Internet Appendix for "Leader-Follower Dynamics in Shareholder Activism"

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Contents

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I Repeated trades (Proof of Proposition 2 and Figure 2)

We begin by analyzing the model where both players trade simultaneously in two periods, as in Proposition 2. In Section I.D, we turn to the the hybrid model where the leader trades alone first and then both players trade in the second period, as in Figure 2.

We focus on linear equilibria in which players follow symmetric strategies; player i's trade in period j is denoted θ_j^i, where

$$\theta_1^i = \alpha_1 X_0^i + \delta_1 \mu, \tag{IA.1}$$
$$\theta_2^i = \alpha_2 (X_1^i - \tilde{M}_1) + \beta_2 (X_0^i - M_1), \tag{IA.2}$$

where X_0^i is the initial position, $X_1^i = X_0^i + \theta_1^i$ is the updated position after the first period, $\tilde{M}_1$ is the MM's mean posterior belief (after period 1) about either player's updated position, and M_1 is the MM's mean posterior belief about either player's initial position.

To illustrate the manipulation incentive in the first period suppose by way of contradiction that there is an equilibrium in which $\delta_1 = -\alpha_1$. Consider trader i of type $X_0^i = \mu > 0$, so that player i's expectations of player j's trade in period 1 and both players' trades in period 2 are all 0, and moreover, player i's conjectured optimal trade in period 1 is 0. The right hand side of the first order condition in period 1 evaluated at 0 reduces to

$$\underbrace{-(\beta_2 + \alpha_2(1+\alpha_1)) \frac{\partial M_1}{\partial \Psi_1} X_0^i}_{\text{value of manipulation}}, \tag{IA.3}$$

since $\frac{\partial \tilde{M}_1}{\partial \Psi_1} = (1+\alpha_1)\frac{\partial M_t}{\partial \Psi_1}$. Now $X_0^i = \mu > 0$, and in the equilibrium we construct, $\beta_2 + \alpha_2(1+\alpha_1) > 0$. Furthermore, $\frac{\partial M_1}{\partial \Psi_1} = \frac{\alpha_1(\phi+\rho)}{2\alpha_1^2(\phi+\rho)+\sigma^2} > 0$. Together, these inequalities imply the value of manipulation is negative. Thus, the first order condition is not satisfied at the conjectured strategy if $\delta_1 = -\alpha_1$, and in particular, player i could do better with a small negative trade. Importantly, this argument holds independently of the sign of ρ (provided $\rho \neq -\phi$), in contrast to the baseline model. Intuitively, a lower trade in period 1 reduces the MM's belief about both players' initial (and current) position, reducing the price in period 2 and leading to a larger period 2 trade by the other player. In contrast, in the baseline model, the analogous value of manipulation depends on the MM's belief about the second trader alone, and the response of that belief to the first period trade depends on the sign of ρ.

We begin by characterizing learning, pricing, and optimality conditions. We then prove existence of equilibrium for small $\sigma > 0$; for technical reasons, we analyze the cases $\rho = \phi$ and $\rho \in [0, \phi)$ separately.[1]

I.A Learning, pricing, and optimality

Players begin with the usual prior mean and variance about each other's positions

$$Y_0^i = \mu + \frac{\rho}{\phi}(X_0^i - \mu), \tag{IA.4}$$

$$\nu_0 = \phi - \frac{\rho^2}{\phi}. \tag{IA.5}$$

Since the second period strategies have the gap form, the prior price is

$$P_0 = 2\mu(1 + \alpha_1 + \delta_1). \tag{IA.6}$$

After observing period 1 order flow Ψ_1, the MM's updated beliefs are

$$M_1 = \mathbb{E}[X_0^i|\Psi_1] = \mu + \frac{\alpha_1(\phi+\rho)}{2\alpha_1^2(\phi+\rho)+\sigma^2}\{\Psi_1 - 2(\alpha_1+\delta_1)\mu\}, \tag{IA.7}$$

$$\gamma_1 = \mathrm{Var}(X_0^i|\Psi_1) = \frac{\alpha_1^2(\phi-\rho)(\phi+\rho)+\phi\sigma^2}{2\alpha_1^2(\phi+\rho)+\sigma^2}, \tag{IA.8}$$

$$\tilde{M}_1 = \mathbb{E}[X_1^i|\Psi_1] = (1+\alpha_1)M_1 + \delta_1\mu, \tag{IA.9}$$

$$\tilde{\gamma}_1 = \mathrm{Var}(X_1^i|\Psi_1) = (1+\alpha_1)^2\gamma_1, \tag{IA.10}$$

$$\rho_1 = \mathrm{Cov}(X_0^i, X_0^j|\Psi_1) = \frac{-\alpha_1^2(\phi-\rho)(\phi+\rho)+\rho\sigma^2}{2\alpha_1^2(\phi+\rho)+\sigma^2}, \tag{IA.11}$$

$$\tilde{\rho}_1 = \mathrm{Cov}(X_1^i, X_1^j|\Psi_1) = (1+\alpha_1)^2\rho_1. \tag{IA.12}$$

[1] In the first case, α_2/σ and β_2/σ have positive, finite limits as $\sigma \to 0$, while in the second case, they diverge.

The MM's price in period 1 is simply

$$P_1 = 2\tilde{M}_1, \tag{IA.13}$$

since it anticipates gap strategies in period 2. Price impact is therefore

$$\Lambda_1 = \frac{2(1+\alpha_1)\alpha_1(\phi+\rho)}{2\alpha_1^2(\phi+\rho)+\sigma^2}. \tag{IA.14}$$

Each player also updates its belief about the other's initial and updated position based on the period 1 residual order flow:

$$Y_1^i = \mathbb{E}_i[X_0^j|\Psi_1] = Y_0^i + \frac{\alpha_1\nu_0}{\alpha_1^2\nu_0+\sigma^2}\{\Psi_1 - \theta_1^i - \alpha_1 Y_0^i - \delta_1\mu\}, \tag{IA.15}$$

$$\tilde{Y}_1^i = \mathbb{E}_i[X_1^j|\Psi_1] = (1+\alpha_1)Y_1^i + \delta_1\mu. \tag{IA.16}$$

In period 2, the MM's pricing formula is

$$P_2 = P_1 + \frac{\mathrm{Cov}(X_2^i + X_2^j, \Psi_2|\Psi_1)}{\mathrm{Var}(\Psi_2|\Psi_1)}\Psi_2, \qquad \text{where} \tag{IA.17}$$

$$\mathrm{Cov}(X_2^i + X_2^j, \Psi_2|\Psi_1) = (2\gamma_1 + 2\rho_1)[(1+\alpha_2)(1+\alpha_1)+\beta_2](\alpha_2(1+\alpha_1)+\beta_2), \tag{IA.18}$$

$$\mathrm{Var}(\Psi_2|\Psi_1) = (2\gamma_1+2\rho_1)(\alpha_2(1+\alpha_1)+\beta_2)^2 + \sigma^2. \tag{IA.19}$$

Player i's objective is

$$\mathbb{E}\Big[-(P_0 + \Lambda_1[\theta_1^i + \theta_1^j - 2\mu(\alpha_1+\delta_1)])\theta_1^i - (P_1 + \Lambda_2[\theta_2^i + \theta_2^j])\theta_2^i \\ + \frac{(X_0^i + \theta_1^i + \theta_2^i)^2}{2} + (X_0^j + \theta_1^j + \theta_2^j)(X_0^i + \theta_1^i + \theta_2^i)|X_0^i\Big].$$

The FOC wrt θ_2^i, given Ψ_1, is

$$0 = -(P_1 + \Lambda_2[\theta_2^i + \mathbb{E}_i[\theta_2^j|\Psi_1]]) - \Lambda_2\theta_2^i + (X_0^i + \theta_1^i + \theta_2^i) + \mathbb{E}_i[X_0^j + \theta_1^j + \theta_2^j|\Psi_1], \tag{IA.20}$$

where $\theta_1^i = X_1^i - X_0^i$ and where P_1 and expectations $\mathbb{E}_i[\cdot|\Psi_1]$ can be written as functions of M_1 and $\tilde{M}_1$ by first writing Ψ_1 in terms of $\tilde{M}_1$ and μ, and then writing $\mu = [\tilde{M}_1 - (1+\alpha_1)M_1]/\delta_1$. This equation is linear in X_1^i and X_0^i. Matching coefficients on X_1^i and X_0^i then yields two equations in $(\alpha_1, \alpha_2, \beta_2)$ (δ_1 does not appear here); it is easy to check that if these two equations are satisfied, then the equations associated with the coefficients on M_1 and $\tilde{M}_1$ are also satisfied, so the first order condition with respect to θ_2^i as a whole is satisfied.

The FOC wrt θ_1^i (using that the second period trade is at an optimum) is

$$0 = -(P_0 + \Lambda_1(\theta_1^i + \mathbb{E}_i[\theta_1^j] - 2\mu(\alpha_1 + \delta_1))) - \Lambda_1\theta_1^i - (\Lambda_1 + \Lambda_2[-\alpha_2\frac{\partial \tilde{M}_1}{\partial \Psi_1} - \beta_2\frac{\partial M_1}{\partial \Psi_1}])\mathbb{E}_i[\theta_2^i]$$
$$+ (X_0^i + \theta_1^i + \mathbb{E}_i[\theta_2^i]) + \mathbb{E}_i[X_0^j + \theta_1^j + \theta_2^j] + [-\alpha_2\frac{\partial \tilde{M}_1}{\partial \Psi_1} - \beta_2\frac{\partial M_1}{\partial \Psi_1}](X_0^i + \theta_1^i + \mathbb{E}_i[\theta_2^i]),$$

where the terms are (i) marginal cost of the first period trade, (ii) price impact in first period, (iii) price impact (from first period trade) on second period trade (via both P_1 and expected period-2 trade by other player), (iv) value of marginal share from own contribution, (v) value of marginal share from other's contribution, and (vi) value of change in other player's effort (due to altered second period trade) applied to expected terminal holdings. This equation is linear in (X_0^i, μ). Setting the coefficient on X_0^i to 0 gives a third equation in $(\alpha_1, \alpha_2, \beta_2)$, and the coefficient on μ yields an equation in $(\alpha_1, \delta_1, \alpha_2, \beta_2)$.

I.B Existence for small σ: $\rho = \phi$

We characterize the limits of $(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\beta}_2) := (\alpha_1/\sigma, \alpha_2/\sigma, \beta_2/\sigma)$ as $\sigma \to 0$.

Denote the equations derived from the X_1^i and X_0^i terms of the second-period first order condition by (FOC2-X_1^i) and (FOC2-X_0^i). Likewise, denote the equations derived from the X_0^i and μ term of the first period FOC by (FOC1-X_0^i) and (FOC1-μ).

We use the implicit function theorem. Taking $\sigma \to 0$ in the first order conditions yields

$$0 = 1 + 4\tilde{\alpha}_1^2\phi - 4\tilde{\alpha}_2^2\phi + 4\tilde{\beta}_2^2\phi, \tag{IA.21}$$
$$0 = 1 + 4\tilde{\alpha}_1^2\phi - 8\tilde{\alpha}_2\tilde{\beta}_2\phi - 8\tilde{\beta}_2^2\phi, \tag{IA.22}$$
$$0 = F(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\beta}_2, \phi), \tag{IA.23}$$
$$0 = G(\tilde{\alpha}_1, \tilde{\delta}_1, \tilde{\alpha}_2, \tilde{\beta}_2, \phi). \tag{IA.24}$$

The full system, and verification of the arguments that follow, can be found in the Mathematica file. The reader can access the full equations and expressions in the Mathematica file `ActivismTwiceRepeated-PerfectCorr.nb` on the authors' websites. We first prove that this system has a solution. Subtracting the second equation from the first and simplifying yields $\tilde{\alpha}_2 = 3\tilde{\beta}_2$ (discarding a solution for which the signs differ). Substituting this back into the first equation yields

$$\tilde{\beta}_2 = \sqrt{\frac{1 + 4\tilde{\alpha}_1^2\phi}{32\phi}}. \tag{IA.25}$$

Using these substitutions, the third equation has only the variable $\tilde{\alpha}_1$. The right hand side is positive at 0 and negative at $\frac{1}{\sqrt{\phi}}$, so by the intermediate value theorem, there is a root in $(0, \frac{1}{\sqrt{\phi}})$, which pins down $\tilde{\alpha}_2$ and $\tilde{\beta}_2$. Note that $\tilde{\alpha}_2, \tilde{\beta}_2 > 0$.

Finally, by adding (FOC1-X_0^i) and (FOC1-μ) and dropping nonzero factors, we obtain a unique characterization for $\tilde{\delta}_1$ given any solution $\tilde{\alpha}_1$:

$$0 = 2(\tilde{\alpha}_1 + \tilde{\delta}_1) + \tilde{\alpha}_2 + \tilde{\beta}_2. \tag{IA.26}$$

Since $\tilde{\alpha}_2$ and $\tilde{\beta}_2$ are positive, $\tilde{\delta}_1 < -\tilde{\alpha}_1$.

Next, we argue that there exists a solution to the system of first order conditions for all sufficiently small $\sigma > 0$. After the change of variables (but before taking $\sigma \to 0$), the system of equations (FOC2-X_1^i), (FOC2-X_0^i), and (FOC1-X_0^i) can be written as $0 = f(\mathbf{x}, \sigma)$, where $\mathbf{x} = (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\beta}_2)$ and where f is of class C^1. Let $\mathbf{x}^*(0)$ denote any solution as identified in the previous step. The Jacobian matrix evaluated at $(\mathbf{x}^*(0), 0)$ has determinant

$$-\frac{32\phi^{\frac{3}{2}}}{27(1+4z^2)^{\frac{7}{2}}}[7\sqrt{2} - 20\sqrt{2}z^2 + 72z\sqrt{1+4z^2}],$$

where $z := \tilde{\alpha}_1\sqrt{\phi} \in (0,1)$, which is easily shown to be negative for all $z \in (0,1)$. Thus there exists an interval $[0, \overline{\sigma})$, some $\overline{\sigma} > 0$, such that for all $\sigma \in [0, \overline{\sigma})$, there is a unique solution $\mathbf{x}^*(\sigma)$ to the system, continuously differentiable in σ. Moreover, (FOC1-μ) is linear in $\tilde{\delta}_1$ with nonzero slope for sufficiently small σ, so the solution extends to the larger system in $(\tilde{\alpha}_1, \tilde{\delta}_1, \tilde{\alpha}_2, \tilde{\beta}_2)$. Since $0 < \tilde{\alpha}_1(0) < \frac{1}{\sqrt{\phi}}$, the same inequalities hold for $\tilde{\alpha}_1(\sigma)$ for sufficiently small σ; reversing the change of variables, we have $0 < \alpha_1 < \alpha^K$. Continuity arguments establish that $\alpha_2, \beta_2 > 0$ and $\delta_1 < -\alpha_1$ for sufficiently small $\sigma > 0$.

The last step is to check that second order conditions are satisfied for sufficiently small σ. In each period $j = 1, 2$, we calculate the second derivative of player i's objective with respect to θ_j^i. For period 1, multiplying by σ, taking $\sigma \to 0$, and suppressing the argument of $\tilde{\alpha}_1(0)$ yields

$$-\frac{8\tilde{\alpha}_1\phi[3\sqrt{1+4\tilde{\alpha}_1^2\phi} - \sqrt{2\tilde{\alpha}_1^2\phi}]}{3(1+4\tilde{\alpha}_1^2\phi)^{\frac{3}{2}}} < 0.$$

The analogous calculation for period 2 yields the limit

$$-\frac{4\sqrt{2\phi}}{3\sqrt{1+4\tilde{\alpha}_1^2\phi}} < 0.$$

Thus, the second order conditions are satisfied, so the coefficients found constitute a PBS equilibrium.

I.C Existence for small σ: general $\rho \in [0, \phi)$

Supporting details for the equations and arguments in this section can be found in the Mathematica file `ActivismTwiceRepeated-ImperfectCorr.nb` on the authors' websites. When $\rho \in [0, \phi)$, the method used for the $\rho = \phi$ case must be modified. In particular, if we repeat the same change of variables and examine the limit of the system as $\sigma \to 0$, the limit system does not admit a real valued solution. Further, numerical analysis for small $\sigma > 0$ indicates that α_2 and β_2 tend to nonzero limits; in other words, the scaled variables α_2/σ and β_2/σ do not have finite limits. However, if we define $\hat{\beta}_2 = \beta_2 + \alpha_2(1 + \alpha_1)$ — the total coefficient on X_0^i in a player's second period trade along the path of play — numerical analysis indicates that $\hat{\beta}_2/\sigma$ does have a finite limit. This coefficient arises if we rewrite the second period trade as

$$\theta_2^i = \alpha_2 \Delta^i + \hat{\beta}_2 (X_0^i - M_1), \tag{IA.27}$$

where $\Delta^i := X_1^i - (1 + \alpha_1) X_0^i - \delta_1 \mu$ is the trader's deviation in the first period and is zero on the path of play. Therefore, for this part of the proof, we adopt the representation (IA.27) and derive the equilibrium conditions accordingly to prove existence analytically.

