Leader-Follower Dynamics in Shareholder Activism*

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Abstract

We propose a theory of coordination and influence among blockholders without explicit agreements. Privately informed activists trade in sequence to control block-acquisition costs. Through this timing of trades, leader activists use order flows to create trading gains that entice other activists to build larger blocks, ultimately causing those followers to bear greater activism costs that add value. Trading activists can thus differ dramatically from "pure insiders" as in Kyle (1985), for whom price impact is the sole disciplinary force: not only do such leaders accumulate shares differently than if acting in isolation—or if firms' true values were exogenous—but their trades end up having *predictability*. We explain how this fundamental departure in the nature of strategic trading relates to free-rider problems affecting governance, and how it produces price abnormalities analogous to those documented empirically. We also uncover how interdependence in some form of private information—activists' blocks, firms' fundamentals, or intervention costs—can be a key catalyst for the mechanism uncovered.

Keywords: activism, insider trading, blockholders, hedge funds

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1 Introduction

The theories of blockholders have proven key to understanding free-rider effects when ownership is dispersed (Shleifer and Vishny, 1986), the impact of market liquidity on governance by voice (Maug, 1998), and the role of disposing of shares in disciplining management (Edmans, 2009; Admati and Pfleiderer, 2009). However, the fundamental question of how activist blockholders gear towards intervening in firms, anticipating that other investors think alike, and can be influenced, is much less understood. This paper proposes a theory of influence dynamics between blockholders: one that operates through market signals.

The empirical relevance of multi-activist interventions is reflected in the increased prevalence of events featuring multiple *hedge funds* engaging with the same target (e.g., Becht et al., 2017). In the U.S., this form of coordination has coincided with three trends in the activism ecosystem: an increase in institutional investors' ownership share of corporations; a regulatory environment that permits substantial communication among shareholders while imposing significant restrictions for formally organized groups; and an increased focus on higher market-capitalization firms as targets, as well as on more complex forms of interventions.[1] As activism costs grow—both to acquire sizable blocks and to perform the actual restructurings—blockholder coordination becomes a key cost-management tool.[2] While the prevailing regulation facilitates a common understanding of potential targets and of the incentives at play, at the same time it discourages the use of explicit agreements.

In this paper, we show how this coordination can be achieved non-cooperatively. Our premise is that, to control costs, activists can have strong incentives to time their trades in sequence; but this means that any "leader activist" will inevitably use market signals to credibly communicate with "follower activists" to acquire a larger share and be more inclined to improve firms. Concretely, two activists decide how much stake to (de-)accumulate in a market structure à la Kyle (1985)—for simplicity, private information is about initial blocks and firm value is determined by effort choices, as in Back et al. (2018). To this baseline setting, we add *block interdependence*: the activists' initial positions exhibit correlation—for instance, if positive, because of similar investment styles. Further, trading is sequential: in the first period, a *leader* (she) activist acts as the unique informed trader, anticipating that a *follower* (he) will play that role in the second period. After the activists finalize their blocks through these trades, both simultaneously exert effort in line with their terminal positions

[1] Brav et al. (2021b) argues that "low-hanging fruits"—targets with easily fixable issues using standard tactics—have been exhausted post financial crisis, giving rise to the use of consultants and advisory boards.

[2] Salesforce was a target in 2022 when its valuation was around \$130B; none of the five activists reached 5%, or \$6.5B—`https://finance.yahoo.com/news/salesforces-activist-investors-who-are-they-and-what-do-they-want-174655497.html`. Away from acquisition costs, Gantchev (2013) finds that activists' campaigns can total \$10M, while Albuquerque et al. (2022) estimate activism costs at \$2.43M.

to determine the firm's share (fundamental) value.

This setup and the variations that we study constitute the first framework for examining block accumulation dynamics when multiple activists intervene in firms and impose externalities on each other. We use our model to offer new insights regarding trading around activism events; to derive new predictions about the quality of corporate governance as orchestrated by leader activists; and to provide new interpretations of existing empirical findings.

Trading In traditional microstructure models such as Kyle (1985), it is price impact that limits an investor's incentive to exploit an informational advantage when trading in a firm's stock. With endogenous fundamentals and multiple traders, there is now a second channel: creating trading gains that entice fellow activists to accumulate more shares, and thus to find it optimal to bear more of the value-generating activism costs.

The follower's trading gains are measured by how far the market price—a belief about the activists' effort choices based on the public order flow—departs from the follower's own belief about the firm's value: this belief depends not only on his initial block, but also on the order flow, which conveys information about the leader's contribution. This difference in beliefs—a form of mispricing—now enters the leader's calculations: she not only balances her trading gains with the cost of driving the price against herself, but she also evaluates how her trades affect the inference made by the follower vis-á-vis that of market makers.

Correlation plays a key role in resolving this trade-off. If positive, an abnormally high first-period order flow indicates to market makers not only a large contribution by the leader, but also that the follower will follow suit; instead, due to his private information, the follower only updates about the leader's effort. Thus, the price is *more responsive* to the first-period order flow than the follower's belief about the firm is, while the opposite occurs with negative correlation: the price increases less because the market makers' updating on each activist's effort move in opposite directions. With prices that are relatively more responsive, buying more aggressively is more costly than when price impact is the only disciplinary force, as such trades now also discourage the follower from building a larger stake—and vice-versa.

Theorem 1 formalizes this logic by uncovering a linear equilibrium in which activists with larger initial blocks accumulate more shares in relative terms than their smaller-block counterparts, as in Kyle-type models where insiders with more optimistic signals about a firm's fundamentals naturally acquire more stock. The novelty of our equilibrium, however, is that the leader's trades are no longer neutral: the leader sells on average when correlation is positive and buys otherwise. That is, being able to influence other blockholders using market signals does not merely translate into more or less aggressive trading while preserving the pervasive property in Kyle (1985) that trades are unpredictable: if activism is at play, the nature of strategic trading fundamentally changes. This finding thus alters the conventional wisdom of insider trading models reinforced over decades following Kyle's seminal work.

Private information Our choice of private initial positions is empirically relevant for three reasons: hedge funds' stakes are typically small; so-called "under the threshold" campaigns have been on the rise; and large-cap firms are becoming more frequent targets.[3] That said, what matters for our result is that private information has interdependence—the activists know more about each other—and not particularly where it comes from. Indeed, if initial blocks are public, qualitatively identical results arise if private information is about exogenous components of firms' values or about the activists' productivity in improving firms.

In turn, a non-trivial degree of interdependence is important solely due to the minimal assumptions imposed on the timing of trades and players' payoffs. In fact, if both activists trade simultaneously over two rounds, or they impose other types of negative externalities on each other (e.g., losing private benefits) the same phenomenon can arise even when initial positions are uncorrelated. Section 3.2 examines these and other robustness checks.

Governance and price abnormalities From an institutional perspective, the way to interpret our finding on average trades is that a leader's accumulation of shares may depart meaningfully from its counterpart when intervening in isolation: in particular, when correlation is positive, a leader with a sufficiently large block will still buy shares, but not as aggressively. Importantly, by building a smaller block than she would otherwise, the leader effectively offloads activism costs on the follower, imposing a form of externality that the follower finds optimal to bear; and if correlation is negative, the leader incentivizes the follower by herself accumulating more shares and bearing greater activism expenses.

Because terminal positions determine effort provision, trading has non-trivial implications for both the quality of corporate governance and stock prices. On the first front, note that since the follower does not change his position on average (due to not having the opportunity to influence any subsequent activist), all non-trivial implications for firm values are linked to the leader's behavior: when correlation is positive (negative) the leader lowers (increases) firm value relative to the counterfactual world in which blocks do not change on average. In other words, with positive interdependence, a first-mover advantage in trading amplifies traditional free-rider effects, but not otherwise. Irrespective of the inefficiencies created, however, we show that multiplayer interactions always deliver more value than their single-player counterparts (Theorem 2).

On the second front, the model naturally delivers measures of abnormality analogous to those documented empirically. The idea is to note that if activism opportunities are absent

[3] We expand on the importance of smaller blocks on Section 2.2. Campaigns with blocks below 5% were majority in the U.S. in 2021 and the targets had higher market capitalization. See `https://www.cnbc.com/2022/01/15/activist-hedge-funds-launched-89-campaigns-in-2021-heres-how-they-fared.html`.

(i.e., fundamentals appear as exogenous), and hence trading is based solely on exploiting informational advantages, trades are expected to be unpredictable: in such "normal times," positions should not change on average. We can then cast our predictions regarding firm value in "price" form: if correlation is positive, prices are predicted to be *abnormally* low on average (and vice-versa) relative to counterfactual periods when activism is not at play. In Section 4.2, we explain how this logic can be used to reinterpret existing empirical findings on price behavior around disclosure events—we also discuss the current evidence that is closest to our proposed mechanism and predictions, and provide directions for how the empirical literature on blockholders and trading can incorporate our results going forward.

First-mover advantages Section 5 examines factors that favor the sequential trading structure studied. As we show, there is a sizable region of correlation levels (both positive and negative) over which both activists are individually better off than if trading simultaneously: coordinating the timing of trades is *mutually beneficial* because acquisition costs are lowered in less competitive settings.[4] With significant negative correlation, however, an activist may prefer to trade simultaneously with a fellow activist because the latter always provides inexpensive liquidity when needed. On the other hand, an increasing positive correlation enhances the leader's ability to influence the follower's trading gains, so moving first is even more desirable—at the expense of the follower nonetheless.

The bottom line is that a leader is more likely to emerge when there is *similarity* among activists, in a block-statistical sense. Further, if the interdependence is positive, the benefit of acting as a leader is enhanced by three factors. First, by having a larger initial block, because this indicates that the follower has a large block too, and hence that he will place sizable trades that exacerbate acquisition costs. Second, by the presence of multiple small followers, because these will aggressively compete to exploit trading gains. Third, by actually being capable of intervening in firms: if the leader is a passive fund that cannot exert effort, she can be trapped into an inferior outcome when trying to influence the follower.

Discussion We conclude the paper with two discussions. In the first, institutionally motivated, we touch on the so-called "wolf pack" activism phenomenon whereby multiple hedge funds attack the same firm in a parallel, seemingly independent, manner after a leader fund acquires a stake. Our model fits many of its features: trading gains matter, in that targets are undervalued firms; blocks are similar, of small to moderate size; behavior is non-cooperative due to the high costs of acting as a formal group; there are followers who do not disclose positions, and hence necessarily have smaller stakes; and there is strong competition at the

[4] This is of great importance for activists, partly because acquisition and activism costs reinforce each other: only after acquiring a maningful block does an incentive to spend resources to change a firm emerge.

moment of trading. But our model can also be used to shed light on the real consequences of this inherently secretive phenomenon, where moves are naturally sequential and hence making correct inferences can be key—the recent 2023 SEC guidance on "tipping" and group formation will likely make market signals an even more powerful coordinating device.

Second, we address the possibility of other equilibria in which the activists trade *against* their initial positions to coordinate with each other in terms of creating or destroying value. Despite this being an interesting theoretical possibility, we argue that these equilibria are less suitable for predicting "positive" activism in practice. Further, we provide conditions under which the equilibrium that we study is the unique prediction within the linear class.

Roadmap We discuss the theoretical literature next. Section 2 present our model, while Section 3 contains our main result. Section 4 is devoted to the model's predictions and connects them with existing empirical work. Section 5 discusses first-mover advantages and the institutional environment, wolf packs included. Section 6 discusses other equilibria and a refinement result. All proofs are in the Appendix or Internet Appendix.

Related literature Our research has been influenced by the "program" proposed by Edmans and Holderness (2017), who highlighted that only models with either one activist building stakes in isolation, or with multiple activists where blocks are fixed, have been studied. Going forward, they suggest considering blocks under 5%; that blockholders interact, imposing externalities on each other; that they can act as informed traders; and that activists' costs and benefits beyond those related to controlling firms matter.

We are not aware of other papers combining these elements in dynamic settings. For instance, in Back et al. (2018)—a fully dynamic *single-activist version* of Kyle (1985)—different activism technologies can have non-trivial implications for market liquidity, but equilibrium trading is always unpredictable. And while there are models involving multiple activists, these feature simultaneous moves among them: in Doidge et al. (2021) activists trade non-cooperatively only once to then act as a coalition when exerting effort; in Edmans and Manso (2011), competition strengthens the threat of disciplinary trading; and in Brav et al. (2021a), reputational motives can lead hedge funds to exert effort to attract funding. Crucially, none of these papers consider the incentives to induce others to develop skin in the game as a means of controlling private costs or increasing private benefits.

Our model is one of activism by "voice"—direct interventions—because effort determines firm value. To contrast with models of "exit," where disposing of shares acts as an ex post disciplinary threat, in our model disposing shares induces other activists to exert voice. This notion of shares' disposal favoring voice relates to models where selling by liquidity (or "noise") traders facilitates block formation and activism (e.g., Maug, 1998; Kahn and

Winton, 1998). Gantchev and Jotikasthira (2018) corroborate this finding for activist hedge funds when institutional investors sell due to negative funding shocks. Instead, in our model, it is an activist who strategically creates favorable market conditions for others.

Activists not only influence firms, but also play a key role in the market for corporate control. In Burkart and Lee (2022), activists who first launch costly campaigns, and then broker takeovers, can mitigate free-rider problems by both target shareholders and activists themselves. Similarly, in Corum and Levit (2019), activists can launch costly proxy contests to lower acquisition costs and trigger ownership transfers that would not have happened otherwise—in our model, the leader both bears direct activism costs (say, launching a campaign) and sacrifices trading gains to induce the follower to improve governance. See also Burkart et al. (2000), where an incumbent wants to sell the majority of her shares to limit a bidder's incentive to extract value-dissipating private benefits after acquiring control.

Finally, our model relates to models focusing on strategies of a more "manipulative nature": steering someone's real action by influencing their beliefs. In models of trading, Goldstein and Guembel (2008) show that short-selling is a profitable strategy for a speculator if it induces a manager to forgo an investment decision; but buy orders are never fruitful there. In Attari et al. (2006), a passive fund may dump shares to insure the value of the remaining block, as activism by a second investor has positive returns only when a firm's fundamentals are low. In Khanna and Mathews (2012), a blockholder instead buys shares to counter a speculator's attempt to lower a firm's value. In contrast to these papers, all of our players directly influence firm values, and both buying or selling can be optimal.[5]

Beyond financial markets, Holmström (1999), Cisternas (2018), Bonatti and Cisternas (2020), Cetemen (2020), Cetemen et al. (2019), Ekmekci et al. (2020) and Cisternas and Kolb (2024) develop models of belief manipulation employing Gaussian fundamentals. A key novelty of our model is that noisier signals (here, order flows) can lead to more manipulation, despite beliefs (here, prices) becoming less responsive. This is because the leader's marginal incentive to manipulate beliefs—captured by her terminal block—is endogenous.

2 Model

2.1 Setup

A *leader* activist (she) and a *follower* counterpart (he) hold initial positions $X_0^L \in \mathbb{R}$ and $X_0^F \in \mathbb{R}$ of shares in a firm, respectively. Each activist's *block* is their private information:

[5]See Yang and Zhu (2021), Boleslavsky et al. (2017), and Ahnert et al. (2020), for models where trading can trigger government interventions, while Chakraborty and Yılmaz (2004), Brunnermeier (2005) and Williams and Skrzypacz (2020) for manipulation in financial markets abstracting from real consequences.

blocks are normally distributed with mean $\mu > 0$, variance $\phi > 0$, and covariance $\rho \in [-\phi, \phi]$.

The model has three periods. In period 1, the leader acts as a single informed trader in a Kyle (1985) market structure. Specifically, she submits an order for $\theta^L \in \mathbb{R}$ units of the firm's stock to a competitive market maker who executes it at a public price P_1 after observing the total order flow of the form

$$\Psi_1 = \theta^L + \sigma Z_1.$$

In this specification, Z_1 is standard normal random variable independent of the initial positions that captures noise traders, and the volatility $\sigma > 0$ is a commonly known scalar.

Having observed P_1, in period 2, the follower replaces the leader as the single informed trader in an identical round of trading: he orders $\theta^F \in \mathbb{R}$ units from the same market maker who in turn executes the order at a (public) price P_2 after observing the total order flow

$$\Psi_2 = \theta^F + \sigma Z_2,$$

where Z_2 is standard normal and independent of (X_0^L, X_0^F, Z_1). Finally, in period 3, the activists simultaneously take actions that determine the firm's fundamentals: activist i exerts effort $W^i \in \mathbb{R}$ at a private cost $\frac{1}{2}(W^i)^2$, $i \in \{L, F\}$, resulting in a *true share value* of

$$W = W^L + W^F.$$

(That is, absent any activism, the share value is common knowledge and normalized to zero.)

Towards stating our players' payoffs, let us use the subscript T to capture *terminal* positions, which for each activist consists of initial positions plus the amount traded:

$$X_T^i = X_0^i + \theta^i, \ i \in \{L, F\}. \tag{1}$$

We also let $(\mathcal{F}_t)_{t=0,1,2}$ denote the public information—which is generated by the prior and the order flows $(\Psi_t)_{t=1,2}$—and we use the indices $t(L) := 1$ and $t(F) := 2$ to link our activists with their corresponding trading periods. Activist $i \in \{L, F\}$ then solves

$$\sup_{\theta^i, W^i} \mathbb{E}\left[\left(W^i + W^{-i}\right) X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(W^i)^2 | X_0^i, \mathcal{F}_{t(i)-1}\right], \tag{2}$$

where the first term is the total value of activist i's holdings, from which trading costs (second term) and activism expenses (third term) are subtracted. Further, because the optimal choice of effort satisfies $W^i = X_T^i,\ i \in \{L, F\}$, the objective (2) can be written as

$$\sup_{\theta^i} \mathbb{E}\left[(X_T^i + X_T^{-i})X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(X_T^i)^2 | X_0^i, \mathcal{F}_{t(i)-1}\right],\ i \in \{L, F\}. \tag{3}$$

Two observations are in order. First, because individual effort is based on an activist's own terminal position, larger blocks translate into a stronger willingness to intervene; but at the same time, there is a collective-action problem because the positive effect of individual effort on the other blockholder's holdings is not internalized. Second, note that the model allows for short positions ($X_0^i < 0$) and negative effort, the latter capturing value destruction or *negative activism*—we discuss this possibility in sections 3.2 and 6.[6] Unless otherwise stated, however, we focus on the opposite situation by assuming $\mu > 0$ and using the cases $X_0^L > 0$ and $X_0^F > 0$ to provide intuition: the activists are initially "long" on the firm and, absent any trading, they would exert positive effort, both conditionally and unconditionally.

Interpretation Our game ending after the third period can be rationalized as the firm's value being revealed after effort is undertaken (which renders subsequent trading unprofitable). Since it takes time to change a firm, one may then wonder whether not allowing for multiple "pre-revelation" rounds of trading is a limitation. Our belief is that this is not the case, for two reasons. First, since in practice activists must reveal their *intended plans* when disclosing positions over 5%, substantial information about plans of action gets revealed well ahead of changes materializing. Second, these disclosures also carry information about trades, revealing that hedge funds trade primarily on the day they cross the 5% threshold—the "trigger date"—or the one after (e.g., Bebchuk et al., 2013 and Collin-Dufresne and Fos, 2015). Thus, trades leading to block completion are not spread out.

Importantly, these trades often happen before the market learns activists' intentions and trades. This is because material adjustments to positions or intentions can be disclosed with a delay—historically, up to 10 days—so trades effectively remain hidden for some time as in our setup.[7] Our model is then best interpreted as taking place in such a pre-disclosure window when the activists have superior information and are gearing up to quickly finalize their positions and attack. A key question is how block completion by a leader hedge fund responds to the possibility of subsequent followers building their own stakes, and what the implications for stock prices are—we will discuss this latter topic in Section 4.

