

Collateral and Capital Structure

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Collateral as the Determinant of Capital Structure

Punchline

- **The need to collateralize loans determines capital structure.**
- Intuition:
 - Enforcement of repayment is limited to tangible assets state-by-state.
 - Thus tangible assets constrain **financing** and **risk management**
 - ... and in turn **investment** itself.
 - **Leasing** is costly collateralized lending relaxing financing constraints.
- Model leads to a unified theory based on collateral constraints of optimal
 - ... investment/capital structure/risk management/leasing.

Stylized Facts on Collateral and Capital Structure

Account for capital structure facts

- Our theory helps account for stylized empirical facts on capital structure.

Tangible assets

- Extensive empirical literature finds tangibility one of few robust determinants of capital structure.
- Key determinant of leverage and explanation of “low leverage puzzle.”

Risk management (puzzle)

- Large/dividend-paying (not small/zero-dividend) firms do risk management.

Leased assets

- Rental leverage quantitatively important and
 - ... reduces fraction of low leverage firms drastically
 - ... changes leverage-size relation.

Model of Dynamic Collateralized Financing

Main elements

- **Collateral constraints** due to limited enforcement
 - Otherwise complete markets
 - Agency based model as in Rampini/Viswanathan (2010)
- **Tangibility**
 - Two types of capital: tangible and intangible capital
- **Dynamic** model
 - Financing is an inherently dynamic problem
- **Leasing** as strong collateralization
 - Eisfeldt/Rampini (2009)

Abridged Literature Review

Dynamic agency based models of the capital structure

- **Limited enforcement**

- Theory: Albuquerque/Hopenhayn (2004), Hopenhayn/Werning (2007)
- Macro: Cooley/Marimon/Quadrini (2004), Jermann/Quadrini (2008)
- Quantitative: Schmid (2008), Lorenzoni/Walentin (2008)

- **Private information** and/or moral hazard

- Theory: Clementi/Hopenhayn (2006)
- Implementation: DeMarzo/Fishman (2007a, 2007b), Atkeson/Cole (2008)
- Cont. time: DeMarzo/Sannikov (2006), Biais/Mariotti/Plantin/Rochet (2007), Biais/Mariotti/Rochet/Villeneuve (2009)
- Neoclassical: DeMarzo/Fishman/He/Wang (2007)

- Address **challenge**

- Existing models typically predict intricate dynamic optimal contracts
- Impediment to empirical implementation

Collateralized Financing: Aggregate Perspective

Panel A: Liabilities (% of tangible assets)

Sector	Debt (% of tangible assets)	Total liabilities (% of tangible assets)
(Nonfinancial) corporate businesses	48.5%	83.0%
(Nonfinancial) noncorporate businesses	37.8%	54.9%
Households and nonprofit organizations		
Total tangible assets	45.2%	47.1%
Real estate	41.2%	
Consumer durables	56.1%	

Panel B: Tangible assets (% of household net worth)

Assets by type	Tangible assets (% of household net worth)
Total tangible assets	79.2%
Real estate	60.2%
Equipment and software	8.3%
Consumer durables	7.6%
Inventories	3.1%

Stylized Facts: Tangible Assets and Debt Leverage

Data

Tangibility quartile	Quartile cutoff (%)	Leverage (%)		Low leverage firms (%) (leverage \leq 10%)
		median	mean	
1	6.3	7.4	10.8	58.3
2	14.3	9.8	14.0	50.4
3	32.2	12.4	15.5	40.6
4	n.a.	22.6	24.2	14.9

Tangibility: Property, Plant, and Equipment – Total (Net) (Item #8) divided by Assets; Assets: Assets – Total (Item #6) plus Price – Close (Item #24) times Common Shares Outstanding (Item #25) minus Common Equity – Total (Item #60) minus Deferred Taxes (Item #74); Leverage: Long-Term Debt – Total (Item #9) divided by Assets.

Stylized facts

- Fact 1: Tangibility important **determinant of debt leverage**.
 - Across tangibility quartiles, leverage varies by factor 2.5-3.
- Fact 2: Tangibility important **explanation of “low leverage puzzle.”**
 - Fraction with leverage \leq 10% much lower in highest tangibility quartile.

Ignored: Rented Capital

Capitalize rented capital

- **Rent** is Jorgenson (1963)'s user cost:

$$\text{Rent} = (r + \delta)k$$

- Capitalize rental expense (Compustat Item #47) using $\frac{1}{r+\delta}$
 - We capitalize by taking 10 times rental expense: $\frac{1}{r+\delta} \approx 10$

- **Jorgensonian user cost**

$$u = r + \delta$$

Rented Capital in Practice

Accounting: constructive capitalization

- Common heuristic capitalization approach: **“8 x rent”**
 - Moody’s rating methodology: 5x, 6x, 8x, and 10x current rent expense.
- Imhoff/Lipe/Wright (1991, 1993): constructive capitalization of operating leases.

Stylized Facts: Rental Leverage

Data

Lease adjusted tangibility quartile	Quartile cutoff (%)	Leverage (%)						Low leverage firms (%) (leverage \leq 10%)		
		Debt		Rental		Lease adjusted		Debt	Rental	Lease adjusted
		median	mean	median	mean	median	mean			
1	13.2	6.5	10.4	3.7	4.2	11.4	14.6	61.7	97.7	46.0
2	24.1	9.8	12.9	6.9	8.1	18.4	21.0	50.1	68.2	16.1
3	40.1	13.1	14.8	8.0	10.5	24.2	25.3	41.7	60.6	12.0
4	n.a.	18.4	20.4	7.2	13.8	32.3	34.2	24.4	57.3	3.7

Lease Adjusted Tangibility: Property, Plant, and Equipment – Total (Net) (Item #8) plus 10 times Rental Expense (#47) divided by Lease Adjusted Assets; Lease Adjusted Assets: Assets – Total (Item #6) plus Price – Close (Item #24) times Common Shares Outstanding (Item #25) minus Common Equity – Total (Item #60) minus Deferred Taxes (Item #74) plus 10 times Rental Expense (#47); Debt Leverage: Long-Term Debt – Total (Item #9) divided by Lease Adjusted Assets; Rental Leverage: 10 times Rental Expense (#47) divided by Lease Adjusted Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage.

Stylized facts

- Fact 3: Rental leverage **reduces fraction of low leverage firms drastically.**
- Fact 2': Lease adjusted tangibility key explanation of “low leverage puzzle.”
 - Fraction with leverage \leq 10% drastically lower with high lease adjusted tangibility.

Stylized Facts: Leverage and Size Revisited

Data

	Size deciles									
Median Leverage	1	2	3	4	5	6	7	8	9	10
Debt	6.0	7.3	7.4	14.1	19.5	22.6	20.6	20.2	21.6	17.8
Rental	21.8	14.6	10.8	11.1	11.2	9.1	9.7	9.1	7.8	7.3
Lease adjusted	30.6	24.2	21.0	28.8	36.4	37.7	33.4	36.6	31.7	26.3

Lease Adjusted Book Assets: Assets – Total (Item #6) plus 10 times Rental Expense (#47); Debt Leverage: Long-Term Debt – Total (Item #9) divided by Lease Adjusted Book Assets; Rental Leverage: 10 times Rental Expense (#47) divided by Lease Adjusted Book Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage.