Eliminating X_1^i in the first order condition using $X_1^i = \Delta^i + (1 + \alpha_1) X_0^i - \delta_1 \mu$, the first order condition in period 2 is linear in Δ^i, X_0^i, and M_1. It is easy to verify that the equation associated with M_1 implies the equation associated with X_0^i. The equations associated with Δ^i and M_1, along with the equations from the first order condition in period 1 associated with X_0^i and μ, yield four polynomial equations in $(\phi, \rho, \sigma, \alpha_1, \delta_1, \alpha_2, \hat{\beta}_2)$. We refer to these equations as (FOC2-Δ^i), (FOC2-M_1), (FOC1-X_0^i), and (FOC1-μ).

After performing a change of variables $\tilde{\alpha}_1 = \alpha_1/\sigma$, $\tilde{\delta}_1 = \delta_1/\sigma$, and $\tilde{\beta}_2 = \hat{\beta}_2/\sigma$ (omitting the hat symbol), we show that the resulting system of equations has a solution at $\sigma = 0$. Evaluating at $\sigma = 0$ yields four equations

$$0 = \tilde{\alpha}_1(\phi - \rho) + 4\alpha_2 \tilde{\beta}_2 \phi + 2\tilde{\alpha}_1^2(\tilde{\alpha}_1 + 2\alpha_2 \tilde{\beta}_2)(\phi^2 - \rho^2), \tag{IA.28}$$

$$0 = 1 - 2\tilde{\beta}_2^2 \phi + 2\tilde{\alpha}_1^2(\phi + \rho) - 2\tilde{\alpha}_1^2 \tilde{\beta}_2^2(\phi^2 - \rho^2), \tag{IA.29}$$

$$0 = F(\tilde{\alpha}_1, \alpha_2, \tilde{\beta}_2, \phi, \rho), \tag{IA.30}$$

$$0 = G(\tilde{\alpha}_1, \tilde{\delta}_1, \alpha_2, \tilde{\beta}_2, \phi, \rho), \tag{IA.31}$$

where F and G are polynomial functions, redefined for the purpose of this proof. Equation

(IA.29) yields a unique positive solution for $\tilde{\beta}_2$ as a function of $\tilde{\alpha}_1$:

$$\tilde{\beta}_2 = \sqrt{\frac{\frac{1}{2} + \tilde{\alpha}_1^2(\rho + \phi)}{\phi + \tilde{\alpha}_1^2(\phi^2 - \rho^2)}}.$$

Solving (IA.28) yields a unique solution for α_2,

$$\alpha_2 = -\frac{\tilde{\alpha}_1(\phi - \rho)(1 + 2\tilde{\alpha}_1^2(\rho + \phi))}{4\tilde{\beta}_2(\phi + \tilde{\alpha}_1^2(\phi^2 - \rho^2))},$$

which is a function of only $\tilde{\alpha}_1$ via the previous solution for $\tilde{\beta}_2$, and which is strictly negative provided that $\tilde{\alpha}_1 > 0$, as we will show. We elaborate on this point in the following remark.

Remark 1. *In contrast to the case where $\rho = \phi$ and σ is small, when correlation is imperfect, $\alpha_2 < 0$ in the equilibrium we construct for small σ. This coefficient captures two forces. The first, of course, is that an upward deviation increases a trader's position. The second is that after an upward deviation, for a fixed public belief, a trader infers a lower trade by the other trader, and in turn, a lower initial position for the other trader. Under perfect correlation, the latter channel is absent, and there we find an equilibrium with $\alpha_2 > 0$. However, whenever there is imperfect correlation, the latter channel is present, and it dominates when noise is very small; hence, a trader buys less in the second period after an upward deviation. Small noise implies that equilibrium trades are small relative to positions, and thus the negative inference about the other trader's position is large for an upward deviation of fixed size.*

We now show that there exists $\tilde{\alpha}_1 \in (0, \frac{1}{\sqrt{\phi}})$ solving (IA.30) after plugging in the expressions for $(\alpha_2, \tilde{\beta}_2)$ above. Defining $\tilde{\rho} = \rho/\phi \in [0, 1)$, (IA.30) reduces to $0 = F(\tilde{\alpha}_1, \tilde{\rho})$ (relabeling F). Direct calculation shows that $F(0, \tilde{\rho}) = \phi(1 + \tilde{\rho})(2 + \tilde{\rho}) > 0$, while $F(\frac{1}{\sqrt{\phi}}, \tilde{\rho})$ is proportional to a function of $\tilde{\rho}$ alone that is negative for all $\tilde{\rho} \in [0, 1)$. Specifically,

$$\begin{aligned} F\left(\frac{1}{\sqrt{\phi}}, \tilde{\rho}\right) &\propto -2\tilde{\rho}^4\sqrt{6 + 4\tilde{\rho}} + \tilde{\rho}^3(3\sqrt{6 + 4\tilde{\rho}} + 4\sqrt{2 - \tilde{\rho}^2}) + \tilde{\rho}^2(10\sqrt{6 + 4\tilde{\rho}} + 2\sqrt{2 - \tilde{\rho}^2}) \\ &\quad - \tilde{\rho}(11\sqrt{6 + 4\tilde{\rho}} + 18\sqrt{2 - \tilde{\rho}^2}) - 19\sqrt{6 + 4\tilde{\rho}} - 18\sqrt{2 - \tilde{\rho}^2} \\ &\le 0 + \tilde{\rho}^3(3\sqrt{6 + 4} + 4\sqrt{2}) + \tilde{\rho}^2(10\sqrt{6 + 4} + 2\sqrt{2}) \\ &\quad - \tilde{\rho}(11\sqrt{6} + 18\sqrt{2 - 1}) - 19\sqrt{6} - 18\sqrt{2 - 1}, \end{aligned}$$

where the inequality follows from using $\tilde{\rho} \in [0, 1)$ term by term on the right hand side. The resulting cubic has second derivative $6\tilde{\rho}(3\sqrt{10} + 4\sqrt{2}) + 20\sqrt{10} + 4\sqrt{2}$ which is positive for all $\tilde{\rho} \ge 0$. Moreover, it is straightforward to see that the cubic is negative at 0 and 1. Thus, it is negative for all $\tilde{\rho} \in [0, 1)$, and the same is true for $F\left(\frac{1}{\sqrt{\phi}}, \tilde{\rho}\right)$. this confirms that a solution

$\tilde{\alpha}_1$ to (IA.30) exists in $(0, \frac{1}{\sqrt{\phi}})$.

Before characterizing $\tilde{\delta}_1$, we show that the Jacobian matrix for the system (IA.28)-(IA.30) has nonzero determinant. Evaluating the determinant using the expressions for α_2 and $\tilde{\beta}_1$ and changing variables $z = \tilde{\alpha}_1\sqrt{\phi}$ and $\tilde{\rho} = \frac{\rho}{\phi}$ yields an expression that is proportional to

$$\begin{aligned} f(z) &:= (5+2\tilde{\rho})\sqrt{1+2z^2(1+\tilde{\rho})} - 2z[3+9\tilde{\rho}+5\tilde{\rho}^2+\tilde{\rho}^3]\sqrt{2+2z^2(1-\tilde{\rho}^2)} \\ &\quad + z^2(1+\tilde{\rho})(60+\tilde{\rho}(13+\tilde{\rho}))\sqrt{1+2z^2(1+\tilde{\rho})} + \sum_{i=3}^{9} z^i A_i(z), \end{aligned}$$

where $A_i(z) \geq 0$ for all $i \in \{3, \ldots, 9\}$, all $\tilde{\rho} \in [0,1)$ and all $z \in [0,1]$ (where $z \in [0,1]$ follows from $\tilde{\alpha}_1 \in (0, \frac{1}{\sqrt{\phi}})$). We show that $f(z) \geq 0$.[2] For $(z, \tilde{\rho}) \in [0,1] \times [0,1)$,

$$\begin{aligned} f(z) &\geq (5+2\tilde{\rho})\sqrt{1+2z^2(1+\tilde{\rho})} - 2z[3+9\tilde{\rho}+5\tilde{\rho}^2+\tilde{\rho}^3]\sqrt{2+2z^2(1-\tilde{\rho}^2)} \\ &\quad + z^2(1+\tilde{\rho})(60+\tilde{\rho}(13+\tilde{\rho}))\sqrt{1+2z^2(1+\tilde{\rho})} \\ &\quad \geq (5+2\tilde{\rho}) - 2z[3+9\tilde{\rho}+5\tilde{\rho}^2+\tilde{\rho}^3]\sqrt{2+2(1-\tilde{\rho}^2)} \\ &\quad + z^2(1+\tilde{\rho})(60+\tilde{\rho}(13+\tilde{\rho})) \\ &\quad =: f_2(z), \end{aligned}$$

where we have used $z \in [0,1]$ to bound the square root in each term. We claim that $f_2(z) \geq 0$. Since $f_2(z)$ is quadratic and positive at $z = 0$, it suffices to show that its discriminant is always negative. That discriminant is

$$\begin{aligned} &-4(264+269\tilde{\rho}-210\tilde{\rho}^2-243\tilde{\rho}^3+52\tilde{\rho}^4+152\tilde{\rho}^5+82\tilde{\rho}^6+20\tilde{\rho}^7+2\tilde{\rho}^8) \\ \leq &-4(264+269\tilde{\rho}-210\tilde{\rho}^2-243\tilde{\rho}^3) =: d(\tilde{\rho}). \end{aligned}$$

Now $d(\tilde{\rho})$ has one sign change and thus exactly one positive real root. It is easy to verify that $d(1) < 0 < d(2)$, so the root lies between 1 and 2 and therefore $d(\tilde{\rho}) < 0$ for all $\tilde{\rho} \in [0,1)$. We conclude that $f(z) \geq f_2(z) \geq 0$. Thus, for all sufficiently small $\sigma > 0$, there exists a solution $(\tilde{\alpha}_1, \alpha_2, \tilde{\beta}_2)$ to the system of first order conditions (FOC2-Δ^i), (FOC2-M_1), and (FOC1-X_0^i), continuously differentiable in σ.

Turning to $\tilde{\delta}_1$, note that adding (IA.30) and (IA.31) yields the identity $2(\tilde{\alpha}_1+\tilde{\delta}_1)+\tilde{\beta}_2 = 0$, from which we conclude $\tilde{\alpha}_1 + \tilde{\delta}_1 < 0$. As in the proof of of existence for the $\rho = \phi$ case, the equation (FOC1-μ) for $\tilde{\delta}_1$ is linear in $\tilde{\delta}_1$ with nonzero slope for sufficiently small $\sigma > 0$. Thus, there exists a solution to the full system of first order conditions that is continuously

[2] The difference in sign of the determinant from the proof of existence for the $\rho = \phi$ case is an artifact of selecting the coefficient on M_1 rather than that on X_0^i in the second-period first order condition.

differentiable in σ.

Second-order conditions. The second derivative of the second-period objective is simply $1 - 2\Lambda_2$. As $\sigma \to 0$, the value of $\sigma(1 - 2\Lambda_2)$ at our constructed solution converges to $-\frac{4(\phi+\rho)\tilde{\beta}_2}{1+2(\rho+\phi)(\tilde{\alpha}_1^2+\tilde{\beta}_2^2)} < 0$. Thus, the second-period second order condition is satisfied for sufficiently small $\sigma > 0$.

For the first-period second order condition, we scale by σ and substitute the expressions for α_2 and $\tilde{\beta}_1$, and we again use $z = \tilde{\alpha}_1\sqrt{\phi}$ and $\tilde{\rho} = \frac{\rho}{\phi}$. The limit as $\sigma \to 0$ can be written

$$A(z)B(z),$$

where

$$B(z) = \frac{z(1+\tilde{\rho})\sqrt{\phi}}{4(1+2z^2(1+\tilde{\rho}))^{\frac{3}{2}}(2+\tilde{\rho}+z^2(1-\tilde{\rho}^2))\sqrt{1+z^2(1-\tilde{\rho}^2)}} > 0$$

and

$$\begin{aligned} A(z) = {} & 4\sqrt{2}z^5(1-\tilde{\rho}^2)^2 + 4\sqrt{2}z^3(3+\tilde{\rho})(1-\tilde{\rho}^2) - 16z^2(1-\tilde{\rho}^2)\sqrt{1+2z^2(1+\tilde{\rho})}\sqrt{1+z^2(1-\tilde{\rho}^2)} \\ & + z\sqrt{2}(3+\tilde{\rho})^2 - 16(2+\tilde{\rho})\sqrt{1+2z^2(1+\tilde{\rho})}\sqrt{1+z^2(1-\tilde{\rho}^2)}. \end{aligned}$$

We show that $A(z) < 0$. Bounding term by term, we have

$$\begin{aligned} A(z) &\le A_2(z) \\ &:= 4\sqrt{2}(1-\tilde{\rho}^2)^2 + 4\sqrt{2}z^2(3+\tilde{\rho})(1-\tilde{\rho}^2) - 16z^2(1-\tilde{\rho}^2) + z\sqrt{2}(3+\tilde{\rho})^2 - 16(2+\tilde{\rho}). \end{aligned}$$

Since $\tilde{\rho} \in [0,1)$, $A_2(z)$ is a convex quadratic with

$$\begin{aligned} A_2(0) &= -16(2+\tilde{\rho}) + 4\sqrt{2}(1-\tilde{\rho}^2)^2 < 0, \\ A_2(1) &= 4\sqrt{2}\tilde{\rho}^4 - 4\sqrt{2}\tilde{\rho}^3 - (19\sqrt{2}-16)\tilde{\rho}^2 - (8-5\sqrt{2})\tilde{\rho} - (48-25\sqrt{2}). \end{aligned}$$

Note that $A_2(1)$ has one sign change, so it has exactly one positive real root. Moreover, by inspection, $A_2(1)$ is negative when $\tilde{\rho} = 0$ and when $\tilde{\rho} = 1$. Thus, $A_2(1) < 0$ for all $\tilde{\rho} \in [0,1)$. We conclude that for all $(z, \tilde{\rho}) \in [0,1] \times [0,1)$, $A_2(z) < 0$ and therefore $A(z) < 0$. Hence, the first-period second order condition is satisfied in the limit $\sigma \to 0$, and by continuity, it is satisfied for sufficiently small $\sigma > 0$.

I.D Hybrid model: leader trades, then both trade

Consider a hybrid of the baseline model and the twice repeated model, where the leader trades alone in the first period and then trades again in the second period, simultaneously with the follower. It is natural to look for a linear equilibrium in which players trade according to strategies of the form

$$\theta_1^L = \alpha_L X_0^L + \delta_L \mu, \tag{IA.32}$$

$$\theta_2^L = \xi_1(X_0^L - \mathbb{E}[X_0^L|\mathcal{F}_1]) + \xi_2(X_1^L - \mathbb{E}[X_1^L|\mathcal{F}_1]), \tag{IA.33}$$

$$\theta^F = \alpha_F(X_0^F - M_1^F), \tag{IA.34}$$

with $\alpha_F > 0$. Both players here follow "gap" strategies in the second period. In θ_2^L, the first gap encodes the leader's informational advantage about the follower's initial position, while the second gap encodes her informational advantage about her own position entering period two.

We argue that the leader's incentive to deviate from a Kyle gap strategy in the first period is robust to this extension of the model. Note that the leader's objective function is

$$\mathbb{E}\Big[-(P_0 + \theta_1^L\Lambda_1)\theta_1^L \tag{IA.35}$$

$$-(P_1 + (\theta^F + \theta_2^L - \mathbb{E}[\Psi_2|\mathcal{F}^1])\Lambda_2)\theta_2^L \tag{IA.36}$$

$$+\frac{(X_0^L + \theta_1^L + \theta_2^L)^2}{2} + (X_0^F + \mathbb{E}[\theta^F|X_0^L, \Psi_1])(X_0^L + \theta_1^L + \theta_2^L)|X_0^L\Big]. \tag{IA.37}$$

Suppose by way of contradiction that the leader's first period trade is a gap strategy: $\alpha_L = -\delta_L > 0$. Consider the mean type $X_0^L = \mu > 0$. Then (i) $\mathbb{E}[\theta_1^L|X_0^L] = \mathbb{E}[\theta_2^L|X_0^L] = \mathbb{E}[\theta^F|X_0^L] = 0$, (ii) $\mathbb{E}[\mathbb{E}[\Psi_2|\mathcal{F}^1]|X_0^L] = 0$, (iii) $\mathbb{E}[\mathbb{E}[\theta^F|X_0^L, \Psi_1]|X_0^L] = 0$. Also, $\alpha_L = -\delta_L$ implies $P_0 = 2\mu$. The derivative of the leader's payoff with respect to θ_1^L under the candidate equilibrium strategy is thus

$$\begin{aligned}
&- P_0 + X_0^L + \mathbb{E}[X_0^F + 0|X_0^L] + X_0^L \frac{d}{d\theta_1^L}\mathbb{E}[\mathbb{E}[\theta^F|X_0^L, \Psi_1]|X_0^L] \\
&= \mu \frac{d}{d\theta_1^L}\mathbb{E}[\mathbb{E}[\theta^F|X_0^L, \Psi_1]|X_0^L] \\
&= \mu(-\alpha_F \frac{dM_1^F}{d\Psi_1}) < 0.
\end{aligned}$$

Hence, the leader could strictly benefit by deviating downward relative to the conjectured Kyle strategy.