[6] See Bliss et al. (2019) for examples of negative activism, and Appel and Fos (2023) for short campaigns run by hedge funds. Refer to `https://www.cnbc.com/2019/12/13/reliving-the-carl-icahn-and-bill-ackman-herbalife-feud-on-cnbc.html` for a famous case in which investors took opposite positions.

[7] Recently, the traditional disclosure requirement for activists to file a 13D form within 10 days after crossing the 5% threshold has been shortened to 5 business days, while material amendments must be filed within 2 business days: `https://www.sec.gov/news/press-release/2023-219`.

Linear Strategies and Equilibrium Concept As is traditional in the literature following Kyle (1985) we will look for equilibria in *linear* strategies. This means two things. First, our leader will condition on her type X_0^L and the prior mean μ (used by market makers to set the firm's price) in a linear manner, while our follower can, in addition, condition on the observed first-period price. That is, we seek strategies of the form

$$\theta^L = \alpha_L X_0^L + \delta_L \mu \quad \text{and} \quad \theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu, \tag{4}$$

where the coefficients $(\alpha_L, \delta_L, \alpha_F, \beta_F, \delta_F)$ are scalars. Second, the price $P_{t(i)}$ set by market makers is affine in the order flow $\Psi_{t(i)}$, $i = L, F$. In an equilibrium of this kind, (i) the trading strategies are mutual best-responses given the pricing rule, and (ii) the linear prices satisfy $P_{t(i)} = \mathbb{E}[W^L + W^F | \mathcal{F}_{t(i)}]$ when the expectation is computed using the activists' strategies (4). In what follows, we omit the linear qualifier when referring to an equilibrium.

We focus on equilibria exhibiting $\alpha_L > 0$ and $\alpha_F > 0$, or *positive block sensitivity* (PBS), both for conceptual and institutional reasons. On the first front, this type of equilibrium conforms with the literature following Kyle (1985), where an insider trades in proportion to her informational advantage: or "trading gains," as measured by the difference between her private and the public information before trading, which is a measure of mispricing. Thus, such trades are zero conditional on the public information, which for our leader would amount to $\alpha_L = -\delta_L$. This form of *unpredictability* is a pervasive finding in this literature, which treats fundamentals as exogenous.[8] But it also holds with endogenity if there are multiple rounds of trading in single-player setups (e.g., Back et al., 2018), or if there are multiple players in static settings (e.g., Doidge et al., 2021). Our model, combining endogenous fundamentals, multiple players, and dynamics, will prove fundamentally different—the notion of PBS equilibrium is the appropriate one for making this distinction.

Economically, in this equilibrium, larger blockholders acquire more stock, or de-accumulate less, than their smaller counterparts: trading solidifies their a priori stronger willingness to intervene, leading to more value added in relative terms. While equilibria placing a negative weight on the initial block can also exist sometimes (Section 6), they exhibit two key features: (i) strong position reversals can happen and (ii) these reversals can be driven by monetizing value destruction. From an institutional viewpoint then, such equilibria are less suited for capturing positive activism—the more prevalent phenomenon and our main focus.

[8]See Back et al., 2018 for a discussion on this topic, where the term "inconspicuous insider trading" is used. This property holds for Gaussian exogenous fundamentals: with any number of traders and degree of correlation in private information (e.g., Foster and Viswanathan, 1996 and Back et al., 2000); that evolve over time (e.g., Caldentey and Stacchetti, 2010); or where volatility is stochastic (e.g., Collin-Dufresne and Fos, 2016). But it can also arise if the fundamentals are time-invariant and non-Gaussian (e.g., Back, 1992).

2.2 Discussing Our Assumptions

Private information and interdependence Blocks below 5% need not be disclosed, and hence can constitute private information.[9] Hedge fund ownership does fluctuate around this threshold: Brav et al. (2021b) find that the median stake for this type of fund is 6.6% upon disclosure, while Collin-Dufresne and Fos (2015) state that, to complete their blocks (e.g., to reach 6.6%), hedge funds purchase around 1% of shares on the day that the 5% threshold is crossed—of course, these numbers do not include all the (smaller) blocks that are not disclosed. Blockholders in the 1%–5% range can have substantial power: Lewellen and Lewellen (2022) document that this segment collectively owns around 22% of shares in the average firm compared to an aggregate 20% for blockholders above 5%. In line with our assumptions, the authors also show that smaller blockholders are more likely to trade.

That said, what really matters is that the activists have (i) some form of private information that (ii) has interdependence—not necessarily linked to initial blocks. When the first requirement is met and trades are hidden, an activist's terminal position will be their private information; but terminal positions determine the firm's value. This means that the activists effectively have long-term private information regarding the firm's value, and can trade on it, even if their initial blocks are public.[10] Section 3.2 formalizes this idea by examining two extensions that deliver the same predictions as our baseline model: the activists are privately informed about an exogenous component of the firm's value, or about their cost of effort.

The correlation requirement allows the leader to influence the follower using the order flow despite market makers also responding to it. While we will discuss this topic extensively in the next section, two observations are in order here. First, correlation is necessary only because of our minimal assumptions on the structure of trading and players' payoffs. To make this point, Section 3.2 also presents two variations in which initial blocks are uncorrelated, yet the mechanism uncovered continues to be at play: two rounds of trading with both activists placing orders in each round, and an example in which the leader's effort imposes a negative externality on the follower (such as the loss of private benefits).

Second, assuming block correlation can be a useful avenue for testing the predictions of our model. Indeed, because both the sign and magnitude of the correlation have non-trivial effects on prices, existing empirical work on blockholder interdependence could be redirected towards contrasting the model's prediction regarding prices with the empirical evidence on price abnormality documented in several studies. We elaborate on this in Section 4.2.

[9] An exception is when a fund holds more than 100 million in shares of publicly traded firms, in which case a form 13F must be filed, even if there is no intention to intervene. As this form is filed quarterly, blocks in this category can be hidden over even longer horizons (e.g., Puckett and Yan, 2011).

[10] In the words of Collin-Dufresne and Fos (2015), the activists have private information about their *willingness to intervene*; in our case, this is an intensive margin.

Payoffs Assuming that efforts are perfect substitutes in the fundamentals' technology is a natural benchmark for examining how the well-known free-rider problems that arise when ownership is dispersed are affected by an activist's first-mover advantage. Further, as Burkart and Lee (2022) note, our choice of a continuous effort variable can be seen as capturing different types of interventions available that in turn require different levels of activist engagement.[11] And while many outcomes can be binary, our model can be seen as a linearized version of such settings where the probability of success increases in total effort.

On the other hand, our choice of quadratic activism costs is in line with the tradition of trading costs in Kyle-type models. However, as Back et al. (2018) have recently shown, moving away from this case can yield new insights through the implied convexity/concavity of effort as a function of an activist's terminal block. Because our main results will be about mean variables (e.g., expected firm value), we expect them to hold as long as such an effort function continues to be increasing, the linear case being the simplest. In fact, a key takeaway of our analysis is that we uncover a novel finding regarding the nature of strategic trading (Theorem 1) that does not rely on technological considerations, but instead on a natural property: past orders and future terminal positions across players are *strategic complements*. This can be seen clearly in the value of the leader's holdings, or $(X_T^L + X_T^F)X_T^L$. In particular, the higher the leader's terminal position, the more she benefits from inducing a higher position by the follower, because the extra value is applied to more shares.

Sequential trades It is well-known that activists act fast once reaching 5% to avoid block acquisition becoming too costly. For instance, Brav et al. (2008) argue that "[other] hedge funds frequently acquire significant stakes in targets within hours of learning that an initial fund has taken a position" (p.1757). In turn, Wong (2020) shows that in events where activist hedge funds complete their blocks on the trigger date, there is 36% more abnormality in trading by other investors on the same day—a correlation between competition and rapid completion. These threats come not only from hedge funds, but also insiders (e.g., Chabakauri et al., 2025) and other investors through brokers (e.g., Di Maggio et al., 2019).

Competitive effects and the resulting incentives to move first will be studied in Section 5. What matters for now is that our earlier discussion regarding (i) an average of 1% shares outstanding being purchased during the disclosure window, and (ii) most of these purchases happening on the trigger date, is in line with the sequentiality assumption: hedge funds' substantial purchases make evading competition of utmost importance, and such purchases are indeed completed fast.[12]

[11] Non-trivial intensive margins are also at play when activism focuses on reallocating resources: see Brav et al. (2015) and Brav et al. (2018) in the case of production plants and patents, respectively.

[12] A goal of 6% ownership once crossing 5% facilitates quick block completion. While smaller blocks reflect

3 Activist Trading

In this section we derive the equilibrium trading strategies for our activists. We note that finding equilibria in environments exhibiting strategic block accumulation and endogenous firm values is in general a difficult task—this issue has been noted before in the literature, and is presumably behind the scarcity of results in the area when it comes to multiplayer analyses.[13] To these features, we are adding interdependent private information and trades that occur in sequence, which make the environment asymmetric across traders.

3.1 Equilibrium Construction and Main Result

The first step for finding the coefficients in the activists' strategies is to compute the players' beliefs about each other under the assumption that their counterparties are following linear trading strategies: market makers' beliefs about the firm's value $X_T^L + X_T^F$ determine prices, and hence the activists' costs of block acquisition; in turn, each activist needs to predict how the other activist trades—based on a private block—to correctly assess their trading gains. With normally distributed blocks and noise, as well as linear strategies, these beliefs are routine applications of the traditional projection theorem for Gaussian random variables.

Lemma 1. *Suppose that the activists follow (4). Prior to trading, the firm's share price is*

$$P_0 = \frac{\mu(2 + \alpha_L + \alpha_F + \delta_L + \delta_F)}{1 - \beta_F}, \tag{5}$$

while each activist updates their belief about the other's initial position as $Y_0^i := \mathbb{E}[X_0^{-i}|X_0^i] = \mu + \frac{\rho}{\phi}(X_0^i - \mu)$ *and* $\nu_0^i := \mathrm{Var}(X_0^{-i}|X_0^i) = \phi - \frac{\rho^2}{\phi}$, $i \in \{L, F\}$. *Subsequently:*

- *At $t = 1$, the leader's trade is executed at a price P_1 which obeys*

$$P_1 = P_0 + \Lambda_1 \left[\Psi_1 - (\alpha_L + \delta_L)\mu\right], \text{ where} \tag{6}$$

$$\Lambda_1 := \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \times \frac{1 + \alpha_L + \rho(1 + \alpha_F)/\phi}{1 - \beta_F}. \tag{7}$$

Meanwhile, the follower updates his belief about the leader's terminal position using Ψ_1

hedge funds' desire to only influence firms, this choice is also driven by important ownership costs above 10%: the "short swing rule" (Section 16(b) of the Securities Act) can force a hedge fund to return any profits from reversal trades over a 6 month period; also, insider trader rules put limitations on trading.

[13] See Edmans and Holderness (2017, pp. 579, 625) on the importance of making trades depend on block size in blockholder models, and how the binary firm values typically assumed for tractability in corporate finance models implicitly restrict trades to exogenous amounts independent of initial blocks.

too. We denote this belief $Y_1^F := \mathbb{E}[X_T^L | X_0^F, \Psi_1]$*, which satisfies*

$$Y_1^F = (1+\alpha_L)Y_0^F + \delta_L\mu + \frac{(1+\alpha_L)\alpha_L\nu_0^F}{\alpha_L^2\nu_0^F + \sigma^2}\left\{\Psi_1 - (\alpha_L Y_0^F + \delta_L\mu)\right\}. \tag{8}$$

- *At* $t = 2$ *the follower's trade is executed at a price* P_2 *which obeys*

$$P_2 = P_1 + \Lambda_2[\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F\mu], \text{ where} \tag{9}$$

$$\Lambda_2 := \frac{\alpha_F\gamma_1^F}{\alpha_F^2\gamma_1^F + \sigma^2} \times [1 + \alpha_F + \rho_1(1+\alpha_L)/\gamma_1^F]. \tag{10}$$

In (9)–(10), M_1^F *and* γ_1^F *denote the mean and variance of the market maker's belief about the follower's initial position given* Ψ_1*, while* ρ_1 *denotes the contemporaneous updated covariance of initial positions (see* (A.3) *and* (A.4) *in the Appendix).*

The initial price P_0 is what the leader is *quoted* before placing an order: it results from the market maker's forecast of the activists' terminal positions under (4) and the prior belief about initial blocks absent any trading.[14] Its denominator encodes the feedback from the financial market to the firm's fundamentals: from a time-zero perspective, if higher first-period prices lead to more purchases by the follower, the firm becomes more valuable, which further reinforces the price, and so forth. And due to the interdependence at play, before any trading happens the activists also adjust their prior beliefs about each other using their own blocks: the new mean and variance are denoted Y_0^i and ν_0^i, respectively, $i \in \{L, F\}$.

At $t = 1$, the quoted price P_0 is adjusted in response to the realized order flow Ψ_1, resulting in the *execution price* P_1, given by (6), that the leader must pay. As usual, the price updates in the direction of the unanticipated order flow from the market makers' perspective, or $\Psi_1 - \mathbb{E}[\Psi_1|\mathcal{F}_0] = \Psi_1 - (\alpha_L + \delta_L)\mu$. The intensity of the response, Λ_1, or *price impact*, is deterministic, and computed using the regression coefficient formula

$$\Lambda_t = \frac{\text{Cov}(X_T^L + X_T^F, \Psi_t)}{\text{Var}[\Psi_t]}, \; t = 1, 2 \tag{11}$$

when Ψ_1 is driven by $\theta^L = \alpha_L X_0^L + \delta_L\mu$. Readers familiar with Kyle (1985) will recognize the first ratio in the right-hand side of (7) as the price impact expression if the firm's true value were exogenous according to the leader's initial position. The second ratio is due to the firm's fundamentals being endogenous via trading: the numerator encodes the leader's contribution ($1 + \alpha_L$ term) and, to what extent, depending on the correlation, her trades signal a commensurate contribution by the follower ($\rho(1+\alpha_F)/\phi$ term); in turn, the

[14] Use that $P_0 = \mathbb{E}[(1+\alpha_L)X_0^L + \delta_L\mu + (1+\alpha_F)X_0^F + \beta_F P_1 + \delta_F\mu|\mathcal{F}_0]$ and $\mathbb{E}[P_1|\mathcal{F}_0] = P_0$.

denominator encodes the aforementioned feedback effect.

Importantly, the follower must also update about the leader at this stage to correctly forecast the firm's value. This is encoded in the follower's belief Y_1^F in (8), which revises his initial forecast of the leader's terminal position, $(1+\alpha_L)Y_0^F + \delta_L \mu$, using the realized first-period order flow Ψ_1; we will return shortly to this belief, which plays an important role in our analysis. As the follower enters the second period with this estimate, he is quoted a price P_1 per share, which then updates in response to the realized second-period order flow Ψ_2: this results in the execution price P_2, as in (9), that the follower pays. The logic is as before. First, the unanticipated order flow now requires predicting the follower's trade: market makers resort to (M_1^F, γ_1^F) that estimates the follower's initial block given Ψ_1.[15] Second, price impact Λ_2 is again derived from (11) using the follower's strategy driving Ψ_2 and updated covariance ρ_1 and variance γ_1 terms: the first ratio in (10) is the analog of the first ratio in (7), while the second term relates to the firm's value being affected by trading—the absence of a denominator is due to the leader not trading again in this baseline model (i.e., the feedback channel operates only through Ψ_1 influencing the follower).[16]

Equipped with prices and activists' beliefs we can set up best-response problems for our activists. To this end, recall that activist i's ex post payoff is given by

$$\underbrace{(X_T^i + X_T^{-i})X_T^i}_{\text{total value of block}} - \underbrace{P_{t(i)}\theta^i}_{\text{trading costs}} - \underbrace{\frac{1}{2}(X_T^i)^2}_{\text{activism costs}}, \quad i \in \{L, F\}.$$

Each activist will then decide how much to trade taking as given (i) its counterparty's trading strategy and (ii) prices as in Lemma 1. Letting $\mathbb{E}_i[\cdot]$ denote the expectation operator of activist i at the moment they decide how to trade, and using that $\Psi_{t(i)} = \theta^i + \sigma Z_{t(i)}$ in the expressions for prices (6) and (9) when activist i places an order θ^i, $i \in \{L, F\}$, the activists' optimal trades satisfy the following first-order conditions (FOCs):

$$\begin{aligned} \textit{follower}: \quad & \theta^F \Lambda_2 = \mathbb{E}_F[X_T^F + X_T^L] - \mathbb{E}_F[P_2], \\ \textit{leader}: \quad & \theta^L \Lambda_1 = \mathbb{E}_L[X_T^F + X_T^L] - \mathbb{E}_L[P_1] + X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L}. \end{aligned}$$

Consider the first condition. The follower's optimal trading strategy equates the cost of having market power—the left-hand side, encoding that as the price responds with sensitivity Λ_2, all inframarginal units become more expensive—with the per-unit trading gain from his perspective—the right-hand side, capturing the net gain on each unit purchased due to the

[15] This belief differs from the prior (μ, ϕ) if and only if there is correlation (i.e., if $\rho \neq 0$).

[16] All these expressions hold on and off the equilibrium path, as an activist's trades are hidden from others.

follower's superior information.[17] Importantly, it is easy to conclude from here that, in equilibrium, the follower must trade in an unpredictable way from the viewpoint of market makers: his order is expected to be zero, or $\mathbb{E}[\theta^F|\mathcal{F}_1] = 0$, as is usual in Kyle-type models.[18]

Inspection of the leader's FOC reveals that any departure from this canonical way of trading by the leader must be driven by the last term there, or

$$X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L}, \tag{12}$$

which is a non-trivial continuation value capturing that the leader's incentives to trade are also influenced by the possibility of inducing the follower to build a larger terminal block—and hence, to add more effort. In what follows, we refer to (12) as the *value of manipulation* because it encodes the value associated with influencing a real action—effort provision—by strategically affecting beliefs, which shape the follower's gains from trading.

Towards understanding this latter channel, insert $X_T^F = \theta^F + X_0^F$ and $P_2 = P_1 + \Lambda_2[\theta^F - \mathbb{E}[\theta^F|\mathcal{F}_1] + \sigma Z_2]$ in the follower's FOC. Solving for θ^F in this condition, while using that the follower's trades are unpredictable in equilibrium, we obtain

$$\theta^F = \frac{Y_1^F + X_0^F - P_1}{2\Lambda_2 - 1}, \tag{13}$$

where Y_1^F is the follower's expectation of the leader's terminal position from Lemma 1. The trade-off that the leader faces is clear. On the one hand, a larger trader, by creating a greater order flow, indicates that more effort by the leader is coming, which increases the follower's motive to trade through Y_1^F. But market makers understand this logic and increase the price P_1 quoted to the follower, which weakens his incentive to trade. On top of this, the leader considers her own trading gains and price-impact costs present in her FOC.

Altogether, imposing that the optimal strategies from the FOCs coincide with the linear strategies (4) leads to fixed-point equations for the coefficients $(\alpha_L, \delta_L, \alpha_F, \beta_F, \delta_F)$. The system is complex not only because of the asymmetry in the coefficients (due to the timing of moves), but also because the system must be augmented to check non-trivial second-order conditions (SOCs) stemming from the endogeneity of the fundamentals:

$$\textit{follower}: \quad 0 > 1 - 2\Lambda_2, \tag{14}$$

$$\textit{leader}: \quad 0 > 1 - 2\Lambda_1(1 - \beta_F). \tag{15}$$

[17]Note that here and in the leader's condition, the change in firm value due to a marginally larger terminal block is absent due to effort choices being at an optimum.

[18]Indeed, this follows from market makers having correct beliefs in equilibrium and the law of iterated expectations, namely that $\mathbb{E}[\mathbb{E}_F[X_T^L + X_T^F - P_2]|\mathcal{F}_1] = \mathbb{E}[X_T^L + X_T^F - \mathbb{E}[X_T^L + X_T^F|\mathcal{F}_2]|\mathcal{F}_1] = 0$.

