are invariant in q. Therefore, S_{D1} is invariant in q in the FP equilibrium. In the PD equilibrium, $\bar{\beta} = \mathcal{B}$ is invariant in q, however, $S_r = \frac{R^2-f_r^2}{2\gamma}$ is a function of q and we have $\frac{dS_r}{dq} = -\frac{f_r}{\gamma}\frac{df_r}{dq} < 0$. Thus, S_{D1} is decreasing in q in the PD equilibrium. In the LC equilibrium, $S_r = \frac{3R^2}{8\gamma}$ is invariant in q, while $\bar{\beta}$ is decreasing in q. Therefore, S_{D1} is decreasing in q in the LC equilibrium. Finally, in the CC equilibrium, as $\bar{\beta} = 0$, S_{D1} is invariant in q.

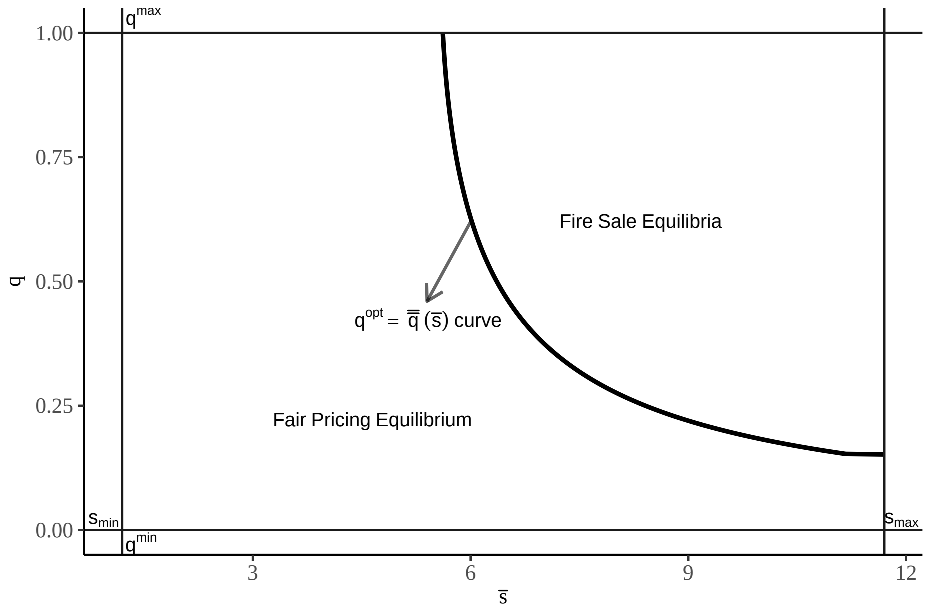
Combining the above results, we observe that S_{Total} is invariant in q in the FP equilibrium and strictly decreasing in the PD and LC equilibria before it again becomes invariant in q in the CC equilibrium. Thus, when $\bar{s}$ is binding across the entire range of q, S_{Total} is maximized in the FP region and $q^{opt} = [0, \bar{\bar{q}}]$ where $\bar{\bar{q}}$ is the value of q at which the system transitions from FP equilibrium to PD equilibrium.

On the other hand, when $\bar{s}$ becomes binding at some internal value of $q = q_b$, two cases can occur – a) $q_b \geq \bar{\bar{q}}$, or b) $q_b < \bar{\bar{q}}$. We already know from Proposition (3) that when $\bar{s}$ is not binding, $\frac{dS_{Total}}{dq} > 0$ in the FP region and $\frac{dS_{Total}}{dq} \leq 0$ in the other three equilibrium regions. Therefore, when $q_b \geq \bar{\bar{q}}$, capital requirements are non-binding in the FP equilibrium and S_{Total} is maximized at $\bar{\bar{q}}$ and strictly decreasing thereafter (till it reaches CC equilibrium at $\hat{q}$, after which S_{Total} is again invariant in q). Consequently, $q^{opt} = \bar{\bar{q}}$ in this case. When $q_b < \bar{\bar{q}}$, capital requirements are partially binding in the FP equilibrium. Therefore, S_{Total} increases with q for $q \in [0, q_b]$ and invariant in q for $q \in (q_b, \bar{\bar{q}}]$, strictly decreasing in q for $q \in (\bar{\bar{q}}, \hat{q}]$ and invariant in q for $q \in (\hat{q}, 1]$. Consequently, $q^{opt} = (q_b, \bar{\bar{q}}]$ in this case. Combining the above results for the two cases, we obtain a general expression for optimal value of q which maximizes S_{Total} as follows: $q^{opt} = [min(q_b, \bar{\bar{q}}), \bar{\bar{q}}]$.[37]

Further, as the cases where $\bar{s}$ is not binding or where $\bar{s}$ is always binding can be seen as subsets of the case where $\bar{s}$ is partially binding, we have in general $q^{opt}(\bar{s}) = \Big[min\Big(q_b(\bar{s}), \bar{\bar{q}}(\bar{s})\Big), \bar{\bar{q}}(\bar{s})\Big]$.

[37] Strictly speaking, $q^{opt} = \big[min\big(max(0, q_b), max(0, \bar{\bar{q}}), 1\big), min\big(max(0, \bar{\bar{q}}), 1\big)\big]$ as the mathematical solutions for q_b and $\bar{\bar{q}}$ are not necessarily bound between 0 and 1. However, the simple expression for q^{opt} is sufficient if we replace any negative values by 0 and values exceeding 1 by 1.

Figure **8**: q^{opt} **variation with** $\bar{s}$ **in the presence of capital controls.** Optimal bankruptcy exemption parameter ($q^{opt} = \bar{\bar{q}}(\bar{s})$) curve displayed as $\bar{s}$ varies. Parameter configuration is the same as that used in Figure 4 (i.e., $k = 1.5$, $\theta^l = 0.35$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.)



A15.2. Variation of $\bar{\bar{q}}(\bar{s})$ *with* $\bar{s}$ *in the* $q - \bar{s}$ *Space*

$\bar{\bar{q}}(\bar{s})$ is given by the locus of points at which $\lambda_{PD}\left(\bar{\bar{q}}(\bar{s}), \bar{s}\right) = k$. Differentiating it with respect to $\bar{s}$, we get:[38],[39]

$$\frac{\partial \lambda_{PD}}{\partial \bar{s}} + \frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\bar{s})} \frac{d\bar{\bar{q}}(\bar{s})}{d\bar{s}} = 0$$

$$\frac{d\bar{\bar{q}}(\bar{s})}{d\bar{s}} = -\frac{\partial \lambda_{PD}/\partial \bar{s}}{\partial \lambda_{PD}/\partial q} < 0 \qquad \text{(A39)}$$

Note that $\bar{\bar{q}}$ is a function of $\bar{s}$ and decreasing in $\bar{s}$ when capital controls are binding. Figure (**8**) displays the variation in $\bar{\bar{q}}(\bar{s})$ with $\bar{s}$ for the same parameter configuration used in Figure **4**. Essentially, as capital controls are tightened (i.e., $\bar{s}$ is reduced), leverage in the economy at Date 0 reduces and consequently, a higher level of bankruptcy exemption (q) is required for the system to go into the fire-sale equilibria at Date 1.

[38] The final result follows from noting that $\frac{d\lambda_{PD}}{dq} < 0$ and $\frac{d\lambda_{PD}}{d\bar{s}} < 0$.

[39] Similarly, we can also show that $\frac{d\bar{q}(\bar{s})}{dq} = -\frac{\partial \bar{\beta}/\partial \bar{s}}{\partial \bar{\beta}/\partial q} < 0$ and $\frac{d\hat{q}(\bar{s})}{dq} = -\frac{\partial \lambda_{CC}/\partial \bar{s}}{\partial \lambda_{CC}/\partial q} < 0$.

A15.3. Optimal $q - \bar{s}$ Combination

To find the optimal combination of q and $\bar{s}$ that maximizes surplus, we take a two-step approach. We first establish optimal level of q for a given $\bar{s}$ and then compare S_{Total} at $q^{opt}(\bar{s})$ across $\bar{s}$. First, consider a level of $\bar{s}$ such that capital requirements are not binding for the system at any q. Then, based on Proposition (3), $q^{opt} = \bar{\bar{q}} \in [0,1]$ and the system is in the FP region at q^{opt}. Let $\bar{s}_0$ be the level of $\bar{s}$ such that the controls are just binding at $q = \bar{\bar{q}}$.