The benefit to the mean type of leader of shading down her trade to reduce the price in period 2 lies in increasing the follower's trade and subsequent value creation applied to her original μ shares. Specifically, since the leader's own trade has the gap form in the second period, the mean type has an expected trade in the second period of size zero, so the benefit of the downward deviation does not operate through the channel of improving the price for her own second-period trade. However, this shading down is dampened relative to our baseline leader-follower model since the presence of the leader in the second period reduces the trading of the follower.

II Passive leader (Proof of Proposition 3)

We divide the proof into two parts: one for the model with private initial positions, and the other for the model with private signals of exogenous components of firm value. Supporting details for both parts can be found in the Mathematica file `PassiveLeader.nb` on the authors' websites.

II.A Private initial positions

We prove the following claims. If $\rho > 0$, then a PBS equilibrium exists, and moreover, in any PBS equilibrium, $0 < \alpha_L < \alpha^K$, and the leader sells on average. If $\rho < 0$, there exists an equilibrium in which $\alpha_L < -\alpha^K$, $0 < \delta_L < \alpha^K$, and the leader still sells on average; and there is no equilibrium in which $\alpha_L > 0$. In both cases, the follower plays a gap strategy.

Assume $\rho \neq 0$. The objectives of the activists are now

$$\begin{aligned}
&\text{Leader:} \qquad \sup_{\theta^L} \mathbb{E}[X_T^F X_T^L - P_1\theta^L | X_0^L, \theta^L] \\
&\text{Follower:} \qquad \sup_{\theta^F} \mathbb{E}[X_T^F X_T^F - P_2\theta^F - \frac{1}{2}(X_T^F)^2 | X_0^F, \mathcal{F}_1, \theta^F].
\end{aligned}$$

For any conjectured linear strategies, price impacts are now

$$\Lambda_1 = \frac{\alpha_L \rho (1+\alpha_F)}{(\alpha_L^2\phi + \sigma^2)(1-\beta_F)}, \tag{IA.38}$$

$$\Lambda_2 = \frac{\alpha_F(1+\alpha_F)\gamma_1^F}{\alpha_F^2\gamma_1^F + \sigma^2}, \tag{IA.39}$$

which differ from (7) and (10) only in that the component associated with the leader's terminal position is absent.

The follower's FOC is

$$0 = -\mathbb{E}_F[P_1 + \Lambda_2\{\Psi_2 - \mathbb{E}[\Psi_2|\mathcal{F}_1]\}] - \Lambda_2\theta^F + X_0^F + \theta^F \tag{IA.40}$$

$$= -P_1 - \Lambda_2(\theta^F - [\alpha_F M_1^F + \beta_F P_1 + \delta_F \mu]) - \Lambda_2\theta^F + X_0^F + \theta^F, \tag{IA.41}$$

and the leader's FOC is

$$\begin{aligned} 0 = &-\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1 + \mathbb{E}_L[X_T^F|\theta^L] \\ &+ (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L}. \end{aligned} \tag{IA.42}$$

Familiar arguments show that the strategy $\theta^F = \alpha_F(X_0^F - M_1^F)$, where $\alpha_F = \alpha_{F,1}(\alpha_L) = \sqrt{\frac{\sigma^2}{\gamma_1^F}}$, still satisfies the follower's FOC; that the follower's strategy has this characterization in any PBS equilibrium; and that the follower's strategy has a gap form in any linear equilibrium. Moreover, in this model, $M_1^F = P_1$, and $\beta_F = -\alpha_F$, and $\delta_F = 0$. It is easy to show that the leader's FOC implies the identity

$$\alpha_L = \frac{\sigma^2}{\phi\alpha_L} - \frac{\alpha_F}{1 + \alpha_F}, \tag{IA.43}$$

where $\alpha_F = \alpha_{F,1}(\alpha_L)$, and the identity (A.11) for δ_L.

The SOCs reduce to

$$0 > 1 - 2\Lambda_2 = -\frac{1}{\alpha_F}, \tag{IA.44}$$

$$0 > -2\Lambda_1(1 - \beta_F) = -2\frac{\alpha_L\rho(1 + \alpha_F)}{\alpha_L^2\phi + \sigma^2}, \tag{IA.45}$$

where again $\alpha_F = \alpha_{F,1}(\alpha_L)$.

The remainder of the proof analyzes separately the two cases $\rho > 0$ and $\rho < 0$.

$\rho > 0$ **case:** We first claim that there exist a $\alpha_L^+ \in (0, \alpha^K)$ solving (IA.43) and that it pins down a PBS equilibrium. As $\alpha_L \downarrow 0$, the RHS of (IA.43) tends to $+\infty$, and at $\alpha_L = \alpha^K$, the RHS is strictly less than α^K. Thus, by the intermediate value theorem, there exists a solution with $\alpha_L \in (0, \alpha^K)$. Moreover, there is no solution with $\alpha_L \geq \alpha^K$, since this would imply the RHS of (IA.43) is strictly less than $\frac{\sigma^2}{\phi\alpha_L} \leq \alpha_L$. Thus, for $\rho > 0$, $\alpha_L \in (0, \alpha^K)$ in any PBS equilibrium.

The follower's SOC (IA.44) is satisfied since $\alpha_F > 0$. The leader's SOC (IA.45) is also satisfied since $\alpha_L^+, \rho > 0$. Thus, the strategies characterized by α_L^+ and $\alpha_F = \alpha_{F,1}(\alpha_L^+)$ (along with $\beta_F = -\alpha_F$, $\delta_F = 0$, and δ_L as in (A.11) are part of a PBS equilibrium. The leader's expected trade $\mu(\alpha_L+\delta_L) = \mu(\alpha_L^+ - (\alpha^K)^2/\alpha_L^+)$ is negative since $\alpha_L^+ \in (0, \alpha^K)$, so the leader sells on average.

$\rho < 0$ **case:** We claim that there exists $\alpha_L^- \in (-\infty, -\alpha^K)$ solving (IA.43) and that it pins down a linear equilibrium. When $\alpha_L = -\alpha^K$, the RHS of (IA.43) equals $-\alpha^K - \frac{\alpha_F}{1+\alpha_F} < -\alpha_K$; and as $\alpha_L \to -\infty$, the RHS tends to a finite limit. Thus, by the intermediate value theorem, there exists $\alpha_L^- \in (-\infty, -\alpha^K)$ solving (IA.43). The follower's SOC (IA.44) is satisfied for the same reason as before, and the leader's SOC (IA.45) is satisfied since $\alpha_L^- < 0$. In such an equilibrium, the leader's expected trade is $\mu(\alpha_L+\delta_L) = \mu(\alpha_L^- - (\alpha^K)^2/\alpha_L^-) < 0$ since $\alpha_L^- \in (-\infty, -\alpha^K)$, so the leader still sells on average. Note that in this case it is impossible to have $\alpha_L > 0$, since it would not satisfy (IA.45).

II.B Private signals of exogenous components of firm value

Consider the extension from Proposition 1(a), where activist i's private information V^i is an exogenous component of firm value, and where initial positions are public. Now suppose that the leader is passive, only trading but not able to influence the firm's value through activism. That is, realized firm value is $V^L + V^F + X_T^F$. The objectives are now

$$\text{Leader:} \qquad \sup_{\theta^L} \mathbb{E}[(V^L + V^F + X_T^F)X_T^L - P_1\theta^L | V^L, \theta^L]$$

$$\text{Follower:} \qquad \sup_{\theta^F} \mathbb{E}[(V^L + V^F + X_T^F)X_T^F - P_2\theta^F - \frac{1}{2}(X_T^F)^2 | V^F, \mathcal{F}_1, \theta^F].$$

For ρ positive or not too negative, we characterize an equilibrium in which trades are

$$\begin{aligned}
\theta^L &:= \alpha_L(V^L - \mu) + \eta_L, \\
\theta^F &:= \alpha_F V^F + \beta_F P_1 + \delta_F \mu + \eta_F = \alpha_F(V^F - M_1^F),
\end{aligned}$$

where $M_1^F := \mathbb{E}[V^F|\mathcal{F}_1]$ (see below) and where $\alpha_L, \alpha_F > 0$.

The ex ante expectation of firm value is

$$P_0 = \mathbb{E}[X_0^F + V^L + V^F + \theta^F] = X_0^F + 2\mu.$$

Given Ψ_1, the MM's updated belief about V^L is

$$M_1^L := \mathbb{E}[V^L|\mathcal{F}_1] = \mu + \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2}\{\Psi_1 - \eta_L\}. \tag{IA.46}$$

And the MM's updated belief about V^F is

$$M_1^F := \mathbb{E}[V^F|\mathcal{F}_1] = \mu + \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \left\{\Psi_1 - \eta_L\right\}. \tag{IA.47}$$

Since the MM expects the follower to trade 0 conditional on first period order flow,

$$\begin{aligned} P_1 &= X_0^F + \mathbb{E}[V^L + V^F + \theta^L|\Psi_1] \\ &= X_0^F + M_1^L + M_1^F \\ &= P_0 + \Lambda_1 \left\{\Psi_1 - \eta_L\right\}, \end{aligned}$$

where $\Lambda_1 := \frac{\alpha_L(\rho+\phi)}{\alpha_L^2\phi+\sigma^2}$.

The follower's posterior belief about the leader's component V^L is

$$Y_1^F := Y_0^F + \frac{\alpha_L \nu_0^F}{\alpha_L^2 \nu_0^F + \sigma^2} \underbrace{\left\{\frac{P_1 - P_0}{\Lambda_1} + \alpha_L(\mu - Y_0^F)\right\}}_{=\Psi_1 - (\alpha_L(Y_0^F - \mu) - \eta_L}.$$

Let $\begin{pmatrix} \gamma_1^L & \rho_1 \\ \rho_1 & \gamma_1^F \end{pmatrix}$ denote the posterior covariance matrix of the market maker's beliefs about (V^L, V^F) after period one. We have

$$\gamma_1^L = \frac{\phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \gamma_1^F = \frac{\alpha_L^2[\phi^2 - \rho^2] + \phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \rho_1 = \frac{\rho\sigma^2}{\alpha_L^2\phi + \sigma^2}, \tag{IA.48}$$

where again the only difference from before is a missing $(1+\alpha_L)^2$ in γ_1^L and a missing $1+\alpha_L$ in ρ_1 since the updating is about V^L.

After seeing Ψ_2, the market maker again updates beliefs about V^L and V^F:

$$\begin{aligned} M_2^F &:= M_1^F + \frac{\alpha_F \gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} \Psi_2, \\ M_2^L &:= M_1^L + \frac{\alpha_F \rho_1}{\alpha_F^2 \gamma_1^F + \sigma^2} \Psi_2. \end{aligned}$$

The price is then

$$\begin{aligned} P_2 &= X_0^F + \mathbb{E}[V^L + V^F + \alpha_F(V^F - M_1^F)|\mathcal{F}_2] \\ &= X_0^F + M_2^L + M_2^F + \alpha_F(M_2^F - M_1^F) \\ &= P_1 + \Psi_2 \left[\frac{\alpha_F \gamma_1^F}{\alpha_F^2 \gamma_1^F + \sigma^2} + (1+\alpha_F)\frac{\alpha_F \rho_1}{\alpha_F^2 \gamma_1^F + \sigma^2}\right] \end{aligned}$$

$$= P_1 + \Psi_2 \underbrace{\frac{\alpha_F[\rho_1 + (1+\alpha_F)\gamma_1^F]}{\alpha_F^2\gamma_1^F + \sigma^2}}_{=:\Lambda_2}.$$

The first order conditions are

$$\begin{aligned}
\text{Follower:} \quad & 0 = Y_1^F + V^F + X_0^F + \theta^F - P_1 - 2\Lambda_2\theta^F \\
\text{Leader:} \quad & 0 = Y_0^L + V^L + X_0^F + \mathbb{E}[\theta^F|V^L,\theta^L] - (P_0 + \Lambda_1(\theta^L - \eta_L)) - \Lambda_1\theta^L + (X_0^L + \theta^L)\Lambda_1\beta_F.
\end{aligned}$$

The second order conditions are

$$0 > 1 - 2\Lambda_2, \tag{IA.49}$$

$$0 > -2\Lambda_1(1-\beta_F) = -2\frac{\alpha_L[\rho(1+\alpha_F)+\phi]}{\alpha_L^2\phi + \sigma^2}, \tag{IA.50}$$

By substituting in the conjectured strategies and matching coefficients, it is easy to verify that the FOCs are satisfied by

$$\begin{aligned}
\theta^L &= \alpha_L(V^L - \mu) + \eta_L, \\
\theta^F &= \alpha_F(V^F - M_1^F),
\end{aligned}$$

where $\alpha_L = \sqrt{\sigma^2/\phi}$, $\alpha_F = \sqrt{\sigma^2/\gamma_1^F}$, and $\eta_L = -X_0^L \frac{\rho\alpha_F}{\rho(1+\alpha_F)+\phi}$. The follower's SOC is satisfied by inspection, while the leader's SOC is satisfied for ρ positive or not too negative. It continues to hold that the leader's average trade is η_L, which implies $\text{sign}(\mathbb{E}[\theta^L|\mathcal{F}_0]) = -\text{sign}(\rho)$, as desired.

III Follower friendly to firm (Proof of Proposition 4)

Let the follower's cost of effort now be $\frac{1}{2}(W^F)^2 + \kappa W^F \mathbb{E}_F[W^L]$, where by convention $\mathbb{E}_i[\cdot]$ is the expectation operator for player i at the moment they trade, and where $\kappa \in (0,1)$. Note that $\mathbb{E}_F[W^L]$ and Y_1^F, already defined as $\mathbb{E}_F[X_T^L]$, are equivalent. Thus, the follower's optimal effort becomes $X_T^F - \kappa Y_1^F$. By the law of iterated expectations, the market price in each period is the MM's expectation of $X_T^F + (1-\kappa)X_T^L$.

It follows that the price impact coefficients adjust to

$$\Lambda_1 = \frac{\alpha_L\phi}{\alpha_L^2\phi + \sigma^2} \times \frac{(1+\alpha_L)(1-\kappa) + \rho(1+\alpha_F)/\phi}{1-\beta_F}, \tag{IA.51}$$

$$\Lambda_2 = \frac{\alpha_F\gamma_1^F}{\alpha_F^2\gamma_1^F + \sigma^2} \times [1 + \alpha_F + (1-\kappa)\rho_1/\gamma_1^F]. \tag{IA.52}$$

The follower's optimal strategy has exactly the same coefficients and gap form as before.

The leader's first order condition is now

$$\begin{aligned}0 &= -\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1 + (X_0^L + \theta^L) + \mathbb{E}_L[W^F|\theta^L] \\ &\quad + (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[W^F|\theta^L]}{\partial \theta^L} \\ &= -\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1 + (X_0^L + \theta^L) + \mathbb{E}_L[X_T^F|\theta^L] \\ &\quad + (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L} - \mathbb{E}_L[\kappa Y_1^F|\theta^L] - (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[\kappa Y_1^F|\theta^L]}{\partial \theta^L}.\end{aligned}$$

Matching coefficients in the usual way yields the identity $\delta_L = -\frac{\sigma^2}{\phi\alpha_L}$ and an equation involving α_L and α_F (after eliminating β_F, δ_F, and δ_L as in the baseline model). For the case $\rho = 0$, this equation reduces to

$$\begin{aligned}\alpha_L &= \frac{\alpha_L\phi(\alpha_L(1-\kappa) - \kappa) + \sigma^2}{\alpha_L\phi(2+\alpha_L) - \sigma^2} \\ \iff\ 0 &= (1+\alpha_L)[\alpha_L(\alpha_L + \kappa)\phi - \sigma^2].\end{aligned}$$

The right hand side is strictly increasing in α_L on $[0,\infty)$ and has exactly one positive root, and it satisfies $\alpha_L < \alpha^K$. This implies $\delta_L = -\frac{\sigma^2}{\phi\alpha_L} < -\alpha^K$, so the leader's expected trade is $(\alpha_L + \delta_L)\mu < 0$. Moreover, right hand side of the equation above is increasing in κ for $\alpha_L > 0$, this root is decreasing in κ, as is the leader's expected trade. The second order conditions are easy to check.

Figure 1 shows the effect of introducing $\kappa > 0$ on equilibrium trading strategy coefficients. Most importantly, the leader's coefficient α_L decreases at all ρ. In particular, this results in $\alpha_L < \alpha^K$ at $\rho = 0$, and by the identity for δ_L above, this implies that the leader sells on average, with $\mu(\alpha_L + \delta_L) < 0$, even for $\rho = 0$. A consequence of the reduction in α_L is that the market maker's variance γ_1^F about the follower is reduced, and therefore follower's coefficient $\alpha_F = \sigma/\sqrt{\gamma_1^F}$ increases in response as shown in the figure.

The intuition for the drop in α_L comes from a new channel: the leader would like to manipulate the follower's belief about the leader's terminal position to directly influence the follower's effort. This channel operates as long as the follower is not already certain about the leader's position due to perfect correlation in initial positions; in other words, it is active even when the original manipulation effect is shut down by setting $\rho = 0$.