The right-hand side of (14) appears in the denominator of (13): in equilibrium then, increasing Y_1^F (or P_1) moves the follower's trades in the direction discussed. Crucially, the leader's SOC is non-trivial because of the term $1 - \beta_F$, which reflects how the leader's *effective cost of trading* is affected by the feedback at play: if large trades cause the follower to acquire a bigger block, these trades are less costly for the leader than in a setting with exogenous fundamentals where price impact is the only disciplining force. This can happen when β_F—the weight that the follower attaches to P_1 in his strategy—is below but close to 1.[19]

Towards our main result, let

$$\alpha^K := \sqrt{\frac{\sigma^2}{\phi}}$$

denote the well-known slope coefficient in the traditional Kyle insider trading strategy.

Theorem 1. *There is $\underline{\rho} \in (-\phi, 0)$ such that for all $\rho \in (\underline{\rho}, \phi]$ a PBS equilibrium exists.*

(i) If $\rho > 0$, the leader sells on average: $-\delta_L > \alpha^K > \alpha_L > 0$, so $\mathbb{E}[\theta^L|\mathcal{F}_0] < 0$;

(ii) If $\rho < 0$, the leader buys on average: $\alpha_L > \alpha^K > -\delta_L > 0$, so $\mathbb{E}[\theta^L|\mathcal{F}_0] > 0$.

By contrast, $\mathbb{E}[\theta^F|\mathcal{F}_1] = 0$ for all ρ. Further, the equilibrium coefficients in the follower's strategy satisfy: $\alpha_F = \sqrt{\sigma^2/\gamma_1^F}$; $\delta_F < 0$; and $\beta_F < 1$, with sign$(\beta_F) = -$sign(ρ). Finally, it is only when $\rho = 0$ that $\alpha_L = -\delta_L = \alpha^K$: in this case, $\theta^i = \alpha^K(X_0^i - \mu)$, $i = L, F$.[20]

From the result, predictability is a *generic property* of the leader's trading: $\mathbb{E}[\theta^L|\mathcal{F}_0] \neq 0$ if and only if $\rho \neq 0$. This property admits two interpretations. From a governance perspective, when activists know more about each other than the public knows, a leader's accumulation of shares departs meaningfully from that arising when she acts in isolation, in which case the value of manipulation would be absent—as we will show, this can have non-trivial implications on firm values and prices (Section 4.2). From a theoretical perspective, the result is proof that the strategic motive of a trading activist is different from that of insider traders who do not directly influence firms. We emphasize this average measure because it is the cleanest outcome variable through which our stylized model speaks to both governance and strategic considerations. In particular, while there is selling pressure on average when $\rho > 0$, leaders with large blocks do acquire more shares as long as $\alpha_L X_0^L + \delta_L \mu > 0$.

It may seem intuitive that the leader's block-building incentives can be weakened, as less-aggressive trades can be used to lower the price P_1 that the follower is quoted, a force that

[19]The scalar 1 in (14)–(15) reflects a convexity linked to trades affecting firm value via effort choices.

[20]More generally, one can show that $\theta^F = \sqrt{\frac{\sigma^2}{\gamma_1^F}}(\theta^F - M_1^F)$, where (M_1^F, γ_1^F) as in Lemma 1. We can prove uniqueness of PBS equilibria analytically for $\rho \in (\rho_0, \phi]$, where $\rho_0 \in (\underline{\rho}, 0)$. Numerically, uniqueness within the PBS class seems to hold for $\rho \in (\underline{\rho}, \rho_0]$. The threshold $\underline{\rho}$ depends on parameters.

would increase his trading gains. The issue is that, by placing a smaller or negative order, the follower also becomes more pessimistic about the leader's contribution to the firm's value (i.e., Y_1^F can fall). To make the matter more stark, consider what happens when $\rho < 0$. In this case, the leader buys on average, driving P_1 up, yet the follower buys more shares as P_1 increases: from the Theorem, $\beta_F > 0$ in the follower's strategy when $\rho < 0$.

At the center of our finding is the *differential sensitivity* of the follower's and market makers' beliefs resulting from the interdependence at play: as the informational content of order flows varies across the follower and market makers, market signals can be used to communicate with, and influence, others. To see how this mechanism operates, use the expression for the follower's order (13) and that $\Psi_1 = \theta^L + \sigma Z_1$ to write the value of manipulation (12) as

$$X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L} = \frac{X_T^L}{2\Lambda_2 - 1} \left[\frac{\partial Y_1^F}{\partial \Psi_1} - \frac{\partial P_1}{\partial \Psi_1} \right], \tag{16}$$

where the bracket encodes how the follower's informational advantage, embedded in her perceived trading gains, is affected by the first-period order flow. To the follower then, an unexpectedly large Ψ_1 is a signal that only the leader will exert more effort, because the follower privately knows what his own willingness to intervene is. But since market makers are uncertain about both the *leader's and follower's* contributions, correlation matters. If $\rho > 0$, market makers infer that the follower will add more value too, so P_1 reacts more strongly to Ψ_1 than Y_1^F (to be established shortly). As (16) becomes negative—reflecting that block acquisition by the follower is discouraged after a positive surprise in Ψ_1—the leader's incentives to acquire stock fall. Conversely, if $\rho < 0$, P_1 reacts less strongly to Ψ_1 than Y_1^F: for market makers, signals that indicate larger contributions by the leader are offset by a perception of smaller contributions by the follower. With relatively less sensitive prices, the value of manipulation is now positive: the leader buys more aggressively to increase the follower's trading gains through inducing more mispricing that can be exploited.

Only when $\rho = 0$ do the discussed sensitivities coincide: absent any interdependence, both the follower and market makers only update about the leader's effort from Ψ_1, and with the same intensity given the common prior. In this knife-edge case, the usual trading strategy in "gap" form $\alpha^K(X_0^i - \mu)$, $i \in \{L, F\}$, emerges, as the final part of the Theorem shows. The left panel in Figure 1 illustrates typical coefficients in the leader's strategy; in turn, the right panel plots the sensitivities $\partial Y_1^F / \partial \Psi_1$ and $\partial P_1 / \partial \Psi_1 = \Lambda_1$.

We conclude with two observations related to this figure. First, in the left panel, the departures from the levels $\pm\alpha^K$ capture the extent of manipulation by the leader: if $\rho > 0$, the leader underweighs the importance of her block in her strategy in favor of the prior

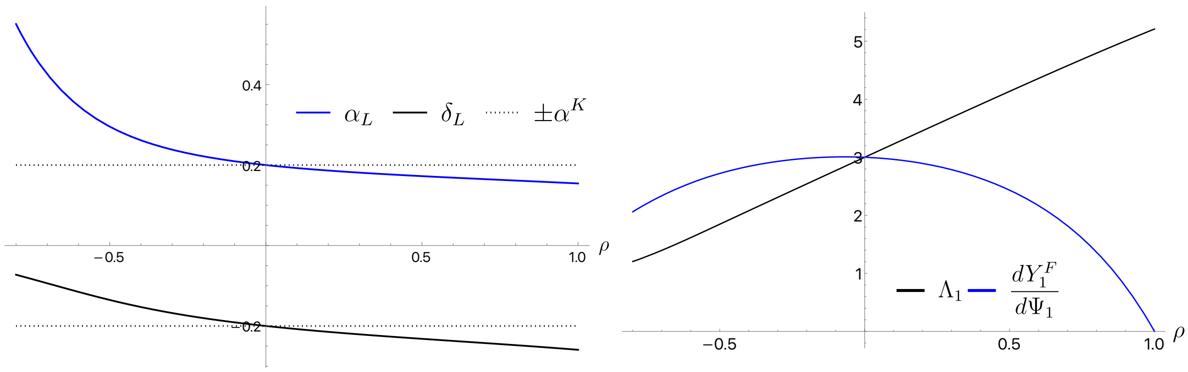


Figure 1: Right panel: leader's strategy coefficients, along with $\alpha^K := \sqrt{\sigma^2/\phi}$. Left panel: sensitivities of P_1 and Y_1^F with respect to Ψ_1. Parameters values: $\phi = 1$, $\sigma = .2$.

mean μ to generate downward pressure on prices. In practice, this means leaders with larger blocks distort their purchases more in absolute terms, because the gains from influencing the follower are applied to more shares. Further, as $|\rho|$ grows, the deviation is more acute because the first-period order flow becomes statistically more informative (for better or worse) about the follower's contribution.[21] The observed asymmetry in the departures for positive and negative ρ is due the differential effect of the feedback channel on the convexity of the leader's problem—it relates to the threshold $\underline{\rho} < 0$ for existence and is discussed in Section 6.

Second, related to the right panel, a direct corollary of the Theorem is that

$$\text{sign}\left(\frac{\partial Y_1^F}{\partial \Psi_1} - \frac{\partial P_1}{\partial \Psi_1}\right) = \text{sign}(\beta_F) = -\text{sign}(\rho),$$

and so the sensitivities of P_1 and Y_1^F to Ψ_1 rank as we anticipated.[22] This also explains why Y_1^F does not need to be carried independently in the follower's strategy: the contributions of Y_1^F and P_1 are subsumed in $\beta_F P_1$, because Y_1^F is linear in Ψ_1, and hence affine in P_1.

3.2 Robustness

Let us briefly discuss model variations that deliver qualitatively identical results.

Other forms of private information As argued, the fact that private information is about initial blocks is not essential when it comes to the type of strategic behavior uncovered.

Proposition 1. *Suppose that the activists' initial blocks are public, and consider the following variations of our model (in each case, the rest of the assumptions remain unchanged):*

[21] The observed decreasing patterns are established in Proposition A.6 in the Appendix.

[22] The last equality comes from Theorem 1. As for the first, use (12) under $\theta^F = \alpha_F X_0^F + \beta_F P_1 + \delta_F \mu$ to obtain $X_T^L \frac{\partial \mathbb{E}_L[X_T^F]}{\partial \theta^L} = X_T^L \beta_F \Lambda_1$ and equate with (16). The result then follows from the SOCs holding and that $\beta_F < 1$ (also from Theorem 1; which, through the leader's SOC, implies that $\Lambda_1 > 0$).

(a) *Exogenous components of firm value: the firm's (share) value is* $V^L + V^F + W^L + W^F$, *where* $V^i \sim \mathcal{N}(\mu, \phi)$ *is exogenous and is activist* i*'s private information,* $i \in \{L, F\}$.

(b) *Activist productivity: Activist* i*'s cost of effort is* $\frac{(W^i)^2}{2} - \zeta_i W^i$, *where* $\zeta^i \sim \mathcal{N}(\mu, \phi)$ *is exogenous and is activist* i*'s private information,* $i \in \{L, F\}$.

Let $\mathrm{Cov}(\xi^L, \xi^F) = \rho \in [-\phi, \phi]$, $\xi \in \{V, \zeta\}$. *If* $X_0^L > 0$, *then in both (a) and (b) there is a linear equilibrium with* $\mathbb{E}[\theta^F|\mathcal{F}_1] \equiv 0$, *while sign*$(\mathbb{E}[\theta^L|\mathcal{F}_0]) = -$*sign*$(\rho)$.

Both variations capture "activist expertise": in (a), the activists can be seen as each having superior information about a different division of the target; in (b), ζ can be seen as an activist's ability to unlock firm value at lower private costs.[23] In either case, the same logic explains why the relationship between the underlying correlation and the leader's average trade is preserved: terminal positions continue to be private information and, as they increase in X_0^L, leaders with larger initial blocks benefit more from the follower's effort.[24]

The leader trades again Because the leader continues to have relevant private information in the second period, she may benefit from trading once again along with the follower. Figure 2 contrasts the leader's average trade in the first period of such a model with that in our baseline model as a function of ρ, showing that a similar distortion from a neutral trade arises too. The magnitude of the departure is smaller though, due to a competition effect: the follower scales back in response to the presence of the leader, which in turn reduces the value of manipulation for the leader activist in the first period. Section 5 expands on variations of this theme (e.g., by varying the number of followers); the bottom line is that our choice of model is purely driven by tractability reasons, bringing us to the next point.

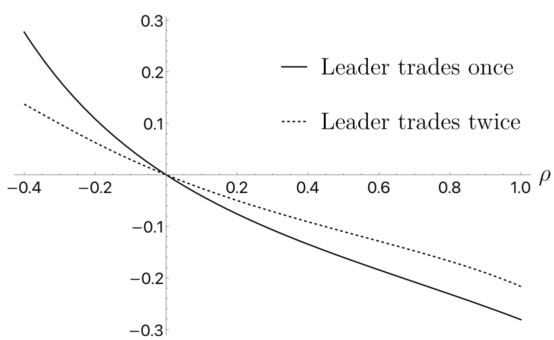


Figure 2: Leader's expected period-one trade. Parameter values: $\mu = \phi = 1$, $\sigma = .8$.

[23] Brick et al. (2024) studies how hedge funds' industry experience affect activism.

[24] In these variations, leaders with larger (now public) initial blocks can buy less shares than smaller blockholder counterparts: see (A.30) which features a negative weight on X_0^L if $\rho > 0$—while this force was present in a PBS equilibrium when blocks were private, it was counteracted by a block being used as a source of informational advantage. See also (A.31) for the leader's terminal position being increasing in X_0^L.

Repeated trades with both activists With activists that trade in sequence, and only once each, the presence of correlation is key to generating the intertemporal effect studied. But this is not needed with multiple rounds of trading featuring both activists in all rounds.

Proposition 2. *Suppose that both activists trade in each of the two rounds prior to the effort stage, and that initial blocks are as in the baseline model with $\rho \geq 0$. If $\sigma > 0$ is sufficiently small, there is a linear equilibrium in which both activists (strictly) sell on average at $t = 1$.*

Here, each activist can learn about their counterparty's contribution to the firm while market makers cannot disentangle the individual contributions, leading our mechanism to emerge even if there is no correlation. Indeed, while market makers rely on the total order flow to learn about the firm, the activists can construct a residual signal net of their own trading to learn about their counterparty, because they know how their own trades add to the order flow. As the former signal has more fundamental uncertainty—because it carries two pieces of unknown information—prices are relatively more sensitive even when $\rho = 0$. This model is considerably more complex, so we prove existence around $\sigma = 0$, which simplifies the equilibrium conditions. See Section I in the Internet Appendix for details.

The key conclusion from this exercise is that, for our mechanism to emerge, the activists just need to have a better ability to filter information about each other than market makers do—the initial correlation in the baseline model being one possibility. We discuss related evidence on this topic in Section 5.3, where we take a more institutionally oriented view.

Passive leader If the leader instead does not exert effort, thus behaving like a passive fund, the same mechanism ensues. We prove the following in Internet Appendix Section II.

Proposition 3. *Consider our baseline model but with only the follower exerting effort. Then, the leader sells on average for all $\rho \neq 0$. Instead, if private information is about exogenous components of the firm as in Proposition 1(a), sign$(\mathbb{E}[\theta^L|\mathcal{F}_0]) = -$sign$(\rho)$.*

If the leader does not intervene, there is nothing to learn about her input to the firm; but as long as blocks are correlated, Ψ_1 informs market makers about the follower's contribution. Thus, Y_1^F ceases to be payoff relevant for the follower, while P_1 responds to Ψ_1 when $\rho \neq 0$, causing the leader to sell on average. In turn, buy orders reemerge if private information is about exogenous fundamentals and negatively correlated: the follower can now learn about the firm's value from the first-period order flow, so Y_1^F—his forecast of the firm's exogenous component known by the leader—now responds to Ψ_1, and the usual logic applies.

Friendliness towards the firm Some blockholders are friendly to firms, resisting the change brought about by activists. To capture such a conflict, we add $\kappa X_T^L W^F$, $\kappa \in (0, 1)$, to the follower's quadratic effort costs: effort W^F by the follower—for example, in supporting the leader's campaign—entails losses that grow with the leader's terminal block—for example, losing more private benefits as the leader's degree of control grows. For simplicity, we keep the leader's payoffs unchanged: she is "benevolent" in that she is purely motivated to unlock value that benefits all shareholders, which comes at the expense of the follower.

The follower's effort now becomes $X_T^F - \kappa Y_1^F$, with the second term encoding *negative activism*: engaging in actions that lower firm value, or that oppose the leader's value-enhancing change, in proportion to his expectation of the leader's contribution. In this situation, trading aggressively is costly to the leader not only because of price impact, but also because the firm's *true* value $X_T^L + X_T^F - \kappa Y_1^F$ moves against the leader due to the follower's counteraction. This channel is non-trivial because it may seem that a drop in fundamentals due to a marginal increase in Ψ_1 boosting Y_1^F is perfectly offset by an identical change in the price $P_1 = \mathbb{E}[X_T^L + X_T^F - \kappa Y_1^F | \mathcal{F}_1]$. However, there is an important difference: price changes apply to newly acquired shares only, while negative activism applies to the entire block. To protect the value of her *initial block* from the follower's negative activism then, the leader reduces her block acquisition, thereby signaling to the follower that only moderate change is coming. Importantly, this phenomenon is most significant when $\rho = 0$, a situation where it is not possible to affect the follower's behavior through influencing his purchases.

Proposition 4. *When facing a follower that is friendly to the firm as above, the leader always sells on average in the first period when $\rho = 0$.*

See Section III in the Internet Appendix. Figure 1 there shows how this new effect amplifies the mechanism already present in the baseline model, that is, the leader's benefit of reducing her purchases when correlation is positive.

4 Predictions

The predictability of trades determines the extent to which initial blocks are expected to change, and hence it speaks to the question of whether ex ante trading promotes or suppresses activist interventions via costly "voice." Because stock prices reflect the market's expectation of firms' true values, our model can link block interdependence, via the implied predictability of trades, with average prices during activism events. We first present theoretical results pertaining to market outcomes in our model, and then connect these findings with the existing empirical literature.

4.1 Market Outcomes

Returning to our baseline model, we will examine market outcomes from an ex ante perspective, that is, averaging across all possible blocks for the leader and follower.[25] To simplify notation, we write $\mathbb{E}[\cdot]$ for $\mathbb{E}[\cdot|\mathcal{F}_0]$ (which averages using the prior distribution of blocks), and note that $\mathbb{E}[X_T^F] = \mathbb{E}[X_0^F] = \mu$ because the follower's trades are neutral on average—thus, only the leader ends up non-trivially affecting the firm's value through her trading. It is easy to see then that the firm's ex ante value and price satisfies $\mathbb{E}[W^L + W^F] = \mathbb{E}[P_1] = \mathbb{E}[P_2] = (2 + \alpha_L + \delta_L)\mu$. Recall that $\mu > 0$ is used for interpretations.

Theorem 2. *In any PBS equilibrium:*

(i) Governance and interdependence: $\mathbb{E}[W^L + W^F] \leq 2\mu$ *if and only if* $\rho \geq 0$ *(with strict inequality if* $\rho \neq 0$*). Further, ex ante firm value monotonically decreases with* ρ*.*

(ii) Efficacy of multiplayer attacks: $\mathbb{E}[W^L + W^F] > \mu$ *for all* ρ *such that a PBS equilibrium exists (i.e.,* $\rho > \underline{\rho}$*, where* $\underline{\rho}$ *is as in Theorem 1).*

(iii) Effect of market liquidity: Fix $\rho > 0$*:*

(iii.1) Both $\lim_{\sigma \to +\infty} \mathbb{E}[\theta^L]$ *and* $\lim_{\sigma \to +\infty} \{\alpha_L - \sqrt{\sigma^2/\phi}\}$ *exist and take a negative value.*

(iii.2) $\lim_{\sigma \to 0} \mathbb{E}[\theta^L] = 0$ *and* $\lim_{\sigma \to 0} \{\alpha_L - \sqrt{\sigma^2/\phi}\} = 0$*.*

The first part of the theorem illustrates how the leader's behavior operates to amplify or mitigate the static free-riding incentives that are inherent in multiplayer engagements. Concretely, absent any trading, ex ante firm value amounts to $\mathbb{E}[X_0^L + X_0^F] = 2\mu$ due to each activist exerting effort according to their own block. When correlation is positive and the leader sells on average, firm value falls below this benchmark—the leader effectively offloads activism costs on the follower, and the extent of free riding grows. Conversely, when correlation is negative, the leader is inevitably forced to bear more of the activism costs and develop more skin in the game to entice the follower to build his block—remarkably, the manipulation at play now mitigates the extent of free riding. The last part of (i) simply says that we can analytically show that the inefficiencies grow as ρ increases.