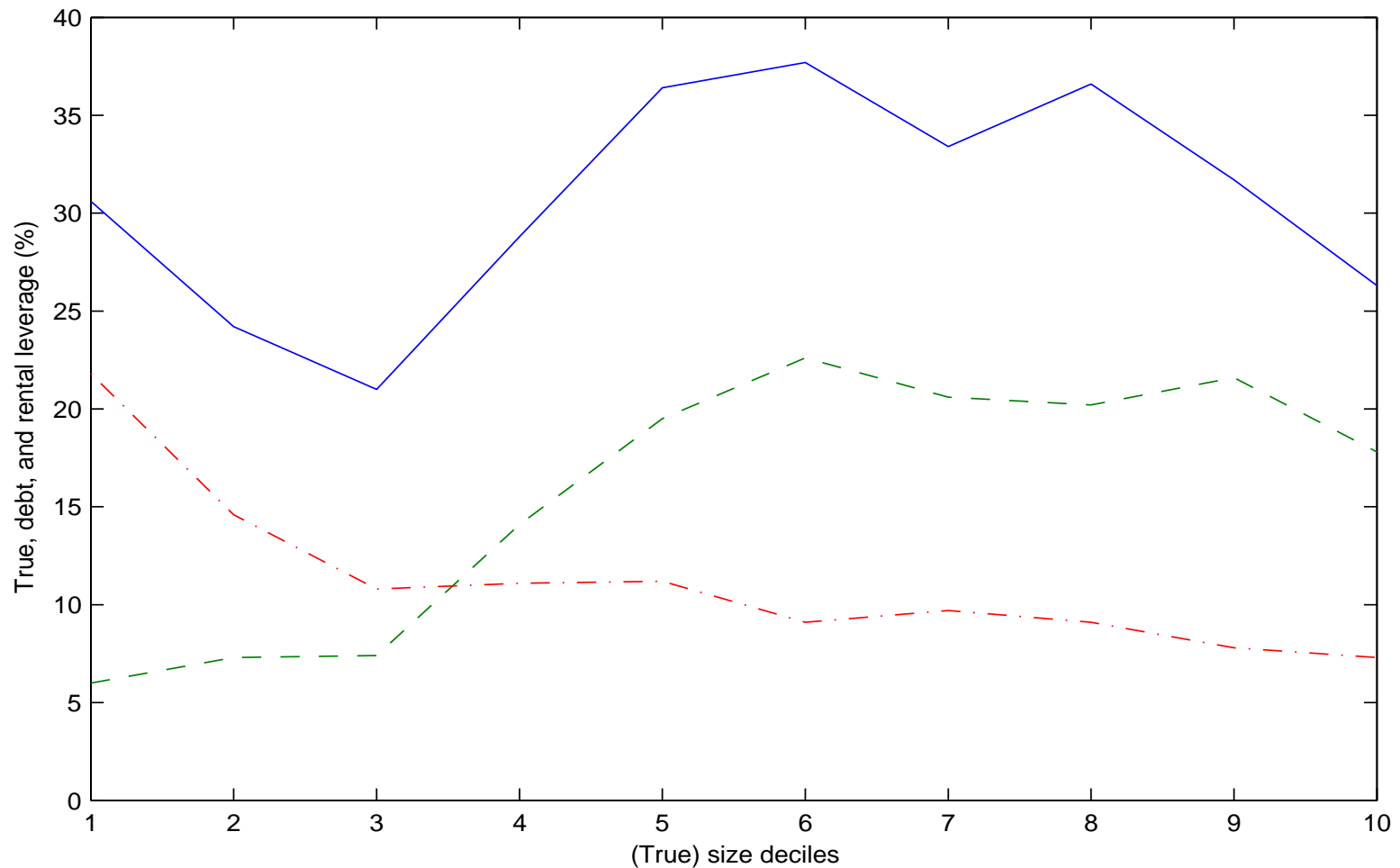
Stylized facts

- Fact 4: Small firms have lower debt leverage but much higher rental leverage.
 - **Total leverage approximately constant across size deciles.**

Debt, Rental, and Lease Adjusted Leverage versus Size

Leverage versus size

Debt leverage (dashed), rental leverage (dash dotted), and lease adjusted leverage (solid) across size deciles (for Compustat firms).



Model

Borrower

- Risk neutral, limited liability, discount future payoffs at $\beta < 1$
- Two types of capital:
 - **Physical** (tangible) **capital**
 - **Intangible capital** k_i
 - Fixed proportions: fraction φ of total capital k is physical capital
 - All capital depreciates at rate δ
 - No adjustment costs
- Standard neoclassical production function:
 - Cash flows $A(s')f(k)$ where $A(s')$ is (stochastic) productivity with Markov process $\Pi(s, s')$

Model (cont'd)

Lenders

- Risk neutral, discount future payoffs at $R^{-1} > \beta$; let $R \equiv 1 + r$

Model (cont'd)

Financing subject to collateral constraints

- Borrower can abscond with cash flows, all intangible capital, and $1 - \theta$ of (owned) physical capital
 - Limited enforcement as in Rampini/Viswanathan (2010): no exclusion
- Collateral constraints: fraction θ of resale value of (purchased) physical capital
 - Related: Kiyotaki/Moore (1997)
- No leasing (for now)

Borrower's Problem

Dynamic program

- Borrower solves

$$V(w, s) \equiv \max_{\{d, k, w'(s'), b(s')\} \in \mathbb{R}_+^{2+S} \times \mathbb{R}^S} d + \beta \sum_{s' \in S} \Pi(s, s') V(w'(s'), s')$$

subject to the budget constraints

$$\begin{aligned} w + \sum_{s' \in S} \Pi(s, s') b(s') &\geq d + k \\ A(s') f(k) + k(1 - \delta) &\geq w'(s') + Rb(s'), \quad \forall s' \in S, \end{aligned}$$

and the **collateral constraints**

$$\theta \varphi k(1 - \delta) \geq Rb(s'), \quad \forall s' \in S.$$

Borrower's Problem (cont'd)

Comments

- **State variables: net worth** w and productivity s
- State-contingent borrowing $b(s')$ allows **risk management**

Dividend Policy

Optimal policy

- **Cutoff dividend policy**

- Dividends paid when net worth exceeds (state-dependent) cutoff $\bar{w}(s)$.
- Firms which pay dividends are not unconstrained
 - ... multiplier on collateral constraint for s' , $\lambda(s')$, positive for some s' ,
 - ... although marginal value of net worth is 1 above this cutoff.

- Investment policy

- Investment constant when net worth exceeds cutoff.

Tangibility and Capital Structure

Industry variation in tangibility

- Determinant of industry variation in leverage
- Determinant of industry variation in financial constraints
 - Firms in industries with lower tangibility more constrained/constrained for longer.

User Cost of Capital

User cost of purchased physical capital u_p

- Premium on internal funds ρ implicitly defined using firm's stochastic discount factor as

$$1/(1 + r + \rho) \equiv \sum_{s' \in S} \Pi(s, s') \beta \mu(s') / \mu.$$

- User cost u_p exceeds Jorgensonian user cost

$$u_p \equiv r + \delta + \frac{\rho}{R + \rho} (1 - \theta)(1 - \delta)$$

where $\rho/(R + \rho) = \sum_{s' \in S} \Pi(s, s') R \beta \lambda(s') / \mu.$

Weighted Average User Cost of Capital

User cost of purchased physical capital u_p

- User cost in **weighted average cost of capital** form

$$u_p = \frac{R}{R + \rho} \left(\underbrace{(r + \rho) (1 - R^{-1}\theta(1 - \delta))}_{\text{Cost of internal funds}} + \underbrace{r (R^{-1}\theta(1 - \delta))}_{\text{Cost of external funds}} + \delta \right)$$

- Wedge in cost of funds: $\rho > 0$ as long as multiplier on collateral constraint $\lambda(s') > 0$, for some $s' \in S$.