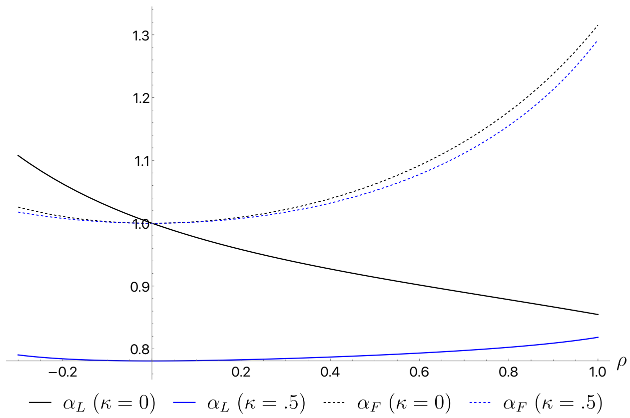

Figure 1: The leader's coefficient α_L and follower's coefficient α_F for $\kappa = 0$ (baseline model) and $\kappa = .5$ while varying ρ. The other parameter values are $\sigma = \phi = 1$.

IV Proofs for Section 5

IV.A Proof of Proposition 6: asymmetric productivity

We first prove the part of the proposition about asymmetric productivity, where one player has productivity parameter $\zeta > 0$ and the other is an unproductive or passive investor who cannot influence firm value through effort. It is immediate that the productive player's optimal effort is ζ times its terminal position. Assume $\rho = \phi$.

As in our baseline model, there is a unique PBS equilibrium for sufficiently small $\sigma > 0$. Let $V_{i,prod}(\sigma)$ denote the expected payoff of the productive trader when their role is $i \in \{L, F\}$ (with the passive investor having the opposite role), and similarly, let $V_{i,pass}(\sigma)$ denote the payoff for the passive investor in role i, with the productive trader having the opposite role. As in the analysis of small σ in the paper, trading disappears as $\sigma \to 0$, and thus, based on the initial positions and optimal effort for the produtive player,

$$\lim_{\sigma\to 0} V_{L,prod}(\sigma) = \lim_{\sigma\to 0} V_{F,prod}(\sigma) = V^0_{prod} := \zeta(\mu^2 + \phi)/2$$
$$\lim_{\sigma\to 0} V_{L,pass}(\sigma) = \lim_{\sigma\to 0} V_{F,pass}(\sigma) = V^0_{pass} := \zeta(\mu^2 + \phi),$$

where $\mu^2 + \phi = \mathbb{E}[X^L_0 X^F_0]$ given the perfect correlation. Since these limiting payoffs are independent of the timing (i.e., assignment of roles), we instead examine rates of convergence by calculating the limits

$$\bar{V}_{i,j} := \lim_{\sigma\to 0} \frac{V_{i,j}(\sigma) - V^0_j}{\sigma}, \qquad i \in \{L, F\}, j \in \{prod, pass\},$$

and we prove the proposition by showing that

$$\bar{V}_{L,prod} > \bar{V}_{F,prod} \qquad \text{and} \qquad \bar{V}_{F,pass} > \bar{V}_{L,pass}.$$

First consider the case where the passive investor trades first, and where the follower has productivity $\zeta > 0$. The players' objectives thus reduce to

$$\begin{aligned}
&\text{Leader:} \qquad \sup_{\theta^L} \mathbb{E}[\zeta X_T^F X_T^L - P_1\theta^L | X_0^L, \theta^L] \\
&\text{Follower:} \qquad \sup_{\theta^F} \mathbb{E}[(\zeta X_T^F) X_T^F - P_2\theta^F - \frac{1}{2\zeta}(\zeta X_T^F)^2 | X_0^F, \mathcal{F}_1, \theta^F].
\end{aligned}$$

It is easy to establish our familiar result that in a PBS equilibrium, the follower trades according to $\theta^F = \alpha_F(X_0^F - M_1^F)$, where $\alpha_F = \frac{\sigma}{\sqrt{\gamma_1^F}}$. It also continues to be the case that $\delta_L = -\sigma^2/(\phi\alpha_L)$. Hence, strategies are characterized by (α_L, α_F).

It is useful to perform a change of variables $r_i := \alpha_i/\sigma$, $i \in \{L, F\}$. Immediately, $r_F = \sqrt{\frac{1+r_L^2\phi}{\phi}}$. The X_0^L-component of the leader's first order condition, pinning down r_L, reduces to

$$\zeta \frac{1 + r_F\sigma - r_L(r_F + r_L + r_F r_L \sigma)\phi}{1 + r_L^2\phi} = 0.$$

Taking $\sigma \to 0$ and solving the resulting system of equations in (r_L, r_F) gives

$$\lim_{\sigma\to 0} (r_L, r_F) = (r_{L,pass}^0, r_{F,prod}^0) := \left(\frac{1}{\sqrt{3\phi}}, \frac{2}{\sqrt{3\phi}}\right).$$

Plugging arbitrary coefficients (r_L, r_F) into the the players' objectives yields the expected payoffs

$$V_{L,pass}(\sigma) = \zeta\left[\mu^2 + \phi + \sigma(1 + r_L\sigma)\sqrt{\frac{\phi}{1 + r_L^2\phi}} + \frac{r_L\sigma\phi}{1 + r_L^2\phi}\right], \tag{IA.53}$$

$$V_{F,prod}(\sigma) = \frac{\zeta}{2}\left[\mu^2 + \frac{\phi(-r_F^4\sigma^2\phi + r_F^2(\sigma^2 + \phi)(1 + r_L^2\phi) + (1 + r_L^2\phi)^2 + 2r_F\sigma(1 + r_L^2\phi))}{(1 + r_L^2\phi)(1 + \phi(r_F^2 + r_L^2))}\right]. \tag{IA.54}$$

To calculate $\bar{V}_{L,pass}$ and $\bar{V}_{F,prod}$, subtract V_{pass}^0 and V_{prod}^0 from (IA.53) and (IA.54), respectively; divide through by σ; and take $\sigma \to 0$ (using $(r_L, r_F) \to (r_{L,pass}^0, r_{F,prod}^0)$). This yields

$$(\bar{V}_{L,pass}, \bar{V}_{F,prod}) = \left(\zeta\frac{3\sqrt{3\phi}}{4}, \zeta\frac{\sqrt{3\phi}}{4}\right).$$

Next, consider the case where the player with productivity $\zeta > 0$ is the leader and the passive investor is the follower. A similar analysis to that above establishes that

$$\lim_{\sigma \to 0} (r_L, r_F) = (r^0_{L,prod}, r^0_{F,pass}) := \left(\frac{1}{\sqrt{\phi}}, \sqrt{\frac{2}{\phi}} \right).$$

Hence, taking the limit as $\sigma \to 0$ of the appropriate expected payoff expressions yields

$$\left(\bar{V}_{L,prod}, \bar{V}_{F,pass} \right) = \left(\zeta \frac{\sqrt{\phi}}{2}, \zeta \frac{(4+\sqrt{2})\phi}{4} \right).$$

By inspection, $\bar{V}_{L,prod} > \bar{V}_{F,prod}$ and $\bar{V}_{F,pass} > \bar{V}_{L,pass}$, concluding the proof.

IV.B Proof of Proposition 6: symmetric productivity

We now prove that if both players have the same productivity parameter ζ, then when noise is sufficiently small, the leader's payoff is higher than the follower's. For sufficiently small $\sigma > 0$, there is a unique PBS equilibrium. Let $V_i(\sigma)$ denote the expected payoff for player $i \in \{L, F\}$ in this equilibrium. We have

$$\lim_{\sigma \to 0} V_L(\sigma) = \lim_{\sigma \to 0} V_F(\sigma) = V^0 := \frac{3}{2} \zeta (\mu^2 + \phi),$$

where the fraction $\frac{3}{2} = 1 + \frac{1}{2}$ is due to a player enjoying the full benefit of the other's effort, but incurring the cost associated with its own effort.

Since the limit is the same for both players, we compare rates of convergence and prove the proposition by showing that

$$\bar{V}_L > \bar{V}_F, \qquad \text{where} \qquad \bar{V}_i := \lim_{\sigma \to 0} \frac{V_i(\sigma) - V^0}{\sigma}.$$

As in the asymmetric case, we use the change of variables $r_i = \alpha_i / \sigma$. As $\sigma \to 0$, $(r_L, r_F) \to (r^0_L, r^0_F)$, where

$$r^0_L = \sqrt{\frac{9 - \sqrt{33}}{6\phi}} \in \left(0, \frac{1}{\sqrt{\phi}} \right), \tag{IA.55}$$

$$r^0_F = \sqrt{\frac{1 + (r^0_L)^2 \phi}{\phi}}. \tag{IA.56}$$

It is straightforward to calculate

$$\bar{V}_L = \zeta \frac{(r_F^0 + 2r_L^0)\phi}{1 + (r_L^0)^2\phi},$$

$$\bar{V}_F = \zeta \left[-\mu^2 \frac{1-(r_L^0)^2\phi}{r_L\phi} + r_L\phi + \frac{2r_F^0}{1+((r_F^0)^2+(r_L^0)^2)\phi} \right].$$

Using (IA.56) and simplifying,

$$\bar{V}_L - \bar{V}_F = \zeta(1-(r_L^0)^2\phi) \left[\frac{\mu^2}{r_L^0\phi} + \frac{r_L^0\phi}{1+(r_L^0)^2\phi} \right] > 0$$

where $1-(r_L^0)^2\phi > 0$ due to the upper bound in (IA.55).

IV.C N followers (Proof of Proposition 7)

Fix μ, σ, ϕ, ρ. Let μs_μ denote the prior mean for each follower, ϕs_ϕ the variance, and ρs_ρ the covariance between the leader and each follower, where s_μ, s_ϕ, s_ρ will vary with N. The setup described in Section 5.2 is captured by $s_\mu = 1/N$, $s_\phi = 1/N^2$, and $s_\rho = 1/N$.

Define $\gamma_1^{\text{sum}} = N^2\gamma_1^F$, the market maker's posterior variance of the sum of all followers' positions. In any PBS equilibrium, the followers play gap strategies and their FOC yields $\alpha_F = \sqrt{\frac{N\sigma^2}{\gamma_1^{\text{sum}}}} = \sqrt{\frac{\sigma^2}{N\gamma_1^F}}$. Incorporating this into the leader's FOC then yields the following equation generalizing (A.14):

$$\frac{(N\rho s_\rho + \phi + \alpha_L\phi)(\sigma^2 - \alpha_L^2\phi)}{N\rho s_\rho[\alpha_L(1+\alpha_L)\phi - \sigma^2]} = \sqrt{\frac{\sigma^4 + \sigma^2\alpha_L^2\phi}{N[\phi s_\phi\sigma^2 + \alpha_L^2(\phi^2 s_\phi - (\rho s_\rho)^2)]}}. \tag{IA.57}$$

Arguments similar to those earlier show that for $\rho > 0$, (IA.57) has a solution α_L in $(\hat{\alpha}, \alpha^K)$, there is no other solution for $\alpha_L \geq 0$, and SOCs are satisfied. The FOC also implies that the coefficient on μ is $\delta_L = -\frac{\sigma^2}{\phi\alpha_L}$. Hence, we have characterized the unique PBS equilibrium.

We now turn to comparative statics wrt N. After plugging in our values for (s_μ, s_ϕ, s_ρ), (IA.57) reduces to

$$\frac{(\rho + \phi + \alpha_L\phi)(\sigma^2 - \alpha_L^2\phi)}{\rho[\alpha_L(1+\alpha_L)\phi - \sigma^2]} = \sqrt{\frac{N(\sigma^4 + \sigma^2\alpha_L^2\phi)}{\phi\sigma^2 + \alpha_L^2(\phi^2 - \rho^2)}}. \tag{IA.58}$$

When these intersect at $\alpha_L \in (\hat{\alpha}, \alpha^K)$, the left hand side crosses the right hand side from above. Then since the right hand side is increasing in N, the equilibrium value of α_L is decreasing in N. It is also straightforward to show that the left side of (IA.58) is decreasing in α_L on $(\hat{\alpha}, \infty)$, so each side of (IA.58) is increasing in N. Since the right hand side is precisely α_F, this establishes that α_F is increasing in N. Note that while the decay in α_L raises γ_1^F in $\alpha_F = \sqrt{\frac{\sigma^2}{N\gamma_1^F}}$ all else equal, this effect does not overturn the direct downward effect that larger N has on γ_1^F, as $\gamma_1^F \leq \phi/N^2$ for any linear strategy of the leader.

Since the followers play gap strategies, ex ante firm value is still $(2 + \alpha_L + \delta_L)\mu = (2+\alpha_L - \sigma^2/(\phi\alpha_L))\mu$ for all N. Since α_L is decreasing in N, ex ante firm value is decreasing in N.

For later use, we show that $\lim_{N\to\infty} \alpha_L = \hat{\alpha} > 0$, where $\hat{\alpha}$ was defined earlier as the positive root of $\alpha_L(1+\alpha_L)\phi - \sigma^2$. As $N \to \infty$, the right hand side of (IA.58) explodes as the rest of the expression in the square root is bounded. Thus, the left hand side must also explode, which requires its denominator to vanish. Given that $\alpha_L > 0$, this implies that α_L converges to $\hat{\alpha}$.

We now turn to the asymptotic result. The leader's expected payoff is

$$\mathbb{E}\left[-P_1\theta^L + \frac{(X_0^L + \theta^L)^2}{2} + (X_0^L + \theta^L)N(X_0^F + \alpha_F(X_0^F - M_1^F))\right]. \tag{IA.59}$$

We simplify (IA.59) one term at a time. The first term equals

$$\begin{aligned}
&- \mathbb{E}[(P_0 + \Lambda_1[\Psi_1 - (\alpha_L + \delta_L)\mu])\theta^L] \\
&= -\mathbb{E}[P_0(\alpha_L X_0^L + \delta_L\mu) + \Lambda_1\alpha_L(X_0^L - \mu)(\alpha_L X_0^L + \delta_L\mu)] \\
&= -[(2 + \alpha_L + \delta_L)(\alpha_L + \delta_L)\mu^2 + \Lambda_1\alpha_L^2\phi] =: S_1.
\end{aligned} \tag{IA.60}$$

Since α_L and δ_L have finite limits as $N \to \infty$, and $\Lambda_1 = \frac{\alpha_L(\rho+\phi(1+\alpha_L))}{\sigma^2+\alpha_L^2\phi}$ also has a finite limit, this term overall is therefore uniformly bounded in N.

The expectation of the second term in (IA.59) equals

$$S_2 := \frac{1}{2}\mathbb{E}\left[(X_0^L(1+\alpha_L) + \delta_L\mu)^2\right] = \frac{1}{2}[(1 + \alpha_L + \delta_L)^2\mu^2 + \phi(1+\alpha_L)^2], \tag{IA.61}$$

which is also uniformly bounded in N.

Using that $\mathbb{E}[X_0^F - M_1^F] = 0$ by the law of iterated expectations, the third term in (IA.59) simplifies as:

$$\begin{aligned}
&\mathbb{E}[(X_0^L(1+\alpha_L) + \delta_L\mu)N(X_0^F + \alpha_F(X_0^F - M_1^F))] \\
&= (1+\alpha_L)(1+\alpha_F)N\mathbb{E}[X_0^L X_0^F] + \delta_L N\mu^2 s_\mu - \mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F M_1^F] \\
&= (1+\alpha_L)(1+\alpha_F)N(\mu^2 s_\mu + \rho s_\rho) + \delta_L N\mu^2 s_\mu - \mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F M_1^F]
\end{aligned}$$

$$
\begin{aligned}
&= (1+\alpha_L)(1+\alpha_F)N(\mu^2 s_\mu + \rho s_\rho) + \delta_L N \mu^2 s_\mu \\
&\quad - \mathbb{E}[X_0^L(1+\alpha_L)N\alpha_F \left\{ \mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2\phi + \sigma^2}[\alpha_L X_0^L + \delta_L \mu - (\alpha_L+\delta_L)\mu] \right\}.
\end{aligned} \tag{IA.62}
$$

We now simplify the last term in (IA.62):

$$
\begin{aligned}
&\mathbb{E}\left[X_0^L(1+\alpha_L)N\alpha_F \left\{ \mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2\phi + \sigma^2}[\alpha_L X_0^L + \delta_L \mu - (\alpha_L+\delta_L)\mu] \right\} \right] \\
&= \mathbb{E}\left[X_0^L(1+\alpha_L)N\alpha_F \left\{ \mu s_\mu + \frac{\alpha_L \rho s_\rho}{\alpha_L^2\phi + \sigma^2}\alpha_L(X_0^L - \mu) \right\} \right] \\
&= (1+\alpha_L)N\alpha_F \mu^2 s_\mu + (1+\alpha_L)N\alpha_F \frac{\alpha_L \rho s_\rho}{\alpha_L^2\phi + \sigma^2}\alpha_L \mathbb{E}[X_0^L(X_0^L-\mu)] \\
&= (1+\alpha_L)N\alpha_F \mu^2 s_\mu + (1+\alpha_L)N\alpha_F \frac{\alpha_L \rho s_\rho}{\alpha_L^2\phi + \sigma^2}\alpha_L \mathrm{Var}(X_0^L) \\
&= (1+\alpha_L)\alpha_F \mu^2 + (1+\alpha_L)\alpha_F \frac{\alpha_L \rho}{\alpha_L^2\phi + \sigma^2}\alpha_L \phi.
\end{aligned}
$$

Incorporating this in (IA.62), the third term of (IA.59) equals

$$
\begin{aligned}
S_3 &:= (1+\alpha_L)(1+\alpha_F)(\mu^2+\rho) + \delta_L\mu^2 - \left[(1+\alpha_L)\alpha_F\mu^2 + (1+\alpha_L)\alpha_F \frac{\alpha_L^2\rho\phi}{\alpha_L^2\phi+\sigma^2} \right] \\
&= (1+\alpha_L)(\mu^2+\rho) + \delta_L\mu^2 + \alpha_F\rho(1+\alpha_L)\frac{\sigma^2}{\alpha_L^2\phi+\sigma^2},
\end{aligned} \tag{IA.63}
$$

where we have canceled N with $1/N$ in s_μ and s_ρ.

