A financial signaling model with multiple signals and private information about future cash flows is analyzed. The model differs from the current literature in that it allows for risky debt and has both rights and outside equity issues as potential signals. A benefit–cost criterion is shown to characterize optimal signal choice; the benefit is the correlation between the signal and the unknown private information and the cost is the marginal cost of signaling. The optimal signaling schedule is derived and the choice between rights and outside equity issues characterized. Information effects for both current debt and equity holders are explored. Consistent with the empirical evidence of Eckbo and Masulis (1990), outside equity issues are shown to be more negative news than rights issues. Also, the empirically documented ambiguous effects of dividend changes on debtholders by Handjinicoloau and Kalay (1984) are rationalized.

The idea that a firm's financial policy can signal managerial information was first formalized by Bhattacharya (1979) and Ross (1977). These and most subsequent
models restrict exogenously the financial signal used by the firm to be either debt or dividends or some other univariate signal. This paper provides a model where managers choose between a variety of signals to reveal their private information about future cash flows and to face personal bankruptcy costs. To signal their private information, managers can use debt, outside equity issues, rights issues, dividends, and investments.

The manager's objective depends on the current and future market value of the firm and on a personal bankruptcy cost. This bankruptcy cost is the monetary value of the reputation loss that occurs to the manager if the firm goes bankrupt. Since the firm has existing debt, the manager faces a strictly positive probability of bankruptcy. The manager of a firm with higher expected cash flows has a lower expected probability of bankruptcy. Such a manager may signal this favorable private information by using a variety of instruments—debt, outside equity issues, rights issues, dividends, and investments. For example, such a firm could reduce its investment and pay dividends if the market interprets this as good news. Alternatively, the firm could increase its debt and pay dividends if the market interprets this as good news.

The choice among signals depends on a benefit–cost criterion. The benefit measures the credibility of the signal, while the cost is the marginal cost of signaling. The benefit is closely related to two intuitive criteria. The first is the marginal amount by which the signal constrains future cash flows: intuitively, the more the future cash flows are constrained, the more credible the belief that the firm has favorable information. The second is the marginal reduction in the dilution of the manager's holdings caused by the signal: a signal that reduces the dilution increases the manager's shareholdings in the firm and thus the signal's credibility.

Using this benefit–cost criterion, the signaling equilibrium is characterized. The choice between the various signals is as follows:

1. When the manager subscribes to a portion of the rights issues using his own capital, rights issues dominate outside equity issues.
2. When the manager does not subscribe to any portion of a rights issue using his own capital, he is indifferent between outside equity issues and rights issues.
3. The optimal rights underpricing is zero.
4. Initially, the signaling equilibrium involves investment reductions. For reasonable technologies, both investment and debt are eventually used.
5. Rights issues generate less negative market reactions that than do outside equity issues. This is consistent with the empirical evidence of Eckbo and Masulis (1990).
6. Increasing the firm's financial leverage increases the value of private information and thus the informativeness of dividends.
7. The effect on the existing debtholders of dividend increases depends on the marginal costs of signaling. When costs are low, dividend increases lead to
a reduction in the value of the debt. When costs are high, dividend increases lead to an increase in the value of the debt. This is consistent with the empirical evidence in Handjinicolou and Kalay (1984).

The intuition for (1), (2), and (3) is as follows: As long as the manager subscribes to a portion of the rights issue and does not sell all his associated rights, the dilution of his holdings, at the margin, is less for a rights issue than for an outside equity issue. The lower the dilution, the more credible the signal, and so rights issues dominate. When the manager sells all his rights corresponding to the shares he holds, the dilution is the same under both rights and outside equity issues. Thus, he is indifferent between rights issues and outside equity issues. Finally, greater rights underpricing implies higher marginal dilution to the manager’s holdings, hence a lower marginal signal benefit and thus an inefficient signaling schedule. This implies that the optimal rights underpricing is zero.

Initially, the signaling-equilibrium involves investment reductions because the marginal cost of investment (as a signal) is zero. The choice between investment and debt depends on which of the two signals reduces future cash flows less (at the margin). As investment falls, further marginal reductions in investment reduce future cash flows even more. At the same time, the reduction in investment increases the bankruptcy risk and thus the marginal face value needed to service an incremental dollar of debt. For technologies like the Cobb–Douglas with “relative risk aversion” parameter less than 1/2, the first effect is greater than the second and leads to the eventual use of debt.

The intuition for result (5) is as follows. In the model, rights issues dominate outside equity issues due to their lower dilution effect. Hence, outside equity issues are used only when current equity holders or inside managers do not have sufficient capital. Consequently, in two otherwise identical firms, the equilibrium involving outside equity issues leads to a greater reduction in value than rights issues. As noted, this is consistent with the empirical results of Eckbo and Masulis (1990).

The reasoning behind result (7) relates to the information effects versus wealth transfer issue empirically studied by Handjinicolou and Kalay (1984). When marginal costs of signaling are low, too much dividend is paid out to signal the firm’s private information. The wealth transfer effect swamps the information effect and the value of existing debt falls. When marginal costs of signaling are high, much less dividend is paid out to signal the firm’s private information. Consequently, the information effect dominates the wealth transfer effect and the firm’s debtholders gain.

Ambarish, John, and Williams (1987) and Williams (1988) have presented multiple-signaling models focusing on dividends and investments as signals. Similarly, Ofer and Thakor (1987) develop a multiple-signaling model where there is a choice between repurchases and dividends. These models differ from the model in this paper in their focus and in the intuition for signaling. In this paper, there is a larger signal space and both rights and outside equity issues are potential signals.
Finally, this model has existing risky debt. Consequently, this paper answers questions about the choice between rights issues and outside equity issues and the effect of the firm’s signaling decisions on its existing debtholders.\(^1\) Hence, the issue of whether a dividend increase lowers bondholder value because of the wealth transfer effect or increases bondholder value because of the information effect is analyzed. These issues have been empirically studied by Handjinicoloau and Kalay (1984); the results in this paper are consistent with their empirical results.

Section I presents the basic model and analyzes the full-information case. Section II characterizes the signaling equilibrium. Section III presents the empirical implications, while Section IV concludes.

**I. THE MODEL WITH FULL INFORMATION**

The model under full information is first analyzed. There are two dates: today (date 0) and tomorrow (date 1). The firm currently has debt of face value \(G\). This debt matures at date 1. Today the firm can undertake an investment \(I\) in a project that pays

\[
Y = f(I) + \delta + q
\]

at date 1.\(^2\) The production function \(f(I)\) satisfies \(f'(\cdot) > 0, f''(\cdot) < 0, f'(0) = \infty\) and \(f'(\infty) = 0\). The shock \(\delta\) is distributed on \([0, \overline{\delta}]\) while the private information \(q\) has support \([q_-, q_+]\).

The following notation is introduced:

- \(a\) = cash on hand with firm (known),
- \(b\) = market value of new debt issued,
- \(B\) = face value of new debt issued,
- \(I\) = investment in production,
- \(M\) = financial investment,
- \(r\) = dollar amount of rights issues to current shareholders,
- \(e\) = dollar amount of equity issues to outside shareholders and
- \(d\) = dividends paid to current shareholders.

Since \(d = a + r + e + b - I - M\), one of the variables is determined by the cash flow equation. Therefore, let the signal vector be \(s = (I, M, b, r, e)\). We do not explicitly include the cash on hand, \(a\), as a signal as it is known ex ante and thus is not a choice variable. However, the signaling equilibrium that results depends critically on \(a\). Often, to reduce the notational burden, we will suppress this dependence.

Following Ross (1977), bankruptcy costs are incurred. These costs are imposed on the manager and represent the reputation loss that the manager suffers if the firm goes bankrupt. This bankruptcy cost is denoted by \(c(l)\), where \(l\) is the shortfall in
cash flows. It is assumed that \( c(0) = 0, c'() > 0, c'(0) > \alpha, \) and \( c''() > 0. \) The parameter \( \alpha \) represents the fraction of the firm owned initially by the manager.

The objective of the firm’s manager is now specified. The manager chooses the signal vector \( s \) to maximize

\[
\lambda \alpha \left( \frac{1}{1 + \eta(s)} \right) V(s, q(s)) + (1 - \lambda) \alpha \left( \frac{1}{1 + \eta(s)} \right) V(s, q) - C(s, q) - \alpha r + \alpha d. \tag{2}
\]

The factor \( 1/[1 + \eta(s)] \) represents the net effect of the dilution that occurs when outside equity is issued. \( V(s, q) \) is the true equity value after signaling decisions are undertaken. \( V(s, q(s)) \) is the perceived market valuation of equity after signaling decisions are undertaken, and \( C(s, q) \) is the manager’s expected bankruptcy cost. Initially, the manager owns a fraction \( \alpha \) of the firm’s shares. Of this, he sells a fraction \( \lambda \) for consumption purposes. Thus, \( (1 - \lambda)\alpha V(s, q(s)) \) is the fraction of the firm that the manager holds until date \( 1. \) As Brennan and Copeland (1987) point out, such an objective is virtually canonical. By definition,

\[
\begin{align*}
\delta V(s, q) &= \int [z(s) + q + \delta] g(\delta) d\delta, \\
\delta C(s, q) &= \int c[-z(s) - q - \delta] g(\delta) d\delta,
\end{align*}
\tag{3}
\]

where \( z(s) = f(l) + M - G - B. \) Also, \( \delta(z, q) \) is defined by

\[
f(l) + M - G - B + q + \delta(z, q) = z(s) + q + \delta(z, q) = 0.
\]