Risk Management Interpretation

Equivalent to state contingent debt $b(s')$

- **Uncontingent debt**

- ... $R^{-1}\theta(1 - \delta)$ per unit of owned physical capital.

- ... and **hedging** by purchasing state s' Arrow securities with payoff

$$h(s') \equiv \theta\varphi k(1 - \delta) - Rb(s')$$

to keep **financial slack**.

- Collateral constraints:

$$h(s') \geq 0, \quad \forall s' \in S$$

Optimal Absence of Risk Management

Absence of risk management

- **Proposition:**

- Firms with sufficiently low net worth do not engage in risk management, that is, $\exists \underline{w}_h > 0$, such that $\forall w \leq \underline{w}_h$ and any state s , all collateral constraints bind.

- Intuition:

- **Need to finance investment overrides hedging concern.**

Optimal Absence of Risk Management: Proof

Proof

- Investment Euler equation

$$\begin{aligned} 1 &\geq \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{[A(s') f_k(k) + (1 - \theta\varphi)(1 - \delta)]}{1 - R^{-1}\theta\varphi(1 - \delta)} \\ &\geq \Pi(s, s') \beta \frac{\mu(s')}{\mu} \frac{A(s') f_k(k)}{1 - R^{-1}\theta\varphi(1 - \delta)} \end{aligned}$$

- As net worth $w \rightarrow 0$, investment $k \rightarrow 0$, and marginal product of capital $f_k(k) \rightarrow +\infty$.
- Hence, relative marginal value of net worth $\frac{\mu(s')}{\mu} \rightarrow 0$ and collateral constraint multipliers

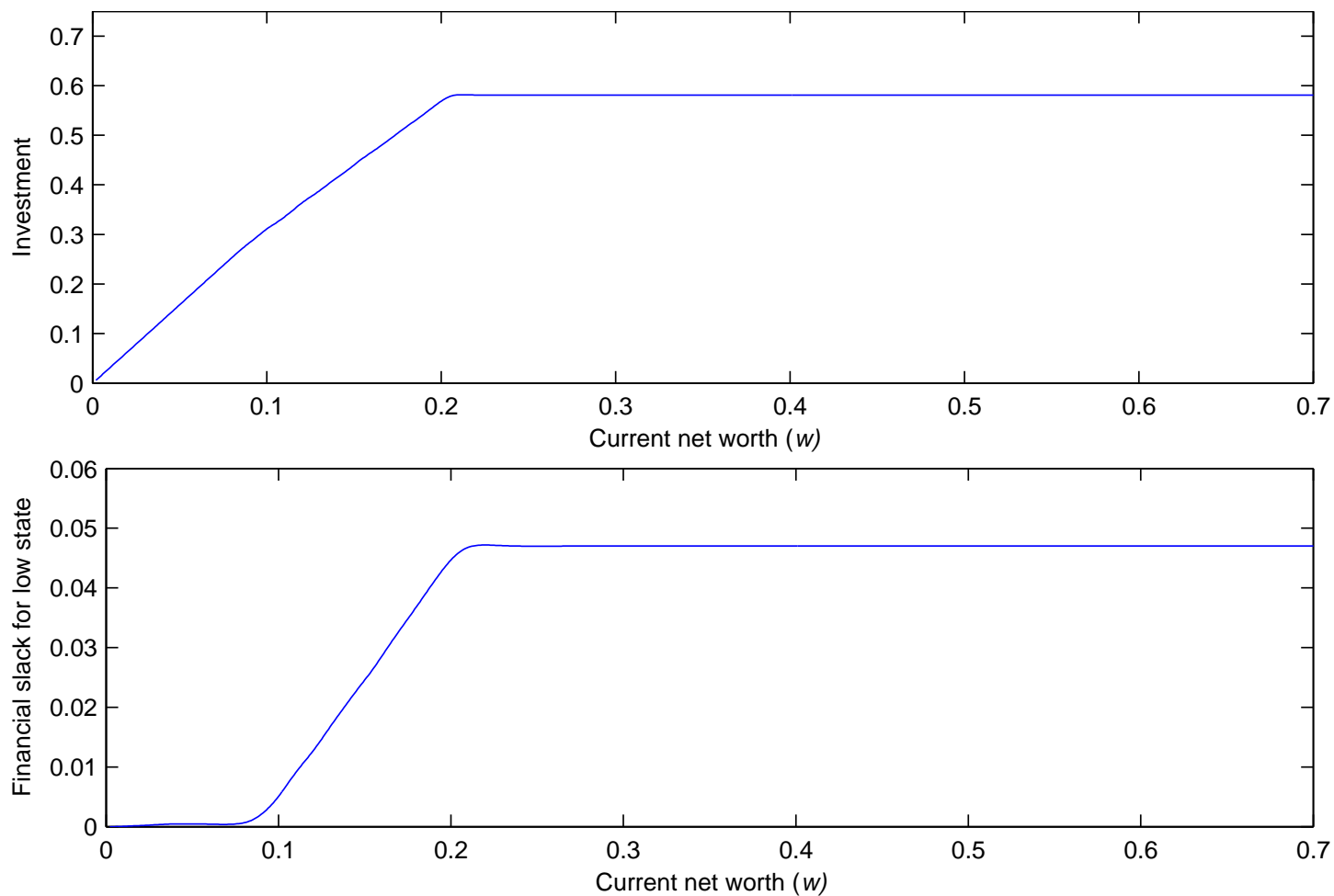
$$\frac{\lambda(s')}{\mu} = (R\beta)^{-1} - \frac{\mu(s')}{\mu} \rightarrow (R\beta)^{-1} > 0, \quad \forall s' \in S.$$

- All collateral constraints bind!

Optimal Risk Management Policy: Example

Investment and financial slack

Investment k (top panel) and financial slack in the low state $h(s'_1) = \theta\varphi k(1 - \delta) - Rb(s'_1)$ (bottom panel) as a function of current net worth w .



Risk Management under Stationary Distribution

Absence of risk management

- **Proposition:**

- Suppose $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$, and $m = +\infty$.
- There exists a unique stationary distribution of firm net worth.
- Under stationary distribution, **firm abstains from risk management with positive probability.**

- Dynamics:

- Sufficiently long sequence of low cash flows renders firm so constrained, that it discontinues risk management.

Reconsidering Risk Management

Evidence

- Smaller (and low dividend paying) firms hedge less.

Risk management puzzle?

- Received theory of risk management (Froot/Scharfstein/Stein (1993))
 - Firms with concave production function and subject to (convex) financing costs are effectively risk averse and have incentive to hedge.