The leader's payoff is the sum of (IA.60), (IA.61), and (IA.63): $\Pi_L = S_1 + S_2 + S_3$. To show that the rate of growth is $\sqrt{N}$, we calculate

$$
\begin{aligned}
\lim_{N\to\infty} \frac{\Pi_L}{\sqrt{N}} &= \lim_{N\to\infty} \frac{S_1}{\sqrt{N}} + \lim_{N\to\infty} \frac{S_2}{\sqrt{N}} + \lim_{N\to\infty} \frac{S_3}{\sqrt{N}} \\
&= 0 + 0 + \lim_{N\to\infty} \frac{S_3}{\sqrt{N}} \\
&= \left(\lim_{N\to\infty} \frac{\alpha_F}{\sqrt{N}} \right) \left(\lim_{N\to\infty} (1+\alpha_L)\rho \frac{\sigma^2}{\alpha_L^2\phi+\sigma^2} \right),
\end{aligned}
$$

where we have used that in S_3, $(1+\alpha_L)(\mu^2+\rho)+\delta_L\mu^2$ is uniformly bounded in N. To take limits in the last line, we use the fact that for $\rho \in (0,\phi]$, $\lim_{N\to\infty}\alpha_L = \hat{\alpha} > 0$, as shown earlier in the proof. The two factors in the product then have limits

$$
\lim_{N\to\infty} \frac{\alpha_F}{\sqrt{N}} = \lim_{N\to\infty} \sqrt{\frac{(\sigma^4 + \sigma^2\alpha_L^2\phi)}{\phi\sigma^2 + \alpha_L^2(\phi^2-\rho^2)}} = \sqrt{\frac{(\sigma^4+\sigma^2\hat{\alpha}^2\phi)}{\phi\sigma^2+\hat{\alpha}^2(\phi^2-\rho^2)}},
$$

$$\lim_{N\to\infty}(1+\alpha_L)\rho\frac{\sigma^2}{\alpha_L^2\phi+\sigma^2} = (1+\hat{\alpha})\rho\frac{\sigma^2}{\hat{\alpha}^2\phi+\sigma^2}.$$

Since these limits are positive and finite, so is their product, and we conclude that Π_L grows asymptotically at rate $\sqrt{N}$.

The following lemma formalizes the last statement of the proposition.

Lemma IA.1. *Assume $\rho = \phi$, and let Π_L^{seq} and Π_L^{sim} denote the leader's payoff in the sequential- and simultaneous-move games, respectively. When N is sufficiently large, the leader's payoff advantage from going first is increasing in N. Specifically, Π_L^{seq} and Π_L^{sim} grow at rate $\sqrt{N}$ asymptotically, and* $\lim_{N\to\infty}\frac{\Pi_L^{seq}-\Pi_L^{sim}}{\sqrt{N}} > 0$.

Proof. Proposition 7 characterizes the asymptotics of Π_L^{seq}, so consider the simultaneous-move game. The FOCs lead to the following system of equations: $\alpha_L = \frac{1-\frac{\rho}{\phi}\Lambda\alpha_F+\frac{\rho}{\phi}(1+\alpha_F)}{2\Lambda-1}$, $\alpha_F = \frac{N(1-\frac{\rho}{\phi}\Lambda\alpha_L+\frac{\rho}{\phi}(1+\alpha_L))}{(N+1)\Lambda-N}$, where $\Lambda = \frac{(1+\alpha_L)(\phi\alpha_L+\rho\alpha_F)+(1+\alpha_F)(\phi\alpha_F+\rho\alpha_L)}{\phi(\alpha_L^2+\alpha_F^2)+2\alpha_L\alpha_F\rho+\sigma^2}$.

For the case $\rho = \phi$, we obtain $(\alpha_L,\alpha_F) = \left(\frac{\sigma}{\sqrt{(N+1)\phi}}, \frac{N\sigma}{\sqrt{(N+1)\phi}}\right)$. The leader's payoff is again of the order $\sqrt{N}$, with coefficient $\lim_{N\to\infty}\frac{\alpha_F}{\sqrt{N}}(1+\alpha_L)\mathrm{Cov}(X_0^L, X_0^F) = \lim_{N\to\infty}\frac{\alpha_F}{\sqrt{N}}(1+\alpha_L)\phi = \sigma\sqrt{\phi}$. To complete the proof, we show that this is strictly less than the corresponding coefficient in the sequential-move game, namely $\sqrt{\frac{(\sigma^4+\sigma^2\hat{\alpha}^2\phi)}{\phi\sigma^2}}(1+\hat{\alpha})\phi\frac{\sigma^2}{\hat{\alpha}^2\phi+\sigma^2}$. By routine simplifications,

$$\begin{aligned}
\sigma\sqrt{\phi} &\le \sqrt{\frac{(\sigma^4+\sigma^2\hat{\alpha}^2\phi)}{\phi\sigma^2}}(1+\hat{\alpha})\phi\frac{\sigma^2}{\hat{\alpha}^2\phi+\sigma^2}\\
\iff 1 &\le \sqrt{\sigma^2+\hat{\alpha}^2\phi}(1+\hat{\alpha})\frac{\sigma}{\hat{\alpha}^2\phi+\sigma^2}\\
\iff \sqrt{\sigma^2+\hat{\alpha}^2\phi} &\le (1+\hat{\alpha})\sigma\\
\iff \sigma^2+\hat{\alpha}^2\phi &\le (1+\hat{\alpha})^2\sigma^2 \qquad \text{(since both sides are positive)}\\
\iff 0 &\le \hat{\alpha}[\hat{\alpha}(\sigma^2-\phi)+2\sigma^2].
\end{aligned}$$

Since $\hat{\alpha}$ solves $\sigma^2 - \hat{\alpha}(1+\hat{\alpha})\phi = 0$, the right hand side is

$$\hat{\alpha}[\hat{\alpha}(\sigma^2-\phi)+2\sigma^2] = \hat{\alpha}[\hat{\alpha}\sigma^2+\hat{\alpha}^2\phi-\sigma^2+2\sigma^2] = \hat{\alpha}[\hat{\alpha}\sigma^2+\hat{\alpha}^2\phi+\sigma^2] \ge 0,$$

establishing the inequality. $\square$

V Results and proofs for Section 6

This section analyzes non-PBS linear equilibria of the baseline model, as described in Section 6, and contains a proof of Proposition 8.

V.A Non-PBS linear equilibria

The results in the following proposition were referred to in Section 6.

Proposition IA.1. *(i) Positive correlation: If $\rho > 0$, then for sufficiently large $\sigma > 0$, there exists a linear equilibrium with $\alpha_L, \alpha_F < -\sqrt{\sigma^2/\phi} < 0$.*

(ii) Perfect negative correlation: If $\rho = -\phi$, there is no linear equilibrium in which α_L and α_F have the same sign. A linear equilibrium in which $\alpha_L < 0 < \alpha_F$ exists for all $\sigma > 0$.

Proof. For part (i), we prove that for sufficiently large σ, there is a solution to (A.15) with $\alpha_L < 0$. We then check the conditions (A.12), (A.13), and $\phi(1+\alpha_L)+\rho \neq 0$ and apply the "converse" part of Proposition A.1.

Recall from Proposition A.1 that (A.15) is the equation for α_L associated with $\alpha_F = -\sqrt{\sigma^2/\gamma_1^F}$, the negative root of (A.7). Since $\gamma_1^F < \phi$, this immediately implies $\alpha_F < -\sqrt{\sigma^2/\phi}$. After a change of variables $x = \alpha_L/\sigma$ in (A.15),

$$-\sqrt{\frac{1+x^2\phi}{\phi + x^2(\phi^2-\rho^2)}} = \frac{\left(\frac{\rho+\phi}{\sigma} + \phi x\right)(x^2\phi - 1)}{\rho[1 - x\phi/\sigma - x^2\phi]}. \tag{IA.64}$$

When $x = -1/\sqrt{\phi}$, the right hand side vanishes, while the left hand side is strictly negative. Now choose σ sufficiently large that $\left(\frac{\rho+\phi}{\sigma} + \phi x\right) < 0$ for all $x \leq -1/\sqrt{\phi}$. Define $\alpha^\dagger$ to be the negative root of $\alpha_L(1+\alpha_L)\phi - \sigma^2$, and define $x^\dagger = \alpha^\dagger/\sigma < -1/\sqrt{\phi}$ to be the unique negative root of the denominator of (IA.64), where $x^\dagger \uparrow -1/\sqrt{\phi}$ as $\sigma \uparrow \infty$. The right hand side of (IA.64) is well-defined and continuous on $(x^\dagger, -1/\sqrt{\phi}]$ and moreover, it has limit $-\infty$ as $x \downarrow x^\dagger$. Thus, by the intermediate value theorem, there exists a solution x_L to (IA.64) in $(x^\dagger, -1/\sqrt{\phi})$. By reversing the change of variables, we recover $\alpha_L = \sigma x_L < -\sqrt{\sigma^2/\phi}$ solving the leader's FOC. Moreover, by the squeeze theorem, $\lim_{\sigma\uparrow\infty} x_L = -1/\sqrt{\phi}$. Note that as $\sigma \uparrow \infty$, $x_F := \alpha_F/\sigma = -\sqrt{\frac{1+x^2\phi}{\phi+x^2(\phi^2-\rho^2)}} \to -\sqrt{\frac{2}{2\phi-\rho^2/\phi}} =: x_F^\infty$

To verify (A.12), note that this is equivalent to the condition $1 - x_L^2\phi - 2x_L\left(\frac{\rho+\phi}{\sigma} + \rho x_F\right) \leq 0$. As $\sigma \uparrow +\infty$, the left hand side has limit $1 - 1 - 2(-1/\sqrt{\phi})\rho x_F^\infty = 2\rho x_F^\infty/\sqrt{\phi} < 0$, so (A.12) is satisfied for sufficiently large σ.

As for (A.13), using that $\alpha_{F,2} < 0$, it suffices to show that

$$\sigma^2[x_L^2(\phi^2 - \rho^2) + x_L\sigma\rho + (\phi + \rho)] \leq 0.$$

Recall that x_L has finite limit as $\sigma \to +\infty$, so the dominating term is $\sigma^3 x_L \rho < 0$. We conclude that (A.13) is satisfied for sufficiently large σ.

Finally, observe that since the left side of (IA.64) is nonzero, at our solution the right side is also nonzero, and thus $\frac{\rho+\phi}{\sigma} + \phi x_L = \frac{1}{\sigma}[\phi(1+\alpha_L) + \rho] \neq 0$. Hence Proposition A.1 applies, giving us existence for large σ.

For part (ii), we begin with the observation that for $\rho = -\phi$, (A.13) becomes

$$\sigma^2 \phi \alpha_F \alpha_L \leq 0. \tag{IA.65}$$

Hence, there is no equilibrium in which α_F and α_L are both strictly positive or both strictly negative, and (14)-(15) imply $\alpha_L \neq 0$ and $\alpha_F \neq 0$.

We now establish the existence of an equilibrium with $\alpha_L < 0 < \alpha_F$. Note that for $\rho = -\phi$, as long as $\alpha_L \neq 0$ (which must hold in any equilibrium), the condition $\phi(1+\alpha_L) + \rho \neq 0$ is satisfied. When $\rho = -\phi$ and $\alpha_F = \alpha_{F,1}$, (A.14) simplifies to

$$\sqrt{\sigma^2/\phi + \alpha_L^2} = \alpha_L \frac{\alpha_L^2 \phi - \sigma^2}{\alpha_L(1+\alpha_L)\phi - \sigma^2}. \tag{IA.66}$$

In particular, an equilibrium with $\alpha_F = \alpha_{F,1}$ exists if and only if there exists α_L satisfying (IA.66) such that both SOCs are satisfied. Now the left hand side of (IA.66) is positive, while the right hand side vanishes at $\alpha_L = -\sigma/\sqrt{\phi}$, has limit $+\infty$ as $\alpha_L \downarrow \alpha^\dagger$, and is continuous on $(\alpha^\dagger, -\sigma/\sqrt{\phi})$, where $\alpha^\dagger$ was previously defined as the negative root of $\alpha_L(1+\alpha_L)\phi - \sigma^2$, and recall that $\hat{\alpha}$ is the positive root. Thus, (IA.66) has a solution in this interval. We finally check (A.12), which is now $\sigma^2 - \alpha_L^2\phi + 2\alpha_L\phi\alpha_F \leq 0$. This is satisfied since $\alpha_L < -\sigma/\sqrt{\phi}$ implies $\sigma^2 - \alpha_L^2\phi < 0$, and clearly $2\alpha_L\phi\alpha_F < 0$. Since α_F and α_L have opposite signs, (A.13) is satisfied. Hence, existence follows from Proposition A.1. $\square$

V.B Existence and uniqueness for small σ (Proof of Proposition 8)

Since Proposition A.2 establishes existence and uniqueness for all $\sigma > 0$ when $\rho = 0$, assume $\rho \neq 0$. We will show that for sufficiently small $\sigma > 0$, there is a unique pair (α_L, α_F) satisfying (A.7), (A.23), (A.12), and (A.13). Further, we will show that $\phi(1+\alpha_L) + \rho \neq 0$, so existence follows from Proposition A.1.

In any equilibrium, (α_L, α_F) must solve (A.23). By squaring both sides of this equation, using (A.7), and multiplying through by the nonzero denominator, we get (A.26). Now as $\sigma \to 0$, the coefficients of the polynomial Q converge to those of

$$Q^{\sigma=0}(\alpha_L) := -\alpha_L^6 \phi^2[\rho + \phi + \alpha_L \phi]^2(\phi^2 - \rho^2), \tag{IA.67}$$

which has a root of multiplicity 6 at 0 and of multiplicity 2 at $-\frac{\rho+\phi}{\phi}$.

By Lemma A.2, for any $\epsilon > 0$, there exists $\delta > 0$ such that if $\sigma \in (0, \delta)$, Q has 6 complex roots within distance ϵ of 0 and 2 complex roots within ϵ of $-\frac{\rho+\phi}{\phi}$. For ϵ sufficiently small that these neighborhoods do not intersect, and δ chosen accordingly, let $\alpha_1, \ldots, \alpha_6$ denote the 6 roots near 0, and let α_7 and α_8 denote the roots near $-\frac{\rho+\phi}{\phi}$. We maintain these assumptions on ϵ and δ throughout the proof.

The following lemma rules out α_7 and α_8 from being part of an equilibrium.

Lemma IA.2. *For sufficiently small $\sigma > 0$, each of α_7 and α_8 is either complex or otherwise fails (A.12).*

Proof. The left side of (A.12) is continuous in (σ, α_L) at $\left(0, -\frac{\rho+\phi}{\phi}\right)$, where it evaluates to $(\phi + \rho)^2/\phi > 0$. Hence, choosing $\epsilon > 0$ sufficiently small, and $\delta > 0$ sufficiently small as described before the lemma, if either α_7 or α_8 is real, it fails (A.12). □

Remark 2. *Having ruled out α_7 and α_8, note that if σ is sufficiently small, then for any real $\alpha_L \in \{\alpha_1, \ldots, \alpha_6\}$, $\rho + \phi + \alpha_L\phi \neq 0$. This fact is useful two fold: (i) this criterion appears in the sufficiency part of Proposition A.1, and (ii) due to (A.23), using that $\rho \neq 0$ and $\alpha_{F,1} \neq 0$ and $\alpha_{F,2} \neq 0$ for α_L real, we have $\sigma^2 - \alpha_L(1 + \alpha_L) \neq 0$ for sufficiently small σ for α_L real. Thus, any real solution to (A.26) solves (A.25).*

We can now rule out equilibria in which $\alpha_F = \alpha_{F,2}$, as these fail the follower's second order condition when σ is sufficiently small. To do so, we use asymptotic properties of the roots of (A.26) as $\sigma \to 0$.

It is useful to define a change of variables $z = \alpha_L/\sigma$ in (A.26) and divide through the resulting equation by σ^6, obtaining an equivalent equation

$$0 = \tilde{Q}(z, \sigma) := \sigma H(z) + F(z), \tag{IA.68}$$

where $H(z)$ is a polynomial of degree 8 and where $F(z)$ is a polynomial independent of σ that has the form $c_6 z^6 + c_4 z^4 + c_2 z^2 + c_0$.[3] For each $i \in \{1, 2, \ldots, 6\}$, define $z_i = \alpha_i/6$.

[3] In particular, $F(z) = -z^6(\phi - \rho)\phi^2(\phi + \rho)^3 + z^4\phi[-2\rho^4 - 4\rho^3\phi + 2\rho\phi^3 + \phi^4] + z^2(\rho^2 + \rho\phi + \phi^2)^2 - \phi(\rho + \phi)^2$.