For the problem to be meaningful, bankruptcy must be possible. It is assumed that \( f(\hat{\delta}) + \bar{q} - G < 0, \) where \( \hat{\delta} \) is given by \( f'(\hat{\delta}) = 1. \)

Under full information, the objective collapses to

\[
\alpha \left( \frac{1}{1 + \eta(s)} \right) V(s, q) - C(s, q) - \alpha r + \alpha d. \tag{4}
\]

Substituting the constraint that the market value of shares issued is equal to the capital raised,

\[
\left( \frac{\eta(s)}{1 + \eta(s)} \right) V(s, q) = e, \tag{5}
\]

we obtain

\[
\alpha V(s, q) - C(s, q) - \alpha(e + r) + \alpha d, \tag{6}
\]

the manager’s objective under full information. The first-order condition for the full information investment is
At the investment level \( \hat{I} \), the left-hand side of Equation (7) is greater than 1. If financial investment is not allowed, the optimal decision is to increase the investment to a level \( I^* > \hat{I} \). At this investment level \( I^* \), the probability of bankruptcy is positive. This follows because \( I^* > \hat{I} \) and a zero probability of bankruptcy imply that Equation (7) is not satisfied. If financial investment is allowed, the optimal strategy is to set \( I^* = \hat{I} \) and raise capital to invest in securities. In this region, the return to securities dominates the return to real investment. The optimal decision is to buy financial securities until the probability of bankruptcy is zero. In what follows, financial investment is disallowed. This changes none of the results in this paper, but drastically reduces the number of cases to be considered. Finally, using the envelope condition, \( I'(q) \) decreases as \( q \) increases.

Under full information, the presence of outstanding debt may make it optimal for the manager to issue more debt and pay out dividends. For such additional debt issues not to be optimal, the marginal benefit from exploiting current debt holders must be less than the marginal cost of bankruptcy. On simplifying, this condition is

\[
f'(I^*) \left( \delta \int g(\delta) d\delta + \frac{1}{\alpha} \int \delta(s,q) c'[-z(s) - q - \delta] g(\delta) d\delta \right) = 1. \tag{7}
\]

The assumption that \( c'(0) > \alpha \) ensures that Equation (8) is satisfied. New debt is not optimal because the marginal cost of bankruptcy exceeds the incremental gains from exploiting current bondholders.

II. THE MODEL UNDER ASYMMETRIC INFORMATION

A. Definition of the Signaling Equilibrium

Under asymmetric information, managers may communicate their private information by signaling. The signaling equilibrium is now defined. The definition is similar to Spence's definition of equilibrium.

**Definition 1.** A fully separating signaling equilibrium at date 0 is

(a) A set of strategies \( s: [\bar{q},q] \rightarrow S \) where \( s \) is a vector function whose values are denoted by \( s(q) \) and \( S \) is the signal space;

(b) A set of beliefs \( \sigma: S \rightarrow \Pi \) about the private information that the firm has, where
\( \sigma(s) \in \Pi = \{ \Gamma \mid \Gamma \text{ is a probability on } [q, \bar{q}] \}; \)

such that

1. Full separation occurs:
   \[ \sigma(s(q)) = 1_{\{q'=q\}}(q'). \]  \hfill (9)

2. The firm's signaling strategy maximizes the manager's objective,
   \[ s(q) \in \arg \max_s O(s, q(s), q) \]
   \[ = \lambda \frac{1}{1 + \eta(s)} V(s, q(s)) + (1 - \lambda) \frac{1}{1 + \eta(s)} V(s, q) - \frac{1}{\alpha} C(s, q) - r + d. \] \hfill (10)

Using the revelation principle (Myerson, 1979), the manager solves the equivalent direct revelation problem,
   \[ q \in \arg \max_{\hat{q}} O(s(\hat{q}), \hat{q}, q) \]
   \[ = \lambda \frac{1}{1 + \eta(s(\hat{q}))} V(s(\hat{q}), \hat{q}) + (1 - \lambda) \frac{1}{1 + \eta(s(\hat{q}))} V(s(\hat{q}), q) \]
   \[ - \frac{1}{\alpha} C(s(\hat{q}), q) - r(\hat{q}) - d(\hat{q}). \] \hfill (11)

This yields the first-order condition\(^9\)
   \[ O_1(s, q(s), s)s(q) + O_2(s, q(s), s) = 0 \] \hfill (12)

and the envelope condition
   \[ \frac{dO(s(q), q, q)}{dq} = O_3(s(q), q, q). \] \hfill (13)

For the first-order condition to be sufficient, the inequality
   \[ O(s(\hat{q}), \hat{q}, \hat{q}) \geq O(s(q), q, q), \text{ for all } q, \] \hfill (14)

must be satisfied for all \( \hat{q} \). Using the envelope condition [Equation (13)], Equation (14) simplifies to
   \[ \hat{q} > q \Rightarrow \int_q^{\hat{q}} O_3(s(t), t, t) dt > \int_q^{\hat{q}} O_3(s(q), q, t) dt. \] \hfill (15)

A related necessary condition for incentive compatibility (the Spence condition) is\(^{10}\)
A sufficient condition that implies Equation (14) is
\[ O_{13}(s(q), q, q) \dot{s}(q) + O_{23}(s(q), q, q) \geq 0. \] (16)

In the context of this model, there is a simple local condition that ensures that Equation (17) holds.

**Lemma 1:** If the signaling equilibrium given by the first-order approach is differentiable and satisfies \( \eta(q) < 0 \) and \( \dot{z}(q) < 0 \), it is globally incentive compatible.

**Proof:** See Appendix.

**B. The Pareto Optimal Signaling Equilibrium**

Any credible signaling equilibrium must satisfy Equations (13) and (15). The multiplicity of potential signals implies that there is more than one possible signaling equilibrium. One approach is to pick signals schedules that yield Pareto-efficient signaling. This Pareto-efficient signaling schedule has been shown by Ramey (1989) to be the unique equilibrium that satisfies the D1 refinement. The D1 refinement is a generalization of Cho and Kreps’s (1986) intuitive criterion to models with continuum of types. Further, the previous work by Engers (1987) and Riley (1979) has focused on this equilibrium. The next lemma proves that if a separating signaling equilibrium exists, a Pareto-dominant separating signaling equilibrium exists.

**Lemma 2:** In the class of separating signaling equilibria, if a separating signaling equilibrium exists, then a Pareto-dominant separating signaling equilibrium exists.

**Proof:** See Appendix.

While this equilibrium need not be unique, each \( q \) gets the same equilibrium objective value in all of the Pareto-dominant signaling equilibria.

We now characterize the Pareto-dominant signaling equilibrium (the D1 equilibrium) and explain how to solve for it. The reader who is interested in the characterization of the equilibrium and the empirical implications may skip to Section III at this point.

A Pareto-dominant separating signaling equilibrium must satisfy a necessary condition that is derived next. Consider the Pareto-dominant separating equilibrium \( S^* \) and a separating signaling equilibrium \( S' \) that agrees with \( S^* \) up to \( q \) and is dominated by \( S^* \) for \( q' > q \). Since both the signaling equilibria are incentive compatible at \( q \), Equation (13) implies that,
\[
\frac{dO(s(q),q,q)}{dq} \bigg|_{s^{'},q} = \frac{dO(s(q),q,q)}{dq} \bigg|_{s^*} = O_3(s(q),q,q). 
\]  
(18)

As well, the fact that \(S'\) is dominated by \(S^*\) implies that
\[
\frac{d^2O(s(q),q,q)}{dq^2} \bigg|_{s^{'},q} \geq \frac{d^2O(s(q),q,q)}{dq^2} \bigg|_{s^*}. 
\]  
(19)

Expanding this second derivative,
\[
\frac{d^2O(s(q),q,q)}{dq^2} = O_{13}(s(q),q,q) \hat{s}(q) + O_{23}(s(q),q,q) + O_{33}(s(q),q,q). 
\]  
(20)

To understand what Equations (19) and (20) imply, define the terms
\[
\mu(q)' = O_1(s(q),q,q), \quad \beta(q)' = O_{13}(s(q),q,q), \quad \phi(q) = O_2(s(q),q,q). 
\]  
(21)

Since \(O_{33}(s(q),q,q)\) has the same value at \(q\) for both \(S^*\) and \(S'\), only the first two terms of Equation (20) are relevant. Notice that these terms are identical to those in Equation (16). The optimal signal maximizes the Spence constraint.