Our model: Fundamental financing – risk management trade-off

- Resolution of risk management puzzle
 - More constrained firms hedge less, since the need to finance investment overrides hedging concerns.
- See also: Rampini/Viswanathan (2010) “Collateral, risk management, and the distribution of debt capacity” (2 period version of model)

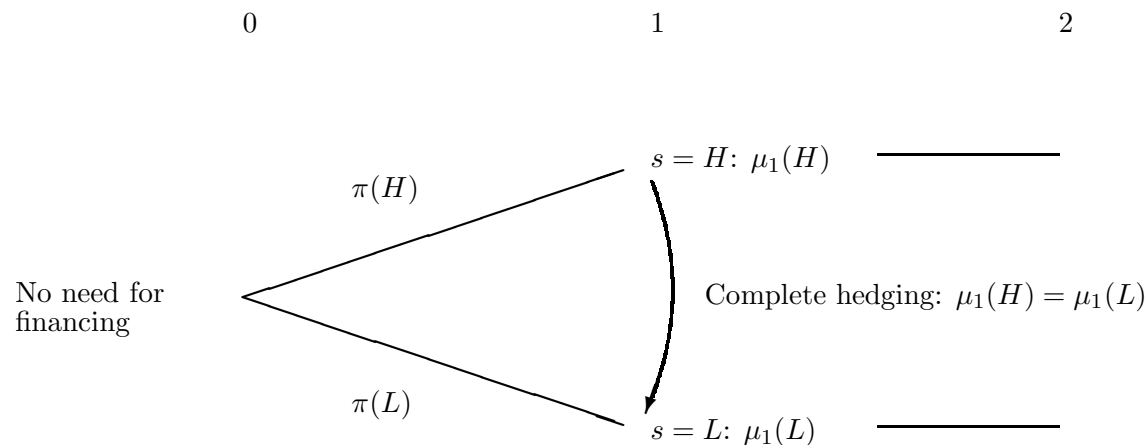
Reconsidering Risk Management (cont'd)

Received theory of risk management

- Froot/Scharfstein/Stein (1993) assume
 - ... complete markets, perfect enforcement at $t = 1$, & no financing need at $t = 0$

and show that optimal hedging policy implies “**full hedging**”

- ... and equalizes marginal value of net worth across states at $t = 1$.



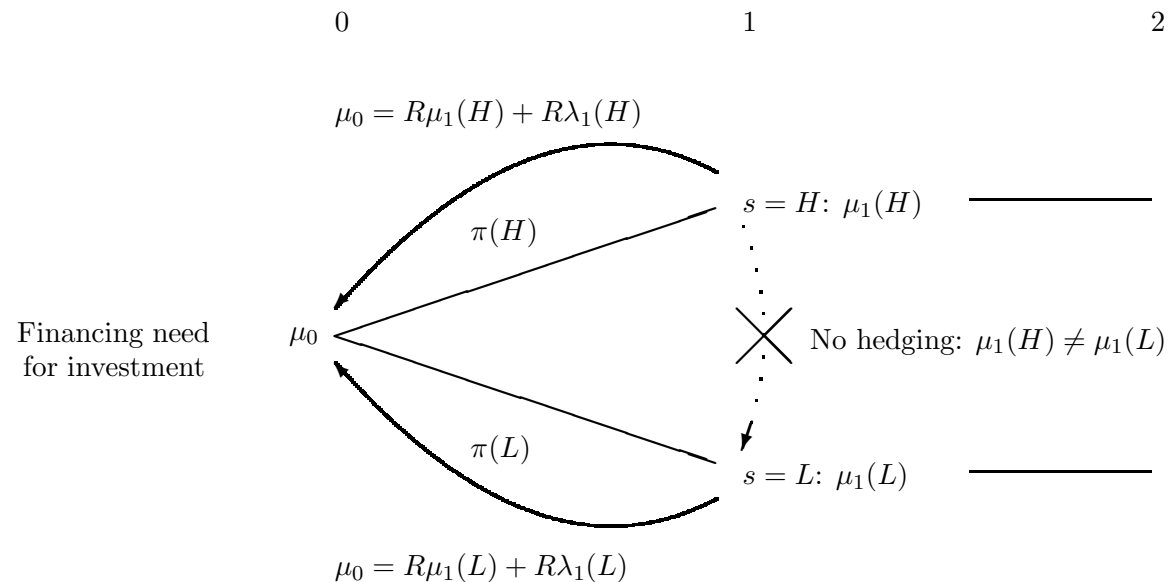
Reconsidering Risk Management (cont'd)

Financing and risk management subject to collateral constraints

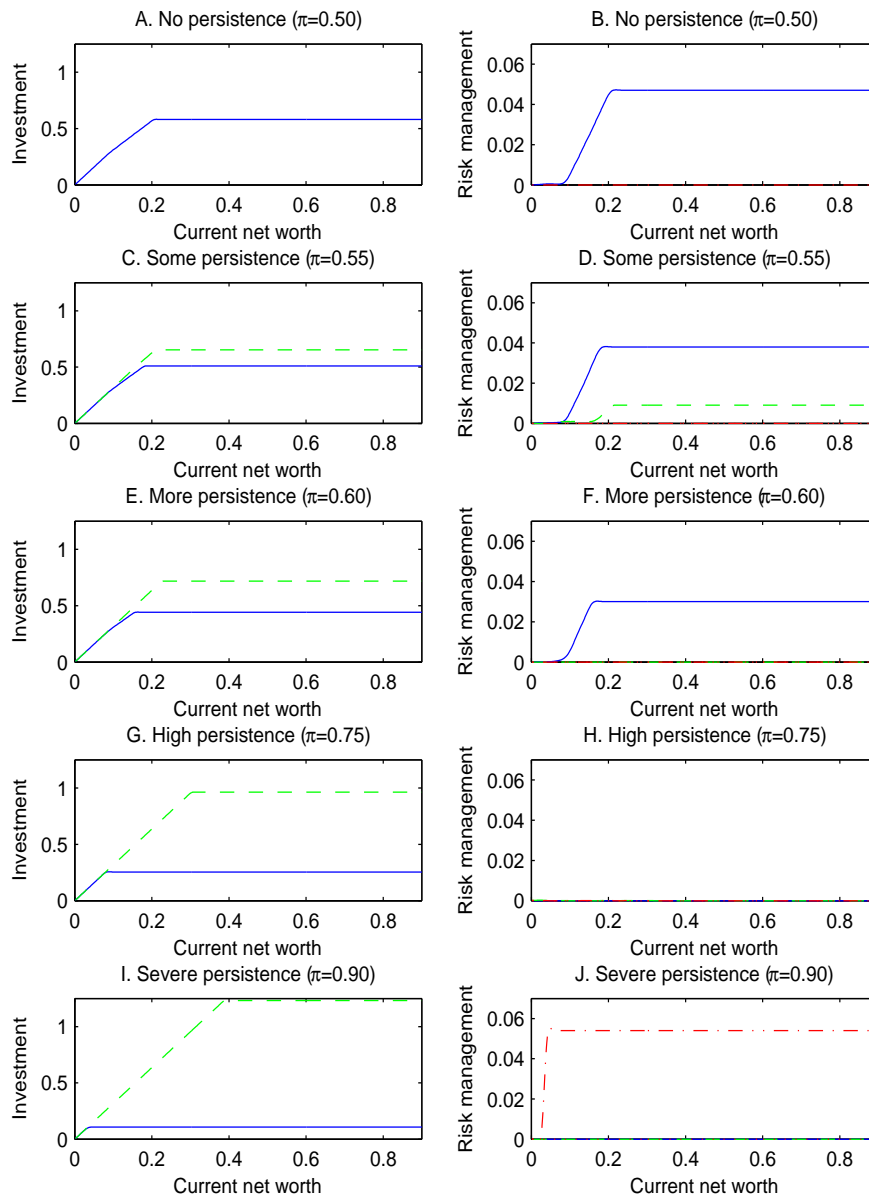
- Our model assumes
 - ... complete markets subject to collateral constraints and financing need at time 0

and implies that

- ... **financing need can override hedging concern.**



Risk Mgmt.: Stochastic Investment Opportunities



Model with Leasing

Lessors

- Risk neutral, discount future payoffs at $R^{-1} > \beta$; let $R \equiv 1 + r$
- Similar: lenders