Lemma IA.3. *F has 6 distinct roots, denoted $\hat{z}_1, \ldots, \hat{z}_6$, of which exactly two are positive, two are negative, and two are complex. As $\sigma \to 0$, $z_1, \ldots, z_6$ converge to $\hat{z}_1, \ldots, \hat{z}_6$.*

Proof. We first characterize the roots of F. Consider the cubic polynomial $G(y) = c_6 y^3 + c_4 y^2 + c_2 y + c_0$, where $F(y) = G(y^2)$. We have $G(0) < 0$ and $\lim_{y \to -\infty} G(y) = +\infty$, so G has a negative root. Also, we have $\lim_{y \to +\infty} G(y) = -\infty$ and $G(1/\phi) = 2\rho^2 \phi > 0$, so G has two distinct positive roots: one in $(0, 1/\phi)$ and one in $(1/\phi, +\infty)$. Since G is cubic, there are no other roots (real or complex). Now the negative root of G corresponds to two distinct complex roots of F, and the positive roots of G each correspond to both one positive and one negative root of F, all distinct.

We now turn to the convergence claim in the lemma. Next, set $K = 1 + \max_{i \in \{1,\ldots,6\}} |\hat{z}_i|$, and define a compact set $\mathcal{K} = \{z \in \mathbb{C} : |z| \leq K\}$. By definition, all roots of F lie in $\mathcal{K}$. Further, note that on $\mathcal{K}$, for any sequence $(\sigma_n)_{n \in \mathbb{N}}$ with $\sigma_n \downarrow 0$, the sequence $(\tilde{Q}(\cdot, \sigma_n))_{n \in \mathbb{N}}$ of functions defined on $\mathcal{K}$ is equicontinuous and converges pointwise to F since $\sigma H(z)$ vanishes; thus, by the Arzela-Ascoli theorem, the sequence converges uniformly to F on $\mathcal{K}$.

Choose $\overline{\eta} > 0$ less than 1 and less than the minimum distance between any $\hat{z}_i$ and $\hat{z}_j$, where $i, j \in \{1, \ldots, 6\}$ and $i \neq j$. Then for all $\eta \in (0, \overline{\eta})$, for each $i \in \{1, \ldots, 6\}$, 0 is the unique value of $t \in (1 - \eta, 1 + \eta)$ such that $0 = F(t\hat{z}_i)$. Further, $F(t\hat{z}_i)$ takes opposite signs at $t = 1 + \eta$ and $t = 1 - \eta$. By uniform convergence, for each such η, it holds that for all sufficiently small $\sigma > 0$, and for all $i \in \{1, \ldots, 6\}$, $\tilde{Q}((1 + \eta)\hat{z}_i, \sigma)$ and $\tilde{Q}((1 - \eta)\hat{z}_i, \sigma)$ have the same signs as $F((1 + \eta)\hat{z}_i)$ and $F((1 - \eta)\hat{z}_i)$, respectively; thus, for all sufficiently small $\sigma > 0$, there exists $t_i(\sigma)$ in $(1 - \eta, 1 + \eta)$ such that $\tilde{Q}(t_i(\sigma)\hat{z}_i, \sigma) = 0$, and therefore, $\{z_1, \ldots, z_6\} = \{t_1(\sigma), \ldots, t_6(\sigma)\}$. Relabelling so that $z_i = t_i(\sigma)$, we have $z_i \to \hat{z}_i$ for each $i \in \{1, \ldots, 6\}$. □

We now analyze the follower's SOC.

Lemma IA.4. *If $\sigma > 0$ is sufficiently small, then (i) there is no equilibrium in which $\alpha_F = \alpha_{F,2}$, and (ii) for $\alpha_F = \alpha_{F,1}$, (A.13) is satisfied for all real roots of Q among $a_1, \ldots, a_6$.*

Proof. Having ruled out equilibria in which $\alpha_L \in \{\alpha_7, \alpha_8\}$ (when $\sigma > 0$ is small), we show that for $\alpha_F = \alpha_{F,2}$ and for sufficiently small $\sigma > 0$, (A.13) fails for all real roots among $\alpha_1, \ldots, \alpha_6$. By Lemma IA.3, each α_i/σ, $i \in \{1, \ldots, 6\}$, converges to a finite nonzero limit $\hat{z}_i$. Hence, for sufficiently small $\sigma > 0$, if $\alpha_L = \alpha_i$, for some $i \in \{1, \ldots, 6\}$ is real, the factor in square brackets in (A.13) is bounded below by

$$\alpha_i^2(\phi^2 - \rho^2) + \sigma^2(\phi + \rho) - |\alpha_i \rho| \sigma^2 \geq \alpha_i^2(\phi^2 - \rho^2) + \sigma^2(\phi + \rho) - |\rho z_i| \sigma^3$$

$$= \sigma^2(z_i^2(\phi^2 - \rho^2) + \phi + \rho - |\rho z_i|\sigma),$$

where $z_i^2(\phi^2 - \rho^2) + \phi + \rho - |\rho z_i|\sigma \to \hat{z}_i(\rho^2 - \rho^2) + \phi + \rho > 0$. Since $-\alpha_{F,2} > 0$, this implies that (A.13) fails.

For $\alpha_F = \alpha_{F,1}$, the same bound above holds, but since $-\alpha_{F,1} < 0$, (A.13) is satisfied. $\square$

From the proof of Proposition A.4, any equilibrium value of α_L must solve (A.14) (with $\alpha_F = \alpha_{F,1}$) or (A.15)(with $\alpha_F = \alpha_{F,2}$). By Lemma IA.4 part (i), α_L must solve (A.14).

We now turn to the leader's SOC.

Lemma IA.5. *If $\sigma > 0$ is sufficiently small, then (i) there is no equilibrium in which $\alpha_L \leq 0$, and (ii) if $\alpha_L > 0$ is a real root of (A.26) and $\alpha_F = \alpha_{F,1}$, then (A.12) is satisfied.*

Proof. For part (i), we only need to consider the roots $\alpha_1, \ldots, \alpha_6$, since for sufficiently small σ α_7 and α_8 cannot be part of an equilibrium by Lemma IA.2. By Lemma IA.4, we further only need to consider $\alpha_F = \alpha_{F,1}$, for which (A.12) becomes

$$\sigma^2 - \alpha_L^2\phi - 2\alpha_L\left(\rho + \phi + \rho\sigma\sqrt{\frac{\sigma^2 + (\alpha_L/\sigma)^2\sigma^2\phi}{\phi + (\alpha_L/\sigma)^2(-(\rho)^2 + (\phi)^2)}}\right) \leq 0. \tag{IA.69}$$

Clearly, this is violated if $\alpha_L = 0$. And since $\alpha_L \to 0$ in proportion to σ by Lemma IA.3, for small σ, the dominating term is $-2\alpha_L(\rho + \phi)$, which is positive (violating (IA.69)) if $\alpha_L < 0$.

For part (ii), we again only need to consider the roots $\alpha_1, \ldots, \alpha_6$, since for sufficiently small σ, α_7 and α_8 are not positive real numbers as they converge to $-\frac{\rho+\phi}{\phi}$. Following the same calculation above, for sufficiently small σ, the left hand side of (A.12) has the same sign as $-2\alpha_L(\rho + \phi)$, which is negative for $\alpha_L > 0$, satisfying (A.12). $\square$

In light of Lemma IA.5, we use Lemma IA.3 to show that for sufficiently small $\sigma > 0$, there is exactly one positive solution to (A.14), and thus one equilibrium candidate. We establish this in the following lemma:

Lemma IA.6. *For sufficiently small $\sigma > 0$, equation (A.26) has exactly two positive roots, one solving (A.14) and the other solving (A.15).*

Proof. Any (positive) solution to (A.14) or (A.15) must be a (positive) root of (A.26). From the proof of Proposition A.4, (A.26) has *at least* two positive roots, one for each equation (A.14) and (A.15), so it suffices to show that these are the only two positive roots of (A.26). Using the change of variables $z = \alpha_L/\sigma$, $\tilde{Q}(\cdot, \sigma)$ has at least two positive real roots for all sufficiently small σ. But $\tilde{Q}(\cdot, \sigma)$ cannot have more than two positive roots for all sufficiently small σ. To see this, recall that for small σ, α_7 and α_8 are complex or negative, so any positive roots must be among $\alpha_1, \ldots, \alpha_6$. And if there were more than two such positive roots, then by Lemma IA.3, F would have more than two nonnegative roots, a contradiction. Mapping back to $\alpha_L = z\sigma$, this implies that (A.26) has exactly two roots for sufficiently small σ, (A.14) and (A.15) each have exactly one. □

From Lemmas IA.4, IA.5, and IA.6, for sufficiently small $\sigma > 0$, there is exactly one pair (α_L, α_F) solving (A.7), (A.10), (A.13), and (A.12), and thus at most one equilibrium. By Remark 2, we can invoke the "converse" part of Proposition A.1, establishing existence.

VI Endogenizing initial positions

In this section, we analyze an extension of the model with pre-game trading and show numerically that it can endogenize perfect positive correlation and imperfect negative correlation, as mentioned in Section 7.

VI.A Setup

There are two identical traders $i = 1, 2$ who start with no ownership of the firm's stock and simultaneously place orders θ^i. A market maker (MM) observes total order flow

$$\Psi_0 = \theta_1 + \theta_2 + \sigma Z_0,$$

where $\sigma > 0$ is a known constant (the same as in the leader-follower game) and $Z_0 \sim N(0, 1)$, and executes at a price P_{pre}. Suppose the firm's value has an exogenous additive component $v \sim N(0, \sigma_v^2)$, and the each agent observes a noisy signal

$$s_i = v + \epsilon_i$$

where ϵ_1, ϵ_2 are jointly normal with mean 0 and the following covariance matrix: $\begin{pmatrix} \sigma_\epsilon^2 & \rho_\epsilon \sigma_\epsilon^2 \\ \rho_\epsilon \sigma_\epsilon^2 & \sigma_\epsilon^2 \end{pmatrix}$.

After this round of trading, we assume that v is publicly revealed.[4] Then, with probability q, it is publicly revealed that there are activism opportunities at the target firm, meaning that our leader-follower game is played, and firm value (per share) is the sum of v and the players' efforts. In Sections VI.B and VI.C, we assume that the roles of leader and follower

[4]We consider the perfect revelation of the exogenous component not because we cannot carry two forms of private information (block sizes and fundamental value), but because it simplifies the task of generating positive/negative correlation while fitting 100% in our baseline model. Even if we consider two-dimensional private information, activists using linear strategies for the both pieces of private information; hence this does not effect our point of generating the initial correlation structure.

are assigned to the players with equal probabilities; in Section VI.D, we allow roles to be assigned with asymmetric exogenous probabilities. Finally, with complementary probability $1 - q$ the game ends (the leader-follower sub-game does not arise).

In Sections VI.B and VI.C, we focus on *symmetric linear equilibria*, where

(i) Traders trade in the pregame according to symmetric strategies αs_i;

(ii) The MM uses a linear pricing rule $P_{pre} = \phi + \Lambda_0 \Psi_0$ in the pregame;[5]

(iii) Traders follow optimal (role-specific) strategies in the leader-follower game, and the MM uses a linear pricing rule.

In Section VI.D, we look for asymmetric linear equilibria where , with respect to (i) the pregame strategy coefficients are player-specific; and with respect to (iii) the strategies and pricing rule in the leader-follower game further depend on the players' identities $i \in \{1, 2\}$.

Overview of results We show that this framework can produce both positive and negative correlation by adjusting the correlation in the noise of the players' pregame signals.

- Positive correlation (Section VI.B): We specialize to the case $\rho_\epsilon = 1$ and numerically establish the existence of an equilibrium. From the perspective of the market maker, players' initial positions entering the leader-follower game have perfect positive correlation. The restriction to $\rho_\epsilon = 1$ is purely to simplify the expressions involved; by continuity the result extends to ρ_ϵ in a neighborhood of 1.

- Negative correlation (Section VI.C): We specialize to the case $\rho_\epsilon = 0$, i.e. players receive conditionally i.i.d. signals of v, and establish the existence of an equilibrium. From the perspective of the MM, conditional on the pregame order flow and v, players' positions now have (imperfect) negative correlation.

- Asymmetric role assignments (Section VI.D): Assuming $\rho_\epsilon = 1$, we show how a change in the probability of that a player becomes the leader affects trading strategies in the pregame. This in turn affects the leader-follower game strategies, since the variances MM's beliefs depend on the pregame strategies.

The solution of this model with pre-game trading is more complicated, mainly due to two additional forces. First, deviations in the pre-game lead to private information that can be payoff-relevant in the sequential game for our players—the continuation game changes after

[5] We use P_{pre} to distinguish from P_0, the expected firm value at the beginning of the leader-follower game (after v has been revealed).

deviations. Specifically, with perfectly correlated signals, players must use both their signal and their actual position resulting from pre-game trading to best respond in the continuation game; they use the signal to forecast the other player's position (who they assume is on path), but after deviations in the pre-game, the player's own position is decoupled from the signal. With conditionally i.i.d. signals, however, the pre-game signals are only needed in the pre-game, since after the revelation of v, a player's own signal becomes irrelevant for forecasting the other's signal or position—the player's own position is the only relevant private information in the continuation game, on and off path.

Second, there is a non-trivial fixed point at play: the coefficient in the trading strategy in the pre-game shapes the degree of correlation in initial blocks in our sequential game which, via continuation payoffs, in turn matters for the determination of the aforementioned coefficient itself in pregame. This fixed point problem is particularly complex in the asymmetric role assignments extension, as the strategies in the leader-follower game when the leader is player 1 affect the analogous strategies when the leader is player 2, and vice versa.

VI.B Inducing positive correlation

In this section, assume $\rho_\epsilon = 1$. We reduce the problem of existence of a (symmetric linear) equilibrium to a fixed point equation in α, which we solve numerically. This equilibrium generates perfectly correlated positions from the perspective of the market maker.

Belief updating in the pregame Under perfectly correlated signals, player i knows $s_{-i} = s_i$. Now given conjectured equilibrium strategies and order flow Ψ_0, the market maker's updated beliefs are

$$\mu_v := \mathbb{E}[v|\Psi_0] = \frac{2\alpha\sigma_v^2}{4\alpha^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma^2}\Psi_0,$$
$$\mu_\theta := \mathbb{E}\left[\theta^i|\Psi_0\right] = \frac{2\alpha^2(\sigma_v^2+\sigma_\epsilon^2)}{4\alpha^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma^2}\Psi_0.$$

Recall that in the leader follower game, with prior mean μ, expected firm value is $(2 + \alpha_L + \delta_L)\mu$. Hence, given Ψ_0, the MM sets price

$$P_{pre} = \mu_v + q(2+\alpha_L+\delta_L)\mu_\theta.$$

At the end of the pregame, players update based on (Ψ_0, v) due to v becoming public. Because of the presence of noise traders, deviations are hidden and hence each player correctly assumes the other is on path; thus player i believes $X_0^{-i} = \alpha s_i$ with probability 1.

The MM assumes both players are on path and have identical positions. The MM's posterior mean given (Ψ_0, v) is

$$\begin{aligned}\mu_X := \mathbb{E}[X_0^i|v,\Psi_0] &= \alpha v + \frac{\mathrm{Cov}(\Psi_0, X_i|v)}{\mathrm{Var}(\Psi_0|v)}(\Psi_0 - 2\alpha v) \\ &= \alpha v + \frac{\mathrm{Cov}(2\alpha(v+\epsilon)+\sigma Z_0, \alpha(v+\epsilon)|v)}{\mathrm{Var}(2\alpha(v+\epsilon)+\sigma Z_0|v)}(\Psi_0 - 2\alpha v) \\ &= \alpha v + \frac{2\alpha^2\sigma_\epsilon^2}{4\alpha^2\sigma_\epsilon^2+\sigma^2}(\Psi_0 - 2\alpha v).\end{aligned}$$

The posterior variance is

$$\phi := \mathrm{Var}(X_0^i|v,\Psi_0) = \frac{\alpha^2\sigma_\epsilon^2\sigma^2}{4\alpha^2\sigma_\epsilon^2+\sigma^2}. \tag{IA.70}$$

Due to perfectly correlated signals, the traders do not use (Ψ_0, v) to update beliefs about each other's signals and positions.

Best response problems in the leader-follower game In this subsection, we solve the players' best response problems in the leader-follower game after arbitrary histories of the pregame.

In a conjectured equilibrium, the relevant state variables entering the leader-follower game are (X_0^i, s_i, μ_X, v), where $\mu_X := \mathbb{E}[X_0^i|\Psi_0, v]$. A few comments are in order:

1. Although the prior expectation of s_i is just v, the public posterior expectation "μ_s" about s_i given (Ψ_0, v) is not v; higher Ψ_0 is indicative of higher errors ϵ_i.
2. However, on the path of play of the pre-game, $X_0^i = \alpha s_i$, and the MM assumes players are on path, so μ_X is a sufficient statistic for μ_s: $\mu_s = \mu_X/\alpha$.
3. Also on the path of play, X_0^i is a sufficient statistic for s_i, but since players can deviate in the pre-game, s_i is a relevant state entering the leader-follower game.
4. All first-order beliefs and higher-order beliefs about (X_0^i, s_i) can be written in terms of (X_0^i, s_i, μ_X, v).