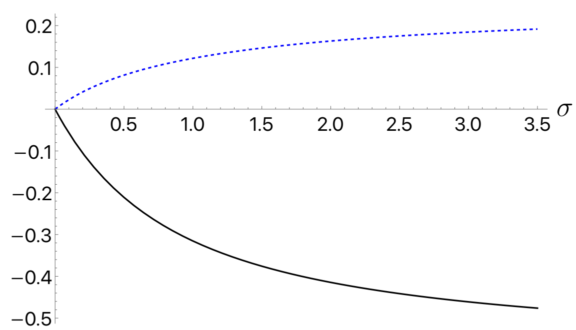
Turning to (ii), note that when only one activist is present—and hence the manipulation motive is trivially absent—trades are unpredictable and ex ante firm value is μ. The theorem

[25] While selection effects can be at play in activism events, this measure is not an unreasonable approximation. On the one hand, while small blockholders may be perceived as less relevant, they are gaining prominence: Brav et al. (2021b) documents an example of a hedge fund owning 0.02 percent of outstanding stock and yet obtaining important concessions. On the other hand, the largest blockholder in a firm typically is a passive fund; further, the largest blockholders are less likely to trade (e.g., Lewellen and Lewellen, 2022).

then asserts that multiplayer engagements always deliver more value than single-player attacks. By always we mean irrespective of the value of ρ, and hence independent of whether the free-riding motive is exacerbated. This is a reflection of the leader not reversing her initial position on average (i.e., $\text{sign}(\mathbb{E}[X_0^L]) = \text{sign}(\mathbb{E}[X_T^L])$), which would be needed for the firm's value to fall below the single-player case. In other words, despite the manipulation, the leader is effectively engaged in positive activism in this type of equilibrium.

Finally, part (iii) explores the effect of order flow volatility when $\rho > 0$; we use limiting values of σ to obtain analytical comparisons. As the market becomes more liquid when σ grows, the price P_1 is less responsive to order flow, suggesting that less manipulation is optimal. But fundamentals are endogenous: as the leader trades more aggressively due to her limited ability to move prices, she builds a larger terminal block. Through this channel—the leader's marginal incentive to manipulate, captured by X_T^L, being endogenous—the value of manipulation grows, despite the lower price impact of trades. This is what part (iii.1) states: as σ grows large, there is a non-trivial degree of manipulation in that $|\alpha_L - \sqrt{\sigma^2/\phi}| \neq 0$ in the limit, a situation where the leader sells a finite amount. Conversely, when $\sigma \searrow 0$ and the market is infinitely illiquid, the leader naturally ceases to trade at all (part iii.2).

Figure 3 extends (iii) for intermediate values of σ, and for a negative ρ: the leader's expected trade (also a measure of manipulation), $(\alpha_L + \delta_L)\mu$, increases in magnitude monotonically as σ grows.



— $\alpha_L + \delta_L(\rho = 1)$ ····· $\alpha_L + \delta_L(\rho = -.2)$

Figure 3: The leader's expected trade as a function of σ for two levels of correlation: $\rho = 1$ (solid) and $\rho = -.2$ (dashed). When $\rho > 0$, the limit is finite as $\sigma \to \infty$. Parameter values: $\phi = \mu = 1$.

4.2 Connection with the Empirical Evidence

The empirical literature on activism focuses on examining trading volume and prices around disclosure events to assess the real effects of this practice. The challenge with multiplayer attacks, however, is that not all of the activists involved have to disclose their positions.

Thus, the analyses on this topic either study events featuring multiple funds with publicly known positions or resort to indirect methods to infer properties of multi-activist interactions.

In the first category, most relevant to us is the work by Becht et al. (2017) who examine activism events featuring disclosing hedge funds in attacks in Asia, Europe and the United States. Of over 1,700 events in their sample, a quarter involve multiple activists targeting the same firm. A key finding is that these events perform "strikingly better" (p. 2933) than their single-player counterparts: their accumulated total stake as a whole is larger, and so are the abnormal returns observed. These findings are consistent with our robust finding in Theorem 2 (ii) that multiplayer engagements add more value for all ρ as in Theorem 1.

In terms of indirect methods, the closest evidence in favor of our mechanism is provided by Flugum et al. (2023), who identify institutional investors that access EDGAR files of specific activists' targets in the days prior to 13D filings.[26] There are three key findings about trading patterns in the presence of such "informed followers." First, disclosing activists tend to acquire more shares before these followers access files, with over 95% of an activist's purchases being free from these followers' competitive threat—i.e., trades happen in sequence. Second, these followers are more likely to increase their holdings before filing occurs, consistent with informational advantages driving block formation. Third, when these followers are present, activists are less likely to increase their stakes beyond those reported in the initial filing. This finding, while pertaining to block acquisition after disclosure, resembles the substitution effect arising when $\rho \geq 0$ in our model: as followers develop skin in the game, an activist's need to continue growing her block—say, to ensure winning a campaign—is weakened.

Let us conclude this section by discussing one possible avenue that the empirical literature can pursue building on our work. The starting point is that our model delivers price patterns analogous to abnormal buy-and-hold returns observed around disclosure events. Indeed, observe that if activism is not at play—i.e., fundamentals are exogenous from the activists' perspective—trades should be neutral on average because they must respond to informational advantages only. As blocks should not change on average, in such "normal" times the average price would be 2μ. But our findings point to average prices departing from this benchmark when activism is possible: by Theorem 2 (i), prices should be *abnormally low when* $\rho > 0$*, and vice versa*; or, in relative terms, *abnormal returns should fall as* ρ *grows.*

The empirical challenge is then to identify characteristics of target firms with enough underlying variation in block interdependence to test the above hypothesis. Market capitalization can be one such variable, which can be exploited by leveraging the work on blockholder interdependence by Hadlock and Schwartz-Ziv (2019). Concretely, while these authors document an overall negative interdependence among blockholders of a variety of

[26]This is done through identifying institution-owned IP addresses that download such information.

classes, this conclusion reverses for a subset of "strategic investors" including hedge funds and private equity: the likelihood of observing a block belonging to this investor type increases when a block from the same category is present at a firm. Further, the authors show that this positive interdependence is stronger as the associated blocks become smaller[27]—at the same time, however, Lewellen and Lewellen (2022) document that in the U.S., the average (institutionally owned) block size falls with capitalization, at least from mid- to large-cap firms in the U.S.[28] A second characteristic could be to account for the presence or absence of short positions, as a mix of blockholders with long and short positions in our model is more likely when correlation is negative. Furthermore, consistent with the previous discussion, highly shorted stocks tend to come from small-cap firms (e.g., Asquith et al., 2005), and also exhibit more disagreement about their prospects (e.g., Diether et al., 2002).

Put together then, these facts can be taken as suggestive of positive (negative) interdependence increasing (weakening) as market capitalization grows, or as short positions are less frequently observed, which in our model would mean gradually lower abnormal returns. This is exactly what Brav et al. (2021b) find for activist hedge funds between 1994 and 2018 in the U.S.: around disclosure and trigger dates, there is substantially more abnormality for small-cap firms, followed by mid-cap, and lastly for the largest firms (even featuring negative abnormality in this latter case). Our prediction would also be in line with Li et al. (2022), who show that when target firms feature investors with large short positions, the abnormal returns are higher than those observed when these investors are absent.

That being said, it is important to acknowledge that these studies do not perfectly fit our setup: for example, hedge funds and private equity are commingled in Hadlock and Schwartz-Ziv (2019), and the firms analyzed need not be targets of activist campaigns; meanwhile, in Li et al. (2022) the investors holding negative positions may not actively try to take actions that undermine value (as would occur in our model). However, further investigating this line of inquiry for "trading blockholders"—hedge funds in particular, whose relevance will be discussed in Section 5.3—is an important endeavor because it would confirm a fundamental dichotomy at play: their ability to overcome collective-action problems may be substantial in smaller firms, but less so in larger ones, purely for strategic reasons.

This conclusion may have meaningful consequences if we expect groups of activists to conglomerate more frequently precisely around large firms—Artiga González and Calluzzo (2019) confirm this type of clustering for hedge funds that are in geographic proximity, and

[27]See Table C.1 in their Appendix, for blocks above 5% and 10%.

[28]From Table 3 in their paper, which encompasses institutional investors beyond hedge funds, (i) the largest, (ii) top 2 and 3 and (iii) the 4-10 blockholders in mid-cap firms have larger fractional holdings on average than their counterparts in large-cap firms. Brav et al. (2008) argues that blocks are smaller in large-cap firms due to the funds needed to acquire a sizable stake in this segment growing considerably.

argue that it is consistent with cost-sharing motives. Moreover, as we have already argued, activists' interest in these types of firms is growing; in 2024, for instance, mega-cap firms constituted 30% of major activists' targets.[29]

5 First-Mover Advantages and Wolf Packs

5.1 Coordination in the timing of trades

To assess the benefit of acting as a leader, we compare trading strategies and payoffs in our model with those in a one-shot trading game in which both activists trade simultaneously.

Proposition 5. *In a symmetric PBS equilibrium of a one-shot interaction with simultaneous moves, the activists trade according to* $\theta^i = \sqrt{\frac{\sigma^2}{2\phi}}(X_0^i - \mu)$, $i = L, F$. *Also, there is a region around* $\rho = 0$ *in which both traders get a higher ex ante payoff if they move sequentially.*[30]

Consider the last part of the result first, which is about a *mutually advantageous coordination* of trades: both the leader and the follower can be better off by building blocks in sequence. In Figure 4, this happens over a wide range of interdependence. To the right of that region, only being a leader pays off: the benefit of acting as such increases with ρ because it is easier to influence market markers' beliefs and hence more activism costs are offloaded on the follower; instead, the follower does not have this ability and faces the downside of encountering more informed market makers. To the left, being a leader ceases to be profitable: if trading simultaneously, the other activist's likely opposite position means access to liquidity at low prices. But being a follower would pay off, because the leader bearing more activism costs offsets the loss from market makers being better informed.

The previous coordination result, stating a preference for sequential trading, is important because there is a channel through which the stronger competition that arises with simultaneous moves could a priori benefit the activists: namely, that the firm's value is endogenous. To illustrate, consider the first part of the proposition stating that the traders scale back when another activist with market power is present (which we anticipated in Section 3.2): the slope $\sqrt{\sigma^2/2\phi}$ is smaller than in the single-player version $\alpha^K := \sqrt{\sigma^2/\phi}$. However, with enough symmetry, their combined order is still larger than the single-player benchmark: if $X_0^L = X_0^F > \mu$ for instance, $2\sqrt{\frac{\sigma^2}{2\phi}}(X_0 - \mu) > \sqrt{\frac{\sigma^2}{\phi}}(X_0 - \mu)$, implying that the competition effect at play delivers a more pronounced impact on the firm's ex post value. The problem

[29] See Barclays 2024 Review of Shareholder Activism.

[30] Here, there is a negative threshold level of correlation above which a symmetric PBS equilibrium exists and is unique. Indeed, price impact becomes very small when $\rho \ll 0$ due to the two activists' opposing trades, and it cannot offset the convexity from endogenous fundamentals in the players' SOCs.

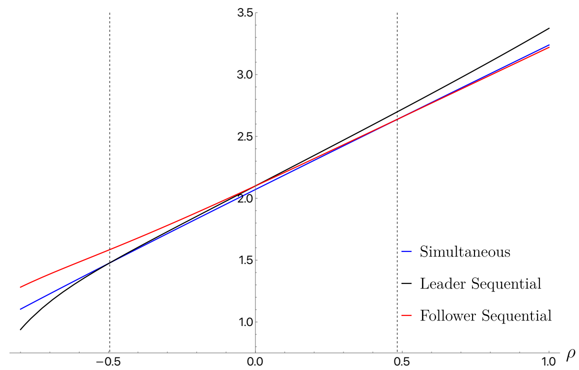


Figure 4: Leader's and follower's payoffs under sequential vs. simultaneous moves. Between the dashed vertical lines, both players prefer sequential moves. Parameters: $\mu = \phi = 1$, $\sigma = .2$.

is that such an outcome need not be stable: to maximize profits, an activist may favor a lower firm value in exchange for lower acquisition and activism costs. This demonstrates the importance of examining how blockholders' *private* benefits and costs from interventions can affect governance, as Edmans and Holderness (2017) emphasize.

5.2 Other Factors Favoring Leader-Type Behavior

Block size Do larger blockholders benefit from acting as leaders? To explore this question, Figure 5 plots the expected payoff of a first (top curve) and second (lower curve) mover conditional on a block X_0^i (horizontal axis), net of the payoff of moving simultaneously with the counterparty; correlation is positive and blocks weakly above average ($\mu = 1$). As blocks grow past a threshold close to the mean, the net benefit of acting as a monopolist in any period is increasing in block size—and being a leader is always preferred to being a follower.

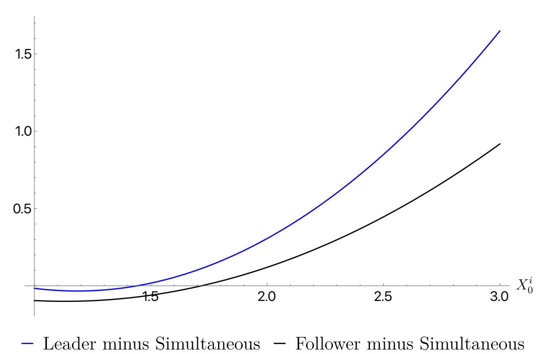


Figure 5: Expected payoff of $i = L, F$ conditional on X_0^i net of simultaneous-move counterpart. Parameter values: $\mu = \phi = 1$, and $(\rho, \sigma) = (0.5, 1)$.

The key to this finding is how the competition effects just discussed play out conditional on block size. Specifically, when correlation is positive, owning a larger block is a signal of the other activist having a large block too (recall Y_0^i, $i \in \{L, F\}$, from Lemma (1)). Thus, it is a signal of high acquisition costs when trading simultaneously, so there are increasing gains from trading in isolation as blocks grow. But this need not be the case for very small blockholders, such as those around μ in the figure. Indeed, since these activists do not change their positions much, nor expect much competition either, acquisition costs are less relevant: the positive effect that competition has on firm value can dominate slightly for them.

Activist productivity Do more productive activists benefit from acting as leaders? Consider now an extreme asymmetric version of our baseline model: there is a productive activist with effort costs $\frac{1}{2\zeta}W^2$, where $\zeta > 0$ is commonly known scalar, and a second activist who cannot affect the firm's value (e.g., $\zeta \searrow 0$ for this player). As this latter activist acts like the passive fund from Section 3.2, we can use this example to uncover the extent to which the benefits associated with being able to manipulate beliefs are linked to the ability to affect fundamentals. To maximize the scope for the former channel, we assume perfect correlation.

Proposition 6. *Suppose that $\rho = \phi$. For sufficiently small $\sigma > 0$, if the productive activist leads and the unproductive activist follows, their respective ex ante payoffs are higher than in the opposite configuration. But if instead both blockholders are equally productive according to $\zeta > 0$, leading is always better than following for $\sigma > 0$ small.*

The productive activist prefers to lead rather than to follow: despite not being able to manipulate in either case (since the other activist is unproductive), her informational advantage is larger when leading. Interestingly, the unproductive blockholder becomes *worse off* when he leads: being able to manipulate traps him in an inferior outcome—at least when σ is small (and hence so are their trades), which makes our analytical results simpler.

This means that being able to directly intervene in the firm can be key for monetizing market signals as a tool to influence others. Indeed, by the final part of the proposition, the first-mover advantage reappears when the productivity of the originally passive activist is restored. The difference lies in the value of holdings for this player when he leads, which takes value $\zeta(X_T^L + X_T^F)X_T^L$ if productive, but only $\zeta X_F^L X_T^L$ if unproductive: in the first case, being able to add value in line with the block limits the extent of the manipulation—because the forgone trading gains are larger—in a way that is not detrimental to overall profits.

Multiple followers Let us quickly explore the effect of varying the number of followers. Concretely, our original follower is split among N individuals: each has an identical initial block $X_0^F \sim \mathcal{N}(\mu/N, \phi/N^2)$, with $\text{Cov}(X_0^L, X_0^F) = \rho/N$. As this specification keeps fixed the total amount of both (i) follower-associated uncertainty and (ii) follower effort absent any trading, any change in outcomes must come from a change in the strategic behavior by the followers. The firm's value continues to be the sum of all of the activists' terminal positions.

Motivated by the notion of similarity attributed to "wolf pack activism"—which we discuss shortly—we set $\rho > 0$. We use $M_1^F := \mathbb{E}[X_0^F|\mathcal{F}_1]$ and $\gamma_1^F := \mathbb{E}[(X_0^F - M_1^F)^2|\mathcal{F}_1]$ for the market makers' posterior about each individual follower's position given Ψ_1.

Proposition 7. *Fix $\rho \in (0, \phi]$. In the unique PBS equilibrium, each follower trades via $\theta^F = \alpha_F(X_0^F - M_1^F)$, where $\alpha_F = \sqrt{\frac{\sigma^2}{N\gamma_1^F}}$. Also, α_F is increasing in N; both α_L and the firm's ex ante value decrease in N; and the leader's ex ante payoff grows $\sim \sqrt{N}$ for N large. If $\rho = \phi$, the leader's gain from moving first also grows $\sim \sqrt{N}$ for N large.*

That α_F increases with N reflects stronger competitive effects: as each follower possesses a smaller fraction of the total existing private information—captured in $\gamma_1^F \propto 1/N^2$—the individual contribution to price impact falls, and trading more aggressively is optimal.[31] The value of manipulation then grows, causing both α_L and the firm's ex ante value to fall with N. In turn, the leader's ex ante payoff grows at a rate of $\sqrt{N}$ for N large, due to the interaction term $\mathbb{E}[X_T^L N X_T^F]$: the followers' more aggressive trading leads to terminal positions that covary more strongly with the leader's. Two additional points are in order. First, if correlation is perfect we can show analytically that the leader's expected payoff net of the simultaneous-move benchmark has the same growth rate (last part of the proposition). Second, related to the covariance effect, Figure 6 shows that N and ρ are complements: as ρ grows, the leader benefits more from having additional followers because their increased trading intensity is more in line with the leader's. This benefit is likely less important when initial blocks are negatively correlated due to the risk of efforts becoming misaligned.

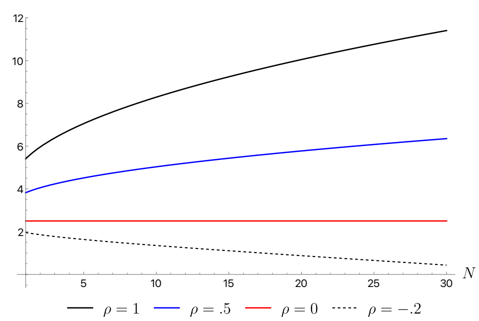


Figure 6: Leader's expected payoff as a function of the number of followers, for various levels of covariance. Other parameter values: $\phi = \mu = \sigma = 1$.

5.3 Wolf Packs

Our model builds on the following hypotheses:

[31] See Edmans and Manso (2011) and Kyle (1989) for results relying on an identical logic.

H1. *Sensitivity to mispricing*: The activists have a strong tendency towards monetizing prevailing trading gains, accumulating more shares as they become more underpriced;

H2. *Non-cooperative behavior:* the activists do not employ formal agreements; rather, they maximize their own profits, understanding their counterparties' incentives and how trading and the price mechanism can be used to their own advantage;

H3. *Similarity*: the activists hold similar stakes in a statistical sense, in that blocks are not too negatively correlated—this favors the emergence of a leader. For intermediate levels of interdependence, coordinating the timing of trades is mutually beneficial;

H4. *Moderate stakes*: since in practice there is a fixed number of shares, similarity (in the above sense) requires the activists to have small to moderate stakes. Otherwise, the possibility of sequential trades is undermined from the perspective of market makers;

H5. *Multiple small followers.* If there is enough similarity (i.e., $\rho > 0$), competition effects associated with the presence of multiple followers make it increasingly profitable for a hypothetical leader to emerge; this effect is reinforced if a leader has a larger block.