For any $\bar{s} \geq \bar{s}_0$, system dynamics at $q = \bar{\bar{q}}$ are not affected by the choice of $\bar{s}$ and $q^{opt} = \bar{\bar{q}}(\bar{s}_0)$.[40] Consequently, maximum value of S_{Total} for any $\bar{s} \geq \bar{s}_0$ is given by $S_{Total}(\bar{s}_0, \bar{\bar{q}}(\bar{s}_0))$. When $\bar{s}$ is reduced from this level and capital controls are tightened, based on Equation (A39), $\bar{\bar{q}}(\bar{s})$ increases. However, from the results of Section (A15.1), we know that $\bar{\bar{q}}(\bar{s}) \in q^{opt}$ and $S_{Total}(\bar{s}, \bar{\bar{q}}(\bar{s}))$ is the maximum value of S_{Total} for a given $\bar{s} < \bar{s}_0$. Comparing $S_{Total}(\bar{s}, \bar{\bar{q}}(\bar{s}))$ with $S_{Total}(\bar{s}_0, \bar{\bar{q}}(\bar{s}_0))$ when $\bar{s} < \bar{s}_0$, we have $S_{D1}(\bar{s}, \bar{\bar{q}}(\bar{s})) = S_{D1}(\bar{s}_0, \bar{\bar{q}}(\bar{s}_0)) = r\mathcal{B}R + (1-r)\mathcal{B}$ as at both points the system is in the FP equilibrium. At the same time, S_{D0} is an increasing function of $\bar{s}$ and invariant in q in the FP equilibrium. Therefore, $S_{D0}(\bar{s}, \bar{\bar{q}}(\bar{s})) < S_{D0}(\bar{s}_0, \bar{\bar{q}}(\bar{s}_0))$ as $\bar{s} < \bar{s}_0$ and by extension, $S_{Total}(\bar{s}, \bar{\bar{q}}(\bar{s})) < S_{Total}(\bar{s}_0, \bar{\bar{q}}(\bar{s}_0))$. Thus, S_{Total} is maximized at $(\bar{s}^{opt}, q^{opt})$ such that $\bar{s}^{opt} = \bar{s}_0$ and $q^{opt} = \bar{\bar{q}}(\bar{s}_0)$.[41,42]

[40] Do note that in this case, if $\bar{q} < 1$ (i.e., system transitions from the PD equilibrium into the LC equilibrium at some higher q), capital controls become binding at some value of $q > \bar{\bar{q}}$ at some $\bar{s} > \bar{s}_0$. However, as established in Section (A15.1), q^{opt} continues to remain at $\bar{\bar{q}}$ even when $\bar{s}$ is partially binding.

[41] Strictly speaking any $\bar{s} > \bar{s}_0$ is also equally optimal and increasing $\bar{s}$ beyond $\bar{s}_0$ has no impact on q^{opt}.

[42] There is also a corner case when $\mathcal{B} > \bar{\beta}(q = 0)$ and the system is in either LC or CC equilibria for any q. In such case $q^{opt} = 0$ where the system is in the LC equilibrium at q^{opt}. Further, $\bar{\beta}(q^{opt}) = \frac{\phi^2}{2\Delta s_{max}} < \mathcal{B}$ and the system continues to be in the LC equilibrium for any level of capital controls. Introduction of capital controls only affects the level of liquidation of assets in the economy and has no impact on the surplus liquidity in the system which is given by $\frac{\phi^2}{2\Delta s_{max}}$. At $q = 0$, as there is no liquidation, all surplus liquidity is diverted towards the Date 1 loan market and $\bar{\beta}(q = 0) = \frac{\phi^2}{2\Delta s_{max}}$ and it is invariant in $\bar{s}$. Thus, $q^{opt}(\bar{s}) = 0$ for any level of $\bar{s}$ and we have $S_{D1}(\bar{s}, q^{opt}(\bar{s})) = (1+r)\mathcal{B}\frac{R}{2}$ is invariant in $\bar{s}$. S_{D0} is an increasing function of $\bar{s}$ when capital controls are binding and invariant in $\bar{s}$ when capital controls are not binding. Therefore, S_{D0}, and by extension S_{Total}, are maximized at the highest possible value of $\bar{s}$ which is binding at q^{opt}. Let $\bar{s}_1$ be the level of $\bar{s}$ at which capital controls become just binding at $q = 0$. Then we have that S_{Total} is maximized at $(\bar{s}^{opt}, q^{opt})$ such that $\bar{s}^{opt} = \bar{s}_1$ and $q^{opt} = 0$. Strictly speaking any $\bar{s} > \bar{s}_1$ is also equally optimal and increasing $\bar{s}$ beyond $\bar{s}_1$ has no impact on q^{opt}.

Appendix B: Internet Appendix

B1. Risk-Shifting Motivation for Funding Illiquidity (k)

The section presents a motivation of how funding illiquidity can arise in a market with financial frictions.

More specifically, we consider the friction arising from a moral hazard problem at the financial intermediary which can switch to an alternative asset (henceforth, the risk-shifting asset) that has higher risk at Date 1, post rollover of its debt. Consider the low state of the world (θ^l). The risk-shifting asset has a payoff of y_1 with a probability of θ_1 and a payoff of 0 with a probability of $(1-\theta_1)$, such that $y_1 > y^l$, $\theta_1 < \theta^l$, and $\theta_1 y_1 \leq \theta^l y^l$. Thus, while the risk-shifting alternative has a higher payoff in the non-default state, it experiences a higher likelihood of the default state. More importantly, it is riskier in that it has a lower expected payoff as compared to the safer alternative (i.e., $\theta_1 y_1 \leq \theta^l y^l$) and has a higher variance per unit expected payoff compared to second asset (i.e. $(1-\theta_1)y_1 > (1-\theta^l)y^l$). Following Acharya and Viswanathan [2011], we also assume that risk-shifting is cost-less to implement, and that assets are financial sector specific (such as money-market funds) and cannot be redeployed by financiers in case they choose not to roll over financing at Date 1, i.e., they must be liquidated to other intermediaries.

The funding liquidity of an asset at Date 1 is the amount of rollover debt that can be raised by pledging the asset. Since the risk-shifting payoff leads to a negative value investment, financiers would want to set the face value (f) in such a way that the borrower has no incentives to risk shift. This requires $\theta^l(y^l - f) > \theta_1(y_1 - f)$, which implies that $f < f^* \equiv \frac{\theta^l y^l - \theta_1 y_1}{\theta^l - \theta_1}$. The funding liquidity (ρ^*) of the financial asset is given by the loan amount that financiers would be able to finance, is equal to $\theta^l f^*$, which can also be represented as $\theta^l y^l - k$. With rearrangement, we obtain $k = \theta^l y^l - \rho^* = \frac{\theta^l \theta_1 (y_1 - y^l)}{(\theta^l - \theta_1)}$. Differentiating k with respect to θ^l and with respect to y^l, we get:

$$\frac{dk}{d\theta^l} = -\frac{\theta_1^2 (y_1 - y^l)}{(\theta^l - \theta_1)^2} < 0 \quad \text{(B1)}$$

$$\frac{dk}{dy^l} = -\frac{\theta^l \theta_1}{\theta^l - \theta_1} < 0 \quad \text{(B2)}$$

In the high state of the world, we assume that there is no risk-shifting alternative, therefore $k(\theta^h) = 0$. In essence, we assume that the risk-shifting alternative's characteristics converge to those of the safe asset in the high state of the world.

B2. Proof of Existence and Uniqueness of Equilibrium Solution

We can rewrite Equation (A11) that describes the dynamics of the supply and demand for financial assets as follows:[43]

$$\int_{\rho_{min}}^{\rho^*} \frac{(\rho^* - \rho)}{(p - \rho^*)} g(\rho) d\rho - \frac{\bar{\beta}(p)}{(p - \rho^*)} = \int_{\rho^*}^{\bar{p}} \frac{(\rho - \rho^*)}{(p - \rho^*)} g(\rho) d\rho + \int_{\bar{p}}^{\rho_{max}} q g(\rho) d\rho \tag{B3}$$

$$where \quad \bar{\beta}(p) = \begin{cases} 0, & \text{if } p < \rho^* + \lambda \\ \bar{\beta}(p) \mid \bar{\beta}(p) \in [0, \mathcal{B}], & \text{if } p = \rho^* + \lambda \\ \mathcal{B}, & \text{if } p > \rho^* + \lambda \end{cases} \tag{B4}$$

The left hand side of Equation (B3) reflects the aggregate demand for financial assets from surplus-liquidity intermediaries, net of their origination of mortgage loans in the Date 1 loan market ($\bar{\beta}$). We denote this aggregate demand as $D(p)$. On the other side, the aggregate supply of financial assets by credit-constrained intermediaries in the financial asset market, denoted $S(p)$, is given by the right hand side of Equation (B3). The excess demand, $ED(p) = D(p) - S(p)$, when set equal to 0, yields the financial asset market price (p).