Write the expanded strategies of the players in the leader-follower game as

$$\begin{aligned}\theta^L &= \hat{\alpha}_L X_0^L + \delta_L\mu + \hat{\nu}_L s_L, \\ \theta^F &= \hat{\alpha}_F(X_0^F - M_1^F) + \hat{\nu}_F(s_F - M_1^F/\alpha) = \hat{\alpha}_F X_0^F + \hat{\nu}_F s_F + \beta_F(P_1 - v) + \delta_F\mu,\end{aligned}$$

where we abbreviate μ_X to μ, and where $M_1^F/\alpha = \mathbb{E}[s_F|\Psi_1, \Psi_0, v]$.

These will coincide with the on-path equilibrium strategies we already know:

$$\alpha_L = \hat{\alpha}_L + \frac{\hat{\nu}_L}{\alpha},$$
$$\alpha_F = \hat{\alpha}_F + \frac{\hat{\nu}_F}{\alpha}.$$

The follower's objective is

$$\sup_{\theta^F} \mathbb{E}[(v + X_0^L + \theta^L + X_0^F + \theta^F)(X_0^F + \theta^F) - (P_1 + \Lambda_2\Psi_2)\theta^F - \frac{1}{2}(X_0^F + \theta^F)^2 | X_0^F, s_F, \mathcal{F}_1, \theta^F],$$

where $\mathcal{F}_1$ is the sigma-algebra generated by (Ψ_0, Ψ_1, v). The first order condition is

$$\begin{aligned} 0 &= \mathbb{E}[v + X_0^L + \theta^L + X_0^F + \theta^F - P_1 - 2\Lambda_2\theta^F | X_0^F, s_F, \mathcal{F}_1, \theta^F] && \text{(IA.71)} \\ &= v + \alpha s_F(1 + \alpha_L) + \delta_L\mu + X_0^F + \theta^F - P_1 - 2\Lambda_2\theta^F. && \text{(IA.72)} \end{aligned}$$

Plugging in the extended strategy and matching coefficients yields

$$\hat{\alpha}_F = \frac{1}{2 + \alpha_L}\alpha_F,$$
$$\hat{\nu}_F = \frac{\alpha(1 + \alpha_L)}{2 + \alpha_L}\alpha_F,$$

and indeed, on path, we have $\hat{\alpha}_F X_0^F + \hat{\nu}_F s_F = \hat{\alpha}_F X_0^F + \hat{\nu}_F X_0^F/\alpha = \alpha_F X_0^F$. Intuitively, the private state s_F informs the follower about the contribution to firm value of $(1 + \alpha_L)X_0^L$ in the leader's terminal position, while the private state X_0^F informs him about his own contribution X_0^F, and $X_0^F = X_0^L$ on path as we are assuming perfect correlation in the signals.

The leader's first-order condition is

$$\begin{aligned} 0 = \mathbb{E}\big[&v + X_0^L + \theta^L + X_0^F + \theta^F - (P_0 + \Lambda_1\{\Psi_1 - (\alpha_L + \delta_L)\mu\}) - \theta^L\Lambda_1 \\ &+ (X_0^L + \theta^L)\Lambda_1\beta_F | X_0^L, s_L, \mathcal{F}_0, \theta^L\big] && \text{(IA.73)} \\ = v &+ X_0^L + \theta^L + \alpha s_L(1 + \alpha_F) + \delta_F\mu + (\beta_F - 1)\mathbb{E}\big[P_1 | X_0^L, s_L, \mathcal{F}_0, \theta^L\big] - \theta^L\Lambda_1 + (X_0^L + \theta^L)\Lambda_1\beta_F, && \text{(IA.74)} \end{aligned}$$

where $\mathcal{F}_0$ is generated by (Ψ_0, v) and where

$$\mathbb{E}\big[P_1 | X_0^L, s_L, \mathcal{F}_0, \theta^L\big] = P_0 + \Lambda_1\big\{\theta^L - (\alpha_L + \delta_L)\mu\big\}.$$

Matching coefficients on X_0^L and s_L yields expressions for $\hat{\alpha}_L$ and $\hat{\nu}_L$ in terms of the already known (baseline model) equilibrium coefficients:

$$\hat{\alpha}_L = \frac{1+\beta_F\Lambda_1}{2(1-\beta_F)\Lambda_1 - 1},$$
$$\hat{\nu}_L = \frac{\alpha(1+\alpha_F)}{2(1-\beta_F)\Lambda_1 - 1},$$

and thus $\frac{\hat{\alpha}_L}{\hat{\nu}_L} = \frac{1+\beta_F\Lambda_1}{\alpha(1+\alpha_F)}$.

Outline of remaining steps to establish fixed point numerically

1. From the optimal extended strategies in the leader-follower game, we obtain the players' expected payoffs (immediately after leader-follower roles are assigned) from arbitrary histories in the pregame as quadratic functions $V_L(X_0^i, s_i, v, \mu_X), V_F(X_0^i, s_i, v, \mu_X)$, for any conjectured α, where ϕ determined by (IA.70).

2. Using these continuation payoffs, we write down trader i's maximization problem in the pre-game:

$$\sup_{\theta^i} \mathbb{E}[-P_{pre}\theta^i + (1-q)v\theta^i + \frac{q}{2}(V_L(\theta^i, s_i, v, \mu_X) + V_F(\theta^i, s_i, v, \mu_X))].$$

3. Next, we obtain a fixed point equation for α by imposing the first-order condition with respect to θ^i and then evaluate at the conjectured equilibrium strategy $\theta^i = \alpha s_i$.

4. Numerically, we show that this equation has a solution $\alpha^* > 0$; see left panel of Figure 2. Moreover, α^* homogeneous of degree 0 in $(\sigma, \sigma_v, \sigma_\epsilon)$.

For details, the reader can access the Mathematica file `inducingpositivecorrelation-symmetric.nb` on the authors' websites.

VI.C Inducing negative correlation

Throughout this section, assume that $\rho_\epsilon = 0$, so that pre-game signals are uncorrelated conditional on v. We again reduce the problem of existence of an equilibrium to a fixed point equation in α and solve it numerically. This equilibrium generates negatively correlated positions from the perspective of the market maker conditional on the public information (Ψ_0, v).

Belief updating in the pregame Given s_i, player i's beliefs about s_{-i} and v are as follows:[6]

- $s_{-i}|s_i \sim N\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}s_i, \frac{\sigma_\epsilon^2(\sigma_\epsilon^2+2\sigma_v^2)}{\sigma_v^2+\sigma_\epsilon^2}\right)$,
- $v|s_i \sim N\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}s_i, \frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2}\right)$.

Let $\mu_\theta := \mathbb{E}\left[\frac{\theta_1+\theta_2}{2}|\Psi_0\right]$, $\mu_v := \mathbb{E}[v|\Psi_0]$, and $\mu_X := \mathbb{E}\left[\frac{\theta_1+\theta_2}{2}|\Psi_0, v\right]$. As $\mathbb{E}[\Psi_0] = 0$, we have

$$\mu_\theta = \frac{\text{Cov}\left(\frac{\theta_1+\theta_2}{2}, \Psi_0\right)}{\text{Var}\left(\Psi_0\right)}\Psi_0 = \frac{\alpha^2(2\sigma_v^2+\sigma_\epsilon^2)}{2\alpha^2(2\sigma_v^2+\sigma_\epsilon^2)+\sigma^2}\Psi_0, \tag{IA.75}$$

$$\mu_v = \frac{\text{Cov}\left(v, \Psi_0\right)}{\text{Var}\left(\Psi_0\right)}\Psi_0 = \frac{2\alpha\sigma_v^2}{2\alpha^2(2\sigma_v^2+\sigma_\epsilon^2)+\sigma^2}\Psi_0. \tag{IA.76}$$

As in the previous section, the MM sets price

$$P_{pre} = \mu_v + q(2+\alpha_L+\delta_L)\mu_\theta,$$

now with μ_v and μ_θ given by (IA.75)-(IA.76).

After v is publicly revealed, the MM's beliefs update as follows:

$$\mu_X := \mathbb{E}[X_0^i|\Psi_0, v] = \alpha v + \frac{\alpha^2\sigma_\epsilon^2}{2\alpha^2\sigma_\epsilon^2+\sigma^2}\Psi_0, \tag{IA.77}$$

$$\phi := \text{Var}(\theta^i|\Psi_0, v) = \alpha^2\sigma_\epsilon^2\left[1-\frac{\alpha^2\sigma_\epsilon^2}{2\alpha^2\sigma_\epsilon^2+\sigma^2}\right] = \frac{\alpha^2\sigma_\epsilon^2(\alpha^2\sigma_\epsilon^2+\sigma^2)}{2\alpha^2\sigma_\epsilon^2+\sigma^2}, \tag{IA.78}$$

$$\rho := \text{Cov}(\theta_1, \theta_2|\Psi_0, v) = -\frac{\alpha^4\sigma_\epsilon^4}{2\alpha^2\sigma_\epsilon^2+\sigma^2}. \tag{IA.79}$$

Note that in the numerical solution we find, $\alpha \neq 0$, so indeed $\rho < 0$; that is, the pregame induces negatively correlated positions.

Unlike in the perfect correlation case, the players must also use Ψ_0 and v to update about the other's positions entering the pregame. Players assume each other are on path. Given X_0^i—which is θ^i from the pre-game—and v, players' private beliefs, on and off path, entering the leader-follower game are $X_0^{-i}|X_0^i \sim N(Y_0^i, \nu_0^i)$, where

$$Y_0^i = \mathbb{E}[\alpha S^{-i}|\Psi_0, \theta^i, v] = v + \frac{\alpha\sigma_\epsilon^2}{\alpha^2\sigma_\epsilon^2+\sigma^2}(\Psi_0 - \theta^i - \alpha v),$$

$$\nu_0^i = \frac{\sigma_\epsilon^2\sigma^2}{\sigma^2+\alpha^2\sigma_\epsilon^2}.$$

[6] The posterior covariance is not needed.

While players could use v to form better estimates of each other's signals, since those signals are payoff irrelevant in the continuation game, and thus this exercise is unnecessary. (Indeed, as above, signals are not used to forecast the other's position.)

The leader-follower continuation game From the preceding discussion, players' only relevant private information in the leader-follower game is their position X_0^i. Expected payoffs will depend on v, but v does not affect the players' strategies (when the follower's strategy is written as $\theta^F = \alpha_F(X_0^F - M_1^F)$), since it is a public additive component of firm value. Let $V_L(x, \mu, v)$ and $V_F(x, \mu, v)$ (quadratic functions) denote the players' expected payoffs from the leader-follower continuation game after roles are assigned, with the information structure parameters ϕ and ρ given by (IA.77)-(IA.79) given a conjectured coefficient α.

Player i's best response problem in the pregame is now

$$\sup_{\theta^i} \mathbb{E}[-P_{pre}\theta^i + (1-q)v\theta^i + \frac{q}{2}(V_L(\theta^i, \mu_X, v) + V_F(\theta^i, \mu_X, v))].$$

The first order condition yields a fixed point equation for α, and we show numerically that it has a solution $\alpha^* > 0$, inducing negatively correlated positions; see the right panel of Figure 2. This solution is again homogeneous of degree zero in $(\sigma, \sigma_v, \sigma_\epsilon)$.

Remark 3. *If we further specialize to $\sigma_v = 0$, then players' signals are no longer payoff relevant even in the pre-game, and the model effectively induces a mixed strategy equilibrium, with the signals serving as independent randomization devices. Since signals are then payoff irrelevant, the first order condition is the same for each signal, and this means that players are indifferent over all possible trades.*

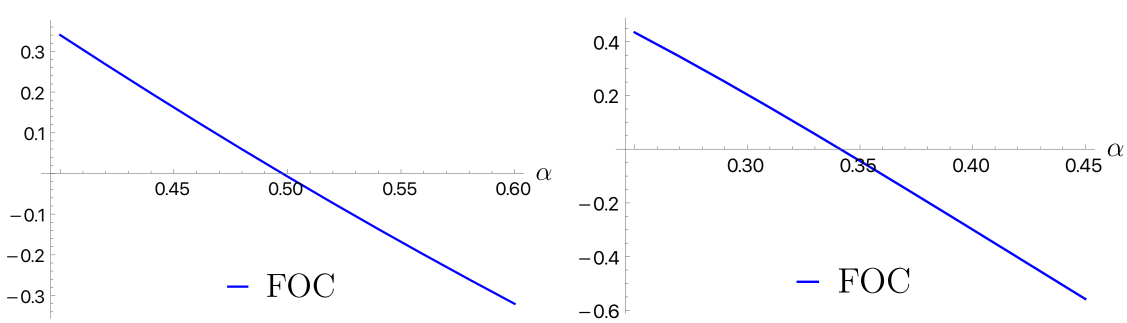

Figure 2: First order condition (FOC) in pregame evaluated at the conjectured strategy α, as a function of α. Parameter values: $\sigma = \sigma_v = \sigma_\epsilon = q = 1$, $\rho_\epsilon = 1$ (left) and $\rho_\epsilon = 0$ (right).

The left and right panels of Figure 1 show the fixed points leading to perfect positive correlation and (imperfect) negative correlation, respectively. The fixed point in the perfect positive correlation case is larger, reflecting more intense trading in the pre-game; this is consistent with the "rat race" phenomenon in dynamic trading models with correlated private information (Foster and Viswanathan, 1996). For details on the negative correlation case, the reader can access the Mathematica file `inducingnegativecorrelation.nb` on the authors' websites.

VI.D Asymmetric random assignment of leader and follower roles

In our extended model that endogenizes initial positions with pregame trading, the leader and follower roles are assigned randomly with equal probability, independent of the first period outcome, making the pre-game symmetric. Let us now suppose that the leader and follower roles (in the sequential trading game that follows the pre-game trading round) are randomly assigned according to a parameter $r \in [1/2, 1]$ before the pre-game, but maintain the other features of the setup. To reduce the notational burden, we assume that the leader-follower game will take place with probability $q = 1$. We assume r is the probability that player 1 is selected to be the leader (and player 2 the follower), while $1 - r$ is the probability that player 2 is selected to be the leader (and player 1 the follower). For simplicity, assume perfectly correlated pre-game signals. Pre-game trades occur simultaneously. Note that for $r > 1/2$, the game is no longer symmetric unless $r = \frac{1}{2}$, therefore players have different incentives in the pre-game. We now look for linear equilibria as follows:

- In the pregame, trader $i \in \{1, 2\}$ trades according to $\chi_i s$.
- The MM uses a linear pricing rule $P_{pre} = \phi + \Lambda_0 \Psi_0$ in the pregame
- In the leader-follower game, the players follow expanded strategies

$$\theta^{L,i} = \hat{\alpha}_{L,i} X_0^{L,i} + \delta_{L,i}\mu_{X_i} + \hat{\nu}_{L,i} s_{L,i},$$
$$\theta^{F,i} = \hat{\alpha}_{F,i}(X_0^{F,i} - M_1^{F,i}) + \hat{\nu}_{F,i}(s_{F,i} - M_1^{F,i}/\chi_{F,i}) = \hat{\alpha}_{F,i} X_0^{F,i} + \hat{\nu}_{F,i} s_{F,i} + \beta_{F,i}(P_1 - v) + \delta_{F,i}\mu_{X_i},$$

which, along the path of play, will have the same form as in the baseline model: $\theta^{L,i} = \alpha_{L,i} X_0^{L,i} + \delta_{L,i}\mu_{X_i}$ and $\theta^{F,i} = \alpha_{F,i} X_0^{F,i} + \beta_{F,i} P_1 + \delta_{F,i}\mu_{X_j}$. (Note that μ_{X_i} and μ_{X_j} are proportional by a constant.) Since problem is no longer symmetric we need to keep track of the identity of the leader and the follower.

Belief updating in the pregame Given the conjectured equilibrium strategies and order flow Ψ_0, the market maker's updated beliefs are

$$\mu_v := \mathbb{E}[v|\Psi_0] = \frac{(\chi_1 + \chi_2)\sigma_v^2}{(\chi_1 + \chi_2)^2(\sigma_v^2 + \sigma_\epsilon^2) + \sigma^2}\Psi_0,$$

$$\mu_{\theta^i} := \mathbb{E}\left[\theta^i|\Psi_0\right] = \frac{\chi_i(\chi_1+\chi_2)(\sigma_v^2+\sigma_\epsilon^2)}{(\chi_1+\chi_2)^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma^2}\Psi_0.$$

Hence, given Ψ_0, the MM sets price

$$P_{pre} = \mu_v + r\left((1+\alpha_{L,1}+\delta_{L,1})\mu_{\theta^1}+\mu_{\theta^2}\right) + (1-r)\left((1+\alpha_{L,2}+\delta_{L,2})\mu_{\theta^2}+\mu_{\theta^1}\right).$$

At the end of the pregame, players update based on (Ψ_0, v) due to v becoming public. Because of the presence of noise traders, deviations are hidden and hence each player correctly assumes the other is on path; thus player i believes $X_0^{-i} = \chi_j s_i$ with probability 1. The MM assumes both players are on path and have perfectly correlated positions. The MM's posterior mean given (Ψ_0, v) is

$$\mu_{X_i} := \mathbb{E}[X_0^i|v,\Psi_0] = \chi_i v + \frac{\text{Cov}(\Psi_0, X_0^i|v)}{\text{Var}(\Psi_0|v)}(\Psi_0-(\chi_1+\chi_2)v) \tag{IA.80}$$

$$= \chi_i v + \frac{\text{Cov}((\chi_1+\chi_2)(v+\epsilon)+\sigma Z_0, \chi_i(v+\epsilon)|v)}{\text{Var}((\chi_1+\chi_2)(v+\epsilon)+\sigma Z_0|v)}(\Psi_0-(\chi_1+\chi_2)v) \tag{IA.81}$$

$$= \chi_i v + \frac{\chi_i(\chi_1+\chi_2)\sigma_\epsilon^2}{(\chi_1+\chi_2)^2\sigma_\epsilon^2+\sigma^2}(\Psi_0-(\chi_1+\chi_2)v). \tag{IA.82}$$

The posterior variance is

$$\phi_i := \text{Var}(X_0^i|v,\Psi_0) = \frac{\chi_i^2\sigma_\epsilon^2\sigma^2}{(\chi_1+\chi_2)^2\sigma_\epsilon^2+\sigma^2}. \tag{IA.83}$$

Due to perfectly correlated signals, the traders do not use (Ψ_0, v) to update beliefs about each other's signals and positions. We define P_0 as the MM's expectation of firm value after v and identity of the leader are revealed but before the leader-follower game starts. Assume agent i is assigned as the leader,

$$P_0 = v + \mu_{X_i}(1+\alpha_{L,i}+\delta_{L,i}) + \mu_{X_j}. \tag{IA.84}$$

Note that P_{pre} is the MM's expectation of P_0 after the MM observes Ψ_0 but just before v and the identity of leader is revealed.