Equation (21) suggests the following approach to characterizing the Pareto optimal separating signaling schedule. Let \(\hat{s}(q) = u(q)y(q)\), where \(u(q)\) is a \(K \times 1\) vector such that \(\|u(q)\| = 1\) and \(y(q)\) is a scalar. No loss of generality is involved as determining the optimal \(u(q)\) and \(y(q)\) is equivalent to determining the optimal \(\hat{s}(q)\). Restate the first-order conditions as
\[
[\mu(q)u(q)]y(q) = -\phi(q). 
\]  
(22)

Given \(u(q), y(q)\) is determined by this differential equation. In what follows, note that \(u(q) = [u_I(q), u_b(q), u_r(q), u_e(q)]'\). Using Equations (19) and (22) and the fact that \(O_{23}(s(q),q,q) = 0\) for this specific problem [see Equation (11)], one concludes that the Pareto-dominant signaling schedule solves the program
\[
\max_{u(q)} \frac{\beta(q)'u(q)}{\mu(q)'u(q)} \phi(q), 
\]
subject to
\[
\|u(q)\| = 1, \quad \frac{\beta(q)'u(q)}{\mu(q)'u(q)} \phi(q) \geq 0, 
\]
restrictions of the form
\[
u_i(q)y(q) \geq 0 \text{ or } \geq \infty, \quad \text{for } i = I, b, r, e, 
\]  
(23a)
or
\[
-u_i(q)\frac{\phi(q)}{u(q)'u(q)} \geq 0 \text{ or } \geq \infty, \quad \text{for } i = I, b, r, e, 
\]  
(23b)
and restrictions of the form
\[
[u_{b(q)} + u_{c(q)} + u_{r(q)} - u_I(q)]y(q) \geq 0 \text{ or } \geq -\infty, 
\]
or

\[- [u_b(q) + u_e(q) + u_r(q) - u_f(q)] \frac{\phi(q)}{\mu(q)'u(q)} \geq 0 \text{ or } \geq -\infty.\]

The program does not contain \(y(q)\) as it has been substituted out using the first-order condition. The third set of constraints represents the feasibility constraints on the signals (imposed by conditions such as \(I(q) = 0, I(q') \geq 0\), for all \(q' > q\) etc.). The fourth constraint is that implied by \(d(q) = 0, d(q') > 0\), for all \(q' > q\). Clearly, these constraints are relevant only when the signal levels are at the boundary of the signal space (the \(\geq 0\) case). Otherwise they are not relevant (the \(\geq -\infty\) case).

The second constraint is the local credibility constraint or the Spence condition for the model. As demonstrated, the left-hand side of this constraint is the objective value of the program. If this program is maximized and its objective value is greater than zero, this constraint is automatically satisfied. Hence, a relaxed program called (P1) is solved and the solution is checked for a positive objective value. The program (P1) is

\[
\max_{u(q)} \frac{\beta(q)'u(q)}{\mu(q)'u(q)} \phi(q),
\]

subject to \(|u(q)| = 1,

\[- u_i(q) \frac{\phi(q)}{\mu(q)'u(q)} \geq 0 \text{ or } \geq -\infty, \quad \text{for } i = l, b, r, e, \quad (24)\]

and

\[
[u_b(q) + u_e(q) + u_r(q) - u_f(q)] \frac{\phi(q)}{\mu(q)'u(q)} \geq 0 \text{ or } -\infty.
\]

This approach ensures that the local credibility constraint is satisfied. From Lemma 1, if this locally credible equilibrium satisfies \(\dot{\eta}(q) < 0\) and \(\dot{z}(q) < 0\), it is globally incentive compatible.

The first-order conditions of the above program are (suppressing the dependence on \(q\))

\[- \frac{\beta y \phi}{\mu'} + \frac{\beta' u}{(\mu')^2} \phi \mu_i - v_i \left( \frac{\phi}{\mu'} + \frac{\phi \mu_i}{(\mu')^2} \right) - \omega \text{ sign}(u_i) \left( \frac{\phi}{\mu'} + \frac{\phi \mu_i}{(\mu')^2} \right) = 0, \quad (25)\]

plus the constraints themselves. \(\text{Sign}(u_i)\) is the sign of the coefficient of \(u_i\) in the last constraint of program (P1). The constraint on \(|u(q)|\) does not enter as it is a scaling constraint and the value of the objective is not affected by this constraint. Hence, the Lagrange multiplier corresponding to this scaling constraint must be zero. From the first-order condition, the ratio \(\beta_i/\mu_i\) plays an intuitive role in the
Multiple-Signaling Model

choice of signal. The optimal signal must be the signal with the highest value of this ratio. The following theorem is now proved.

**Theorem 1:** Define the set \( I = \{ i | i \) is a signal maximizing the ratio \( \beta_i / \mu_i \}. \) Let \( I' = \{ i | i \notin I \}. \) If:

a. If all signals in the set \( I' \) have a resource constraint binding, the efficient signal involves the set \( I. \)

b. If some signal in the set \( I' \) does not have any resource constraint binding, there is a discontinuity in the signaling schedule. The efficient signal involves the set \( I \) plus the signals in \( I' \) with all resource constraints slack.

**Proof:** See Appendix.

Given the above discussion, one solves for the equilibrium signaling schedule as follows:

1. Solve for the optimal full information decisions of the lowest type \( q. \) In the signaling equilibrium, these will continue to be the allocation that type \( q \) receives.

2. Solve for the optimal program (P1) as follows. Find the signals that maximize the benefit–cost ratio \( \beta_i / \mu_i \) and follow Theorem 1 to construct the optimal signaling schedule. Prove that the chosen signals are the unique solutions to the first-order condition.

3. Ensure global incentive compatibility by checking that Equation (17) holds.

**C. Characterization of the Signaling Equilibrium**

To begin, the marginal costs and marginal benefits of each signal are:

<table>
<thead>
<tr>
<th>Signal</th>
<th>( \mu(q) )</th>
<th>( \beta(q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>( f'(l) f(s,q) )</td>
<td>( f'(l) H(s,q) )</td>
</tr>
<tr>
<td>Debt</td>
<td>( dB / dB H(s,q) )</td>
<td>( dB / dB H(s,q) )</td>
</tr>
<tr>
<td>Pure rights issue</td>
<td>(-1 - \frac{\partial \eta / \partial r}{[1 + \eta(s)]^2} V(s(q),q))</td>
<td>( -(1 - \lambda) \frac{\partial \eta / \partial r}{[1 + \eta(s)]^2} V_q(s(q),q))</td>
</tr>
<tr>
<td>Outside equity</td>
<td>(-\frac{\partial \eta / \partial e}{[1 + \eta(s)]^2} V(s(q),q))</td>
<td>( -(1 - \lambda) \frac{\partial \eta / \partial e}{[1 + \eta(s)]^2} V_q(s(q),q))</td>
</tr>
<tr>
<td>Dividends</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

where

\[
J(s,q) = \frac{1}{1 + \eta(s)} V_z - \frac{1}{\alpha} C_z
\]
\( H(s,q) = \frac{1 - \lambda}{1 + \eta(s)} V_{zq} - \frac{1}{\alpha} C_{zq} < 0, \)

\[
\begin{align*}
V(z,q) &= \int_{\delta(s,q)}^\delta [z(s) + q + \delta]g(\delta) \, d\delta, \\
C(z,q) &= \int_{\delta(s,q)}^0 c[-z(s) - q - \delta]g(\delta) \, d\delta.
\end{align*}
\]

(26)

In addition, \( \eta(s) \) and \( b(s) \) are defined by

\[
\frac{\eta(s(q))}{1 + \eta(s(q))} V(s(q),q) = \epsilon,
\]

(27)

\[
b(s) = \int_{\delta(s,q)}^\delta B g(\delta) \, d\delta + \int_0^B \frac{B}{B + G} (z + q + \delta)g(\delta) \, d\delta.
\]

The marginal benefits and marginal costs are obtained by appropriate differentiation of the manager’s objective. The cost of outside equity is the dilution in the manager’s holdings due to the issue of an incremental dollar of capital. The cost of a rights issue is the direct marginal cost to shareholders ($1) plus the marginal dilution in the holdings of existing outside equityholders. The derivation of these marginal costs and benefits is shown in the Appendix.

This information is used to analyze the signal choice. Two distinct cases are considered. In the first, \( a < \Gamma'(q) \) and the lowest-type firm raises capital. In the second, \( a > \Gamma'(q) \) and the lowest-type firm pays out dividends.

**CASE 1.** \( a < \Gamma'(q) \).

Under this case, the lowest-type firm finances its full information investment \( \Gamma'(q) \) by raising capital of \( \Gamma'(q) - a \). This investment can be financed through a pure rights issue or an outside equity issue.\(^{20}\) If rights issues are feasible, then the signaling equilibrium involving rights issues dominates outside equity issues. The first step is to note that:

**Lemma 3:** The Pareto optimal equilibrium has rights issues by type \( q \).\(^{21}\)

**Proof:** Comparing the slopes of the signaling schedule under the two alternatives [using Equation (13)],

\[
\left. \frac{dO(s(q),q,q)}{dq} \right|_{q=0} = (1 - \lambda) \frac{1}{1 + \eta(s)} V_q - \frac{1}{\alpha} C_q < \left. \frac{dO(s(q),q,q)}{dq} \right|_{q=0}
\]
Multiple·Signaling Model 13
(28)
Since the only difference between the two alternatives is the extent of
dilution, the manager captures more of the marginal value of information
with a rights issue. Thus rights issues dominate outside equity issues for type
$q > q$ and arbitrarily close to $q$. Hence the lowest-type firm chooses a rights
issue.

Since the lowest-type firm raises funds with a rights issue, dividends are not paid.
More importantly, no type of firm issues outside equity.\textsuperscript{22} The result that no type
will make an outside equity issue follows from the observation that if both rights
and outside equity issues are positive, signaling must involve reducing outside
equity issues. If the lowest type makes a rights issue and does not make an outside
equity issue, reducing outside equity issues is impossible. Thus no outside equity
issues occur.

**Lemma 4**: The Pareto optimal equilibrium has rights issues by any type $q$.

**Proof**: The proof is shown later in the paper when we analyze the case where
the lowest type makes both an outside equity issue and an rights issue due
to resource constraints faced by the firm's managers. In particular, Equations
(32), (33), and (34) and the discussion that follows prove this theorem.