Natural candidates to satisfy H1–H5 are hedge funds: in particular, the so-called *wolf-pack activism* phenomenon, whereby multiple hedge funds of small to moderate size attack a firm in parallel—and in a seemingly non-cooperative manner—after a leader fund has built a stake in the target.[32] Starting with H1, these funds—the quintessential example of exploitation of mispricing opportunities—have followed a "value investor" approach to activism by targeting firms underpriced relative to their potential, as measured by a large book-to-market value ratio or a low Tobin's q (Brav et al., 2008; Brav et al., 2021b). In our model, the intensive margin of intervention increases—through a more aggressive block-building—as the firm is more underpriced.

Regarding H2, there are substantial costs associated with being perceived as a "group" in the U.S.[33] The key issue is that an organized set of activists is treated as a single entity with a block equal to the sum of its components. In this situation, there are potential legal fees if the target firm alleges a violation of disclosure requirements (e.g., not disclosing when the aggregate block surpasses 5%), which would be absent if the activists were individually below the 5% threshold and acted non-cooperatively. On the other hand, complying with disclosure rules means that a group necessarily invites undesired competition *before* achieving a desired block size, thereby making block acquisition more costly. Additionally, the target

[32]See Becht et al. (2017), Brav et al. (2021a), Briggs (2007) and Coffee Jr and Palia (2016).

[33]Section 13(d)(3) of the Securities Exchange Act. See for instance, Coffee Jr and Palia (2016), pp. 24–26 for an expanded discussion on this topic.

firm may bar the acquisition of more shares by the group members—the identities of which are revealed upon disclosure—which may preclude the success of any engagement.[34]

With explicit agreements being risky (and indeed rarely seen, as Becht et al. (2017) argue), activists can rely on their common understanding of the environment to act in parallel—for the U.S., two factors have heightened the role of *strategic considerations* in recent decades. First, ownership has become more concentrated. Second, changes in SEC regulation allow activists to communicate in a limited manner without this being characterized as insider trading or trading as a group, or without triggering costly filing requirements.[35] The result is that there are fewer relevant players present, each with the power to influence market outcomes, and with communication channels that favor the development of implicit, more tacit, agreements: one possibility being to immediately attack after others do, with the common knowledge of it triggering leader behavior in the way that we propose.

Regarding similarity (H3), hedge funds' niche business strategies strongly suggest block similarity in a statistical sense: this is in line with the findings on strategic investors by Hadlock and Schwartz-Ziv (2019), who argue that positive interdependence is indeed indicative of similar investment styles. Similar trading strategies also translates into similar research, and hence into an overlap in potential targets that reinforces block interdependence. And as we argued, hedge funds stakes are relatively small (H4), consistent with a goal to influence firms but not exert control (e.g., Brav et al., 2021b): yet, this blockholder category is argued to be the only one with a proven record of significantly affecting firms (Brav et al., 2008).

While obtaining direct evidence on wolf packs is difficult due to their inherently secretive nature, indirect evidence of a potential wolf pack orchestration by a leader is provided by Wong (2020). Regarding competition effects (H5), he shows that on the trigger date of campaigns featuring a single 13D filer, the trades by this hypothetical leader only explain 25% of the turnover observed, with the unexplained component averaging 240% of that in normal times; further, he shows that investors who have a prior relationship with the leader in past campaigns are more likely to buy shares. We note that leaders in such attacks must necessarily have bigger blocks, simply because no other follower discloses.

The SEC's recent guidance on shareholder communication in the context of "tipping" argues that when a blockholder who will have to make a disclosure communicates this non-

[34]Similar restrictions apply in other jurisdictions. Under European law for instance, activists "acting in concert" would be treated as one under the Transparency Directive (crossing a given shareholder ownership threshold) and under the Acquisitions Directive (crossing a given shareholder ownership threshold in financial firms after a proposed acquisition). See Ghetti (2014) for more discussion on the European case.

[35]See pp.1–2 in Lewellen and Lewellen (2022) for how Rule 14a–2(b)(2) of the Securities Exchange Act permits shareholder communication regarding voting intentions (and reasons) at minimal costs as long as proxies are not solicited or votes coordinated. The authors also document that to reach 25% of shares only 5 shareholders need to be contacted on average, while the 50% threshold requires around 27 institutions.

public information to other investors so that they purchase shares, and these purchases happen, the actors involved could be classified as a group.[36] As such, the importance for activists of market signals in the way that we propose is likely to grow, so long as activists do have a superior ability to filter information about each other's actions as we explained in Section 3.2. In this line, Chabakauri et al. (2025) provide evidence that such an ability to read market signals is indeed present among blockholders nowadays, in the case of corporate insiders precisely detecting activists' trades from order flows: during activism events, these investors engage in abnormal purchases following activists' trades before interventions go public. As the authors argue, such blockholders are unlikely tippees, because they have strong incentives to counter activists and defend their private benefits from control.

6 Other Equilibria and Refinement

Equilibria in which at least one of our activists attaches a negative weight to the initial block can arise due to a *coordination motive* in value creation or destruction. Suppose that the activists start "long" on the firm (i.e., $X_0^L, X_0^F > 0$) and that the leader expects the follower to switch to a negative position. In the expectation of a negative effort by the follower, the leader may then want to build a negative stake too, as a positive surplus would indeed emerge if both players exert negative effort—and similarly for the follower.

Negative weights on initial blocks operationalize this logic, which we formalize through two results in our Internet Appendix (Section V.A). First, we show that given $\rho > 0$, if $\sigma > 0$ is sufficiently large, there is an equilibrium with both α_L and α_F taking negative values: since $\rho > 0$ implies that the activists' initial blocks likely have the same sign, this is consistent with a motive to "meet on the same side." In fact, a larger order flow volatility σ facilitates such coordination because, by reducing price impact, it can induce the follower to trade so aggressively that a *position reversal happens*. Indeed, we can show that the follower's equilibrium coefficients must always satisfy $\alpha_F = \pm\sqrt{\sigma^2/\gamma_1^F}$, so in the negative root case, the fact that $\alpha_F < -\sqrt{\sigma^2/\phi}$ implies that α_F can be arbitrarily negative as σ grows; in turn, this means that initial blocks past a threshold are necessarily reversed, and the coordination mechanism described can be self-fulfilling.[37]

Order flow volatility also relates to the lower bound $\underline{\rho} < 0$ in Theorem 1, which helps explain our second coordination result, now for negative ρ. The key is the 'effective cost of trading' mentioned in the SOCs of Section 3.1: with positive correlation, more aggressive trades are more costly than if fundamentals were exogenous because they lower the follower's

[36] See pp. 130–139 in https://www.sec.gov/files/rules/final/2023/33-11253.pdf

[37] Our result also shows that in the leader's strategy, $\alpha_L < -\sqrt{\sigma^2/\phi}$.

effort; but with negative correlation such trades now add value, a force going against price impact.[38] As the leader's problem gains concavity when $\rho > 0$, a PBS equilibrium always exists in that region. But this concavity weakens when $\rho < 0$, to the point that PBS equilibria will cease to exist if ρ becomes sufficiently negative for fixed σ: the leader's SOC (15) cannot be satisfied by positive (α_L, α_F) pairs, explaining $\underline{\rho} < 0$ in our Theorem. Our second coordination result is a proof of concept of this logic: if $\rho = -\phi$, there is no equilibrium in which α_F and α_L have the same sign; but one with $\text{sign}(\alpha_L) \neq \text{sign}(\alpha_F)$ exists for all $\sigma > 0$.

Lower order flow volatility can then play a dual role: by making manipulation easier, it can make deviations from candidate coordination equilibria more profitable when $\rho > 0$; and by increasing price impact, it can restore concavity in the leader's problem when $\rho < 0$. Thus, market illiquidity can refine the PBS equilibrium within the linear class. We prove the following in Section V.B of the Internet Appendix.

Proposition 8. *Suppose that $\rho \in (-\phi, \phi)$. Then for sufficiently small but positive σ, a PBS equilibrium exists and is the unique equilibrium within the linear class.*

Coordination equilibria are not unreasonable because they rely on negative fundamentals, as our model and many others in the literature allow: after all, it is well-known that acquiring a negative position can be profitable if it triggers a mechanism that lowers a firm's value (Goldstein and Guembel, 2008). When it comes to positive activism, however, it is the feature of revising one's initial choices so radically simply due to the expectation of what others will do that seems stark: such an unwinding before activism occurs means going against the information acquisition and research that in reality leads to the choice of an initial block. Brav et al. (2021b) provide evidence precisely undermining this possibility: hedge funds' average duration of investment in a target is over 530 days, meaning that more than a year and a half passes between disclosure of a position and a major divestiture happen.

7 Conclusions

We developed a theory of influence among blockholders at the center of which are cost-management motives: trading in sequence to control acquisition costs, and then using market signals to influence others to bear intervention costs. Through our analysis, we have provided new insights regarding blockholders as informed traders vis-à-vis traditional insider traders; explained how externalities that activists impose on others via trading can shape corporate governance; and derived measures of price abnormality that permit new interpretations of the empirical evidence on the topic. Altogether, the model that we have proposed, along

[38] To see this tension, refer to the numerator in the second ratio of (7) for period-1 price impact Λ_1.

with its variations, constitutes a fresh approach to a highly understudied topic: how activist investors may coordinate their actions in non-cooperative ways.

We have already mentioned some lines of inquiry that future empirical work can explore using our results. On the theoretical front, there are three clear directions. The first is developing a multiplayer, fully dynamic, repeated trade version of Proposition 2. For example, one could start from the uncorrelated case to shed light on time-horizon effects: on the one hand, the usual insider "splitting trades" logic would say that trades should be small away from the end-game; but at the same time, that is when beliefs are most responsive, which would favor the dampening of trades. The second is examining cost structures beyond the quadratic case: while we argued that our findings are mainly driven by effort being increasing in terminal positions—the linear case being the simplest—building on the innovative work by Back et al. (2018) could provide a more informed view regarding the interplay between market liquidity and the distortion of trades we have uncovered.

Finally, while we have examined factors supporting both block accumulation and first-mover advantages, developing a theory that endogeneizes the emergence of leaders and followers, and their blocks, is key. While this is ultimately a matter of signals observed, constructing a general theory model seems challenging. In Section VI of our Internet Appendix, we choose a middle ground by endogeneizing the activists' initial positions while keeping the identities exogenous, but possibly random. Our generalization adds an exogenous component of firm value (as in Proposition 1(a)) and a pre-round of trading based on private signals, just like in traditional microstructure models. By varying the degree of correlation of the latter signals and allowing for some interim information revelation about firm value, we generate early trades—hence "initial" blocks—that exhibit both types of interdependence.

This early round can be used by prospective leaders to ease the tension between exploiting trading gains and manipulating others that arises when finalizing a block. As one activist becomes increasingly likely to be the leader then, she trades less aggressively on her private signal in the pre-round, resulting in a smaller block and hence in smaller trading gains to be given up when influencing others, all else equal. At the same time, however, a smaller footprint means a reduced informational advantage block-wise, and hence it encodes lower price impact: the likely leader then ends up building a block more aggressively in the first period of the leader-follower as a result. Altogether, through this initial block optimization, a greater anticipation of monopoly and manipulation power leads to block-accumulation dynamics featuring purchases that slow down early on, but that can then accelerate to adequately exploit trading gains more while not discouraging others to add value.

A Appendix: Proofs

A.1 Proof of Lemma 1

We make repeated use of the traditional projection theorem for Gaussian random variables. Concretely, if (X, Y) are jointly Gaussian, then $X|Y$ is also Gaussian with $\mathbb{E}[X|Y] = \mathbb{E}[X] + \frac{\text{Cov}[X,Y]}{\text{Var}[Y]}(Y - \mathbb{E}[Y])$ and $\text{Var}[X|Y] = \text{Var}[X] - \frac{\text{Cov}^2[X,Y]}{\text{Var}[Y]}$.

Prior to trading, players use their own positions to update beliefs about the other player's position. Applying the projection theorem to the pair (X_0^i, X_0^{-i}) yields Y_0^i and ν_0^i as stated in the lemma. Using the conjectured linear strategies, P_0 satisfies

$$P_0 = \mathbb{E}[(1 + \alpha_L)X_0^L + \delta_L\mu + (1 + \alpha_F)X_0^F + \beta_F P_1 + \delta_F\mu]. \tag{A.1}$$

Using that $\mathbb{E}[P_1] = P_0$ to eliminate P_1 yields an equation for P_0 with solution (5), where the denominator is nonzero due to the leader's second order condition (15).

In period 1, after observing Ψ_1, the market maker updates beliefs about X_0^L and X_0^F. By the projection theorem, $\begin{pmatrix} X_0^L \\ X_0^F \end{pmatrix} \sim N\left(\begin{pmatrix} M_1^L \\ M_1^F \end{pmatrix}, \begin{pmatrix} \gamma_1^L & \rho_1 \\ \rho_1 & \gamma_1^F \end{pmatrix}\right)$, where

$$M_1^L := \mathbb{E}[X_0^L|\mathcal{F}_1] = \mu + \frac{\alpha_L\phi}{\alpha_L^2\phi + \sigma^2}\left\{\Psi_1 - \mu(\alpha_L + \delta_L)\right\}, \tag{A.2}$$

$$M_1^F := \mathbb{E}[X_0^F|\mathcal{F}_1] = \mu + \frac{\alpha_L\rho}{\alpha_L^2\phi + \sigma^2}\left\{\Psi_1 - \mu(\alpha_L + \delta_L)\right\}, \tag{A.3}$$

$$\gamma_1^L = \frac{\phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \gamma_1^F = \frac{\alpha_L^2[\phi^2 - \rho^2] + \phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \rho_1 = \frac{\rho\sigma^2}{\alpha_L^2\phi + \sigma^2}. \tag{A.4}$$

The MM forecasts terminal positions based on the conjectured strategies:

$$\begin{aligned} \tilde{M}_1^L := \mathbb{E}[X_T^L|\mathcal{F}_1] &= (1 + \alpha_L)M_1^L + \delta_L\mu, \\ \mathbb{E}[X_T^F|\mathcal{F}_1] &= (1 + \alpha_F)M_1^F + \beta_F P_1 + \delta_F\mu. \end{aligned}$$

We use $\tilde{\rho}_1 := (1+\alpha_L)\rho_1$ to denote the MM's posterior covariance of X_T^L and X_0^F (the current positions at the end of the first period). The first period price P_1 is obtained by solving the equation $P_1 = \mathbb{E}[X_T^L + X_T^F|\mathcal{F}_1]$ for P_1, which yields (6)-(7). After observing Ψ_1, the follower forms an updated mean belief Y_1^F about X_T^L by first using the projection theorem to form an updated belief about X_0^L (from his prior Y_0^F) and then forecasting the terminal position based on the conjectured strategy for the leader, resulting in (8).

In period 2, given Ψ_2, the market maker's updated mean beliefs about (X_T^L, X_T^F) follow from the projection theorem and conjectured strategies:

$$\begin{aligned} M_T^F &:= \mathbb{E}[X_T^F|\mathcal{F}_2] \\ &= (1+\alpha_F)M_1^F + \beta_F P_1 + \delta_F\mu + \frac{\alpha_F\gamma_1^F(1+\alpha_F)}{\alpha_F^2\gamma_1^F+\sigma^2}[\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F\mu], \qquad \text{(A.5)} \\ M_T^L &:= \mathbb{E}[X_T^L|\mathcal{F}_2] = \tilde{M}_1^L + \frac{\alpha_F\tilde{\rho}_1}{\alpha_F^2\gamma_1^F+\sigma^2}[\Psi_2 - \alpha_F M_1^F - \beta_F P_1 - \delta_F\mu]. \qquad \text{(A.6)} \end{aligned}$$

The second period price is then simply $P_2 = M_T^L + M_T^F$, which produces (9)-(10).

A.2 Preliminaries for Equilibrium Construction

In this section, we state and prove a proposition, to be used in proving our main results, that characterizes equilibria via a system of equations and inequality conditions derived from the players' first and second order conditions and the pricing equations. The first half of the proposition below provides necessary conditions for equilibrium. The second half of the proposition is a strong converse: it shows that we can focus on the system of equations for the signaling coefficients (α_F, α_L); these coefficients determine price impact and therefore pin down the remaining coefficients.

Proposition A.1. *The tuple $(\alpha_F, \beta_F, \delta_F, \alpha_L, \delta_L)$ with a pricing rule defined by (6)-(7) and (9)-(10) characterize an equilibrium only if $\Lambda_1 \neq 0$, $\Lambda_2 > \frac{1}{2}$, $\beta_F \neq 1$, $\phi(1+\alpha_L)+\rho \neq 0$, and*

$$\begin{aligned} \alpha_F^2 &= \sigma^2/\gamma_1^F, && \text{(A.7)} \\ \beta_F &= -\frac{\rho}{\phi(1+\alpha_L)+\rho}\alpha_F, && \text{(A.8)} \\ \delta_F &= \frac{(\alpha_L+\delta_L)\rho - \alpha_L\phi - (\phi-\rho)}{\phi(1+\alpha_L)+\rho}\alpha_F, && \text{(A.9)} \\ \alpha_L &= \frac{\sigma^2}{\phi\alpha_L} - \frac{\rho\alpha_F}{\phi(1+\alpha_L)+\rho(1+\alpha_F)}, && \text{(A.10)} \\ \delta_L &= -\frac{\sigma^2}{\phi\alpha_L}, && \text{(A.11)} \\ 0 &\geq \sigma^2 - \alpha_L^2\phi - 2\alpha_L[\rho(1+\alpha_F)+\phi], && \text{(A.12)} \\ 0 &\geq -\alpha_F[\sigma^2(\phi+\rho(1+\alpha_L)) + \alpha_L^2(\phi^2-\rho^2)]. && \text{(A.13)} \end{aligned}$$

Further, if $\rho \neq 0$, one of the following conditions must hold:

$$\alpha_F = \alpha_{F,1}(\alpha_L) := \sqrt{\frac{\sigma^4 + \alpha_L^2\sigma^2\phi}{\sigma^2\phi + \alpha_L^2(\phi^2-\rho^2)}} = \frac{(\rho+\phi+\phi\alpha_L)(\alpha_L^2\phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L(1+\alpha_L)\phi]} \ \ \textit{or} \qquad \text{(A.14)}$$

$$\alpha_F = \alpha_{F,2}(\alpha_L) := -\sqrt{\frac{\sigma^4 + \alpha_L^2\sigma^2\phi}{\sigma^2\phi + \alpha_L^2(\phi^2 - \rho^2)}} = \frac{(\rho + \phi + \phi\alpha_L)(\alpha_L^2\phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]}. \tag{A.15}$$

Conversely, suppose (α_F, α_L) *satisfy* (A.12) *and* (A.13)*, either* (A.14) *or* (A.15)*, and* $\phi(1 + \alpha_L) + \rho \neq 0$*. Then (i)* $(\beta_F, \delta_F, \delta_L)$ *are well defined via* (A.8), (A.9)*, and* (A.11)*, with* $\beta_F \neq 1$*; (ii)* $\Lambda_1 \neq 0$ *and* $\Lambda_2 \neq 0$ *are well defined via* (7) *and* (10)*; and (iii) the associated strategies and pricing rule constitute an equilibrium.*