For $p = \rho^*$, $S(p)$ is finite, while $D(p)$ is infinite, and therefore, $ED(p)$ is positive.[44] At the other end, for $p > \theta^l y^l$, $D(p)$ is 0 while $S(p)$ is positive, and therefore, $ED(p)$ is negative.[45] Consequently, there always exists at least one solution to $ED(p) = 0$ that corresponds to a price in the range ρ^* to $\theta^l y^l$. Below, we present a concise expression for excess demand ($ED(p)$), which can also be inferred from Equation (12):

$$ED(p) = \frac{\int_{\rho_{min}}^{\bar{p}} G(\rho) d\rho - q(p - \rho^*) G(\rho_{max}) - \bar{\beta}}{(p - \rho^*)} \tag{B5}$$

If $\frac{d}{dp}[ED(p)] < 0 \ \forall p \in (\rho^*, \theta^l y^l)$, it would imply that the solution to $ED(p) = 0$ in the range $(\rho^*, \theta^l y^l)$ is unique. However, as the denominator of $ED(p)$ in Equation (B5) is always positive for $p \in (\rho^*, \theta^l y^l)$, it suffices to show that the numerator of $ED(p)$ in Equation (B5) is monotonically decreasing in $p \ \forall p \in (\rho^*, \theta^l y^l)$ to establish that the excess demand curve intersects the x-axis only once over the interval $(\rho^*, \theta y^l)$. We establish this result using $(\hat{G}(\rho))$, the endogenous distribution of

[43] The restrictions on $\bar{\beta}$ in Equation (B4) arise from the cross-market arbitrage conditions in Lemma (A17). A lower price than $\rho^* + \lambda$ would cause the return from investing in the financial asset market to exceed that from investing in the Date 1 loan market, resulting in a market shut down in the real sector ($\bar{\beta} = 0$). On the other hand if the price is greater than $\rho^* + \lambda$, the return in the Date 1 loan market can match any feasible return in the financial asset market and the return in the financial asset market is decreasing in the amount of liquidity supplied to it. Therefore, surplus-liquidity intermediaries exhaust all lending opportunities in the Date 1 loan market before supplying to the financial asset market ($\bar{\beta} = \mathcal{B}$).

[44] At $p = \rho^*$, the cost of acquiring a financial asset is 0. Therefore, even a small number of surplus liquidity firms have the potential to acquire an infinity of financial assets.

[45] When $p > \theta^l y^l$, the return on acquiring a financial asset is negative, and therefore, the demand for financial assets is 0.

leverage that takes into account ex-post dynamics in the economy (see Lemma (5)).[46] Differentiating the Equation (B5) with respect to p, we get

$$\begin{aligned} \frac{d}{dp}\left[NUM(ED(p))\right] &= q\hat{G}(\bar{p}) - q\left[\hat{G}(\rho_{max}) + \frac{(1-r)q(p-\rho^*)}{\Delta s_{max}}\right] - \frac{d\bar{\beta}}{dp} \\ &= -\frac{rq\left[\theta^h y^h - \bar{p}\right]}{\Delta s_{max}} - \frac{(1-r)q^2(p-\rho^*)}{\Delta s_{max}} - \frac{d\bar{\beta}}{dp} < 0 \end{aligned} \tag{B6}$$

Note that the first two terms in Equation (B6) are negative, but the sign of the third term depends on the sign of $\frac{d\bar{\beta}}{dp}$. It can be seen from Equation (B4), $\bar{\beta}$ is a step function of p. Therefore, $\frac{d\bar{\beta}}{dp}$ is 0 for all p not equal to $\rho^* + \lambda$ and is equal to the Dirac Delta function (which is positive) at $p = \rho^* + \lambda$. In short, $\frac{d\bar{\beta}}{dp} \geq 0$.

It follows that $\frac{d}{dp}\left[NUM(ED(p))\right] < 0 \ \forall p \in (\rho^*, \theta^l y^l)$. Hence the excess demand curve intersects the x-axis only once. This result establishes the existence and uniqueness proof.

B3. Shortfall (s) financed for a given face value (ρ)

Table (4) maps the investment shortfall ($s(\rho)$) that can be financed for a given ρ. Since the payoff potential depends on ρ, the investment shortfall that can be financed changes in specific form over different intervals of ρ, as can be seen in the different rows of Table (4), but is a piece-wise linear function of ρ.

ρ	Default States	Non-default States	Investment Shortfall That is Financed by Debt ($s(\rho)$)
$\rho_{min} \leq \rho \leq \bar{p}$	$\varnothing$	Ω_1, Ω_2, Ω_3	ρ
$\bar{p} < \rho \leq p(\theta^h)$	Ω_2, Ω_3	Ω_1	$r\rho + (1-r)\bar{p}(\theta^l)$
$p(\theta^h) < \rho$	Ω_1, Ω_2, Ω_3	$\varnothing$	$rp(\theta^h) + (1-r)\bar{p}(\theta^l)$

Table 4: **Mapping of the Face Value of Liability (ρ).** This table presents mapping between the face value of repo contract ρ and the corresponding investment shortfall, $s(\rho)$, that can be financed at that level of ρ. $s(\rho)$ is equal to the expected ex-ante payoff (at Date 0) that the financiers would receive for face value ρ.

[46] The same result can be obtained when $G(\rho)$ is exogenously specified. In this case, $\frac{d}{dp}[NUM(ED(p))] = -q(G(\rho_{max} - G(\bar{p}))) - \frac{d\bar{\beta}}{p} < 0$.

B4. Price in the Price Discrimination and Credit Crunch Equilibria

In the Price Discrimination Equilibrium, we solve Equation (B11) and substitute for λ_{PD} in $p|_{PD} = \rho^* + \lambda_{PD}$ to obtain:[47]

$$p|_{PD} = \rho^* + \frac{-r(\theta^h y^h - \rho^*) + \sqrt{r^2(\theta^h y^h - \rho^*)^2 + (1-2r)(\phi^2 - 2\mathcal{B}\Delta s_{max})}}{(1-2r)q} \tag{B7}$$

Similarly, in the Credit Crunch Equilibrium, we solve Equation (B16) and substitute for λ_{CC} in $p|_{CC} = \rho^* + \lambda_{CC}$ to obtain:

$$p|_{CC} = \rho^* + \frac{-r(\theta^h y^h - \rho^*) + \sqrt{r^2(\theta^h y^h - \rho^*)^2 + (1-2r)\phi^2}}{(1-2r)q} \tag{B8}$$

B5. Expression for $\bar{\beta}$ in the Ex-Ante Equilibrium

In the ex-ante equilibrium, we use the endogenous distribution of debt obtained in Lemma (5) along with Equation (A13) to solve for $\bar{\beta}$ in the LC equilibrium. Denoting $\hat{s} - s_{min} = \Delta\hat{s}$ and $s_{max} - s_{min} = \Delta s_{max}$ and noting that $p - \rho^* = \lambda$ in the LC equilibrium, we obtain:

$$\bar{\beta} = -q\lambda \frac{\Delta\hat{s}}{\Delta s_{max}} + \int_{\rho_{min}}^{\bar{\rho}} \frac{\rho - s_{min}}{\Delta s_{max}} d\rho$$

Notating $\rho^* - s_{min} = \phi$ and $r(\theta^h y^h - p) = \pi$ and noting that $\bar{p} = \bar{\rho} = \rho^* + q\lambda$, we get:[48]

$$\bar{\beta} = -q\lambda \frac{\Delta\hat{s}}{\Delta s_{max}} + \frac{(\phi + q\lambda)^2}{2\Delta s_{max}} \tag{B9}$$

$$= \frac{(\theta^l y^l - k - s_{min})^2 - q\lambda\left[2r(\theta^h y^h - p) + (q + 2r - 2qr)\lambda\right]}{2(s_{max} - s_{min})} \tag{B10}$$

B6. Proof of Footnote (23):

B6.1. Price Discrimination Equilibrium (PD)

Price in the PD region is obtained by using the endogenous distribution of debt from Lemma (5) in Equation (18) and solving for p. Using earlier notations of $\Delta s_{max} = s_{max} - s_{min}$, $\Delta\hat{s} = \hat{s} - s_{min}$, $\phi = \rho^* - s_{min}$ and denoting $p - \rho^* = \lambda_{PD}$, we obtain:

$$\mathcal{B} = -q\lambda_{PD} \frac{\Delta\hat{s}}{\Delta s_{max}} + \int_{\rho_{min}}^{\bar{\rho}} \frac{\rho - s_{min}}{\Delta s_{max}} d\rho$$

[47]Note that the other root of the quadratic can be ignored as for that root, $\lambda_{PD} < 0$ when $r < 1/2$ and $\lambda_{PD} > k$ when $r > 1/2$. When $r = 1/2$, Equation (B11) is linear in λ_{PD} and can be solved to obtain $\lambda_{PD} = \frac{\phi^2 - 2\mathcal{B}\Delta s_{max}}{(\theta^h y^h - \rho^*)q}$.