Changes in the amount of rights issues must involve changes in the investment
level or the amount of additional debt. Using Theorem 1, the choice between these
two signals (underinvestment and reducing rights issues, substitution of debt for equity) reduces to\textsuperscript{23}

\[
- \frac{\beta_I}{\mu_I + \mu_r} \geq - \frac{\beta_b}{\mu_b - \mu_r},
\]

\[
\iff -f'(I)H(s,q) \geq -(dB/db)H(s,q) \iff \frac{f'(I)}{f'(I)J(s,q) - 1} \geq \frac{(dB/db)J(s,q) - 1}{(dB/db)J(s,q) - 1},
\]

\[
\iff f''(I) \leq dB/db.
\] (29)

At $I^*(q)$, $b(q) = 0$, and hence

\[
1 - J(s,q) \frac{dB}{db} \bigg|_{B=0} < 0 = J(s,q) f'(I^*(q)) - 1.
\] (30)

Thus $f''(I^*(q)) < dB/db$ and reducing investment and rights issues is the optimal
signal.
Whether debt is eventually used depends on the technology. One sufficient condition that ensures the use of debt is \( f'(I) f(I) \rightarrow \infty \) as \( I \rightarrow 0 \). For example, if \( f(I) = I^{\gamma}, \gamma < 1/2 \), this condition is satisfied. If this restriction on technology holds,

\[
\left. f'(I) \frac{d\delta}{dB} \right|_{B=0} = f'(I) \int_{\delta(z,q)}^{\bar{\delta}} \delta(z,q) g(\delta) d\delta + \int_{0}^{\delta(z,q)} \frac{f(I) + q + \delta}{G} g(\delta) d\delta. \tag{31}
\]

goess to \( \infty \) as \( I \) goes to zero. Eventually both debt and investment are used as signals.

The intuition for Equations (30) and (31) is as follows. Using Equation (29), the choice of signal depends on how marginal reductions in investment and marginal increases in debt constrain future cash flows. The signal that constrains future cash flows less is more efficient. Thus the choice of signal is determined by differences in marginal costs, which dominate differences in marginal benefits. Initially, investment dominates since it reduces future cash flows by a lower amount. As investment is reduced, further marginal reductions in investment constrain future cash flows even more. At the same time, the lower investment increases the bankruptcy risk and thus the marginal face value needed to service an additional unit of debt. For the technologies satisfying the above sufficient condition, the first effect dominates the second, leading to the eventual use of debt.

This completes the analysis for the case in which it is feasible for the lowest-type firm to make a pure rights issue. If shareholders do not have any capital, the firm cannot raise any money via rights (the sale of rights is not allowed). In this case, the lowest-type firm finances with a outside equity issue. The choice of financing between underinvestment and debt reduces to exactly the same condition as in the pure rights issue case. Thus, the same conclusions hold. The proof of this is in the Appendix.

In intermediate cases, the lowest-type firm finances its investment through a mixture of rights and outside equity issues. The firm has the choice of reducing rights issues or reducing outside equity issues. This comparison involves

\[
- \frac{\beta_I + \beta_e}{\mu_I + \mu_e} > - \frac{\beta_I + \beta_r}{\mu_I + \mu_r}. \tag{32}
\]

The costs of both signals are the same. To see this, differentiate Equation (27) to obtain

\[
\frac{1}{[1 + \eta(s)]^2} \frac{\partial \eta}{\partial e} V + \frac{\eta(s)}{[1 + \eta(s)]} \left( V_I + V_q \frac{dq}{de} \right) = 1,
\]

\[
\frac{1}{[1 + \eta(s)]^2} \frac{\partial \eta}{\partial r} V + \frac{\eta(s)}{[1 + \eta(s)]} \left( V_I + V_r \frac{dq}{dr} \right) = 0. \tag{33}
\]
Substituting for $\partial \eta(s)/\partial e$ and $\partial \eta(s)/\partial r$ in the definition of the marginal costs of the two signals,

$$
\mu_e = f'(I) \left(V_{z} - \frac{1}{\alpha} C_{z}\right) - 1 + \frac{\eta(s)}{[1 + \eta(s)]} V_q \frac{dq}{de},
$$

$$
\mu_r = f'(I) \left(V_{z} - \frac{1}{\alpha} C_{z}\right) - 1 + \frac{\eta(s)}{[1 + \eta(s)]} V_q \frac{dq}{dr} .
$$

If $dq/de$ and $dq/dr$ are equal, then the two expressions are identical.

Using the first-order condition one can show that $dq/de$ and $dq/dr$ are equal. The costs of the two signals are equal and the benefits determine the choice of signal. The benefits are higher for the outside equity if $\partial \eta/\partial e > \partial \eta/\partial r$. This follows from Equation (33) and the fact that $dq/de$ and $dq/dr$ are identical. The optimal signal is to reduce outside equity since it is the more credible signal. The signaling equilibrium is characterized by the low-type firms undertaking a mixture of rights and outside equity issues while higher-type firms use rights issues. The analysis of whether these reductions are financed by investment or by debt remains the same.

Suppose the sale of rights is allowed. The manager sells a fraction $\lambda$ of his shares cum rights and the buyers of these shares do not care about the price of rights since they buy both the right and the current shares. The manager may sell up to a fraction $1 - \lambda$ of his rights if he lacks capital. Suppose the manager sells a fraction $k$ of these rights. Let the owners of all rights get a fraction $\xi(s)$ of the shares. The buyers of these particular rights receive

$$
\frac{(1 - \lambda) k \alpha \xi(s)}{1 + \xi(s)} V(s(q), q, q) = (1 - \lambda) k \alpha r + w,
$$

where $w$ is the price paid for the rights that the manager sells and $r$ is the dollar amount received by the firm from the rights issue. For the lowest-type firm, $r(q) = a - I^*(q)$. Also, the higher $w$, the higher is $\xi(s)$. Hence the greater the underpricing of rights, the higher is the dilution.

**Lemma 5:** The optimal underpricing is zero, i.e., $w(q) = 0$.

**Proof:** The manager’s objective is

$$
\lambda V(s, q(s)) + (1 - \lambda)(1 - k)V(s, q) + (1 - \lambda)k \frac{1}{1 + \xi(s)} V(s, q)
$$

$$
- \frac{1}{\alpha} C(s, q) + \frac{1}{\alpha} w(s(q)) - [\lambda + (1 - \lambda)(1 - k)]r.
$$

Equation (36) yields that the slope of the signaling schedule at $q$ is
\[
\frac{dO(s(q),q,q)}{dq} = (1 - \lambda) \frac{1 + (1 - k)\xi(s)}{1 + \xi(s)} V_q - \frac{1}{\alpha} C_q. \tag{37}
\]

Equation (37) is maximized at the lowest dilution as \(V_q\) and \(C_q\) are functions only of \(I'(q)\) and thus are fixed.

If \(k < 1\), the optimal dilution under a rights issue of size \(\xi(s)\) is less than that obtained under an outside equity issue. This is clear from Equation (37), which is decreasing in \(k\); a pricing problem occurs only to the extent that rights were sold. Rights issues thus dominate outside equity issues for type \(q\). This is stated as

**Lemma 6:** If the manager provides capital and subscribes to a portion of the rights issue, rights issues dominate outside equity issues for type \(q\).

The analysis proceeds as before except that one has to allow for the possibility that rights are underpriced. Since \(w(q) = 0\) and \(w(q) \geq 0\), the only possibility is that firm types close to but greater than \(q\) must have higher underpricing. Suppose

\[
\alpha \frac{(1 - \lambda)k\xi(s)}{1 + \xi(s)} V(s(q),q,q) = \alpha(1 - \lambda)k[r(q) - v] + w(v), \tag{38}
\]

where \(w(v)\) represents the manager's proceeds from the sale of rights when the firm raises \(v\) dollars less from a rights issue. Then

\[
\frac{\partial \xi}{\partial I} \frac{\alpha(1 - \lambda)k}{[1 + \xi(s)]^2} V + \frac{\alpha(1 - \lambda)k\xi(s)}{1 + \xi(s)} \left( Vf'(I) + V_q \frac{dq}{dl} \right) = \alpha(1 - \lambda)k - \frac{\partial w}{\partial v}. \tag{39}
\]

Let \(\partial w/\partial v \bigg|_2 > \partial w/\partial v \bigg|_1\). The comparison of two levels of underpricing turns on the marginal benefits as the marginal costs are equal. The greater is the underpricing, the greater is the dilution. This greater marginal dilution reduces the credibility of the signal as efficient signaling involves higher-type firms diluting as little as possible to discriminate themselves from low types. Thus \(\partial w/\partial v = 0\) is optimal. By induction, the argument that works when \(w(q) = 0, q' > q\), also works when \(w(q) = 0, q' > q\). Hence zero underpricing is optimal.

**Lemma 7:** The optimal underpricing sets \(w(q) = 0\).

**Proof:** See Appendix.

The final step is to show that when \(k < 1\), outside equity is never issued by any type, not just type \(q\) (Lemma 8). The proof of this lemma mimics the case where sale of rights is not allowed. First, it is proved that when rights issues and outside equity issues are positive, the optimal signal involves reducing outside equity issues. This and the fact that the lowest-type firm makes a rights issue imply that outside equity issues are suboptimal for all types.
CASE 1: $a_0 < I^*(q)$

Rights issues are used by the lowest type to finance investment

Both rights and outside equity issues are used by the lowest type to finance investment

Figure 1. The equilibrium signaling schedule
CASE 2: $a_0 > I^*(q)$

**Figure 1.** (Continued)

**Lemma 8:** If the manager provides capital and subscribes to a portion of the rights issue, rights issues dominate outside equity issues for all $q$.