Model with Leasing (cont'd)

Leasing as financing

- Leased physical capital k_l
 - ... requires monitoring cost $m > 0$
 - ... agent cannot abscond with leased capital
- Competitive lessor charges **user cost of leased capital** u_l in advance

$$u_l \equiv r + \delta + m,$$

that is, firm pays $R^{-1}u_l$ per unit of leased capital up front

- Equivalent: faster depreciation $\delta_l \equiv \delta + m$ as in Eisfeldt/Rampini (2009)

Borrower's Problem with Leasing

Dynamic program

- The borrower solves

$$V(w, s) \equiv \max_{\{d, k, k_l, w'(s'), b(s')\} \in \mathbb{R}_+^{3+S} \times \mathbb{R}^S} d + \beta \sum_{s' \in S} \Pi(s, s') V(w'(s'), s') \quad (1)$$

subject to the budget constraints

$$w + \sum_{s' \in S} \Pi(s, s') b(s') \geq d + k - (1 - R^{-1}u_l)k_l \quad (2)$$

$$A(s')f(k) + (k - k_l)(1 - \delta) \geq w'(s') + Rb(s'), \quad \forall s' \in S, \quad (3)$$

the **collateral constraints**

$$\theta(\varphi k - k_l)(1 - \delta) \geq Rb(s'), \quad \forall s' \in S, \quad (4)$$

and the constraint that only physical capital can be leased

$$\varphi k \geq k_l. \quad (5)$$

Lease or Buy?

Leasing decision

- Using first order conditions and user cost definitions

$$u_l = u_p - R\bar{\nu}_l/\mu + R\underline{\nu}_l/\mu.$$

- Straight **comparison of user cost**

- $u_l > u_p$ (or $\underline{\nu}_l > 0$): purchase all physical capital.
- $u_l < u_p$ (or $\bar{\nu}_l > 0$): lease all physical capital.
- $u_l = u_p$: indifferent between leasing and purchasing capital at margin.

- (Sufficiently) **constrained firms lease!**

- $u_l = r + \delta + m$
 - constant
- $u_p = r + \delta + \sum_{s' \in S} \Pi(s, s') R \beta \frac{\lambda(s')}{\mu} (1 - \theta)(1 - \delta)$
 - increasing with $\sum_{s' \in S} \Pi(s, s') \frac{\lambda(s')}{\mu}$

Optimality of Leasing

(Sufficiently) constrained firms lease

- **Assumption 1** *Leasing is neither dominated nor dominating, that is,*

$$(1 - \theta)(1 - \delta) > m > (1 - R\beta)(1 - \theta)(1 - \delta).$$

- **Proposition:**

- Firms with sufficiently low net worth lease all physical capital.

- Intuition:

- **Financing need makes higher debt capacity worthwhile.**

Optimality of Leasing: Proof

Proof

- Similar to the proof of the optimal absence of risk management

- As net worth $w \rightarrow 0$,

$$\frac{\lambda(s')}{\mu} = (R\beta)^{-1} - \frac{\mu(s')}{\mu} \rightarrow (R\beta)^{-1} > 0, \quad \forall s' \in S,$$

that is, all collateral constraints bind.

- User cost of owned physical capital exceeds user cost of leasing

$$u_p \rightarrow r + \delta + (1 - \theta)(1 - \delta) > r + \delta + m = u_l$$

Deterministic Capital Structure Dynamics with Leasing

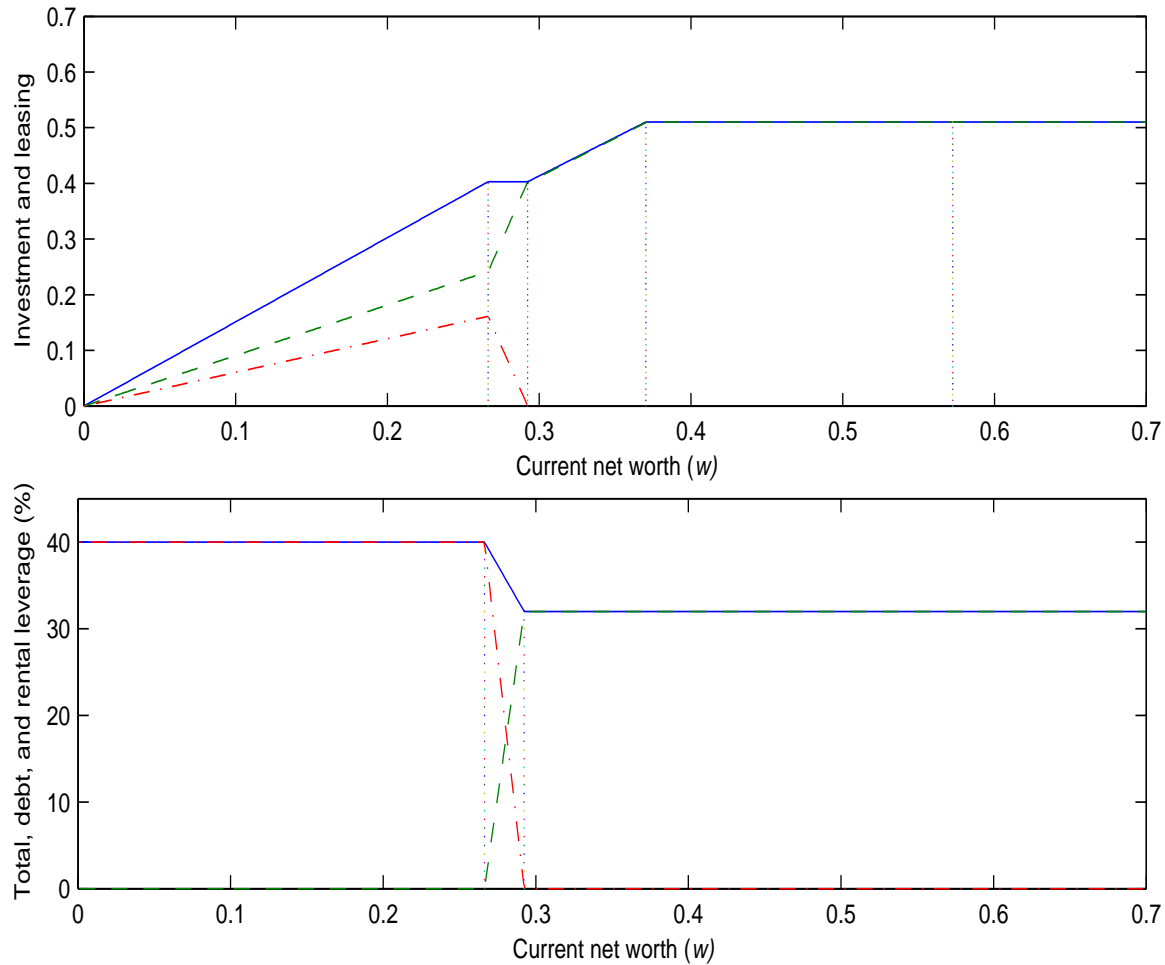
(Sufficiently) constrained firms lease

- Firm life cycle in four phases depending on net worth:
 - $w \leq \underline{w}_l$: Lease all physical capital, accumulate net worth, no dividends.
 - $\underline{w}_l < w < \bar{w}_l$: Substitute to purchased physical capital.
 - $\bar{w}_l \leq w \leq \bar{w}$: Purchase all capital, accumulate net worth, no dividends.
 - $w > \bar{w}$: Constant investment and positive dividends.