[48]$\Delta\hat{s} = r\theta^h y^h + (1-r)\rho^* + (1-r)q(p-\rho^*) - s_{min} = r(\theta^h y^h - \rho^* - (p - \rho^*)) + \big(r + (1-r)q\big)(p - \rho^*) + \rho^* - s_{min} = \pi + m(p - \rho^*) + \phi$ where $\pi = r(r\theta^h y^h - p)$, $\phi = \rho^* - s_{min}$ and $m = r + (1-r)q$. This general result is valid across equilibrium regions. In the PD, LC and CC regions, $p - \rho^*$ is replaced by λ_{PD}, λ and λ_{CC}, respectively.

$$\Rightarrow 2\mathcal{B}\Delta s_{max} = -2q\lambda_{PD}\Delta\hat{s} + (\phi + q\lambda_{PD})^2 \tag{B11}$$

The above quadratic in λ_{PD} can be solved to obtain λ_{PD} which can be used to obtain $p = \rho^* + \lambda_{PD}$. To evaluate the impact of q, we note that $\frac{d\bar{p}}{dq} = \frac{d(q\lambda_{PD})}{dq} = \lambda_{PD} + q\frac{d\lambda_{PD}}{dq}$. Further, $\frac{d\hat{s}}{dq} = (1-r)\frac{d\bar{p}}{dq} = (1-r)\frac{d(q\lambda_{PD})}{dq}$. Differentiating Equation (B11) with respect to q and noting that $\Delta\hat{s} = \pi + \phi + m\lambda_{PD}$, we obtain:

$$\begin{aligned}
0 =& -2\left[\frac{d\bar{p}}{dq}\Delta\hat{s} + q\lambda_{PD}(1-r)\frac{d\bar{p}}{dq}\right] + 2(\phi + q\lambda_{PD})\frac{d\bar{p}}{dq} \\
\Rightarrow\ 0 =& -[\pi + (m - rq)]\frac{d\bar{p}}{dq} \\
\Rightarrow\ \left.\frac{d\bar{p}}{dq}\right|_{PD} =& \ 0 \quad as \quad [\pi + (m - rq)] > 0 && \text{(B12)} \\
\Rightarrow\ \left.\frac{dp}{dq}\right|_{PD} =& \ \frac{d\lambda_{PD}}{dq} = \frac{1}{q}\left[\left.\frac{d\bar{p}}{dq}\right|_{PD} - \lambda_{PD}\right] = -\frac{\lambda_{PD}}{q} < 0 && \text{(B13)}
\end{aligned}$$

f_r in the PD region is a function of p and therefore varies with q. Differentiating Equation (17) with respect to q while noting that $\frac{4\gamma k}{R^2} = \lambda$, we obtain:

$$\begin{aligned}
\left.\frac{df_r}{dq}\right|_{PD} =& -\frac{1}{4}\left[R^2 - \frac{4\gamma k}{p - \rho^*}\right]^{-\frac{1}{2}}(-4\gamma k)\left[\frac{-1}{(p-\rho^*)^2}\right]\left[\left.\frac{dp}{dq}\right|_{PD}\right] \\
=& \left[R^2 - \frac{4\gamma k}{\lambda_{PD}}\right]^{-\frac{1}{2}}\left[\frac{\gamma k}{\lambda_{PD}^2}\right]\left[\frac{\lambda_{PD}}{q}\right] \\
=& \frac{\lambda R}{4q\lambda_{PD}}\left[1 - \frac{\lambda}{\lambda_{PD}}\right]^{-\frac{1}{2}} > 0 && \text{(B14)}
\end{aligned}$$

Finally, as $\bar{\beta} = \mathcal{B}$ in the PD region, $\left.\frac{d\bar{\beta}}{dq}\right|_{PD} = 0$.

B6.2. Liquidity Crunch Equilibrium (LC)

In the LC Equilibrium, $p = \rho^* + \lambda$, $\bar{p} = \rho^* + q\lambda$, $f_r = R/2$ and $\bar{\beta}$ is given by Equation (B9). Therefore, $\left.\frac{dp}{dq}\right|_{LC} = 0$, $\left.\frac{d\bar{p}}{dq}\right|_{LC} = \lambda > 0$ and $\left.\frac{df_r}{dq}\right|_{LC} = 0$. We differentiate Equation (B9) with respect to q, noting that $\frac{d\hat{s}}{dq} = (1-r)\lambda$ in the LC equilibrium, to obtain (using the notational simplifications of m, π, ϕ, Δs_{max} and $\Delta\hat{s}$ developed earlier):

$$\begin{aligned}
\frac{d\bar{\beta}}{dq} =& -\frac{\lambda\left[\Delta\hat{s} + (1-r)q\lambda\right]}{\Delta s_{max}} + \frac{2(\phi + q\lambda)\lambda}{2\Delta s_{max}} \\
\Rightarrow \frac{d\bar{\beta}}{dq} =& -\frac{\Big(\pi + (m - rq)\lambda\Big)\lambda}{\Delta s_{max}} < 0 && \text{(B15)}
\end{aligned}$$

As π, $(m - r)$ and λ are all positive, $\frac{d\bar{\beta}}{dq} < 0$.

B6.3. Credit Crunch Equilibrium (CC)

Price in the CC region is obtained by using the endogenous distribution of debt from Lemma (5) in Equation (22) and solving for p. We obtain an expression similar to Equation (B11) with $\mathcal{B} = 0$; a quadratic in λ_{CC} which can be solved to obtain $p = \rho^* + \lambda_{CC}$:

$$0 = -2q\lambda_{CC}\Delta\hat{s} + (\phi + q\lambda_{CC})^2 \tag{B16}$$

Differentiating Equation (B16) with respect to q, we get results similar to Equations (B12) & (B13):

$$0 = [\pi + (m - rq)] \left.\frac{d\bar{p}}{dq}\right|_{CC} \Rightarrow \left.\frac{d\bar{p}}{dq}\right|_{CC} = 0 \quad as \quad [\pi + (m - rq)] > 0 \tag{B17}$$

$$\Rightarrow \left.\frac{dp}{dq}\right|_{CC} = \frac{d\lambda_{CC}}{dq} = \frac{1}{q}\left[\left.\frac{d\bar{p}}{dq}\right|_{CC} - \lambda_{CC}\right] = -\frac{\lambda_{CC}}{q} < 0 \tag{B18}$$

Further, as $\bar{\beta} = 0$ and $f_r = R/2$ in the CC equilibrium, it follows that $\left.\frac{d\bar{\beta}}{dq}\right|_{CC} = 0$ and $\left.\frac{df_r}{dq}\right|_{CC} = 0$

B7. $\left.\frac{dp}{d\theta^l}\right|_{LC} > 0$ *and* $\left.\frac{d\bar{p}}{d\theta^l}\right|_{LC} > 0.$

In the LC Equilibrium, p is given by $p = \rho^* + \lambda$, and therefore, $\left.\frac{dp}{d\theta^l}\right|_{LC} = y^l - (1-\omega)\frac{dk}{d\theta^l} > 0$ as $\frac{dk}{d\theta^l} < 0$. Further, as $\bar{p} = \rho^* + q\lambda$, we have $\left.\frac{d\bar{p}}{d\theta^l}\right|_{LC} = y^l - (1 - q\omega)\frac{dk}{d\theta^l} > 0.$

B8. $\left.\frac{dp}{d\theta^l}\right|_{CC} > 0$ *and* $\left.\frac{d\bar{p}}{d\theta^l}\right|_{CC} > 0.$