**Proof:** See Appendix.

If the manager sells all his rights due to the lack of funds, $k = 1$. In this case, the distinction between outside equity issues and rights issues disappears. Thus equilibria involving rights issues and outside equity issues are identical and the optimal signaling equilibrium can involve either. This is demonstrated in Lemma 9.

**Lemma 9:** When the manager does not subscribe to the rights issue and sells the rights corresponding to the fraction $(1 - \lambda)$, equilibria involving rights issues and outside equity issues are identical and thus the manager is indifferent between the two methods.

**Proof:** See Appendix.

This completes the treatment of Case 1. Figure 1 shows the constructed signaling equilibrium that follows from the analyses.
CASE 2: \( a \geq I'(q) \).

Now the lowest type pays out a dividend \( d(q) = a - I'(q) \). The results for this case follow immediately from those in Case 1. The comparison of investment versus debt is the same as it was for a pure rights issue. Investment is used in the optimal signal if

\[
f'(I) \equiv dB/db.
\]

Mimicking the analysis in Case 1, the equilibrium initially involves dividend payments and the reduction of investment. For technologies satisfying our sufficient condition, the equilibrium eventually involves the use of debt financing. The constructed signaling equilibrium is shown in Figure 1 for technologies satisfying the sufficient condition.

III. EMPIRICAL IMPLICATIONS OF THE SIGNALING EQUILIBRIUM

The evidence that higher dividends are good news and higher issues of stock are bad news is now accepted. Aharony and Swary (1980) found that dividend announcements are accompanied by significant average residuals on announcement of 0.35% (\( t \)-value 2.35) for increases in dividends and −1.46% (\( t \)-value −1.88) for decreases (this is for the subsample where earnings announcements precede dividend announcements, the case relevant to this model.). Similarly, Asquith and Mullins (1986) and Masulis and Korwar (1986) show that the issue of shares is bad news—the average residuals on announcement of these issues (in the Asquith and Mullins study) equal −3.0% (\( t \)-value −12.5). The signaling equilibrium presented here is consistent with this evidence.

**Theorem 2.** Higher dividends are good news, higher rights and outside issues of shares are bad news. While the issue of debt is good news, higher-debt issues do not necessarily mean higher value.

**Proof:** See Appendix.

This theorem implies that the issue of debt is good news. However, higher-debt issues are not necessarily better news.\(^{30}\) The empirical evidence due to Dann and Mikkelsen (1984) indicates that the announcement effect for the issue of debt is −0.37% (\( t \)-value −1.76). Subsequently, Eckbo (1986), using a larger sample, finds average insignificant residuals for of −0.11% (\( t \)-value −0.96). Thus, relative to equity, debt issues are good news. The model in this paper predicts not only that debt issues are good news relative to equity but that debt issues are good news in absolute terms.\(^ {31}\) One conclusion is that the empirical evidence is not entirely consistent with the above signaling equilibrium.
However, the indirect evidence on debt increasing transactions strongly supports the view that higher debt is good news in absolute terms. Exchange offers of common stock for debt are bad news, with the excess returns on announcement equal to −7.44% [see Smith (1986) for a survey of the empirical evidence]. Similarly, exchange offers of debt for common stock are good news, as they are accompanied by positive excess returns on announcement of 10.52%. The reason for the difference in information effects of debt issues for cash and exchange offers is that debt issues for cash involve information revelations about current cash flows. In contrast, debt for equity exchange offers do not offer information about current cash flows. Modeling this second kind of uncertainty may resolve this difference in information effects.

The model also implies that equilibria involving rights issues dominate equilibria involving outside equity issues. Outside equity issues are used only when the firm’s managers will not exercise any of their rights or face a shortage of capital (i.e., they either sell their shares or their rights). Rights issues are used when the firm’s managers exercise some of their rights. Since the dilution is lower in the second case, it follows that:

**Theorem 3.** Equilibria with rights issues dominate equilibria with outside equity. Thus

\[ V(s(q), q, q) \bigg|_{\text{rights issue observed}} > V(s(q), q, q) \bigg|_{\text{outside equity observed}} \]

This implies that rights issues are less negative news than outside equity issues. The results of Eckbo and Masulis (1990) strongly support this distinction. Eckbo and Masulis find that the average announcement effect is weakest for uninsured rights (−1.39%, z-value −1.56) and for standby rights offerings (−1.04%, z-value −2.04). The announcement effect for firm commitment offerings to outsiders is 3.34% (z-value −21.48). These empirical results are strongly supportive of the model in this paper.

A unique feature of the model is the presence of existing debt. Consequently, the implications of the signaling equilibrium for the current debtholders are explored next.

**Theorem 4.** The effect of signaling on the value of existing debt is ambiguous and depends on the value of information and the marginal costs of signaling. For example, if dividends are financed by investments, then

\[ \frac{dm}{dq} > 0 \iff (1 - \lambda) V_z f'(I) - \frac{1}{\alpha} C_z - 1 > 0, \]

where \( m(q) \) is the market value of the existing debt of the firm with information \( q \).


**Proof:** See Appendix.

The expression in Equation (40) is closely related to the marginal costs of signaling. When marginal costs are low, a large dividend is paid to signal each unit of information. This wealth transfer effect overwhelms the positive effects of the good news. Conversely if the marginal costs of signaling are high, a small dividend is paid out to signal a unit of information. Thus, the good news effect dominates the wealth transfer effect.

Theorem 4 is consistent with the evidence of Handjinicoloau and Kalay (1984). They find that when dividends are increased, debtholders do not gain (the average residuals are 0.065 with a z-value of −0.734). On the other hand, when dividends are reduced, debtholders lose (the average residuals are −0.392 with a z-value of −2.521). Firms that reduce dividends are firms that are in trouble. Such firms are likely to have high marginal costs of signaling. As a result, reductions in dividends are bad news for debtholders.

A related question is the informativeness of dividends when the leverage changes. If leverage increases, one expects the value of information to go up. This is true even when signaling occurs. Thus the informativeness of the dividend signal goes up for high-leverage firms. This is stated in the following theorem.

**Theorem 5:** Suppose the initial level of debt changes from G to G', G' > G. Then,

(i) \[ O_2(s(q),q,q) \left| G' > O_2(s(q),q,q) \right| G \]

(ii) \[ \text{Var}[O(s(q),q,q) \left| G'] > \text{Var}[O(s(q),q,q) \left| G] \]

**Proof:** See Appendix.

Handjinicoloau and Kalay (1984) present some indirect tests of Theorem 5. They find that unexpected dividend increases are associated with higher information effects for high-leverage firms. The average residuals for these firms are 0.510 (z-statistic 4.615) versus average residuals of 0.298 (z-statistic 2.328) for low-leverage firms. Unfortunately, this finding is not true for dividend decreases. The average residuals are 0.099 (z-statistic 0.449) for high-leverage firms versus −1.181 (z-statistic −5.538) for low-leverage firms. There is partial support for the model predictions. It is possible that the smaller sample size of dividend decreases makes these estimates unreliable. A direct variance test, as suggested in Theorem 5, would provide a more powerful test of the model in this paper.

**IV. CONCLUSIONS**

The paper analyzed a model where firms have multiple financial instruments to signal their private information. The choice between signaling instruments was
characterized. In particular, the choice of signals was determined by a benefit-cost ratio where the benefit measures the credibility of the signal and the costs are the marginal costs of signaling. The credibility of a signal was related to (1) the marginal reduction in future cash flows due to the signal and (2) the marginal reduction in dilution due to the signal. Intuitively, the more a signal constrains future cash flows, the higher its credibility. Also, the greater the marginal reduction in dilution, the more the manager is concerned about future cash flows and the more credible the signal.

Using this characterization, the choice between rights and outside equity issues was characterized. When the firm's managers subscribe to a portion of the rights issue with their own capital, rights dominate outside equity issues as they dilute the firm less. However, when the firm's managers sell all their rights, they are indifferent between rights issues and outside equity issues. Provided the manager has some capital to subscribe to rights issues, the market value drop due to rights issues is less than that due to outside equity issues. This is consistent with the empirical evidence in Eckbo and Masulis (1990).

Initially, signaling involves the reduction of investment since the marginal cost of investment is zero. The choice between investment and debt depends on the marginal effect of investment reductions and debt increases on future cash flows. The signal with the lower marginal reduction of future cash flow is more efficient. Investment is the signal used initially. As investment declines, it constrains future cash flow more, yielding a less efficient signal under this criterion. The reduction in investment also increases the bankruptcy risk and thus the marginal face needed for an incremental dollar of debt. For reasonable technologies, the first effect is stronger and the firm eventually uses debt.

The effect of signaling on current bondholders depends on the marginal costs of signaling. When the marginal costs are high, bondholders gain since the signal change and hence the wealth transfer effect is small and the information effect dominates. When the marginal costs are low, the reverse occurs. A large signal change occurs and the negative effect of this wealth transfer effect swamps the information effect. Finally, the value of the manager's private information increases when leverage increases. When the probability of bankruptcy increases, future cash flows and thus the manager's information is more valuable. The evidence of Handjinicoloau and Kalay (1984) is supportive of these empirical implications.