Capital Structure Dynamics with Leasing (cont'd)

(Sufficiently) constrained firms lease: deterministic case

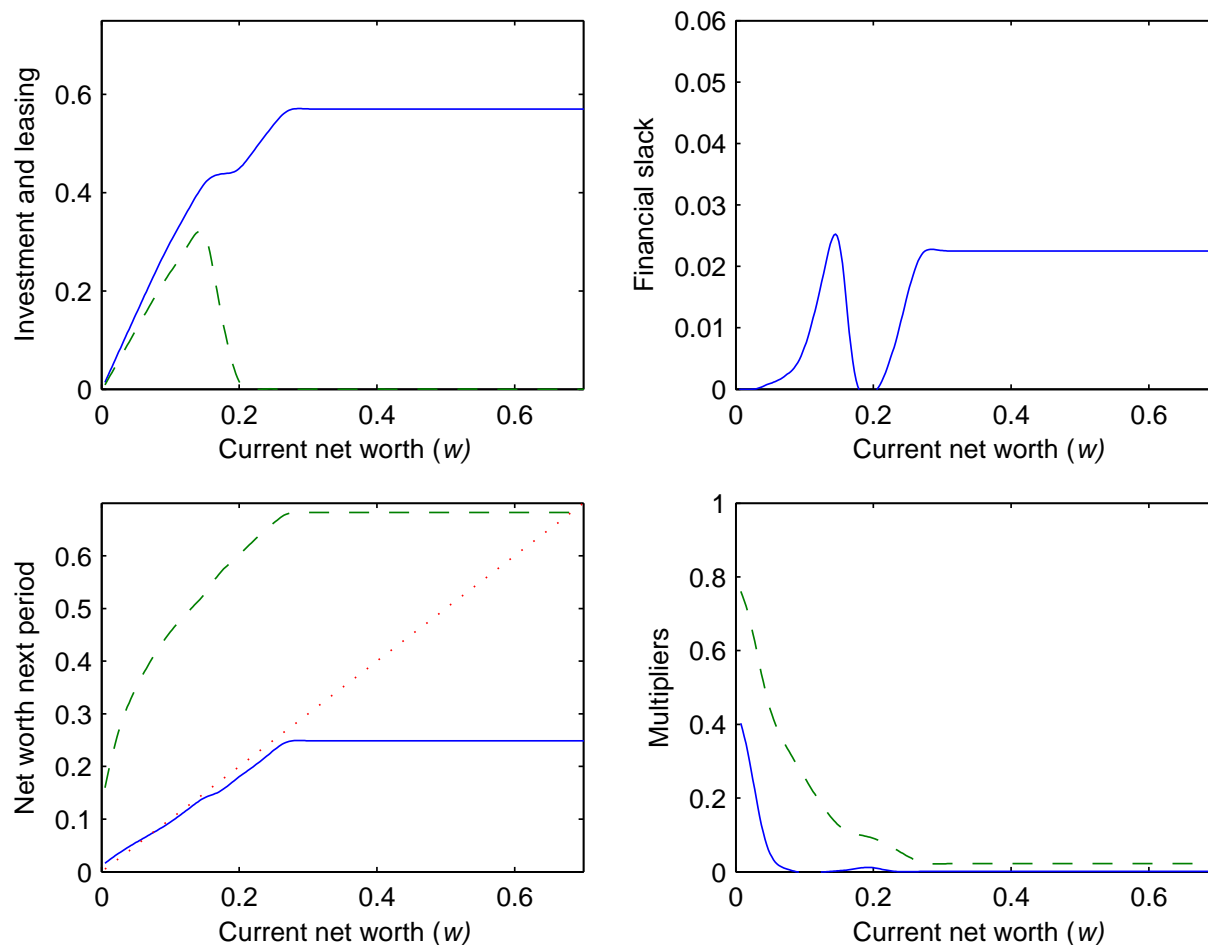
Total capital k (solid), leased capital k_l (dash dotted), and purchased capital $k - k_l$ (dashed) vs. current net worth (w).



Leasing and Risk Management Interaction

Leasing, leverage, and risk management

Two state Markov process without persistence $\Pi(s_1, s_1) = \Pi(s_2, s_2) = 0.5$.



- Note: Sale-and-leaseback transactions under stationary distribution

Conclusions

Collateral key determinant of capital structure

- **Tangible assets** determine debt and hence leverage.
 - Crucial aspect: leased tangible capital
- Dynamic model with **limited enforcement** yields
 - ... predictions for capital structure, including leasing
 - ... implications for risk management.
- **Framework to study dynamic corporate finance** questions
 - ... theoretically, quantitatively, and empirically.

Revisiting stylized “facts”

- Low leverage firms are low tangibility firms.
- Leverage and size relation: flat **not** increasing!

Dynamic Programming

Assumptions

- **Assumption 2** For all $\hat{s}, s \in S$ such that $\hat{s} > s$, **(i)** $A(\hat{s}) > A(s)$, and **(ii)** $A(s) > 0$.
- **Assumption 3** f is strictly increasing and strictly concave, $f(0) = 0$ and $\lim_{k \rightarrow 0} f_k(k) = +\infty$.

Well behaved dynamic program

- Let $x \equiv [d, k, k_l, w'(s'), b(s')]'$; $\Gamma(w, s)$ set of $x \in \mathbb{R}_+^{3+S} \times \mathbb{R}^S$ s.t. (2)-(5) satisfied; operator T : $(Tf)(w, s) = \max_{x \in \Gamma(w, s)} d + \beta \sum_{s' \in S} \Pi(s, s') f(w'(s'), s')$.
- **Proposition 1** **(i)** $\Gamma(w, s)$ is convex, given (w, s) , and convex in w and monotone in the sense that $w \leq \hat{w}$ implies $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$. **(ii)** The operator T satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point V . **(iii)** V is strictly increasing and concave in w . **(iv)** Without leasing, $V(w, s)$ is strictly concave in w for $w \in \text{int}\{w : d(w, s) = 0\}$. **(v)** Assuming that for all $\hat{s}, s \in S$ such that $\hat{s} > s$, $\Pi(\hat{s}, s')$ strictly first order stochastically dominates $\Pi(s, s')$, V is strictly increasing in s .

Characterization

First order conditions

- Multipliers

- ... on (2), (3), (4), and (5): μ , $\Pi(s, s')\beta\mu(s')$, $\Pi(s, s')\beta\lambda(s')$, and $\bar{\nu}_l$
- ... on $k_l \geq 0$ and $d \geq 0$: $\underline{\nu}_l$ and ν_d