As $\left.\frac{d\hat{s}}{d\theta^l}\right|_{CC} = (1-r)\left[y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{CC}}{d\theta^l}\right]$, we rearrange and differentiate Equation (B16) with respect to θ^l to obtain:

$$2q\left[\Delta\hat{s}\frac{d\lambda_{CC}}{d\theta^l} + (1-r)\lambda_{CC}\left(y^l\frac{dk}{d\theta^l} + q\frac{d\lambda_{CC}}{d\theta^l}\right)\right] = 2(\phi + q\lambda_{CC})\left[y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{CC}}{d\theta^l}\right]$$

$$\Rightarrow \frac{d\lambda_{CC}}{d\theta^l} = \frac{(y^l - \frac{dk}{d\theta^l})(\phi + rq\lambda_{CC})}{q\left[\pi + (m - rq)\lambda_{CC}\right]} > 0 \tag{B19}$$

We also have:[49]

$$\left.\frac{dp}{d\theta^l}\right|_{CC} = \frac{d\rho^*}{d\theta^l} + \frac{d\lambda_{CC}}{d\theta^l} = y^l - \frac{dk}{d\theta^l} + \frac{d\lambda_{CC}}{d\theta^l} > 0 \tag{B20}$$

$$\left.\frac{d\bar{p}}{d\theta^l}\right|_{CC} = \frac{d\rho^*}{d\theta^l} + q\frac{d\lambda_{CC}}{d\theta^l} = y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{CC}}{d\theta^l} > 0 \tag{B21}$$

[49] We also obtain that $\left.\frac{dp}{d\theta^l}\right|_{CC} > \left.\frac{dp}{d\theta^l}\right|_{LC}$ as $\left.\frac{dp}{d\theta^l}\right|_{CC} - \left.\frac{dp}{d\theta^l}\right|_{LC} = \frac{d\lambda_{CC}}{d\theta^l} - \omega\frac{dk}{d\theta^l} > 0$. Similarly, we obtain that $\left.\frac{d\bar{p}}{d\theta^l}\right|_{CC} > \left.\frac{d\bar{p}}{d\theta^l}\right|_{LC}$ as $\left.\frac{d\bar{p}}{d\theta^l}\right|_{CC} - \left.\frac{d\bar{p}}{d\theta^l}\right|_{LC} = \frac{d\lambda_{CC}}{d\theta^l} - q\omega\frac{dk}{d\theta^l} > 0.$

B9. $\frac{d\bar{\beta}}{d\theta^l} > 0$

As $\frac{d\hat{s}}{d\theta^l}\Big|_{LC} = (1-r)[y^l - (1-q\omega)\frac{dk}{d\theta^l}]$, differentiating Equation (B9) with respect to θ^l, yields:

$$2\frac{d\bar{\beta}}{d\theta^l}\Delta s_{max} + 2\bar{\beta}(1-r)y^l = 2q\Delta\hat{s}(-\omega\frac{dk}{d\theta^l}) - 2q\lambda(1-r)\left[y^l - (1-q\omega)\frac{dk}{d\theta^l}\right] + 2(\phi+q\lambda)\left[y^l - (1-q\omega)\frac{dk}{d\theta^l}\right]$$

Rearranging and simplifying, we obtain:[50]

$$\frac{d\bar{\beta}}{d\theta^l} = \frac{[\phi + qr\lambda - (1-r)\bar{\beta}]y^l - [\phi + qr\lambda + q\omega(\pi + (m-qr)\lambda)]\frac{dk}{d\theta^l}}{\Delta s_{max}} > 0 \tag{B22}$$

B10. Proof: $\frac{d\bar{\theta}^l(q)}{dq} > 0$

$\bar{\theta}^l(q)$, the boundary between PD and LC equilibria is defined by the following equation:

$$\bar{\beta}\big(q, \bar{\theta}^l(q)\big) = \mathcal{B} \tag{B23}$$

Differentiating Equation (B23) with respect to q yields:[51]

$$\frac{\partial\bar{\beta}}{\partial q} + \frac{\partial\bar{\beta}}{\partial\bar{\theta}^l(q)}\frac{d\bar{\theta}^l(q)}{dq} = 0$$

$$\frac{d\bar{\theta}^l(q)}{dq} = -\frac{\partial\bar{\beta}/\partial q}{\partial\bar{\beta}/\partial\theta^l} > 0 \tag{B24}$$

B11. Proof: $\frac{d\hat{\theta}^l(q)}{dq} > 0$

$\hat{\theta}^l(q)$, the boundary between the LC and CC regions is defined by the following equation:

$$\lambda_{CC}\big(q, \hat{\theta}^l(q)\big) = \lambda \tag{B25}$$

Differentiating Equation (B25) with respect to q yields:[52]

$$\frac{\partial\lambda_{CC}}{\partial q} + \frac{\partial\lambda_{CC}}{\partial\hat{\theta}^l(q)}\frac{d\hat{\theta}^l(q)}{dq} = 0$$

$$\frac{d\hat{\theta}^l(q)}{dq} = -\frac{\partial\lambda_{CC}/\partial q}{\partial\lambda_{CC}/\partial\theta^l} > 0 \tag{B26}$$

[50] The result in Equation (B22) follows as $\bar{\beta} \leq \bar{\beta}(q=0) < \phi$.

[51] The final result follows as $\frac{\partial\bar{\beta}}{\partial q} \leq 0$ (see Footnote (23)) and $\frac{\partial\bar{\beta}}{\partial\theta^l} > 0$ (see Section (B9)).

[52] The final result follows as $\frac{\partial\lambda_{CC}}{\partial q} \leq 0$ (see Footnote (23)) and $\frac{\partial\lambda_{CC}}{\partial\theta^l} > 0$ (see Section (B8)).

B12. q^{opt} when $E_\theta[\rho^*(\theta)] \leq \theta^l y^l$

When $E_\theta[\rho^*(\theta)] \leq \theta^l y^l$, it can be shown that $\left.\frac{dS_{Total}}{dq}\right|_{LC} > 0$ for $q < \breve{q} = \frac{2\lambda - 5r\lambda - 3\pi}{3\lambda - 6r\lambda + 2(1-r)\omega\lambda}$. See Figure (4) for the definitions of $\bar{q}$, $\bar{\bar{q}}$, and $\hat{q}$. Three possible cases arise.

(i) $\breve{q} \leq \bar{q}$: In this case, S_{Total} is always decreasing with q in the LC equilibrium and therefore $q^{opt} = \bar{\bar{q}}$ (i.e., the border between the FP and PD equilibria).

(ii) $\bar{q} < \breve{q} < \hat{q}$: In this case, S_{Total} first increases with q in the LC equilibrium till it reaches a local maxima at $q = \breve{q}$, after which it decreases with q. Consequently $q^{opt} = \arg\max_q(S_{Total}(\bar{\bar{q}}), S_{Total}(\breve{q}))$.

(iii) $\hat{q} \leq \breve{q}$: In this case, S_{Total} increases with q across the LC equilibrium, reaching a local maximum value at $\hat{q}$ (i.e., the border of the LC and CC equilibria). Consequently $q^{opt} = \arg\max_q(S_{Total}(\bar{\bar{q}}), S_{Total}(\hat{q}))$. Note that when $q^{opt} = \hat{q}$, as S_{Total} is invariant with q in the CC equilibrium, $q^{opt} = (\hat{q}, 1)$.

Essentially, when $E_\theta[\rho^*(\theta)] \leq \theta^l y^l$, q^{opt} is one of the following: $\bar{\bar{q}}$, $\breve{q}$, $(\hat{q}, 1)$.