**APPENDIX**

Second-Order Condition on Manager's Objective Function

In the full-information case, differentiating the first-order condition with respect to \( I \) a second time,
\[ f''(I^*) \left\{ g(\delta) \, d\delta + \frac{1}{\alpha} \int_0^{\delta(s,q)} c'(-z(s) - q - \delta)g(\delta) \, d\delta \right\} \]
\[ + f'(I^*) \left\{ g(\delta(s,q))f''(I^*) - \frac{1}{\alpha} c'(0)g(\delta(s,q))f'(I^*) \right\} \]
\[ - \frac{1}{\alpha} f'(I^*) \int_0^{\delta(s,q)} c''(-z(s) - q - \delta)g(\delta) \, d\delta \right\}. \]  
\[ (A1) \]

Now,
\[ g(\delta(s,q))f'(I^*) - \frac{1}{\alpha} c'(0)g(\delta(s,q))f''(I^*) \]
\[ - \frac{1}{\alpha} f'(I^*) \int_0^{\delta(s,q)} c''(-z(s) - q - \delta)g(\delta) \, d\delta < 0 \]  
\[ (A2) \]

if \( c'(0) > \alpha. \)

**Proof of Lemma 1.** Suppose the signaling equilibrium satisfies the conditions stated in the lemma. Then \( \hat{q} > q \Rightarrow z(\hat{q}) < z(q) \) and \( \eta(\hat{q}) < \eta(q) \). Thus
\[ (1 - \lambda) \frac{1}{1 + \eta(q)} V_q(z(q),t) - \frac{1}{\alpha} C_q(z(q),t) \]
\[ > (1 - \lambda) \frac{1}{1 + \eta(q)} V_q(z(q),t) - \frac{1}{\alpha} C_q(z(q),t) \]  
\[ (A3) \]

since \( V_q(z,t) \) is a monotone decreasing function of \( z \) and \( C_q(z,t) \) is a monotone increasing function of \( z \).

**Proof of Lemma 2.** Let \( s'(q) \) be the allocation received by \( q \) in the separating signaling equilibrium \( S' \) and
\[ s^*(q) = \arg \max_{s(q)} O(s(q),q,q), \]  
\[ (A4) \]

where \( O(s'(q),q,q) \) is the objective value that \( q \) gets from the allocation \( s'(q) \). The \( s^*(q) \) form a Pareto-dominant separating signaling equilibrium. First incentive compatibility is shown. Consider \( q,q' \). By definition \( \exists \) a sequence of equilibria \( S^n(q) \) s.t. \( s^n(q) \rightarrow s^*(q) \). Then,
\[ O(s^*(q'),q',q') \geq O(s^n(q'),q',q') \geq O(s^n(q),q,q'). \]  
\[ (A5) \]
Since this holds for each \( n \), it holds in the limit by continuity, i.e.,

\[
O(s^*(q'), q', q') \geq O(s^*(q), q, q'),
\]

which proves incentive compatibility. To prove separation, assume not, i.e., \( s^*(q) = s^*(q') \) and \( q' < q \). Then,

\[
O(s^*(q'), q', q') \geq O(s^*(q), q, q') = O(s^*(q'), q', q') > O(s^*(q'), q', q'),
\]

which is a contradiction.

**Proof of Theorem 1.** The proof is by inspection. Ideally, one would like to be long in the signals in set \( I \) and short in the signals in set \( I' \). If resource constraints are binding for all signals in set \( I' \), this is not feasible and the optimal signal involves signals in \( I \). If all resource constraints are not binding for some elements in \( I' \), some signal in \( I' \) can be shorted. Suppose \( -\beta_1/\mu_1 > -\beta_2/\mu_2 \). Create a zero cost signal as follows. Let

\[
\mu_1 u_1 + \mu_2 u_2 = 0,
\]

where \( u_1 \) has the same sign as \( \mu_1 \). Then

\[
-\beta'u = -\beta_1 u_1 - \beta_2 u_2 = -\beta_1 u_1 - \beta_2 \left( -\frac{\mu_1}{\mu_2} \right) u_1
\]

\[
= \mu_1 u_1 \left[ -\frac{\beta_1}{\mu_1} + \frac{\beta_2}{\mu_2} \right] > 0.
\]

Thus a zero cost credible signal is available. By continuity, \( -\beta_1/\mu_1 > -\beta_2/\mu_2 \) over a region \( q \). But the signaling schedule moves infinite amounts when \( \mu'u = 0 \).

Then,

\[
\dot{y} = -\phi/\mu'u = -\infty.
\]

Thus \( s_i = u_i \dot{y} \) is also infinite. If the slope is infinite over a finite interval, the signaling schedule moves infinite amounts. This implies that there is a discontinuity at \( q \). Signal 2 is reduced until a resource constraint is reached. Signal 1 is increased to keep the incentive constraints binding.

**Derivation of Marginal Costs and Benefits of Various Signals**

Note that
\[ O(s,q(s),q) = \frac{\lambda}{1 + \eta(s)} V(s,q(s)) + (1 - \lambda) \frac{1}{1 + \eta(s)} V(s,q) - \frac{1}{\alpha} C(s,q) - r, \]
\[ O_3(s,q(s),q) = \frac{1 - \lambda}{1 + \eta(s)} V_q(s,q) - \frac{1}{\alpha} C_q(s,q). \]

To compute the marginal cost of investment, note that
\[ O(s,q(s),q) = V(s,q(s)) f'(I) \]
\[ + (1 - \lambda) \frac{1}{1 + \eta(s)} V_q(s,q) f'(I) - \frac{1}{\alpha} C(s,q) f'(I). \]

Imposing the equilibrium condition that \( q(s(q)) = q \), the marginal cost is given by
\[ \mu_I = O_I(s(q),q,q) \]
\[ = \left[ \frac{1}{1 + \eta(s(q))} V_q(s(q),q) - \frac{1}{\alpha} C_q(s(q),q) \right] f'(I) \]
\[ = J(s,q) f'(I). \]

The marginal benefit is given by
\[ \beta_I = \left[ \frac{1 - \lambda}{1 + \eta(s(q))} V_{q}(s,q) - \frac{1}{\alpha} C_{q}(s,q) \right] f'(I) \]
\[ = H(s,q) f'(I). \]

A similar derivation holds for debt.
For outside equity,
\[ O_e(s,q(s),q) = -\lambda \frac{\partial \eta/\partial e}{[1 + \eta(s)]^2} V(s,q(s)) - (1 - \lambda) \frac{\partial \eta/\partial e}{[1 + \eta(s)]^2} V(s,q). \]

Imposing the constraint that \( q(s(q)) = q \), the marginal cost is given by
\[ \mu_e = O_e(s(q),q,q) = -\frac{\partial \eta/\partial e}{[1 + \eta(s(q))]^2} V(s(q),q). \]

The marginal benefit is given by
\[ \beta_e = -(1 - \lambda) \frac{\partial \eta/\partial e}{[1 + \eta(s(q))]^2} V_q(s(q),q). \]

For a rights issue, similar expressions lead to
\[
\mu_r = \eta_t(s(q), q, q) = -\frac{\partial \eta/\partial r}{[1 + \eta(s(q))]^2} V(s(q), q) - 1,
\]

\[
\beta_r = -(1 - \lambda) \frac{\partial \eta/\partial r}{[1 + \eta(s(q))]^2} V\eta(s(q), q). \quad \text{(A18)}
\]

When there is no outside equity issue, an incremental rights issue does not change the dilution as \(\eta(s) = 0\). Then \(\partial \eta/\partial r = 0\) and the marginal cost and benefit of a rights issue simplify to \(-1\) and 0.

This completes the derivation of the marginal costs and benefits.

Conditions for Choice of Investment versus Debt in Financing Reductions in Outside Equity

Investment is the efficient signal if

\[
-\frac{\beta_I + \beta_e}{\mu_I + \mu_e} > \frac{\beta_b - \beta_e}{\mu_b - \mu_e}. \quad \text{(A19)}
\]

Let

\[
J(s,q) = \frac{1}{1 + \eta(s)} V_z - \frac{1}{\alpha} C_z, \quad c(s,q) = \frac{1}{[1 + \eta(s)]^2} V,
\]

\[
H(s,q) = \frac{1 - \lambda}{1 + \eta(s)} V_{zq} - \frac{1}{\alpha} C_{zq}, \quad d(s,q) = \frac{1 - \lambda}{[1 + \eta(s)]^2} V_{q} \quad \text{(A20)}
\]

The comparison simplifies to

\[
-f'(I)H(s,q) \frac{d\eta}{db} J(s,q) - \frac{\partial \eta}{\partial I} f'(I)H(s,q)c(s,q)
\]

\[
+ \frac{\partial \eta}{\partial I} \frac{dB}{db} d(s,q)J(s,q) + \frac{\partial \eta}{\partial I} \frac{\partial \eta}{\partial I} c(s,q)d(s,q)
\]

\[
> - \frac{dB}{db} \frac{\partial}{\partial I} f'(I)J(s,q) + \frac{\partial \eta}{\partial I} \frac{dB}{db} H(s,q)c(s,q)
\]

\[
- \frac{\partial \eta}{\partial I} f'(I)d(s,q)J(s,q) + \frac{\partial \eta}{\partial I} \frac{\partial \eta}{\partial I} c(s,q)d(s,q), \quad \text{(A21)}
\]

or

\[
\frac{\partial \eta}{\partial b} f'(I)H(s,q)c(s,q) + \frac{\partial \eta}{\partial I} \frac{dB}{db} d(s,q)J(s,q)
\]
or
\[
\frac{\partial \eta}{\partial I} \frac{dB}{db} [d(s,q)J(s,q) - H(s,q)c(s,q)] > \frac{\partial \eta}{\partial b} f'(I)[d(s,q)J(s,q) - H(s,q)c(s,q)].
\] (A23)