- First order conditions

$$\mu = 1 + \nu_d$$

$$\mu = \sum_{s' \in S} \Pi(s, s')\beta \{ \mu(s') [A(s')f_k(k) + (1 - \delta)] + \lambda(s')\theta\varphi(1 - \delta) \} + \bar{\nu}_l\varphi$$

$$(1 - R^{-1}u_l)\mu = \sum_{s' \in S} \Pi(s, s')\beta \{ \mu(s')(1 - \delta) + \lambda(s')\theta(1 - \delta) \} + \bar{\nu}_l - \underline{\nu}_l$$

$$\mu(s') = V_w(w'(s'), s'), \quad \forall s' \in S,$$

$$\mu = \beta\mu(s')R + \beta\lambda(s')R, \quad \forall s' \in S,$$

- Envelope condition

$$V_w(w, s) = \mu.$$

Deterministic Capital Structure Dynamics

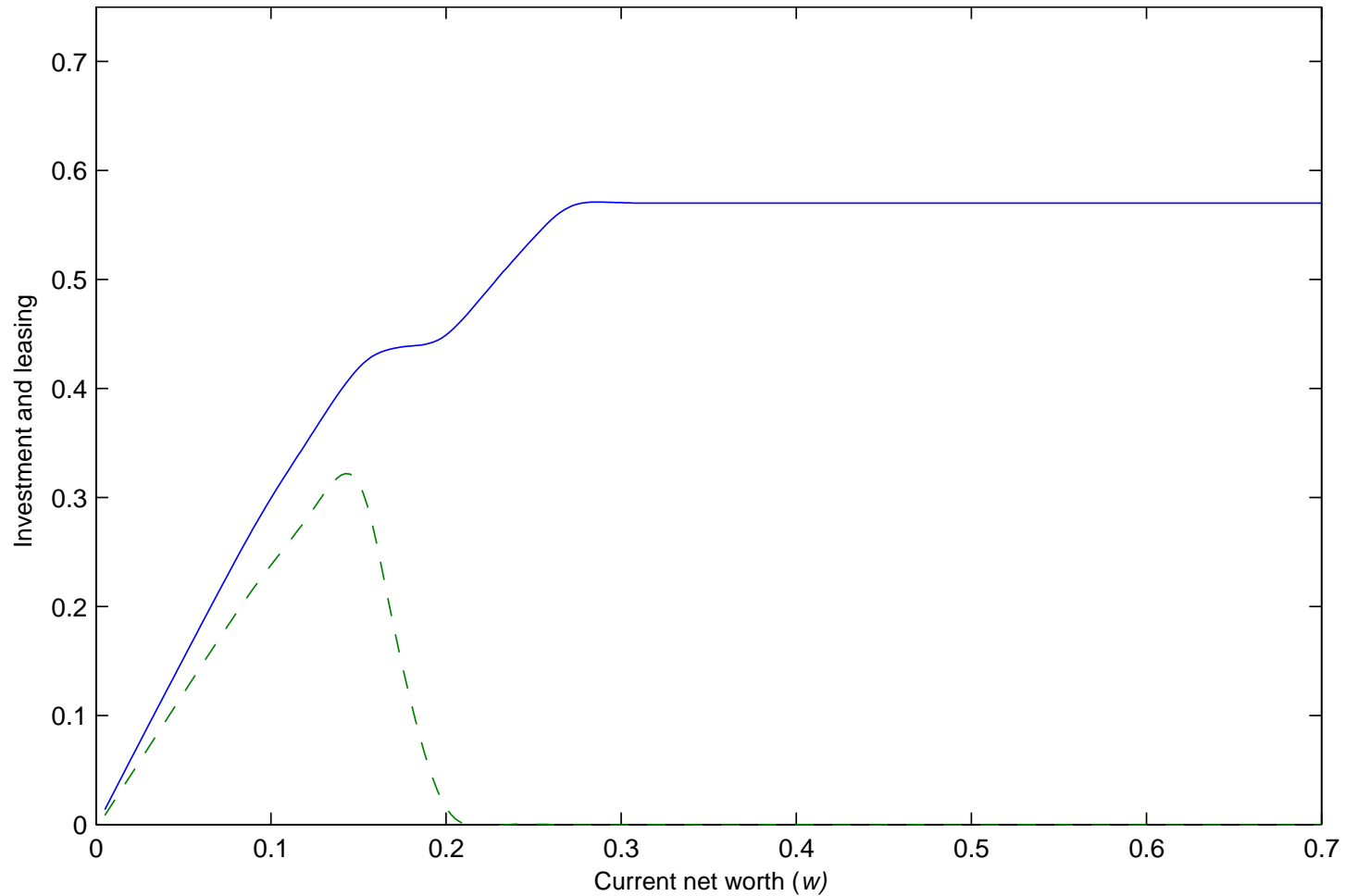
Deterministic dynamics without leasing

- Firm life cycle in two phases:
 - Net worth $w \leq \bar{w}$: Invest all funds, accumulate net worth, no dividends.
 - Net worth $w > \bar{w}$: Constant investment and positive dividends.

Capital Structure Dynamics with Leasing (cont'd)

(Sufficiently) constrained firms lease: stochastic case

Total capital k (solid) and leased capital k_l (dashed) vs. current net worth (w).



Leasing and Firm Growth

Leverage for growth

- Under Assumption 1, leasing allows higher leverage:

$$1/(1 - \varphi + R^{-1}u_l\varphi) > 1/(1 - R^{-1}\theta\varphi(1 - \delta))$$

- **Corollary 1 (Leasing and firm growth)** *Leasing enables firms to grow faster.*