B13. Proof: $\frac{d\lambda_{PD}}{d\theta^l} > 0$

We evaluate the impact of θ^l on λ_{PD}. We have $\frac{d\hat{s}}{d\theta^l} = (1-r)\frac{d\bar{p}}{d\theta^l} = (1-r)(y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{PD}}{d\theta^l})$. Differentiating Equation (B11) with respect to θ^l to obtain:[53]

$$\begin{aligned}
2\mathcal{B}(1-r)y^l =& -2(1-r)q\lambda_{PD}(y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{PD}}{d\theta^l}) - 2q\Delta\hat{s}\frac{d\lambda_{PD}}{d\theta^l} + 2(\phi + q\lambda_{PD})(y^l - \frac{dk}{d\theta^l} + q\frac{d\lambda_{PD}}{d\theta^l}) \\
\Rightarrow\ q\left[\pi + (m - rq)\lambda_{PD}\right]\frac{d\lambda_{PD}}{d\theta^l} =& (\phi + rq\lambda_{PD})(y^l - \frac{dk}{d\theta^l}) - (1-r)\mathcal{B}y^l \\
\Rightarrow\ \frac{d\lambda_{PD}}{d\theta^l} =& \frac{(\phi + rq\lambda_{PD})(y^l - \frac{dk}{d\theta^l}) - (1-r)\mathcal{B}y^l}{q\left[\pi + (m - rq)\lambda_{PD}\right]} > 0 \qquad \text{(B27)}
\end{aligned}$$

B14. Proof: $\frac{\partial\lambda_{PD}}{\partial k} < 0$

Noting that $\frac{d\Delta\hat{s}}{dk} = (1-r)\frac{d\bar{p}}{dk} = (1-r)(q\frac{d\lambda_{PD}}{dk} - 1)$, we differentiate Equation (B11) with respect to k to obtain:

$$\begin{aligned}
2q\Delta\hat{s}\frac{d\lambda_{PD}}{dk} =& -2q\lambda_{PD}(1-r)\left(q\frac{d\lambda_{PD}}{dk} - 1\right) + 2(\phi + q\lambda_{PD})\left[-1 + q\frac{d\lambda_{PD}}{dk}\right] \\
\Rightarrow \frac{d\lambda_{PD}}{dk} =& -\frac{\phi + rq\lambda_{PD}}{q[\pi + (m - rq)\lambda_{PD}]} < 0 \qquad \text{(B28)}
\end{aligned}$$

[53]The result in Equation (B27) obtains as the numerator of the fraction in Equation (B27) is positive in the PD region. PD region exists at a given θ^l for some q only if $\mathcal{B} < \bar{\beta}(q = 0)$ which implies $\mathcal{B} < \phi$.

B15. Proof: $\frac{d\lambda_{PD}}{d\mathcal{B}} < 0$

Noting that $\frac{d\hat{s}}{d\mathcal{B}} = (1-r)\frac{d\bar{p}}{d\mathcal{B}} = (1-r)q\frac{d\lambda_{PD}}{d\mathcal{B}}$ while differentiating Equation (B11) with respect to $\mathcal{B}$, we obtain:

$$\frac{d\lambda_{PD}}{d\mathcal{B}} = -\frac{\Delta s_{max}}{q\left[\pi + (m - rq)\lambda_{PD}\right]} < 0 \qquad \text{(B29)}$$

B16. Discussion of Proposition (4)

B16.1. Impact of θ^l on q^{opt}

As the severity of the economic shock increases, fire-sale effects are triggered at lower levels of q and the optimal q decreases. Figure (**4**) illustrates this situation. Consider the case of a severe economic shock ($\theta^l = \theta_{severe} = 0.30$). In this case, there is an acute shortage of funding liquidity due to the severity of the economic shock. The economy will be in a Liquidity Crunch Equilibrium even at the lowest feasible value of $q = 0$ (which induces the least amount of ex-post liquidation). The solid curve representing the boundary of the Fair Pricing region and the Fire Sale region (depicted by the $\bar{\bar{q}}(\theta^l)$ *curve*) does not arise in the vertical line drawn at $\theta = 0.30$, i.e., both the Fair Pricing region and the Price Discrimination region vanish for the given level of economic shock. For such a severe economic shock, the economy is always in the Fire Sale region for the entire range of feasible $q \in (0, 1)$. This situation arises because the financial market cannot clear without reducing the supply of loans to the real sector, i.e., the system will always be in the Liquidity Crunch region, and there will some unmet demand in the real sector ($\bar{\beta} < \mathcal{B}$). The system transitions to a Credit Crunch Equilibrium at higher values of q. Interestingly, the ex-ante optimal q^{opt} is equal to 0.
Figure (**4**) also depicts the situation in which the optimal bankruptcy exemption can be equal to 1. Consider the case of a mild economic shock ($\theta^l = \theta_{mild} = 0.75$). In this case, there is sufficient liquidity in the economy that there are no ex-post fire-sale effects. Both the financial asset and the Date 1 loan trade at fair value for any level of q. Since there is no negative externality of ex-post liquidation, it is optimal to employ full bankruptcy exemption, which facilitates ex-ante lending that maximizes total surplus in the economy.

B16.2. Impact of k on q^{opt}

In Panel A of Figure **9**, we map the equilibria in the system in the (k, q) space, which is defined over $k \in [k_{min}, k_{max}]$ and $q \in [0, 1]$. Similar to the analysis behind Figure **4**, the $\bar{\bar{q}}(k)$ curve in Panel A of Figure **9** divides the feasible (k, q) space into two regions (the Fair Pricing and the Fire Sale region) for any given (k, q) combination. Based on Proposition (3), the $\bar{\bar{q}}(k)$ curve represents the q^{opt} for a given k. Note that the curve representing the border of the Fair Pricing and Fire Sale regions is downward sloping in the feasible (k, q) space. If collateral quality is sufficiently high, the optimal q can be as high as 1 (see $k = 0.5$ in Panel A of Figure **9**). On the other hand, for low quality collateral, the optimal q is 0 (see $k = 2.5$ in Panel A of Figure **9**).

B16.3. Impact of $\mathcal{B}$ on q^{opt}

In general, as the size of the real sector $\mathcal{B}$ increases, it is less likely that the Date 1 loan market will be fully satiated, but the extent to which the real sector loans are offered depends on the liquidity in the economy. In Panel B of Figure **9**, we map the Fair Pricing and the Fire Sale boundary (shown by the $q^{opt}(\mathcal{B} = 0)$ *curve*) in the (θ, q) space for different values of $\mathcal{B}$. As $\mathcal{B}$ increases, the border of the Fair Pricing region and the Fire Sale regions shifts downward (and to the right). This shift causes the optimal q to decrease with $\mathcal{B}$.

At the extreme, when $\mathcal{B}$ is sufficiently high, even at $q = 0$ when there is no ex-post liquidation, the spare liquidity is insufficient to satisfy the Date 1 loan demand. Consequently, the system always lies in the Liquidity Crunch region. This can be seen in Panel B of Figure (**9**), where for $\theta^l = 0.55$ and for $\mathcal{B} = 1.3$, $q^{opt} = 0$. For any higher $\mathcal{B}$, the optimal q for the given economic shock ($\theta^l = 0.55$) will continue to be 0.

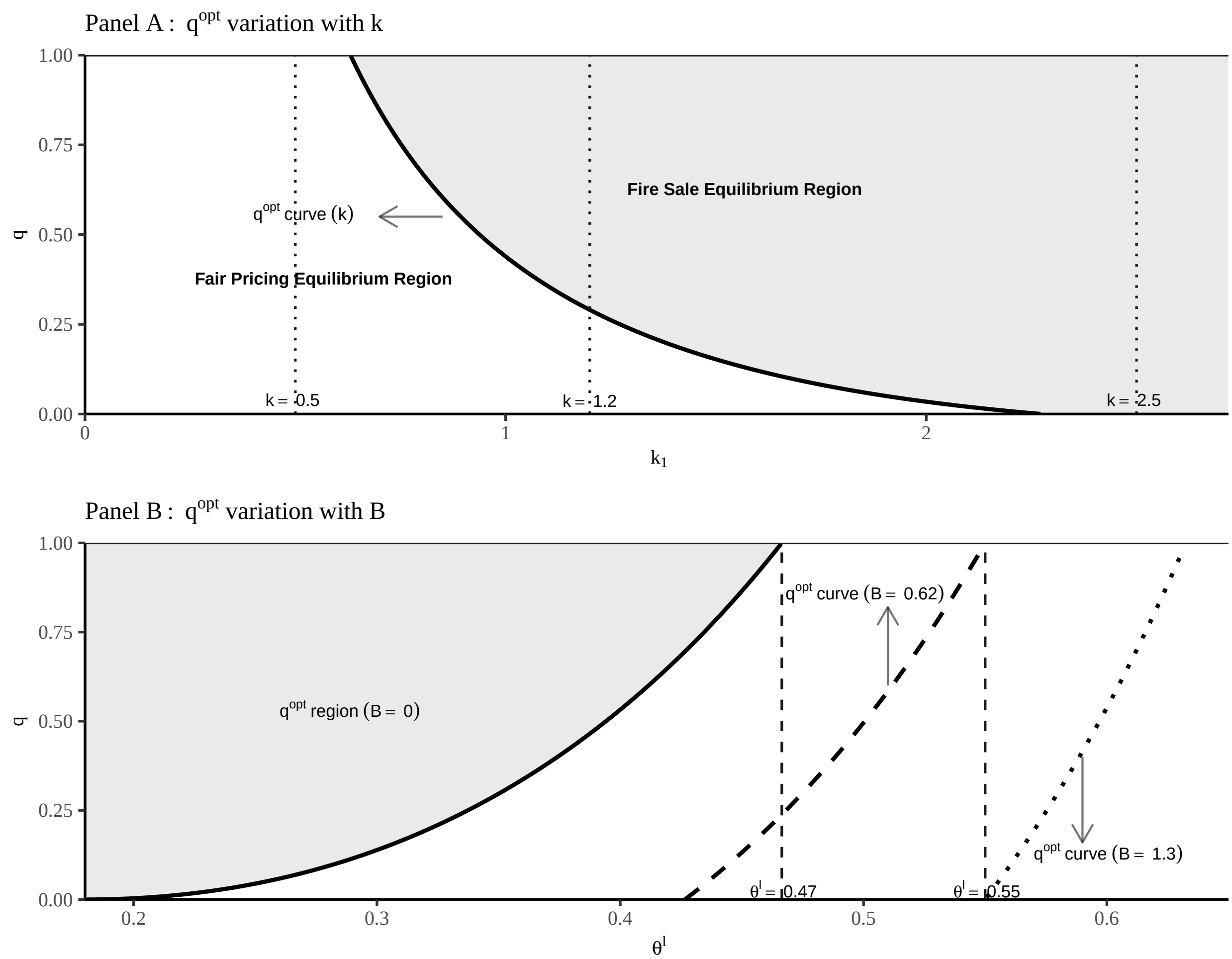
Conversely, as $\mathcal{B}$ decreases, the curve moves toward the northwest of (q, θ^l) space. However, this leftward movement is bounded when $\mathcal{B}$ hits 0, i.e., when the real sector is absent. This situation corresponds to the special case of the model examined in Acharya and Viswanathan (2011). with q assumed to be 1. However, in our model, the optimal q could range from an interior value to 1, as can be seen from the $q^{opt}(\mathcal{B} = 0)$ *region* in Panel B of Figure (**9**). The specific details can be seen in Appendix A14.3. The combined effect of the economic shock and the size of real sector is discussed in Appendix B17.