Since the term in square brackets is positive, this reduces to
\[
\frac{\partial \eta}{\partial I} \frac{dB}{db} > -\frac{\partial \eta}{\partial b} f'(I).
\] (A24)

Using the definition of \( \eta(s) \),
\[
\frac{\partial \eta}{\partial I} \frac{1}{[1 + \eta(s)]^2} \left[ V + \frac{\eta(s)}{1 + \eta(s)} \left( V_z f'(I) + V_q \frac{dq}{dI} \right) \right] = 1,
\]
\[
-\frac{\partial \eta}{\partial b} \frac{1}{[1 + \eta(s)]^2} \left[ V - \frac{\eta(s)}{1 + \eta(s)} \left( V_z \frac{dB}{db} + V_q \frac{dq}{dB} \right) \right] = 1.
\] (A25)

Thus the choice of signal condition reduces to
\[
\left[ 1 - \frac{\eta(s)}{1 + \eta(s)} \left( V_z f'(I) + V_q \frac{dq}{dI} \right) \right] \frac{dB}{db} > f'(I) - \frac{\eta(s)}{1 + \eta(s)} V_q \frac{dq}{db} f'(I),
\] (A26)

which simplifies to
\[
\frac{dB}{db} - \frac{\eta(s)}{1 + \eta(s)} V_q \frac{dq}{dI} \frac{dB}{db} > f'(I) - \frac{\eta(s)}{1 + \eta(s)} V_q \frac{dq}{db} f'(I).
\] (A27)

To simplify this, consider the first-order condition for investment,
\[
\frac{1}{1 + \eta(s)} V_z f''(I) - \frac{1}{\alpha} C_z f'(I) - \frac{\partial \eta}{\partial I} \frac{1}{[1 + \eta(s)]^2} \left[ V + \lambda \frac{1}{1 + \eta(s)} V_q \frac{dq}{dI} \right] = 0.
\] (A28)

Using
\[
\frac{\partial \eta}{\partial I} \frac{1}{[1 + \eta(s)]^2} V + \frac{\eta(s)}{1 + \eta(s)} \left( V_z f'(I) + V_q \frac{dq}{dI} \right) = 1,
\] (A29)

one obtains
\[ (V_z f'(I) - \frac{1}{\alpha} C_z f'(I) - 1) + \frac{dq}{dl} \left( \frac{\eta(s)}{1 + \eta(s)} + \frac{\lambda}{1 + \eta(s)} \right) V_q = 0. \]  

Let \( R(s,q) = V_z - (1/\alpha)C_z \). Then,

\[ V_q \frac{dq}{dl} = -\frac{f'(I)R(s,q) - 1}{\eta(s) + \frac{\lambda}{1 + \eta(s)}}. \]  

Similarly

\[ V_q \left( -\frac{dq}{db} \right) = -\frac{(dB/db)R(s,q) - 1}{\eta(s) + \frac{\lambda}{1 + \eta(s)}}. \]

Thus,

\[ \frac{\eta(s)}{1 + \eta(s)} V_q \frac{dB}{dl} = -\phi(\eta(s))[f'(I)R(s,q) - 1] \frac{dB}{db}. \]  

and

\[ \frac{\eta(s)}{1 + \eta(s)} V_q \frac{-dq}{db} f'(I) = -\phi(\eta(s)) \left( \frac{dB}{db} R(s,q) - 1 \right) f'(I). \]

Using this, the comparison simplifies to

\[ [1 - \phi(\eta(s))] \frac{dB}{db} > [1 - \phi(\eta(s))] f'(I). \]  

Since \( \phi(\eta(s)) < 1 \), this is equivalent to

\[ \frac{dB}{db} > f'(I). \]  

This completes the characterization of the choice of investment versus debt.

**Proof of Lemma 7.** From Equation (36), the marginal cost of signaling is given by

\[ \lambda V_z f'(I) + (1 - \lambda) \frac{1 + (1 - k) \xi}{1 + \xi} V_z f'(I) - \frac{1}{\alpha} C_z f'(I) \]

\[ -\lambda - (1 - \lambda)(1 - k) - (1 - \lambda) \frac{\partial \xi}{\partial l} \frac{k}{(1 + \xi)^2} V + \frac{1}{\alpha} \frac{\partial w}{\partial l} \]

\[ = \lambda (V_z f'(I) - 1) - \frac{1}{\alpha} C_z f'(I) + (1 - \lambda) \left( \frac{1 + (1 - k) \xi}{1 + \xi} V_z f'(I) - (1 - k) \right) \]
Multiple-Signaling Model

\[ + (1 - \lambda) \left[ \frac{k_\xi}{1 + \xi} (V_z f'(I) + V_q \frac{dq}{dI}) - k + \frac{1}{1 - \lambda} \frac{1}{\alpha} \frac{\partial w}{\partial I} \right] + \frac{1}{\alpha} \frac{\partial w}{\partial I} \]

\[ = \left( V_z f'(I) - 1 - \frac{1}{\alpha} C_z f'(I) \right) + (1 - \lambda) \left( \frac{k_\xi}{1 + \xi} V_q \frac{dq}{dI} \right). \tag{A37} \]

The marginal costs are the same provided \( dq/dI \) is the same under the two underpricing rules. The first-order condition is

\[ \left( V_z f'(I) - 1 - \frac{1}{\alpha} C_z f'(I) \right) + (1 - \lambda) \left( \frac{k_\xi}{1 + \xi} V_q \frac{dq}{dI} \right) \]

\[ = 0. \tag{A38} \]

Since all the terms except \( dq/dI \) are identical, \( dq/dI \) is the same under the two strategies. Hence the marginal cost is identical under the two strategies.

The marginal benefit increases or decreases with \( \partial w/\partial v \) as the derivative \( \partial \xi/\partial I \) changes with \( \partial w/\partial v \). But

\[ (1 - \lambda) \frac{\partial \xi}{\partial I} k \frac{1}{(1 + \xi)^2} V \]

\[ = k(1 - \lambda) - \left[ \frac{1}{\alpha} \frac{\partial w}{\partial v} + (1 - \lambda) \frac{k_\xi}{1 + \xi} (V_z f'(I) + V_q \frac{dq}{dI}) \right]. \tag{A39} \]

This immediately yields that \( \partial \xi/\partial I \) decreases with \( \partial w/\partial v \). Thus the marginal benefit decreases with \( \partial w/\partial v \).

**Proof of Lemma 8.** This theorem is tedious and mimics the proof in the case where a pure rights issues occurs. It is available from the author.

**Proof of Lemma 9.** Both alternatives have the same slope for the objective at \( q \). Also, both have the same marginal signaling benefit. Thus, the choice of signal has to depend on the slope of the signaling schedule at \( q \). Thus outside equity dominates rights issues if and only if

\[ \frac{dq}{dI} \bigg|_{\text{rights issues}} > \frac{dq}{dI} \bigg|_{\text{outside equity}}. \tag{A40} \]

But

\[ V_q \frac{dq}{dI} \bigg|_{\text{rights issues}} = \frac{V_z - (1/\alpha) C_z - 1}{[(1 - \lambda) \xi / (1 + \xi)] + \lambda} \tag{A41} \]

and
\[ V_q \frac{dq}{dl} \bigg|_{\text{outside equity}} = -\frac{V_z - (1/\alpha)C_z - 1}{[\eta/(1 + \eta)] + [\lambda/(1 + \eta)]}. \] 

(A42)

Since \( \zeta = \eta \), the two denominators are equal. In fact for \( q > q^* \), similar results can be shown to hold by setting \( k = 1 \) in the proof of Lemma 8.

**Proof of Theorem 2.** From Lemmas 3, 4, 6, and 8, we know that reducing rights issues is the optimal signal in Case 1; thus the claim on rights issues follows. For outside equity issues, when resource constraints force outside equity issues, the optimal signal is to reduce outside equity issues (see p. XX); thus the claim of outside equity issues follows. The claim on dividends follows from the discussion of Case 2; instead of reducing rights issues, one pays dividends.

Since initially the signaling equilibrium always involves the reduction of investment (in both Cases 1 and 2), only types higher than some cutoff type issue debt. This implies that the issue of debt is good news. However, the possible nonmonotonicity of the signaling schedule in debt and investment (discussed in note 18) yields the last statement in the Theorem.