Proof. We first establish necessity. It is immediate from the leader's second order condition (15) that $\Lambda_1 \neq 0$ and $\beta_F \neq 0$; likewise, (14) implies $\Lambda_2 > \frac{1}{2}$, and in particular, $\Lambda_2 \neq 0$. Next, we analyze the follower's conditions. The follower's FOC expands as

$$\begin{aligned} 0 &= -\mathbb{E}_F[P_1 + \Lambda_2\{\Psi_2 - \mathbb{E}[\Psi_2|\mathcal{F}_1]\}|\theta^F] - \Lambda_2\theta^F + (X_0^F + \theta^F) + Y_1^F & \text{(A.16)} \\ &= -P_1 - \Lambda_2(\theta^F - [\alpha_F M_1^F + \beta_F P_1 + \delta_F\mu]) - \Lambda_2\theta^F + (X_0^F + \theta^F) + Y_1^F, & \text{(A.17)} \end{aligned}$$

which we impose at the candidate strategy in (4). Since $\Lambda_1 \neq 0$, we can invert (6) to write $\Psi_1 = \mu(\alpha_L + \delta_L) + \frac{P_1 - P_0}{\Lambda_1}$, with P_0 given by (5), which we can use to eliminate Ψ_1 in M_1^F and Y_1^F (see (A.3) and (8)). Recalling that Y_0^F (appearing in Y_1^F) is a linear combination of (X_0^F, μ), the resulting equation is linear in (X_0^F, P_1, μ), and it must be identically zero over $(X_0^F, P_1, \mu) \in \mathbb{R}^3$. Hence, the coefficients on each variable (X_0^F, P_1, μ) must be zero, delivering three equations. The first of these, from the coefficient on X_0^F, is

$$0 = -2\Lambda_2\alpha_F + (1 + \alpha_F) + \frac{\partial Y_1^F}{\partial X_0^F} = \frac{\tilde{\Lambda}_2}{\gamma_1^F}(\sigma^2 - \alpha_F^2\gamma_1^F), \tag{A.18}$$

where $\tilde{\Lambda}_2 := \frac{\gamma_1^F}{\alpha_F^2\gamma_1^F + \sigma^2} \times [1 + \alpha_F + \rho_1/\gamma_1^F]$. The second, from the coefficient on P_1, is

$$\begin{aligned} 0 &= -1 - \Lambda_2\left(-\alpha_F\frac{\partial M_1^F}{\partial P_1}\right) - \Lambda_2\beta_F + \beta_F + \frac{\partial Y_1^F}{\partial P_1} \\ &= -\frac{\tilde{\Lambda}_2}{\gamma_1^F}\left[\frac{\rho\sigma^2(1 - \beta_F)}{\phi(1 + \alpha_L) + \rho(1 + \alpha_F)} + \beta_F\alpha_F\gamma_1^F\right]. & \text{(A.19)} \end{aligned}$$

The third, from the coefficient on μ, is

$$\begin{aligned} 0 &= -\Lambda_2\left(-\alpha_F\frac{\partial M_1^F}{\partial \mu}\right) - \Lambda_2\delta_F + \delta_F + \frac{\partial Y_1^F}{\partial \mu} \\ &= \frac{\tilde{\Lambda}_2}{\gamma_1^F}\left[-\sigma^2 + \frac{(2 + \alpha_F + \alpha_L + \delta_F + \delta_L)\rho\sigma^2}{\phi(1 + \alpha_L) + \rho(1 + \alpha_F)} - \alpha_F\delta_F\gamma_1^F\right], & \text{(A.20)} \end{aligned}$$

where the μ terms of M_1^F and Y_1^F incorporate the elimination of Ψ_1 described above.

We argue that in any linear equilibrium, the right hand sides of (A.18)-(A.20) are well defined and $\tilde{\Lambda}_2 \neq 0$. First, $\gamma_1^F > 0$ for any (finite) α_F. Second, (14) implies $\Lambda_2 \neq 0$, so $\tilde{\Lambda}_2$ is well defined and nonzero. Third, $\Lambda_1 \neq 0$ implies $\phi(1+\alpha_L) + \rho(1+\alpha_F) \neq 0$ in the denominators in (A.19) and (A.20).

We can now derive (A.7)-(A.9) and (A.13). Since $\tilde{\Lambda}_2 \neq 0$ is necessary for equilibrium, (A.18) reduces to (A.7). (Note that this implies $\alpha_F \neq 0$.) Using this fact to write $\alpha_F \gamma_1^F = \sigma^2/\alpha_F$, (A.19) reduces to

$$\begin{aligned} 0 &= -\frac{\tilde{\Lambda}_2}{\gamma_1^F}\left[\frac{\rho\sigma^2(1-\beta_F)}{\phi(1+\alpha_L)+\rho(1+\alpha_F)} + \beta_F\frac{\sigma^2}{\alpha_F}\right] \\ &= -\frac{\tilde{\Lambda}_2\sigma^2}{\gamma_1^F\alpha_F[\phi(1+\alpha_L)+\rho(1+\alpha_F)]}\left[\rho\alpha_F + \beta_F(\phi[1+\alpha_L]+\rho)\right]. \end{aligned} \tag{A.21}$$

We claim that $\phi(1+\alpha_L)+\rho \neq 0$ in equilibrium. By way of contradiction, if $\phi(1+\alpha_L)+\rho = 0$, then (A.21) implies $\alpha_F = 0$ or $\rho = 0$. Equation (A.7) rules out $\alpha_F = 0$. And if $\rho = 0$, we have $\alpha_L = -1$, and thus $\Lambda_1 = 0$, violating the leader's SOC. Hence, $\phi(1+\alpha_L)+\rho \neq 0$, and (A.21) reduces to (A.8). Analogous arguments yield (A.9) from (A.20). Lastly, using (A.7) to eliminate α_F^2 terms, the follower's SOC (14) reduces to (A.13).

Next, we derive the leader's identities (A.10)-(A.11) and condition (A.12). For the leader, the following FOC, evaluated at the conjectured strategy, must hold for all $(X_0^L, \mu) \in \mathbb{R}^2$:

$$\begin{aligned} 0 = &-\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1 + (X_0^L + \theta^L) + \mathbb{E}_L[X_T^F|\theta^L] \\ &+ (X_0^L + \theta^L)\frac{\partial \mathbb{E}_L[X_T^F|\theta^L]}{\partial \theta^L}. \end{aligned} \tag{A.22}$$

Setting the coefficients on these variables to 0 and using (A.7) and (A.8), it is straightforward to show that (A.22) reduces to (A.10)-(A.11) where $\alpha_L \neq 0$ in equilibrium since the leader's SOC implies $\Lambda_1 \neq 0$. The leader's SOC is equivalent to (A.12).

To obtain (A.14) or (A.15), first note that the positive and negative values of α_F solving (A.7) are $\pm\sqrt{\frac{\sigma^4+\alpha_L^2\sigma^2\phi}{\sigma^2\phi+\alpha_L^2(\phi^2-\rho^2)}}$. Next, solve for α_F in (A.10) by multiplying through by the denominators on the right hand side and rearrange terms to obtain

$$\alpha_F\rho[\sigma^2 - \alpha_L(1+\alpha_L)\phi] = [\phi(1+\alpha_L)+\rho](\alpha_L^2\phi - \sigma^2). \tag{A.23}$$

We claim that $\sigma^2 - \alpha_L(1+\alpha_L)\phi \neq 0$ in any solution to (A.23). Indeed, since $\phi(1+\alpha_L)+\rho \neq 0$, $\sigma^2 - \alpha_L(1+\alpha_L)\phi = 0$ would imply $\alpha_L^2\phi - \sigma^2 = 0$, but these two equations cannot hold simultaneously. Thus, if $\rho \neq 0$, (A.23) implies

$$\alpha_F = \frac{(\rho + \phi + \phi\alpha_L)(\alpha_L^2\phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]}.$$

Since the solutions to (A.7) are $\alpha_F = \alpha_{F,1}$ and $\alpha_F = \alpha_{F,2}$, we obtain (A.14) and (A.15).

For the sufficiency half of the proposition, take (α_F, α_L) as in the statement. Clearly, either $\alpha_F = \alpha_{F,1}$ or $\alpha_F = \alpha_{F,2}$ implies (A.7). Now given $\phi(1 + \alpha_L) + \rho \neq 0$, we can multiply through (A.14) or (A.15) by $\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]$ to recover (A.23). To recover (A.10) from (A.23), note that (A.12) can be rewritten as $\sigma^2 + \alpha_L^2\phi - 2\alpha_L[\rho(1+\alpha_F)+\phi(1+\alpha_L)] \leq 0$, which implies $\alpha_L \neq 0$ and $\phi(1 + \alpha_L) + \rho(1 + \alpha_F) \neq 0$; thus, the steps used to obtain (A.23) from (A.10) can be reversed. Given that $\phi(1+\alpha_L)+\rho \neq 0$ by supposition, (β_F, δ_F) are well defined by (A.8)-(A.9). Further, $\phi(1 + \alpha_L) + \rho(1 + \alpha_F) \neq 0$ implies that $1 \neq -\frac{\rho\alpha_F}{\phi(1+\alpha_L)+\rho} = \beta_F$. This establishes (i). Hence, Λ_1 and Λ_2 are well defined by (7) and (10), respectively. Moreover, by construction, (A.12)-(A.13) imply (15)-(14), so $\Lambda_1 \neq 0$ and $\Lambda_2 \neq 0$, establishing (ii).

For part (iii) of the sufficiency claim, observe that since the players' best responses problems are quadratic, it suffices to check first and second order conditions. Given that the inequalities $\Lambda_1 \neq 0$, $\Lambda_2 \neq 0$, $\beta_F \neq 1$, $\phi(1 + \alpha_L) + \rho \neq 0$ are satisfied, the equations (A.7)-(A.11) imply the FOCs (A.16) and (A.22) by construction, and as noted for part (ii), the SOCs (15) and (14) are satisfied. □

A.3 Gap Form of Follower's Strategy

The following result formalizes the claim made about the follower's strategy in footnote 20, which we use to confirm that the follower's trade is unpredictable as anticipated in the main body.

Lemma A.1. *In any linear equilibrium (PBS or otherwise), $\theta^F = \alpha_F(X_0^F - M_1^F)$ for $\alpha_F = \pm\sqrt{\frac{\sigma^2}{\gamma_1^F}}$. Hence, in a PBS equilibrium, $\theta^F = \sqrt{\frac{\sigma^2}{\gamma_1^F}}(\theta^F - M_1^F)$.*

Proof. By Proposition A.1, α_F must satisfy (A.7), so either $\alpha_F = \alpha_{F,1} := \sqrt{\frac{\sigma^2}{\gamma_1^F}}$ or $\alpha_F = \alpha_{F,2} := -\sqrt{\frac{\sigma^2}{\gamma_1^F}}$. Moreover, in a PBS equilibrium (by definition), $\alpha_F > 0$, so $\alpha_F = \alpha_{F,1}$. Note that by the same proposition, (β_F, δ_F) are characterized by (A.8)-(A.9).

Now express M_1^F in terms of P_1 and μ by using (6) to replace the surprise term $\Psi_1 - \mu(\alpha_L + \delta_L)$ in (A.3):

$$M_1^F = \mu + \frac{\alpha_L\rho}{\alpha_L^2\phi + \sigma^2}\frac{P_1 - P_0}{\Lambda_1}, \tag{A.24}$$

where P_0 is linear in μ (see (5)). Substituting (A.24) into $\theta^F = \alpha_{F,i}(X_0^F - M_1^F)$, $i \in \{1, 2\}$, then yields an expression for the follower's strategy in which the coefficient on X_0^F is $\alpha_{F,i}$, and the coefficients on (P_1, μ) equal $(\beta_{F,i}, \delta_{F,i})$ when (A.8)-(A.9) hold. This confirms that the follower's strategy has the stated form. $\square$

A.4 Proof of Theorem 1

We first prove the claims about the follower's strategy up to some inequalities involving the leader's strategy that we establish later in the proof. To prove that $\mathbb{E}[\theta^F|\mathcal{F}_1] = 0$, we simply use the fact that, by Lemma A.1, $\theta^F = \alpha_F(X_0^F - M_1^F)$, where M_1^F was defined as $\mathbb{E}[X_0^F|\mathcal{F}_1]$, and take expectations conditional on $\mathcal{F}_1$. Lemma A.1 also establishes that in a PBS equilibrium $\alpha_F = \sqrt{\sigma^2/\gamma_1^F}$. To sign β_F, recall that $\alpha_F, \alpha_L > 0$ and $|\rho| \leq \phi$, so $sign(\beta_F) = -sign(\rho)$ via (A.8). Similarly, from (A.9), $sign(\delta_F) = sign((\alpha_L + \delta_L)\rho - \alpha_L\phi - (\phi - \rho))$. Since $\alpha_L\phi > 0$ and $\phi \geq \rho$, the sign will be negative if $(\alpha_L + \delta_L)\rho \leq 0$, which we will establish when characterizing the leader's equilibrium strategy. Turning to the inequality $\beta_F < 1$, this is immediate when $\rho \geq 0$, since this implies $\beta_F \leq 0$. For $\rho < 0$, note that by using (A.8), (A.10) can be written as $\alpha_L = \frac{\sigma^2}{\phi\alpha_L} + \frac{\beta_F}{1-\beta_F}$. Our characterization of the leader's strategy will show that $\alpha_L > \alpha^K$, and thus $\alpha_L > \frac{(\alpha^K)^2}{\alpha_L} = \frac{\sigma^2}{\phi\alpha_L}$. It follows that $\frac{\beta_F}{1-\beta_F} > 0$, and thus $\beta_F \in (0, 1)$.

The rest of the proof is divided into four components as follows. First, we first address $\rho = 0$, in which case the unique linear equilibrium can be characterized in closed form (Proposition A.2). Second, we consider $\rho \in (0, \phi]$, for which we establish existence of a PBS equilibrium and uniqueness within the PBS class (Proposition A.3). Third, we show that for all $|\rho| > 0$ sufficiently small (allowing for positive or negative ρ), there exists a unique equilibrium within the whole linear class, and it is a PBS equilibrium (Proposition A.4). For both positive and negative ρ we prove the inequalities stated in the proposition. Fourth, we show that a PBS equilibrium fails to exist if ρ is sufficiently low (Proposition A.5), and we construct $\underline{\rho} \in (-\phi, 0)$ presented in the proposition and ρ_0 mentioned in footnote 20. Recall that $\alpha^K := \sqrt{\frac{\sigma^2}{\phi}}$.

Proposition A.2. *For $\rho = 0$, there is a unique linear equilibrium: for $i \in \{L, F\}$, trader i trades $\theta^i = \alpha^K(X_0^i - \mu)$, and $\mathbb{E}[\theta^L|\mathcal{F}_0] = 0$.*

Proof. For $\rho = 0$, (A.13) becomes $-\alpha_F[\sigma^2\phi + \alpha_L^2] \leq 0$. The only solution to (A.7) satisfying this is $\alpha_F = \sqrt{\frac{\sigma^2}{\gamma_1^F}} = \alpha^K$ (as $\rho = 0$ implies $\gamma_1^F = \phi$). Equation (A.10) then yields $\alpha_L = \pm\alpha^K$. Of these, only $\alpha_L = \alpha^K$ satisfies (A.12). Given $(\alpha_F, \alpha_L) = (\alpha^K, \alpha^K)$, $(\beta_F, \delta_F, \delta_L) = (0, -\alpha^K, -\alpha^K)$ is the unique solution to (A.8), (A.9), and (A.11). These strategies and the pricing rule in (6) and (9) satisfy the first and second order conditions, so they constitute an equilibrium. Moreover, $\mathbb{E}[\theta^L|\mathcal{F}_0] = \mathbb{E}[\alpha^K(X_0^L - \mu)|\mathcal{F}_0] = \alpha^L(\mu - \mu) = 0$. □

In the next two propositions, note that the ranking of α_L and $-\delta_L$ determines the sign of $\mathbb{E}[\theta^L|\mathcal{F}_0] = (\alpha_L + \delta_L)\mu$.

Proposition A.3. *If $\rho \in (0, \phi]$, there is a unique PBS equilibrium, and $0 < \alpha_L < \alpha^K < -\delta_L$.*

Proof. By Proposition A.1, (A.14) is a necessary condition for (α_F, α_L) to be part of PBS equilibrium. Let $L(\alpha_L)$ and $R(\alpha_L)$ denote the left and right sides of (A.14). Define $\hat{\alpha} := \frac{-\phi+\sqrt{\phi^2+4\sigma^2\phi}}{2\phi} > 0$ to be the positive root of the denominator on the right side of (A.14). Note that $\alpha^K > \hat{\alpha}$.

L is positive and strictly increasing in α_L for $\alpha_L \geq 0$. Meanwhile, R is continuous on $[0, \hat{\alpha}) \cup (\hat{\alpha}, +\infty)$ and satisfies $R(\hat{\alpha}-) = -\infty$, $R(\hat{\alpha}+) = +\infty$, and $R(\alpha^K) = 0$. Further, for $\alpha_L \in [0, \hat{\alpha}) \cup (\hat{\alpha}, +\infty)$,

$$R'(\alpha_L) = -\phi \frac{(\alpha_L^2\phi - \sigma^2)^2 + (\rho + \phi)(\alpha_L^2 + \sigma^2) + 2\alpha_L^3\phi^2}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]^2},$$

which is unambiguously strictly negative when $\rho > 0$. Thus, R is strictly decreasing on $[0, \hat{\alpha})$ and on $(\hat{\alpha}, +\infty)$. These facts imply that there exists a solution to (A.14) on $(\hat{\alpha}, \alpha^K)$ and this is the only solution on $(\hat{\alpha}, +\infty)$. Since L is increasing for $\alpha_L \geq 0$ with $L(0) > 0$, while R is decreasing on $[0, \hat{\alpha})$ with $R(0) = -(\rho + \phi)/\rho < 0 < L(0)$ (given $\rho > 0$), there is no solution on $[0, \hat{\alpha})$, so the solution just found is unique among $\alpha_L \geq 0$. And by (A.11), $\alpha_L < \alpha^K$ implies $\alpha^K < -\delta_L$ (and $\delta_L < 0$).

Given a unique candidate for PBS equilibrium, we now verify SOCs. For the leader, note that since $\alpha_L, \alpha_F > 0$, (A.12) is bounded above by $\sigma^2 - \alpha_L^2\phi - \alpha_L\phi$, which is negative since $\alpha_L > \hat{\alpha}$. For the follower, (A.13) holds by inspection for $\rho > 0$ since $\alpha_L > 0$ and $\alpha_F > 0$. □

Next, we turn to $|\rho| > 0$ close to 0.

Proposition A.4. *If $|\rho| > 0$ is sufficiently small, there exists a unique linear equilibrium, and it is a PBS equilibrium. If $\rho > 0$, $\alpha_L < \alpha^K < -\delta_L$, and if $\rho < 0$, $\alpha_L > \alpha^K > -\delta_L > 0$.*

Proof. Assume throughout that $\rho \neq 0$. Let us call any pair (α_L, α_F) satisfying (A.14) or (A.15) a *candidate signaling pair*. We construct two candidate signaling pairs (α_L^*, α_F^*) and $(\alpha_L^\flat, \alpha_F^\flat)$. We then show that for small $|\rho|$, there are no other candidate signaling pairs satisfying the leader's second order condition, and of these two pairs, only (α_L^*, α_F^*) satisfies the follower's SOC. We then invoke the converse part of Proposition A.1 to establish existence of a unique equilibrium based on (α_L^*, α_F^*).