Optimality of Incomplete Hedging

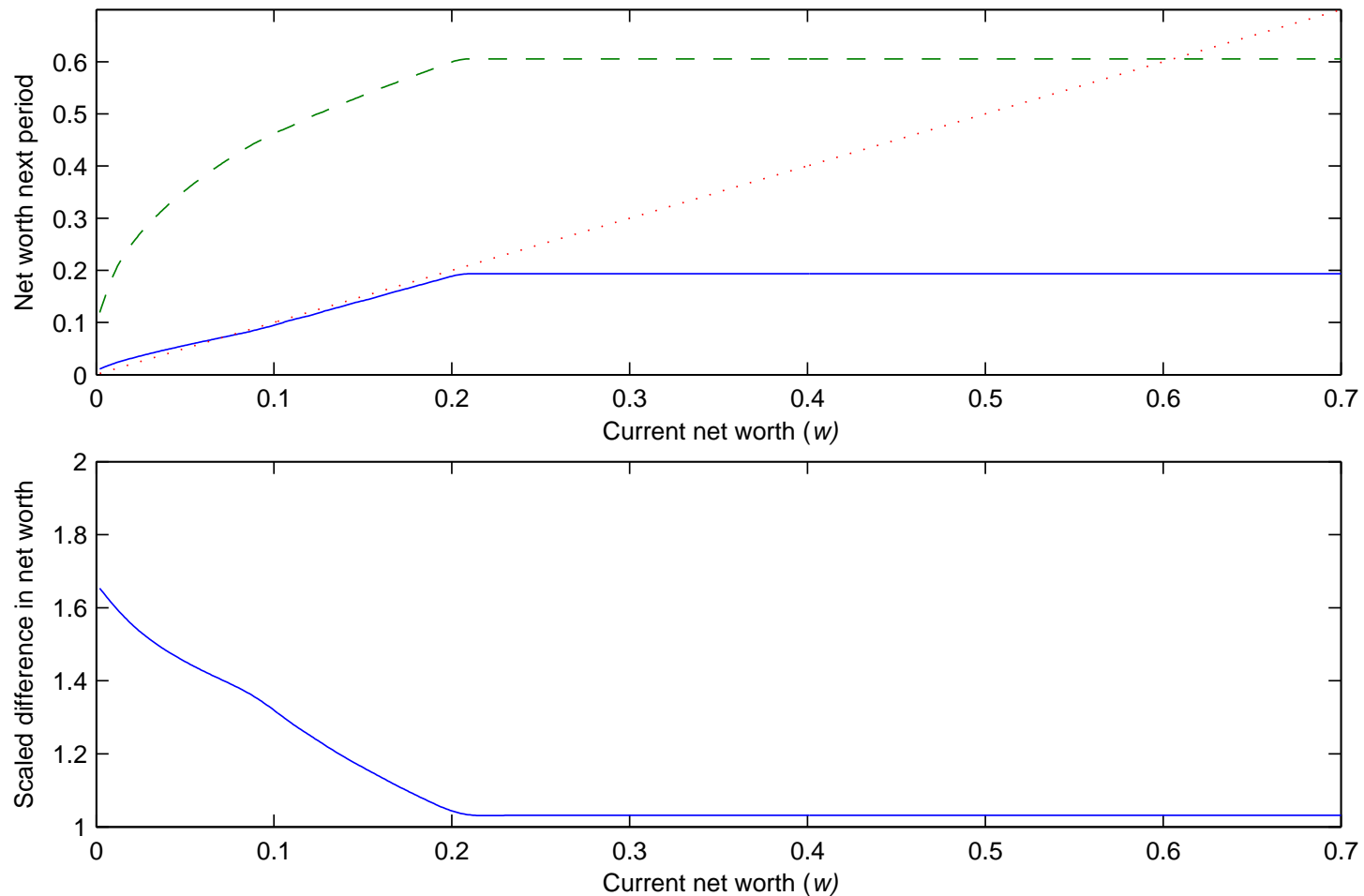
Hedging and value of internal funds

- Assumption: independence: $\Pi(s, s') = \Pi(s'), \forall s, s' \in S,$
- Marginal value of net worth
 - ... (weakly) decreasing in net worth
 - ... (weakly) decreasing in cash flow.
 - Multipliers on the collateral constraints
 - ... higher for states with higher cash flow next period.
- **Incomplete hedging is optimal**
 - $\exists s' \in S$ such that $\lambda(s') > 0$
 - Indeed, net worth not same across all states next period
 - Never hedge highest state
 - Hedge lowest states, if at all
- Not generally exhaust ability to pledge; **conserve debt capacity.**

Optimal Risk Management Policy: Example (cont'd)

Differences in net worth next period

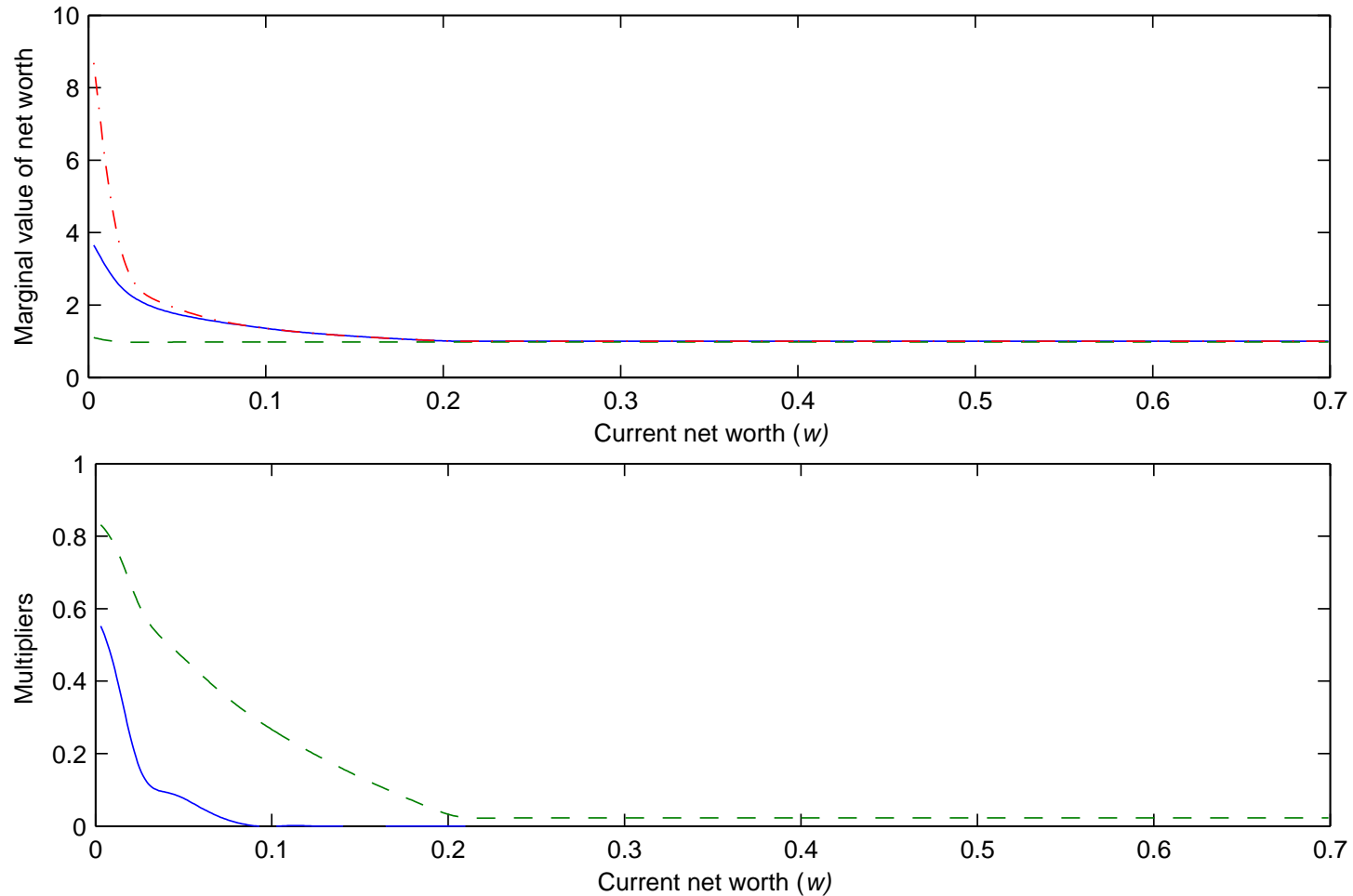
Top panel: Net worth in low state next period $w'(s'_1)$ (solid) and in high state next period $w'(s'_2)$ (dashed). Bottom panel: Difference between net worth in high state and low state next period scaled by expected net worth next period, that is, $(w'(s'_2) - w'(s'_1)) / \sum_{s' \in S} \Pi(s, s') w'(s')$.



Optimal Risk Management Policy: Example (cont'd)

Marginal value of net worth and collateral constraint multipliers

Top panel: Current marginal value of net worth $\mu = V_w(w)$ (dotted), scaled marginal value of net worth in the low state next period $R\mu(s'_1) = R\beta V_w(w'_1)$ (solid) and in the high state next period $R\mu(s'_2) = R\beta V_w(w'_2)$ (dashed). Bottom panel: Multipliers on the collateral constraint for the low state next period $\lambda(s'_1)$ (solid) and for the high state next period $\lambda(s'_2)$ (dashed).



Reconsidering Risk Management (cont'd)

Hedging, collateral constraints, and investment

- Rewriting Froot/Scharfstein/Stein (1993)
 - Problem in our notation

$$\max_{\{k_1(s), b_1(s)\}} \sum_{s \in S} \pi(s) \{A_2(s)f(k_1(s)) + (1 - \theta)(1 - \delta)k_1(s)\}$$

subject to

$$w_1(s) \geq (1 - R^{-1}\theta(1 - \delta))k_1(s) - Rb_1(s), \quad \forall s \in S,$$
$$\sum_{s \in S} \pi(s)b_1(s) \geq 0,$$

- Full hedging: $\mu_1(s) = \mu_1(\hat{s}), \forall s \in S$.
- Collateral constraints $0 \geq Rb_1(s), \forall s \in S$, limit hedging.
- Financing need for investment (k_0) may override hedging concern.