**Proof of Theorem 4.** By definition,

\[ m(q) = \int Gg(\delta) \, d\delta + \int [f(I) + q + \delta]g(\delta) \, d\delta, \] 

(A43)

where \( m(q) \) is the market value of existing debt. Thus

\[ \frac{dm}{dq} = \left( \int g(\delta) \, d\delta \right) \left( f'(I) \frac{dl}{dq} + 1 \right). \] 

(A44)

The sign of \( dm/dq \) depends on the second term on the RHS. Now

\[ \frac{dl}{dq} = -\frac{\lambda V_z}{f'(I) V_z - (1/\alpha)C_z - 1}. \] 

(A45)

Thus

\[ f'(I) \frac{dl}{dq} + 1 > 0. \]
\[ \iff -f'(l)\lambda V_z + f'(l)V_z - \frac{1}{\alpha} C_z - 1 \geq 0 \]
\[ \iff (1 - \lambda) f'(l)V_z - \frac{1}{\alpha} C_z - 1 > 0. \] (A46)

**Proof of Theorem 5.** For the lowest-type firm \( q \), using the envelope theorem

\[ \frac{dO(s(q),q,q)}{dG} = \frac{\partial O(s(q),q,q)}{\partial G} < 0. \] (A47)

Thus it must be true that \( z(q,G') < z(q,G) \) as \( I(q,G') > I(q,G) \). It is claimed that this is true for all \( q \). Suppose not, i.e., there exists \( \hat{q} \) such that the reverse is true. By continuity there exists \( q \) such that \( z(q,G') = z(q,G) \). Since \( G' > G \), \( I(q,G') > I(q,G) \). Then \( f''(I(G',q)) < f''(I(G,q)) \) and

\[ -\frac{dI}{dq} \bigg|_{g'} < -\frac{dI}{dq} \bigg|_{g}. \] (A48)

This contradicts \( z(q,G') = z(q,G) \) as \( z(q,G') \) must intersect \( z(q,G) \) from below and the left for equality to occur. Thus this cannot happen. Hence, \( z(q,G') < z(q,G) \) and

\[ O_3(s(q),q,q) \bigg|_{G'} > O_3(s(q),q,q) \bigg|_{G}. \] (A49)

To prove the second part, define

\[ y(q) = O(s(q),q,q) \bigg|_{G'} - O(s(q),q,q) \bigg|_{G} \]
\[ z(q) = O(s(q),q,q) \bigg|_{G'} + O(s(q),q,q) \bigg|_{G} \]
\[ -E[O(s(q),q,q)] \bigg|_{G'} - E[O(s(q),q,q)] \bigg|_{G}. \] (A50)

Also, \( y'(q) > 0 \), by Equation (A49) and \( z'(q) > 0 \) from the monotonicity of the signaling schedule in \( q \). Then,

\[
\text{Var}[O(s(q),q,q)\big|_{G'} - O(s(q),q,q)\big|_{G}]
= E_q[(y(q) - E(y(q))z(q))]
= E_q[y(q)z(q)] \text{ using } E[z(q)] = 0.
\]
\[ z^{-1}(q) \]
\[ \bar{q} \]
\[ q \]
\[ z^{-1}(q) \]
\[
= \int_{q} y(q)z(q)h(q) \, dq + \int_{\bar{q}} y(q)z(q)h(q) \, dq
\]
\[
\begin{align*}
\int_{\mathcal{I}(0)} z^{-1}(0) & > y(z^{-1}(0)) \int_{\mathcal{I}(0)} z(q) h(q) \, dq + y(z^{-1}(0)) \int_{\mathcal{I}(0)} \bar{q} z(q) h(q) \, dq \\
& = 0. 
\end{align*}
\]  

This completes the proof of the theorem.

**ACKNOWLEDGMENTS**

I am indebted to Ramsastry Ambarish, Doug Foster, Larry Glosten, Milt Harris, Ravi Jagannathan, Kose John, Nancy Keeshan, Bob Korajczyk, Bob McDonald, Kevin McCardle, Steve Matthews, Mike Moore, Dan Siegel, Chester Spatt, and seminar participants at Brown University, Columbia University, University of Chicago, Duke University, University of Michigan, Massachusetts Institute of Technology, New York University, University of Wisconsin, University of California at Los Angeles, and University of Southern California for detailed comments. The usual disclaimer applies.

**NOTES**

1. None of the above-mentioned papers uses debt as a potential signal. Ofer and Thakor (1987) do not consider equity issues at all. In this paper, both rights and outside equity issues are potential signals. This contrasts with Ambarish, John, and Williams (1987) and Williams (1988), where only outside equity issues are a signal. Finally, the sale of rights is explicitly analyzed in this paper. Also, the technical methods for analyzing multiple-signaling problems proposed in this paper are different from those in the other papers.

2. The linear technology is somewhat restrictive. Ambarish, John, and Williams (1987) allow for multiplicative technologies. Their key result is that overinvestment is a possible outcome. A similar result would obtain under the setup of this paper with a multiplicative technology. In such a case, it is difficult to verify the global incentive compatibility constraints. In a different model, Kumar (1988) makes a similar point.

3. For the local approach to work (i.e., inframarginal incentive constraints do not bind), this assumption or some variant is needed. For example, if \( \delta \) is distributed uniformly, the assumption that \( c'(\cdot) > \alpha \) suffices. Lack of some such assumption on the bankruptcy costs may result in binding global incentive constraints. In a different model, Kumar (1988) emphasizes this point.

4. Similar objectives are used by Miller and Rock (1985) and Harris and Raviv (1985).

5. An alternative interpretation, following Ross (1977), is that \( \alpha \lambda V(s,q(s)) \) is the wage paid today and \( \alpha(1 - \lambda) V(s,q) \) is the wage paid tomorrow.

6. The investment that is optimal for the manager is not optimal from the viewpoint of other shareholders. Contracts that bring together the incentives of these two groups are ruled out. Dybvig and Zender (1991) discuss this issue in greater detail in the context of the Myers-Majluf asymmetric information model.

7. We are assuming that \( \hat{r}(q) \) can be raised in capital. Obviously, if current shareholders can monitor capital raised, they may not allow an investment greater than \( \hat{r} \). This may lead to different investment levels depending on whether the capital used is internal or external capital. We avoid these issues by assuming that \( \hat{r}'(q) \) in capital can be raised. Allowing for such complications does not change the main conclusions we arrive at but increases the number of cases to be considered.
8. The results for the case where positive financial investment is optimal are available from the author.
9. This assumes differentiability almost surely of the relevant schedules. See Viswanathan (1987) for further discussion of this issue.
10. This condition is necessary for Equation (14) to hold. Unlike the one-signal case, it is not sufficient.
11. Equation (14) can be derived from Equation (17) by integration.
12. To reduce the technical complexity of the paper, we do not explicitly introduce the refinement that leads to the D1 equilibrium. The interested reader is referred to Ramey (1989) for discussion.
14. This constraint holds if \( d(q) = 0 \). Then \( d(q') \geq 0 \), for all \( q' > q \), implies that
\[
\frac{dd(q)}{dq} = \frac{db(q)}{dq} + \frac{de(q)}{dq} + \frac{dr(q)}{dq} - \frac{dl(q)}{dq} \geq 0.
\]
This equation simplifies to the fourth constraint of expression 23.
15. These marginal costs and benefits are computed by appropriately differentiating the objective of the manager.
16. If no outside equity is made, \( e(q) = 0 \) and the marginal costs and benefits of a rights issue reduce to \(-1\) and \(0\). Incremental rights issues affect dilution only when there is an outside equity issue, i.e., \( e(q) > 0 \).
17. \( \eta(s) \) does not change as a function of \( I \) or \( b \), because \( I \) or \( b \) can change only when \( e \) or \( r \) changes. Since \( d(q) = 0 \) when \( e(q) > 0 \) or \( r(q) > 0 \), we do not have to worry how \( \eta(s) \) changes as \( d(q) \) changes.
18. For dividends, we assume that \( e(q) = 0 \). As we will see, dividends are always zero when \( e(q) > 0 \) and thus we need not consider this case.
19. The first condition in Equation (27) is the rational expectations requirement that the shares bought by outsiders are worth the price paid.
20. In a pure rights issue, no sale of rights occurs. The sale of rights is discussed later in the paper.
21. Strictly speaking, type \( q \) always receives the full information solution in any signaling equilibrium and thus is indifferent between them. The lemma compares the utility of types greater than \( q \) but arbitrarily close to it in alternative signaling equilibrium; that is why the lemma considers the slope of the signaling schedule at \( q \). Thus Lemmas 3, 5, and 6 are statements about type \( q \) and about types close to \( q \).
22. In this model, a pure rights issue is a negative dividend.
23. Since \( e(q) = 0 \), \( \beta_r = 0 \), \( \mu_r = -1 \).
24. It is emphasized that this is a sufficient condition and not a necessary condition.
25. The technology of the example is well behaved in the following sense. Investment is reduced and debt is increased in the signaling equilibrium. For other technologies, reversals of signal ordering may occur. For example, there may be regions where both investment and debt are decreased.
26. Equations (32), (33), and (34) and the discussion that follows also proves Lemma 4. Given that \( e = 0 \), there is no way to reduce the amount of outside equity.
27. Eckbo and Masulis (1990) also present a model where the fraction of rights shares subscribed by insiders plays an important role.
28. Strictly speaking, there is still a distinction. Outside equity issues involve dilution of the fraction \( \lambda \) of the shares that are sold by the manager. In the rights issues case, the fraction \( \lambda \) is sold cum rights and thus dilution is not relevant. However, since the proceeds from sale are the same in the two cases, this does not matter. What is important is the dilution on the \((1 - \lambda)\) fraction that is retained. This is identical for both outside equity issues and rights issues when \( k = 1 \).
29. This result is reminiscent of Myers and Majluf's Section 4.1.
30. In the region where both debt and investment are used, reversals of signal ordering are possible for one of the two signals. For example, regions where higher types undertake lower debt and lower investment may occur.

31. This is because the lowest types do not use debt.

32. Thus debt is good news relative to equity but the issue of capital is itself bad news.

33. An analysis similar to the above can be done for equity issues to explain the empirical results of Kalay and Shimrat (1987).

REFERENCES


Multiple-Signaling Model