In the special case when $\mathcal{B} = 0$, it can be seen from Equation (29) that $\bar{\bar{q}} > 0$ as $\phi > 0$. However, in this case, the system directly transitions from the Fair Pricing region to the Credit Crunch region. Therefore, S_{Total} is invariant in q beyond $\bar{\bar{q}}$ when $\mathcal{B} = 0$, rendering $q^{opt} = (\bar{\bar{q}}, 1)$. Further, denoting the value of θ^l for which $\bar{\bar{q}} = 1$ when $\mathcal{B} = 0$ as $\theta^{l,B0}$, we obtain $\theta^{l,B0} = \frac{s_{min}+(1-r)k+\sqrt{(1-r)^2k^2+2rk(\theta^h y^h - s_{min})}}{y^l}$.[54]

Now, as $\bar{\bar{q}}$ is increasing in θ^l, for any $\theta^l < \theta^{l,B0}$, $\bar{\bar{q}} < 1$ and $q^{opt} = (\bar{\bar{q}}, 1)$. In addition, as $\bar{\bar{q}}$ is

[54] To obtain $\theta^{l,B0}$, we solve Equation (A28) for θ^l after setting $q = 1$ and $\mathcal{B} = 0$.

Figure **9**: q^{opt} **variation with** k **and** $\mathcal{B}$**.**
Panel A shows the typical demarcation of the feasible $k - q$ space into the Fair Pricing (FP) and Fire Sale (FS) equilibria. The plot is obtained by evaluating the model for assets varying in their collateral quality (k). The solid $q^{opt}(k)$ curve represents the boundary between the two equilibrium regions. For a moderate quality asset, indicated by $k = 1.2$, as q is increased from 0, the system transitions from FP equilibrium to FS equilibrium at $q = 0.47$. For a high quality asset indicated by $k = 0.5$, the system remains in FP equilibrium for any q. For a low quality asset, indicated by $k = 2.5$, the system remains in FS equilibrium for any q. $k = 0.5$, $k = 0.1.2$ and $k = 2.5$ are indicated by the three thin vertical dashed lines. Parameter Configuration: $\theta^l = 0.35$, $\theta^h = 1$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.
Panel B shows the optimal bankruptcy exemption parameter (q^{opt}) curve for three different levels of $\mathcal{B}$. The solid curve shows q^{opt} for $\mathcal{B} = 0$, the dashed curve shows q^{opt} for $\mathcal{B} = 0.62$ and the dotted curve shows q^{opt} for $\mathcal{B} = 1.3$. The vertical dashed line at $\theta^l = 0.47$ indicates the value of θ^l at which $q^{opt}(\mathcal{B} = 0) = 1$. The values of $\mathcal{B}$ used to obtain the dashed and dotted q^{opt} curves are chosen such that for $\theta^l = 0.55$ (indicated by the second vertical dashed line), we have $q^{opt}(\mathcal{B} = 0.62) = 1$ and $q^{opt}(\mathcal{B} = 1.3) = 0$. Parameter configuration is the same as that used in Figure 4 (i.e., $k = 1.5$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.)



decreasing in $\mathcal{B}$, $q^{opt} = \bar{\bar{q}} < 1$ for any $\mathcal{B} > 0$ for any $\theta^l < \theta^{l,B0}$.

For $\theta^l \geq \theta^{l,B0}$, we denote the value of $\mathcal{B}$ at which $\bar{\bar{q}} = 1$ as $\mathcal{B}_1$ and the value of $\mathcal{B}$ at which $\bar{\bar{q}} = 0$ as $\mathcal{B}_2$. Again as q^{opt} is decreasing in $\mathcal{B}$, for a given $\theta^l \geq \theta^{l,B0}$, we conclude that:

(i) $q^{opt} = 1$ for $\mathcal{B} \leq \mathcal{B}_1$

(ii) $0 < q^{opt} < 1$ for $\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$

(iii) $q^{opt} = 0$ for $\mathcal{B} \geq \mathcal{B}_2$

We obtain $\mathcal{B}_1 = \frac{\phi^2 - k^2 - 2rk(\theta^h y^h - \theta^l y^l)}{2\Delta s_{max}]}$ and $\mathcal{B}_2 = \frac{\phi^2}{2\Delta s_{max}]}$.[55]

B17. The Combined Effect of Economic Shock and Size of Real Sector

The table 5 presents the possible range of q^{opt} for different ranges of θ^l and $\mathcal{B}$.

θ^l Range	$\mathcal{B}$ Range	Implication for q^{opt}
$\theta^l_{min} \leq \theta^l < \theta^{l,B0}$	$\mathcal{B} = 0$	$q^{opt} \in (\bar{\bar{q}}, 1)$
	$\mathcal{B} > 0$	$0 \leq q^{opt} < 1$
$\theta^{l,B0} \leq \theta^l \leq \theta^l_{max}$	$0 \leq \mathcal{B} \leq \mathcal{B}_1$	$q^{opt} = 1$
	$\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$	$0 < q^{opt} < 1$
	$\mathcal{B}_2 \leq \mathcal{B}$	$q^{opt} = 0$