Best response problems Suppose the agent i becomes the follower and agent j is the leader, then follower's objective in the leader-follower game is

$$\sup_{\theta^{F,i}} \mathbb{E}[(v+X_0^{L,j}+\theta^{L,j}+X_0^{F,i}+\theta^{F,i})(X_0^{F,i}+\theta^{F,i}) - (P_1+\Lambda_2\Psi_2)\theta^{F,i} - \frac{1}{2}(X_0^{F,i}+\theta^{F,i})^2|X_0^{F,i}, s_{F,i}, \mathcal{F}_1, \theta^{F,i}],$$

where $\mathcal{F}_1$ is the sigma-algebra generated by (Ψ_0, Ψ_1, v). The first order condition is

$$\begin{aligned} 0 &= \mathbb{E}[v + X_0^{L,j} + \theta^{L,j} + X_0^{F,i} + \theta^{F,i} - P_1 - 2\Lambda_2\theta^{F,i} | X_0^{F,i}, s_F, \mathcal{F}_1, \theta^{F,i}] \\ &= v + \chi_j s_F(1+\alpha_{L,j}) + \delta_{L,j}\mu_{X_j} + X_0^{F,i} + \theta^{F,i} - P_1 - 2\Lambda_2\theta^{F,i}, \end{aligned} \tag{IA.85}$$

where $\mu_{X_j} = \mu_{X_i}\chi_j/\chi_i$. Plugging in the extended strategy and matching coefficients yields

$$\begin{aligned} \alpha_{F,i} &= \sigma/\sqrt{\gamma_1^{F,i}}, \qquad \text{(IA.86)} \\ \hat{\alpha}_{F,i} &= \frac{\alpha_{F,i}}{1 + (1+\alpha_{L,j})\chi_j/\chi_i}, \\ \hat{\nu}_{F,i} &= \chi_j(1+\alpha_{L,j})\hat{\alpha}_{F,i}, \\ \beta_{F,i} &= -\hat{\alpha}_{F,i}, \\ \delta_{F,i} &= \delta_{L,j}\hat{\alpha}_{F,i}\chi_j/\chi_i. \end{aligned}$$

The leader's first-order condition (agent j) is

$$\begin{aligned} 0 = \mathbb{E}\Big[&v + X_0^{L,j} + \theta^{L,j} + X_0^{F,i} + \theta^{F,i} - (P_0 + \Lambda_1\left\{\Psi_1 - (\alpha_{L,j} + \delta_{L,j})\mu_{X_j}\right\}) - \theta^{L,j}\Lambda_1 \\ &+ (X_0^{L,j} + \theta^{L,j})\Lambda_1\beta_{F,i} | X_0^{L,j}, s_{L,j}, \mathcal{F}_0, \theta^{L,j}\Big] \\ = {}& v + X_0^{L,j} + \theta^{L,j} + \chi_i s_L(1+\alpha_{F,i}) + \delta_{F,i}\mu_{X_i} + (\beta_{F,i} - 1)(P_0 + \Lambda_1\left\{\theta^{L,j} - (\alpha_{L,j} + \delta_{L,j})\mu_{X_j}\right\}) \\ &- \beta_{F,i}v - \theta^{L,j}\Lambda_1 + (X_0^{L,j} + \theta^{L,j})\Lambda_1\beta_{F,i}, \end{aligned}$$

where $\mathcal{F}_0$ is generated by (Ψ_0, v) and $\mu_{X_i} = \mu_{X_j}\chi_i/\chi_j$. Matching coefficients on $X_0^{L,j}$, s_L, and μ_{X_j}[7] yields

$$\begin{aligned} \hat{\alpha}_{L,j} &= \frac{1 + \beta_{F,i}\Lambda_1}{2(1-\beta_{F,i})\Lambda_1 - 1}, \\ \hat{\nu}_{L,j} &= \frac{\chi_i(1+\alpha_{F,i})}{2(1-\beta_{F,i})\Lambda_1 - 1}, \\ \delta_{L,j} &= -\frac{\sigma^2}{\alpha_{L,j}\phi_{L,j}}. \end{aligned}$$

On the path of play, the weight on $X_0^{L,j}$ is

$$\alpha_{L,j} = \hat{\alpha}_{L,j} + \hat{\nu}_{L,j}/\chi_j = \frac{1 + \beta_{F,i}\Lambda_1 + (\chi_i/\chi_j)(1+\alpha_{F,i})}{2(1-\beta_{F,i})\Lambda_1 - 1}, \tag{IA.87}$$

[7]The v-terms in the FOC already cancel out, so there is no coefficient on v in the leader's strategy.

which is an equation in $(\alpha_{L,j}, \alpha_{F,i})$ given our earlier characterization of $\beta_{F,i}$. By the same arguments as before, there is a pair of positive real values of $(\alpha_{L,j}, \alpha_{F,i})$ that solves the system (IA.86) and (IA.87).

Given their pregame trades (i.e., positions), realized pregame order flow Ψ_0, the revealed v, and the leader/follower role assignments, the players obtain quadratic expected continuation payoffs for the leader-follower game: $V_{L,i}(\theta^i, s, v, \mu_{X_i}, \mu_{X_j})$ and $V_{F,i}(\theta^i, s, v, \mu_{X_j}, \mu_{X_i})$.

The first order condition for player i's pregame trade is

$$0 = \frac{\partial}{\partial \theta^i}\mathbb{E}[-P_{pre}\theta^i + rV^{L,i}(\theta^i, s, v, \mu_{X_i}, \mu_{X_j}) + (1-r)V^{F,i}(\theta^i, s, v, \mu_{X_j}, \mu_{X_i})]. \tag{IA.88}$$

Together, the first order conditions (IA.88) for player $i = 1, 2$ in the pregame and the first order conditions for player $i = 1, 2$ as leader in the leader-follower game (IA.87) yield a system of four equations in $(\chi_1, \chi_2, \alpha_{L,1}, \alpha_{L,2})$, as all other strategy coefficients can be written in terms of these coefficients. We solve this system numerically by solving all four equations simultaneously. Although $(\alpha_{L,1}, \alpha_{L,2})$ can be solved numerically in terms of (χ_1, χ_2) alone, this would still leave a system in (χ_1, χ_2), and it would still not be possible to plot an analog of Figure 2. The reader can access the full equations and the solution in the Mathematica file `includingpositivecorrelation-asymmetricroleassignments.nb` on the authors' websites.

Equilibrium and intuition Let us examine how player 1's equilibrium behavior varies with $r \in [1/2, 1]$ from a time-0 perspective, but after learning her type s. To this end, recall that on the path of play activist i places a pre-game trade of

$$\chi_i s. \tag{IA.89}$$

Meanwhile, if she happens to become the leader, she completes her block according to the strategy

$$\alpha_{L,i} X_0^{L,i} + \delta_{L,i}\mu_{X_i}, \tag{IA.90}$$

where $X_0^{L,i} = \chi_i s$ and $\mu_{X_i} = \mathbb{E}[X_0^{L,i}|v, \Psi_0]$. Averaging $\mu_{L,i}$ over (Ψ_0, v) delivers activist i's expected trade conditional on being a leader of type s:

$$\begin{aligned}
\mathbb{E}[\theta^{L,i}|s] &= \chi_i\alpha_{L,i}s + \delta_{L,i}\mathbb{E}\big[\mathbb{E}\left[X_0^{L,i}|v, \Psi_0\right]|s\big] \qquad \text{(IA.91)}\\
&= \chi_i\alpha_{L,i}s + \delta_{L,i}\mathbb{E}\left[\chi_i v + \frac{\chi_i(\chi_1+\chi_2)\sigma_\epsilon^2}{(\chi_1+\chi_2)^2\sigma_\epsilon^2 + \sigma^2}(\Psi_0 - (\chi_1+\chi_2)v)\Big|s\right]\\
&= \chi_i(\alpha_{L,i} + \frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}\delta_{L,i})s + \delta_{L,i}\frac{\chi_i(\chi_1+\chi_2)^2\sigma_\epsilon^2}{(\chi_1+\chi_2)^2\sigma_\epsilon^2+\sigma^2}\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_v^2}s
\end{aligned}$$

$$= \chi_i \left(\alpha_{L,i} + \delta_{L,i} \left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2} + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_v^2} \frac{(\chi_1 + \chi_2)^2 \sigma_\epsilon^2}{(\chi_1 + \chi_2)^2 \sigma_\epsilon^2 + \sigma^2} \right) \right) s. \tag{IA.92}$$

The first equality follows from $X_0^{L,i} = \chi_i s$; the second uses equation (IA.82); and the final two use that $\mathbb{E}[v|s] = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2} s$, together with the fact that noise trading is independent of s .

Equipped with this, player i's terminal position conditional on being a leader of type s reads

$$\mathbb{E}[X_T^{L,i}|s] = \chi_i s + \chi_i \left(\alpha_{L,i} + \delta_{L,i} \left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2} + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_v^2} \frac{(\chi_1 + \chi_2)^2 \sigma_\epsilon^2}{(\chi_1 + \chi_2)^2 \sigma_\epsilon^2 + \sigma^2} \right) \right) s. \tag{IA.93}$$

The following figure shows how the players' equilibrium coefficients, as well as their expected trades and terminal positions, vary with r.

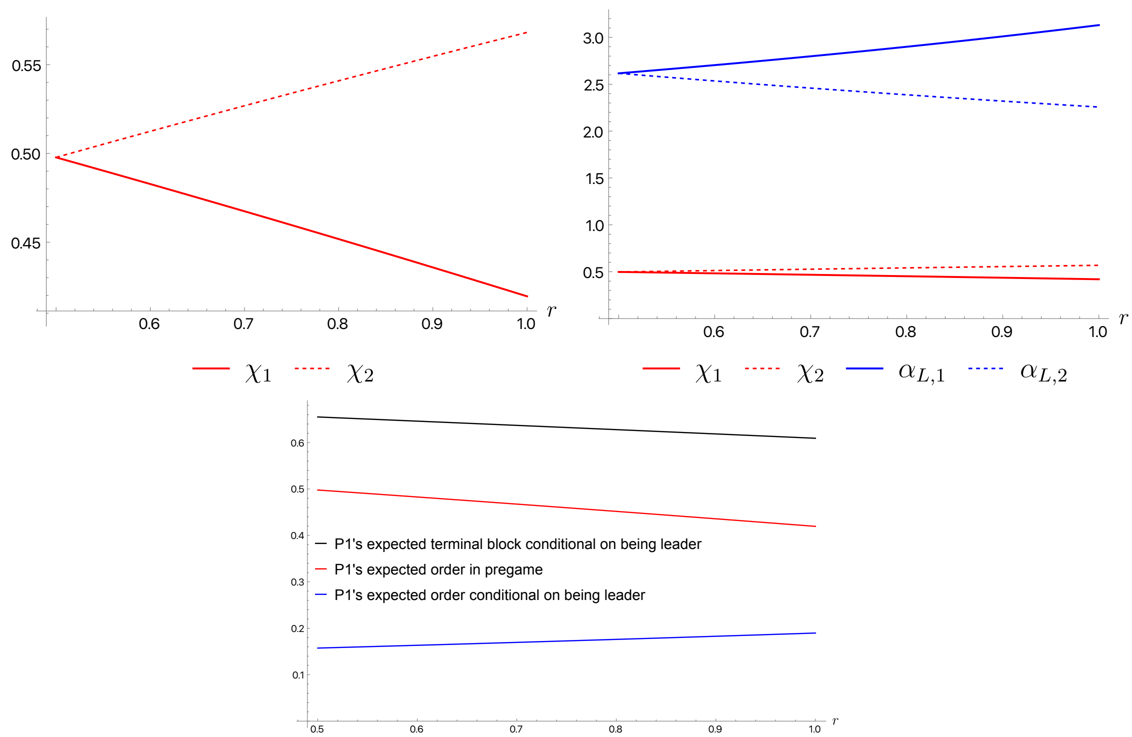

Figure 3: Coefficients as a function of r, fixing $\sigma = \sigma_\epsilon = \sigma_v = 1$. Upper left panel: $(\chi_1, \chi_2, \alpha_{L,1}, \alpha_{L,2})$; Upper right panel: $(\chi_1, \chi_2, \chi_{\text{static}})$. Lower panel: Player 1's expected trades and terminal position conditional on $s = 1$.

The top left panel displays the strategy coefficient in the pre-round: χ_1 in continuous red for player 1, and in dashed the counterpart for player 2, both as a function of r. The main message is the decreasing pattern of player 1's coefficient: *(1) player 1 scales down her purchases as r grows and she is more likely to be the leader* (player two responds with a coefficient χ_2 that is increasing as a result). The panel on the right then plots the coefficient on the now initial block $\chi_i s$ for each of our players: $\alpha_{L,1}$ in continuous blue, while $\alpha_{L,2}$ in dashed (the original χ_i coefficients are at the bottom for comparison), showing now that *(2) player 1, if becoming a leader, would attach more weight to her initial block as r increases.*

Whether this maps into more or less buying depends on the "public" part of the strategy. For this we look at the bottom panel for player 1's expected trades conditioning only on s: the middle curve is her pre-round trade $\chi_1 s$ (same as in the first panel); the bottom curve is her expected trade if chosen a leader, $\mathbb{E}[\theta^{L,i}|s]$ as in (IA.92); and at the top is the sum of both, or player 1's expected terminal position $\mathbb{E}[X_T^{L,i}|s]$ as given in (IA.93). There are two additional messages here: first, *(3) player 1 does buy more aggressively as r grows when finalizing her block*; second, *(4) conditional on being a leader, player 1's average terminal position falls with r.*[8]

The takeaways (1)–(4) are interesting. First, (4) is consistent with our model's message that having an opportunity to move first would yield a weaker incentive to build a block—now we have a version of that logic using the intensive margin $r \in [1/2, 1]$. Equipped with this, how exactly (1), (2) and (3) are linked is noteworthy. As player 1 trades less aggressively in the first round and builds a smaller block, she knows that she constitutes a smaller fraction of the total uncertainty related to the firm (i.e., $\text{Var}(\chi_1 s) = \chi_1^2 \text{Var}(s)$ is increasing in χ_1), which limits her price impact. With prices that move less, the manipulation motive is weakened in favor of exploiting trading gains, leading to both $\alpha_{L,1}$ and player 1's expected trade conditional on being a leader to grow with r, explaining (2) and (3). But why would the likely leader trade less in the pre-round, our point (1)? One possibility is that she anticipates the trading losses that she incurs when acting as a leader, which are encoded in the deviation from the "Kyle optimum." Optimizing those losses from a time zero perspective would likely require choosing an initial block size that is smaller, because this reduces the trading gains that are given up at the moment of acting as a leader. This, in turn, eases the tension between manipulating the follower and making trading profits.

Altogether, this variation reveals that while a higher likelihood of being a leader reduces the overall incentive to acquire shares—as measured by player 1's terminal position—it also triggers more aggressive block-building conditional on being a leader. This has interesting implications on the dynamics of block-building. Concretely, an activist leader initially places increasingly small trades—reminiscent of trying to camouflage her buildup—as she expects a higher chance of enjoying both monopoly and manipulation rents in the future. When this time comes, her smaller footprint in the market leads the activist to build her block with more confidence while not discouraging others to add value. Smaller initial blocks for

[8] Player 1 buys when she is a leader because her type s is larger than the mean, which we have normalized to zero. On the other hand, the blue vs. red ranking in the bottom panel depends on parameters: for instance, by setting $\sigma_\epsilon = 10$, blue can cross red from below. Indeed, for σ_ϵ large the expected value of $\mathbb{E}[X_0^{L,i}|\Psi_0, v]$ conditional on s is close to 0, therefore muting the contribution of a negative $\delta_{L,i}$ in the leader's strategy.

leaders—understood as the blocks that are owned before gearing towards an attack—then constitute an *equilibrium property* of this broader leader-follower theory of activism.

References

Foster, F. D. and S. Viswanathan (1996): "Strategic trading when agents forecast the forecasts of others," *The Journal of Finance*, 51, 1437–1478.