We claim that if $\rho < 0$, there exists $\alpha_L^* \in (\alpha^K, \infty)$ solving (A.14) and $\alpha_L^\flat \in (\hat{\alpha}, \alpha^K)$ solving (A.15). Analogous arguments for the case $\rho > 0$ establish the existence of $\alpha_L^* \in (\hat{\alpha}, \alpha^K)$ and $\alpha_L^\flat \in (\alpha^K, \infty)$; we omit this case for brevity. In either case, we will ultimately show that α_L^* is the unique equilibrium value of α_L for small $|\rho|$. As before, let $R(\alpha_L)$ denote the right hand side common to (A.14) and (A.15). Note that R is continuous on $(\hat{\alpha}, \infty)$, and it has the properties $\lim_{\alpha_L \to +\infty} R(\alpha_L) = +\infty$, $\lim_{\alpha_L \downarrow \hat{\alpha}} R(\alpha_L) = -\infty$, and $R(\alpha^K) = 0$. The left hand side of (A.14) is strictly positive and bounded, so by the intermediate value theorem (IVT), there exists a solution $\alpha_L^* \in (\alpha^K, \infty)$ to (A.14). Similarly, the left hand side of (A.15) is strictly negative and bounded, so by the IVT, there exists a solution $\alpha_L^\flat \in (\hat{\alpha}, \alpha^K)$ to (A.15).

Define $\alpha_F^* := \alpha_{F,1}(\alpha_L^*)$ and define $\alpha_F^\flat = \alpha_{F,2}(\alpha_L^\flat)$. By definition, both (α_L^*, α_F^*) and $(\alpha_L^\flat, \alpha_F^\flat)$ are candidate signaling pairs.

To assess other candidate signaling pairs, we derive a polynomial equation such that (α_L, α_F) is a candidate signaling pair only if α_L is a root of this equation. By squaring either (A.14) or (A.15), we obtain a necessary condition

$$\frac{\sigma^4 + \alpha_L^2 \sigma^2 \phi}{\sigma^2 \phi + \alpha_L^2(-(\rho)^2 + (\phi)^2)} = \left(\frac{(\rho + \phi + \phi \alpha_L)(\alpha_L^2 \phi - \sigma^2)}{\rho[\sigma^2 - \alpha_L(1 + \alpha_L)\phi]} \right)^2, \tag{A.25}$$

and by cross multiplying, an eighth-degree polynomial equation

$$0 = Q(\alpha_L; \rho) = \sum_{i=0}^{8} A_i \alpha_L^i, \qquad \text{where} \tag{A.26}$$

$A_8 = \phi^4(\rho^2 - \phi^2)$, $\quad A_7 = 2(\rho - \phi)\phi^3(\rho + \phi)^2$,

$A_6 = \phi^2(\rho^2 - \phi^2)[\rho^2 + 2\rho\phi + \phi(-\sigma^2 + \phi)]$, $\quad A_5 = 2\sigma^2\phi^2[-2\rho^3 - \rho^2\phi + \rho\phi^2 + \phi^3]$,

$A_4 = \sigma^2\phi[-2\rho^4 - 4\rho^3\phi + 2\rho\phi^3 + \phi^3(\sigma^2 + \phi)]$, $\quad A_3 = 2\sigma^4\phi[\rho^3 + \rho^2\phi + \rho\phi^2 + \phi^3]$,

$A_2 = \sigma^4[\rho^4 + 2\rho^3\phi + 2\rho\phi^3 + \phi^3(-\sigma^2 + \phi) + \rho^2\phi(-\sigma^2 + 3\phi)]$, $\quad A_1 = -2\sigma^6\phi[\rho^2 + \phi\rho + \phi^2]$,

$A_0 = \sigma^6[\rho^2(\sigma^2 - \phi) - 2\rho\phi^2 - \phi^3]$.

Being an eighth-degree polynomial, $Q(\cdot; \rho)$ has exactly eight complex roots, counting multiplicity; two of these are α_L^* and $\alpha_L^\flat$.

We now show that of all candidate signaling pairs, when $|\rho|$ is sufficiently small, only (α_L^*, α_F^*) satisfies both activists' SOCs. To that end, it is useful to approximate all of the roots of (A.26) for small $|\rho|$. We will make use of a standard result on the continuous dependence of the (complex) roots of a polynomial on its coefficients:

Lemma A.2 (Uherka and Sergott (1977)). *Let $p(x) = x^n + \sum_{i=1}^n a_i x^{n-i}$ and $p^*(x) = x^n + \sum_{i=1}^n a_i^* x^{n-i}$ be two degree-n polynomials. Suppose λ^* is a root of p^* with multiplicity m and*

$\epsilon > 0$. *Then for* $|a_i - a_i^*|$ *sufficiently small* ($i = 1, \ldots, n$), p *has at least* m *roots within* ϵ *of* λ^*.

For a proof, see Uherka and Sergott (1977) or the references therein.

We apply this lemma to the polynomial Q indexed by ρ. (While Lemma A.2 assumes a leading coefficient of 1, we can divide through our polynomial $Q(\cdot;\rho)$ in (A.26) by A_8, which is bounded away from 0 provided that $|\rho| < |\phi|$, allowing us to apply the lemma.) In the limit as $\rho \to 0$, the polynomial is

$$Q(\alpha_L; 0) = -(1+\alpha_L)^2\phi^3(\sigma^2 - \alpha_L^2\phi)^2(\sigma^2 + \alpha_L^2\phi).$$

By inspection, $Q(\cdot;0)$ is nonpositive and has double roots at -1 and $\pm\alpha^K$, and it has complex roots at $\pm\alpha^K i$.

Lemma A.2 then has two important implications about candidate signaling pairs. We state the first one as a corollary.

Corollary A.1. *As* $\rho \to 0$, $\alpha_L^* \to \alpha^K$, $\alpha_L^\flat \to \alpha^K$, $\alpha_F^* \to \alpha^K$, *and* $\alpha_F^\flat \to -\alpha^K$.

Since $\alpha_L^*, \alpha_L^\flat \geq 0$, these can only converge to α^K (among the roots of $Q(\cdot;0)$); the corresponding limits of α_F^* and $\alpha_F^\flat$ are then immediate. The second implication of Lemma A.2 is that for any $\epsilon > 0$, there exists $\overline{\rho} > 0$ such that for all ρ with $0 < |\rho| < \overline{\rho}$ all of the other six roots of $Q(\cdot;\rho)$ lie within ϵ of -1, $-\alpha^K$, or $\pm\alpha^K i$. Hence, for such ρ, α_L^* and $\alpha_L^\flat$ are roots with multiplicity 1, and they are uniquely defined.

We can now check SOCs: for the leader in Lemma A.3 and the follower in Lemma A.4.

Lemma A.3. *For* $|\rho| > 0$ *sufficiently small, the candidate signaling pairs* (α_L^*, α_F^*) *and* $(\alpha_L^\flat, \alpha_F^\flat)$ *satisfy* (A.12) *and are the only candidate signaling pairs that do.*

Proof. First, we show that (α_L^*, α_F^*) satisfy (A.12) for sufficiently small $|\rho| > 0$. As $\rho \to 0$, the right hand side of (A.12) tends to $\sigma^2 - (\alpha^K)^2\phi - 2\alpha^K\phi = -2\sigma\sqrt{\phi} < 0$, where we have used that $\alpha_L^* \to \alpha^K$ by Corollary A.1. A nearly identical calculation shows $(\alpha_L^\flat, \alpha_F^\flat)$ also satisfy (A.12) for sufficiently small $|\rho| > 0$.

The remaining candidates for equilibria are associated with the real roots of (A.26) other than $\alpha_L^*, \alpha_L^\flat$. By Lemma A.2, as $\rho \to 0$, these roots must converge to the other roots of $Q(\cdot;0)$, namely -1, $-\alpha^K$, or $\pm\alpha^K i$. Any root of $Q(\cdot;\rho)$ that is in a sufficiently small neighborhood of $\pm\alpha^K i$ has a nonzero complex component and is not an equilibrium candidate. Therefore, we need only consider candidates in neighborhoods of -1 or $-\alpha^K$. In the first case, for any $\alpha_F \in \{\alpha_{F,1}, \alpha_{F,2}\}$, the right hand side of (A.12) converges to $\sigma^2 - (-1)^2\phi - 2(-1)\phi = \sigma^2 + \phi > 0$. In the second case, for any $\alpha_F \in \{\alpha_{F,1}, \alpha_{F,2}\}$, the right hand side of (A.12) converges to $\sigma^2 - \left(-\alpha^K\right)^2 \phi - 2\left(-\alpha^K\right)\phi = 2\sigma\sqrt{\phi} > 0$. Thus, for $|\rho| > 0$ sufficiently small, all roots of $Q(\cdot;\rho)$ other than α_L^* and $\alpha_L^\flat$ violate the leader's SOC. $\square$

Lemma A.4. *For $|\rho| > 0$ sufficiently small, the candidate signaling pair (α_L^*, α_F^*) satisfies (A.13), while the pair $(\alpha_L^\flat, \alpha_F^\flat)$ does not.*

Proof. For the pair (α_L^*, α_F^*), the right hand side of (A.13) tends to $-\alpha^K[\sigma^2\phi + (\alpha^K)^2\phi^2] < 0$ as $\rho \to 0$. For the pair $(\alpha_L^\flat, \alpha_F^\flat)$, it tends to $\alpha^K[\sigma^2\phi + (\alpha^K)^2\phi^2] > 0$, violating (A.13). $\square$

To conclude the proof of Proposition A.4, from Lemmas A.3 and A.4, we have that for $|\rho| > 0$ sufficiently small, (α_L^*, α_F^*) is the unique candidate signaling pair satisfying both (A.12) and (A.13). Hence, in any linear equilibrium, (α_L, α_F) must equal (α_L^*, α_F^*). As $\rho \to 0$, $\phi(1 + \alpha_L^*) + \rho \to \phi(1 + \alpha^K) > 0$, allowing us to apply the "converse" part of Proposition A.1 when $|\rho|$ is sufficiently small, giving us existence. Since we have already shown that $0 < \alpha_L^* < \alpha^K$ if $\rho > 0$, (A.11) implies $-\delta_L > \alpha^K$ in this case, and likewise when $\rho < 0$, we have $\alpha_L^* > \alpha^K$ which implies $0 < -\delta_L < \alpha^K$. $\square$

By the results above, a unique PBS equilibrium exists if ρ is (i) positive or (ii) sufficiently close to zero. Thus, $\underline{\rho} := \inf\{\rho' \in [-\phi, \phi] : \text{a PBS equilibrium exists for all } \rho \in [\rho', \phi]\} < 0$ and $\rho_0 := \inf\{\rho' \in [-\phi, \phi] : \text{a unique PBS equilibrium exists for all } \rho \in [\rho', \phi]\} < 0$, where $\rho_0 \geq \underline{\rho}$ is obvious. To show that $\underline{\rho} > -\phi$, we invoke the following result.

Proposition A.5. *Fix $\sigma, \phi > 0$. There exists $\hat{\rho} \in (-\phi, 0)$ such that if $\rho < \hat{\rho}$, there is no PBS equilibrium.*

Proof. The proof is based on the following two lemmas.

Lemma A.5. *There is no $[-\phi, \phi]$-valued sequence $(\rho_n)_{n\in\mathbb{N}}$ that converges to $-\phi$ and has the property that there is an associated sequence of PBS equilibria such that $(\alpha_{F,n})_{n\in\mathbb{N}}$ is bounded.*

Proof. Suppose by way of contradiction that there exists such a sequence with associated PBS equilibria indexed by n. We claim that $(\alpha_{L,n})_{n\in\mathbb{N}}$ is bounded. To see this, take n sufficiently large that $\rho_n \neq 0$, and note that the right hand side of (A.14) must be bounded, since it equals $\alpha_{F,n}$ which we have supposed is bounded. Since the numerator on the right hand side is cubic while the denominator is quadratic, it must be that $(\alpha_{L,n})_{n\in\mathbb{N}}$ is bounded.

Given that $(\alpha_{F,n})_{n\in\mathbb{N}}$ and $(\alpha_{L,n})_{n\in\mathbb{N}}$ are both bounded, we can pass to a subsequence such that $\alpha_{F,n} \to \overline{\alpha}_F \geq 0$ and $\alpha_{L,n} \to \overline{\alpha}_L \geq 0$, where the inequalities follow from $\alpha_{F,n}, \alpha_{L,n} \geq 0$ in PBS equilibria by definition. Then taking limits in (A.14), we have

$$\overline{\alpha}_F = \sqrt{\frac{\sigma^2}{\phi} + \overline{\alpha}_L^2} > \overline{\alpha}_L. \tag{A.27}$$

The right hand side of (A.12) then has limit

$$\sigma^2 + \overline{\alpha}_L^2\phi - 2\overline{\alpha}_L[-\phi(1+\overline{\alpha}_F) + \phi(1+\overline{\alpha}_L)] = \sigma^2 + \overline{\alpha}_L^2\phi + 2\overline{\alpha}_L\phi(\overline{\alpha}_F - \overline{\alpha}_L) > 0, \tag{A.28}$$

where $\overline{\alpha}_F - \overline{\alpha}_L > 0$ by (A.27). But since (A.12) is satisfied for all n, this limit must be nonpositive, a contradiction. □

Lemma A.6. *There is no $[-\phi, \phi]$-valued sequence $(\rho_n)_{n\in\mathbb{N}}$ that converges to $-\phi$ and has the property that there is an associated sequence of PBS equilibria such that $(\alpha_{F,n}) \to +\infty$.*

Proof. Suppose by way of contradiction that there were such a sequence. From the expression for $\alpha_{F,n}$ in (A.14), it must be that $\alpha_{L,n} \to +\infty$. We claim that $\frac{\alpha_{F_n}}{\alpha_{L,n}} \to 1$. To obtain this, divide through (A.14) by $\alpha_{L,n}$ to get

$$\frac{\alpha_{F_n}}{\alpha_{L,n}} = \frac{(\rho_n + \phi + \phi\alpha_{L,n})(\alpha_L^2\phi - \sigma^2)}{\rho_n\alpha_{L,n}\left[\sigma^2 - \alpha_{L,n}(1+\alpha_{L,n})\phi\right]} \to 1.$$

We now show that (A.12) eventually fails. The right hand side of (A.12) rearranges to

$$\sigma^2 + \alpha_{L,n}^2\phi - 2\alpha_{L,n}[\phi + \rho_n + \alpha_{L,n}(\rho_n\alpha_{F,n}/\alpha_{L,n} + \phi)]. \tag{A.29}$$

Since $\phi + \rho_n \to 0$ and $\frac{\alpha_{F,n}}{\alpha_{L,n}} \to 1$, for any $\epsilon > 0$, the expression in square brackets in (A.29) is less than $\epsilon\alpha_{L,n}$ for sufficiently large n. Hence, (A.29) is eventually greater than $\sigma^2 + \alpha_{L,n}^2\phi - 2\epsilon\alpha_{L,n}^2$, which is positive for $\epsilon < \phi/2$, violating (A.12), contradicting equilibrium. □

The existence of $\hat{\rho} > -\phi$ then follows immediately from Lemmas A.5 and A.6, since if there is no such $\hat{\rho}$ there would exist a sequence $(\rho_n)_{n\in\mathbb{N}}$ with $\rho_n \to -\phi$ and an associated sequence of PBS equilibria such that either (i) $\alpha_{F,n} \to +\infty$ along some subsequence (which is ruled out by Lemma A.6) or (ii) $(\alpha_{F,n})_{n\in\mathbb{N}}$ is bounded (ruled out by Lemma A.5). Since Proposition A.4 shows that a PBS equilibrium exists for some $\rho < 0$, we have $\hat{\rho} < 0$. □

For any $\hat{\rho}$ as in Proposition A.5, $\underline{\rho} \geq \hat{\rho} > -\phi$. This concludes the proof of Theorem 1.

A.5 Monotonicity of leader's strategy coefficients

The following result establishes the decreasing patterns of α_L and δ_L with respect to ρ shown in Figure 1. Note that Proposition A.2 established that when $\rho = 0$, $\alpha_L = \alpha^K = -\delta_L$.

Proposition A.6. *Suppose $\rho > \rho_0$, where $\rho_0 < 0$ was defined in the proof of Theorem 1. Then in the unique PBS equilibrium, α_L and δ_L are decreasing in ρ.*

Proof. Due to the identity (A.11), it is sufficient to prove the claim for α_L. First suppose $\rho > 0$. The right hand side of (A.14) crosses the left hand side from above at α_L. Moreover, when $\rho > 0$, the right hand side is positive and decreasing in ρ at α_L while the left hand side is increasing in ρ. Hence, α_L is decreasing in ρ. In turn, when $\rho < 0$, the right hand side of (A.14) crosses the left hand side from below; the left hand side is decreasing in ρ; and the right hand side is increasing in ρ at α_L. Hence, again, α_L is unambiguously decreasing in ρ. The result then follows since α_L is continuous in ρ at $\rho = 0$ by Corollary A.1. $\square$

A.6 Proof of Proposition 1

For both parts (a) and (b), we focus on equilibria with positive weight on private information, i.e. linear equilibria in which

$$\begin{aligned} \theta^L &:= \alpha_L \xi^L + \delta_L \mu + \eta_L, \\ \theta^F &:= \alpha_F \xi^F + \beta_F P_1 + \delta_F \mu + \eta_F = \alpha_F(\xi^F - M_1^F), \end{aligned}$$

for $\xi \in \{V, \zeta\}$, where $M_1^F := \mathbb{E}[\xi^F | \mathcal{F}_1]$ (see below), and where $\alpha_L, \alpha_F > 0$. The reader can access the full equations and expressions in the Mathematica file `OtherPrivateInfo.nb` on the authors' websites. A straightforward adaptation of the steps from the baseline analysis can be used to show that the follower's strategy must be a "gap strategy" in any linear equilibrium, i.e. a strategy of the form $\theta^F = \alpha_F(\xi^F - \mathbb{E}[\xi^F|\mathcal{F}_1])$ which implies $\mathbb{E}[\theta^F|\mathcal{F}_1] = 0$. Thus, in what follows, we assume this form.

We show specifically that there exists a unique equilibrium within this class whenever ρ is not too negative. The leader trades according to

$$\theta^L = \alpha^K(\xi^L - \mu) + \eta_L, \tag{A.30}$$

where $\alpha^K = \sigma/\sqrt{\phi}$ and $\eta_L = X_0^L \frac{\beta_F}{1-\beta_F}$. Moreover, we show that $\beta_F < 1$ and $\text{sign}(\beta_F) = -\text{sign}(\rho)$, and thus $\text{sign}(\mathbb{E}[\theta^L|\mathcal{F}_0]) = \text{sign}(\eta_L) = -\text{sign}(\rho)$. An implication is that the leader's expected terminal position,

$$X_0^L + \mathbb{E}[\theta^L|\mathcal{F}_0] = X_0^L + \eta_L = \frac{X_0^L}{1 - \beta_F}, \tag{A.31}$$

is increasing in her initial position X_0^L.

A.6.1 Part (a)

Since the effort technology is unchanged, it continues to be optimal to choose effort equal to the number of shares held; thus the firm's final value will be $V^L + V^F + X_T^L + X_T^F$, where $X_T^i = X_0^i + \theta^i$ as before. Hence, the objective of activist i reduces to

$$\sup_{\theta^i} \mathbb{E}[(V^L + V^F + X_T^i + X_T^{-i})X_T^i - P_{t(i)}\theta^i - \frac{1}{2}(X_T^i)^2 | V^i, \mathcal{F}_{t(i)-1}, \theta^i].$$

Learning and pricing Conjecturing linear strategies (with a gap strategy for the follower), the ex ante expectation of firm value is

$$P_0 = X_0^L + X_0^F + \eta_L + (2 + \alpha_L + \delta_L)\mu,$$

where we have used that the follower's expected trade is 0 from an ex ante perspective. Since the type distribution is unchanged, the players' private prior beliefs about each other's initial positions have the same form as in the baseline model, with V^L and V^F playing the role of X_0^L and X_0^F, respectively.