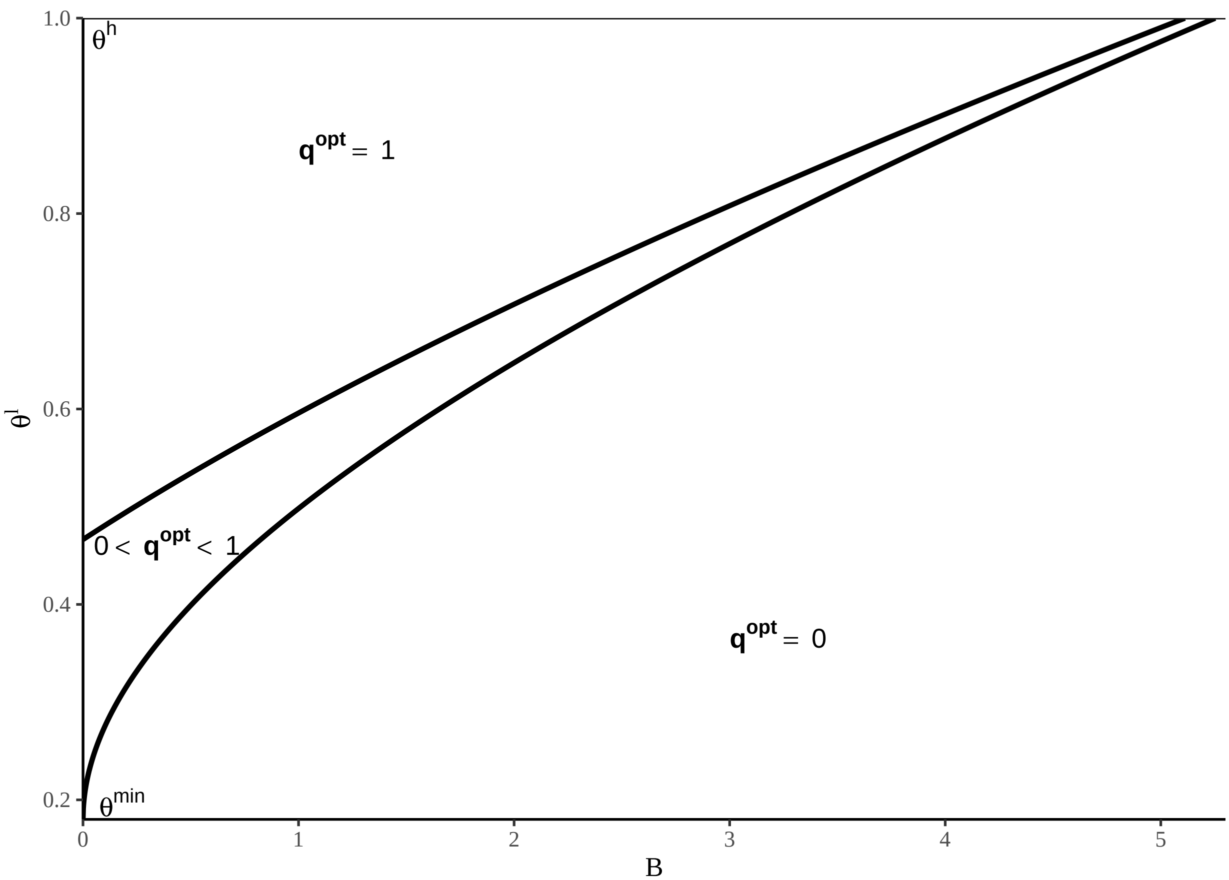
Table 5: **Impact of $\mathcal{B}$ on q^{opt}**. Implication of the size of the real sector ($\mathcal{B}$) on the optimal bankruptcy exemption parameter (q^{opt}) for a given level of the economic shock (θ^l) is presented.

Figure (**10**) displays the results of Table 5 in graphical form by presenting the joint impact of the level of the magnitude of the economic shock (θ^l) and the size of the real sector ($\mathcal{B}$) on q^{opt}. We consider the $(\mathcal{B}, \theta)$ space and map the three regions of optimal q ($q^{opt} = 0$, an interior q^{opt}, and $q^{opt} = 1$). We see that when the magnitude of the economic shock is mild and the size of the real sector market is small (i.e., top left corner of Fig(**10**)), $q^{opt} = 1$. As the size of real sector market increases or the severity of the economic shock increases, q^{opt} falls below 1 and moves towards 0 (i.e., bottom right corner of Fig(**10**)).[56]

[55] To obtain $\mathcal{B}_1$ and $\mathcal{B}_2$, we solve Equation (A28) for $\mathcal{B}$ for which $\bar{\bar{q}} = 1$ and $\bar{\bar{q}} = 0$, respectively, at a given θ^l.

[56] Note that in Fig(**10**), for $\mathcal{B} = 0$ the chart plots the value of $\bar{\bar{q}}$, the lower end of the range for q^{opt} as shown in Table 5.

Figure **10**: q^{opt} **in** $\mathcal{B} - \theta^l$ **space.** Demarcation of the $\mathcal{B} - \theta^l$ space into regions where $q^{opt} = 0$, $0 < q^{opt} < 1$ and $q^{opt} = 1$. Parameter Configuration used: $k = 1.5$, $\theta^h = 1$, $y^l = 15$, $y^h = 16$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.



B18. List of Proofs

Main Appendix

No.	Description of Proof	Appendix Reference	Main Text Reference
1	Effort Lemma	Appendix A, SS A2	Lemma (1)
2	Strategic Default Lemma	Appendix A, SS A3	Lemma (2)
3	Equilibrium Restrictions on f_r, γ, and p	Appendix A, SS A4	Section 4 SS 4.4
4	Optimization Lemma	Appendix A, SS A5	Lemma (3)
5	Market Clearing Lemma	Appendix A, SS A6	Lemma (4)
6	Ex Post: Equilibrium p, f_r and $\bar{\beta}$ in FP region	Appendix A, SS A7	Proposition (1)
7	Ex Post: Equilibrium p, f_r and $\bar{\beta}$ in FS region	Appendix A, SS A8	Proposition (2)
8	Ex Ante: Derived Distribution of Debt	Appendix A, SS A9	Section 5, Lemma (5)
9	Ex Ante: Model Restrictions	Appendix A, SS A10	Section 5, SS 5.3
10	Ex Ante: S_{D1} Dynamics	Appendix A, SS A11	Proposition (3)
11	Ex Ante: S_{D0} Dynamics	Appendix A, SS A12	Proposition (3)
12	Ex Ante: q^{opt}	Appendix A, SS A13	Proposition (3)
13	Ex Ante: q^{opt} is increasing in θ^l	Appendix A, SS A14.1	Proposition (4)
14	Ex Ante: q^{opt} is decreasing in k	Appendix A, SS A14.2	Proposition (4)
15	Ex Ante: q^{opt} is decreasing in $\mathcal{B}$	Appendix A, SS A14.3	Proposition (4)
16	Optimal $q - \bar{s}$ combination	Appendix A, SS A15	Proposition (5)

Internet Appendix

No.	Description of Proof	Appendix Reference	Main Text Reference
1	Funding Illiquidity and Risk-Shifting	Appendix B, SS B1	Section 4 SS 4.1
2	Existence and Uniqueness of Solution	Appendix B, SS B2	Section 5, SS 5.3
3	s for a given ρ	Appendix B, SS B3	Section 5, Lemma (5)
4	Expression for $p\|_{PD}$ and $p\|_{CC}$	Appendix B, SS B4	Footnote (23)
5	Ex Ante: Equilibrium $\bar{\beta}$	Appendix B, SS B5	Footnote (23)
6	Ex Ante: PD region Dynamics	Appendix B, SS B6.1	Footnote (23)
7	Ex Ante: LC region Dynamics	Appendix B, SS B6.2	Footnote (23)
8	Ex Ante: CC region Dynamics	Appendix B, SS B6.3	Footnote (23)
9	Ex Ante: $\left.\frac{dp}{d\theta^l}\right\|_{LC} > 0$ and $\left.\frac{d\bar{p}}{d\theta^l}\right\|_{LC} > 0$	Appendix B, SS B7	Footnote (23)
10	Ex Ante: $\left.\frac{dp}{d\theta^l}\right\|_{CC} > 0$ and $\left.\frac{d\bar{p}}{d\theta^l}\right\|_{CC} > 0$	Appendix B, SS B8	Footnote (23)
11	Ex Ante: $\frac{d\bar{\beta}}{d\theta^l} > 0$	Appendix B, SS B9	Footnote (23)
12	Ex Ante: $\frac{d\bar{\theta}^l(q)}{dq} > 0$	Appendix B, SS B10	Section 5, SS 5.3
13	Ex Ante: $\frac{d\hat{\theta}^l(q)}{dq} > 0$	Appendix B, SS B11	Section 5, SS 5.3
14	q^{opt} when $\theta^h y^h - \theta^l y^l < \left[\frac{8\gamma(1-r)}{3rR^2} - 1\right] k$	Appendix A, SS B12	Proposition (3)
15	Ex Ante: $\frac{d\lambda_{PD}}{d\theta^l} > 0$	Appendix B, SS B13	Proposition (4)
16	Ex Ante: $\frac{\partial\lambda_{PD}}{\partial k} < 0$	Appendix B, SS B14	Proposition (4)
17	Ex Ante: $\frac{d\lambda_{PD}}{d\mathcal{B}} < 0$	Appendix B, SS B15	Proposition (4)
18	Ex Ante: Discussion of Proposition (4)	Appendix B, SS B16	Proposition (4)
19	Ex Ante: Combined Effect of θ^l and $\mathcal{B}$	Appendix B, SS B17	Proposition (4)

Table 6: **List of Proofs in Appendices**