Given Ψ_1, the MM's updated belief about V^L is

$$M_1^L := \mathbb{E}[V^L|\mathcal{F}_1] = \mu + \frac{\alpha_L \phi}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \right\}.$$

And the MM's updated belief about V^F is

$$M_1^F := \mathbb{E}[V^F|\mathcal{F}_1] = \mu + \frac{\alpha_L \rho}{\alpha_L^2 \phi + \sigma^2} \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \right\}.$$

Since the MM expects the follower to trade 0 conditional on first period order flow,

$$\begin{aligned} P_1 &= X_0^L + X_0^F + \eta_L + \mathbb{E}[V^L + V^F + \theta^L|\Psi_1] \\ &= X_0^L + X_0^F + \eta_L + M_1^L(1 + \alpha_L) + \delta_L\mu + M_1^F \\ &= P_0 + \Lambda_1 \left\{ \Psi_1 - \mu(\alpha_L + \delta_L) - \eta_L \right\}, \end{aligned}$$

where $\Lambda_1 := \frac{\alpha_L[\rho + (1+\alpha_L)\phi]}{\alpha_L^2\phi + \sigma^2}$. This is identical (up to a possibly different value of α_L) to Λ_1 in the baseline model, using the identity (A.8) that β_F satisfies in a gap strategy. Note that $\Lambda_1 > 0$ for any $\alpha_L > 0$, since $\rho + \phi \geq 0$.

The MM's posterior belief about (V^L, V^F) has covariance matrix $\begin{pmatrix} \gamma_1^L & \rho_1 \\ \rho_1 & \gamma_1^F \end{pmatrix}$, where

$$\gamma_1^L = \frac{\phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \gamma_1^F = \frac{\alpha_L^2[\phi^2 - \rho^2] + \phi\sigma^2}{\alpha_L^2\phi + \sigma^2}, \qquad \rho_1 = \frac{\rho\sigma^2}{\alpha_L^2\phi + \sigma^2}.$$

The follower's mean posterior belief about the leader's component V^L is

$$Y_1^F := Y_0^F + \frac{\alpha_L \nu_0^F}{\alpha_L^2\nu_0^F + \sigma^2} \underbrace{\left\{\frac{P_1 - P_0}{\Lambda_1} + \alpha_L(\mu - Y_0^F)\right\}}_{=\Psi_1 - (\alpha_L Y_0^F + \delta_L\mu) - \eta_L}.$$

After seeing Ψ_2, the market maker again updates beliefs about V^L and V^F:

$$M_2^F := M_1^F + \frac{\alpha_F\gamma_1^F}{\alpha_F^2\gamma_1^F + \sigma^2}\Psi_2 \quad \text{and} \quad M_2^L := M_1^L + \frac{\alpha_F\rho_1}{\alpha_F^2\gamma_1^F + \sigma^2}\Psi_2.$$

The price is then

$$P_2 = P_1 + \Psi_2 \underbrace{\frac{\alpha_F[(1+\alpha_L)\rho_1 + (1+\alpha_F)\gamma_1^F]}{\alpha_F^2\gamma_1^F + \sigma^2}}_{=:\Lambda_2}.$$

This Λ_2 is equivalent to the one in the baseline model.

Optimality conditions and equilibrium characterization The follower's first order condition is

$$\begin{aligned} 0 &= \mathbb{E}[V^L + \theta^L | V^F, \mathcal{F}_1] + V^F + X_0^L + X_0^F + \theta^F - P_1 - 2\Lambda_2\theta^F \\ \implies \theta^F &= \frac{Y_1^F(1+\alpha_L) + \delta_L\mu + \eta_L + V^F + X_0^L + X_0^F - P_1}{2\Lambda_2 - 1}. \end{aligned}$$

It is straightforward to check that the RHS is equivalent to $\alpha_F(V^F - M_1^F)$ for $\alpha_F = \sqrt{\sigma^2/\gamma_1^F}$. The remaining coefficients are $\beta_F = -\frac{\rho\alpha_F}{\phi(1+\alpha_L)+\rho}$ and $\delta_F = \frac{(\alpha_L+\delta_L)\rho - \alpha_L\phi - (\phi-\rho)}{\phi(1+\alpha_L)+\rho}\alpha_F$, and $\eta_F = -\beta_F(X_0^L + X_0^F + \eta_L)$. Note that β_F has the opposite sign of ρ.

The leader's FOC is

$$0 = -\mathbb{E}_L[P_0 + \Lambda_1\{\Psi_1 - \mathbb{E}[\Psi_1]\}|\theta^L] - \theta\Lambda_1$$

$$+\underbrace{(X_0^L+\theta^L)+V^L+Y_0^L+\mathbb{E}_L[X_T^F|\theta^L]}_{=\mathbb{E}[\text{firm value}|V^L,\theta^L]}+\underbrace{(X_0^L+\theta^L)\overbrace{\frac{\partial\mathbb{E}_L[X_T^F|\theta^L]}{\partial\theta^L}}^{=\Lambda_1\beta_F}}_{\text{value of manipulation}} \tag{A.32}$$

$$=-\mathbb{E}_L[P_1|\theta^L]-\theta^L\Lambda_1+V^L+Y_0^L(1+\alpha_F)+\beta_F\mathbb{E}_L[P_1|\theta^L]+\delta_F\mu+\eta_F+\left(X_0^L+\theta\right)(1+\beta_F\Lambda_1).$$

Matching coefficients on V^L and μ and the intercept yields three equations. After substituting in the follower's strategy derived above, it is easy to verify that the coefficients $(\alpha_L,\delta_L)=(\alpha^K,-\alpha^K)$ solve the V^L- and μ- components of the FOC, and these are the only solutions with $\alpha_L>0$ when ρ is positive or sufficiently close to 0.[39] The last equation, derived from the intercept, yields $\eta_L=-X_0^L\frac{\rho\alpha_F}{\rho(1+\alpha_F)+\phi(1+\alpha_L)}=X_0^L\frac{\beta_F}{1-\beta_F}$. The second order conditions are the same as before:

$$\begin{aligned}1-2\Lambda_1(1-\beta_F)<0,\ \text{for}\ \ i=L,\\ 1-2\Lambda_2<0,\ \text{for}\ \ i=F.\end{aligned}$$

By direct substitution of our closed form solution for η_L, these can be rewritten in terms of (ϕ,ρ,σ), and it is easy to check that they are satisfied whenever $\rho\geq\underline{\rho}$, for some $\underline{\rho}\in[-\phi,0)$. Also, recalling that $\Lambda_1>0$ by inspection, the leader's SOC implies $\beta_F<1$. Since $\text{sign}(\beta_F)=-\text{sign}(\rho)$, it follows that $\text{sign}(\mathbb{E}[\theta^L|\mathcal{F}_0])=\text{sign}(\eta_L)=-\text{sign}(\rho)$: the leader sells (buys) on average when correlation is positive (negative).

A.6.2 Part (b)

Given the cost function, trader i's optimal effort is $X_T^i+\zeta^i$. Hence, trader i's objective is

$$\sup_{\theta^i}\mathbb{E}\left[(X_T^i+X_T^{-i}+\zeta^i+\zeta^{-i})X_T^i-P_{t(i)}\theta^i-\frac{1}{2}(X_T^i+\zeta^i)^2+\zeta^i(X_T^i+\zeta^i)\middle|\zeta^i,\mathcal{F}_{t(i)-1},\theta^i\right].$$

For each trader, this objective is the same as in the variation from part (a) of the proposition, with ζ^i in place of V^i, except for a $\frac{(\zeta^i)^2}{2}$ term which is not strategically relevant. The information structure is also the same. Thus, the equilibria are the same as in part (a), and the leader trades according to (A.30).

[39] Note that in the baseline model, for positive correlation, $\alpha_L<\alpha^K$. The greater sensitivity to type here comes from the fact that in the original model, higher types had a greater benefit of manipulation since they, by definition, had more initial shares.

A.7 Proof of Theorem 2

Part (i) Ex ante expected firm value is $\mathbb{E}[W^L + W^F] = \mathbb{E}[X_0^L + \theta^L + X_0^F + \theta^F] = 2\mu + \mathbb{E}[\theta^L]$, where we have used that terminal efforts coincide with terminal positions and that $\mathbb{E}[\theta^F] = 0$. The inequality $\mathbb{E}[W^L + W^F] \leq 2\mu$ is therefore equivalent to $\mathbb{E}[\theta^L] \leq 0$, which holds iff $\rho \leq 0$ (with strict inequality if $\rho \neq 0$) by Theorem 1. Moreover, since $\mathbb{E}[\theta^L] = (\alpha_L + \delta_L)\mu$, we have $\mathbb{E}[W^L + W^F] = (2 + \alpha_L + \delta_L)\mu$, which is monotone decreasing in ρ by Proposition A.6.

Part (ii) We show that $\alpha_L + \delta_L > -1$. Using (A.11), we have $\alpha_L + \delta_L = \alpha_L - \frac{\sigma^2}{\phi\alpha_L} =: h(\alpha_L)$. Note that h is increasing in α_L for $\alpha_L > 0$, and from the proof of Proposition A.3, $\alpha_L > \hat{\alpha}$. By direct calculation, $h(\hat{\alpha}) = -1$, so we are done.

Part (iii) Fix $\rho > 0$. For part (iii.1), we begin with some useful preliminary observations. Recall from the proof of Proposition A.3 that $\hat{\alpha} < \alpha_L < \alpha^K$. But $\lim_{\sigma\to+\infty} \frac{\hat{\alpha}}{\sigma} = 1/\sqrt{\phi} = \lim_{\sigma\to+\infty} \frac{\alpha^K}{\sigma}$, so $\lim_{\sigma\to+\infty} \frac{\alpha_L}{\sigma} = 1/\sqrt{\phi}$. Then by (A.11), $\lim_{\sigma\to+\infty} \frac{\delta_L}{\sigma} = -1/\sqrt{\phi}$. These limits imply $\lim_{\sigma\to+\infty} \alpha_L = +\infty$ and $\lim_{\sigma\to+\infty} \delta_L = -\infty$. Let $x_L := \alpha_L/\sigma$ and $x_F := \alpha_F/\sigma$.

For the first limit in part (iii.1), recall from above that x_L converges to a positive constant as $\sigma \to +\infty$. Using the expression for α_F in (A.14), it is easy to see that x_F also converges to a positive constant as $\sigma \to +\infty$. Now $\mathbb{E}[\theta^L] = \mu(\alpha_L + \delta_L)$, and from (A.10) and (A.11), $\alpha_L + \delta_L = -\frac{\rho\alpha_F}{\phi(1+\alpha_L)+\rho(1+\alpha_F)} = -\frac{\rho x_F}{(\rho+\phi)/\sigma+\phi x_L+\rho x_F}$, which converges to a negative constant as $\sigma \to +\infty$ since both x_L and x_F converge to positive constants.

For the second limit in part (iii.1), note that $\alpha_L - \alpha^K = \frac{\alpha_L}{\alpha_L+\alpha^K}\left(\alpha_L - \frac{(\alpha^K)^2}{\alpha_L}\right)$. The first factor is $\frac{x_L}{x_L+1/\sqrt{\phi}}$, which has a finite positive limit as $\sigma \to +\infty$, and the second equals $\alpha_L + \delta_L$ which, as just argued, converges to a finite negative limit. Hence $\lim\limits_{\sigma\to+\infty}\{\alpha_L - \alpha^K\} \in (-\infty, 0)$.

For part (iii.2), from the proof of Proposition 8, in the PBS equilibrium, α_L/σ converges to a positive constant as $\sigma \to 0$, so it follows that $\lim_{\sigma\to 0} \alpha_L = 0$. By (A.11), $\delta_L/\sigma = -1/(\phi\alpha_L/\sigma)$ converges to a negative constant, and thus $\lim_{\sigma\to 0} \delta_L = 0$. Therefore, $\lim_{\sigma\to 0} \mathbb{E}[\theta^L] = \lim_{\sigma\to 0}\{(\alpha_L + \delta_L)\mu\} = 0$, and $\lim_{\sigma\to 0}\{\alpha_L - \sqrt{\sigma^2/\phi}\} = 0 - 0 = 0$.

A.8 Proof of Proposition 5

We consider symmetric linear strategies of the form

$$\theta^i = \alpha X_0^i + \beta\mu. \tag{A.33}$$

We begin by characterizing belief updating and pricing, and then we use these to set up the best-response problem of either trader. We show that in any symmetric PBS equilibrium, $\alpha = \frac{\sigma}{\sqrt{2\phi}}$, and then we show that there exists $\rho_0^{\text{sim}} \in (-\phi, 0)$ such that for all $\rho \in [\rho_0^{\text{sim}}, \phi]$, there exists a unique symmetric PBS equilibrium.

After observing the total order flow, the market maker updates her beliefs about the activists' positions. Given the form of strategies and symmetry, it is sufficient for the market maker to only estimate the sum of initial positions. By the projection theorem,

$$\begin{aligned}\mathbb{E}[X_0^i + X_0^j|\mathcal{F}_1] &= 2\mu + \frac{\mathrm{Cov}(X_0^i + X_0^j, \Psi_1)}{\mathrm{Var}(\Psi_1)}\left\{\Psi_1 - \underbrace{[2\alpha\mu + 2\beta\mu]}_{=\mathbb{E}[\theta^i+\theta^j]}\right\} \\ &= 2\mu + \frac{2\alpha\,(\phi+\rho)}{2\alpha^2\,(\phi+\rho)+\sigma^2}\left\{\Psi_1 - 2\mu(\alpha+\beta)]\right\}.\end{aligned}$$

Hence, P_1 is equal to

$$\begin{aligned}P_1 &= \mathbb{E}[W|\mathcal{F}_1] = \mathbb{E}[X_T^i + X_T^j|\mathcal{F}_1] = (1+\alpha)\mathbb{E}[X_0^i + X_0^j|\mathcal{F}_1] + 2\mu\beta \\ &= P_0^S + \Lambda_1^S\left\{\Psi_1 - 2\mu(\alpha+\beta)]\right\},\end{aligned}$$

where $P_0^S := 2\mu(1+\alpha+\beta)$ is the ex ante expected firm value and $\Lambda_1^S := (1+\alpha)\frac{2\alpha(\phi+\rho)}{2\alpha^2(\phi+\rho)+\sigma^2}$ is Kyle's lambda.

Each activist then maximizes

$$\sup_{\theta^i}\mathbb{E}\left[\frac{(X_0^i+\theta^i)^2 + 2X_T^{-i}(X_0^i+\theta^i)}{2} - P_1\theta^i\big|X_0^i,\theta^i\right]. \tag{A.34}$$

The FOC is $\frac{2(X_0^i+\theta^i)+2\mathbb{E}[X_T^{-i}|X_0^i]}{2} - \theta^i\frac{\partial P_1}{\partial\theta^i} - P_1 = 0$. Plugging in the expression for Λ_1^S, evaluating at the conjectured strategy (A.33), and setting the coefficient on X_0^i to 0 yields an equation for α with the following three roots:

$$\alpha = \frac{\sigma}{\sqrt{2\phi}}, \quad -\frac{\sigma}{\sqrt{2\phi}}, \quad -1. \tag{A.35}$$

Similarly, setting the coefficient on μ to 0, we can pin down β from α via the following equation

$$\beta = \frac{\sigma^2}{2\sigma^2 - 4\alpha(1+\alpha)\phi}. \tag{A.36}$$

Since the second and third roots are negative, we have a unique candidate for a symmetric PBS equilibrium.

<u>Existence and uniqueness:</u> For existence, we must check the SOC: $1 - 2\Lambda_1^S \leq 0$. Plugging in

$\alpha = \frac{\sigma}{\sqrt{2\phi}}$, this condition is equivalent to the inequality

$$\sigma^2 - 2\alpha(2+\alpha)(\rho+\phi) = \sigma^2 - 2\frac{\sigma}{\sqrt{2\phi}}\left(2 + \frac{\sigma}{\sqrt{2\phi}}\right)(\phi+\rho) \leq 0.$$

The left hand side is decreasing and continuous in ρ, and it is strictly negative when $\rho = 0$, so there exists $\rho_0^{\text{sim}} \in (-\phi, 0)$ such that the inequality is satisfied, and in turn a unique PBS equilibrium exists, whenever $\rho \in [\rho_0^{\text{sim}}, \phi]$.

Payoff comparison: To compare payoffs to those in the sequential-move game, first consider $\rho = 0$. The equilibrium is characterized in Proposition A.2, and $\alpha_L = \alpha_F = \sqrt{\frac{\sigma^2}{\phi}}$. The coefficient in the simultaneous-move game is $\alpha_S := \sqrt{\frac{\sigma^2}{2\phi}}$ (see (A.35)), where $\alpha_L = \alpha_F > \alpha_S$.

To calculate the players' expected payoffs in the sequential case (which are the same given $\rho = 0$), plug the equilibrium strategies into (3) to obtain

$$\begin{aligned}\mathbb{E}\Bigg[&\tfrac{1}{2}\left(X_0^L\left(1+\sqrt{\tfrac{\sigma^2}{\phi}}\right) - \sqrt{\tfrac{\sigma^2}{\phi}}\mu\right)^2 + \left(X_0^F + \sqrt{\tfrac{\sigma^2}{\phi}}(X_0^F-\mu)\right)\left(X_0^L + \sqrt{\tfrac{\sigma^2}{\phi}}(X_0^L-\mu)\right)\\ &- \left(P_0 + \Lambda_1\left(\sqrt{\tfrac{\sigma^2}{\phi}}(X_0^L-\mu) + \sigma Z_1\right)\right)\sqrt{\tfrac{\sigma^2}{\phi}}(X_0^L-\mu)\Bigg].\end{aligned}$$

Opening up the expectation and simplifying we can write the first line as $\frac{1}{2}\left(\mu^2 + (\sigma+\sqrt{\phi})^2\right) + \mu^2$ and second line as $-\frac{\sigma(\sigma+\sqrt{\phi})}{2}$. Hence, each trader's total expected payoff when $\rho = 0$ is

$$\frac{1}{2}\left[3\mu^2 + \phi + \sigma\sqrt{\phi}\right]. \tag{A.37}$$

Following similar steps for the simultaneous case, we can write the equilibrium payoff of player i ($i = 1, 2$) as

$$\begin{aligned}\mathbb{E}\Bigg[&\tfrac{1}{2}\left(X_0^i\left(1+\sqrt{\tfrac{\sigma^2}{2\phi}}\right) - \sqrt{\tfrac{\sigma^2}{2\phi}}\mu\right)^2 + 2\left(X_0^j + \sqrt{\tfrac{\sigma^2}{2\phi}}(X_0^j-\mu)\right)\left(X_0^i + \sqrt{\tfrac{\sigma^2}{2\phi}}(X_0^i-\mu)\right)\\ &- \left(P_0^S + \Lambda_1^S\left(\sqrt{\tfrac{\sigma^2}{2\phi}}(X_0^i-\mu) + \epsilon_i\right)\right)\sqrt{\tfrac{\sigma^2}{2\phi}}(X_0^i-\mu)\Bigg].\end{aligned}$$

Opening up the expectation, the first line simplifies to $\frac{1}{2}\left(\mu^2 + \frac{(\sigma+\sqrt{2\phi})^2}{2}\right) + \mu^2$, while the second line simplifies to $-\frac{\sigma(\sigma+\sqrt{2\phi})}{4}$, for a total expected payoff of

$$\frac{1}{2}\left[3\mu^2 + \phi + \frac{\sigma\sqrt{2\phi}}{2}\right]. \tag{A.38}$$

Subtracting (A.38) from (A.37) yields $\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)\sigma\sqrt{\phi}$, which is strictly positive. Therefore, the both players unambiguously prefers the sequential-move game when $\rho = 0$.

The same comparison extends to $|\rho| >$ sufficiently small by continuity. Specifically, Proposition A.4 and the results above, establish existence and uniqueness for small $|\rho|$. For such $|\rho|$, α_L and α_F in the sequential-move game are continuous in ρ at $\rho = 0$ by Corollary A.1. After using (A.8), (A.9), and (A.11) to eliminate $(\beta_F, \delta_F, \delta_L)$, the players' payoffs can be written as continuous functions of $(\rho, \alpha_L, \alpha_F)$ and are therefore continuous in ρ at $\rho = 0$.[40] For the simultaneous-move case, the equilibrium trading coefficients are independent of ρ as shown earlier, and payoffs are clearly continuous in ρ. Figure 4 illustrates.

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[40] Full expressions for general ρ are available from the authors upon request.

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